SOLVED EXAMPLES

Example 7.1 : *A large tank filled with a mixture of gases A and B at 101 kPa and 298 K (25 °C) is connected to another large tank filled with a mixture of A and B of different compositions of A and B at 101 kPa and 298 K (25 °C). The tanks are connected by a tube of inner diameter of 50 mm and is 150 mm long. Calculate the steady state rate of transport of A through the tube when the concentration of A in one tank is 90 mole % and in the other is 5 mole % assuming uniformity in the composition in each tank and transfer takes by molecular diffusion. The diffusivity of A in B is* 4.3×10^{-3} *m²/s.*

Solution : Mole fraction of A in tank–1 $= x_{A1} = \frac{90}{100} = 0.90$

Mole fraction of A in tank–2 $= x_{A2} = \frac{5}{100} = 0.05$

$$D_{AB} = 4.3 \times 10^{-3} \text{ m}^2/\text{s}$$

$$z = 150 \text{ mm} = 0.15 \text{ m} = \text{length of diffusion path}$$

$$\text{Area} = \frac{\pi}{4} D^2 = \frac{\pi}{4} \times (0.05)^2 = 1.963 \times 10^{-3} \text{ m}^2$$

$$p_{A1} = x_{A1} \,.\, P = 0.9 \times 101 = 90.9 \text{ kPa}$$

$$p_{A2} = x_{A2} \,.\, P = 0.05 \times 101 = 5.05 \text{ kPa}$$

$$R = 8.31451 \text{ m}^3\text{. kPa/(kmol.K)}$$

$$\text{Rate of transport of A} = N_A \cdot A = \frac{D_{AB}\,(p_{A1} - p_{A2})}{RTz} \times A$$

$$= \frac{4.3 \times 10^{-3} \times (90.9 - 5.05) \times 1.963 \times 10^{-3}}{8.31451 \times 298 \times 0.15}$$

$$= 1.95 \times 10^{-6} \text{ kmol/s}$$ ... Ans.

Example 7.2 : *In an oxygen-nitrogen gas mixture at 101.3 kPa and 298 K, the concentrations of oxygen at two planes 2 mm apart are 20 and 10% by volume respectively. Calculate the flux of diffusion of oxygen for the cases where :*

(i) nitrogen is non-diffusing

(ii) there is equimolar counter diffusion of the two gases. Diffusivity of O_2 *in* N_2 *is* 1.81×10^{-5} *m²/s.*

Solution : 1. For the diffusion of A through a non-diffusing B, the flux is given by

$$N_A = \frac{D_{AB}\,P}{RTz\,p_{B,M}}\,(p_{A1} - p_{A2}) \qquad \text{... (1)}$$

Total pressure, P = 101.3 kPa

For an ideal gas, volume % = mole %. Therefore, mole % of O_2 = volume % of O_2

$$x_{A1} = \frac{20}{100} = 0.20$$

∴

$$x_{A2} = \frac{10}{100} = 0.10$$

Partial pressure of A = Molefraction of A × Total pressure

$$p_{A1} = x_{A1} P = 0.20 \times 101.3 = 20.26 \text{ kPa}$$

$$p_{A2} = x_{A2} P = 0.10 \times 101.3 = 10.13 \text{ kPa}$$

We have,

$$P = p_{A1} + p_{B1}$$

∴

$$p_{B1} = 101.3 - 20.26 = 81.04 \text{ kPa}$$

$$p_{B2} = 101.3 - 10.13 = 91.17 \text{ kPa}$$

$$p_{B,M} = (p_{B2} - p_{B1}) / \ln (p_{B2} / p_{B1})$$

$$= (91.17 - 81.04) / \ln (91.17 / 81.04) = 86 \text{ kPa}$$

$$z = 2 \text{ mm} = 2 \times 10^{-3} \text{ m}, \; D_{AB} = 1.81 \times 10^{-5} \text{ m}^2/\text{s}$$

$$R = 8.31451 \text{ m}^3.\text{kPa} / (\text{kmol.K}), \; T = 298 \text{ K}$$

Substituting the values, Equation (1) gives

$$N_A = \frac{1.81 \times 10^{-5} [20.26 - 10.13] \times 101.3}{8.31451 \times 298 \times 2 \times 10^{-3} \times 86}$$

