

Density :

Density (ρ) or mass density of a fluid is *the mass of the fluid per unit volume*. In the SI system, it is expressed in kg/m^3 . The density of pure water at 277 K (4 °C) is taken as 1000 kg/m^3 .

Weight Density :

Weight density of a fluid is *the weight of the fluid per unit volume*. In the SI system, it is expressed in N/m^3 . Specific weight or weight density of pure water at 277 K (4 °C) is taken as 9810 N/m^3 .

The relation between mass density and weight density is

$$w = \rho g$$

where g is the acceleration due to gravity (9.81 m/s^2).

Specific Volume :

Specific volume of a fluid is *the volume of the fluid per unit mass*. In the SI system, it is expressed in m^3/kg .

Specific Gravity :

The specific gravity of a fluid is *the ratio of the density of the fluid to the density of a standard fluid*. For liquids, water at 277 K (4 °C) is considered/chosen as a standard fluid and for gases, air at NTP (0°C and 760 torr) is considered as a standard fluid.

Vapour Pressure :

The vapour pressure of a pure liquid is defined as *the absolute pressure at which the liquid and its vapour are in equilibrium at a given temperature* or The pressure exerted by the vapour (on the surface of a liquid) at equilibrium conditions is called as the vapour pressure of the liquid at a given temperature. Pure air free water exerts a vapour pressure of 101.325 kPa (760 torr) at 373.15 K (100 °C).

Surface Tension :

The property of liquid surface film to exert tension is called as the surface tension. It is the force required to maintain a unit length of film in equilibrium. It is denoted by the symbol σ (Greek sigma) and its SI unit is N/m .

Viscosity :

- A fluid undergoes continuous deformation when subjected to a shear stress. The resistance offered by a fluid to its continuous deformation (when subjected to a shear stress/force) is called viscosity
- The viscosity of a fluid at a given temperature is a measure of its resistance to flow.
- The viscosity of a fluid (gas or liquid) is practically independent of the pressure for the range that is normally encountered in practice. However, it varies with temperature. For gases, viscosity increases with an increase in temperature, while for liquids it decreases with an increase in temperature.

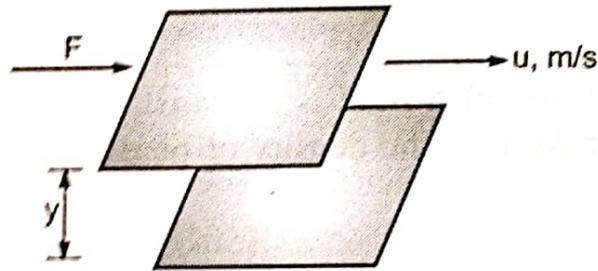


Fig. 7.1 : Definition of viscosity

Consider two layers of a fluid 'y' cm apart as shown in Fig. 7.1. Let the area of each of these layers be $A \text{ cm}^2$. Assume that the top layer is moving parallel to the bottom layer at a velocity of 'u' cm/s relative to the bottom layer. To maintain this motion, i.e., the velocity 'u' and to overcome the fluid friction between these layers, for any actual fluid, a force of 'F' dyne (dyn) is required.

Experimentally it has been found that the force F is directly proportional to the velocity u and area A , and inversely proportional to the distance y .

Therefore, mathematically it becomes

$$F \propto u.A/y \quad \dots (7.1)$$

Introducing a proportionality constant μ (Greek 'mu'), Equation (7.1) becomes

$$F = \mu u A/y \quad \dots(7.2)$$

$$F/A = \mu.u/y \quad \dots (7.3)$$

Shear stress, τ (Greek 'tau') equal to F/A between any two layers of a fluid may be expressed as

$$\tau = F/A = \mu.u/y \quad \dots (7.4)$$

The above equation in a differential form becomes

$$\tau = \mu \cdot \frac{du}{dy} \quad \dots (7.5)$$

(The ratio u/y can be replaced by the velocity gradient du/dy .)

In the SI system, the shear stress τ is expressed in N/m^2 and the velocity gradient/shear rate or rate of shear deformation is expressed in $1/\text{s}$ or s^{-1} .

Equation (7.5) is called Newton's law of viscosity. In the rearranged form, it serves to define the proportionality constant as

$$\mu = \frac{\tau}{du/dy} \quad \dots (7.6)$$

which is called as the coefficient of viscosity, or the dynamic viscosity (since it involves force), or simply the viscosity of a fluid. Hence, the dynamic viscosity μ , may be defined as the shear stress required to produce unit rate of shear deformation (or shear rate).

Viscosity is the property of a fluid and in the SI system it has the units of $(\text{N.s})/\text{m}^2$ or Pa.s or $\text{kg}/(\text{m.s})$.

As the unit $(\text{N.s})/\text{m}^2$ is very large for most of the fluids, it is customary to express viscosity as $(\text{mN.s})/\text{m}^2$ or mPa.s , where mN is millinewtons, i.e., 10^{-3} N and mPa is millipascal, i.e., 10^{-3} Pa .

