HEAT TRANSFER EQUIPMENT DESIGN.

Heat exchangers are ~~extent~~ extensively used in the chemical process industries and other engineering processes. Chemical engineers and mechanical engineers, in particular, ought to have a good understanding of the procedures of heat transfer design.

The design of a heat exchangers may be divided into two parts - 1. The process design part &

2. The mechanical design part.

Here, we shall discuss process design, more commonly called the thermal design, of heat exchangers only with the objective of sizing a heat exchanger. Within the constraints of allowable pressure drops, thermal design of a heat exchanger broadly includes the

1. Estimation of the heat transfer area
2. Determination of the tube diameter.
3. Number and length of tubes
4. Tube layout
5. Shell diameter and the TEMA shell type
6. Number of shell-side and tube-side passes of fluids.
7. Number and size of baffles.
8. Shell side and tube side pressure drops. etc

~~Mechanical~~

Mechanical design includes the

1. Selection of the materials of construction depending upon the process characteristics and conditions,
2. Determination of the thickness of the shell, the tube sheets, flanges, channels, baffles, etc.
3. Selection of gaskets nozzle connections, design of the support etc.

Mechanical design calculations are done according to a 'standard' or 'stipulated' heat exchanger design code.

The most widely used code is that of Tubular Exchanger Manufacturers Association (TEMA). This is a US code and is used together with ASME (American Society of Mechanical Engineers) Section VIII (for the design of unfired pressure vessels). Thickness of the shell, bonnet, channel etc are calculated using the ASME code.

The Indian code for heat exchanger design is IS 4503 and British code is BS 3274.

Design Procedure for Double-Pipe Heat Exchanger.

1. From the known terminal temperatures, calculate the log mean driving force (temp difference), LMTD.

$$\Delta T_{LMTD} = \frac{\Delta T_1 - \Delta T_2}{\ln\left(\frac{\Delta T_1}{\Delta T_2}\right)}$$

2. Select the diameters of the inner and the outer pipes. There are no rules for this purpose. It primarily depends upon the flow rates of the streams. If a small diameter pipe is selected, the fluid velocity will be large, i.e the Reynolds number will also be large. This will no doubt provide a larger heat transfer co-efficient, but give rise to a larger pressure drop as well. Conversely, if a pipe of bigger diameter is used, though the pressure drop will be less, the heat transfer coefficient will be small. Therefore, a judicious choice of pipe diameters will strike a balance between these opposing factors. If the allowable pressure drops for the individual streams are given, these may provide the basis for selection of the pipe diameters.

iii. Calculate the inner fluid Reynolds number; estimate the heat transfer coefficient h_i from the Dittus-Boelter equation or Sider-Tate equation if (μ/μ_w) is substantially different from unity or from the j_H-factor chart.

$$N_{Re} = \frac{d_i V_i \rho_i}{\mu_i}$$

$$N_{Pr} = \frac{C_p \mu}{k}$$

If $N_{Re} < 2100$, then sider-tate equation

$$\frac{h_i d_i}{k} = N_{Nu} = 1.86 \left[\frac{(N_{Re})(N_{Pr})}{(L/d_i)}\right]^{1/3} \left[\frac{\mu}{\mu_w}\right]^{0.14}$$

If $N_{Re} > 2100$, then Dittus - Boelter equation.

$$\frac{h_i d_i}{k} = N_{Nu} = 0.023 (N_{Re})^{0.8} (N_{Pr})^n$$

L - length of the pipe

d_i - Inner dia of inner pipe

h_i - inside heat transfer co-efficient

* It is applicable when $0.48 < N_{Pr} < 16,700$
* The viscosity ratio is with in the range of $0.0044 < \mu/\mu_w < 9.75$
* $N_{Nu} = 0.023 (N_{Re})^{0.8} (N_{Pr})^n$

$n = 0.4$ for heating

$n = 0.3$ for cooling

iv. Calculate the Reynolds number of the outer fluid flowing through the annulus. Use the equivalent diameter of the annulus. Estimate the outside heat transfer coefficient h_o using the equations ~~or~~ the ~~chart~~ j_H-factor chart.

Cross-sectional area, $A_o = \frac{\pi}{4} d_o^2 - \frac{\pi}{4} d_i^2$

$$d_e = 4 r_H$$

$$r_H = \frac{C/s \text{ area}}{\text{Wetted perimeter}}$$

Wetted perimeter $= \pi (d_i + d_o)$

$$N_{Re} = \frac{D_e \rho_o V_o}{\mu_o}$$

If $N_{Re} < 2100$ - Sider Tate equation

$$\frac{h_o D_e}{k} = N_{Nu} = 1.86 \left[\frac{(N_{Re})(N_{Pr})}{(L/D_e)} \right]^{1/3} \left(\frac{\mu}{\mu_w} \right)^{0.14}$$

If $N_{Re} > 2200$:- Dittus-Boelter equation

$$\frac{h_o D_e}{k} = N_{Nu} = 0.023 \, (N_{Re})^{0.8} \, (N_{Pr})^n$$

μ - Viscosity of fluid at bulk temp

μ_w - viscosity of fluid at wall temp.

V. Calculate the clean overall heat transfer coefficient (this is usually expressed on the basis of the outside diameter of inner pipe/tube). Calculate the design overall heat transfer coefficient U_d from the equation using a suitable value of the dirt factor.

$$\frac{1}{U_d} = \frac{1}{h_i} + \frac{1}{h_o} \quad - \text{clean}$$

$$\frac{1}{U_d} = \frac{1}{U} + R_d \quad - \text{Design}$$

VI. Calculate the heat transfer area A. [Note that for a counterflow DPHE, the question of LMTD correction does not arise, take $F_T = 1$). Determine the length of the pipe, that will provide the required heat transfer area. Use a number of hairpins in series, if the length is large

$$Q = \dot{M}_h C_{ph} (T_{h_1} - T_{h_2}) = \dot{M}_c C_{pc} (T_{c_2} - T_{c_1})$$

$$Q = U_d A \Delta T$$

Heat transfer area, $A = \dfrac{Q}{U_d \, \Delta T_{LMTD}}$

$$L = \frac{A}{\pi d_i}$$

No. of tubes, $n = \frac{A}{\pi d_i L}$

viii) Calculate the pressure drops of the fluids.

$$\Delta P = \frac{f G_t^2 L n}{2 g \rho_t d_i \phi_t}$$

f - friction factor from chart.