$$= \mathbf{4.356 \times 10^{-5} \text{ kmol/(m}^2\text{.s)}}$$ **... Ans.**

2. The flux for the equimolar counter diffusion is given by

$$N_A = \frac{D_{AB}}{RTz} (p_{A1} - p_{A2})$$

$$= \frac{1.81 \times 10^{-5} \times (20.26 - 10.13)}{8.31451 \times 298 \times 2 \times 10^{-3}}$$

$$= \mathbf{3.7 \times 10^{-5} \text{ kmol/(m}^2\text{.s)}}$$ **... Ans.**

Example 7.3 : *Methane diffuses at steady state through a tube containing helium. At point 1, the partial pressure of methane is 55 kPa and at point 2 it is 15 kPa. The points 1 and 2 are 30 mm apart. The total pressure is 101.3 kPa and temperature is 298 K (25 °C). Calculate the flux of CH_4 at steady state for equimolar counter diffusion. The diffusivity of methane at the prevailing conditions is 6.75×10^{-5} m²/s.*

Solution : The flux for the equimolar counter diffusion is given by

$$N_A = \frac{D_{AB}}{RTz} (p_{A1} - p_{A2})$$

where,

$$D_{AB} = 6.75 \times 10^{-5} \text{ m}^2/\text{s}, \; R = 8.31451 \text{ m}^3.\text{kPa/(kmol.K)}$$

$$T = 298 \text{ K}, \; z = 30 \text{ mm} = 0.03 \text{ m}$$

$$p_{A1} = 55 \text{ kPa}, \; p_{A2} = 15 \text{ kPa}$$

$$N_A = \frac{6.75 \times 10^{-5}(55 - 15)}{8.31451 \times 298 \times 0.03}$$

$$= \mathbf{3.63 \times 10^{-5} \ kmol/(m^2.s)}$$

... Ans.

Example 7.4 : *Ammonia gas (A) diffuses through a non-diffusing nitrogen gas (B) under steady state conditions. The partial pressure of A at location 1 is 1.5×10^4 Pa and that at location 2 is 5×10^3 Pa (Pascal). The locations 1 and 2 are 0.15 m apart. The total pressure is 1.103×10^5 Pa and temperature is 298 K. Calculate the flux of diffusion of ammonia. Also calculate the flux of diffusion for equimolar counter diffusion considering that nitrogen is also diffusing. The diffusivity of ammonia at the prevailing conditions is 2.30×10^{-5} m²/s.*

Solution : 1. The flux equation for the diffusion of A through a non-diffusing B is given by

$$N_A = \frac{D_{AB} P}{RTz \, p_{B,M}} (p_{A1} - p_{A2})$$

where

$$D_{AB} = 2.30 \times 10^{-5} \text{ m}^2\text{/s}$$

$$P = 1.103 \times 10^5 \text{ Pa}$$

$$p_{A1} = 1.5 \times 10^4 \text{ Pa}$$

$$p_{A2} = 5 \times 10^3 \text{ Pa}$$

$$p_{B1} = P - p_{A1} = 1.103 \times 10^5 - 1.5 \times 10^4 = 8.63 \times 10^4 \text{ Pa}$$

$$p_{B2} = P - p_{A2} = 1.103 \times 10^5 - 5 \times 10^3 = 9.63 \times 10^4 \text{ Pa}$$

$$p_{B,M} = \frac{p_{B2} - p_{B1}}{\ln (p_{B2} / p_{B1})}$$

$$= \frac{(9.63 \times 10^4 - 8.63 \times 10^4)}{\ln (9.63 \times 10^4 / 8.63 \times 10^4)}$$

$$= 9.121 \times 10^4 \text{ Pa}$$

$$T = 298 \text{ K}, \quad z = 0.15 \text{ m}$$

$$R = 8.31451 \text{ m}^3\text{. kPa/(kmol.K)} = 8314.51 \text{ m}^3\text{.Pa / (kmol.K)}$$

$$N_A = \frac{2.30 \times 10^{-5} \times 1.103 \times 10^5 (1.5 \times 10^4 - 5 \times 10^3)}{8314.51 \times 298 \times 0.15 \times 9.121 \times 10^4}$$

$$= \mathbf{7.484 \times 10^{-7} \ kmol/(m^2.s)}$$

... Ans.