In the C.G.S. system, viscosity may be expressed in poise (P) (the unit poise is named after the French scientist Poiseuille) or centipoise (cP).

$$\begin{aligned}
 1 \text{ poise} &= 1 \text{ P} = 1 \text{ gm}/(\text{cm.s}) \\
 &= 0.10 \text{ kg}/(\text{m.s}) \\
 &= 0.10 \text{ (N.s)}/\text{m}^2 \text{ or Pa.s} \\
 &= 100 \text{ cP}
 \end{aligned}$$

In many problems involving viscosity, there appears a term kinematic viscosity.

The kinematic viscosity of a fluid is defined as *the ratio of the viscosity of the fluid to its density* and is denoted by the symbol ν (Greek 'nu').

$$\nu = \mu/\rho \quad \dots (7.7)$$

In the SI system, ν has the units of m^2/s . The C.G.S. unit of kinematic viscosity is termed as stoke and is equal to $1 \text{ cm}^2/\text{s}$.

NEWTONIAN AND NON-NEWTONIAN FLUIDS

For most commonly known fluids, a plot of τ v/s du/dy results in a straight line passing through the origin and such fluids are called as Newtonian fluids.

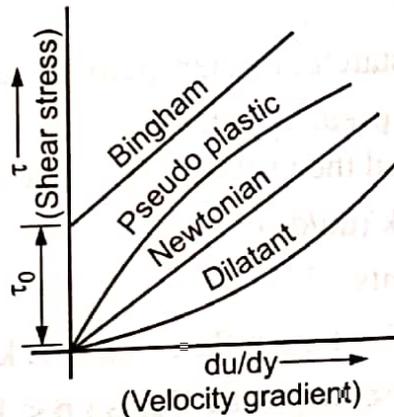


Fig. 7.2 : Shear stress v/s velocity gradient for Newtonian and Non-Newtonian fluids

Fluids that obey Newton's law of viscosity, i.e., the fluids for which the ratio of the shear stress to the rate of shear or shear rate is constant, are called as **Newtonian fluids**. This is true for all gases and for most pure liquids.

Examples of Newtonian Fluids : All gases, air, liquids, such as kerosene, alcohol, glycerine, benzene, hexane ether etc., solutions of inorganic salts and of sugar in water.

Fluids for which the ratio of the shear stress to the shear rate is not constant but is considered as a function of rate of shear, i.e., fluids which do not follow Newton's law of viscosity are called as **non-Newtonian fluids**. Generally, liquids particularly those containing a second phase in suspension (solutions of finely divided solids and liquid solutions of large molecular weight materials) are non-Newtonian in behaviour.

Examples of Non-Newtonian Fluids : Tooth pastes, paints, gels, jellies, slurries and polymer solutions.

A Newtonian fluid is one that follows Newton's law of viscosity. If viscosity is independent of rate of shear or shear rate, the fluid is said to be Newtonian and if viscosity varies with shear rate, the fluid is said to be non-Newtonian.

There are three common types of non-Newtonian fluids.

(a) **Bingham Fluids or Bingham Plastics** : These fluids resist a small shear stress indefinitely but flow linearly under the action of larger shear stress, i.e., these fluids do not deform, i.e., flow unless a threshold shear stress value (τ_0) is not exceeded.

These fluids can be represented by

$$\tau = \tau_0, \quad du/dy = 0, \quad \tau > \tau_0, \quad \tau = \tau_0 + \eta \cdot du/dy$$

where τ_0 is the yield stress / threshold shear stress and η is commonly called as the coefficient of rigidity.

Examples : Tooth paste, jellies, paints, sewage sludge and some slurries.

(b) **Pseudoplastic Fluids** : The viscosity of these fluids decreases with increase in velocity gradient, i.e., shear rate.

Examples : Blood, solution of high molecular weight polymers, paper pulp, muds, most slurries and rubber latex.

(c) **Dilatant Fluids** : The viscosity of these fluids increases with an increase in velocity gradient.

Examples : Suspensions of starch in water, pulp in water, and sand filled emulsions.

The experimental curves for pseudoplastic as well as dilatant fluids can be represented by a power law, which is also called the Ostwald-de-Waele equation.

$$\tau = k (du/dy)^n \quad \dots (7.8)$$

where k and n are arbitrary constants.

Newtonian fluids : $n = 1, k = \mu$

Pseudoplastic fluids : $n < 1$

Dilatant fluids : $n > 1$

Pseudoplastics are said to be shear-rate-thinning and dilatant fluids are said to be shear-rate-thickening.

