CONVECTION

A large majority of practical applications of heat transfer in the chemical process industries involve either heat transfer to a fluid or heat transfer from a fluid.

The heat flow mechanisms in solid is by conduction, whereas the heat flow mechanism in fluids is due to convection. Convection is the transfer of heat from one point to another point within a fluid by mixing of hot and cold portions of the fluid. It occurs in the presence of a fluid medium.

Heat transfer by convection occurs as a result of the movement of a fluid on a macroscopic scale in the form of circulating currents. The circulating currents may be set up either by heat transfer process itself or by some external agency. It is restricted to the flow of heat in fluids.

In the case of convective heat transfer, the physical mixing of the hot and cold portions of a fluid is responsible for the flow of heat from one place to another with in the fluid.

There are two types of convection:

i) Free or Natural convection

ii) Forced convection

Free or Natural convection – is the mode of heat transfer when the circulating currents arise from the heat transfer process itself i.e, from the density difference due to temperature differences / gradients within a fluid mass.

Examples:

1. Heating of a vessel containing liquid by means of a gas flame situated underneath. – The liquid at the bottom of the vessel gets heated & expands & rises because its density has become less than that of the remaining liquid. Cold liquid of higher density takes its place & a circulating current is set up.

2. The flow of air across a heated radiator/ heating of a room by means of a steam radiator.
3. Heating of water using an immersed heating coil.
4. Transfer of heat from the surface of a pipe to ambient air.

Forced Convection - is the mode of heat transfer when the circulating currents are produced by an external agency such as an agitator in a reaction vessel, pump, fan or blower. Here the fluid motion is independent of density-gradients.

Examples:

1. Heat flow to a fluid pumped through a heated pipe.

In general, higher rates of heat transfer are obtained in forced convection as compared to natural convection owing to greater magnitude of circulation in the forced circulation.

In the case of convective heat transfer taking place from a surface to a fluid, the circulating currents die out in the immediate viscinity of the surface and a film of the fluid, free of turbulence, covers the surface. Heat transfer through this film takes place by thermal conduction. Since the thermal conductivity of most fluids is low, the main resistance to heat transfer lies in the film. Therefore, an increase in the velocity of the fluid over the surface results in improved heat transfer mainly because of reduction in the thickness of the film.

If the resistance to heat transfer is considered as lying within the film covering the surface, the rate of heat transfer Q is given by,

$$Q = kA\,\Delta T/x$$

The effective thickness x is not generally known and therefore the equation is usually rewritten ~~as~~ in the form.

$$Q = hA\,\Delta T$$

This is the basic equation for the rate of heat transfer by convection under steady-state conditions.

h - is called the film heat transfer co-efficient or surface coefficient or simply film co-efficient.

The value of 'h' depends upon the properties of the fluid within the film region, hence it is called the film heat transfer co-efficient. It also depends upon the various properties of the fluid, linear dimension of the surface and fluid velocity (i.e. the nature of flow).

Numerically, heat transfer coefficient (h) is the quantity of heat transformed in unit time through unit area at a temperature difference of one degree between the surface and surroundings. The SI unit of h is $W/m^2.K$. The term $1/h$ is called as the thermal resistance.

The process of transfer of heat from a hot fluid to cold fluid through a metal wall is very common in the chemical process industry. The heat transferred may be latent heat of a phase change, E.g., condensation, vaporisation etc. or may be sensible heat. In all process equipments, e.g., heater, cooler, condenser, reboiler etc. heat is transferred by both conduction and convection.

Individual and Overall Heat Transfer Coefficients:

Consider that a hot fluid is flowing through a circular pipe and a cold fluid is flowing on the outside of the pipe. The heat will flow from the hot fluid to the cold fluid through a series of resistances. Generally, the velocity of the fluid may be considered to be zero at the solid surface and it rapidly increases as we move away from the wall surface. It is found that even in turbulent flow where convective heat flow occurs from a surface to a fluid, the thin film of the fluid free of turbulence (viscous sublayer) exist at the wall surface. This thin film of fluid covering the surface is of great importance in determining the rate of heat transfer as all the heat reaching the bulk of cold fluid must pass through the film of fluid by conduction. The thermal conductivities of the fluids are very low so that the resistance offered by the film to the heat flow is very large even though the film is thin. Beyond this film the turbulence present brings about rapid equalisation temperature.

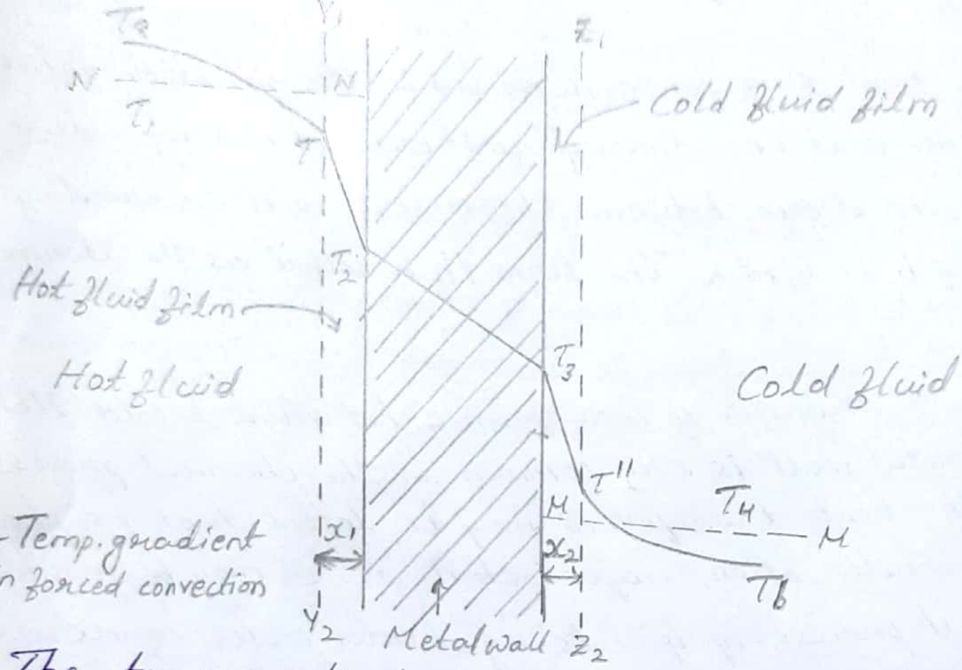


Fig- Temp. gradient in forced convection

The temp. gradients for the situation under consideration are shown in fig.

The dotted lines Y_1Y_2 and Z_1Z_2 represent the boundaries of thin films (hot and cold fluid films). The flow of fluid to the left of Y_1Y_2 and right of Z_1Z_2 is turbulent. The temperature gradient from the bulk of the hot fluid to the metal wall is represented by $T_a T' T_2$, where

T_a - is the maximum temp. of the hot fluid.

T' - is the temp. at the boundary b/n turbulent & viscous regions

T_2 - is the temp. at the actual interface b/n fluid + solid.

