MODULE-III

FLOW OF FLUID PAST IMMERSED BODIES:

Drag, Drag coefficient, Pressure drop – Kozeny-Carman equation, Blake-Plummer, Ergun equation, Fluidization, conditions for fluidization, Minimum fluidization velocity, Pneumatic conveying.

MOTION OF PARTICLES THROUGH FLUIDS:

Mechanics of particle motion, Equation for one dimensional motion of particles through a fluid in gravitational and centrifugal field, Terminal velocity, Drag coefficient, Motion of spherical particles in Stoke's region, Newton's region, and Intermediate region, Criterion for settling regime, Hindered settling, Modification of equation for hindered settling, Centrifugal separators, Cyclones and Hydro cyclones.

MODULE - III

The size distribution is important for finding the average size of mixture of particles. The variations of settling velocities of particles because of difference are size and of difference in densities make it easy for the separation of particles in to different sizes. Therefore knowledge of settling of particles in mediums is important to calculate the particle velocity. Also settling velocities of particles will be helpful in classification of particles and also in clarification.

MOTION OF PATICLES THOUGH FLUIDS [SETTLING]

When a particle is moving through a fluid, it is acted upon by three forces

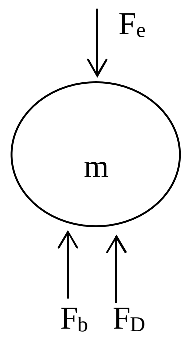
i. The external force (gravitational or centrifugal)

ii. The buoyant force – acts parallel to external force but in the opposite direction.

iii. The drag force – appears whenever there is a relative motion between solid and fluid. The drag force opposes the motion of the particle.

Equation for one dimensional motion of particle through fluid:

Consider a particle of mass ‘m’ moving through a fluid under the action of an external force F_e. Let the velocity of the particle relative to the fluid be u. Let the buoyant force on the particle be F_b and let the drag force be F_d. The resultant force on the particle will be $F_e – F_b - F_D$. This gives momentum to the particle and the particle moves down with an acceleration of du/dt . Then



Mass m

$$m \cdot \frac{du}{dt} = F_e - F_b - F_D \qquad (1)$$

The external force can be expressed as a product of mass and acceleration of the particle from this force.

$$\text{i.e } F_e = ma_e \qquad (2)$$

The buoyant force is equal to the product of mass of liquid displaced and acceleration due to gravity.

$$F_b = \left(\frac{m}{\rho_p}\right) \rho\, a_e \qquad (3)$$

$\rho \rightarrow$ Density of fluid

$\rho_P \rightarrow$ Density of solid

The drag force is given by

$$F_D = \frac{C_D u^2 \rho A_p}{2} \qquad (4) \qquad\qquad C_D = \frac{F_D / A_P}{\rho\, u^2 / 2} \qquad \text{Where}$$

C_D – Dimension less drag co-efficient

A_P – Projected area of particle measured in a plane perpendicular to the direction of motion of particle.

Substituting (2) (3) (4) in (1)

$$m \cdot \frac{du}{dt} = ma_e - \frac{m}{\rho_p} .\rho a_e - \frac{C_D u^2 \rho\, Ap}{2}$$

$$\Rightarrow \frac{du}{dt} = a_e - \frac{\rho}{\rho_p} a_e - \frac{C_D u^2 \rho\, A_p}{2m}$$

$$= a_e \frac{(\rho_p - \rho)}{\rho_p} - \frac{C_D u^2 \rho\, A_p}{2m}$$

If external force is gravitational force, $a_e = g$

$$\frac{du}{dt} = \frac{(\rho_p - \rho)}{\rho_p} g - \frac{C_D u^2 \rho\, A_p}{2m} \qquad (5)$$

Terminal Velocity:

In gravitational settling, ‘g’ is constant. The drag always increases with velocity. According to equation (5), the acceleration decreases with time and approaches zero. The particles quickly reach a constant velocity, which is the maximum attainable under the circumstances and is called the terminal velocity [terminal settling velocity]. The equation for terminal settling velocity u_t is found for gravitational settling by taking $\frac{du}{dt} = 0$, then from (5)

$$0 = \frac{\rho_P - \rho}{\rho_P}\, g - \frac{C_D u_t^{\,2} \rho A_P}{2m}$$

$$= \frac{C_D u_t^{\,2} \rho A_P}{2m} = \frac{\rho_p - \rho}{\rho_p} \cdot g$$

$$u_t = \sqrt{\frac{2g\dfrac{(\rho_p - \rho)}{\rho_p} m}{C_D\, \rho\, A_p}} \qquad (6)$$

Free and Hindered settling:

When a particle is at sufficient distance from the boundary of the container and from other particles so that its fall is not affected by them, the process is called free settling. If the motion is impended by other particles which will happen when the particles are near to each other even though they may not actually be colliding, the process is called hindered settling. The drag co-efficient in hindered settling is greater than in free settling.

Motion of spherical Particles: (under free settling):

For spherical Particles of dia. D_P, $m = \dfrac{\pi D_P^3}{6} \int_P$ and $A_P = \dfrac{\pi}{4} D_P^2$

Substituting for m and A_P in (6)

A_P – Projected area of particle measured in a plane perpendicular to the direction of motion of particle.

m- Mass of the particle

$$u_t = \sqrt{\frac{2g(\rho_P - \rho)}{C_D \rho_P} \times \cancel{\pi} \frac{DP^3}{6} \times \rho_P \cdot \frac{4}{\cancel{\pi}} \frac{1}{D_P^2}}$$

$$= \sqrt{\frac{4}{3} \cdot \frac{g D_P (\rho_p - \rho)}{C_D \rho}} \qquad (7)$$

For particles moving with constant velocity under gravitational force, the drag co-efficient C_D is related to the N_{Rep} by the relation – $C_D = \dfrac{b_1}{N_{\text{Rep}}^n}$ where b_1 and n are constants and are given in the following table for different settling range.

Range	b_1	n
Stoke's	24	1
Intermediate	18.5	0.6
Newton's	0.44	0

N_{ReP} is defined as $N_{ReP} = \dfrac{D_P^2 u_t \rho}{\mu}$

Substituting for C_D in (7)

$$u_t = \sqrt{\frac{4}{3} \cdot \frac{g D_P (\rho P - \rho)}{\frac{b_1}{N_{\mathrm{Re}\,P}^n} \rho}}$$

$$= \sqrt{\frac{4}{3} \frac{g D_P}{b_1 \rho} (\rho_P - \rho) N_{\mathrm{Re}\,P}^n} = \sqrt{\frac{4}{3} \frac{g D_{P(\rho_P - \rho)}}{b_1 \rho} \frac{D_P^n u_t^n \rho^n}{\mu_1^n}}$$

$$u_t^2 = \frac{4}{3} \frac{g D_P (\rho_P - \rho)}{b_1 \rho} \frac{D_n^P u_t{}^n \rho^n}{\mu^n}$$

$$u_t^{2-n} = \frac{4}{3} \frac{g D_P^{n+1} (\rho_p - \rho)}{b_1 \mu^n \rho^{1-n}}$$

$$u_t = \left[\frac{4}{3} \frac{g D_P^{n+1} (\rho_p - \rho)}{b_1 \mu^n \rho^{1-n}} \right]^{1/2-n} \qquad (8)$$

equation (8) is the general eqn. for settling of spherical particles under free settling and gives the terminal velocity in terms of dia. of particle and properties of solid and fluid.

Stoke's Law: Substituting $b_1 = 24$, $n = 1$ in (8)

$$u_t = \left[\frac{4}{3} g \frac{D_P^{1+1} (\rho_p - \rho)}{24 \mu^1 \rho^{1-1}} \right]^{1/2-1}$$

$$= \frac{4}{3} \frac{g D_P^2 (\rho_p - \rho)}{24 \mu}$$

$$= \frac{g D_P^2 (\rho_p - \rho)}{18 \mu} \qquad (9)$$

This is Stoke's law equation for free settling. N_{ReP} 0 – 1

Newton's range: Substituting $b_1 = 0.44$, $n = 0$ in (8)

$$u_t = \left[\frac{4}{3} g \frac{D_P^{1+0}}{0.44} \frac{(\rho_p - \rho)}{\mu^0 \rho^{1-0}}\right]^{1/(2-0)}$$

$$= \left[\frac{4}{3} g \frac{D_P(\rho_p - \rho)}{0.44\rho}\right]^{1/2}$$

$$= 1.75 \sqrt{\frac{gD_P(\rho_p - \rho)}{\rho}} \qquad (10)$$

This is the Newton's law equation for free settling.

N_{ReP} range from 1000 – 200000.

Intermediate range: Substituting $b_1 = 18.5$, $n = 0.6$ in (8)

$$u_t = \frac{0.153 g^{0.71} D_P^{1.14} (\rho_p - \rho)^{0.71}}{g^{0.29} \mu^{0.43}}$$

N_{ReP} range 1 to 1000

Criterion equation for selecting equation:

If the terminal velocity of the particle of known dia. is desired and if the Reynolds's number is unknown, a choice of equation cannot be made. To identify the range in which the motion of particle lies, the velocity term is eliminated from N_{ReP} by substituting u_t from equation (9) for Stoke's law range to give

$$N_{ReP} = \frac{D_P u_t \rho}{\mu}$$

$$= \frac{D_P \rho}{\mu} \frac{g D_P^2 \rho(\rho_p - \rho)}{18\mu} = \frac{D_P^3 \rho(\rho_p - \rho) g}{18\mu^2} \qquad (11)$$

To apply Stoke's law, N_{ReP} should be between 0 and 1. To provide a convenient criterion K, let

$$K = D_P \left[\frac{g(\rho_p - \rho)\rho}{\mu^2}\right]^{1/3} \qquad (12)$$

Comparing (11) and (12), $N_{ReP} = \frac{K^3}{18}$

Settling $N_{ReP} = 1$, $K^3 = 18$ $K = 2.62$

If the size of particle is known, K can be calculated from the equation (12). If the value of K so calculated is less than 2.62, the Stoke's law is valid.

Substituting for u_t from equation (11)

$$N_{ReP} = \frac{D_P \rho}{\mu} \times 1.75 \sqrt{\frac{g D_P (\rho_p - \rho)}{\rho}} \qquad (14)$$

Comparing (12) and (14)

$N_{ReP} = 1.75\, K^{1.5}$

Setting up $N_{ReP} = 1000$, $1000 = 1.75\, K^{1.5}$

$$K = \left[\frac{1000}{1.75}\right]^{\frac{1}{1.5}} = 68.89.$$

If K is > 68.9, the particle settles under Newton's range.

If K is between 2.62 and 68.9, the particle settles under intermediate region.

Problem:1

A steel ball 3 mm in dia. falls through glycerin and covers a distance of 25 cm in 10 seconds. The density of steel and glycerin are – 7.8 gm/cc and 1.26 gm/cc respectively. Calculate the viscosity of glycerin justify the selection of the equation used here.

Assume that the particle moves under stokes region.

$$u_t = \frac{g D_P^2 (\rho_p - \rho)}{18 \mu} \Rightarrow \mu = \frac{g D_P^2 (\rho_p - \rho)}{18 u_t}$$

$$u_t = \frac{distance\ covered}{time} = \frac{25 cm}{10 \sec} = 2.5 \times 10^{-2} \text{ m/sec.}$$

$D_P = 3 \text{ mm} = 3 \times 10^{-3} \text{ m}$

$$\rho_P = 7.8 \text{ gm / cc} = \frac{7.8 \times 10^{-3}}{(10)^3} \text{ kg/m}^3 = 7.8 \times 10^3 \text{ kg/m}^3$$

ρ = 1.26 g/cc = 1.26 × 10^3 kg / m^3

$$\mu = \frac{9.81\times(3\times10^{-3})^2(7.8\times10^{+3}-1.26\times10^3)}{18\times2.5\times10^{-2}}$$

$$= 1.28 \ \frac{N-s}{m^2}$$

Check for N_{Re}

$$N_{Re} = \frac{D_P u t \rho}{\mu} = \frac{3\times10^{-3}\times2.5\times10^{-2}\times1.26\times10^3}{1.28}$$

= 0.0738.

and

$$K = D_P\left[\frac{g(\rho_p-\rho)\rho}{\mu^2}\right]^{1/3}$$

$$= 3\times10^{-3}\left[\frac{9.81(7.8\times10^3-1.26\times10^3)1.26\times10^3}{(1.28)^2}\right]^{1/3}$$

= 1.1

Since N_{Re} = 0.0738 < 1 and K = 1.1 < 2.62, the selected equation [assumption] is correct.

