The Bernoulli equation is a special form of a mechanical energy balance.

where P_1, P_2 are the pressures at stations 1 and 2, respectively in N/m²,

α_1 and α_2 are the kinetic energy conversion factors,

Z_1 and Z_2 are the heights of stations 1 and 2 from some arbitrarily chosen datum level respectively in m,

u_1 and u_2 are the average velocities at stations 1 and 2, respectively in m/s,

h_f is the total frictional loss of energy due to friction between stations 1 and 2 in J/kg.

The term h_f indicates the friction generated per unit mass of fluid that occurs in the fluid between stations 1 and 2.

Each term involved in the Bernoulli equation [Equation (7.66)] has the units of J/kg.

PUMP WORK IN BERNOULLI EQUATION

A pump is installed in a flow system for increasing the mechanical energy of the fluid to maintain its flow.

Assume that a pump is installed in the flow system between the stations 1 and 2 as shown in Fig. 7.12.

Let W_p be the work done by the pump per unit mass of fluid.

Let h_{fp} be the total friction in the pump per unit mass of fluid (friction in bearings, seals or stuffing box.).

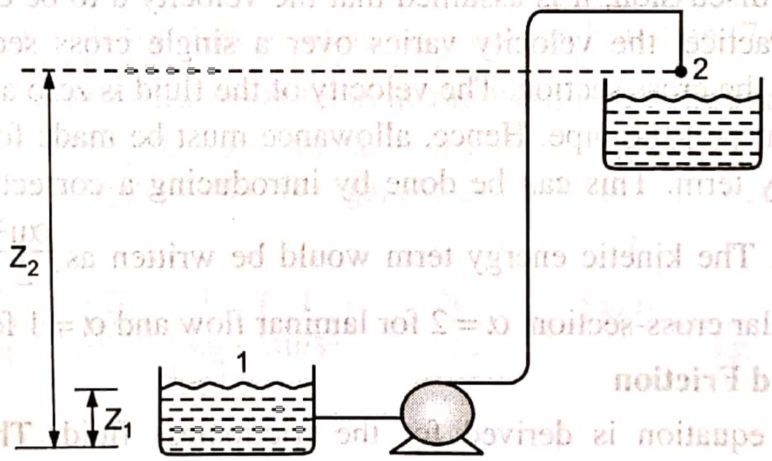

Fig. 7.12 : Pump work in Bernoulli equation

The net mechanical energy delivered to the flowing fluid is the difference between the mechanical energy supplied to the pump and frictional losses within the pump. i.e., $W_p - h_{fp}$. But to obtain the net mechanical energy (net work) delivered to the fluid, instead of using h_{fp}, a pump efficiency designated by the symbol η is used. It is defined as

$$W_p - h_{fp} = \eta \, W_p \quad \ldots (7.67)$$

$$\eta = \frac{W_p - h_{fp}}{W_p} \quad \ldots (7.68)$$

Since η is always less than one, the mechanical energy delivered to the fluid (ηW_p) is less than the work done by the pump. The Bernoulli equation corrected for the pump work between stations 1 and 2 is thus given by

$$\frac{P_1}{\rho} + g Z_1 + \frac{\alpha_1 u_1^2}{2} + \eta W_p = \frac{P_2}{\rho} + g Z_2 + \frac{\alpha_2 u_2^2}{2} + h_f \quad \text{... (7.69)}$$

REYNOLDS' EXPERIMENT

This experiment is still used to demonstrate the distinction between the two types of flow, namely laminar flow and turbulent flow. It was first conducted by Sir Osborne Reynolds in 1883.

The experimental set up is shown in Fig. 7.13. It consists of a horizontal glass tube with a flared entrance immersed in a glass walled constant head tank filled with water. The flow of water through the glass tube can be adjusted to any desired value by means of a valve provided at the outlet of the tube. A capillary tube connected to a small reservoir containing water soluble dye is provided at the centre of the flared entrance of the glass tube for injecting a dye solution in the form of a fine or thin filament into the stream of water.