Similarly, the temp. gradient in the cold fluid is represented by the line $T_3 T'' T_b$. The heat transfer calculations for convenience, the average temp. of the fluid is usually used rather than the maximum temp. @ temp. at the outer surface of the film.

The average temperature (T_1) of the hot fluid is represented by the line marked NN and similiarly the average temp. (T_4) of the cold fluid is represented by the line marked MM.

The temp. change from T_1 to T_2 is taking place in the hot fluid film of thickness x_1. The rate of heat transfer through this film by conduction is given by,

$$Q = \frac{k_1 A_1 (T_1 - T_2)}{x_1} \quad \text{——— ①}$$

The effective film thickness x_1 depends upon the nature of flow and nature of the surface, and is generally not known. Therefore eqn. ① is rewritten as,

$$Q = h_i A_i (T_1 - T_2) \quad \text{——— ②}$$

where, h_i – is known as the inside heat transfer co-efficient (or) surface (or) simply film coefficient.

The film co-efficient is a measure of the rate of heat transfered for unit temp. difference and unit surface of heat transfer and it indicates the rate (or) speed of transfer of heat by a fluid having a variety of the physical properties under varying degrees of agitation. It has the units of $W/m^2.K$.

The overall resistance to heat flow from a hot fluid to a cold fluid is made up of three resistances in series. They are:

1. Resistance offered by the film of the hot fluid
2. Resistance offered by the metal wall &
3. Resistance offered by the film of the cold fluid.

The rate of heat transfer through metal wall is given by,

$$Q = \frac{k A_w (T_2 - T_3)}{x_w} \quad \text{——— ③}$$

where,

A_w – Log mean area of the pipe
x_w – Thickness of the pipe wall
k – Thermal conductivity of the pipe material

The rate of heat transfer through the cold fluid film is given by,

$$Q = h_o A_o (T_3 - T_4) \quad \text{---} ④$$

where,

h_o - is the outside film coefficient or individual heat transfer coefficient.

Eqn ② can be rearranged as,

$$T_1 - T_2 = \frac{Q}{h_i A_i} \quad \text{---} ⑤$$

Similarly, Eqn ③ & ④ can be rearranged as,

$$T_2 - T_3 = \frac{Q}{(k A_w / x_w)} \quad \text{---} ⑥$$

$$T_3 - T_4 = \frac{Q}{h_o A_o} \quad \text{---} ⑦$$

Adding equations ⑤, ⑥ & ⑦, we get

$$(T_1 - \cancel{T_2}) + (\cancel{T_2} - \cancel{T_3}) + (\cancel{T_3} - T_4) = Q\left[\frac{1}{h_i A_i} + \frac{1}{(k A_w / x_w)} + \frac{1}{h_o A_o}\right] \quad \text{---} ⑧$$

$$\therefore \quad (T_1 - T_4) = Q\left[\frac{1}{h_i A_i} + \frac{1}{(k A_w / x_w)} + \frac{1}{h_o A_o}\right] \quad \text{---} ⑨$$

where,

T_1 & T_4 are the average temps. of the hot & cold fluid respectively.

Therefore, an equation similar to eqn. ① in terms of overall heat transfer coefficients can be written as,

$$Q = U_i A_i (T_1 - T_4) \quad \text{---} ⑩$$

$$Q = U_o A_o (T_1 - T_4) \quad \text{---} ⑪$$

where, U_i & U_o - are the overall heat transfer coefficients based on the inside & outside area, respectively.

Eqn ⑩ & ⑪ state that the rate of heat transfer is a product of three factors, namely, the overall heat transfer coefficient, the area of heating surface and the temp. drop.

Eqn. ⑪ can be rearranged as.

$$(T_1 - T_4) = \frac{Q}{U_o A_o} \quad \text{---} ⑫$$

Comparing eqns. (9) & (12), we get

$$\frac{1}{U_o A_o} = \frac{1}{h_i A_i} + \frac{1}{(k A_w / x_w)} + \frac{1}{h_o A_o} \quad \text{—(13)}$$

$$\frac{1}{U_o} = \frac{1}{h_i}\left(\frac{A_o}{A_i}\right) + \frac{x_w}{k}\left(\frac{A_o}{A_w}\right) + \frac{1}{h_o} \quad \text{—(14)}$$

where,

A_o - Area of heat transfer based on the outside diameter i.e. outside area of the tube

A_i - Area of heat transfer based on the inside diameter i.e the inside area of the tube

We have,

$A_i = \pi D_i L$ (where L is the length of the pipe)

$A_o = \pi D_o L$

$$\therefore \frac{A_o}{A_i} = \frac{D_o}{D_i} \quad \text{—(15)}$$

Similarly, $\frac{A_o}{A_w} = \frac{D_o}{D_w}$ —(16)

where, D_w — Logarithmic mean diameter

$D_w = 2 \cdot r_m$

r_m - is the Logarithmic mean radius.

Substituting the values of area ratios in Eqn. (14) we get

$$\frac{1}{U_o} = \frac{1}{h_i}\left(\frac{D_o}{D_i}\right) + \frac{x_w}{k}\left(\frac{D_o}{D_w}\right) + \frac{1}{h_o} \quad \text{—(17)}$$

Similiarly,

$$\frac{1}{U_i} = \frac{1}{h_i} + \frac{x_w}{k}\left(\frac{D_i}{D_w}\right) + \frac{1}{h_o}\left(\frac{D_i}{D_o}\right) \quad \text{—(18)}$$

For thin walled tubes, the inside & outside radii are not much different from each other & hence the overall heat transfer coefficient U_o or U_i may be replaced simply by U & is written in terms of h_i, h_o etc.

$$\frac{1}{U} = \frac{1}{h_i} + \frac{x_w}{k} + \frac{1}{h_o} \quad \text{—(19)}$$

$$\frac{1}{U} = \frac{1}{h_i} + \frac{1}{(k/x_w)} + \frac{1}{h_o} \quad \text{—(20)}$$

(or)

$$U = \frac{1}{1/h_i + x_w/k + 1/h_o}$$

When the metal wall resistance is very small in comparison with the resistances of fluid films, then Eqn (19) reduces to

$$\frac{1}{U} = \frac{1}{ho} + \frac{1}{hi} \quad \text{— (21)}$$

$$\frac{1}{U} = \frac{ho + hi}{ho.hi} \quad \text{— (22)}$$

Fouling Factor:

When the heat transfer equipment is put into service, after sometime, scale, dirt and other solids deposit on both sides of the pipe wall, providing two more resistances to the heat flow. The added resistances must be taken into account in the calculation of the overall heat transfer coefficient. The additional resistances reduce the original value of U. & the required amount of heat is no longer transferred by the original heat transfer surface. Hence, heat transfer equipments are designed by taking into account the deposition of dirt & scale by introducing a resistance Rd known as the fouling factor. (It is a thermal resistance due to scale).

