

PRESSURE DROP IN FLOW THROUGH POROUS MEDIA

Porous medium : It is a continuous solid phase with many pores or void spaces in it. Typical examples of porous media include sponges, filters and packed beds used for absorption and distillation, etc.

Porous media which are not having interconnected pores are said to be impermeable to fluid flow. In such cases, there is no possibility of a fluid to flow through them, e.g., foamed polystyrene drinking cups.

Porous media which are having interconnected pores are said to be permeable to fluid flow. In such cases, there is a flow of fluid through them, e.g., a bed of granular solids.

The flow of fluids through permeable porous media is of great practical importance in filters, packed columns used for absorption and distillation and fixed-bed catalytic reactors.

Consider the flow of a homogeneous fluid through a bed of spherical particles of uniform size. If the average velocity at any cross-section perpendicular to the flow is based on the entire cross-sectional area of the pipe/tower/bed (i.e., empty tower/bed cross-section), then it is called as the superficial velocity and is given by

$$u_0 = Q/A$$

... (7.137)

On the other hand, the average velocity may be based on the area actually available for the flowing fluid, in which case it is called as the interstitial velocity. It is given by

$$u = \frac{u_0}{e} = \frac{Q}{e.A} \quad \dots (7.138)$$

where e is the porosity, or void fraction or voidage of the bed.

$$\begin{aligned} e &= \frac{\text{total volume of bed} - \text{volume of solids in bed}}{\text{total volume of bed}} \\ &= \frac{\text{free volume available for flow}}{\text{total volume of fixed bed}} \quad \dots (7.139) \end{aligned}$$

If D is the diameter of a pipe/bed/tower, h is the height of a bed in the pipe/tower, n is the number of solids in the bed and v_p is the volume of one solid particle then

$$e = \frac{\pi/4 D^2 h - n v_p}{\pi/4 D^2 h} \quad \dots (7.140)$$

From the theoretical point of view, the interstitial velocity is more important since it determines the kinetic energy, the fluid forces and nature of the flow (i.e., whether the flow is laminar or turbulent). From the practical point of view, the superficial velocity is usually more useful.

For a packed bed, the hydraulic radius varies along the bed height and is given by

$$r_H = \frac{\text{Wetted cross-sectional area normal/perpendicular to fluid flow}}{\text{Wetted perimeter}}$$

Multiplying both the numerator and the denominator by the length of the bed / the height of the bed, we get

$$r_H = \frac{\text{Volume open to flow/available for flow}}{\text{Total wetted surface area of packing}}$$

For a porous medium made of spherical particles of uniform size, the hydraulic radius is given by

$$r_H = \frac{\text{Total volume of bed} \times e}{\text{Number of particles} \times \text{Surface area of one particle}} \quad \dots (7.141)$$

The number of solid particles in the bed are given by

$$\text{Number of particles} = \frac{\text{Volume of bed} \times (1 - e)}{\text{Volume of one particle}} \quad \dots (7.142)$$

Therefore, the hydraulic radius becomes

$$\begin{aligned} r_H &= \frac{e}{(1 - e) \times \left(\frac{\text{surface area}}{\text{volume}} \right) \text{ of a spherical particle}} \\ &= \frac{e}{(1 - e) (\pi d_p^2 / \pi/6 d_p^2)} \\ r_H &= \frac{d_p}{6} \left(\frac{e}{1 - e} \right) \quad \dots *(7.142) \end{aligned}$$

where d_p is the diameter of the spherical particle.

For flow through packed beds, the Reynolds number and the friction factor become

$$N_{Re} = D_e u \rho / \mu \\ = (4 r_H) (u_o/e) \rho / \mu$$

Substituting for r_H gives

$$N_{Re} = 4 \frac{d_p}{6} \times \frac{e}{1-e} \times \frac{u_o}{e} \times \frac{\rho}{\mu} \\ N_{Re} = \frac{2 d_p u_o \rho}{3 \mu (1-e)} \quad \dots (7.143)$$