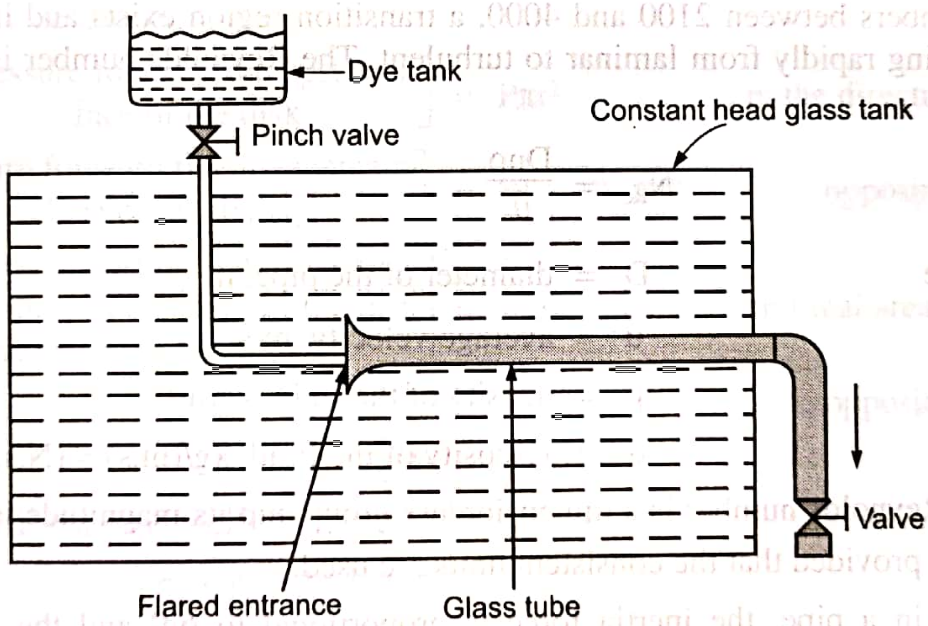

Fig. 7.13 : Reynolds experimental set up for demonstrating type of flow

By introducing a water soluble dye into the flow of water, the nature of flow could be observed. At low flow rates (i.e., at low water velocities), the filament/thread of coloured water flowed along with the stream of water in a thin, straight line without any lateral mixing. This indicated that the water was flowing in the form of parallel streams which did not interfere with each other (i.e., the water was flowing in parallel, straight lines). This type of flow pattern is called *streamline or laminar*. The laminar flow is characterised by the absence of bulk movement at right angles to the main stream direction (lateral movement). As the water flow rate was increased, a definite velocity called the critical velocity was reached, oscillations appeared in the coloured filament/thread and the thread of coloured water became wavy, gradually disappeared and the entire mass of water in the tube became uniformly coloured. In other words, the individual particles instead of flowing in an orderly manner parallel to the axis of the tube, moved erratically in the form of cross-currents and eddies which resulted into complete mixing. This type of flow pattern is known as *turbulent*. The turbulent flow is characterised by the rapid movement of fluid in the form of eddies in random directions across the tube. In between these two types of flow, there exists a transition region wherein the oscillations in the flow were unstable and any disturbance would quickly disappear.

The velocity at which the flow changes from laminar to turbulent is known as the critical velocity.

Reynolds further found that the critical velocity for the transition from laminar flow to turbulent flow depends on the diameter of the pipe, the average velocity of the flowing fluid, the density and viscosity of the fluid.

He grouped these four variables into a dimensionless group, $Du\rho/\mu$. This dimensionless group is known as the Reynolds number and found that the transition from laminar to turbulent flow occurred at a definite value of this group. The Reynolds number is a basic tool to predict the flow pattern in a conduit and is of a vital importance in the study of fluid flow. When the value of the Reynolds number is less than 2100, the flow is always laminar and when the value of the Reynolds numbers is above 4000, the flow is always turbulent. For Reynolds numbers between 2100 and 4000, a transition region exists and in this region the flow is changing rapidly from laminar to turbulent. The Reynolds number is denoted by the symbol N_{Re}.

$$N_{Re} = \frac{Du\rho}{\mu} \quad \ldots (7.70)$$

where D = diameter of the pipe, m

u = average velocity, m/s

ρ = density of the fluid, kg/m³

and μ = viscosity of the fluid, kg/(m.s) = (N.s)/m² = Pa.s

∴ The Reynolds number is a dimensionless group and its magnitude is independent of the units used, provided that the consistent units are used.

For flow in a pipe, the inertia force is proportional to ρu^2 and the viscous force is proportional to $\mu . u/D$.

$$\frac{\text{Inertia force}}{\text{Viscous force}} = \frac{\rho u^2}{\mu\, u/D}$$

$$= \frac{Du\rho}{\mu} = N_{Re}$$

Thus, Reynolds number is the *ratio of the inertia force to the viscous force*. This is an important physical significance of the Reynolds number.

Reynolds number is a useful tool to determine the nature of flow, whether laminar or turbulent.