2. The flux equation for the equimolar counter diffusion is given by

$$N_A = \frac{D_{AB}}{RTz} (p_{Ai} - p_{A2})$$

$$= \frac{2.30 \times 10^{-5} (1.5 \times 10^4 - 5 \times 10^3)}{8314.51 \times 298 \times 0.15}$$

$$= \mathbf{6.19 \times 10^{-7} \ kmol/(m^2.s)}$$

... Ans.

Example 7.5 : *Hydrochloric acid (A) at 283 K diffuses through a thin film of water (B). The film is 4 mm thick. The concentration of A at location 1, on one boundary of the film, is 12 weight % (density ρ_1 = 1060.7 kg/m³) and at location 2, on the other boundary, is 4 weight % (density ρ_2 = 1020.15 kg/m³). The diffusivity of HCl in water is 2.5×10^{-9} m²/s. Calculate the flux of diffusion of A assuming water to be stagnant (i.e., non-diffusing).*

Solution : Molecular weight of HCl = 36.5, Molecular weight of H_2O = 18

At location 1 : Mole fraction of HCl (A)

$$x_{A1} = \frac{12/36.5}{12/36.5 + (100-12)/18}$$

$$= 0.063$$

We have : $x_{A1} + x_{B1} = 1$

$\therefore$ $x_{B1} = 1 - 0.063$

$$= 0.937$$

Average molecular weight at location-1 is

$$M_1 = \frac{100}{12/36.5 + (100-12)/18}$$

$$= 19.166 \text{ kg/kmol}$$

OR : In 100 kg of HCl – water mixture, there are 12 kg of HCl and 100 – 12 = 88 kg of H_2O at location-1 (as at location-1, HCl concentration is 12 weight %).

moles of HCl $= \dfrac{12}{36.5} = 0.3287$ kmol, moles of $H_2O = \dfrac{88}{18} = 4.8888$ kmol

Total moles of the mixture $= 0.3287 + 4.8888 = 5.2175$ kmol

$$\therefore \quad x_{A1} = \frac{\text{Moles of HCl}}{\text{Total moles}} = \frac{0.3287}{5.2175} = 0.063$$

$$M_1 = \text{kg of mixture / kmol of mixture}$$

$$= \frac{100}{5.2175} = 19.166 \text{ kg/kmol}$$

At location–2, the molefraction of HCl is

$$x_{A2} = \frac{4/36.5}{(4/36.5) + (100-4)/18}$$

$$= \frac{0.1096}{0.1096 + 5.3333} = 0.0201$$

$$x_{B2} = 1 - 0.0201$$

$$= 0.9799$$

Average molecular weight at location-2 is

$$M_2 = \frac{100}{4/36.5 + 96/18}$$

$$= 18.3728 \text{ kg/kmol}$$

$$C_{avg} = \frac{\rho_1/M_1 + \rho_2/M_2}{2}$$

$$= \frac{(1060.7/19.166) + (1020.15/18.3728)}{2}$$

$$= 55.4336 \text{ kmol/m}^3$$

$$x_{B,M} = (x_{B2} - x_{B1}) / \ln (x_{B2} / x_{B1})$$

$$= (0.9799 - 0.937) / \ln (0.9799/0.937)$$

$$= 0.958$$

$$z = 4 \text{ mm} = 0.004 \text{ m}$$

The flux equation for the diffusion A through a non-diffusing B is given by

$$N_A = \frac{D_{AB}\, C_{avg}}{x_{B,M} \cdot z} (x_{A1} - x_{A2})$$

$$= \frac{2.5 \times 10^{-9} \times 55.4336 (0.063 - 0.0201)}{0.958 \times 0.004}$$

Flux of HCl $= \mathbf{1.55 \times 10^{-6}}$ **kmol/(m².s)** ... **Ans.**

Example 7.6 : *Calculate the rate of diffusion of acetic acid (A) across a film of non-diffusing water (B) which is 1 mm thick at 290 K if the concentrations of acetic acid on the opposite sides of the film are 9% and 3% respectively. The densities of 9% and 3% solutions are 1012 kg/m³ and 1003.2 kg/m³ respectively. The diffusivity of acetic acid in water is 0.95 × 10⁻⁹ m²/s.*

Solution : $D_{AB} = 0.95 \times 10^{-9}$ m²/s, $z = 1$ mm $= 1 \times 10^{-3}$ m

At location 1, on one side of the film :

$\rho_1 = 1012$ kg/m², Acetic acid = 9% by weight

Mol. Wt. of acetic acid (CH_3COOH) = 60, Mol. Wt. of water (H_2O) = 18.