G_t - mass velocity of the tube fluid - $kg/m^2 \cdot s^{-1}$

L - length of tubes.

g - gravitational acceleration - $9.81\ m/s^2$

ρ_t - density of tube fluid

d_i - inside dia of tube

ϕ_t - dimensionless viscosity ratio.

Example 8.1 Benzene from the condenser at the top of a distillation column is cooled at a rate of 1000 kg/h from 75 °C to 50 °C in a countercurrent double-pipe heat exchanger (Fig. 8.16). The construction of the heat exchanger is hairpin type with an effective length of 25 m. The inner tube

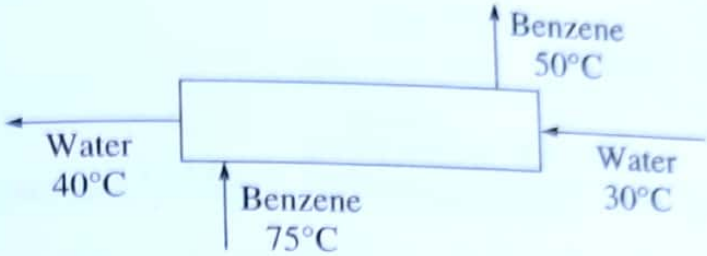

Fig. 8.16 Countercurrent double-pipe heat exchanger (Example 8.1).

is of carbon steel, 25 mm o.d., 14 BWG. The outer pipe is schedule 40, 1½ inch nb (nominal bore). Benzene flows through the annulus. Water which flows through the inner tube, entering at 30 °C and leaving at 40 °C, is the coolant.

(a) Calculate the heat duty of the exchanger and the water flow rate.

(b) Calculate the individual film coefficients and the overall coefficient based on both inside and outside areas.

(c) Do you think that the tube walls have gathered scale and have been fouled? If so, estimate the fouling factor.

The following data are available:

Inner tube: i.d. = 21 mm; o.d. = 25.4 mm; wall thickness = 2.2 mm; thermal conductivity of the tube wall = 74.5 W/m K.

Outer pipe: i.d. = 41 mm; o.d. = 48 mm.

Thermophysical properties of benzene at the average temperature (62.5 °C): specific heat = 1.88 kJ/kg °C; viscosity = 0.37 cP; density = 830 kg/m^3; thermal conductivity = 0.154 W/m K.

Properties of water at the average temperature (35 °C): viscosity = 0.8 cP; thermal conductivity = 0.623 W/m K, specific heat = 4.187 kJ/kg °C.

SOLUTION (a) 1000 kg of benzene is cooled from 75°C to 40°C per hour. Therefore,

$$\text{Heat duty} = (1000\ \text{kg/h})(1.88\ \text{kJ/kg°C})(75 - 50)\text{°C} = \boxed{47{,}000\ \text{kJ/h}}$$

Water gets heated from 30°C to 40°C.

Therefore,

$$\text{Water rate} = \frac{47{,}000}{(4.187)(10)} = \boxed{1122\ \text{kg/h}}$$

(b) *Tube-side (water) calculations*

Specific heat of water = 4.187 kJ/kg °C; $d_i = 21$ mm = 21×10^{-3} m; μ for water = 0.8 cp = 8×10^{-4} kg/m s

$$\text{Flow area} = (\pi/4)(21 \times 10^{-3})^2 = 3.46 \times 10^{-4}\ \text{m}^2$$

$$\text{Flow rate} = 1122\ \text{kg/h} = 1.122\ \text{m}^3/\text{h}$$

$$\text{Velocity} = \frac{1.122}{(3.46 \times 10^{-4})(3600)} = 0.9\ \text{m/s}$$

$$\text{Reynolds number, Re} = \frac{(21 \times 10^{-3})(0.9)(1000)}{8 \times 10^{-4}} = 23{,}625$$

$$\text{Prandtl number, Pr} = \frac{c_p\mu}{k} = \frac{(4.187)(1000)(8 \times 10^{-4})}{0.623} = 5.37$$

Use of Dittus-Boelter equation to calculate h_i

$$\text{Nu} = \frac{h_i d_i}{k} = 0.023(\text{Re})^{0.8}(\text{Pr})^{0.3} = (0.023)(23{,}625)^{0.8}(5.37)^{0.3} = 120$$

Thus,

$$h_i = (120)\left(\frac{k}{d_i}\right) = \frac{(120)(0.623)}{(21)(10^{-3})} = 3560\ \text{W/m}^2\ °\text{C}$$

Outer side (benzene) calculation

Flow area = inner cross-section of the pipe – outer cross-section of the tube

$$= (\pi/4)(41 \times 10^{-3})^2 - (\pi/4)(25.4 \times 10^{-3})^2 = 8.13 \times 10^{-4}\ \text{m}^2$$

$$\text{Wetted perimeter} = \pi(d_i + d_o) = \pi(0.041 + 0.0254) = 0.2086\ \text{m}$$

Hydraulic diameter of the annulus,

$$d_H = \frac{(4)(\text{area})}{\text{wetted perimeter}} = \frac{(4)(8.13 \times 10^{-4})}{0.2086} = 0.0156\ \text{m}$$

$$\text{Benzene flow rate} = 1000\ \text{kg/h} = (1000)/(860)\ \text{m}^3/\text{h} = 1.163\ \text{m}^3/\text{h}$$

$$\text{Velocity} = \frac{1.163}{(8.13 \times 10^{-4})(3600)} = 0.397\ \text{m/s}$$