We have, $\frac{\Delta P}{\rho} = \frac{4 f L u^2}{2 D}$

Replacing D by D_e and rearranging, we get

$$f = \left(\frac{\Delta P}{\rho L} \right) \left(\frac{D_e}{2 u^2} \right)$$

Replacing D_e by $4 r_H$ gives

$$f = \left(\frac{\Delta P}{\rho L} \right) \frac{(4 r_H)}{2 (u_o/e)^2}$$

Substituting for r_H gives

$$f = \left(\frac{\Delta P}{\rho L} \right) 4 \frac{d_p}{6} \times \frac{e}{1-e} \times \frac{e^2}{2 u_o^2} \\ f = \frac{1}{3} \left(\frac{\Delta P}{\rho L} \right) \frac{d_p}{u_o^2} \times \frac{e^3}{1-e} \quad \dots (7.144)$$

Hence, the modified friction factor and the Reynolds number for flow through packed beds are defined as

$$f_m = \left(\frac{\Delta P}{\rho L} \right) \frac{d_p}{u_o^2} \times \frac{e^3}{1-e} \quad \dots (7.145)$$

and $N_{Re,m} = \frac{d_p u_o \rho}{\mu (1-e)} \quad \dots (7.146)$

The Ergun equation for the flow of homogeneous fluids through packed beds relating f_m and $N_{Re,m}$ is

$$f_m = \frac{150}{N_{Re,m}} + 1.75 \quad \dots (7.147)$$

Equation (7.147) fits experimental data well for $N_{Re,m}$ between 1 and 2000.

For $N_{Re,m} < 10$ (i.e., when the flow is laminar/streamline through packed beds), the viscous forces control and Equation (7.147) reduces to

$$f_m = \frac{150}{N_{Re,m}} \quad \dots (7.148)$$

Substituting the values of f_m and $N_{Re,m}$, we get

$$\left(\frac{\Delta P}{\rho L}\right) \frac{d_p}{u_0} \times \frac{e^3}{1-e} = \frac{150}{d_p u_0 \rho / \mu (1-e)}$$

$$\therefore \left(\frac{\Delta P}{\rho}\right) = \frac{150 \mu u_0 L}{d_p^2 \rho} \times \frac{(1-e)^2}{e^3} \quad \dots (7.149)$$

Equation (7.149) is known as the Blake-Kozeny equation, or Kozeny-Carman equation.

The flow of fluids through industrial filters is usually streamline/laminar and thus Equation (7.149) applies.

For $N_{Re,m} > 1000$ (i.e., when the flow is turbulent), the effect of viscous forces is negligible and inertial forces control and Equation (7.147) reduces to

$$f_m = 1.75 \quad \dots (7.150)$$

$$\text{or } \frac{\Delta P}{\rho} = 1.75 \frac{u_0^2 L}{d_p} \times \frac{(1-e)}{e^3} \quad \dots (7.151)$$

Equation (7.151) is known as the Burke-Plummer equation.

For non-spherical particles, the diameter d_v of a non-spherical particle may be defined as the diameter of a sphere having the same volume as that of the particle.

For non-spherical particles, the shape factor or sphericity ϕ_s is defined as below :

$$\phi_s = \frac{\text{Surface area of a sphere having the same volume as that of the particle}}{\text{Surface area of a particle}} \quad \dots (7.152)$$

For spheres : $\phi_s = 1$ and for all other particle shapes : $0 < \phi_s < 1$

For beds of non-spherical granular solids, d_p in Equations (7.142) and (7.151) is to be replaced by $\phi_s d_v$ so that the Ergun Equation (7.147) becomes

$$\frac{\Delta P}{\rho L} = 150 \frac{\mu u_0}{\rho (\phi_s d_v)^2} \frac{(1-e)^2}{e^3} + 1.75 \frac{u_0^2}{\phi_s d_v} \times \frac{1-e}{e^3} \quad \dots (7.153)$$

PRESSURE DROP IN FLUIDISATION

When a fluid (gas or liquid) is passed up through a bed of solid particles at very low velocity, the particles do not move and the pressure drop is given by the Ergun equation. If the fluid velocity is steadily increased, the pressure drop and drag on the individual particles increase, and eventually the particles start to move and become suspended in the fluid stream.

An operation by which fine granular solids are transformed into a fluid-like state through contact with a fluid is known as fluidisation. The term fluidised bed is used to describe the condition of fully suspended particles in a fluid stream.