In 100 kg mixture there are 9 kg of acetic acid and 91 kg of water

$$\therefore \quad x_{A1} = \frac{\text{moles of A}}{\text{Total moles}} = \frac{9/60}{9/60 + (100 - 9)/18}$$

$$= \frac{0.15}{0.15 + 5.05}$$

$$= \frac{0.15}{5.2} = 0.0288$$

M_1 = Average molecular weight of solution $= \dfrac{\text{kg of solution}}{\text{kmol of solution}} = \dfrac{100}{5.2} = 19.21$ kg/kmol

Since

$$x_{A1} + x_{B1} = 1$$

$$x_{B1} = 1 - 0.0288 = 0.9712$$

At location-2, on the other / opposite side of the film :

Weight % of acetic acid = 3%, ρ_2 = 100.3 kg/m³

$$x_{A2} = \frac{3/60}{3/60 + (100-3)/18} = 0.0092$$

$\therefore$

$$x_{B2} = 1 - 0.0092 = 0.9908$$

and

$$M_2 = \frac{100}{3/60 + (100-3)/18} = 18.40$$

$$C_{avg.} = \frac{\rho_1/M_1 + \rho_2/M_2}{2}$$

$$= \frac{(1012/19.21) + (1003.2/18.40)}{2}$$

$$= 53.6 \text{ kmol/m}^3$$

$$x_{B,M} = (x_{B2} - x_{B1}) / \ln (x_{B2}/x_{B1})$$

$$= (0.9908 - 0.9712) / \ln (0.9908/0.9712)$$

$$= 0.980$$

The flux of acetic acid is

$$N_A = \frac{D_{AB} \cdot C_{avg}}{z . x_{B,M}} (x_{A1} - x_{A2})$$

$$= \frac{0.95 \times 10^{-9}}{1 \times 10^{-3} \times 0.98} (0.0288 - 0.0092)$$

$$= \mathbf{1.018 \times 10^{-6} \text{ kmol/(m}^2\text{.s)}}$$ **... Ans.**

Example 7.7 : *Hydrogen gas at 202.6 kPa (2 atm) and 298 K (25°C) flows through a pipe made of unvulcanised neoprene rubber with i.d. and o.d. 25 and 50 mm, respectively. The diffusivity of hydrogen through rubber is 1.8 × 10⁻⁶ cm²/s. Calculate the rate of loss of hydrogen by diffusion per meter length of the pipe.*

The solubility of hydrogen is 0.053 cm³ (NTP)/cm³·atm.

Solution : Consider 1 m of the pipe,

$$z = r_2 - r_1 = \frac{\text{o.d.} - \text{i.d.}}{2} = \frac{50-25}{2} = 12.5 \text{ mm} = 0.0125 \text{ m}$$

$$D_A = 1.8 \times 10^{-6} \text{ cm}^2/\text{s} = 1.8 \times 10^{-10} \text{ m}^2/\text{s}$$

$$L = 1 \text{ m}$$

$$A_{avg.} = \frac{2\pi L (r_2 - r_1)}{\ln (r_2/r_1)} = \frac{2\pi (1)(0.0125)}{\ln (0.025/0.0125)}$$

$$= 0.1133 \text{ m}^2$$

At 202.6 kPa (2 atm) hydrogen pressure, the solubility is

$$= 0.053 \times \frac{2}{1} = 0.106 \text{ cm}^3 \text{(NTP)/cm}^3$$

$$= 0.106 \text{ m}^3 \text{(NTP)/m}^3$$

$\therefore$ C_{A1} at the inner surface of the pipe $= \dfrac{0.106}{22.4} \times 1 = 4.73 \times 10^{-3}$ kmol H_2/m^3

At the outer surface, $C_{A2} = 0$

The rate of loss of hydrogen per 1 m length of the pipe is

$$w = N_A \cdot A_{avg.} = D_A \cdot A_{avg.} [C_{A1} - C_{A2}]/z$$

$$= 1.8 \times 10^{-10} \times 0.1133 \; (4.73 \times 10^{-3} - 0)/0.0125$$

$$= \mathbf{7.72 \times 10^{-2}\ kmol\ H_2/s\ per\ m}$$ **... Ans.**