RADIATION

Heat transfer by radiation usually takes place simultaneously with heat transfer by convection and conduction. The heat transfer by radiation is of much more importance at high temperature levels as compared to the other two mechanisms. Direct-fired kettles, electric heaters, steam boilers, rotary kiln, etc. are examples of chemical process equipments where radiation is a major energy transfer mechanism.

Radiation : It refers to the transport of energy through space by electromagnetic waves.

Radiation is the mode of transport of energy in the form of electromagnetic waves through space, at the speed of light (3×10^8 m/s).

It depends upon the electromagnetic waves as a means for the transfer of energy from a source to a receiver.

Radiant energy is of the same nature as the ordinary visible light. It travels in straight lines and it may be reflected from a surface. The electromagnetic waves with wavelength ranging from 0.5 to 50 μm (microns) are of importance to radiant-heat transfer. [1 μm = 10^{-6} m]. Radiation of a single wavelength is called monochromatic.

Thermal radiation is the energy emitted by a body entirely due to its temperature and we restrict our discussion to this type of radiation.

Typical examples of heat transfer by radiation :

(i) Transfer of heat from the sun to the earth.

(ii) Heat loss from an unlagged steam pipe.

(iii) Use of energy from the sun in solar heaters.

(iv) Heating of a cold room by a radiant electric heater.

In contrast to conduction and convection, radiation heat transfer does not require an intervening medium (material or fluid) and the heat can be transmitted by a radiation mode across an absolute vacuum.

Radiation is the only significant mode of energy/heat transfer when no medium is present (e.g., the heat leakage through the evacuated walls of a thermos flask).

Absorptivity, Reflectivity and Transmissivity :

Any substance receives and gives off/emits energy in the form of electromagnetic waves. When energy emitted by a heated body falls on a second body (i.e., thermal radiation falling on a body), it will be partly absorbed, partly reflected and partly transmitted. It is the only absorbed energy that appears as a heat in the body.

The proportions of the incident energy that are absorbed, reflected and transmitted depend mainly on the characteristics of a receiver and temperature of the receiver (incident radiation = heat absorbed + heat transmitted + heat reflected).

The fraction of the incident radiation on a body that is absorbed by the body is known as the **absorptivity**. It may be denoted by the letter **'a'**.

The fraction of the incident radiation on a body that is reflected by the body is known as the **reflectivity**. It may be denoted by the letter **'r'**.

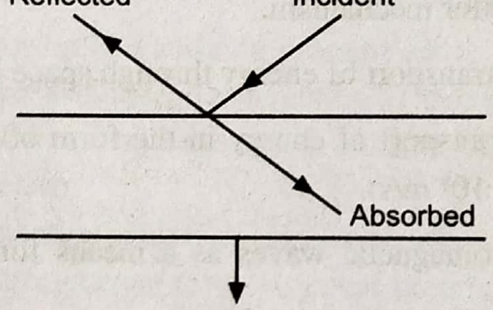


Fig. 4.1 : Reflection, absorption and transmission of radiation

The fraction of the incident radiation on a body that is transmitted through the body is known as the **transmissivity**. It may be denoted by the letter **'τ'**. The energy balance about a body (a receiver) on which the total incident energy falling is unity (the sum of these fractions is unity) is given as :

$$a + r + \tau = 1.0 \qquad \text{... (4.1)}$$

A majority of engineering materials are opaque (i.e., for which the amount transmitted is very negligible, $\tau = 0$) and in such cases, the Equation (4.1) simplifies to :

$$a + r = 1.0 \text{ (as } \tau = 0) \text{ ... for an opaque material/surface} \qquad \text{... (4.2)}$$

If $\tau = 1$, $a = r = 0$, then all the incident energy passes through the body and it is called perfectly transparent., e.g., rock salt (NaCl), quartz and fluorite.

If $r = 1$, $a = \tau = 0$, then all the incident energy is reflected by the body and is called specular. If $a = 0$, $r + \tau = 1$, then the body is called as a perfectly white body, e.g., a piece of white chalk (white body).

(i) $r = 0$ represents a non-reflecting surface.

(ii) $r = 1$ represents a perfect reflector.

(iii) $a = 0$ represents a non-absorbing surface.

(iv) $a = 1$ represents a perfectly absorbing surface or a black surface.

(v) $\tau = 1$ represents a perfectly transparent surface.