Problem :2

A spherical particle is held motionless in water flowing upwards at a velocity of 1.2 cm / sec. The particle dia. is 0.975 mm and the density is 3.8 gm. / cm^3. Viscosity of water is 0.98 Cp. When the particle is released, in what direction and with what velocity will it move?

D_P = 0.975 × 10^{-3} m

1 C_P = 1 centipoise = 0.001 N –s / m^2

μ = 0.98 C_P = 0.00098 N – s / m^2

ρ_P = 3500 kg / m^3

ρ = 1000 kg/m^3

$$k = D_P \left[\frac{g(\rho_p - \rho)\rho}{\mu^2}\right]^{1/3} = 0.975 \times 10^{-3}\left[\frac{9.81(3500-1000)1000}{(0.00098)^2}\right]^{1/3}$$

$$= 28.71$$

Since $K = 28.71 > 2.62$ and less than 68.9, the settling is in intermediate range

$$\Rightarrow u_t = \frac{0.153 g^{0.71} D_p^{1.14} (\rho_P - \rho)^{0.71}}{\rho^{0.29} \mu^{0.43}}$$

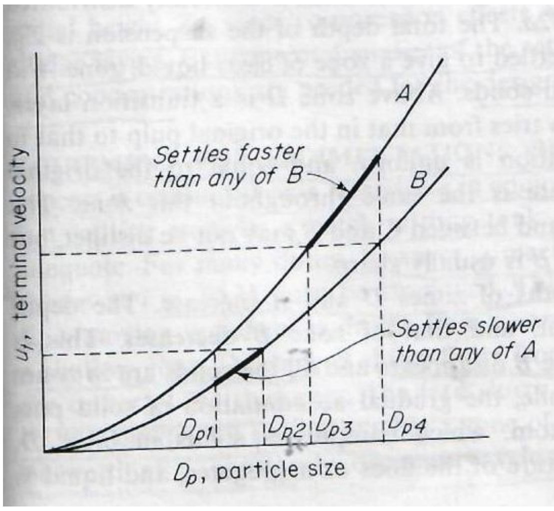
$$= \frac{0.153(9.81)^{0.71}(0.975\times10^{-3})^{1.14}(2500)^{0.71}}{(1000)^{0.29}(0.00098)^{0.43}}$$

$$= 0.196 \text{ m / sec.}$$

The spherical particle moves downward with a velocity of 0.196 – 0.12

$$= 0.076 \text{ m / sec.}$$

Differential settling Methods 'or' Equal settling velocity of particles 'or' Mixed particle separation:



The significance of equal settling velocity of particles in a separation process is shown by the graph, in which u_t Vs D_P are plotted for components A and B. Assume that the diameter range for both substances lies between points D_{P1} and D_{P4} on the size axis. Then, all particles of the light component B having diameters D_{P1} and D_{P2} will settle more slowly than any particles of the heavy substance A and can be obtained as a pure fraction. Likewise, any particles of substance A having diameters between D_{P2} and D_{P4} settle faster than any particles of substance B and can also be obtained as a pure fraction. But any light particles having a diameter between D_{P2} and D_{P4} settles at the same speed as a particle of substance A in the size range between D_{P1} and D_{P3}, and all particles in these size ranges form as mixed fraction.

Equations developed to find diameter ratios show that the sharpness of separation is improved if the density of the medium is increased. It is also clear from the graph that the mixed fraction can be reduced or eliminated by closer sizing of feed i.e. from D_{P3} to D_{P4} in the graph, and complete separation is possible.

Laminar (Stoke's law) region:

For material A,

$$u_{tA} = \frac{gD_P^2 A(\rho_{pA} - \rho)}{18\mu}$$

u_{tA} - Terminal settling velocity of A

ρ_{PA} Density of material A

D_{PA} Dia. of material A

ρ Density of medium

μ Viscosity of medium

Similarlly,

For material B,

$$u_{tB} = \frac{gD_P^2 B(\rho_{pB} - \rho)}{18\mu}$$

For equal settling particles

$u_{tA} = u_{tB}$

$$\frac{gD_{PB}^2(\rho_{pA} - \rho)}{18\mu} = \frac{gD_{PB}^2\,(\rho_{pB} - \rho)}{18\mu}$$

$$\frac{D_{PB}^2}{D_{PA}^2} = \frac{(\rho_{pA} - \rho)}{(\rho_{pB} - \rho)}$$

$$\frac{D_{PB}}{D_{PA}} = \left(\frac{\rho_{PA} - \rho}{\rho_{PB} - \rho}\right)^{1/2}$$

For Intermediate region:

$$u_{tA} = \frac{0.153 g^{0.71} D_{PA}{}^{1.14} (\rho_{PA} - \rho)^{0.71}}{\rho^{0.29} \mu^{0.43}}$$

$$u_{tB} = \frac{0.153 g^{0.71} D_P A^{1.14} (\rho_{PB} - \rho)^{0.71}}{\rho^{0.29} \mu^{0.43}}$$

For equal settling particles, $u_{tA} = u_{tB}$

$$\frac{0.153 g^{0.71} D_{PA}{}^{1.14} (\rho_{PA} - \rho)^{0.71}}{\rho^{0.29} \mu^{0.43}} = \frac{0.153 \times g^{0.71} D_{PB}{}^{1.14} (\rho_{PB} - \rho)^{0.71}}{\rho^{0.29} \mu^{0.43}}$$

$$\frac{(D_{PB})^{1.14}}{(D_{PA})^{1.14}} = \left(\frac{\rho_{PA} - \rho}{\rho_{PB} - \rho}\right)^{0.71}$$

$$\frac{D_{PB}}{D_{PA}} = \left(\frac{\rho_{PA} - \rho}{\rho_{PB} - \rho}\right)^{1.14} = \left(\frac{\rho_{PA} - \rho}{\rho_{PB} - \rho}\right)^{0.623}$$

Newton's region:

$$u_{tA} = 1.75 \left[\frac{g D_{PA} (\rho_{PA} - \rho)}{\rho}\right]^{1/2}$$

$$u_{tB} = 1.75 \left[\frac{g D_{PB} (\rho_{PB} - \rho)}{\rho}\right]^{1/2}$$

For equal settling particles, $u_{tA} = u_{tB}$

$$1.75 \left[\frac{g D_{PA} (\rho_{PA} - \rho)}{\rho}\right]^{1/2} = 1.75 \left[\frac{g D_{PB} (\rho_{PB} - \rho)}{\rho}\right]^{1/2}$$

$D_{PA}^{1/2} \ (\rho_{PA} - \rho)^{1/2} = D_{PB}^{1/2} \ (\rho_{PB} - \rho)^{1/2}$

$$\left(\frac{D_{PB}}{D_{PA}}\right)^{\frac{1}{2}} = \left(\frac{\rho_{PA}-\rho}{\rho_{PB}-\rho}\right)^{\frac{1}{2}} \qquad \text{or} \qquad \left(\frac{D_{PB}}{D_{PA}}\right) = \left(\frac{\rho_{PA}-\rho}{\rho_{PB}-\rho}\right)$$

The general equation for $\frac{D_{PB}}{D_{PA}}$ can be written as

$$\frac{D_{PB}}{D_{PA}} = \left(\frac{\rho_{PA}-\rho}{\rho_{PB}-\rho}\right)^{n}$$

Where n = ½ for Stoke's region

= 0.623 for intermediate region.

= 1 for Newton's region.

Problems: 3

A mixture of silica (sp.gr.2.56) and galena (Sp.g.7.5) particles ranging from sizes of 0.0074 cm to 0.0652 cm are to be separated by a rising stream of water.

a) What velocity of water flow will give an un-contaminated product of galena?

b) What is the size range of product?

c) What is the effect of separation on changing the fluid?

Solution:

To get a pure product of galena, all the silica should be lifted up. For this find the settling velocity of biggest particles of silica.

$D_P = 0.0652$ cm $= 0.0652 \times 10^{-2}$ m

$\rho_P = 2560$ kg/m^3

$\rho = 1000$ kg/ m^3

$\mu = 0.001$ N – s/m^2 (for water)

$$u_t = \frac{gD_P^2(\rho_{PA}-\rho)}{18\mu}$$

$$= \frac{9.81\times(0.0656\times10^{-2})^2(2560-1000)}{18\times0.001}$$

$= 0.361$ m / sec.

Using above u_t, find D_P for galena particles.

$$0.361 = \frac{gD_P^2(\rho_P - \rho)}{18\mu}$$

$$= D_P^2 = \frac{0.361 \times 18\mu}{g(\rho_P - \rho)}$$

$$= \frac{0.361 \times 18 \times 0.001}{9.81(7500 - 1000)}$$

$$= 1.019 \times 10^{-07}$$

$$D_P = 0.319 \times 10^{-3} \text{ m}$$

$$= 0.0319 \text{ cm.}$$

a) Velocity of water flow should be more than 0.361 m/s.

b) The particles of galena which have dia. from 0.0319 to 0.0652 settle down and particles of galena having dia. between 0.0074 and 0.0319 go out along with silica particles.

c) If the fluid is of greater density than the previous fluid [water].

$\frac{\rho_{PA} - \rho}{\rho_{PB} - \rho}$ in the equation $\frac{D_{PB}}{D_{PA}} = \left(\frac{\rho_{PA} - \rho}{\rho_{PB} - \rho}\right)^{1/2}$ decreases.

$\frac{D_{PB}}{D_{PA}}$ decreases and separation is not sharp.

On the other hand if the changed fluid has density less than previous fluid [water], $\frac{\rho_{PA} - \rho}{\rho_{PB} - \rho}$ increases and hence $\frac{D_{PB}}{D_{PA}}$. $\therefore$ sharp separation is achieved.

Problem :4

Determine the ratio of diameter for spherical particles of galena (sp.g.5.39) and quartz (sp.gr 2.64) that have the same terminal settling velocities in water a) for the case of free settling and, b) for the case of hindered settling if the sphericity is 0.7.

