equations for molecular transport as well as for turbulent transport and we can say that we have analogies among these transport processes. A great deal of effort has been taken in developing analogies (similarities) among these three transport processes for turbulent transfer.

General Molecular Transport Equation :

The molecular transport of momentum, heat and mass is characterised by the same general equation of the type :

$$\text{Rate of a transfer process} = \frac{\text{Driving force}}{\text{Resistance}} \qquad \dots (7.95)$$

Molecular transport or molecular diffusion equations for momentum, heat and mass transfer :

These equations are applicable to laminar flow as molecular transport is a characteristic of laminar/stream line flow.

Newton's law for momentum transport, i.e., Newton's equation for molecular diffusion of momentum for constant density is

$$\tau_{zx} = \frac{-\mu}{\rho}\frac{d(u_x \rho)}{dz} \qquad \dots (7.96)$$

$$\tau_{zx} = -\nu\frac{d(u_x \rho)}{dz} \qquad \dots (7.97)$$

where τ_{zx} is the momentum transferred per unit time per unit area, where the momentum has the units of (kg.m)/s. [It is also called as the momentum flux], μ is viscosity and ρ is the density of fluid. ν is the kinematic viscosity and also called as the diffusivity of momentum and has the units of m^2/s.

Fourier's law for heat transport, i.e., Fourier's equation for molecular diffusion of heat for constant ρ and C_p is

$$\frac{q_z}{A} = -\alpha\frac{d(\rho C_p T)}{dz} \qquad \dots (7.98)$$

where $\frac{q_z}{A}$ is the heat transferred per unit time per unit area and is called as heat flux and has the units of W/m^2 $[J/(m^2.s)]$. α is the thermal diffusivity or diffusivity of heat in m^2/s.

Fick's law of molecular mass transport or Fick's equation for molecular diffusion of mass for constant total concentration in a fluid is

$$J_{Az} = -D_{AB}\frac{dC_A}{dz}$$

$$= -D_{AB}\frac{d(\rho_A/M_A)}{dz} \qquad \dots (7.99)$$

where J_{Az} is the molar flux of component A in the z direction due to molecular diffusion in kmol A/(m^2.s), D_{AB} is the molecular diffusivity of molecule A in B or mass diffusivity in m^2/s. C_A is the molar concentration of A in $kmol/m^3$.

These three equation states, respectively, that (a) the momentum transport occurs because of a gradient in momentum concentration, (b) the energy transport occurs because of a gradient in energy concentration, and (c) the mass transport occurs because of a gradient in mass concentration.

Turbulent diffusion equations for momentum, heat and mass transfer :

For combined molecular and eddy transfer, the relations for momentum, heat and mass are :

For turbulent momentum transfer for constant density, we have

$$\tau_{zx} = -(\mu/\rho + \varepsilon_M)\frac{d(u_x\,\rho)}{dz} \qquad \text{... (7.100)}$$

$$\tau_{zx} = -(\nu + \varepsilon_M)\frac{d(\rho\,u_x)}{dz} \qquad \text{... (7.101)}$$

For turbulent heat transfer for constant ρ and C_p, we have

$$\frac{q_z}{A} = -(\alpha + \varepsilon_H)\frac{d(\rho\,C_p\,T)}{dz} \qquad \text{... (7.102)}$$

For turbulent mass transfer for constant total molar concentration, we have

$$J_{Az} = -(D_{AB} + \varepsilon_D)\frac{dC_A}{dz} \qquad \text{... (7.103)}$$

In the above equations, ε_m is the turbulent or eddy diffusivity of momentum in m²/s, ε_H is the turbulent or eddy diffusivity of heat or eddy thermal diffusivity in m²/s and ε_M is the turbulent or eddy diffusivity of mass in m²/s. Again, these equations are quite similar or analogous to each other.