Advantages of fluidisation : (i) good contact of the fluid with all parts of the solid particles, (ii) prevents segregation of the solid particles by thoroughly agitating the bed, (iii) temperature is uniform even in a large reactor due to complete mixing and (iv) high heat and mass transfer rates. **Disadvantages of fluidisation:** (i) greater power requirement, (ii) higher breakage of solid particles, (iii) serious erosion of pipelines and vessels and (iv) necessity of recovery systems.

Fluidisation finds applications in catalytic cracking, drying and transportation of solids, etc.

Consider a vertical tube partially filled with a fine granular particles. The tube is open at the top and has a porous plate to support the bed of particles and to distribute the flow uniformly over the entire cross-section.

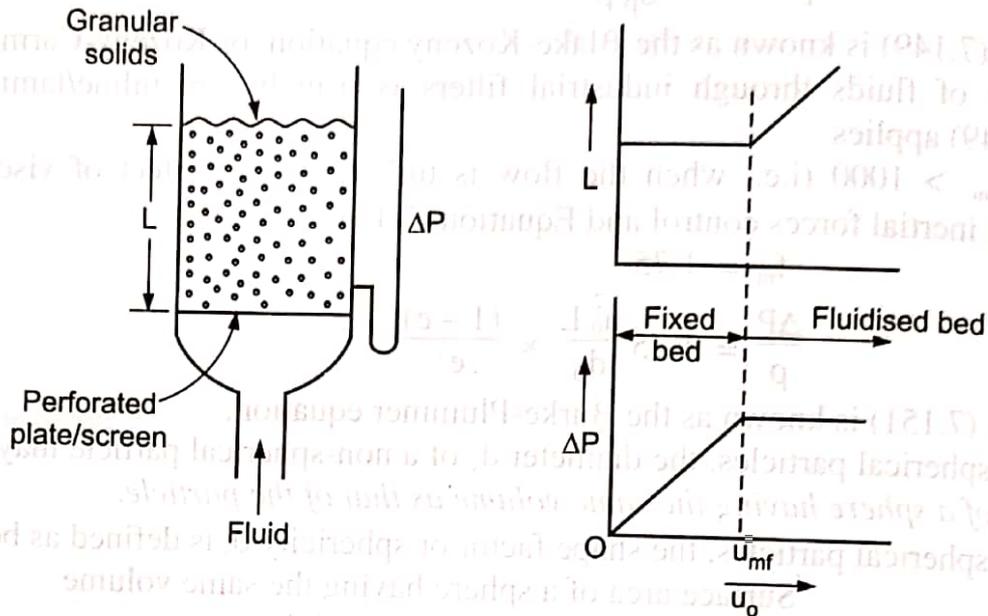


Fig. 7.21 : Pressure drop and bed heights v/s superficial velocity for bed of solids

Further, consider the upward flow of a fluid through a vertical bed of fine particles. At a low flow rate, the fluid simply passes through the void spaces between stationary particles without causing any particle motion. The flow will be laminar and the pressure drop across the bed will be proportional to the superficial velocity u_0 .

As we go on increasing steadily the flow rate of the fluid, a stage will be reached when all the particles are just suspended in the flowing fluid. At this stage, the pressure drop across the bed equalizes the weight of the bed. The bed in this condition is considered to be just fluidised and is termed as an incipiently fluidised bed.

Thus, for a bed at minimum fluidisation, we have

$$\Delta P A = W = A L_{mf} (1 - e_{mf}) (\rho_p - \rho) g \quad \dots (7.154)$$

where A = cross-sectional area of the tube and ρ_p = density of the particle.

Rearranging Equation (7.154) gives

$$\frac{\Delta P}{L_{mf}} = (1 - e_{mf}) (\rho_p - \rho) g \quad \dots (7.155)$$

In many practical applications of fluidisation, since the particles are very small and the fluid velocity is low, the flow is streamline/laminar. Therefore, Equation (7.149) becomes

$$\frac{\Delta P}{L_{mf}} = \frac{150 \mu u_{mf}}{d_p^2} \times \frac{(1 - e_{mf})^2}{e_{mf}^3} \quad \dots (7.156)$$

The superficial fluid velocity at minimum fluidisation conditions is obtained by equating the right hand sides of Equations (7.155) and 7.156). Therefore, we get u_{mf} as

$$u_{mf} = \frac{d_p^2}{150} \times \frac{\rho_p - \rho}{\mu} g \left(\frac{e_{mf}^3}{1 - e_{mf}} \right) \dots (7.157)$$

Once the bed is fluidised, the pressure drop across the bed remains constant but the bed height increases continuously with increasing velocity (i.e., flow).