Solution: Free Settling:

Density of galena particles = ρ_p = 5.39 × 1000

= 5390 kg/ m^3

Density of quartz particles = ρ_P = 2.64 × 1000

= 2640 kg / m^3

Density of water = 1000 kg/ m^3

Assume particles settle under Stoke's region

$$\frac{D_{PB}}{D_{PA}} = \left(\frac{\rho_{PA}-\rho}{\rho_{PB}-\rho}\right)^{1/2} = \left(\frac{5390-1000}{2640-1000}\right)^{1/2} = (2.67)^{1/2} = 1.636$$

For Newton's region,

$$\frac{D_{PB}}{D_{PA}} = \left(\frac{\rho_{PA}-\rho}{\rho_{PB}-\rho}\right) = \frac{5390-1000}{2640-1000} = 2.67$$

Hindered Settling:

u_{tq} = Terminal velocity of galena particles

$$= \left[\frac{gD_{PA}^2(\rho_p-\rho)t}{18\mu/\psi_P}\right]$$

u_{tq} = Terminal velocity of quartz particles.

$$= \left[\frac{gD_{PB}^2(\rho_B-\rho)t}{18\mu/\psi_P}\right]$$

Since ε is same for but n particles , ψ_P is same

For equal settling

$$\frac{gD_{PA}^2(\rho_{PA}-\rho)t}{18\mu/\psi_P} = \frac{gD_{P_B}^2\ (\rho_{PB}-\rho)t}{18\mu/\psi P}$$

$$\frac{D_P q}{D_P q} = \sqrt{\frac{\rho q-\rho}{\rho q-\rho}} = 1.636.$$

GRAVITY SETTLING PROCESSES:

Two types of equipment are available

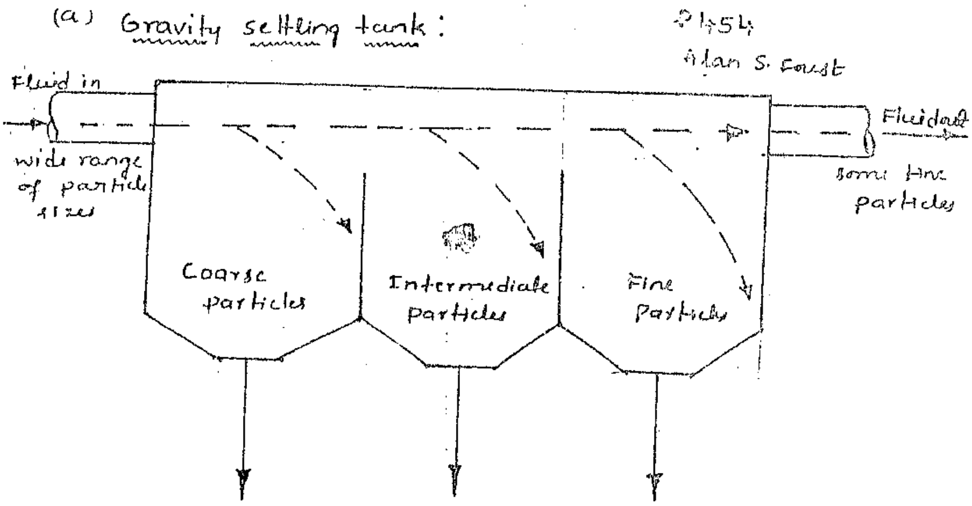
1. A settler that removes virtually all the particles from a liquid is known as a clarifier.

2. A device that separates the solids into two fractions is called a classifier.

The same principle of sedimentation apply to both the kind of equipments.

Types of Classifiers:

(a) Gravity settling tank:



DIFFERENTIAL SETTLING (or CLASSIFICATION)

☞ **Separation of solid particles into several size fractions based upon the settling velocities in a medium.**

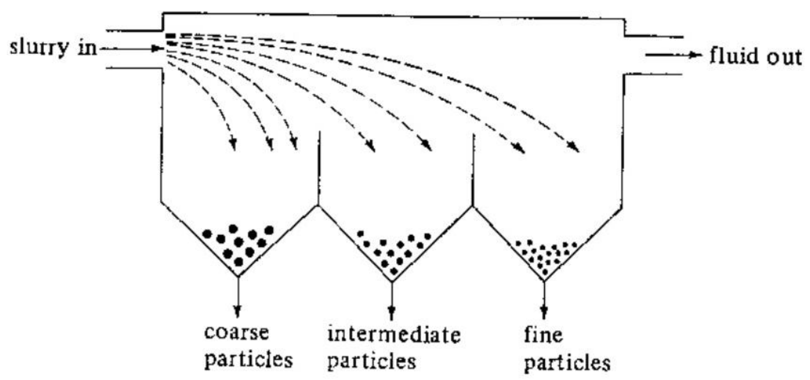


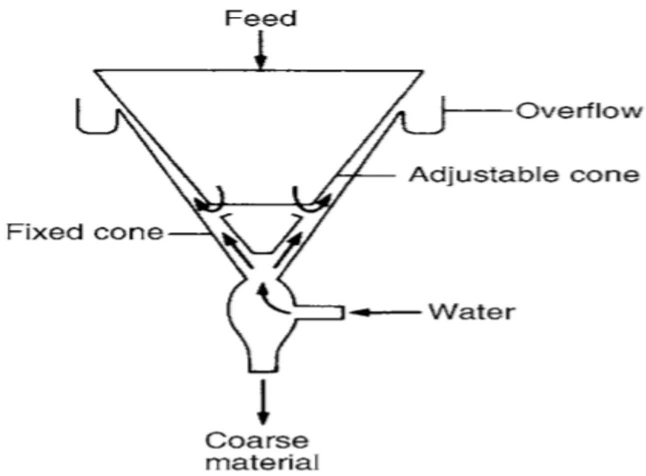
FIGURE 14.3-6. *Simple gravity settling classifier.*

A slurry feed enters the tank through a pipe. Immediately upon entry, the linear velocity of the feed decreases as a result of the enlargement of the cross sectional area. The influence of gravity causes the particles to settle, with the faster settling particles falling to the bottom of the tank near the entrance, the slower settling particles to the bottom. The placing of vertical battles with in the tank allows for the collection of several fractions. The very fine particles will be carried out of the tank with the liquid overflow. The position in the tank at which a certain size particle may be expected can be calculated on the assumption that the particles quickly reach terminal velocities.

The gravity settling tank is referred to as a surface velocity classifier. The resulting separation is not a sharp one, since rather considerable overlapping of size occurs.

Double Core classification:

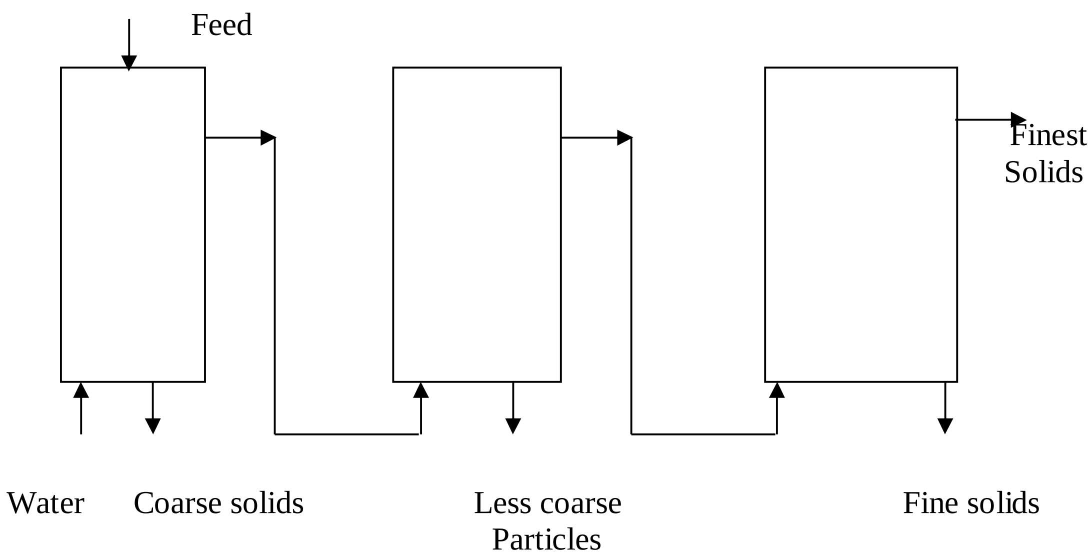
The double core classifier shown in figure is an example of classifiers using hydraulic water



The feed enters the inner cones A, and the hydraulic water is introduced at B. The particles settling from the inner cone meet a rising stream of water at the point C. The fine particles pass upward escape by the peripheral launder, D while the course particles settle into the chamber, E and are drawn off at intervals. The degree of separation accomplished is regulated both by regulating the water supply at B and by regulating the night of the inner cone by the hand wheel F.

(c) Elutriator:

Elutriator is equipment used to separate the particles by making use of the difference on settling velocity of particles.



The feed is fed from the top of the column and water is sent through the bottom of the column. The velocity of water can be adjusted to any varying the volumetric rate of water. The coarse particles which have settling velocities higher than the upward velocity of water settle down and collect at the bottom. The particles which are carried with water in the first column are fed to the second column of bigger size (or bigger dia.).Because of the increase of cross section of the column the upward velocity of water decreases and some particles settle in the second column. Still small particles are carried by the water. This way the particles in a feed can be separated into different fractions by arranging different diameter columns in series.

Hindered Settling Equation:

In hindered settling, the effective density of fluid can be taken as that of the slurry and can be calculated from the composition of the slurry and the densities of the particles and of the fluid.

The effective viscosity for hindered settling is calculated by dividing the actual viscosity - μ by an empirical correction factor ψ_P which depends on the fractional volume of the slurry occupied by the liquid. This is equivalent to the porosity of the aggregation of particles, and is denoted by ε. For settling of spheres in Stoke's range, ψ is given by

$$\psi = e^{-19}(1 - t) = e^{-4.19(1 - t)}$$

Problem:

Particles of sphalerite (sp.g.4) are settling under the force of gravity through a slurry 25% by volume of quartz particles (sp.gr.2.65) and water. The diameter of sphalerite particles is 0.15 mm. The volumetric ratio of sphalerite to slurry is 0.25. What is the terminal settling velocity of sphalerite? The velocity of water 1 Cp = 0.001 N – s / m^2.

Solution:

The density of settling medium (slurry) is calculated from the composition. The specific gravity of composition = $0.25 \times 2.65 + 0.75 \times 1 = 1.41$.

Porosity = $1 - 0.25 = t = 0.75$ (traction of liquid)

Correction factor, $\psi_P = e^{-4.19}(1 - t)$

$= e^{-4.19\,(1-0.75)} = 0.3508$

Criterion equation for Stoke's region.

$$K = D_P \left[\frac{g\rho(\rho P - \rho)}{\left(\mu / \psi P\right)^2} \right]^{1/3}$$

$$= 0.15 \times 10^{-3} \left[\frac{9.81 \times 1410(4000 - 1410) \times 0.3508^2}{(0.001)^2} \right]^{1/3}$$

$= 2.46 \Rightarrow$ Settling comes in stoke's range.

$$\text{Terminal settling velocity, } u_t = \left[\frac{g D_P^2 (\rho P - \rho)\varepsilon}{18\, \mu / \psi P} \right]$$

$$u_t = \left[\frac{9.81 \times (0.15 \times 10^{-3})^2 (4000 - 1410) \times 0.75 \times 0.3508}{18 \times 0.001} \right]$$

$= 8.41 \times 10^{-3}$ m / sec.

Another way of calculating settling velocity for hindered settling:

In hindered settling, the velocity gradients around each particle are affected by the presence of nearby particles, so the normal drag corrections do not apply. Also, the particles in settling displace liquids which flow upward and make the particle velocity relative to the fluid greater than the absolute settling velocity. For a uniform suspension can be settling velocity u_s can estimated from the terminal velocity for an isolated particle using the empirical equation MAUDE and WHITMORE –

$$u_s = u_t\,(t)^n \qquad \text{------------- (1)}$$

The viscosity is also affected by the presence of the dispersed phase. The effective viscosity is may be calculated from the relation

$$\frac{\mu s}{\mu} \quad \frac{1 + 0.5(1-t)}{t^4} \qquad \text{-------------- (2)}$$

Procedure to find us:

1. First calculate μ_s using equation (2) knowing μ and t, the porosity.

2. Substitute μ_s in place of μ fo calculating μt for different ranges.

3. Substitute the obtained μt in (1) to get us.

Knowing N_{ReP} n can be calculated 1 by using the following table given.

N_{ReP}	n
0.1	4.6
1	4.3
10	3.7
10^2	3.0
10^3	2.5

FLOW PAST IMMERSED BODIES – FLOW THROUGH PACKED BEDS

Application:

In many technical processes, liquids or gases flow through beds of solids particles. Examples are the unit operation-filtration, two phase counter current flow of liquid and gas through packed towers (Extraction (or) absorption), other processes, such as ion exchange (or) catalytic reactors a single fluid (liquid (or) gas) flows through a bed of granular solids. The present treatment is restricted to the flow of a single fluid phase through a column of stationary solid particles.

Definitions:

Drag: The force in the direction of flow exerted by the fluid on the solid is called drag.

