FLOW OF INCOMPRESSIBLE FLUIDS IN PIPES

Shear-stress distribution in a cylindrical tube

Consider the steady flow of a fluid through a horizontal tube. Imagine a disk-shaped element of fluid, of radius r and length dL, concentric with the axis of the tube as shown in Fig. 7.14. Let P and P + dP be the fluid pressures on the upstream and downstream faces of the disk respectively. Assume fully developed flow and the density of fluid to be constant.

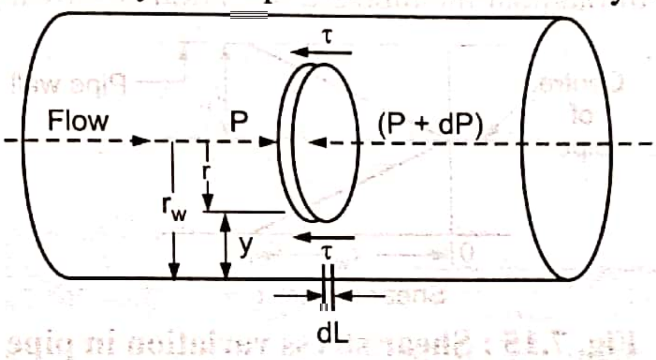

Fig. 7.14 : Fluid element (of radius r and length dL) in steady flow through the pipe

The sum of all the forces acting on this disk shaped element of fluid is equal to zero as the flow is fully developed.

$$\begin{bmatrix}\text{Pressure force on the upstream}\\ \text{face of the disk}\end{bmatrix} = P\pi r^2 \quad \text{... in the direction of flow (+ve)}$$

$$\begin{bmatrix}\text{Pressure force on the downstream}\\ \text{face of the disk}\end{bmatrix} = (P + dP)\,\pi\, r^2 \quad \text{... opposing the flow (−ve)}$$

$$\begin{bmatrix}\text{Shear force opposing flow}\\ \text{acting at the outer surface of the disk}\\ \text{due to the viscosity of fluid}\end{bmatrix} = \text{shear stress} \times \text{cylindrical area}$$

$$= \tau\,.\,(2\pi r\, dL) \quad \text{... opposing the flow (−ve)}$$

We have :
$$\sum F = 0 \quad \text{... (7.71)}$$

$$\therefore \quad +\pi r^2 P - \pi r^2 (P + dP) - 2\pi r\, dL\, \tau = 0 \quad \text{... (7.72)}$$

$$\therefore \quad -\pi r^2\, dP - 2\pi r\, dL\, \tau = 0 \quad \text{... (7.73)}$$

$$\pi r^2\, dP + 2\pi r\, dL\, \tau = 0 \quad \text{... (7.74)}$$

Dividing by $\pi r^2\, dL$, we get

$$\frac{dP}{dL} + \frac{2\tau}{r} = 0 \quad \text{... (7.75)}$$

In steady-state laminar or turbulent flow, the pressure at any given cross-section of a stream tube is constant, therefore dP/dL is independent of r. For the entire cross-section of the tube, Equation (7.75) can be written by taking $\tau = \tau_w$ at $r = r_w$, where τ_w is the shear stress at the wall of the tube and r_w is the radius of the tube.

$$\therefore \quad \frac{dP}{dL} + \frac{2\tau_w}{r_w} = 0 \quad \text{... (7.76)}$$

Subtracting Equation (7.75) from Equation (7.76), we get

$$\frac{\tau_w}{r_w} = \frac{\tau}{r} \quad \dots (7.77)$$

For r = 0, τ = 0 (i.e., shear stress is zero at the centreline of the tube). The simple linear relation between τ and r throughout the tube cross-section is shown graphically in Fig. 7.15.

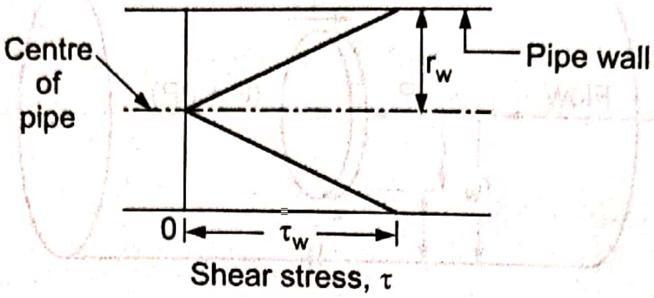

Fig. 7.15 : Shear stress variation in pipe

Relation between Skin friction and Wall shear

The Bernoullis equation (Equation 7.66) can be written over a definite length, ΔL for the complete stream.

Let P_1 be the pressure at the upstream of length ΔL and P_2 be the pressure at the downstream of length ΔL such that $P_1 > P_2$, so that ΔP is the pressure drop over the length ΔL.