For benzene: μ = 0.37 cP = 3.7×10^{-4} kg/m s; c_p = 1.88 kJ/kg °C; k = 0.154 W/m K

$$\text{Reynolds number, Re} = \frac{(0.0156)(0.397)(830)}{3.7 \times 10^{-4}} = 13893$$

$$\text{Prandtl number, Pr} = \frac{(1880)(3.7 \times 10^{-4})}{0.154} = 4.5$$

Calculate h_o from the Dittus-Boelter equation (use $Pr^{0.4}$ for cooling)

$$Nu = (0.023)(13893)^{0.8}(4.51)^{0.4} = 86.7$$

Equivalent diameter of the annulus for heat transfer [form Eq. (4.13c)]

$$\frac{1}{d_i}(d_o^2 - d_i^2) = \frac{(41)^2 - (25.4)^2}{25.4} = 0.0408 \text{ m}$$

$$h_o = (86.7)\left(\frac{k}{d_{e,h}}\right) = \frac{(86.7)(0.154)}{0.0408} = \boxed{336.4 \text{ W/m}^2\,{}^\circ\text{C}}$$

Calculation of clean overall heat transfer coefficient, outside area basis

Use Eq. (8.28),

$$\frac{1}{U_o} = \frac{1}{h_o} + \left(\frac{A_o}{A_m}\right)\left(\frac{r_o - r_i}{k_w}\right) + \frac{A_o}{A_i}\left(\frac{1}{h_o}\right)$$

Values of various quantities:

$A_o = (\pi)(0.0254)(l)$

$A_i = (\pi)(0.021)(l)$

$$A_m = \frac{(0.0254 - 0.021)(\pi l)}{\ln(0.0254/0.021)} = (0.0231)(\pi l)$$

$$\frac{A_o}{A_m} = \frac{0.0254}{0.0231} = 1.098$$

$$\frac{A_o}{A_i} = \frac{0.0254}{0.021} = 1.21$$

Therefore,

$$\frac{1}{U_o} = \frac{1}{336.4} + (1.098)\left(\frac{0.0254 - 0.021}{74.5}\right) + \frac{1.21}{3560} = 0.003345$$

$$U_o = \boxed{299 \text{ W/m}^2\text{K}}$$

[Note that the wall resistance is negligible and the tube-side resistance is small]

Calculation of U_i

$$U_oA_o = U_iA_i$$

i.e.,

$$U_i = U_o\left(\frac{A_o}{A_i}\right) = (299)(1.21) = \boxed{361.8 \text{ W/m}^2\text{K}}$$

Calculation of LMTD

$$\Delta T_1 = 75 - 40 = 35$$

$$\Delta T_2 = 50 - 30 = 20^\circ\text{C}$$

Therefore,

$$\text{LMTD} = \frac{35 - 20}{\ln(35/20)} = 26.8\,°\text{C}$$

Now calculate the area required from, $Q = U_o A_o \Delta T_m$.

Now,
$$Q = \frac{(47{,}000)(1000)}{3600} = 13{,}055 \text{ W}$$

Therefore,
$$A_o = \frac{Q}{U_o \Delta T_m} = \frac{13{,}055}{(299)(26.8)} = 1.64 \text{ m}^2 = \text{required area.}$$

$$\text{Tube length necessary, } l = \frac{1.64}{(\pi)(0.0254)} = 20.5 \text{ m}$$

(c) The actual length of the tube in the hairpin is 25 m, i.e. the cooling process needs an area considerably larger than the theoretical value. In other words, the actual heat transfer coefficient in the existing hairpin is less than that computed. The exchanger surface must have been fouled.

Calculation of the fouling factor, R_d

$$\text{Heat transfer area of the hairpin} = (\pi)(0.0254)(25) = 2.0 \text{ m}^2$$

Overall heat transfer coefficient with dirt factor,

$$U_{do} = \frac{13{,}055}{(2.0)(26.8)} = 243.6 \text{ W/m}^2\,°\text{C}$$

Using Eq. (8.2),

$$R_{do} = \frac{1}{U_{do}} - \frac{1}{U_o} = \frac{1}{243.6} - \frac{1}{299} = \boxed{0.00076 \text{ m}^2\,°\text{C/W}}$$

[A] Calculation of a Double Pipe Heat Exchanger :

The calculation procedure consists simply of computing h_o and h_i to obtain U_C. U_D is obtained taking into account fouling resistances. Knowing U_D, the surface area can be found with the help of the following equation

$$Q = U_D \cdot A \cdot \Delta Tlm$$

In the stepwise procedure of calculation given below, the hot and cold fluid temperatures are represented by the upper and lower case letters respectively. All the fluid properties are indicated by the lower case letters.

Process conditions required :

Hot fluid : T_1, T_2, $\dot{m}_h$, C_p, μ, k, ΔP, R_{do} or R_{di}

Cold fluid : t_1, t_2, $\dot{m}_c$, C_p, μ, k, ΔP, R_{di} or R_{do}

The diameter of each pipe must be given or assumed. A convenient order of the calculation is as given below :

1. Check the heat balance, Q, from T_1, T_2, t_1, t_2 using C_p at T_{mean} (T_m) and t_{mean} (t_m).

   Calculate Q using the following equation :

$$Q = \dot{m}_h C_{p_h} (T_1 - T_2) = \dot{m}_c \, C_{p_c} \, (t_2 - t_1)$$

$$T_1 > T_2 \text{ and } t_2 > t_1$$

$$T_m = T_1 + T_2/2, \quad t_m = t_1 + t_2/2$$

2. Calculate the log mean temperature difference LMTD (ΔTlm) for a counterflow arrangement.

$$T_1 \longrightarrow T_2$$

$$t_2 \longleftarrow t_1$$

$$\Delta T_1 = T_1 - t_2, \qquad \Delta T_2 = T_2 - t_1$$

$$\text{L.M.T.D.} = \Delta Tlm = \frac{\Delta T_1 - \Delta T_2}{\ln \dfrac{\Delta T_1}{\Delta T_2}}, \text{ K}$$

3. Evaluate the physical properties of the hot and cold fluid at the arithmetic mean of T_1 and T_2 and t_1 and t_2. For non-viscous fluids, $\left(\dfrac{\mu}{\mu_w}\right)^{0.14}$ may be taken as 1.0 (as assumed below).