Equations (7.97), (7.98) and (7.99) for momentum, heat and mass transfer are similar to each other and to the general molecular transport Equation (7.95). All these equations have a flux on the left hand side which is momentum, heat or mass transferred per unit time per unit area and a diffusivity of momentum, heat and mass (i.e., transport properties, ν, α and D_{AB}) all in m²/s, and a derivative of the concentration of a property with respect to the distance on the right hand side. In all the above cases, the flux is proportional to the driving force. These three molecular transport equations are mathematically identical. Thus, we say that we have an analogy or similarity among them. Even though there is a mathematical analogy among them, the actual physical mechanisms occurring is totally different.

The mass diffusivity D_{AB}, the kinematic viscosity, ν and thermal diffusivity, α are analogous as seen from the above equations.

The similarity in nature of transfer of these three processes are referred to as analogy. Considering similarities between the governing equations of heat, mass and momentum transfer, it is to be expected that the correlations for heat transfer coefficients and mass transfer coefficients would also be similar. Various quantitative relations are available to describe the analogical behaviour. The simplest and oldest is due to Reynolds.

The Reynolds Analogy :

Reynolds was first to note similarities in transport processes and relates turbulent momentum and heat transfer.

The basic assumption of the Reynolds analogy is that the ratio of two molecular diffusivities equals to that of two eddy diffusivities.

$$\frac{\nu}{\alpha} = N_{Pr} = \frac{\varepsilon_M}{\varepsilon_H} \qquad \text{... (7.104)}$$

For turbulent flow conditions, the Reynolds analogy equations are :

The statement of Reynolds analogy between heat and momentum transfer is :

$$\frac{h}{C_p\, u\, \rho} = \frac{h}{C_p\, G} = N_{St} = \frac{f}{2} \qquad \text{... (7.105)}$$

The statement of Reynolds analogy between mass and momentum transfer is :

$$\frac{k_c'}{u} = \frac{f}{2} \qquad \text{... (7.106)}$$

Therefore, the complete Reynolds analogy is

$$\frac{h}{C_p u\, \rho} = \frac{k_c'}{u} = \frac{f}{2} \qquad \text{... (7.107)}$$

$$N_{St} = N_{St_m} = \frac{f}{2} \qquad \text{... (7.108)}$$

where f is the Fanning friction factor (a measure of skin friction), u is the average velocity of fluid, k_c' is the convective mass transfer coefficient, h is the convective heat transfer coefficient.

Equation (7.107) agrees well with the experimental data (correlates data) for gases in turbulent flow if the Schmidt and Prandtl numbers are about unity and only the skin friction is present in a flow past a flat plate or inside pipe. The equations do not correlate the data for liquids in turbulent flow nor for any fluids in laminar flow, i.e., in such cases the analogy is not valid.

Although the Reynolds analogy is of limited utility, the significant conclusion that may be drawn is the mechanisms for momentum, heat and mass are identical at $N_{Pr} = N_{Sc} = 1.0$.

If a measure of the skin friction, i.e., the fanning friction factor is known, the analogy may be used to find the heat transfer coefficient from the mass transfer coefficient and vice-versa.

$$\therefore\ C_{A1} \text{ at the inner surface of the pipe } = \frac{0.106}{22.4} \times 1 = 4.73 \times 10^{-3} \text{ kmol } H_2/m^3$$

At the outer surface, $C_{A2} = 0$

The rate of loss of hydrogen per 1 m length of the pipe is

$$w = N_A \cdot A_{avg.} = D_A \cdot A_{avg.} [C_{A1} - C_{A2}]/z$$

$$= 1.8 \times 10^{-10} \times 0.1133 \ (4.73 \times 10^{-3} - 0)/0.0125$$

$$= \mathbf{7.72 \times 10^{-2} \text{ kmol } H_2/s \text{ per m}} \quad \text{... Ans.}$$

Some Important Definitions :

1. **Gram mole :** It is defined as the *mass in grams of a substance that is numerically equal to its molecular weight.*

In this book, gram mole and kilogram mole are specified as mol and kmol respectively.