Equations (7.155) and (7.156), for a given fluid-solid system, predict that

$$\begin{aligned} \frac{\Delta P}{L(1 - e)} &= k_1 \\ &= k_2 u_0 \frac{(1 - e)}{e^3} \end{aligned} \dots (7.158)$$

where k_1 and k_2 are constants.

$$\begin{aligned} k_1 &= k_2 u_0 \frac{1 - e}{e^3} \\ u_0 &= \frac{k_1}{k_2} \frac{e^3}{1 - e}, \text{ Let } k = k_1/k_2 \\ u_0 &= k \frac{e^3}{1 - e} \end{aligned} \dots (7.159)$$

where k is a constant for the system.

Equation (7.159) gives us idea regarding the variation of the bed porosity (e) with superficial fluid velocity (u_0) in a fluidised bed. This equation has been found to apply equally well for a liquid-solid system for e (bed voidage) less than 0.80.

The relationship between bed heights and bed voidages is given by

$$\frac{L}{L_{mf}} = \frac{1 - e_{mf}}{1 - e} \dots (7.160)$$

Equ. (7.160) is applicable for particle Reynolds numbers less than 1.0 $N_{Re, p} = d_p u_t \rho / \mu$.

MEASUREMENT OF FLUID FLOW

In the chemical process industry, it is desirable to know the amount of a fluid flowing to or from the process equipment. Many different types of flow meters are used industrially to measure the rate at which a fluid is flowing through a pipe or a duct. The *flow measuring devices* or *flow meters* are classified as :

1. Variable head meters, e.g., orifice meter, venturi meter.
2. Variable area meters, e.g., rotameter.
3. Current meters, e.g., cup type current meter.
4. Positive displacement meters, e.g., wet gas meter.
5. Electromagnetic meters, e.g., magnetic meter.

Variable head meters such as venturi meters, orifice meters and pitot tubes and area meters such as rotameters of various designs are widely used for flow measurement.

VENTURI METER

A venturi meter is a variable head meter which is used for measuring the flow rate of a fluid through a pipe. In this meter, the fluid is gradually accelerated to a throat and then gradually retarded in a diverging section where the flow expands to the pipe size. A large portion of the kinetic energy is thus recovered (converted back to pressure energy).

Principle :

The basic principle on which a venturi meter works is that *by reducing the cross-sectional area of the flow passage, a pressure difference is created and the measurement of the pressure difference (between the inlet of the meter and at a point of reduced pressure) enables the estimation of the discharge/flow rate through the pipe.*