PRESSURE

The basic property of a static fluid is pressure. When a certain mass of fluid is contained in a vessel, it exerts forces at all points on the surfaces of the vessel in contact. The forces so exerted always act in the direction normal to the surface in contact. *The normal force exerted by a fluid per unit area of the surface* is called as the **fluid pressure**. If F is the force acting on the area A , then the pressure or intensity of pressure is given by

$$P = F/A \quad \dots (7.9)$$

In a static fluid, the pressure at any given point is the same in all the directions. If the pressure at a given point was not the same in all directions, there would be non-equilibrium and the resultant force should exist. As the fluid is in static equilibrium, there is no net unbalanced force at any point. Hence, the pressure in all directions is the same and thus independent of direction.

Pressure Head :

The vertical height or the free surface above any point in a liquid at rest is called as the pressure head. The pressure head may be expressed as

$$h = \frac{P}{\rho g}, \frac{N/m^2}{kg/m^3 \times m/s^2} = \frac{(kg \cdot m/s^2)/m^2}{(kg/m^3)(m/s^2)} = m \quad \dots (7.10)$$

where P is in N/m^2 , ρ in kg/m^3 and g in m/s^2 . The units of h are m of liquid.

As the pressure at any point in a static liquid depends upon the height of the free surface above the point, it is convenient to express a fluid pressure in terms of pressure head. The pressure head is then expressed in terms of meters of a liquid column.

HYDROSTATIC EQUILIBRIUM

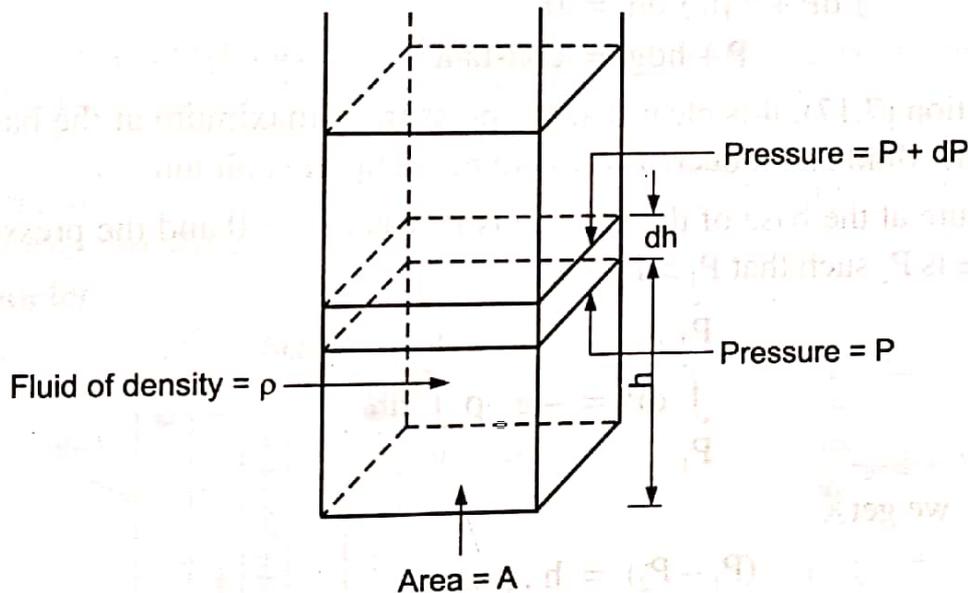


Fig. 7.3 : Hydrostatic equilibrium

Consider the vertical column of a single static fluid shown in Fig. 7.3. In this column of the static fluid, the pressure at any point is the same in all directions. The pressure is also constant at any horizontal plane parallel to the earth's surface, but it varies with the height of the column (it changes along the height of the column). Let the cross-sectional area of the column be A m^2 and the density of the fluid be ρ kg/m^3 . Let ' P ', N/m^2 be the pressure at a height ' h ' (meter) from the base of the column. At a height $h + dh$ from the base of the column, let the pressure be $P + dP$, N/m^2 . The forces acting on a common (another horizontal plane), let the pressure be $P + dP$, N/m^2 . The forces acting on a small element of the fluid of a thickness dh between these two planes are :

- (i) Force $(P + dP)A$ is acting downwards. ... taken as +ve.
- (ii) Force due to gravity is acting downwards and is equal to mass times acceleration due to gravity : $mg = V \rho g = A \cdot dh \cdot \rho \cdot g$... taken as -ve.

where m is the mass of the fluid contained within the two planes.

- (iii) Force PA is acting upwards ... taken as -ve.

As the fluid element is in equilibrium, the resultant of these three forces acting on it must be zero. Thus,

$$+ P.A - A. dh. \rho. g - (P + dP) A = 0 \quad \dots (7.11)$$

$$P.A - A. dh. \rho. g - PA - A.dP = 0 \quad \dots (7.12)$$

$$- A.dh. \rho. g - A.dP = 0 \quad \dots (7.13)$$

$$dP + dh. \rho. g = 0 \quad \dots (7.14)$$

Equation (7.14) is the desired basic equation that can be used for obtaining the pressure at any height. Let us apply it to incompressible and compressible fluids.