(vi) $\tau = 0$ represents an opaque surface.

Black Body :

A black body is an idealised physical body which absorbs all incident electromagnetic radiation. It is a perfect emitter and a perfect absorber of thermal radiation.

A body for which $a = 1$, $r = \tau = 0$, i.e., which absorbs all the incident radiant energy, is called **a black body**. It neither reflects nor transmits, but absorbs all the radiation incident on it, so it is treated as an ideal radiation receiver. It is not necessary that surface of the body be black in colour. The black body radiates maximum possible amount of energy at a given temperature and though perfectly black bodies do not exist in nature, some materials may approach it. Lampblack is the nearest to a black body. It absorbs 96 % of the visible light. Both absorptivity and emissivity of a perfectly black body are unity.

The concept of a black body is an idealisation with which the radiation characteristics of real bodies are compared.

Laws of Black Body Radiation :

Kirchhoff's Law :

This law sets up a relationship between the emissive power of a body/surface to its absorptivity.

Consider that the two bodies are kept into a furnace held at a constant temperature of T K. Assume that, of the two bodies one is a black body and the other is a non-black body, i.e., the body having **'a'** value less than one. Both the bodies will ultimately attain the temperature of T K and the bodies neither become hotter nor cooler than the furnace. At this condition of thermal equilibrium, each body absorbs and emits thermal radiation at the same rate. The rate of absorption and emission for the black body will be different from that of the non-black body.

Let A_1 and A_2 be the areas of the non-black body and black body respectively. Let 'I' be the rate at which radiation falling on bodies per unit area and E_1 and E_b be the emissive powers **(emissive power is the total quantity of radiant energy emitted by a body per unit area per unit time)** of non-black and black body respectively.

At thermal equilibrium, absorption and emission rates are equal. Therefore,

$$I a_1 A_1 = A_1 E_1 \quad \ldots (4.3)$$

$$\therefore \quad I a_1 = E_1 \quad \ldots (4.4)$$

and

$$I a_b A_2 = A_2 E_b \quad \ldots (4.5)$$

$$I a_b = E_b \quad \ldots (4.6)$$

From Equations (4.4) and (4.6), we get

$$\frac{E_1}{a_1} = \frac{E_b}{a_b} \quad \ldots (4.7)$$

where a_1, a_b are the absorptivities of non-black and black bodies respectively.

If we introduce a second body (non-black), then for the second non-black body, we have :

$$I A_3 a_2 = E_2 A_3 \quad \ldots (4.8)$$

$$I a_2 = E_2 \quad \ldots (4.9)$$

where a_2 and E_2 are the absorptivity and emissive power of the second non-black body.

Combining Equations (4.4), (4.6) and (4.9), we get

$$\frac{E_1}{a_1} = \frac{E_2}{a_2} = \frac{E_b}{a_b} = E_b \quad \ldots (4.10)$$

(As the absorptivity of the black body is 1.0)

Statement of Kirchhoff's law :

It states that : *at thermal equilibrium, the ratio of the total emissive power to its absorptivity is the same for all bodies.* Equation (4.10) is the mathematical statement of Kirchhoff's law.

The **emissivity** 'e' of any body is defined as the **ratio of the total emissive power E of the body to that of a black body E_b at the same temperature.** The emissivity depends on the temperature of the body only. The emissivity of a body is a measure of how it emits radiant energy in comparison with a black body at the same temperature.

$$e = \frac{E}{E_b} \qquad \dots (4.11)$$

Since $\frac{E}{a}$ is constant for all bodies,

$$\frac{E}{a} = \frac{E_b}{a_b} \qquad \dots (4.12)$$

$$e = \frac{E}{E_b} = \frac{a}{a_b} \qquad \dots (4.13)$$

But $\quad a_b = 1$ (for black body)

$$\therefore \quad e = a \qquad \dots (4.14)$$

Thus, *when any body is in thermal equilibrium with its surroundings, its emissivity and absorptivity are equal.* Equation (4.14) may be taken as the another statement of **Kirchhoff's law**.

Monochromatic emissive power : It is the radiant energy emitted from a body per unit area per unit time, per unit wavelength about the wavelength λ. It is denoted by the symbol E_λ. It has the units of $W/(m^2 \cdot \mu m)$.