Construction

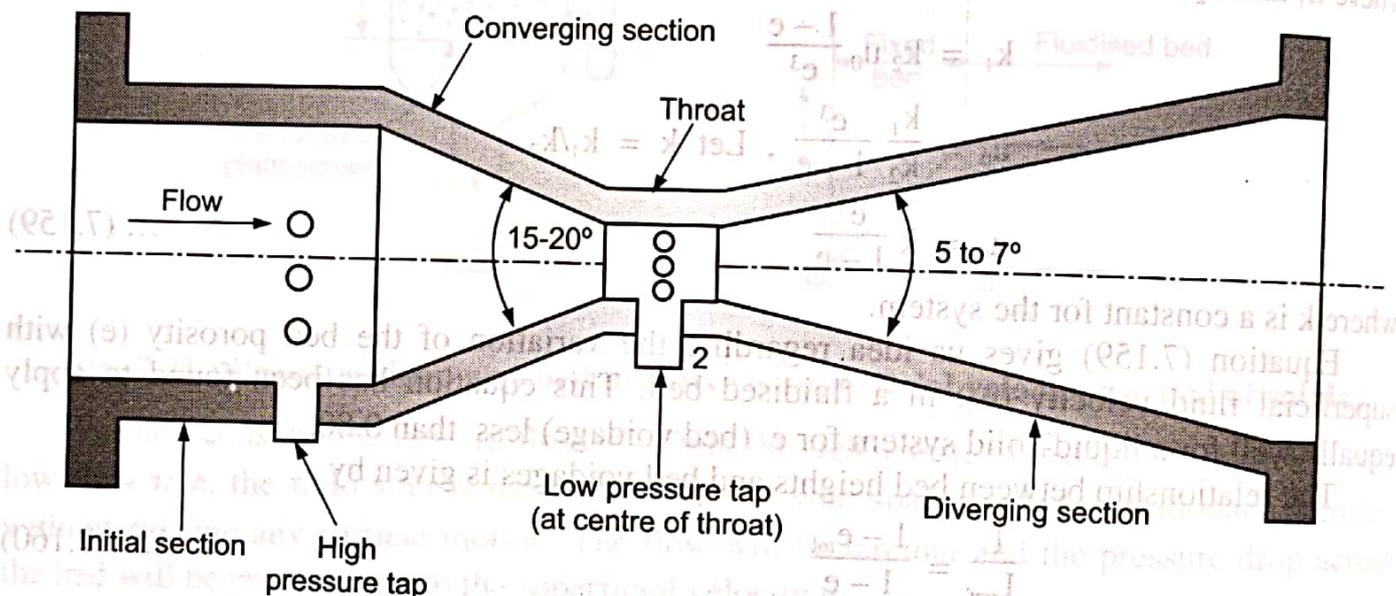


Fig. 7.22 : Venturi meter

A venturi meter consists of

- (i) An inlet section followed by a convergent cone/section. The inlet section of the venturi meter is of the same diameter as that of the pipe line in which it is installed which is followed by the short convergent section with a converging cone angle of 15-20° and its length parallel to the axis is approximately equal to $2.7(D - D_T)$, where D is the pipe diameter and D_T is the throat diameter. In the convergent section, the fluid is accelerated.
- (ii) A cylindrical throat – the section of constant cross-section with its length equal to diameter. The flow area is minimum at the throat. In general, the diameter of throat may vary from 1/3 to 3/4 of the pipe diameter and more commonly the diameter of throat is 1/2 the pipe diameter.
- (iii) A long diverging section/a gradual divergent cone with a cone angle of about 5 - 7° wherein the fluid is retarded and a large portion of the kinetic energy is converted back into the pressure energy.

The convergent section/cone is a short pipe that tapers from the original pipe size to that of the throat of the meter. The divergent cone/section of the venturi meter is a gradually diverging pipe with its cross-sectional area gradually increasing from that of the throat to the original pipe size. At the inlet section and the throat (centre of throat), i.e., at stations 1 and 2 of the venturi meter, pressure taps are provided through the pressure rings/piezometer rings.

A piezometer ring is an annular chamber provided at the pressure taps with small holes drilled from the inside of the tube and is used for averaging out the individual pressures transmitted through the several small holes to a pressure measuring device.

Working :

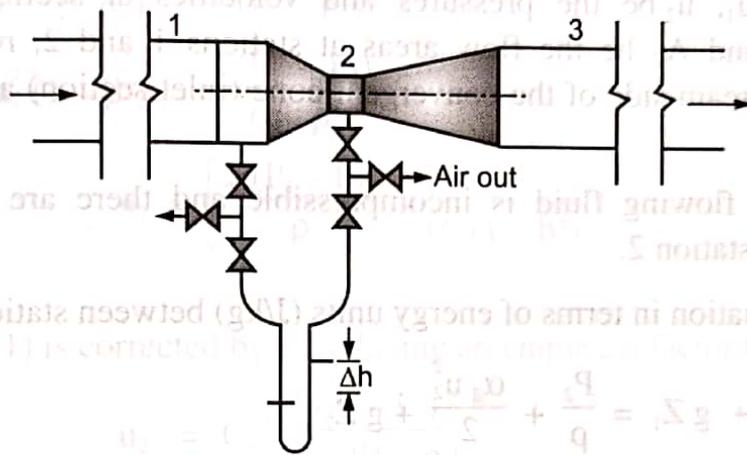


Fig. 7.23 : Venturi meter with U-tube manometer

A venturi meter of known coefficient is installed in the pipeline and the pressure taps are connected to a pressure measuring device. Air pockets, if any, are removed from connecting tubings, measuring device, etc. after starting the flow of fluid through the pipeline in which it is installed for flow measurement. In the meter, the fluid is accelerated in the converging cone and then retarded in the diverging cone gradually. An increase in the flow velocity at the throat (a section of minimum flow area) results in a decrease in the pressure at the throat. Due to this a pressure difference is developed between the inlet section and the throat section which is measured/noted with the help of manometer or pressure gauges (connected to the pressure taps) after the steady state is attained. This pressure difference/drop is then related to the flow rate by a mathematical flow equation for the meter.