Drag Coefficient:

It is the ratio of the shear stress to the product of velocity head and density.

$$C_D = \frac{F_D / A_P}{\int \bar{V}_0^2 / 2}$$

Where C_D = the Drag Coefficient

F_D = Total Drag

A_P = Projected area.

$\bar{V}_0$ = Velocity of approaching stream.

Superficial Velocity: $\bar{V}_0$

The velocity ($\bar{V}_0$) is often called the superficial (or) empty tower velocity, as it is the upward (or) downward velocity, in the open section (or in the empty tower above or below the bed).

Interstitial Velocity: $\left(\bar{V}\right)$**:**

The flow of the fluid through the voids is called interstitial velocity

Various forms of Drag:

The different forms of drag are wall Drag and from Drag. The total drag is the sum of the wall drag and form drag.

1. The drag from wall shear is called the wall Drag.
2. The drag from pressure is called form Drag.

Pressure Drop for flow of fluids through pocked Bed:

Derivation of Ergun, Kozeny-Carman, Blake-Plummer Equations:

Assumptions:

1. Actual channels may be effectively replaced by set of identical parallel conduits each of variable cross section.

2. The mean hydraulic radius of the channels is adequate to account for the variations in channel cross-sectional size and shape.

3. Total drag per unit area of channel wall is the sum of two kinds of drag.

   (i) Viscous drag forces and
   (ii) Inertial forces.

4. Particles are packed at random.

5. Roughness effects are unimportant.

6. All particles are of same size and shape.

7. End and wall effects are negligible.

Drag force acts against the flow of a fluid through a bed of solids. The total drag/unit area is the sum of the viscous drag force and inertial force.

Or $$F_D/A_S = F_\mu/A_S + F_i/A_S \qquad (1)$$

$$F_\mu/A_S = k_1\mu\bar{V} / r_h \qquad (2)$$

Where k_1 is a constant

μ - Viscosity of fluid

$\bar{V}$ = Velocity in channel.

r_h = Hydraulic radius

A_S = area of channel boundary.

$$F_i / A_S = k_2 \rho \bar{V}^2 \qquad (3)$$

Where k_2 is a proportional constant

ρ - Density of fluid.

$$F_D / A_S = \left(\frac{k_1 \mu \bar{V}}{r_h} + k_2 \int \bar{V}^2 \right) \qquad (4) \qquad\qquad \int = \rho$$

The porosity ε is defined as the ratio of volume of voids in the bed to the total volume of bed. The velocity $\bar{V}$ is the average velocity of fluid though the channels in the bed. It is more convenient to use $\bar{V}_O$, the velocity of stream just before it encounters the first layer of solids. The velocity $\bar{v}_O$ is often called the superficial (or) empty tower velocity. The relation between $\bar{V}_O$ and $\bar{V}$ is

$$\bar{V} = \bar{V}_O / \varepsilon \qquad (5)$$

The total area A_S is given by

$A_S = N_P S_P$

Where N_P is the number of particles and S_P is the surface area of one particle.

The number of particles may be calculated from the total volume of solids and volume of one particle.

Or $\quad N_P = \dfrac{S_O L(1-\varepsilon)}{V_P} \qquad (6)$

Where S_O is the cross section area of tower.

L – height of bed.

V_P = volume of one particle.

$$A_S = \frac{S_O L(1-\varepsilon)}{V_P}\; S_P \qquad (7)$$

The hydraulic radius r_H is the ratio of cross section area of the conduit to perimeter of conduit. If the numerator and denominator are multiplied by L, r_h becomes the ratio of the volume of the voids in the bed the total surface area of solids.

$$r_H = \frac{S_O L\varepsilon}{A_S}$$

Substituting for A_S from equation (7)

We get $$r_H = \frac{S_O L\varepsilon}{S_O L(1-\varepsilon)\, S_P / V_P} = \frac{\varepsilon V_P}{(1-\varepsilon) S_P} \qquad (8)$$

Substituting $\bar{V}$ from (5) A_S from (7) and r_H from (8) in equation (4)

$$F_D = \frac{S_O L(1-\varepsilon) S_P}{V_P}\left[k_1 \mu \frac{\bar{V}_O (1-\varepsilon) S_P}{\varepsilon^2 V_P} + k_2 \frac{\rho \bar{V}_O^2}{\varepsilon^2}\right]$$

$$= \frac{S_O L(1-\varepsilon) S_P}{V_P}\;\frac{\rho}{\varepsilon^2}\left[\frac{k_1 \mu \bar{V}_O (1-\varepsilon) S_P}{\rho V_P} + k_2 \bar{V}_O^2\right] \qquad (9)$$

The pressure drop- Δp for the flow of fluid may be written as

$$-\Delta P = F_D / S_O\varepsilon \quad \text{(or)} \quad F_D = -\Delta P \varepsilon S_O \qquad (10)$$

From (9) and (10)

$$-\quad \Delta P \varepsilon S_O = \frac{S_O L(1-\varepsilon) S_P}{V_P}\;\frac{\rho}{\varepsilon^2}\left[\frac{k_1 \mu \bar{V}_O (1-\varepsilon) S_P}{\rho V_P} + k_2 \bar{V}_O^2\right]$$

$$\text{(or)}\; \frac{-\Delta P}{\rho L} = \frac{(1-\varepsilon)}{\varepsilon^3}\;\frac{S_P}{V_P}\left[\frac{k_1 \mu \bar{V}_O (1-\varepsilon) S_P}{\rho V_P} + k_2 \bar{V}_O^2\right] \qquad (11)$$

for a non-spherical particle we know the sphericity

$$\phi_s = \frac{6V_P}{S_P D_P} \qquad \text{or} \qquad \frac{S_P}{V_P} = \frac{6}{\phi_s D_P}$$

Ergun calculated experimental data and found that the constants of k_1 and k_2 in equation (11) are 150/36 and 1.75/6 respectively.

Substituting k_1 and k_2 and S_P/V_P in (11) we get

$$\frac{-\Delta P}{\int L} = \frac{(1-\varepsilon)}{\varepsilon^3}\frac{6}{\phi_S D_P}\left[\frac{150}{36}\frac{\mu \bar{V}_O (1-\varepsilon)6}{\rho \phi_S D_P} + \frac{1.75}{6}\bar{V}_O^2\right] \qquad \int = \rho$$

$$\frac{(1-\varepsilon)}{\varepsilon^3}\frac{\bar{V}_0^2}{\phi_S D_P}\left[\frac{150\mu(1-\varepsilon)}{\phi_S D_P \int \bar{V}_O} + 1.75\right]$$

$$\frac{-\Delta P}{\rho \bar{V}_O^2}\frac{\phi_S D_P}{L}\frac{\varepsilon^3}{1-\varepsilon} = \frac{150(1-\varepsilon)}{\phi_S \dfrac{D_P \bar{V}_O \int}{\mu}} + 1.75 \qquad (12)$$

Equation (12) is the Ergun's Equation:

The L.H.S of Ergun equation is the friction factor for a packed bed denoted by f_P.

$$\therefore f_P = \frac{150(1-\varepsilon)}{\phi_S N_{Re}} + 1.75 \qquad (13)$$

At low Reynolds's Numbers the quantity 1.75 is negligible in comparison with the first term. This implies that viscous force control and inertial forces are unimportant.

For this case equation (13) can be written as

$$\frac{-\Delta P \phi_S^2 D_P^2 \varepsilon^3}{L \bar{V}_O \mu (1-\varepsilon)^2} = 150 \qquad (14)$$

Equation (14) is called the Kozeny-Carman Equation and is applicable only for Laminar flow ($N_{Re} < 10$).

For large Reynolds's numbers the first term on the right hand side of equation (13) fades out as viscous forces are negligible and inertial forces control.
For this case equation (13) can be written as

$$\frac{-\Delta P}{\rho L}\ \frac{\phi_S D_P \varepsilon^3}{\bar{V}_o^2(1-\varepsilon)} = 1.75 \tag{15}$$

Equation 15 is called Blake Plummer Equation and is used for Turbulent flow condition ($N_{Re} > 100$).

Problems:

A gas is at steady flow though a bed of solid particles. Given the following data, calculate the pressure drop. Gas density = 1.28 kg/m^3. Gas viscosity 1.653 × 10^{-5} Ns/m^2 porosity of bed = 3.5, particle diameter = 5 mm. Superficial velocity of gas is 3.05 cm/sec. Bed height is 3.05 m. The friction factor may be estimated from the relationship f = 5/N_{ReP}.

Ans:

ρ = 1.28 kg/m^3

μ = 1.653 × 10^{-5} N-s/m^2

ε = 0.35

D_P = 5mm = 5 × 10^{-3} m.

$\bar{V}_o$ = 3.05 cm/sec = 3.05 × 10^{-2} m/sec.

L = 3.05 m

ΔP = ?

f = 5/ N_{Rep}.

$$N_{ReP} = \frac{D_P \bar{V}_o \rho}{\mu} = \frac{5\times10^{-3}\times3.05\times10^{-2}\times1.28}{1.653\times10^{-5}}$$

$$= 11.83.$$

Note:

Kozeny Caman N_{Rep} 0 – 10 Ergun 10 – 100 Blake plummer > 100

$$f = 5 / N_{Rep} = \frac{5}{11.83} = 0.423.$$

$$0.423 = \frac{\Delta P}{\int \bar{V}_o^2} \phi_S \frac{D_P}{L} \frac{\varepsilon^3}{1-\varepsilon}$$

$$f_P = 0.423 = \frac{150(1-\varepsilon)}{\phi_S \times \frac{D_P \bar{v}_O \int}{\mu}} + 1.75$$

Assume $\phi_S = 1$ for spherical particle.

$$0.423 = \frac{\Delta P \times 1 \times 5 \times 10^{-3} \times (0.35)^3}{1.28 \times (3.05 \times 10^{-2})^r \times 3.05 \times 0.65} = 4.65 \text{ N/m}^2.$$

Problem;

The pressure drop though a particle bed can be used to determine the external surface area and average particle diameter. Data for a bed of crushed particle show $\Delta P/L$ = 19.1 ×10^5 N/m^2/m for air flow at superficial velocity of 0.0045 m/sec. The measured void faction is 0.47 and the estimated sphericity is 0.7. Calculate the average particle size and the surface area per unit mass if the solid has a density of 4100 kg/m^3. Assumed $\int$ = 1.2 kg/m^3 μ = 1.83 × 10^{-5} Ns/ m^2.

Data: $\Delta P/L$ = 19.1 × 10^5 N/m^2/m. $\bar{V}_o$ = 0.0045 m/s , ε = 0.47 , ϕ_S = 0.7

Solution:

Ergun's Equation:

$$-\Delta P/L \; \frac{\phi_S D_P \varepsilon^3}{\rho \bar{v}_o^r (1-\varepsilon)} = \frac{150(1-\varepsilon)}{\phi_S N_{\mathrm{Re}\,P}} + 1.75$$

$$\frac{19.1 \times 10^5 \times 0.7 \times D_P \times (0.47)^3}{1.2(0.0045)^2 (1-0.47)} = \frac{150(1-0.47) \times 1.83 \times 10^{-5}}{0.7 \times D_P \times 0.0045 \times 1.2} + 1.75$$

$$107781.15 \times 10^5 D_P = \frac{38488.095 \times 10^{-5}}{D_P} + 1.75$$

$$107781.15 \times 10^5 \, D_P^2 = 38488.095 \times 10^{-5} + 1.75 \, D_P$$

$$107781.15 \times 10^5 \, D_P^2 - 1.75 \, D_P - 38488.095 \times 10^{-5} = 0.$$

$$D_P = \frac{1.75 \pm \sqrt{(1.75)^2 + 4 \times 107781.15 \times 10^5 \times 38488.095 \times 10^{-5}}}{2 \times 107781.15 \times 10^5}$$

$$D_P = 5.96 \times 10^{-6} \text{ m.}$$

$$A_W = \frac{6}{\phi_S \int_P D_P} = \frac{6}{0.7 \times 4100 \times 5.96 \times 10^{-6}} = 350.77 \text{ m}^2 / \text{kg.}$$

Estimation of Porosity:

The Porosity is usually measured insitu after the bed has been formed, by filling the bed with water, draining and comparing the volume of the bed as calculated from its depth and cross section. The usually anger of valves for 'ε' is from 0.35 to 0.70.