Total emissive power : It is the total quantity of radiant energy of all wavelength emitted by a body per unit area per unit time. It is denoted by the symbol E. The unit of E in the SI system is W/m^2.

The emissive power, E_b, of a black surface is defined as the energy emitted by the surface per unit area per unit time.

For the entire spectrum of radiation from a surface, it is the sum of all the monochromatic radiations from the surface.

$$E = \int_0^\infty E_\lambda \, d\lambda \qquad \dots (4.15)$$

Monochromatic emissivity : *It is the ratio of the monochromatic emissive power of a surface to that of a black surface at the same wavelength.*

$$e_\lambda = \frac{E_\lambda}{E_{b,\lambda}} \qquad \dots (4.16)$$

Grey Body :

A body having the same value of the monochromatic emissivity at all wavelengths is called a grey body.

A grey body is the one of which emissivity is independent of wavelength.

[The adjective monochromatic indicates that the quantity being defined for a particular wavelength / single wavelength. Monochromatic property refers to a single wavelength and the total property is the sum of the monochromatic values of property. Monochromatic values are not important to the direct solution of engineering problems.]

(i) Steafan-Boltzmann Law :

It states that *the total emissive power (total energy emitted per unit area per unit time) of a black body is directly proportional to the fourth power of its absolute temperature*. This law relates the total amount of radiation emitted by a body (object) to its temperature.

$$E_b \propto T^4 \qquad \text{... (4.17)}$$

$$E_b = \sigma \cdot T^4$$

where
T = Temperature in K
σ = Steafan-Boltzmann constant
$= 5.67 \times 10^{-8}\ W/(m^2 \cdot K^4)$

If E_b is in W/m^2, T is in K, then the Steafan-Boltzmann constant has the value of $5.67 \times 10^{-8}\ W/(m.K^4)$ in the SI system.

For a non-black body,

$$\frac{E}{E_b} = e \qquad \text{... (4.18)}$$

$$E = e \cdot E_b \qquad \text{... (4.19)}$$

Combining Equations (4.17) and (4.19), we get

$$E = e \cdot \sigma \cdot T^4 \qquad \text{... (4.20)}$$

where 'e' is the emissivity of the non-black body.

The Steafan-Boltzmann equation is a fundamental relation for all the radiant energy transfer calculations.

(ii) Planck's Law :

This law gives a relationship between the monochromatic emissive power of a black body, absolute temperature and the corresponding wavelength.

$$E_{b,\lambda} = \frac{2\pi\, hc^2 \lambda^{-5}}{e^{hc/k\lambda T} - 1} \qquad \text{... (4.21)}$$

where $E_{b,\lambda}$ is the monochromatic emissive power of the black body / black surface, $W/(m^2 \cdot \mu m)$, h is Planck's constant, k is the Boltzmann constant, c is the speed of light, T is the absolute temperature and λ is the wavelength of radiation. The Planck's constant has the value of 6.625×10^{-3} J.s in SI.

The above equation can be written as,

$$E_{b,\lambda} = \frac{C_1 \lambda^{-5}}{(e^{C_2/\lambda T} - 1)} \qquad \text{... (4.22)}$$

where C_1 and C_2 are constants.

$$C_1 = 3.472 \times 10^{-16}\ W \cdot m^2 \quad \text{and} \quad C_2 = 0.01439\ m \cdot K$$

(iii) Wiens Displacement Law :

It states that the wavelength at which the maximum monochromatic emissive power is obtained (i.e., λ_{max}) is inversely proportional to the absolute temperature, or

$$T\lambda_{max} = C \qquad \text{... (4.23)}$$

where λ_{max} is in micrometers and T is in Kelvins, the value of constant C is equal to 2890.

This law gives a relationship between the wavelength at which maximum emissive power is attained and the absolute temperature.

Heat Transfer by Radiation :

A body having emissivity 'e' at temperature T_1 emits the radiant energy equal to e σT_1^4 per unit area. If the surroundings are black, none of this radiation will be reflected by them and if the surroundings are at temperature T_2, they will emit the radiation equal to σT_2^4. If a body is grey, it will absorb fraction 'e' of this energy, so that the net rate of radiant energy flow from the grey body to the black surroundings is given by the expression

$$\frac{Q}{A} = e \cdot \sigma \left(T_1^4 - T_2^4\right) \qquad \text{... (4.24)}$$

where 'e' = Emissivity of grey body.