Eqn (19) then becomes,

$$\frac{1}{U} = \frac{1}{hi} + \frac{x_w}{k} + \frac{1}{ho} + Rd \quad \text{— (23)}$$

The overall heat transfer coefficient calculated by taking into account Rd is known as the design (or) dirty overall heat transfer co-efficient & the one calculated without taking into account the term Rd is known as the clean overall heat transfer coefficient. [eqn (19)].

The dirty overall heat transfer coefficient [eqn (23)] is always less than the clean overall heat transfer coefficient. Fouling factor Rd is composed of Rdi, the dirt factor for an inner fluid at the inside surface of a pipe & Rdo, the dirt factor for an outer fluid at the outside surface of the pipe; $Rd = Rd_i + Rd_o$. The dirt (or) fouling factor Rd has the units of $m^2.K/w$.

Classification of Individual Heat Transfer Co-efficients:

The problem of predicting the rate of heat transfer from one fluid to another through a metal wall reduces essentially to the problem of predicting the numerical values of the film co-efficients of the fluids involved in the overall process. In practice, we come across a variety of cases & each one must be considered separately. The cases encountered in practice which are considered are given below:

a. Heat transfer to (or) from fluids inside tubes, without phase change

b. Heat transfer to (or) from fluids outside tubes, without phase change

c. Heat transfer from condensing vapours.

d. Heat transfer to boiling liquids.

Heat transfer to fluids without phase change

In most of heat exchange applications, heat is transferred between fluid streams without phase change in the fluids. Examples are:

i. Heat exchange between hot and cold petroleum streams.

ii. Heat transfer from a stream of hot gas to cooling water.

iii. Cooling of a hot liquid stream by cooling water/air.

In such cases, the two streams are separated by a metal wall that forms the heat transfer surface. The heat transfer surface may consist of tubes, (or) other channels of constant cross section. etc

A fluid being heated (or) cooled in a heat exchange equipment may be flowing in laminar flow, in turbulent flow (or) in the transistion region between laminar & turbulent flow.

Flow Arrangements in Heat Exchangers:

A heat exchanger is a device used to exchange heat b/n two fluids that are at different temps.

There are 3 basic flow arrangements:

1. Parallel flow / Co-current flow
2. Counter-current flow &
3. Cross flow.

Consider a double pipe heat exchanger wherein a hot fluid is flowing through the inside pipe and a cold ~~pipe~~ fluid is flowing through the annular space. for the explation of flow arrangements.

Co-current / parallel flow – is the flow when both fluids flow in the same direction from one end of a heat exchanger to the other end through the heat exchanger.

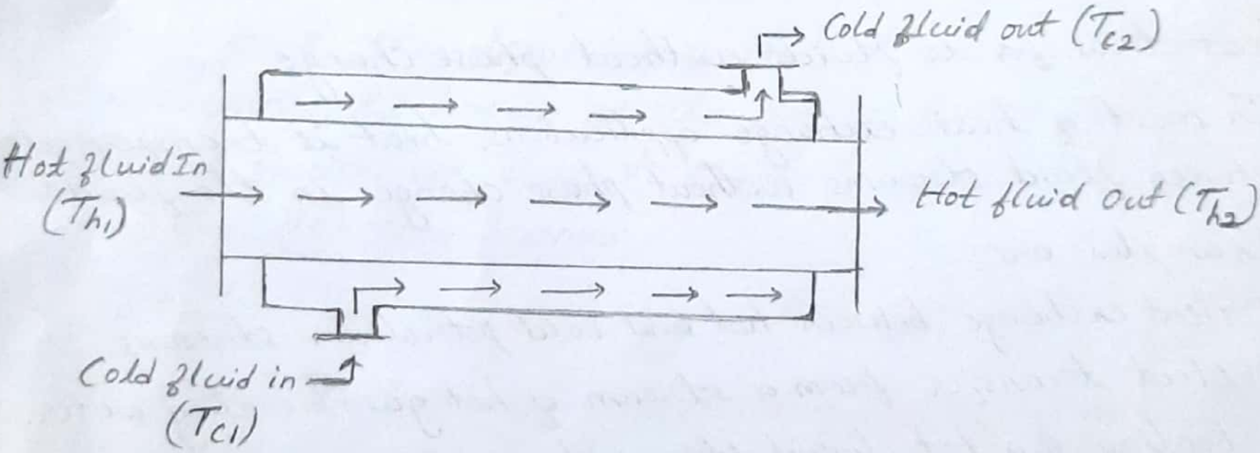
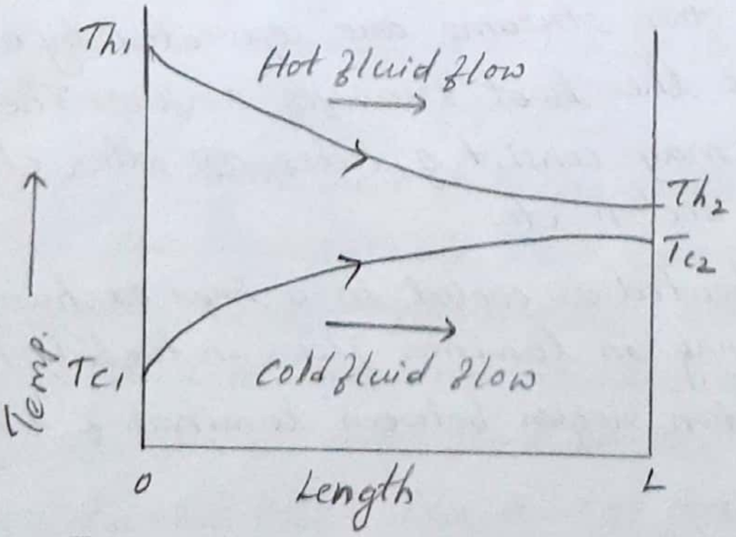


Fig – Co-current / Parallel flow in heat exchanger



$\Delta T_1 = T_{h_1} - T_{c_1}$

$\Delta T_2 = T_{h_2} - T_{c_2}$

Fig – Temp - length curve for parallel flow

Counter-current flow - is the flow when the fluids are flowing through a heat exchanger in opposite directions with respect to each other (i.e. one fluid enters at one end of the heat exchanger and the other fluid enters at the opposite end of the heat exchanger).