Darcy's Law:

The Darcy's law states that the fluid flow is proportional to the pressure drop and inversely proportional to the fluid viscosity. It is given by the equation

$$\frac{\Delta P}{L} = \frac{150 \bar{V}_o \, \mu (1-\varepsilon)^2}{\phi_s^2 D_p^2 \varepsilon^3}$$

FLUIDIZATION

Definition:

When a liquid or gas is passed at very low velocity up through a bed of solid particles, the particles do not move. If the fluid velocity is steadily increased, the pressure drop and the drag on individual particles increase, and eventually the particles start to move and become suspended in the fluid. The term fluidization is used to describe the condition of fully suspended particles, since the solids suspended behaves like a dense fluid.

Types of Fluidization:

1. Particulate Fluidization:

The behavior of mass of fluidized solids depends, on the particle size of the solids and the nature of fluid. With larger particles, fluidized by either a gas or a liquid, fluidization begins as a gentle racking or oscillation of the solid particles. In the fully fluidized bed the particles move in random direction though all parts of the liquid as individuals. This mode of action is called particulate fluidization.

2. Aggregate Fluidization:

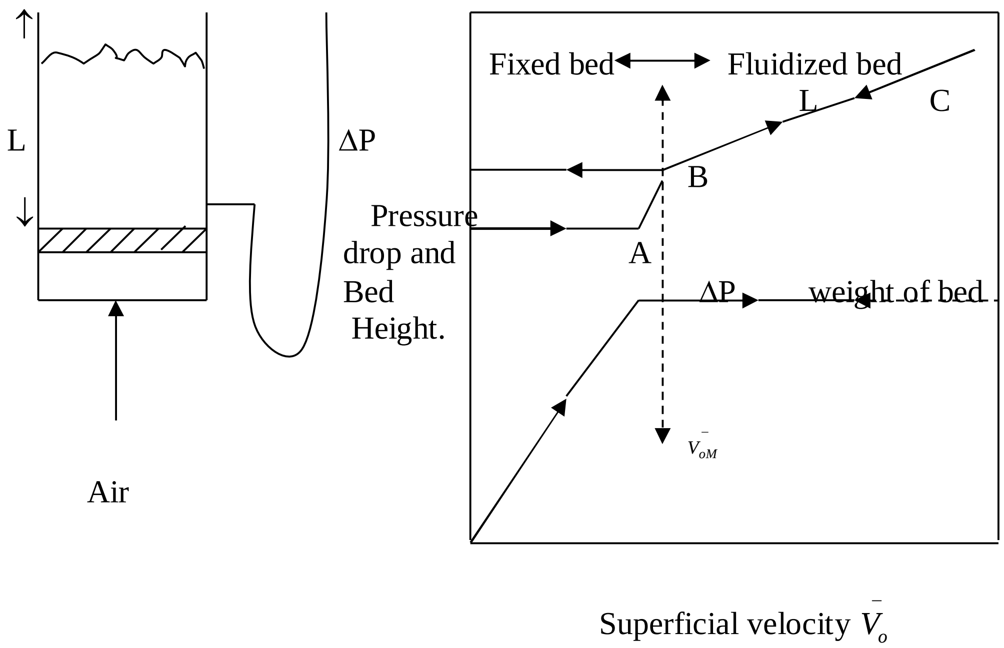
When the fluid is a gas, the action in the bed is strongly influenced by the particles size. Under conditions for good fluidization some of the gas travels through the bed between individual particles, but much of it travels through pockets containing almost no solids. At the bed surface the bubble break splashing individual particles into the space above. In the bed itself the particles move in distinct aggregates, which are lifted by bubbles. A fluidizing action of the kind is known as Aggregate Fluidization.

3. Slugging Fluidization:

When particles are fluidized in a tall narrow vessel, bubbles of gas tend to expand and grow as they rise through the fluidized bed. The rate of growth depends on the size and density of the particles; it is rapid when the particles are large and heavy, slow when they are small and light. If a vessel that is small in diameter contains a deep bed of solids, the bubbles may grow until they fill the entire cross section of the vessel, entire bubbles then travel up the vessel, separated by slugs of solid particles. Fluidizing action of the kinds is called as slugging fluidization.

Mechanism of fluidization:

Consider a vertical tube partly filled with a fine granular material such as catalytic cracking catalyst as shown schematically.



The tube is open at the top and has a porous plate at the bottom to support the bed of catalyst and to distribute the flow uniformly over the entire cross section. Air is admitted below the distributor plate at a low flow rate and passes upward through the bed without causing any particle motion. As the velocity is gradually increased the pressure drop increases, but the particles do not move and the bed height remains the same. At a certain velocity, the pressure drop across the bed counter balances the force of gravity on the particles or the weight of the bed and any further increase in velocity causes the particles to move. This is point A on the graph. Sometimes the bed expands slightly with the grains still in contact, since just a 'slight increase in 'ε' can offset an increase of several percent in $\bar{V}_o$ and keep ΔP constant. With further increase in velocity the particles become separated enough to move about in the bed and true fluidization beings a point (B).

Once the bed fluidized, the pressure drop across the bed stays constant, but the bed height continues to increase with increasing flow. The bed can be operated at quite high velocities with very little or no loss of solids, since the superficial velocity needed to support a bed of particles is much less than the terminal velocity for individual particles.

If the flow rate to the fluidized bed is gradually reduced, the pressure drop remains constant, and the bed height deceases, following the line BC which was observed for increasing velocities. However the final bed height may be greater than the initial value for the fixed bed, since in solids dumped in a tube tend to pack more tightly than solids slowly settling from a fluidized state. The pressure drop at low velocities is then less than in the original fixed bed. On starting up again, the pressure drop offsets the weight of the bed at point B, and this point, rather than Point A, should be considered to give the minimum fluidization velocity $\bar{V}_{OM}$. To measure $\bar{V}_{OM}$, the bed should be fluidized vigorously, allowed to settle with the gas turned off, and the flow rate increased gradually until the bed starts to expand. More, reproducible values of $\bar{V}_{OM}$ can sometimes be obtained from the intersection of the graphs of pressure drop in the fixed bed and the fluidized bed.

Minimum Fluidization Velocity:

It is the superficial velocity of fluid at which the solids starts fluidizing.

Equation for Minimum fluidization velocity:

An equation for the minimum fluidization velocity can be obtained by setting the pressure drop across the bed equal to the weight of the bed per unit cross section, allowing for the buoyant force of the displaced fluid.

$$\Delta PS = SL\,(1-\varepsilon)\,\rho_P g - SL\,(1-\varepsilon)\,\rho g$$

$$\Delta P\,S = SL\,(1-\varepsilon)\,(\rho_P - \rho)\,g$$

$$\Delta P = g\,(1-\varepsilon)\,(\rho_P - \rho)\,L$$

At incipient fluidization, ε is the minimum porosity, ε_m

$$\text{Then, } \frac{\Delta P}{L} = g\,(1-\varepsilon_m)\,(\rho_P - \rho)$$

The Ergun equation for pressure Drop in packed beds can be rearranged to give.

$$\frac{\Delta P}{L} = \frac{150\mu \bar{V}_o (1-\varepsilon)^2}{\phi_s^2 D_P^2 \varepsilon^3} + \frac{1.75\rho \bar{V}_o^2 (1-\varepsilon)}{\phi_S D_P \varepsilon^3}$$

At the point of incipient fluidization

$$\frac{\Delta P}{L} = \frac{150\mu \bar{V}_{oM} (1-\varepsilon m)^2}{\phi_s^2 D_P^2 \varepsilon_m^3} + \frac{1.75\rho \bar{V}_{om}^2 (1-\varepsilon m)}{\phi_S D_P \varepsilon_m^3}$$

Equating $\frac{\Delta P}{L}$ then,

$$\frac{150\mu \bar{V}_{oM} (1-\varepsilon\, m)^r}{\phi_s^r D_\rho^r \varepsilon_m^3} + \frac{1.75\rho \bar{V}_{om}^r (1-\varepsilon m)}{\phi_S D_P \varepsilon_m^3} = (1 - \varepsilon m)\, g\, (\rho_P - \rho)$$

$$\frac{150\mu \bar{V}_{oM} (1-\varepsilon m)}{\phi_s^2 D_P^2 \varepsilon_m^3} + \frac{1.75\rho \bar{V}_{om}^2}{\phi_S D_P \varepsilon_m^3} = g\, (\rho_P - \rho)$$

This is the general form of the equation. We get Quadratic equation for $\bar{V}_{om}$. When $N_{Rep} < 1$, then the 2nd term on L.H.S drop out.

$$\bar{V}_{om} \cong \frac{g(\rho_P - P)\varepsilon_m^3}{150\mu(1-\varepsilon\, m)} \phi_s^2 D_P^2$$

When $N_{ReP} > 10^3$ $\bar{V}_{om} = \left[\frac{\phi_S D_P g(\rho_P - \rho)\varepsilon_m^3}{1.75\rho}\right]^{1/2}$

Expansion of fluidized Beds:

The expanded bed height may be obtained from ε and the values of L at the incipient fluidization.

$$LSA\,(1 - \varepsilon) = L_m SA\,(1 - \varepsilon_m)$$

$$L\,(1 - \varepsilon) = L_m\,(1 - \varepsilon_m)$$

$$L = L_m \frac{(1-\varepsilon_m)}{(1-\varepsilon)}$$

Application of fluidization:

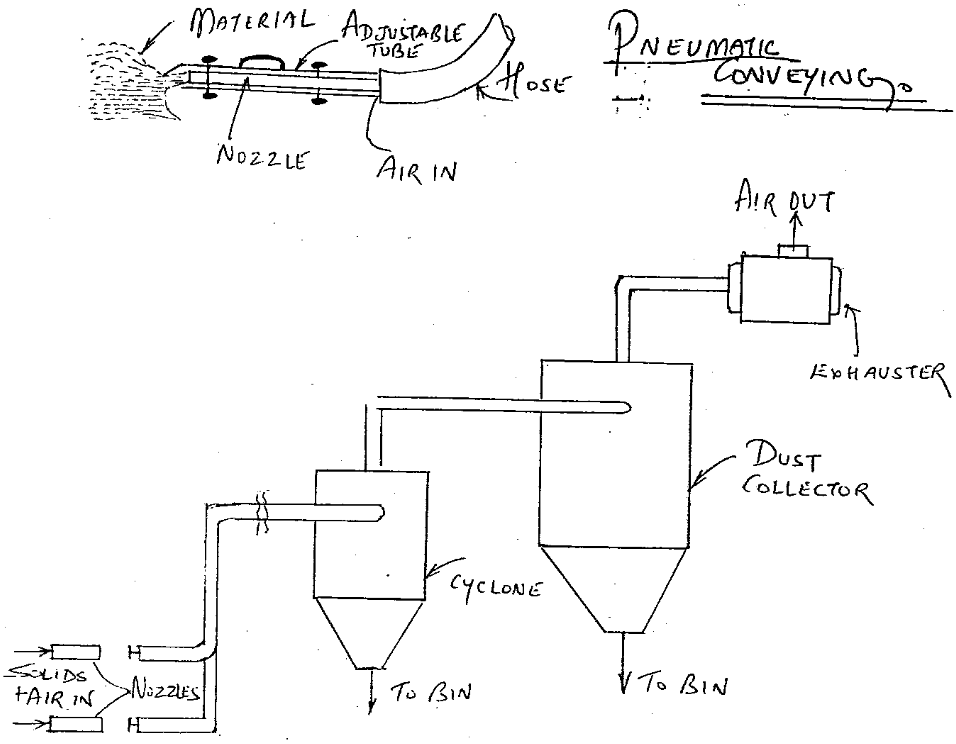
1. Extensive use of fluidization began in the petroleum Industry with the development of fluid-bed catalytic cracking.
2. Fluidization is used in other catalytic processes, such as synthesis of acrylonitrile, and for carrying out solid gas reactions.
3. Fluidized beds are also used for roasting ores, drying fine solids, and adsorption of gases.