In the venturi meter, fluid is accelerated in the convergent cone from the inlet section 1 to the throat section 2 and in the divergent cone, it is retarded from the throat section 2 to the end section 3 of the venturi meter. In order to avoid the possibility of flow separation and consequent energy loss, the divergent cone of a venturi meter is made longer with a gradual divergence. Since the separation of flow may occur in the divergent cone of a venturi meter, this portion is not used for measuring the flow rate.

Since there is a gradual reduction in the area of flow, there is no vena contracta and the flow area is minimum at the throat so that the coefficient of contraction is unity.

FLOW EQUATION FOR A VENTURI METER

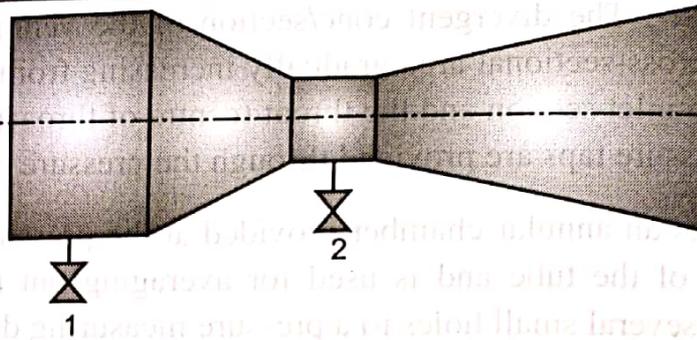


Fig. 7.24 : Schematic view of venturi meter

Let P_1 , P_2 and u_1 , u_2 be the pressures and velocities at section/stations 1 and 2, respectively. Let A_1 and A_T be the flow areas at stations 1 and 2, respectively. Section/station 1 is at the upstream side of the convergent cone (inlet section) and station 2 is at the throat.

Assume that the flowing fluid is incompressible and there are no frictional losses between station 1 and station 2.

The Bernoulli equation in terms of energy units (J/kg) between stations 1 and 2 is

$$\frac{P_1}{\rho} + \frac{\alpha_1 u_1^2}{2} + g Z_1 = \frac{P_2}{\rho} + \frac{\alpha_2 u_2^2}{2} + g Z_2 \quad \dots (7.161)$$

The venturi is connected in a horizontal pipe (Fig. 7.24), so $Z_1 = Z_2$ (or if we assume that a datum is passing through the axis of the venturi meter, then $Z_1 = Z_2 = 0$) and the above equation then reduces to

$$\frac{P_1}{\rho} + \frac{\alpha_1 u_1^2}{2} = \frac{P_2}{\rho} + \frac{\alpha_2 u_2^2}{2} \quad \dots (7.162)$$

From the equation of continuity, we have

$$\dot{m} = \rho u_1 A_1 = \rho u_2 A_T \quad \dots (7.163)$$

where $A_1 = \pi/4 \cdot D^2$ and $A_T = \pi/4 \cdot D_T^2$

in which D = pipe diameter and D_T = throat diameter

$$\therefore u_1 (\pi/4 \cdot D^2) = u_2 (\pi/4 \cdot D_T^2) \quad \dots (7.164)$$

Let $D_T/D = \beta$ = diameter ratio, diameter of throat/diameter of pipe

$$u_1 = (D_T/D)^2 u_2 \quad \dots (7.165)$$

$$u_1 = \beta^2 \cdot u_2 \quad \dots (7.166)$$

Substituting for u_1 from Equation (7.166) into Equation (7.162), we get

$$\frac{P_1}{\rho} + \frac{\alpha_1 (\beta^2 u_2)^2}{2} = \frac{P_2}{\rho} + \frac{\alpha_2 u_2^2}{2} \quad \dots (7.167)$$