Inner pipe :

4. Calculate the flow area : $A_i = \dfrac{\pi}{4} D_i^2$

$$N_{Re} = \frac{D_i u \rho}{\mu} = \frac{D_i G}{\mu}$$

where D_i is in m, u in m/s, ρ in kg/m^3, μ is in kg/(m·s) [N·s/m^2] and G is in kg/(m^2·s),

$$G = \frac{\dot{m}}{A_i}.$$

5. From C_p, μ, k, all obtained at T_m or t_m (the mean temperature), evaluate N_{Pr}.

$$N_{Pr} = \frac{C_p \mu}{k}$$

$$C_p = J/(kg \cdot K)$$

$$\mu = kg/(m \cdot s), \quad k = W/(m \cdot K)$$

6. Obtain the value of h_i-inside heat transfer coefficient using Seider-Tate or Dittus-Boelter equation taking $(\mu/\mu_w)^{0.14} = 1$.

$$\frac{h_i D_i}{k} = 0.023 \, (N_{Re})^{0.8} \, (N_{Pr})^a$$

$$a = 0.4 \text{ for heating, } a = 0.3 \text{ for cooling}$$

7. Convert h_i to h_{io}.

h_{io} = Inside heat transfer coefficient referred to outside diameter

$$h_{io} = h_i \times \frac{D_i}{D_o}, \quad W/(m^2 \cdot K)$$

Annulus :

8. Flow area = $A_a = \pi/4 \left(D_2^2 - D_1^2\right)$, m^2

where D_1 = Outside diameter of inner pipe

D_2 = Inside diameter of outer pipe.

Find the equivalent diameter of the annulus from the following relation :

$$D_e = \frac{4 \times \text{Flow area}}{\text{Wetted perimeter}} = \frac{4 \times \pi/4 \left(D_2^2 - D_1^2\right)}{\pi D_1}$$

$$= \frac{D_2^2 - D_1^2}{D_1}, \text{ m}$$

9. Calculate the mass velocity $G_a = \dot{m}/A_a$, $kg/(m^2 \cdot s)$

10. Obtain μ [kg/(m·s)] at T_m or t_m (take the mean temperature).

Calculate $N_{Re,\,a}$

$$N_{Re,\,a} = \frac{D_e G_a}{\mu}$$

11. Obtain C_p, μ, k at T_m or t_m (take the mean temperature), C_p is in J/(kg·K), μ is in kg/(m·s), k in W/(m·K).

Evaluate N_{Pr} :

$$N_{Pr} = \frac{C_p \mu}{k}$$

12. Obtain h_o either by using Dittus-Boelter or Sieder-Tate equation taking $(\mu/\mu_w)^{0.14}$ equal to 1.0.

$$\frac{h_o D_e}{k} = 0.023 \, (N_{Re})^{0.8} \left(\frac{C_p \mu}{k}\right)^a$$

a = 0.4 for heating, a = 0.3 for cooling

13. Compute U_c, the overall clean heat transfer coefficient, using the following relation

$$\frac{1}{U_c} = \frac{1}{h_o} + \frac{1}{h_{io}}$$

$$U_c = \frac{h_o\, h_{io}}{(h_o + h_{io})}, \quad W/(m^2 \cdot K)$$

14. Compute U_D, the design or dirty overall heat transfer coefficient, using the following relation

$$\frac{1}{U_D} = \frac{1}{U_C} + R_d$$

R_d is the combined dirt factor, $(m^2 \cdot K)/W$

$$R_d = R_{di} + R_{do}$$

where R_{di} is the inside dirt factor and R_{do} is the outside dirt factor.

15. Compute the heat transfer area 'A' from :

$$Q = U_D \cdot A \cdot \Delta Tlm$$

L, length of the pipe is obtained by : $A = \pi D_o L$

Whenever a metal wall resistance is to be considered, then we have to use the following relationship for calculating U_C :

$$\frac{1}{U_C} = \frac{1}{h_o} + \frac{1}{h_{io}} + \frac{x_w}{k_w}\left(\frac{D_o}{D_w}\right)$$

D_w – Log mean diameter $= (D_o - D_i)/\ln (D_o/D_i)$

x_w – Wall thickness, m; k_w = Thermal conductivity of the metal wall

SOLVED EXAMPLES

Example 5.1 : *It is desired to heat 4450 kg/h of cold benzene from 300 K (27°C) to 322 K (49°C) using hot toluene which is cooled from 344 K (71°C) to 311 K (38°C). The specific gravities of benzene and toluene are 0.88 and 0.87 respectively. A fouling factor of 1.60×10^{-4} (m²·K)/W should be provided for each stream. A number of 6 m hairpins of 50 by 31.75 mm IPS pipe are available. How many hairpins are required ?*

Data :

i.d. of inner pipe = 35 mm

o.d. of inner pipe = 42 mm

i.d. of outer pipe = 52.5 mm

For a 31.75 mm IPS standard pipe there is 0.1326 m² of external surface per m length. Benzene flows through the inner pipe in a counter-current fashion to toluene.