Advantages of Fluidization:

1. The chief advantages of fluidization are that the solid is vigorously agitated by the fluid passing through the bed, and the mixing of the solids ensures that they are practically no temperature gradients in the bed.

2. The violent motion of the solids also give high heat transfer to the wall of the cooling tubes immersed in the bed. Because of the fluidity of the solids it is easy to pass solids form one vessel to another.

Pneumatic Conveying:



Vaccum Pressure System

The suspending fluid in a pneumatic conveying is a gas, usually air flowing at velocities between 15 and 30 m/s in pipes ranging from 50 to 400 mm in diameter. There are two principle types of systems, negative pressure (Vacuum) systems and positive pressure system. The negative pressure system is useful for transferring solids from multiple intake points (rail cars, ships etc.) to a single input station and one or more points of delivery.

A typical vacuum system is shown in the above figure. In vacuum systems the mass ratio of solids to gas is usually less than 5; for such suspensions. The critical velocity in meter per second may be estimated from the empirical relation.

$$\bar{V}_C = 132.4 \frac{\rho_P}{\rho_P + 998} D_P^{0.40}$$

Where D_P is the diameter of the largest particle to be conveyed.

Problems:

Estimate the minimum fluidization velocity for a bed of particles fluidized by water. The data given as $D_P = 120 \times 10^{-6}$ m; $\phi_S = 1$. $\rho_P = 2500$ kg/m^3, $\varepsilon_m = 0.45$ $\rho = 1000$ kg/m^3; $\mu = 0.9 \times 10^{-3}$ N-S/m^2. Also calculate the bed voidage and the ratio of the height of the fluidized bed to that of the fixed bed for $\dfrac{\bar{V}_o}{\bar{V}_{om}} = 10$.

Solution:

Assume $N_{ReP} < 1$

$$\bar{V}_{oM} = \frac{g(\rho P - \rho)\varepsilon_M^3 \times \phi_S^r D_p^r}{150\mu \quad 1 - \varepsilon_M}$$

$$= \frac{9.81(2500 - 1000)(0.45)^3 \times 1 \times (120 \times 10^{-6})^2}{150 \times 0.9 \times 10^{-3}(1 - 0.45)}$$

$$= \frac{9.81 \times 1500 \times 0.091 \times 1.44 \times 10^{-8}}{0.074}$$

$$= \frac{1.92 \times 10^{-5}}{0.074} = 25.\ 94 \times 10^5 \text{ m / sec}$$

Check:

$$N_{ReP} = \frac{\rho \bar{V}_{oM} D_P}{\mu} = \frac{2500 \times 25.94 \times 10^{-5} \times 120 \times 10^{-6}}{0.9 \times 10^{-3}}$$

$$= \frac{7.782 \times 10^{-5}}{0.9 \times 10^{-3}} = 0.086.$$

Problem:

A bed of ion-exchange beads 244 cm deep is to be back washed with water to remove dirt. The particles have a density 1.249 / cm^3 and an average size of 1.1 mm. What is the minimum fluidization velocity using water at 20^{o}C, and what velocity is required to expand the bed by 25%? The beads are assumed to be spherical ($\phi_s = 1$) and ε_M is taken as 0.40.

Solution:

The Quantities needed are $\mu = 0.01$ P (P is a poise)

$\Delta\rho$ 0.24 g/cm^3

We know the equation

$$\frac{150\times\mu\bar{V}_{oM}(1-\varepsilon_M)}{\phi_s^2 D_P^2\varepsilon_M^3} + \frac{1.75\rho\bar{V}_{oM}^2}{\phi_S D_P}\times\frac{1}{\varepsilon^3 m} = g\ (\rho_P - \rho)$$

$$\frac{150\times(0.01)\bar{V}_{oM}\,0.6}{(0.11)^2(0.4)^3} + \frac{1.75(1.0)(\bar{V}_{oM})^2}{0.11}\times\frac{1}{(0.4)^3} = 980\ (0.24)$$

$$1162\ \bar{V}_{oM} + 248.6\ \bar{V}^{2\ r}_{oM} = 235.2$$

From Quadratic formula $\bar{V}_{oM}$ = 0.194 Cm/s (or) 1.94 mm/s.

At $\bar{V}_{oM}$ $N_{ReP} = \dfrac{0.11(0.194)(1.24)}{0.01} = 2.65$

From the graph m ≅ 3.09

For 25% expansion L = 1.25 Lm.

$$\left(\frac{\varepsilon}{\varepsilon_m}\right)^{3.9} = \frac{\bar{V}_o}{\bar{V}_{oM}}$$

$1 - \varepsilon = {(1-\varepsilon_M)}/{1.25} = 0.48$; $\varepsilon = 0.52$ $\qquad \bar{V}_o = 5.40$ mm/s.

$$L = L_M\ \frac{1-\varepsilon_M}{1-\varepsilon}$$

$$1.25\ \cancel{L_M} = \cancel{L_M}\ \frac{1-\varepsilon_M}{1-\varepsilon}$$

$$1 - \varepsilon = \frac{1-\varepsilon_M}{1-\varepsilon} = \frac{1-0.40}{1.25} = 0.48.$$

From this ε = 0.52

By the elation

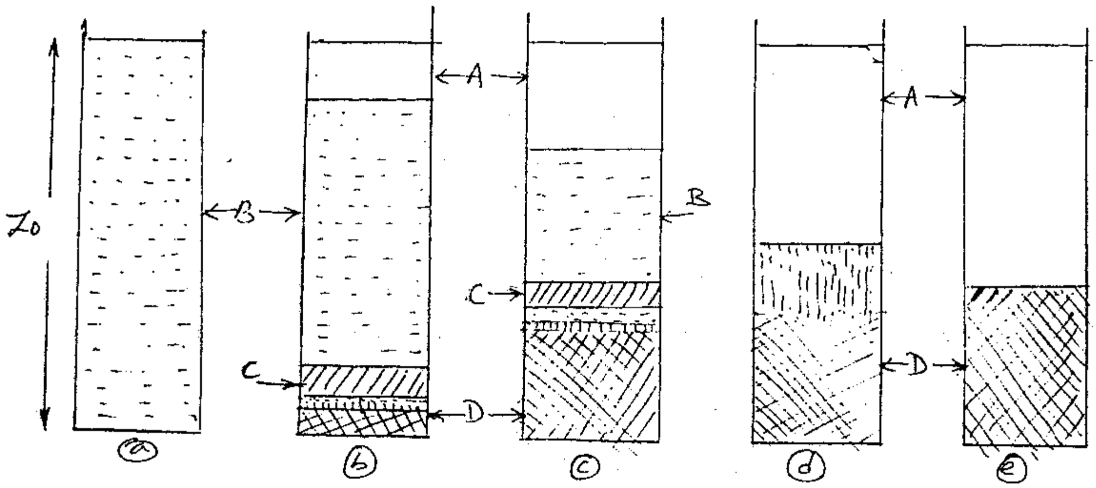
$$\left(\frac{\varepsilon}{\varepsilon_M}\right)^{3.9} = \frac{\bar{V}_o}{\bar{V}_{oM}}$$

$$= \left(\frac{0.52}{0.48}\right)^{3.9} = \frac{\bar{V}_o}{1.94} \quad \Rightarrow \bar{V}_o = 5.40 \text{ mm/s.}$$

SEDIMENTATION

The separation of dilute slurry by gravity settling into a clear liquid and a slurry of higher concentration solid content is called sedimentation. This is also called clarification.

Batch Sedimentation:
These are several stages in the settling of a flocculated suspension, and different zones are formed as sedimentation proceeds. Usually, the concentration of solids is high enough that sedimentation of individual particles or flocs is hindered by other solids to such an extent that all solids at a given level settle at a common velocity.



At first, solid is uniformly distributed in the liquid as shown in fig (a). The total depth of suspension is 'Z_o'. After a short time, the solids have settled to give a zone of clear liquid, zone A in fig (b), and a zone of settled solids D. Above zone D is a transition layer, zone C, in which the solids content varies from that in the original pulp to that in zone D. In zone B, the concentration is uniform and equal to the original concentration, since the settling rate is the same through this zone. The boundaries between zones D and C and between C and B may not be distinct, but the boundary between zones A and B is usually sharp.

As settling continues, the depth of zones D and A increases. The depth of zones C remains nearly constant, and that of zone B decreases. This is shown in fig (c). Eventually zone B disappears and all the solids entering are in zones C and D (fig (d). Meanwhile, the gradual accumulation of solids put stress on the material at the bottom, which compress solids in layer D. Compression breaks down the structure of flocks or aggregates, and liquid is expelled into the upper zones. Sometimes liquid in the flocks spurts out of zone D like small geysers as layer D compresses. Finally, when the weight of the solid is balanced by the compressive strength of the flocks the settling stops, as shown in fig (e). The entire process shown in fig (a) to fig (e) is called Sedimentation.

This laboratory batch-settling (The above said batch process batch sedimentation test is the basis for the design of a continuous thickener).

Rate of Sedimentation:

THEORIES OF SEDIMENTATION: There are two theories.

(i) Coe and Clevenger theory: For a given set of operating conditions (the solid material in the slurry feed, the conditions (the solid material in the slurry feed, the size-frequency distribution of the solid particles, and the liquid properties remain same.)

1. It is assumed that the settling rate is a function only of the solids concentration. (Expressed as volume of solids/ unit volume of slurry).

2. It is also assumed that if batch sedimentation test, were conducted at different initial pulp concentrations, the essential characteristics of the solids (Degree of flocculation) will be unchanged .

(ii) Kynch Theory: First assumption of Coe and Clevenger theory has been taken .

Application of Batch-Sedimentation test for design of continuous Thickeners:

The capacity of a continuous thickener is determined by the fact that the solids initially present in the feed must be able to settle through all zones of slurry concentration from that of initial feed to that of the under flow, at a rate equal to that at which they are introduced into the thickener. If the area provided is not sufficient at some point, the solids will build up through the settling zone and into the clarification zone until finally some solids are discharged in the overflow. Furthermore, it is not known at the start which zone will be the zone of minimum capacity.

Kynch Theory:

For the design of thickener it is assumed that for a given set of operating conditions (the solid material in the slurry feed, the size frequency distributed of the solid particles, and the liquid properties remain constant), the settling rate is a function only of the solids concentration expressed as volume of solids/ unit volume of slurry.

This method is based on the mathematical analysis of batch settling test presented by Kynch, which showed that the settling rate and the concentration of zone that limits the capacity can be determined from a single batch settling test (for a given pulp and temperature of operation). In a batch sedimentation test started with uniform initial concentration of solids, the concentration of solids in the zone C must range between that of the initial slurry concentration in zone B and that of the final slurry in zone D. If the solids handling capacity per unit area is lowest at some intermediate concentration, a zone of such concentration must start building up since the rate at which solids enter this zone will be more than the rate at which they will leave this zone. It has been showed that the rate of upward propagation of such a zone is constant and is a function of the solid concentration.

$$\bar{v} = C.\frac{dv}{dc} - v \qquad (1)$$

Where

$\bar{v}$ - upward velocity of propagation of concentration zone of minimum settling rate with respect to vessel.

v – Settling velocity of solids in concentration zone of minimum settling rate with respect to vessel.

C – Concentration of solids, weight of solids per unit volume of pulp.