T_1 = Absolute temperature of grey body

T_2 = Absolute temperature of surroundings.

The Equation (4.24) is also applicable when a heat source is small as compared to the surroundings (so that none of the heat radiated from the source is reflected to it), i.e. a body radiating to the atmosphere (in the calculation of heat loss from a steam pipe).

Concept of a Black Body :

A black body is the one which absorbs all radiation incident upon it, of whatever wave-length, λ. It is an ideal body that absorbs all incident radiation energy and reflects or transmits none. This means that the black body is perfectly non-reflecting and non-transmitting. Actually no matter with **a = 1** and **τ = r = 0** exists. Even the blackest surfaces occurring in nature still have reflectivity of about 1 per cent (r = 0.01).

Hence, although a black body must be black in colour, this is not a sufficient condition. Kirchhoff, however, conceived the following possibility of making a practically perfect black body. If a hollow body is provided with only one very small opening and is held at a uniform temperature, then any beam of radiation entering through the hole is partly absorbed, and partly reflected inside. The reflected radiation will not find the outlet, but will fall again on the inside of the wall. There it will be only partly reflected (other part of it is absorbed by the walls) and so on. By such a sequence of reflections, the entering radiation will be almost absorbed by the body, and an arrangement of this kind will act just as a perfectly black body as shown in Fig. 4.2.

All substances emit radiation, the quality and quantity depending upon the absolute temperature and the properties of the material composing a radiating body. It may be shown that, at a given temperature, good absorbers of any particular wavelength are also good emitter of that wavelength. Therefore, since by definition, a black body is a complete radiator of all wavelengths it is also the best possible emitter of the thermal radiation, i.e., it is a full radiator.

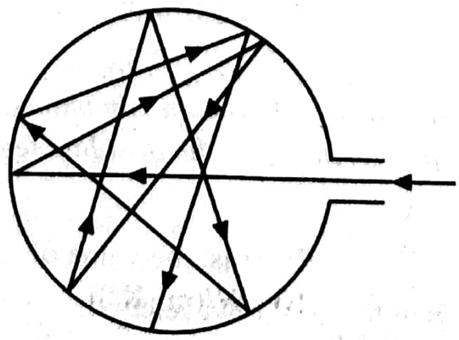


Fig. 4.2 : Black body

Transfer Coefficient for Radiation (Radiative Heat Transfer Coefficient) :

The net heat transfer by radiation from a unit surface area of a grey body at temperature T_1 to the black surroundings at temperature T_2 may be expressed as

$$Q_r = h_r (T_1 - T_2)$$

Therefore,
$$h_r = \frac{Q_r}{(T_1 - T_2)} = \frac{e \cdot \sigma}{(T_1 - T_2)} \left(T_1^4 - T_2^4\right) \quad \text{... (4.25)}$$

where h_r is the radiative heat transfer coefficient. Equation (4.21) is also applicable if the surroundings are not black, the body is small and none of its radiation is reflected back to it.

SOLVED EXAMPLES

Example 4.1 : *Calculate the heat loss by radiation from an unlagged horizontal steam pipe, 50 mm o.d. at 377 K (104°C) to air at 283 K (10°C).*

Data : Emissivity, e = 0.90.

Solution : The heat loss by radiation per unit area is given by

$$\frac{Q_r}{A} = e \cdot \sigma \cdot \left(T_1^4 - T_2^4\right)$$

where
$$e = 0.90$$
$$\sigma = 5.67 \times 10^{-8} \text{ W/(m}^2\cdot\text{K}^4)$$
$$T_1 = 377 \text{ K and } T_2 = 283 \text{ K}$$
$$\frac{Q_r}{A} = 0.90 \times 5.67 \times 10^{-8} \left(\overline{377}^4 - \overline{283}^4\right)$$
$$= \mathbf{704 \text{ W/m}^2} \quad \text{... Ans.}$$

Example 4.2 : *Calculate the rate of heat transfer by radiation from an unlagged steam pipe, 50 mm o.d., at 393 K (120°C) to air at 293 K (20°C).*

Assume emissivity 'e' of 0.9.