Rearranging, we get

$$\frac{\alpha_2 u_2^2}{2} - \frac{\alpha_1 \beta^4 u_2^2}{2} = \frac{P_1 - P_2}{\rho} \quad \dots (7.168)$$

$$\alpha_2 u_2^2 - \alpha_1 \beta^4 u_2^2 = \frac{2(P_1 - P_2)}{\rho}$$

$$\alpha_1 \left[\frac{\alpha_2}{\alpha_1} u_2^2 - \beta^4 u_2^2 \right] = \frac{2(P_1 - P_2)}{\rho} \quad \dots (7.169)$$

Usually $\alpha_2/\alpha_1 = 1.0$. Therefore,

$$\alpha_1 [u_2^2 - \beta^4 u_2^2] = 2(P_1 - P_2) / \rho$$

$$u_2^2 (1 - \beta^4) = \frac{2(P_1 - P_2)}{\alpha_1 \cdot \rho} \quad \dots (7.170)$$

$$u_2 = \left[\frac{2(P_1 - P_2)}{\rho} \times \frac{1}{\alpha_1 (1 - \beta^4)} \right]^{1/2} \quad \dots (7.171)$$

Equation (7.171) is corrected by introducing an empirical factor C_v as

$$u_2 = C_v \left[\frac{2(P_1 - P_2)}{(1 - \beta^4) \cdot \rho} \right]^{1/2} \quad \dots (7.172)$$

where C_v is the coefficient of venturi meter, or the coefficient of discharge of venturi meter, or simply the venturi coefficient and it takes into account the error introduced by assuming no frictional losses between stations 1 and 2 as well as assuming $\alpha_1/\alpha_2 = 1$ and $\alpha_1 = 1.0$ [i.e., it accounts for the frictional loss between stations 1 and 2 and small effects of kinetic energy factors α_1 and α_2].

Volumetric flow rate, Q , is given by

$$Q = u_2 A_T \quad \dots (7.173)$$

where A_T = cross-sectional area of the throat.

Combining Equations (7.172) and (7.173), we get

$$Q = C_v A_T \left[\frac{2(P_1 - P_2)}{(1 - \beta^4) \rho} \right]^{1/2} \quad \dots (7.174)$$

$$Q = \frac{C_v A_T}{\sqrt{1 - \beta^4}} \cdot \sqrt{\frac{2(P_1 - P_2)}{\rho}} \quad \dots (7.175)$$

where Q is the actual discharge / volumetric flow rate

Q_{th} = theoretical discharge and is given by

$$Q_{th} = A_T \left[\frac{2(P_1 - P_2)}{(1 - \beta^4) \cdot \rho} \right]^{1/2} \quad \dots (7.176)$$

$$\therefore C_v = Q / Q_{th} \quad \dots (7.177)$$

The coefficient of discharge of a venturi meter is the *ratio of the actual discharge to the theoretical discharge of the venturi meter*.

Equation (7.176) gives the value of theoretical discharge through the venturi as it has been obtained by considering no frictional losses in the system.

Equation (7.175) is the desired flow equation for a venturi meter.

For a well designed venturimeter, the coefficient of venturi, C_v is about **0.98** for pipe diameters ranging from 50 to 200 mm and about **0.99** for larger sizes.

If U-tube manometer is used for measuring the pressure difference between station 1 and station 2 and Δh is the manometer reading obtained in terms of meters of the manometric fluid, then pressure drop across the meter is given by

$$(P_1 - P_2) = \Delta h (\rho_M - \rho) g \quad \dots (7.178)$$

where ρ_M = density of manometric fluid in kg/m^3

ρ = density of flowing fluid in kg/m^3

and $g = 9.81 \text{ m/s}^2$

Combining Equations (7.175) and (7.178), we get

$$Q = \frac{C_v A_T}{\sqrt{1 - \beta^4}} \cdot \sqrt{\frac{2 \Delta h (\rho_M - \rho) g}{\rho}} \quad \dots (7.179)$$

If ΔH is the pressure difference across the venturi expressed in terms of meters of the flowing fluid, then

$$\Delta H = \Delta h \frac{(\rho_M - \rho)}{\rho} \quad \dots (7.180)$$

Combining Equations (7.179) and (7.180), we get

$$Q = \frac{C_v A_T}{\sqrt{1 - \beta^4}} \cdot \sqrt{2 g \Delta H} \quad \dots (7.181)$$

Equation (7.179) is the desired flow equation for a venturi meter, where the pressure difference is expressed in terms of manometer reading and Equation (7.181) is the desired flow equation where the pressure difference is expressed in terms of meters of the flowing fluid.