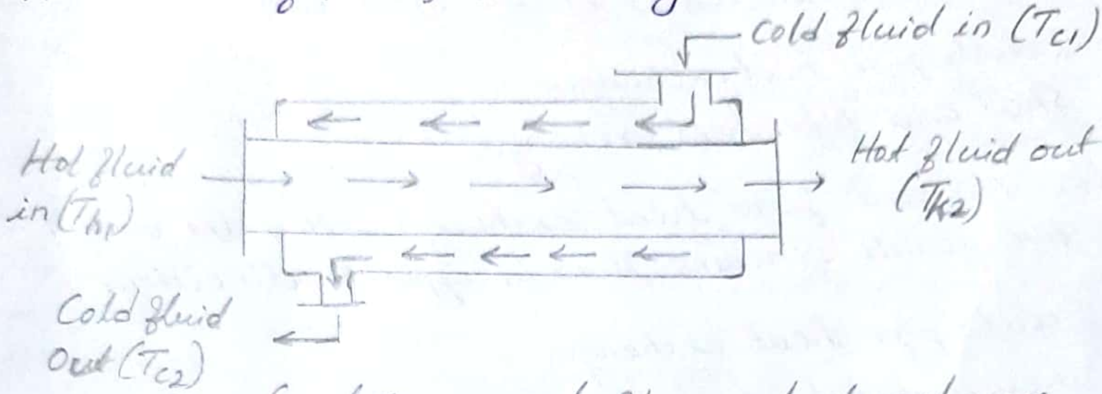
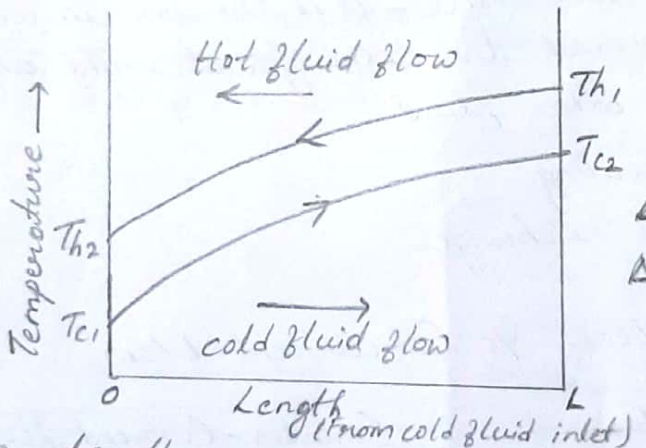


Fig - Counter-current flow in heat exchanger



$\Delta T_1 = T_{h_2} - T_{c_1}$

$\Delta T_2 = Th_1 - T_{c_2}$

Fig - Temp. - length curve for counter-current flow

Cross-Flow - is the flow of fluid when the fluids are directed at right angles to each other through a heat exchanger.

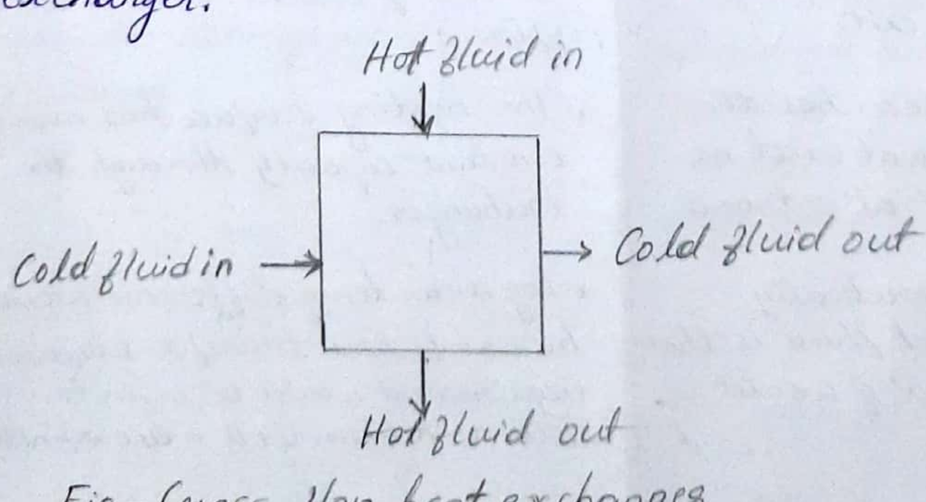


Fig - Cross-flow heat exchanger

Classification of heat exchangers according to the flow arrangement:

1. Parallel flow heat exchanger - It is the one in which two fluid streams enter at one end, flow through it in the same direction and leaves at the other end.

   Ex. - Double pipe heat exchanger
   Shell and tube heat exchanger

2. Counter-current flow heat exchanger - It is the one in which two fluids streams flow in opposite directions.

   Ex - Double pipe heat exchanger
   Shell and tube heat exchanger.

3. Cross-flow heat exchanger - It is the one in which one fluid moves through the exchanger at right angles to the flow path of the other fluid.

   Ex - Air or gas heating
   - Plate-fin heat exchanger.

Counter-current flow V/s Co-current Flow

Co-Current Flow	Counter-Current flow.
* The temp. gradient in case of parallel is maximum at the entrance and continuously decreases towards the exit.	* Temp. gradient is fairly constant over the length of heat exchanger in case of counter-current flow.
* The heating surface has the capacity much less at exist as compared to that at entrance.	* The heating surface has nearly constant capacity through the exchanger.
* Lowest temp. theoretically attainable by a hot fluid is that of the outlet temp. of a cold fluid.	* Log mean temp. difference would be zero & heat transfer surface requirement would be infinite. [$Q = U.A.\Delta T_{lm}$, U & A - are infinite]
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Co-current Flow	Counter-current Flow
With a parallel flow arrangement, it is not possible to bring the hot fluid temp. below outlet temp. of the cold fluid. & thus has marked effect of on the ability of heat exchanger to recover heat. In parallel flow heat exchanger, heat transfer is restricted by the cold fluid outlet temp. rather than the cold fluid inlet temp and hence the counter flow arrangement is very common in heat transfer apparatus. Parallel flow arrangement is used whenever it is necessary to limit the maximum temp. of the cooler fluid.	In counter-current flow, it is possible for the cooling liquid to leave at a high temp. than the heating fluid, & one of the greatest advantages of counter flow is that it is possible to extract a higher proportion of the heat content of the heating fluid. For the same terminal temps, it is imp. to note that the value of logarithmic mean temp. difference for counter flow is appreciably greater than the value for co-current flow. For the same terminal temps. & same heat load, heat transfer area required for a counter flow heat exchanger is less than it for a parallel flow heat exchanger. The rate of heat transfer in a counter-current flow heat exchanger is more than it in a co-current flow arrangement. exchanger.
In a counter-current flow, the temp. difference will show less variation throughout the heat exchanger.	In a co-current flow, temps of the two streams progressively approach one another. Temp. difference will show a more variation throughout the heat exchanger.

Range -

It is the actual rise or fall of temperature of a fluid. If T_{h_1} is the inlet temperature and T_{h_2} is the outlet temp. of a hot fluid, then $T_{h_1} - T_{h_2}$ is the range for the hot fluid.

Similiarly, if T_{c_1} and T_{c_2} are the inlet and outlet temps of a cold fluid, then $T_{c_2} - T_{c_1}$ is the range for the cold fluid

Approach:

It is the terminal point temp. difference between hot and cold fluids. Thus, T_{h_1} is the inlet temp. of a hot fluid and T_{c_2} is the outlet temp. of a cold fluid at one end of a counter-current heat transfer apparatus, then $T_{h_1} - T_{c_2}$ is called the approach.

In a condenser, where the vapour entering it is not superheated and condensate is not subcooled below its boiling temp, the temp throughout the shell side of the condenser is constant as the pressure in the shell space is constant. The temp-length curve for the condenser is shown in fig.