From the assumption that the settling rate is a function only of solid concentration only, i.e., v = f (C)

$$\bar{v} = Cf^1 (C) - f (C) \qquad (2)$$

Suppose C_o and Z_o represent the initial concentration and height respectively of a pulp in a batch settling test, the total weight of solids in this pulp is then C_o A Z_o , where A is the Cross section area of the column of pulp. Consider the test at the instant of time when the layer corresponding to the limiting setting rate has reached the interface between the clear supernatant liquid and the pulp. All the solids in the initial pulp must have passed through this layer-since the layer was propagated upward from the bottom of the column. If the concentration of this layer is C_L and the time instant at which the layer reaches the interface is θ_L, then

$$C_L A\,(v_L + \bar{v}_L)\,\theta_L = C_o A Z_o \qquad (3)$$

Where v_L and $\bar{v}_L$ refer to the respective velocities for a layer having a solid concentration of C_L. Let Z_L correspond to the weight of the interface at time θ_L. Then

$$\bar{v}_L = Z_L/\theta_L \qquad (4)$$

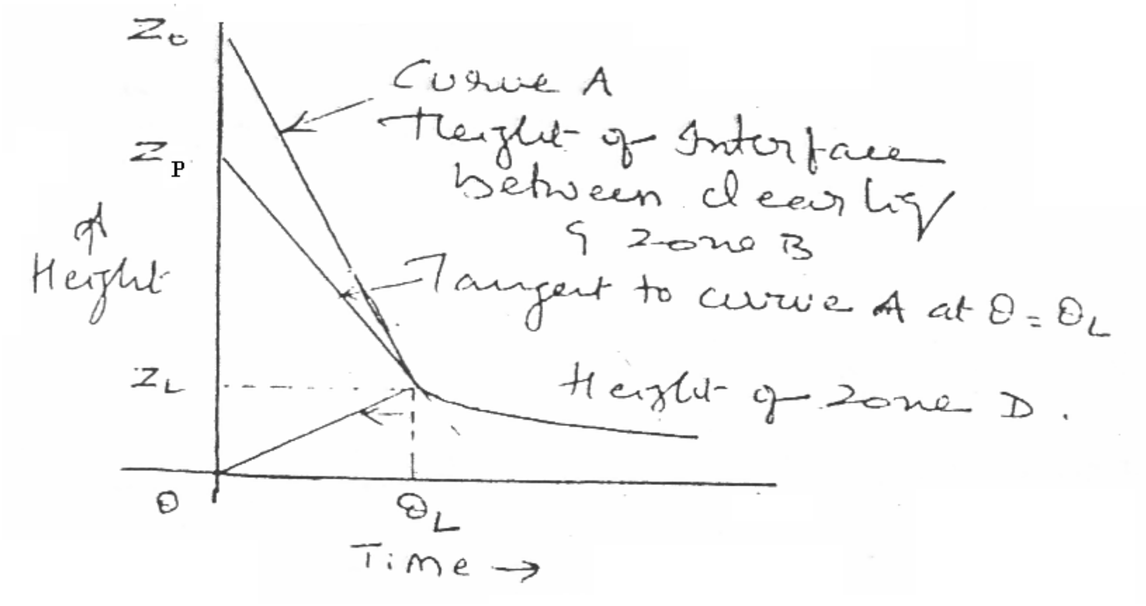
Since from (2) $\bar{v}$ is constant if 'C' is constant, substituting (4) in (3) and simplifying.

$$C_L A \left(V_L + \frac{Z_L}{\theta_L} \right) \theta_L = C_o A Z_o$$

$$\Rightarrow C_L\,(v_L\theta_L + Z_L) = C_o Z_o$$

$$\Rightarrow C_L = \frac{C_o Z_o}{\theta_L v_L + Z_L} \qquad (5).$$

The value of settling velocity V_L is the slope of tangent to the curve at $\theta = \theta_L$.



Determination of setting velocities from Batch-Settling curve:

This tangent intersects the vertical axis as Z = Z_i. The slope of this line is

$$\text{Tan } \alpha \ \frac{Z_i - Z_L}{\theta = \theta_L} = Z_i - Z_L = -\theta_L \tan\alpha = \theta_L V_L$$

$$Z_i = Z_L + \theta_L v_L \qquad (6)$$

Comparing (5) and (6) $\qquad Z_i = \dfrac{C_o Z_o}{C_L}$

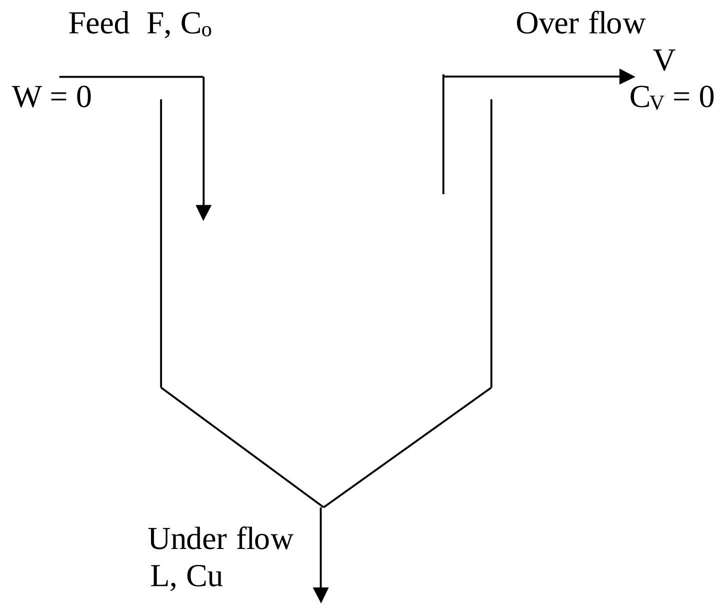
$$C_L Z_i = C_o Z_o \qquad (7)$$

Equation (7) states that Z_i is the length of a uniform slurry concentration C_L which contains the same amount of solids as in the initial slurry.

The setting velocity as a function of concentration may be determined from a single settling test by the use of the above relation ship. Using arbitrarily chosen values of settling time θ, the corresponding tangents to the settling curve are located and the values of slope and intercepts are determined. The values of intercepts and slopes are used in (7) to determine the corresponding concentration. The respective settling rates are given by the corresponding slopes.

Determination of Thickener area:

F is the volume feed rate of slurry per unit time. Co-initial concentration of solids in slurry concentration of solids in underflow. L – underflow volume rate per unit time.



The material balance for solids is:

$$FC_o = LC_u \qquad \text{or} \qquad L = \frac{FC_o}{C_u} \qquad (1)$$

Writing a liquid material balance for the thickener

$$F(1 - C_o) - L(1 - C_u) = V \qquad (2)$$

Where V – the overflow volume / unit time.

Substituting for L from (1) in (2),

$$F(1 - C_o) - F\frac{C_o}{C_u}(1 - C_u) = V$$

$$F - F\cancel{C_o} - \frac{C_o}{C_u} . F + F\cancel{C_o} = V$$

$$F - F . \frac{C_o}{C_u} = V$$

$$FC_o\left(\frac{1}{C_o} - \frac{1}{C_u}\right) = V \qquad (3)$$

If the cross sectional area of the thickener is ‘A’

Dividing equation (3) by the cross sectional area A of thickener.

$$\frac{V}{A} = \frac{FC_o}{A}\left(\frac{1}{C_o} - \frac{1}{C_u}\right) \qquad (4)$$

The term V/A in equation (4) represents the upward propagation velocity in the clarification zone of the thickener. When the thickener is operated at capacity, the lowest settling rate encountered must be equal to or greater than this value, otherwise solids will leave in overflow consequently V/A = v (must be replaced by v).

$$v = \frac{FC_o}{A}\left(\frac{1}{C_o} - \frac{1}{C_u}\right)$$

The above equation may be written interms of concentration of limiting layer which limits the capacity rather them in terms of feed concentration, the rate which is set by this capacity i.e. $FC_o = L_L C_L$

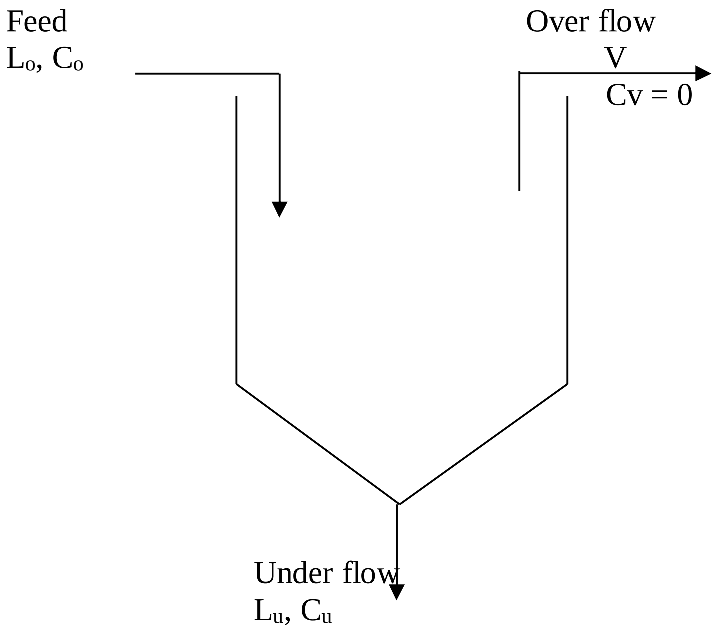
$$v_L = \frac{L_L C_L}{A}\left(\frac{1}{C_L} - \frac{1}{C_u}\right)$$

Or $$\frac{L_L C_L}{A} = \frac{V_L}{\left(1/C_L - \frac{1}{C_u}\right)} \qquad (5)$$

By using settling velocity concentration curve to obtain corresponding values of v_L and C_L, and using these values in (5), the various values of $\frac{L_L C_L}{A}$, the solid handing capacity per unit area may be calculated. The lowest value calculated is to be used in determining the area of thickener.

$$A = \frac{L_L C_L}{\left(v_L / 1/C_L \; 1/C_u\right)_{\min}}$$

Determination of Thickener Area: (Alternative method)



L_o = Volume of feed / Unit time

C_o = Concentration of feed slurry.

V = over flow volume per unit time.

C_v = concentration of solids in slurry in over flow = 0

L_u, C_u respectively volume / unit time and concentration in underflow.

The required thickener area is fixed at the concentration layer requiring the maximum area to pass a unit quantity of solids.

For the thickener shown above,

The material balance for solids is

$$L_oC_o = L_uC_u \qquad (1)$$

$$L_u = \frac{L_oC_o}{C_u} \qquad (2)$$

An over all liquid balance gives;

$$L_o(\rho_o - C_o) = V\rho_w + L_u(\rho_u - C_u) \qquad (3)$$

Eliminating L_u in equation (3) by substitution of (2)

$$L_o (\rho_o - C_o) = V\rho_w + L_o \frac{C_o}{C_u} (\rho_u - C_u)$$

$$\Rightarrow L_o [\rho_o - C_o - \frac{C_o}{C_u} \times \rho u + \frac{C_o}{C_u}] = V \rho w$$

$$V = L_o [\rho_o - \frac{C_o}{C_u} \times \rho u] \frac{1}{\int w}$$

$$= C_o L_o \left[\frac{\rho_o}{C_o} - \frac{\rho_u}{C_u}\right] \frac{1}{\rho w} \qquad (4)$$

Dividing both sides of equation (4) by thickener cross sectional area, S and using ρ_{av} for slurries gives

$$\frac{V}{S} = \frac{L_o C_o}{S} \left[\frac{1}{C_o} - \frac{1}{C_u}\right] \frac{\rho av}{\rho w} \qquad (5)$$

The term V/S is the upward linear velocity of the clarified liquid. In order to keep solids from overflowing, the upward velocity of the liquor must be equal to or less than the settling velocity of the solids, therefore, V/S may be replaced by V.

Equation (5) may be written in terms of the capacity-limiting layer, even though it has not yet been established that a particular value of C_L and corresponding downflow (L_L) represent the true unit; therefore

$L_o C_o$ - $L_L C_L$

And $$\frac{L_L C_L}{S} = \frac{v}{\left[\frac{1}{C_L} - \frac{1}{C_u}\right]} \frac{\rho av}{\rho w}$$

The lowest value of L_L C_L/S determines the minimum thickener area required.