Physical properties of benzene and toluene at the average/mean temperatures are :

Property	Benzene	Toluene
C_p, kJ/(kg·K)	1.779	1.842
k, W/(m·K)	0.147	0.157
μ, kg/(m·s)	4.09×10^{-4}	5.0×10^{-4}

Solution : $Q = \dot{m}_b C_{pb} (t_2 - t_1)$ ... for benzene

where $\dot{m}_b$ = 4450 kg/h

C_{pb} = 1.779 kJ/(kg·K)

t_2 = 322 K

t_1 = 300 K

$Q = \dot{m}_t C_{pt} (T_1 - T_2)$ ... for toluene

where $\dot{m}_t$ = Mass flow rate of toluene, kg/hr

The heat balance is

Heat gained by benzene = Heat removed from toluene

The heat balance is used to calculate $\dot{m}_t$.

$$\dot{m}_t = \frac{Q}{C_{pt}(T_1 - T_2)} = \frac{\dot{m}_b C_{pb} (t_2 - t_1)}{C_{pt} \cdot (T_1 - T_2)}$$

$$= \frac{4450 \times 1.779 \times (322 - 300)}{1.842 \times (344 - 311)}$$

$$\dot{m}_t = 2865 \text{ kg/h}$$

Calculation of Q : The rate of heat transfer from toluene to benzene is

$$Q = 4450 \times 1.779\ (322 - 300)$$

$$Q = 174164\ \text{kJ/h} = 48379\ \text{J/s} \equiv 48379\ \text{W}$$

Calculation of ΔTlm for countercurrent flow :

Hot fluid (toluene)

Cold fluid (benzene)

$$344\ \text{K} \xrightarrow{\text{toluene}} 311\ \text{K}$$

$$322\ \text{K} \xleftarrow{\text{benzene}} 300\ \text{K}$$

$$\Delta T_1 = 22\ \text{K}, \qquad \Delta T_2 = 11\ \text{K}$$

ΔTlm = log mean temperature difference

$$\Delta Tlm = \frac{\Delta T_1 - \Delta T_2}{\ln (\Delta T_1 / \Delta T_2)}$$

$$= \frac{22 - 11}{\ln (22/11)} = 15.87\ \text{K}$$

Cold fluid : Inner pipe, benzene

$$D_i = 3.5\ \text{mm} = 0.035\ \text{m}$$

Flow area :

$$A_i = \frac{\pi}{4} D_i^2$$

$$= \frac{\pi}{4} \times (0.035)^2$$

$$= 9.62 \times 10^{-4}\ \text{m}^2$$

Mass velocity :

$$G_i = \dot{m}_b / A_i$$

$$= \frac{4450}{9.62 \times 10^{-4}}$$

$$= 4625237\ \text{kg/(m}^2\cdot\text{h)}$$

$$= 1285\ \text{kg/(m}^2\cdot\text{s)}$$

$$N_{Re} = \frac{D_i G_i}{\mu}$$

where

$$\mu = 4.09 \times 10^{-4}\ \text{kg/(m}\cdot\text{s)}$$

$$N_{Re} = \frac{0.035 \times 1285}{4.09 \times 10^{-4}} = 109945$$

$$N_{Pr} = \left(\frac{C_p \mu}{k}\right)$$

$$C_p = 1.779 \times 10^3\ \text{J/(kg}\cdot\text{K)}$$

$$\mu = 4.09 \times 10^{-4}\ \text{kg/(m}\cdot\text{s)}$$

$$k = 0.147\ \text{W/(m}\cdot\text{K)}$$

$$N_{Pr} = \frac{1.779 \times 10^3 \times 4.09 \times 10^{-4}}{0.147} = 4.95$$

$$\frac{h_i D_i}{k} = 0.023\,(N_{Re})^{0.8}\,(N_{Pr})^{0.4}$$

$$h_i = \frac{k}{D_i} \times 0.023\ (N_{Re})^{0.8}\,(N_{Pr})^{0.4}$$

$$h_i = \frac{0.147}{0.035} \times 0.023 \times (109945)^{0.8} \times (4.95)^{0.4}$$

$$h_i = 1976\ \text{W/(m}^2\cdot\text{K)}$$

$$h_{io} = h_i \times \frac{D_i}{D_o}$$

$$= 1976 \times \frac{0.035}{0.042}$$

$$= 1646.6\ \text{W/(m}^2\cdot\text{K)}$$

Hot fluid : Annulus, toluene

$$D_1 = \text{outer diameter of inner pipe}$$

$$= 42\ \text{mm} = 0.042\ \text{m}$$

$$D_2 = \text{inner diameter of outer pipe}$$

$$= 52.5\ \text{mm} = 0.0525\ \text{m}$$

$$D_e = \frac{D_2^2 - D_1^2}{D_1}$$

$$= \frac{(0.0525)^2 - (0.042)^2}{0.042} = 0.0236\ \text{m}$$

$$A_a = \text{flow area} = \pi\left(D_2^2 - D_1^2\right)/4$$

$$= \pi\,[(0.0525)^2 - (0.042)^2]/4 = 7.8 \times 10^{-4}\ \text{m}^2$$

Mass velocity,

$$G_a = \dot{m}_t/A_a$$

$$= \frac{2865}{7.8 \times 10^{-4}} = 3676323\ \text{kg/(m}^2\cdot\text{h)}$$

$$= 1021\ \text{kg/(m}^2\cdot\text{s)}$$

$$N_{Re} = \frac{D_e\,G_a}{\mu}$$

$$= \frac{0.0236 \times 1021}{5.01 \times 10^{-4}}$$

$$N_{Re} = 48104$$

$$N_{Pr} = \frac{C_p\,\mu}{k}$$

$$N_{Pr} = \frac{1.842 \times 10^3 \times 5.01 \times 10^{-4}}{0.157} = 5.88$$

$$\frac{h_o D_e}{k} = 0.023 (N_{Re})^{0.8} (N_{Pr})^{0.3}$$

$$h_o = \frac{k}{D_e} \times 0.023 (N_{Re})^{0.8} (N_{Pr})^{0.3}$$

$$= \frac{0.157}{0.0236} \times 0.023 \times (48104)^{0.8} \times (5.88)^{0.3} = 1450 \text{ W/(m}^2\cdot\text{K)}$$