1. Incompressible Fluids :

For incompressible fluids, density is independent of pressure.

Integrating Equation (7.14), we get

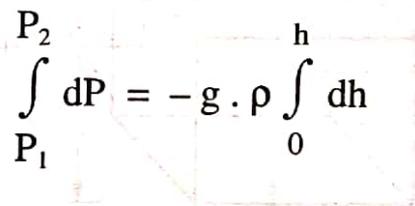
$$dP + g. \rho. dh = 0 \quad \dots (7.15)$$

$$\int dP + g. \rho. \int dh = 0 \quad \dots (7.16)$$

$$\therefore P + h\rho g = \text{constant} \quad \dots (7.17)$$

From Equation (7.17), it is clear that the pressure is maximum at the base of the column or container of the fluid and it decreases as we move up the column.

If the pressure at the base of the column is P_1 where $h = 0$ and the pressure at any height h above the base is P_2 such that $P_1 > P_2$, then



$$\int_{P_1}^{P_2} dP = -g. \rho \int_0^h dh \quad \dots (7.18)$$

Integrating, we get

$$(P_1 - P_2) = h. \rho. g \quad \dots (7.19)$$

where P_1 and P_2 are expressed in N/m^2 , ρ in kg/m^3 , h in m , 'g' in m/s^2 in SI.

With the help of Equation (7.19), the pressure difference in a fluid between any two points can be obtained by measuring the height of the vertical column of the fluid.

2. Compressible Fluids :

For compressible fluids, density varies with pressure. For an ideal gas, the density is given by the relation

$$\rho = \frac{PM}{RT} \quad \dots (7.20)$$

where

P = absolute pressure

M = molecular weight of gas

R = universal gas constant

T = absolute temperature.

Putting the value of ' ρ ' from Equation (7.20) into Equation (7.14),

$$dP + g (PM/RT) dh = 0 \quad \dots (7.21)$$

Rearranging Equation (7.21),

$$\frac{dP}{P} + g \cdot \frac{M}{RT} dh = 0 \quad \dots (7.22)$$

Integrating Equation (7.22), we get

$$\ln P + g \cdot \frac{M}{RT} \cdot h = \text{constant} \quad \dots (7.23)$$

Integrating the above equation between two heights h_1 and h_2 where the pressures acting are P_1 and P_2 , we get

$$\ln \frac{P_2}{P_1} = - g \cdot \frac{M (h_2 - h_1)}{RT} \quad \dots (7.24)$$

$$\frac{P_2}{P_1} = \exp \cdot \left[- g \cdot \frac{M}{RT} (h_2 - h_1) \right] \quad \dots (7.25)$$

Equation (7.25) is known as the barometric equation and it gives us the idea of pressure distribution within an ideal gas for isothermal conditions.

MANOMETERS

Manometers are the simplest pressure measuring devices and are used for measuring low pressure or pressure differences.