The temp. of the coolant used continuously increases as it passes through the tubes of condenser.

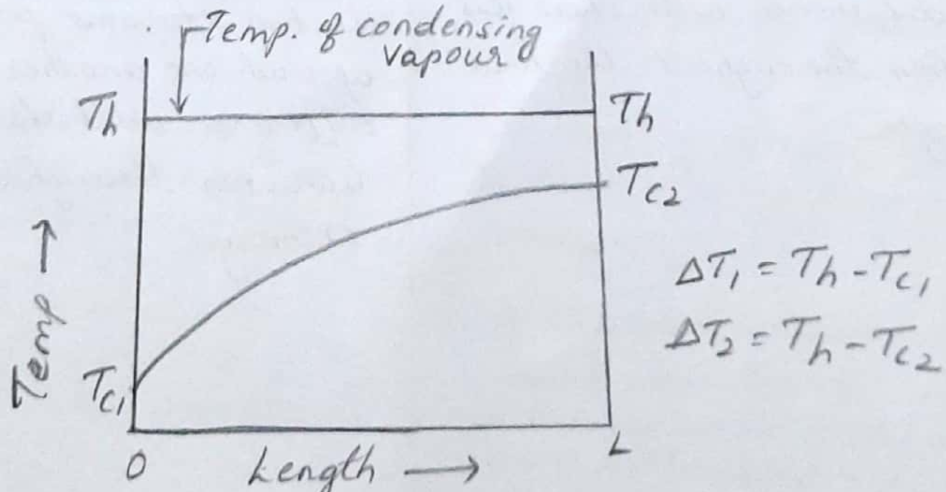

Fig - Temp - length curve for a condenser

Log Mean Temperature Difference [LMTD or ΔT_{lm} or ΔT_{LMTD}]

The temp. difference between two fluids is not constant in a heat exchanger. It changes along the length of the heat exchanger. The arithmetic temp. difference is less accurate way to calculate the mean temp. difference, thus the logarithmic mean temp. difference is used to determine the temp. driving force [LMTD] for heat transfer in heat exchanger. The LMTD is a logarithmic average of the temp. difference between the hot and cold fluids at each end of the heat exchanger. LMTD is always less than the arithmetic mean temp. difference.

The heat transfer flux is directly proportional to a driving force. The driving force for heat flow is taken as $T_h - T_c$, where T_h and T_c are the temps. of hot and cold fluids respectively. As the term $\Delta T = T_h - T_c$ varies along the length of heat exchanger, the flux also varies over the entire length. Consider a differential element of area dA through which a differential amount of heat dQ flows under the driving force of ΔT. Then, dQ/dA is related by the relation given below:

$$\frac{dQ}{dA} = U.(\Delta T) = U.\ (T_h - T_c) \quad \text{—— ①}$$

where, U – is the overall heat transfer co-efficient.

Eqn. ① needs to be integrated for its application to the entire area of heat transfer.

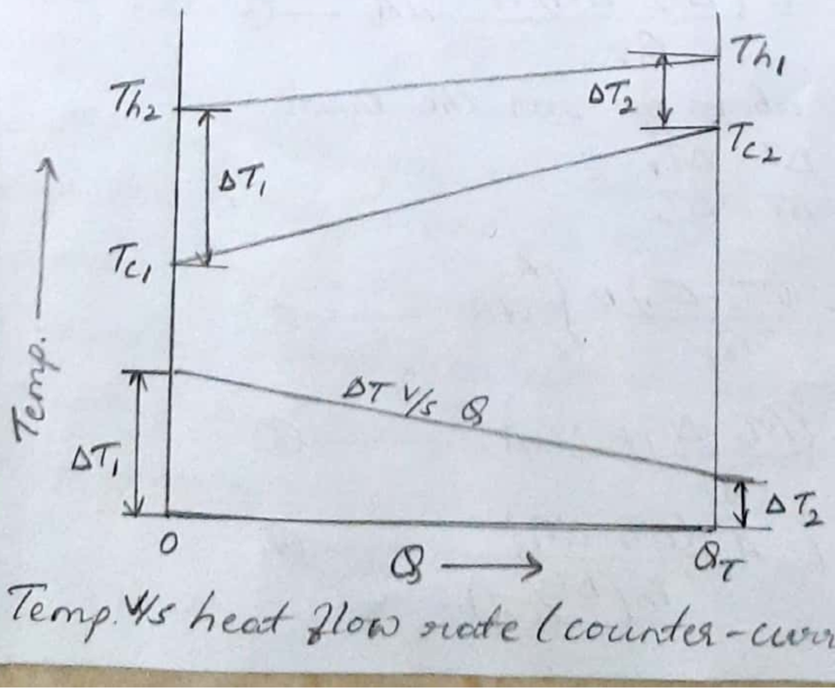

Fig - Temp. V/s heat flow rate (counter-current flow)

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Assumptions to be made for the integration of eqn ①

1. Overall heat transfer coefficient U is constant throughout the exchanger.
2. Specific heats of the hot and cold fluids are constant.
3. Heat flow to and from ambient is negligible &
4. The flow is steady and may be parallel or countercurrent type.
5. Temps. of both the fluids are uniform over a given cross-section and may be represented by their bulk temperatures.

Based upon assumptions ② & ④, we get straight lines if T_c & T_h are plotted against Q & also a plot of ΔT v/s Q is a straight line. Such plots are shown fig.

The slope of the plot of ΔT v/s Q is constant.

$$slope = \frac{d(\Delta T)}{dQ} = \frac{\Delta T_2 - \Delta T_1}{Q_T} \quad \text{——— ②}$$

where,

ΔT_1 & ΔT_2 are the terminal temp. differences (i.e. approaches)

& Q_T – is the rate of heat transfer in the entire heat exchanger.

Substituting the value of dQ from Eqn. ① into eqn ② gives,

$$\frac{d(\Delta T)}{U . \Delta T . dA} = \frac{\Delta T_2 - \Delta T_1}{Q_T} \quad \text{——— ③}$$

Rearranging eqn. ③

$$\frac{d(\Delta T)}{\Delta T} = \frac{(\Delta T_2 - \Delta T_1) . U}{Q_T} . dA \quad \text{——— ④}$$

Integrating the above eqn. over the limits;

At $A = 0$, $\Delta T = \Delta T_1$

& At $A = A$, $\Delta T = \Delta T_2$

$$\int_{\Delta T_1}^{\Delta T_2} \frac{d(\Delta T)}{\Delta T} = \frac{(\Delta T_2 - \Delta T_1) U}{Q_T} \int_0^A dA \quad \text{——— ⑤}$$

$$\ln\left(\frac{\Delta T_2}{\Delta T_1}\right) = \frac{(\Delta T_2 - \Delta T_1)}{Q_T} U . A \quad \text{——— ⑥}$$

$$Q_T = U . A . \frac{(\Delta T_2 - \Delta T_1)}{\ln(\Delta T_2 / \Delta T_1)} \quad \text{——— ⑦}$$

The heat transfer rate in the entire heat exchanger can be denoted by symbol Q.

$$Q = U.A.\frac{(\Delta T_2 - \Delta T_1)}{\ln\left(\frac{\Delta T_2}{\Delta T_1}\right)} \quad \text{——} (8)$$

$$Q = U.A.\Delta T_{lm} \quad \text{——} (9)$$

$$\Delta T_{lm} = \frac{(\Delta T_2 - \Delta T_1)}{\ln\left(\frac{\Delta T_2}{\Delta T_1}\right)} \quad \text{——} (10)$$

ΔT_{lm} is referred to as the logarithmic mean or the log mean temp. difference (LMTD).