Clean overall coefficient, U_C :

$$\frac{1}{U_C} = \frac{1}{h_o} + \frac{1}{h_{io}}$$

$$\frac{1}{U_C} = \frac{1}{1450} + \frac{1}{1646.6}$$

$$U_C = 771 \text{ W/(m}^2\cdot\text{K)}$$

Design heat transfer coefficient, U_D :

$$\frac{1}{U_D} = \frac{1}{U_C} + R_d$$

$$R_d = R_{di} + R_{do}$$

$$R_d = 1.6 \times 10^{-4} + 1.6 \times 10^{-4} = 3.2 \times 10^{-4} \text{ (m}^2\cdot\text{K)/W}$$

$$\frac{1}{U_D} = \frac{1}{U_C} + R_d$$

$$\frac{1}{U_D} = \frac{1}{771} + 3.20 \times 10^{-4}$$

$$U_D = 618.4 \text{ W/(m}^2\cdot\text{K)}$$

Area of heat transfer :

We have :

$$Q = U_D A \,\Delta Tlm$$

$$A = \frac{Q}{U_D \Delta Tlm}$$

$$= \frac{48379}{618.42 \times 15.87} = 4.93 \text{ m}^2$$

Given : External surface per meter length = 0.136 m^2. Therefore

$$\text{Required length} = \frac{\text{Area of heat transfer}}{\text{External surface}} = \frac{4.93}{0.136} = 36.25 \text{ m}$$

Total length of one hairpin of 6 m = 2 × 6 = 12 m

$$\text{Hair pins required} = \frac{36.25}{12} = 3$$

∴ **Required surface is fulfilled by connecting three 6 m hairpins in series.** ... **Ans.**

Example 5.2 : *A chemical plant produces 300 metric tons of sulphuric acid per day. The acid is to be cooled from 333 K (60°C) to 313 K (40°C) by 500 metric tons of water per day (24 hours) which has an initial temperature of 288 K (15°C). A counterflow cooler consisting of concentric pipes 12.5 mm thick, is to be used. The inner pipe through which the acid flows is 75 mm bore and the outer one 125 mm bore. The outside diameter of the inner pipe is 100 mm. The physical properties of the fluid at the mean temperature are as follows :*

	Acid	*Water*
Density, kg/m³	*1800*	*998.2*
Heat capacity, kJ/(kg·K)	*1.465*	*4.187*
Thermal conductivity, W/(m·K)	*0.302*	*0.669*
Viscosity, (N.s)/m² or kg/(m·s)	*0.0112*	*0.0011*

Thermal conductivity of the pipe material is 46.52 W/(m·K).

Use Dittus-Boelter equation to calculate h.

Calculate the length of the pipe required.

Solution : Basis : 300 tonnes of sulphuric acid per day

Inner side fluid ⇒ Sulphuric acid

Annulus fluid ⇒ Water

Mass flow rate of acid, $\dot{m}_a = \dfrac{300 \times 1000}{24} = 12500$ kg/h

Mass flow rate of water $= \dfrac{500 \times 1000}{24} = 20833.3$ kg/h

Calculation of Q :

Q = Rate of heat transfer

$Q = \dot{m}_a C_p (T_1 - T_2)$ ... heat removed from acid

where

$\dot{m}_a = 12500$ kg/h

$C_p = 1.465$ kJ/(kg·K)

$T_1 = 333$ K, $T_2 = 313$ K

$Q = 12500 \times 1.465 \times (333 - 313)$

$Q = 366250$ kJ/h $\equiv 101736$ W

t_1 = Inlet temperature of water

t_2 = Outlet temperature of water

Calculation of t_2 : The heat gained by water is

$Q = \dot{m}_w C_p (t_2 - t_1)$

where

$\dot{m}_w = 20833.3$ kg/h

$C_p = 4.187$ kJ/(kg·K)

$$366250 = 20833.3 \times 4.187\,(t_2 - t_1)$$

$$t_2 = t_1 + 4.2$$

$$t_2 = 288 + 4.2 = 292.2 \text{ K} \quad \ldots \text{ water outlet temperature}$$

Calculation of ΔTlm :

$$333 \text{ K} \xrightarrow{\text{Acid}} 313 \text{ K}$$

$$292.2 \text{ K} \xleftarrow{\text{Water}} 288 \text{ K}$$

$$\Delta T_1 = 333 - 292.2 = 40.8 \text{ K}$$

$$\Delta T_2 = 313 - 288 = 25 \text{ K}$$

$$\Delta Tlm = \frac{40.8 - 25}{\ln(40.8/25)}$$

$$\Delta Tlm = 32.26 \text{ K } (^\circ\text{C})$$

Calculation of h_i and h_o :

Inner pipe (Acid) :

$$\dot{m} = 12500 \text{ kg/h}$$

$$D_i = 75 \text{ mm} = 0.075 \text{ m}$$

$$A_i = \frac{\pi}{4} D_i^2$$

$$= \frac{\pi}{4} (0.075)^2$$

$$= 4.418 \times 10^{-3} \text{ m}^2$$

$$G = \text{Mass velocity} = \frac{\dot{m}_a}{A_i}$$

$$= \frac{12500}{4.418 \times 10^{-3}}$$

$$= 2829421 \text{ kg/(m}^2\cdot\text{h)}$$

$$= 785.95 \text{ kg/(m}^2\cdot\text{s)}$$

$$N_{Re} = \frac{D_i G}{\mu}$$

where $D_i = 0.075$ m, $\mu = 0.0112$

$$N_{Re} = \frac{0.075 \times 785.95}{0.0112}$$

$$N_{Pr} = 5263$$

$$N_{Pr} = \frac{C_p \mu}{k}$$

where

$$C_p = 1.465 \times 10^3 \text{ J/(kg}\cdot\text{K)}$$

$$k = 0.302 \text{ W/(m}\cdot\text{K)}$$

$$\mu = 0.0112 \text{ (N.s)/m}^2 \equiv \text{kg/(m}\cdot\text{s)}$$

$$N_{Pr} = \frac{1.465 \times 10^3 \times 0.0112}{0.302} = 54.25$$

$$\frac{h_i D_i}{k} = 0.023\,(N_{Re})^{0.8}\,(N_{Pr})^{0.3}$$

$$N_{Re} = 5263, \quad N_{Pr} = 54.25$$

$$D_i = 0.075 \text{ m}$$

$$k = 0.302 \text{ W/(m·K)}$$

$$h_i = \frac{0.302}{0.075} \times 0.023 \times (5263)^{0.8} \times (54.25)^{0.3}$$

$$= 291.2 \text{ W/(m}^2\text{·K)}$$

$$h_{io} = h_i \times \frac{D_i}{D_o}$$

$$D_o = 10 \text{ cm} = 0.1 \text{ m}$$

$$h_{io} = 291.2 \times \frac{0.075}{0.1} = 218.4 \text{ W/(m}^2\text{·K)}$$

Calculation of h_o :