U-tube Manometer

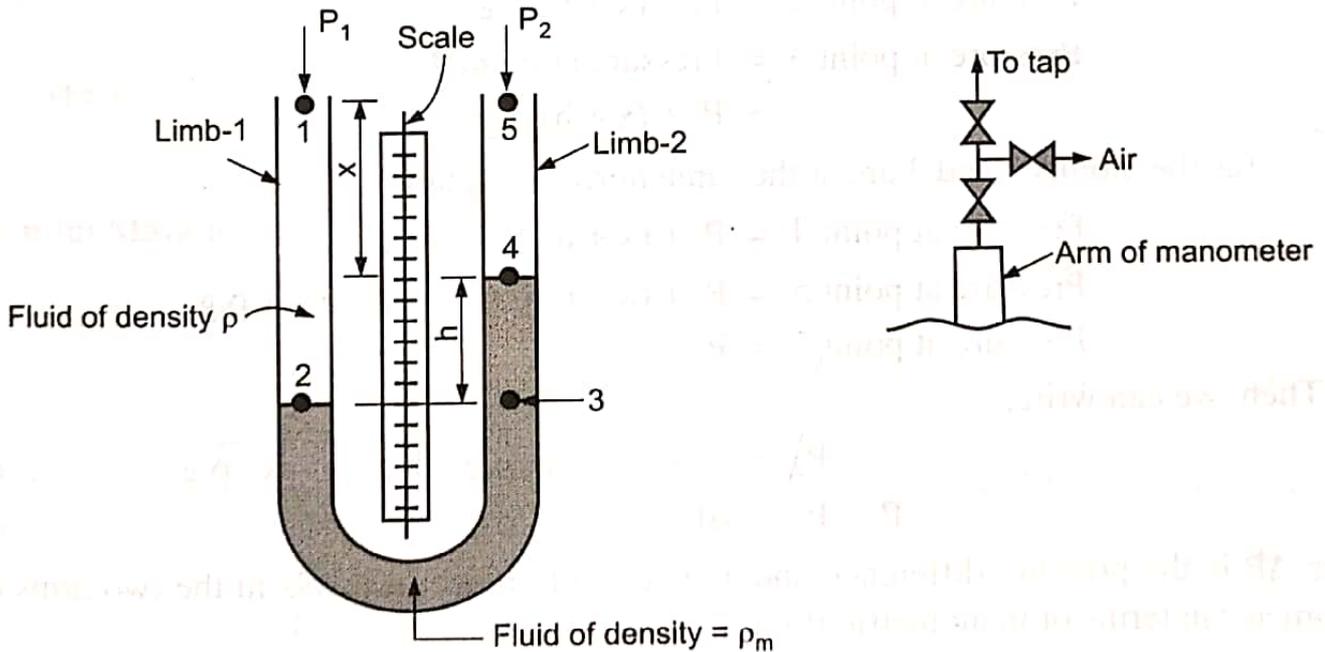


Fig. 7.4 : U-tube Manometer

- U-tube manometer is the simplest form of manometer. It consists of a small **diameter** U-shaped tube of glass. The tube is clamped on a wooden board. Between the two arms or legs of the manometer, a scale is fixed on the same board. The U-tube is partially filled with a manometric fluid which is heavier than the process fluid. The two limbs of the manometer are connected by a tubing to the taps between which the pressure drop is to be measured. Air vent valves are provided at the end of each arm for the removal of trapped air in the arm. The manometric fluid is immiscible with the process fluid. The common manometric fluid is **mercury**.

- U-tube manometer is filled with a given manometric fluid (fluid M) upto a certain height. The remaining portion of the U-tube is filled with the process fluid/flowing fluid of density ρ including the tubings. One limb of the manometer is connected to the upstream tap in a pipeline and the other limb is connected to the downstream tap in the pipeline between which the pressure difference $P_1 - P_2$ is required to be measured. Air, if any, is there in the line connecting taps and manometer is removed. At steady state, for a given flow rate, the reading of the manometer, i.e., the difference in the level of the manometric fluid in the two arms is measured and it gives the value of pressure difference in terms of manometric fluid across the taps (stations). It may then be converted in terms of m of flowing fluid.
- Consider a U-tube manometer as shown in Fig. 7.4 connected in a pipeline. Let pressure P_1 be exerted in one limb of the manometer and pressure P_2 be exerted in the another limb of the manometer. If P_1 is greater than P_2 , the interface between the two liquids in the limb 1 will be depressed by a distance 'h' (say) below that in the limb 2. To arrive at a relationship between the pressure difference ($P_1 - P_2$) and the difference in the level in the two limbs of the manometer in terms of manometric fluid (h), pressures at points 1, 2, 3, 4 and 5 are considered.

$$\text{Pressure at point 1} = P_1$$

$$\text{Pressure at point 2} = P_1 + (x + h) \rho \cdot g$$

$$\begin{aligned} \text{Pressure at point 3} &= \text{Pressure at point 2} \\ &= P_1 + (x + h) \rho \cdot g \end{aligned}$$

(as the points 2 and 3 are at the same horizontal plane).

$$\text{Pressure at point 4} = P_1 + (x + h) \rho \cdot g - h \cdot \rho_M \cdot g$$

$$\text{Pressure at point 5} = P_1 + (x + h) \rho \cdot g - h \cdot \rho_M \cdot g - x \cdot \rho \cdot g$$

$$\text{Pressure at point 5} = P_2$$

Then, we can write,

$$P_2 = P_1 + (x + h) \cdot \rho \cdot g - h \rho_M \cdot g - x \cdot \rho \cdot g \quad \dots (7.26)$$

$$P_1 - P_2 = \Delta P = h (\rho_M - \rho)g \quad \dots (7.27)$$

where ΔP is the pressure difference and 'h' is the difference in levels in the two arms of the manometer in terms of manometric fluid.

If the flowing fluid is a gas, density ρ of the gas will normally be small compared with the density of the manometric fluid, ρ_M and thus Equation (7.27) reduces to

$$\Delta P = P_1 - P_2 = h \cdot \rho_M \cdot g \quad \dots (7.28)$$

Inclined Manometer

- Inclined manometers are used for measuring small pressure differences.
- This type of manometer is shown in Fig. 7.5. One arm of the manometer is inclined at an angle of 5 to 10° with the horizontal so as to obtain a larger reading. (e.g., movement of 7 to 10 mm is obtained for a pressure change corresponding to 1 mm head of liquid.)