In counter-current flow, the hot end approach ΔT_2 may be less than the cold end approach ΔT_1 so the subscripts may be interchanged for eliminating confusion due to negative (–) signs.

LMTD for counter current flow:

$$T_{c_1} \xrightarrow{\text{Cold fluid}} T_{c_2}, \quad \Delta T_1 = T_{h_2} - T_{c_1}$$

$$T_{h_2} \xleftarrow{\text{Hot fluid}} T_{h_1}, \quad \Delta T_2 = T_{h_1} - T_{c_2}$$

LMTD for parallel/co-current flow:

$$T_{c_1} \xrightarrow{\text{Cold fluid}} T_{c_2}, \quad \Delta T_1 = T_{h_1} - T_{c_1}$$

$$T_{h_1} \xrightarrow{\text{Hot fluid}} T_{h_2}, \quad \Delta T_2 = T_{h_2} - T_{c_2}$$

$$LMTD = \Delta T_{lm} = \frac{\Delta T_1 - \Delta T_2}{\ln\left(\frac{\Delta T_1}{\Delta T_2}\right)}$$

In a 1-2 heat exchanger, the tube side fluid flows twice through the exchanger and the shell side fluid flows once through it. Consequently, there is a combination of co-current and counter-current flow patterns in such multipass units. The rate of heat transfer in these units using corrected LMTD is given by,

$$Q = U.A.F_T(\Delta T_{lm}) \quad \text{—} (11)$$

where,

F_T - Correction factor for LMTD & is usually taken as 0.85 - 0.90.

ΔT_{lm} - Log mean temp difference calculated for countercurrent flow from terminal temps.

For cross-flow arrangement, true ΔT_{lm} is calculated as:

ΔT_{lm} for cross flow = $F_T \cdot \Delta T_{lm}$ for countercurrent flow

where,

F_T - is the correction factor for LMTD.

Application of Dimensional Analysis to Heat Transfer by Convection:

An equation for predicting the film co-efficient or the surface co-efficient in any particular case must include all properties of a fluid and conditions of its flow that affect the problem. In a particular case, the factors that might be considered are diameter of pipe, the velocity of flowing fluid, density, viscosity, thermal conductivity, specific heat of fluid, etc. The dimensional analysis is one of the most useful methods to assemble these variables/factors into an equation. This method results in arranging the variables into various dimensionless groups.

It is found that an equation for the film co-efficient for heat transfer to or from a flowing fluid without phase change will probably of the form:

$$N_{Nu} = f\left(N_{Re}, N_{Pr}, N_{Gr}, \frac{L}{D}\right)$$

Dimensional analysis is a method of correlating a number of variables into a single equation, clearly states an effect of the variables. When a particular physical quantity is influenced by a number of factors, then it is impossible to determine their effects by the experimental analysis wherein the variables are arranged in dimensionless groups which are significantly less than the number of factors. Dimensional analysis finds application in fluid flow, heat transfer etc.

Convection Heat Transfer Coefficients

The rate of heat transfer q by convection from the surface of a body to the surrounding fluid is given by,

$$q = h A (T_s - T_\infty)$$

where, T_s - temp. of the surface
T_∞ - temp of the fluid
A - surface area of the body
h - convection heat transfer coefficient.

The general methods for the estimation of convection heat transfer co-efficients are:

1. Dimensional analysis coupled with experimental data.
2. Exact mathematical analysis of the boundary layer equations.
3. Approximate analysis of the boundary layer equations.
4. Analogy between heat, mass and momentum transfer.

Dimensional Analysis.

It is a method of deducing logical groupings of the variables involved in a process. This method combines the variables into dimensionless groups which allow us to extend the range of applicability of experimental data. In practice, the convection heat transfer co-efficients are calculated from empirical equations which are obtained by correlating the experimental data in terms of dimensionless groups.

The limitations of the dimensional analysis are

a. It is necessary to know the variables which influence the phenomena.

b. It does not provide any knowledge of the mechanism.

Applications of Dimensional Analysis.

a. When equations governing a process are known and saluable dimensional analysis suggests logical grouping of quantities for presenting the results.

b. When the mathematical equations governing certain process are unknown or two complex, dimensional analysis lays the foundation of an efficient experimental program for obtaining the results by reducing the number of variables requiring investigation and by indicating a possible form of the semier mperical co-relations that may be formulated.

It should be born in mind however that dimensional analysis by itself can't provide quantitative answers and then can't be a substitute for the exact or the approximate mathematical solutions.

Buckingham π-theorem

The Buckingham π-theorem is used to determine the number of independent dimensionless groups which are necessary to describe the phenomenon in a mathematical expression. The Buckingham π-theorem states that the number of independent dimensionless groups which can be formed by combining the physical variables of a problem is equal to the total number of physical quantities (n) minus the number of primary dimensions (m) needed to express the dimensional formulae of the n physical quantities.

The equation which expresses the relationship along the variables is of the form.

$$F(\pi_1, \pi_2, \pi_3 \dots) = 0$$

where, π_1, π_2, π_3 .... are the dimensionless groups.

Empirical Correlation for forced convection

The convection heat transfer co-efficients for flow of a fluid through a tube have been experimentally determined. Using the Buckingham π-theorem develop an expression for correlating the experimental data.

Solution - Let us list the physical quantities which influence the convective heat transfer co-efficient are.

$$f(D, V, \rho, \mu, k, C_p, h) = 0$$

Physical quantity	Symbol	Dimensional Formula	
1. Tube diameter	D	m	L
2. Velocity of fluid	u, ~~m/s~~	m/s	LT^{-1}
3. Density of fluid	ρ	kg/m^3	ML^{-3}
4. Viscosity of fluid	μ	kg/m.s	$ML^{-1}T^{-1}$
5. Thermal conductivity of fluid	k	W/m.K $\frac{kg.m^2/s}{m^2.K}$	$MLT^{-3}\theta^{-1}$
6. Specific heat of fluid	C_p	J/kg.K $J \cdot \frac{kg.m^2/s^2}{kg.K}$	$L^2T^{-2}\theta^{-1}$
7. Convection heat transfer co-efficient	h	$W/m^2.K$ $\frac{kg.m^2/s^3}{m^2.K}$	$MT^{-3}\theta^{-1}$

The no. of physical quantities, $n = 7$

The no. of primary dimensions needed to express the physical quantity, $m = 4$.