The term ΔP is commonly used for pressure drop, i.e., $P_1 - P_2$ (inlet/upstream pressure - outlet/downstream pressure) and this terminology is used henceforth in this book.

If $P_1 = P$, then $P_2 = P - \Delta P$

since $P_1 - P_2 = \Delta P$

In this case, $Z_1 = Z_2 = 0$ and $u_1 = u_2$. The friction that exists in this case is the skin friction between the wall and the fluid stream (h_{fs}). Skin friction is the tangential friction associated with a fluid flowing over a smooth surface. When a fluid is flowing through a pipe, it is only the skin friction that exists.

Then, the Bernoulli equation over the length ΔL becomes

$$\frac{P}{\rho} = \frac{P - \Delta P}{\rho} + h_{fs} \quad \dots (7.78)$$

$$\therefore \quad h_{fs} = \frac{\Delta P}{\rho} \quad \dots (7.79)$$

In the above equation, each term has the units of J/kg.

If ΔP is expressed in N/m² and ρ in kg/m³, then the frictional loss h_{fs} has the units of J/kg.

The Bernoulli equation, over the length ΔL, when each term in it is expressed as the head (in m of flowing fluid) is

$$\frac{P}{\rho g} = \frac{P - \Delta P}{\rho\, g} + \frac{h_{fs}}{g} \quad \dots (7.80)$$

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[Equation (7.80) is obtained by dividing each term of Equation (7.78) by g]

$$\therefore \quad \frac{h_{fs}}{g} = \frac{\Delta P}{\rho g} \qquad \text{... (7.81)}$$

h_{fs}/g represents the head loss due to friction.

$$\begin{bmatrix}\text{Head loss due to friction} \\ \text{(frictional head loss)}\end{bmatrix} = h'_{fs} = \frac{h_{fs}}{g} = \frac{\Delta P}{\rho\, g} \qquad \text{... (7.82)}$$

Combining Equations (7.76) and (7.79) for eliminating ΔP from these equations, we get

$$h_{fs} = \frac{2\,\tau_w}{\rho_{r_w}}\,\Delta L = \frac{4\,\tau_w}{\rho\, D}\,\Delta L \qquad \text{... (7.83)}$$

where D is the diameter of the pipe ($D = 2r_w$).

THE FANNING FRICTION FACTOR (f)

It is especially useful in the study of turbulent flow. It is defined as *the ratio of the wall shear stress to the product of the kinetic energy of fluid and the density.*

$$f = \frac{\tau_w}{\rho . u^2/2} = \frac{2\,\tau_w}{\rho u^2} \qquad \text{... (7.84)}$$

The unit of f is ⇒

$$\frac{N/m^2}{(kg/m^3)\,(m^2/s^2)} = \frac{kg.m}{s^2.m^2} \times \frac{1}{\frac{kg}{m^3} \times m^2/s^2}$$

$$= \frac{kg.m^4.s^2}{kg.m^4.\,s^2}$$

Since the term $2\tau_w/\rho u^2$ (which defines f) is not having units, f is a dimensionless quantity.

From Equation (7.84), we have

$$\tau_w = f\,\rho\, u^2/2 \qquad \text{... (7.85)}$$

Substituting the value of τ_w from Equation (7.85) into Equation (7.83), we get

$$h_{fs} = \frac{4\,f\,\rho\, u^2\,\Delta L}{2\,\rho\, D} \qquad \text{... (7.86)}$$

$$h_{fs} = \frac{4\,f\,u^2\,\Delta L}{2\,D} \qquad \text{... (7.87)}$$

ΔL can be replaced by L (the length of the pipe).

$$\therefore \quad h_{fs} = \frac{4\,f\,L\,u^2}{2\,D} \qquad \text{... (7.88)}$$

We have,
$$h_{fs} = \frac{\Delta P}{\rho} \quad \ldots (7.89)$$

$$\therefore \quad h_{fs} = \frac{\Delta P}{\rho} = \frac{4 f L u^2}{2 D} \quad \ldots (7.90)$$

Note that each term in Equation (7.90) has the units of energy per unit mass (J/kg).

In Equation (7.90), ΔP is the pressure drop over a length L of the pipe.

Whenever we have to calculate the head loss due to friction, then Equation (7.90) modifies to :

$$h'_{fs} = \frac{h_{fs}}{g} = \frac{\Delta P}{\rho g} = \frac{4 f L u^2}{2 g D} \quad \ldots (7.91)$$

where h'_{fs} or h_{fs}/g represents the head loss due to friction measured in terms of m of a flowing fluid.

From Equation (7.90), we get

$$\Delta P = \frac{4 f \rho L u^2}{2 D} \quad \ldots (7.92)$$

The pressure drop due to friction in a pipe for turbulent flow can be calculated from Equation (7.92) and is known as the *Fanning equation.*