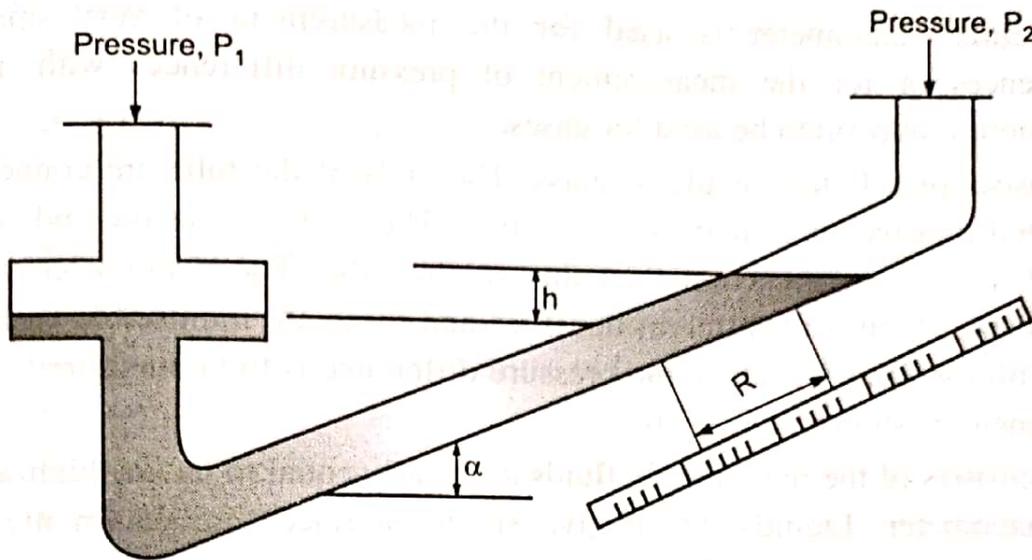


Fig. 7.5 : Inclined Tube Manometer

- In the vertical leg of this manometer an enlargement is provided so that the movement of the meniscus in this enlargement is negligible within the operating range of the manometer. If the reading R (in m) is taken as shown, i.e., distance travelled by the meniscus of the manometric fluid along the tube, then

$$h = R \sin \alpha \quad \dots (7.29)$$

where

α = angle of inclination

$$\text{and } (P_1 - P_2) = R \sin \alpha (\rho_M - \rho) g \quad \dots (7.30)$$

Differential Manometer / Two Liquid Manometer / Multiplying Gauge

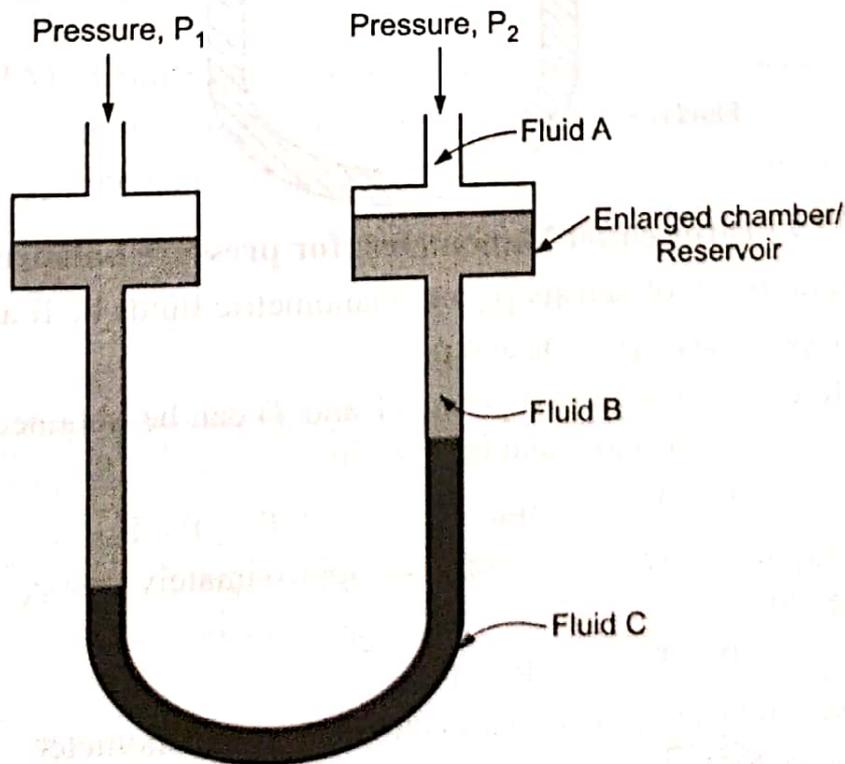


Fig. 7.6 : Differential Manometer

- Differential manometer is used for the measurement of very small pressure differences or for the measurement of pressure differences with a very high precision. It may often be used for gases.
- It consists of a U-tube made of glass. The ends of the tube are connected to two enlarged transparent chambers / reservoirs. The reservoirs at the ends of each arm are of a large cross-section than that of the tube. The manometer contains two manometric liquids of different densities and these are immiscible with each other and with the fluid for which the pressure difference is to be measured. This type of manometer is shown in Fig. 7.6.
- The densities of the manometric fluids are nearly equal to have a high sensitivity of the manometer. Liquids which give sharp interfaces are commonly used, e.g., paraffin oil and industrial alcohol, etc.