2. **Weight fraction :** It is the *ratio of the weight of the individual component to the total weight of the system.* It is denoted by the symbol x'.

For two component system : $x'_A + x'_B = 1.0.$

$$\text{Weight \% of A} = \text{Weight fraction of A} \times 100$$

3. **Mole fraction :** It is the *ratio of the moles of the individual component to the total moles of the system.* It is denoted by the symbol x.

For a binary system of A and B : $x_A + x_B = 1.0.$

Mole % of A = mole fraction of A × 100.

4. **More volatile component :** It is the *component with a lower boiling point or with a higher vapour pressure at a given temperature (in a binary system).* It is also called as the lighter component.

In case of distillation, the compositions of vapour and liquid phases are expressed in terms of mole fraction of the more volatile component.

5. **Less volatile component :** In a binary system, it is the component with a higher boiling point or with a lower vapour pressure at a given temperature. It is also called as the heavier component.

6. **Vapour pressure :** The vapour pressure of a liquid is defined as the *absolute pressure at which the liquid and its vapour are in equilibrium at a given temperature.*

7. **Partial pressure :** The partial pressure of a component gas that is present in a mixture of gases in the pressure that would be exerted by that component if it alone were present in the same volume and at the same temperature as the mixture.

8. **Ideal gas law :** The ideal gas law is given by

$$PV = nRT$$

if P is in kPa, V in m^3, n in kmol and T in K then R will be in $m^3 \cdot kPa/(kmol \cdot K)$.

$$R \text{ (universal gas constant)} = 8.31451 \ m^3 \cdot kPa/(kmol \cdot K)$$

9. **Dalton's law :** Mathematically, Dalton's law is given by

$$P = p_A + p_B + p_C + \ldots$$

where P is the total pressure exerted by a gas mixture and p_A, p_B, p_C ... are the partial pressures of component gases A, B, C, ... present in the mixture.

10. **Raoult's law :** It states that the equilibrium partial pressure of component A is equal to the product of the vapour pressure and the mole fraction of A in the liquid phase.

$$\therefore \quad p_A = p_A^0 \cdot x_A$$

p_A is also related to y_A by the following equation

$$p_A = y_A \cdot P$$

y_A is the mole fraction of A in the gas phase.

11. **Henry's law :** Mathematically Henry's law is given as,

$$p_A = H\,x_A$$

where H is the Henry's law constant.

Henry's law expresses the relationship between the concentration of a gas dissolved in a liquid and the equilibrium partial pressure of the gas above the liquid surface.

12. **Gibb's phase rule :** It is the relationship that governs all heterogeneous equilibria. It is given by

$$F = C - P + 2$$

where C = number of components

P = number of phases

F is the number of degrees of freedom or number of intensive variables (temperature, pressure, composition) that must be specified so that remaining variables will be fixed automatically and the system will be defined completely.

EXERCISES

1. Give the mathematical statement of Fick's law of diffusion and give the meaning of each terms involved in it.
2. Define :
   (i) Mass fraction,
   (ii) Mole fraction,
   (iii) Molar concentration,
   (iv) Mass average velocity, and
   (v) Molar average velocity.
3. Define : Mass flux and Molar flux and give the expressions for Mass and Molar fluxes relative to the mass average velocity and molar average velocity.

4. Define Diffusion, Molecular diffusion, Eddy/turbulent diffusion and explain briefly the role of diffusion in mass transfer.
5. Explain briefly analogy between heat, mass and momentum.
6. Explain briefly Reynolds analogy.
7. Show that for equimolar counter diffusion, $D_{AB} = D_{BA}$.
8. State the Fick's law of diffusion.
9. Give the mathematical expression for analogy between heat, mass and momentum transport for laminar and turbulent flow. Give the meaning of each term.
10. Define mass transfer coefficient. Give its SI unit.
11. What do you mean by interphase mass transfer ?
12. State salient features of two-film theory.
13. Explain the controlling film concept.