Annulus (water) :

$$D_1 = \text{Outer diameter of inner pipe}$$

$$= 100 \text{ mm} = 0.10 \text{ m}$$

$$D_2 = \text{Inner diameter of outer pipe}$$

$$= 125 \text{ mm} = 0.125 \text{ m}$$

$$D_e = \frac{D_2^2 - D_1^2}{D_1} = \frac{(0.125)^2 - (0.10)^2}{0.10} = 0.056 \text{ m}$$

$$A_a = \text{Annulus flow area} = \frac{\pi}{4}\left[D_2^2 - D_1^2\right]$$

$$= \frac{\pi}{4}\left[(0.125)^2 - (0.10)^2\right] = 0.0044 \text{ m}^2$$

$$G_a = \text{Mass velocity in annulus i.e. of water}$$

$$G_a = \dot{m}_w / A_a$$

$$= \frac{20833.3}{0.004} = 4734841 \text{ kg/(m}^2\text{·h)} = 1315.23 \text{ kg/(m}^2\text{·s)}$$

$$N_{Re} = \frac{D_e G_a}{\mu}$$

where $\mu = 0.0011$ kg/(m·s)

$$N_{Re} = \frac{0.056 \times 1315.23}{0.0011} = 66957$$

$$N_{Pr} = \frac{C_p\,\mu}{k}$$

$$k \text{ for water} = 0.669 \text{ W/(m·K)}$$

$$C_p \text{ for water} = 4.187 \times 10^3 \text{ J/(kg·K)}$$

$$= \frac{4.187 \times 10^3 \times 0.0011}{0.669} = 6.89$$

$$\frac{h_o D_e}{k} = 0.023\,(N_{Re})^{0.8}\,(N_{Pr})^{0.4}$$

One can use Dittus-Boelter equation also as is applicable for water.

$$h_o = \frac{k}{D_e} \times 0.023\,(N_{Re})^{0.8}\,(N_{Pr})^{0.4}$$

$$= \frac{0.689}{0.056} \times 0.023 \times (66957)^{0.8} \times (6.89)^{0.4}$$

$$h_o = 4314 \text{ W/(m}^2\text{·K)}$$

Metal wall resistance :

$$x_w = 10 - 7.5/2 = 1.25 \text{ cm} = 0.0125 \text{ m}$$

$$D_w = \frac{D_o - D_i}{\ln (D_o/D_i)}$$

where $D_o = 0.10$ m, $D_i = 0.075$ m

$$D_w = \frac{0.10 - 0.075}{\ln (0.10/0.075)} = 0.0869 \text{ m}$$

Calculation of U_C :

$$\frac{1}{U_C} = \frac{1}{U_o} = \frac{1}{h_o} + \frac{1}{h_{io}} + \frac{x}{k_w}\,\frac{D_o}{D_w}$$

where k_w = Thermal conductivity of metal wall

$$= 46.52 \text{ W/(m·K)}$$

$$\frac{1}{U_C} = \frac{1}{4314} + \frac{1}{218.4} + \frac{0.0125}{46.52}\left(\frac{0.10}{0.0869}\right)$$

$$U_C = 195.32 \text{ W/(m}^2\text{·K)}$$

Calculation of A and L :

As dirt factor values are not given, use U_C in place of U_D in the design equation.

$$U_D = U_C = 195.32 \text{ W/(m}^2\text{·K)}$$

We have : $Q = U_D \cdot A \cdot \Delta Tlm$

$$\therefore \quad A = \frac{Q}{U_D \times \Delta Tlm}$$

$$= \frac{101736}{195.32 \times 32.26} = \mathbf{16.14\ m^2} \quad \text{... Ans.}$$

A is given by

$$\pi D_o \cdot L = A = 16.14 \text{ m}^2$$

$$L = 16.14/\pi \times 0.10 = \mathbf{51.37\ m^2} \quad \text{... Ans.}$$

Example 5.3 : *Calculate the total length of a double pipe heat exchanger required to cool 5500 kg/h of ethylene glycol from 358 K (85°C) to 341 K (68°C) using toluene as a cooling medium which flows in a counter-current fashion. Toluene enters at 303 K (30°C) and leaves at 335 K (62°C).*

Data :

Outer diameter of outer pipe = 70 mm

Outer diameter of inner pipe = 43 mm

Wall thickness of both pipes = 3 mm

Mean properties of two fluids are as given below :

Property	Ethylene glycol	Toluene
Density	1080 kg/m^3	840 kg/m^3
Specific heat	2.680 kJ/(kg·K)	1.80 kJ/(kg·K)
Thermal conductivity	0.248 W/(m·K)	0.146 W/(m·K)
Viscosity	3.4×10^{-3} Pa·s	4.4×10^{-4} Pa·s

Thermal conductivity of the pipe material is 46.52 W/(m·K) and ethylene glycol is flowing through the inner pipe.