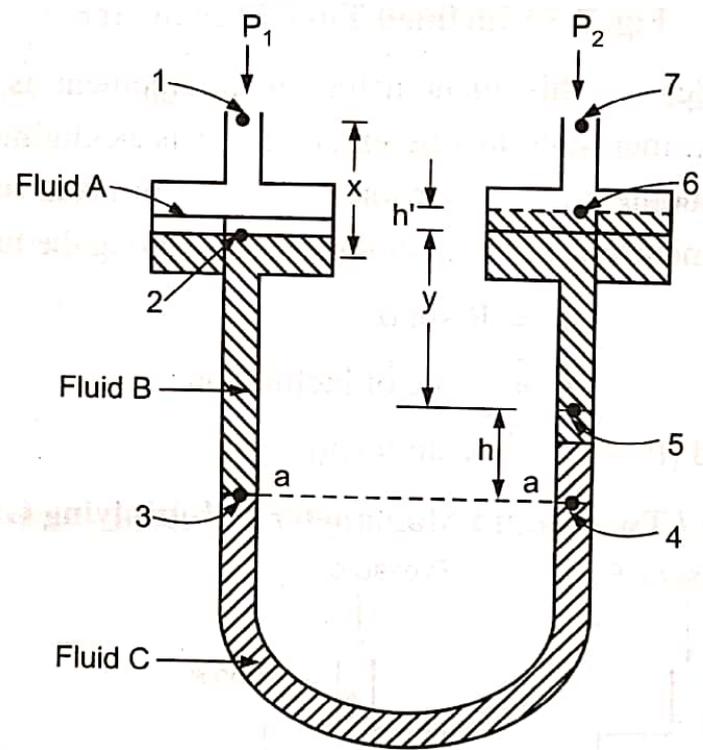


Fig. 7.7 : Differential Manometer (for pressure balance)

Let the flowing fluid be 'A' of density ρ_A and manometric fluids be B and C of densities ρ_B and ρ_C ($\rho_C > \rho_B$), respectively [$\rho_A < \rho_B$ and ρ_C].

The pressure difference between two points (1 and 7) can be obtained by writing down pressures at points 1, 2, 3, 4, 5, 6, and 7 and is given by

$$P_1 - P_2 = h' (\rho_B - \rho_A) g + h (\rho_C - \rho_B) g \quad \dots (7.31)$$

If the level of liquid in two reservoirs is approximately same, then $h' \approx 0$ and Equation (7.31) reduces to

$$P_1 - P_2 = h (\rho_C - \rho_B) g \quad \dots (7.32)$$

where h is the difference in level in the two arms/limbs of the manometer.

When the densities ρ_B and ρ_C are nearly equal [$(\rho_C - \rho_B)$ small], then very large values of h can be obtained for small pressure differences.

Alternately, the pressure at the level a – a in Fig. 7.7 must be the same in each of the limbs and therefore,

$$P_1 + [x.\rho_A + h' \rho_A + y.\rho_B + h.\rho_B] g = P_2 + [x.\rho_A + h' \rho_B + y.\rho_B + h.\rho_C] g \quad \dots (7.33)$$

$$\therefore (P_1 - P_2) = h' (\rho_B - \rho_A) g + h (\rho_C - \rho_B) g \quad \dots (7.34)$$

CONTINUOUS GRAVITY DECANTER

Decantation involves the separation of two immiscible liquids of differing densities from one another. Basically, the difference in densities of two immiscible liquids is responsible for such a separation.

Decanters used for the separation of two immiscible liquids are : (i) gravity decanter and (ii) centrifugal decanter. Decanters utilize either a gravitational force or a centrifugal force to effect the separation.

A gravity decanter is used for the separation of two immiscible liquids when the difference between densities of the two liquids is large. A centrifugal decanter is used for the separation of two immiscible liquids whenever the difference between densities of the two liquids is small. The separating force (centrifugal force) in the centrifugal decanter is much larger than the force of gravity.