$\therefore$ The no. of dimensionless groups $= n - m$
$= 7 - 4$
$= 3$

Let us denote the dimensionless groups as π_1, π_2, & π_3.

$$f(\pi_1, \pi_2, \pi_3) = 0$$

To determine the dimensionless groups, let us write π as a product of the physical quantities, each raised to an unknown power as,

$$\pi_1 = D^{a_1} V^{b_1} \rho^{c_1} K^{d_1} h$$

$$\pi_2 = D^{a_2} V^{b_2} \rho^{c_2} K^{d_2} C_p$$

$$\pi_3 = D^{a_3} V^{b_3} \rho^{c_3} K^{d_3} \mu$$

Solving the above, we get

$$\pi_1 = M^0 L^0 T^0 \theta^0 = [L]^{a_1} [LT^{-1}]^{b_1} [ML^{-3}]^{c_1} [MLT^{-3}\theta^{-1}]^{d_1} [MT^{-3}\theta^{-1}]$$

For $M: 0 = c_1 + d_1 + 1 \Rightarrow c_1 - 1 + 1 \Rightarrow \boxed{c_1 = 0}$

$L: 0 = a_1 + b_1 - 3c_1 + d_1 \Rightarrow a_1 + 0 - 0 - 1 \Rightarrow \boxed{a_1 = 1}$

$T: 0 = -b_1 - 3d_1 - 3 \Rightarrow -b_1 + \cancel{3} - \cancel{3} \Rightarrow \boxed{b_1 = 0}$

$\theta: 0 = -d_1 - 1 \Rightarrow \boxed{d_1 = -1}$

$$\pi_1 = D^1 V^0 \rho^0 K^{-1} h$$

$$\boxed{\pi_1 = \frac{Dh}{k} = N_{Nu}}$$

$$\pi_2 = D^{a_2} V^{b_2} \rho^{c_2} K^{d_2} C_p$$

$$\pi_2 = M^0 L^0 T^0 \theta^0 = [L]^{a_2} [LT^{-1}]^{b_2} [ML^{-3}]^{c_2} [MLT^{-3}\theta^{-1}]^{d_1} [L^2 T^{-2} \theta^{-1}]$$

For, $M: 0 = c_2 + d_2 \Rightarrow c_2 + (-1) \Rightarrow \boxed{c_2 = 1}$

$L: 0 = a_2 + b_2 - 3c_2 + d_2 + 2 \Rightarrow a_2 + \cancel{1} - 3 - \cancel{1} + 2 \Rightarrow \boxed{a_2 = 1}$

$T: 0 = -b_2 - 3d_2 - 2 \Rightarrow -b_2 + 3 - 2 \rightarrow \boxed{b_2 = 1}$

$\theta: 0 = -d_2 - 1 \Rightarrow \boxed{d_2 = -1}$

$$\pi_2 = D^1 V^1 \rho^1 K^{-1} C_p$$

$$= \frac{D.V.\rho.\ C_p}{K} \qquad = m \times \frac{m}{s} \times \frac{kg}{m^3} = \frac{kg}{m \cdot s} = \mu.$$

$$\boxed{\pi_2 = \frac{\mu C_p}{k} = N_{pr}}$$

$$\pi_3 = D^{a_3} V^{b_3} \rho^{c_3} k^{d_3} \mu$$

$$\pi_3 = M^0 L^0 T^0 \theta^0 = [L]^{a_3} [LT^{-1}]^{b_3} [ML^{-3}]^{c_3} [MLT^{-3}\theta^{-1}]^{d_3} [ML^{-1}T^{-1}]$$

For, $M: 0 = c_3 + d_3 + 1 \Rightarrow c_3 + 0 + 1 \Rightarrow \boxed{c_3 = -1}$

$L: 0 = a_3 + b_3 - 3c_3 + d_3 - 1 \Rightarrow a_3 - 1 + 3 + 0 - 1 \Rightarrow \boxed{a_3 = -1}$

$T: 0 = -b_3 - 3d_3 - 1 \Rightarrow -b_3 - 3(0) - 1 \Rightarrow \boxed{b_3 = -1}$

$\theta: 0 = -d_3 \Rightarrow \boxed{d_3 = 0}$

$$\pi_3 = D^{-1} V^{-1} \rho^{-1} k^0 \mu$$

$$\boxed{\pi_3 = \frac{\mu}{DV\rho} = \frac{DV\rho}{\mu} = N_{Re}}$$

then,

$$f(\pi_1, \pi_2, \pi_3) = 0$$

$$f\left(\frac{Dh}{k}, \frac{C_p\mu}{k}, \frac{DV\rho}{\mu}\right) = 0$$

$$f(N_{Nu}, N_{Pr}, N_{Re}) = 0$$

$$N_{Nu} = \phi(N_{Re}) \cdot \psi(N_{Pr})$$

$$\boxed{N_{Nu} = C (N_{Re})^m (N_{Pr})^n}$$

where, $C, m, \& n$ – are constants

Emperical correlation for free (or) Natural Convection

Natural convection is caused due to Buoyancy force which is the effect of decrease in density due to heating. Consider the fluid which is heated and forced up due to Buoyancy force.

Let, ρ_0 – Density of fluid when it was cold

ρ – Density of fluid when it is heated

V – Volume of fluid going/rising up.

Gravitational force acting downwards $= V \cdot \rho \cdot g$.

Buoyant force acting upwards $= V \rho_0 \cdot g$

$\therefore$ The net force acting / causing the flow in upward direction per unit mass of the fluid.

$$F = \frac{(\rho_0 - \rho)}{\rho V} \cdot g \cdot V = \frac{(\rho_0 - \rho)}{\rho} \cdot g \quad — ①$$

If β is the co-efficient of thermal expansion then,

$$\rho_0 = \rho[1 + \beta(\Delta\theta)] \quad — ②$$

$\Delta\theta$ - temp. difference between hot and cold fluid.

Substituting eqn ② in ① we get,

$$F = \left[\frac{\rho_0}{\rho} - 1\right] \cdot g$$

$$= \left[\frac{\rho[1 + \beta(\Delta\theta)]}{\rho} - 1\right] g$$

$$\boxed{F = \beta g \Delta\theta} \quad — ③$$

The rate of heat transfer by free convection to an incompressible fluid travelling at constant mass rate is influenced by the following factors/physical quantities.