Pressure Recovery in Venturi meter

Since the angle of divergence in the recovery cone (i.e., in the divergent cone) is small, the permanent pressure loss from a venturi meter is relatively small. In a well designed meter, the permanent pressure loss is about 10 percent of the venturi differential $P_1 - P_2$ (i.e., 10 percent of the pressure drop across the meter) which means that about 90 percent of the venturi differential is recovered.

The pressure recovery in a venturi meter is very high and thus may be used where only a small pressure head is available though it is expensive.

Advantages of Venturi meter :

1. Low permanent pressure loss and hence high pressure recovery.
2. High accuracy over wide flow ranges.

3. It can be used for flow measurement of compressible and incompressible fluids.
4. It can also be used where only a small pressure head is available.
5. High reproducibility.
6. Less power loss.

Disadvantages of Venturi meter :

1. It is expensive and bulky.
2. It occupies considerable space.
3. Relatively complex in construction.
4. The ratio of throat diameter to pipe diameter cannot be changed.
5. Not suitable for flow measurement of highly viscous slurries.
6. Used only for permanent installations.
7. It cannot be altered once it is installed.

ORIFICE METER

It is a variable head meter used for measuring the discharge / flow rate through a pipe. In this meter, the fluid is accelerated by causing it to flow through a sudden constriction (orifice), the kinetic energy of the fluid increases and the pressure energy therefore decreases. With this meter, the overall pressure drop across the meter is high - a large percentage of the pressure drop across the meter is not recoverable, but is relatively cheap and reliable instrument and its installation requires a small length as compared to the venturi meter. Because of this where the space is limited, the orifice meter may be used for the measurement of discharge (flow rate) through pipes.

Principle :

The basic principle on which an orifice meter works is that *by reducing the cross-sectional area of the flow passage, the fluid is accelerated and a pressure difference is created/developed, and the measurement of the pressure difference (between the inlet of the meter and a point of reduced pressure) enables the determination of the discharge (flow rate) through the pipe*

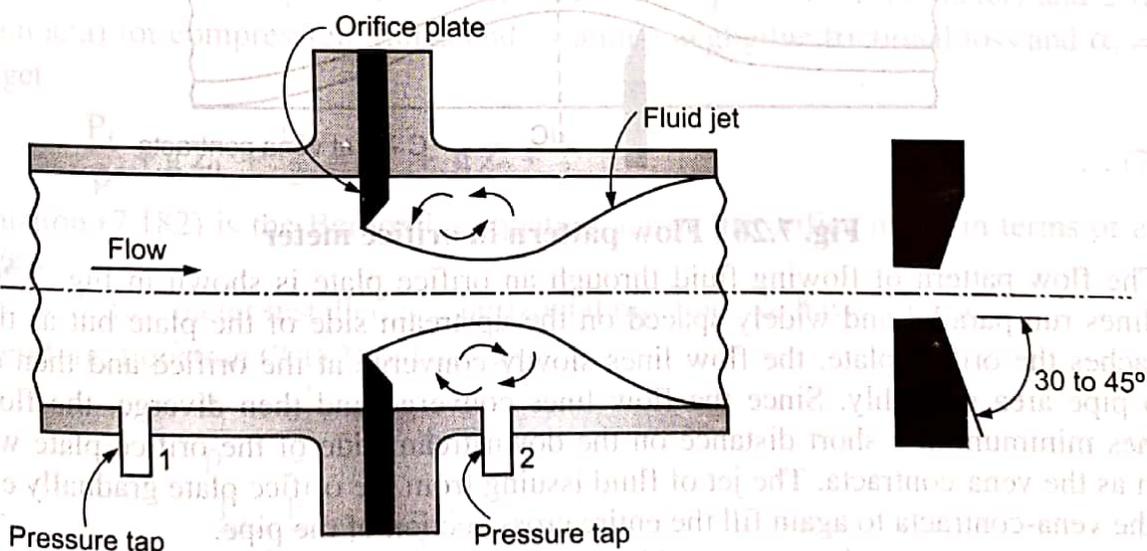


Fig. 7.25 : Orifice meter