Problem:

A single batch settling test was made on a lime stone slurry. The interface between clear liquid and suspended solids were observed as a function of time and the results are tabulated below. The test was made using 236 gm of lime stone per litre of slurry.

Time(Hrs) (θ_b):	0	0.25	0.50	1.0	1.75	3.0	4.75	12	20
Height of inter: Face (Cm)	36	32.4	28.6	21	14.7	12.3	11.55	9.8	8.8

Prepare a curve showing the relationship between settling rate and solids concentration using this, plot a curve of settling velocity V_s solid concentration. Find the thickener area if the slurry is fed at a rate of 50,000 kgs dry solids/ hr to produce a thickeners sludge of 550 gm of lime stone per litre.

Solution:

1. Draw a graph of height of interference (in mts) (on y-axis) Vs time θ (in seconds((on x – axis).

2. Draw tangents to the curve obtained at many points.

3. The point where tangent cuts the Y – axis gives : Z_i and from the point where the tangent cuts the curve move horizontally towards Y – axis to get Z_L.

C_o = 23.6 gm / lt = 236 kg/ m^3 $\qquad$ $Z_o = 36 \times 10^{-2}$ m

Z_i, (m)	Z_L (m)	Θ_L (sec)	$V_L = \dfrac{(Z_i - Z_L)}{\theta_L}$ (m/sec)	$C_L = \dfrac{C_o Z_o}{Z_i}$ kg/m^3
23.0×10^{-2}	16.0×10^{-2}	90	0.0777×10^{-2}	369.39
18.8×10^{-2}	14.8×10^{-2}	110	0.0366×10^{-2}	451.91
17.4×10^{-2}	13.6×10^{-2}	140	0.0271×10^{-2}	488.27
16.6×10^{-2}	12.8×10^{-2}	174	0.0218×10^{-2}	511.80
16.2×10^{-2}	1.24×10^{-2}	180	0.0212×10^{-4}	524.80

θ_L can be found point by moving vertically down from the point where the tangent cuts the curve till y – axis is reached. C_u = 550 gm/wt = 550 kg/m^3.

$\frac{1}{C_L}$ (m^3/kg)	$\frac{1}{C_u}$ (m^3/kg)	$\frac{1}{C_L}$ - $\frac{1}{C_u}$ m^3 / kg	$V_L \Big/ \left(1/C_L - 1/C_u\right)$ kg/m^2 - sec
2.71×10^{-3}	1.82×10^{-3}	0.89×10^{-3}	0.873
2.21×10^{-3}	1.82×10^{-3}	0.39×10^{-3}	0.939
2.05×10^{-3}	1.82×10^{-3}	0.23×10^{-3}	1.178
1.95×10^{-3}	1.82×10^{-3}	0.13×10^{-3}	1.677
1.90×10^{-3}	1.82×10^{-3}	0.08×10^{-3}	2.65

The area of thickener can be found by the formula :

$$A = \frac{L_L C_L}{\left[v_L \Big/ \left(1/C_L - 1/C_u\right) \right]_{\min}}$$

From last column it is clear that $V_L \Big/ \left(1/C_L - 1/C_u\right)$ is in increasing order. Hence 0.873 is taken as minimum value of $\frac{V_L}{1/C_L - 1/C_u}$. If values of $V_L \Big/ \left(1/C_L - 1/C_u\right)$ are in increasing – decreasing order, then a graph of $\frac{L_L C_L}{A}$ i.e. $\left[\frac{V_L}{1/C_L - 1/C_u}\right]$ on y – axis Vs $V_{/L}$ on x-axis is drawn and from the graph, $\left[\frac{V_L}{1/C_L - 1/C_u}\right]_{\min}$ is obtained.

$$A = \frac{50{,}000 kg/hr}{0.873 k/m^3 \sec} \qquad L_L C_L = 50{,}000 \text{ kg/hr.}$$

$$= \frac{50{,}000}{60 \times 60} \times \frac{1}{0.873} \frac{kg \times m^2 \sec}{kg \quad \sec}$$

$$= 15.91 \text{ m}^2.$$

Uses of Thickener:

1. For the production of magnesium from sea – water.

2. In sewage treatment and also in water – purification.

3. They also used in cement manufacture.

General Procedure to design a thickener:

Separation of fine solids from slurry in tonnage is to be carried out in a thickener. The process is best described by batch sedimentation and from this the area of the thickener can be calculated in the operation of thickeners a basic assumption is that the material to be settled contains flocks of uniform size and shape and they settle under hindered settling conditions.

The capacity of a thickener is determined by the fact that the solids initially present in the feed must be able to settle through all zones of slurry concentrations from that of the initial feed equal to that at which they are introduced into the thickener. If the area provided is not sufficient the solids will build up through the settling zone until finally some solids are discharged in the overflow. The method is based on the mathematical analysis proposed by Kynch. The settling time and concentration of the zone that units capacity can be determined through batch sedimentation test. From graph of Z vs θ, settling velocity as a function of concentration, and the solids handling capacity per unit area can be calculated. The lowest value of solid handling capacity is to be used to find the thickener area. The formula used in batch sedimentation are

$$C_L = \frac{C_o Z_o}{Z_i} \qquad (1)$$

Where,

C_L = Concentration of solids in limiting layer kg/m^3

C_o = initial solids concentration kg/ m^3

Z_o = initial height of interface, m

Z = height of interface at time θ, m

Z_i = height of interface at θ_L, m

$$V_L = \frac{Z_i - Z_L}{\theta_L} \qquad (2)$$

Where, V_L = Velocity of limiting layer, m/s.

$$\frac{L_L C_L}{A} = \frac{V_L}{\frac{1}{C_L} - \frac{1}{C_u}} \qquad (3)$$

Where, C_u = underflow solids concentration kg/ m^3.

Take 50 gms of powdered sample and prepare the slurry by mixing it with water and make up to 1 litre in a measuring graduated jar. Note the initial weight Z_o and stir the slurry to get uniform concentration. When stirring is stopped simultaneously start a stopwatch to note the time taken for each cm of height of fall of the interface. Carry on the experiment till the height of the interface remains almost constant tabulate Z and θ as follows:

$Z \times 10^2$, m	
θ, sec	

Draw a graph of Z vs θ, calculate slopes (V_L) using equation (2) and concentration (C_L) using equation (1) and tabulated them as:

Sl.No	$Z_L \times 10^{-3}$ m	C_L kg/ m^3	V_Lm/sec	$\frac{1}{C_L}$ m^3/kg	$\frac{1}{C_u}$ m^3/kg	$\frac{1}{C_L} - \frac{1}{C_u}$ m^3/kg	$\frac{V_L}{\frac{1}{C_L} - \frac{1}{C_u}}$ kg/m^{2}sec

Then draw a graph of the solid handling capacity Vs velocity of the limiting layer i.e, $\left[\frac{V_L}{\frac{1}{C_L} - \frac{1}{C_u}}\right]$ Vs V_L to get the minimum value of the solid handling capacity.

This minimum value is used in equal, to find the area (A) of the thickener.

Thickeners Working:

Equipment for Sedimentation Thickener:

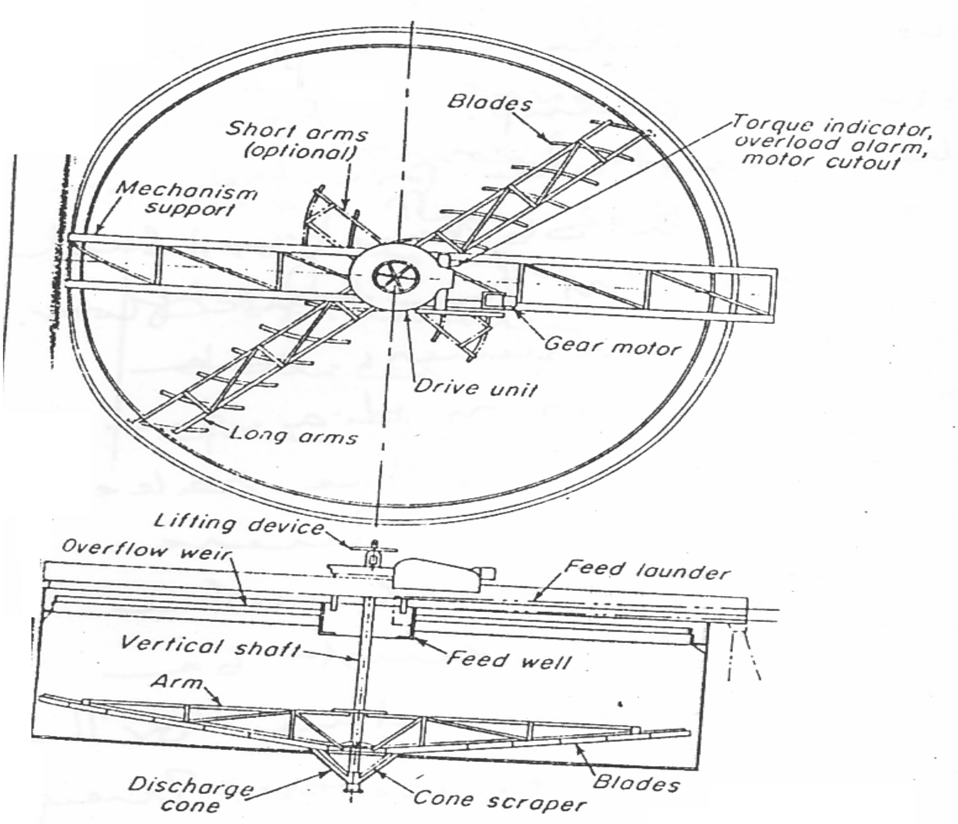


FIGURE 30.34
Gravity thickener. (*Eimco Corp.*)

Industrially thickening is conducted on a large scale in equipments called thickeners. For heavy duties, a mechanically agitated thickeners like that shown are employed

This is a large, fairly shallow tank slow moving radical rakes driven from a central shaft. Its bottom may be flat or a shallow cone. Dilute feed slurry flows from an inclined trough or launder in to centre of the thickener. The feed slurry being more denser than water, tends to flow downward until it reaches a zone of equal density. Then it moves radialy outward at a constantly decreasing velocity and the flow gradually divides between the downward-moving suspension and the upward-moving flow that is nearly free of solids. Liquor moves radialy at a constantly decreasing velocity, allowing the solids to settle to the bottom of the tank. Clear liquor spills over the edge of the tank into a launder. The rake arms gently agitate the sludge and move it to the center of the tank, where it flows through a large from the inlet of a sludge pump. In some designs of thickener the rake arms are pivoted so that they can ride over an obstruction, such as a hard pump of mud, on the tank floor.

Mechanically agitate thickeners are usually large, typically 10 to 100 m in dia. and 2.5 to 3.5 m on deep. In a large thickener the rakes may revolve once every 30 min. These thickeners are especially valuable when large volume of dilute slurry must be thickened as in cement manufacture or the production of magnesium from sea water. They are also used extensively in sewage treatment and in water purification. The feed pulp is admitted at the centerline of the unit at a depth of 1m or so below the surface of the liquid. Above the feed level is a clarification zone in which the liquid is almost free of solids. Below the feed level is a zone of hindered settling and, near the bottom a compression zone in which the solids concentration is high. These sedimentation zones are discussed later in this section.

The volume of clear liquor produced in a unit time by a continuous thickener depends primarily on the cross section area available for settling and in industrial separators is almost independent of the liquid depth. Higher capacities per unit of floor area are therefore obtained by using a multiple-tray thickener, with several shallow settling zones, one above the other, in a cylindrical tank. Rake or scraper agitators move the settled sludge downward from one tray to the next. Multistage counter current displacement washing is possible in these devices. They are considerably smaller in diameter, however, than single-stage thickener.