Physical quantity	Symbol		Dimensional Formula
1. ~~Tube diameter~~ Density of fluid	ρ	kg/m^3	ML^{-3}
2. Characteristic length	L	m	L
3. Viscosity of fluid	μ	$kg/m.s$	$ML^{-1}T^{-1}$
4. Thermal conductivity	K	$W/m.K$ $\frac{kg.m^2/s^3}{m.K}$	$MLT^{-3}\theta^{-1}$
5. Specific heat of fluid	C_p	$J/kg.K$ $\frac{kg.m^2/s^2}{kg.K}$	$L^2T^{-2}\theta^{-1}$
6. Heat transfer coefficient	h	$W/m^2.K$ $\frac{kg.m^2/s^3}{m^2.K}$	$MT^{-3}\theta^{-1}$
7. Buoyant force	F		LT^{-2}

For finding out the heat transfer coefficient consider the following functional groups.

$$h = f(\rho, L, \mu, K, C_p, (\beta g \Delta T))$$

$$f = (\rho, L, \mu, K, h, C_p, (\beta g \Delta T)) = 0$$

The no. of physical quantities, $n = 7$

The no. of fundamental dimensions, $m = 4$

$\therefore$ The no. of dimensionless groups, $\pi = n - m$
$= 7 - 4$
$\pi = 3$

The dimensionless groups are π_1, π, π_3.

$$f = (\pi_1, \pi_2, \pi_3) = 0$$

$\pi_1 = \rho^{a_1} L^{b_1} \mu^{c_1} K^{d_1} h$

$\pi_2 = \rho^{a_2} L^{b_2} \mu^{c_2} K^{d_2} C_p$

$\pi_3 = \rho^{a_3} L^{b_3} \mu^{c_3} K^{d_3} (\beta \cdot g \cdot \Delta\theta)$

Solving above equations,

$\pi_1 = \rho^{a_1} L^{b_1} \mu^{c_1} K^{d_1} h = 0$

$\pi_1 = M^0 L^0 T^0 \theta^0 = [ML^{-3}]^{a_1} [L]^{b_1} [ML^{-1}T^{-1}]^{c_1} [MLT^{-3}\theta^{-1}]^{d_1} [MT^{-3}\theta^{-1}]$

For $M: 0 = a_1 + c_1 + d_1 + 1 \rightarrow \boxed{a_1 = 0}$

$L: 0 = -3a_1 + b_1 - c_1 + d_1 \rightarrow \boxed{b_1 = 1}$

$T: 0 = c_1 - 3d_1 - 3 \Rightarrow \boxed{c_1 = 0}$

$\theta: 0 = -d_1 - 1 \Rightarrow \boxed{d_1 = -1}$

$\pi_1 = \rho^0 L^1 \mu^0 K^{-1} h^1$

$$\boxed{\pi_1 = \frac{hL}{K} = N_{Nu}}$$

$\pi_2 = \rho^{a_2} L^{b_2} \mu^{c_2} K^{d_2} C_p = 0$

$\pi_2 = M^0 L^0 T^0 \theta^0 = [ML^{-3}]^{a_2} [L]^{b_2} [ML^{-1}T^{-1}]^{c_2} [MLT^{-3}\theta^{-1}]^{d_2} [L^2 T^{-2} \theta^{-1}]$

For, $M: 0 = a_2 + c_2 + d_2 \Rightarrow \boxed{a_2 = 0}$

$L: 0 = -3a_2 + b_2 - c_2 + d_2 + 2 \Rightarrow \boxed{b_2 = 0}$

$T: 0 = -c_2 - 3d_2 - 2 \Rightarrow \boxed{c_2 = 1}$

$\theta: 0 = -d_2 - 1 \Rightarrow \boxed{d_2 = -1}$

$\pi_2 = \rho^0 L^0 \mu^1 K^{-1} C_p$

$$\boxed{\pi_2 = \frac{\mu C_p}{K} = N_{Pr}}$$

$$\pi_3 = \rho^{a_3} L^{b_3} \mu^{c_3} K^{d_3} (\beta \cdot g \cdot \Delta\theta) = 0$$

$$\pi_3 = M^0 L^0 T^0 \theta^0 = [ML^{-3}]^{a_3} [L]^{b_3} [ML^{-1}T^{-1}]^{c_3} [MLT^{-3}\theta^{-1}]^{d_3} [LT^{-2}]$$

For, $M: 0 = a_3 + c_3 + d_3 \Rightarrow \boxed{a_3 = 2}$

$L: 0 = -3a_3 + b_3 - c_3 + d_3 + 1 \Rightarrow \boxed{b_3 = 3}$

$T: 0 = -c_3 - d_3 - 2 \Rightarrow \boxed{c_3 = -2}$

$\theta: 0 = -d_3 \Rightarrow \boxed{d_3 = 0}$

$$\pi_3 = \rho^2 L^3 \mu^{-2} K^0 (\beta \cdot g \cdot \Delta\theta)$$

$$\boxed{\pi_3 = \frac{\rho^2 L^3}{\mu^2} (\beta \cdot g \cdot \Delta\theta)} = \frac{L^3 (\beta \cdot g \cdot \Delta\theta)}{\nu^2} = N_{Gr}$$

Since, $\frac{\mu}{\rho} = \nu$, kinematic viscosity

$$\pi_1 = \phi(\pi_2)\,\psi(\pi_3)$$

$$\frac{hL}{K} = \phi\left[\frac{\mu C_p}{K}\right] \psi\left[\frac{L^3(\beta \cdot g \cdot \Delta\theta)}{\nu^2}\right]$$

$$\boxed{N_{Nu} = \phi(N_{Pr})\,\psi(N_{Gr})}$$

In case of pipe, linear dimension is D (inside diameter) & above equation becomes

$$\frac{hL}{K} = \phi\left[\frac{\mu C_p}{K}\right] \psi\left[\frac{D^3(\beta \cdot g \cdot \Delta\theta)}{\nu^2}\right]$$

$$N_{Nu} = \phi[N_{Pr}] \cdot \psi[N_{Gr}]$$

OR

$$N_{Nu} = C[N_{Pr}]^m [N_{Gr}]^n$$

where, C, m & n are constants.

The above equation are used to correlate the experimental data.

Physical significance of the ~~above groups~~ dimensionless groups.

1. Reynolds number, $N_{Re} = \dfrac{\text{Inertia force}}{\text{Viscous Force}} = \dfrac{DV\rho}{\mu}$

2. Grashof Number, $N_{Gr} = \dfrac{\text{Buoyancy forces} \times \text{Inertia Forces}}{(\text{Viscous Forces})^2}$

$$N_{Gr} = \frac{D^3 \beta g \Delta T}{\nu^2}$$

3. Prandtl Number, $N_{Pr} = \dfrac{\text{Molecular diffusivity of momentum}}{\text{Molecular diffusivity of heat}}$

$$N_{Pr} = \frac{C_p \mu}{k}$$

4. Nusselt Number, $N_{Nu} = \dfrac{\text{Wall heat transfer rate}}{\text{Heat transfer by conduction}}$

$$N_{Nu} = \frac{hD}{k}$$

The relationship:

$$N_{Nu} = C[N_{Re}]^a [N_{Pr}]^d [N_{Gr}]^z$$

For natural convection where there is a bouyancy effect, N_{Gr} influences the heat transfer characteristic more than N_{Re} so that N_{Nu} is a function of N_{Gr} and N_{Pr}.

For natural convection;

$$N_{Nu} = f(N_{Pr}, N_{Gr})$$

For forced convection, the Reynolds number influences the heat transfer characteristics and the Grashof number may be omitted. Therfore, for forced convection,

$$N_{Nu} = f(N_{Re}, N_{Pr})$$