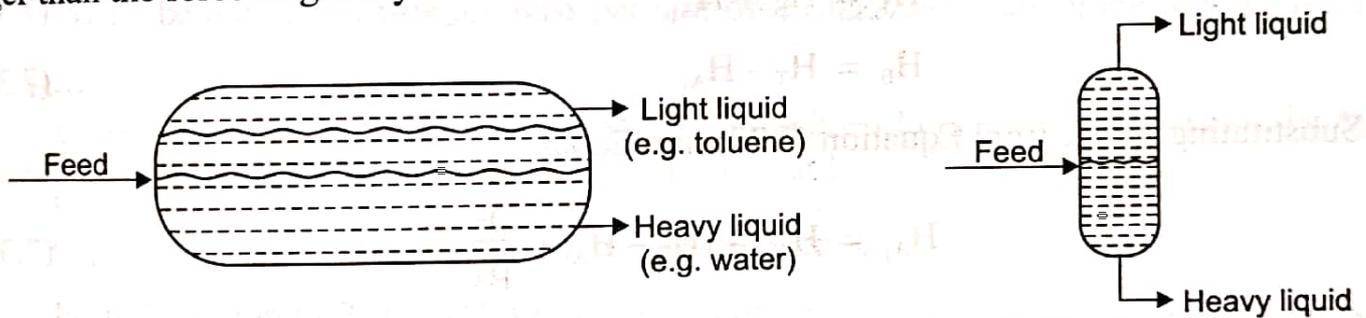


Fig. 7.8 (A) : Continuous Gravity Decanters for immiscible liquids

Separation of two immiscible liquids based on the density difference of the phases involved is commonly encountered in the mass transfer operation such as liquid-liquid extraction.

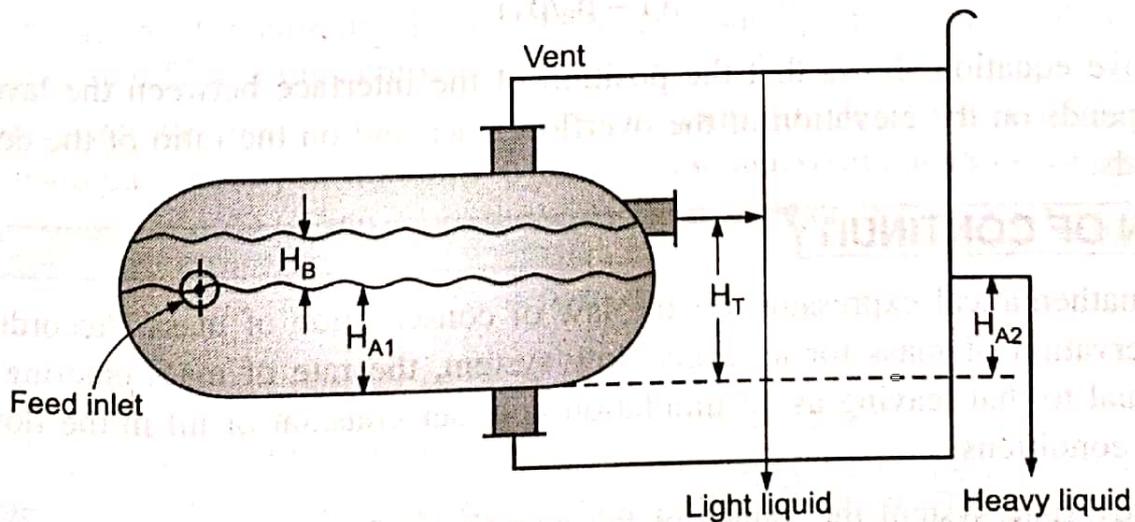


Fig. 7.8 (B) : Continuous Gravity Decanter

Feed enters into the decanter at one end, two immiscible liquids flow slowly, separate into two layers based on the density difference, and then finally the separated layers leave the decanter through the overflow lines at the other end.

Let the densities of the heavy and light liquids be ρ_A and ρ_B respectively. Let the interface between two liquids is at a height H_{A1} from the bottom of the vessel. The total depth of the liquid in the vessel is H_T and the depth of the layer of the light liquid is H_B . The overflow of the light liquid is at a height H_{A2} and that of the heavy liquid is at a height H_{A2} from the bottom of the vessel.

Assume that the frictional losses in the overflow discharge lines are negligible, and the overflow lines and the vessel itself are open to the atmosphere through a vent line.

A hydrostatic balance gives

$$H_B \cdot \rho_B + H_{A1} \cdot \rho_A = H_{A2} \cdot \rho_A \quad \dots(7.35)$$

$$H_{A1} = H_{A2} - H_B \cdot \frac{\rho_B}{\rho_A} \quad \dots (7.36)$$

$$H_T = H_B + H_{A1} \quad \dots (7.37)$$

$$H_B = H_T - H_{A1} \quad \dots(7.38)$$

Substituting for H_B from Equation (7.38) into Equation (7.36), we get

$$H_{A1} = H_{A2} - (H_T - H_{A1}) \cdot \frac{\rho_B}{\rho_A} \quad \dots (7.39)$$

Collecting the terms, we get

$$H_{A1} (1 - \rho_B/\rho_A) = H_{A2} - H_T (\rho_A/\rho_B) \quad \dots (7.40)$$

$$H_{A1} = \frac{H_{A2} - H_T (\rho_A/\rho_B)}{(1 - \rho_B/\rho_A)} \quad \dots (7.41)$$

The above equation shows that the position of the interface between the layers in the separator depends on the elevation of the overflow lines and on the ratio of the densities of the two liquids.