Solution : Ethylene : in the inside pipe

Toluene : in the annulus

For ethylene glycol flowing through the inner pipe :

Calculation of h_i :

$$\dot{m}_e = \text{Mass flow rate of ethylene glycol}$$

$$= 5500 \text{ kg/h} = 1.528 \text{ kg/s}$$

$$\text{O.D. of inner pipe} = 42 \text{ mm}$$

$$\text{I.D. of inner pipe} = 43 - 2 \times 3 = 37 \text{ mm} = 0.037 \text{ m}$$

$$\text{Area of the inner pipe} = A_i = \frac{\pi}{4} D_i^2 = \frac{\pi}{4} \times (0.037)^2$$

$$A_i = 0.001075 \text{ m}^2$$

$$G = \text{Mass velocity} = \dot{m}_e / A_i = \frac{1.528}{0.001075} = 1421.4 \text{ kg/(m}^2\text{·s)}$$

$$N_{Re} = \frac{D_i u \rho}{\mu} = \frac{D_i G}{\mu}$$

where

$$\mu = 3.4 \times 10^{-3} \text{ Pa·s} \equiv 3.4 \times 10^{-3} \text{ kg/(m·s)}$$

$$N_{Re} = \frac{0.037 \times 1421.4}{3.4 \times 10^{-3}} = 15468$$

$$C_p = 2.68 \text{ kJ/(kg·K)} = 2.68 \times 10^3 \text{ J/(kg·K)}$$

$$\mu = 3.4 \times 10^{-3} \text{ kg/(m·s)}, \; k = 0.248 \text{ W/(m·K)}$$

$$N_{Pr} = \frac{C_p \mu}{k} = \frac{2.68 \times 10^3 \times 3.4 \times 10^{-3}}{0.248} = 36.74$$

As $N_{Re} > 10{,}000$, we can use the Dittus-Boelter equation (for cooling) for calculating h_i.

$$N_{Nu} = 0.023 (N_{Re})^{0.8} (N_{Pr})^{0.3}$$

$$= 0.023 \times (15468)^{0.8} \times (36.74)^{0.3} = 152.32$$

$$\frac{h_i D_i}{k} = 152.34$$

$$h_i = \frac{152.34 \times 0.248}{0.037} = 1021 \text{ W/(m}^2\text{·K)}$$

For toluene flowing through the annulus :

$$\dot{m}_t = \text{Mass flow rate of toluene}$$

Calculation of $\dot{m}_t$:

Heat balance :

$$\text{Heat lost by ethylene glycol} = \text{Heat gained by toluene}$$

$$5500 \times 2.68 \times (358 - 341) = \dot{m}_t \times 1.80 \times (335 - 303)$$

$$\dot{m}_t = 4350.35 \text{ kg/h} = 1.21 \text{ kg/s}$$

Calculation of h_o :

$$\text{Inner diameter of the outer pipe} = 70 - 2 \times 3 = 64 \text{ mm} = 0.064 \text{ m}$$

$$D_e = \text{Equivalent diameter for the annulus} = \frac{D_2^2 - D_1^2}{D_1}$$

$$= \frac{(0.064)^2 - (0.043)^2}{(0.043)} = 0.052 \text{ m}$$

$$\text{Area of cross section for flow} = \frac{\pi}{4} [D_2^2 - D_1^2]$$

$$= \frac{\pi}{4} [(0.064)^2 - (0.043)^2]$$

$$A_a = 0.00176 \text{ m}^2$$

$$G_a = \text{Mass velocity through the annulus} = \frac{1.21}{0.00176} = 687.5 \text{ kg/(m}^2\text{·s)}$$

$$N_{Re} = \frac{D_e G_a}{\mu} = \frac{0.052 \times 687.5}{4.4 \times 10^{-4}} = 81250$$

$$N_{Pr} = \frac{C_p \mu}{k} = \frac{1.80 \times 10^3 \times 4.4 \times 10^{-4}}{0.146} = 5.42$$

The Dittus-Boelter equation for heating is

$$N_{Nu} = 0.023\,(N_{Re})^{0.8}\,(N_{Pr})^{0.4}$$

$$= 0.023 \times (81250)^{0.8} \times (5.42)^{0.4} = 383$$

$$\frac{h_o D_e}{k} = 383$$

$$h_o = \frac{383 \times 0.146}{0.052} = 1075.3 \;\; W/(m^2 \cdot K)$$

D_w = Log mean diameter of the inside pipe

$$= \frac{0.043 - 0.037}{\ln(0.043/0.037)} = 0.0399 \text{ m}$$

Calculation of U_o :

$$\frac{1}{U_o} = \frac{1}{h_o} + \frac{1}{h_i} \times \frac{D_o}{D_i} + \frac{x}{k}\,\frac{D_o}{D_w}$$

$$\frac{1}{U_o} = \frac{1}{1075.3} + \frac{1}{1021} \times \frac{0.043}{0.037} + \frac{0.003}{46.52} \times \frac{0.043}{0.0399}$$

$$U_o = 468 \; W/(m^2 \cdot K)$$

Calculation of ΔTlm :

ΔTlm = Log mean temperature difference for counter-current flow

$$358\text{ K} \xrightarrow{\text{E.G.}} 341\text{ K}$$

$$335\text{ K} \xleftarrow{\text{Toluene}} 303\text{ K}$$

$$\Delta T_1 = 38\text{ K} \quad \text{and} \quad \Delta T_2 = 23\text{ K}$$

$$\Delta Tlm = (38 - 23) / \ln(38/23) = 29.87\text{ K}$$

Calculation of Q :

$$Q = \dot{m}\, C_{pe}\,(T_1 - T_2) = 1.528 \times 2.680 \times (358 - 341)$$

$$= 69.61 \text{ kJ/s} = 69.61 \times 10^3 \text{ J/s} \equiv 69.61 \times 10^3 \text{ W}$$

Calculation of A and L of the exchanger :

$$Q = U_o \cdot A_o \cdot \Delta Tlm$$

$$69.61 \times 10^3 = 468 \times A_o \times 29.87$$

$$A_o = 4.98 \text{ m}^2$$

$$A = 4.98 = \pi D \cdot L$$

$$4.98 = \pi\,(0.043)\,L$$

$\therefore$ L = **36.86 m** ... **Ans.**