Solution : Given :
$$e = 0.90$$
$$T_1 = 393 \text{ K}, \quad T_2 = 293 \text{ K}$$
$$\sigma = 5.67 \times 10^{-8} \text{ W/(m}^2\cdot\text{K}^4)$$

The rate of heat transfer by radiation per unit area is

$$\frac{Q_r}{A} = e \cdot \sigma \left(T_1^4 - T_2^4\right)$$
$$= 0.90 \times 5.67 \times 10^{-8} \left(\overline{393}^4 - \overline{293}^4\right)$$
$$= \mathbf{841.2 \text{ W/m}^2} \quad \text{... Ans.}$$

Example 4.3 : *A 50 mm i.d. iron pipe at 423 K (150°C) passes through a room in which the surroundings are at temperature of 300 K (27°C). If the emissivity of the pipe metal is 0.8, what is the net interchange of radiation energy per meter length of pipe ? The outside diameter of the pipe is 60 mm.*

Solution : Length of pipe = 1 m

$e = 0.8$, $\quad \sigma = 5.67 \times 10^{-8}$ W/(m²·K⁴)

$T_1 = 423$ K, $\quad T_2 = 300$ K, $\quad D_o = 60$ mm $= 0.06$ m

Outside surface area per 1 meter length of the pipe is

$$A = \pi D_o L = \pi \times 0.06 \times 1 = 0.189 \text{ m}^2$$

The net radiation rate per 1 meter length of the pipe is

$$Q_r = e\sigma A\left(T_1^4 - T_2^4\right) = 0.8 \times 5.67 \times 10^{-8} \times 0.189\left(\overline{423}^4 - \overline{300}^4\right)$$

$$= \mathbf{205\ W/m}$$ ... **Ans.**

Example 4.4 : *Estimate the total heat loss by convection and radiation from an unlagged steam pipe, 50 mm o.d., at 415 K (142°C) to air at 290 K (17°C).*

Data : Take emissivity, $e = 0.90$

The film coefficient (h_c) to calculate heat loss by natural convection is given by

$$h_c = 1.18\,(\Delta T/D_o)^{0.25},\ \text{W/(m}^2\cdot\text{K)}$$

Solution : Outside area of pipe $= \pi D \cdot L$

Consider 1 m length of the steam pipe.

$$L = 1 \text{ m}$$

$$D_o = 50 \text{ mm} = 0.05 \text{ m} \ \ldots \text{(given)}$$

Outside area per unit length of the pipe $= \pi \times 0.05 \times 1.0 = 0.157$ m²/m

$$\Delta T = T_1 - T_2 = 415 - 290 = 125 \text{ K}$$

Let us calculate h_c.

$$h_c = 1.18\,(\Delta T/D_o)^{0.25} = 1.18\,(125/0.05)^{0.25} = 8.34 \text{ W/(m}^2\cdot\text{K)}$$

The heat loss by convection per 1 m length of the pipe is

$$Q_c = h_c \cdot A\,(T_1 - T_2)$$

$$= 8.34 \times 0.157\,(415 - 290) = 163.7 \text{ W/m}$$

The heat loss by radiation per 1 m length of the pipe is

$$Q_r = e\sigma A\left(T_1^4 - T_2^4\right)$$

$$= 0.9 \times 5.67 \times 10^{-8} \times 0.157\left(\overline{415}^4 - \overline{290}^4\right) = 181 \text{ W/m}$$

∴ The total heat loss by convection and radiation per 1 m length of the pipe is

$$Q_t = Q_c + Q_r$$

$$= 163.7 + 181$$

$$= \mathbf{344.7\ W/m}$$ ... **Ans.**

Example 4.5 : *Calculate the rate of heat loss from a 6 m long horizontal steam pipe, 60 mm o.d. when carrying steam at 800 kN/m². The temperature of the surrounding atmosphere is 290 K.*

Data : Take emissivity, $e = 0.85$ and

$\sigma = 5.67 \times 10^{-8}$ W/(m²·K⁴) – Steafan-Boltzmann constant)

The film coefficient (h_c) for heat loss by natural convection can be calculated by

$$h_c = 1.64\,(\Delta T)^{0.25},\ \text{W/(m}^2\cdot\text{K)}$$

Steam is saturated at 800 kN/m² and 443 K (170°C).

Solution : Neglecting the inside resistance and resistance of the metal wall, it may be assumed that the surface temperature of the pipe is 443 K.

Given : $T_1 = 443$ K, $T_2 = 290$ K, $D_0 = 60$ mm $= 0.06$ m

$$\Delta T = 443 - 290 = 153 \text{ K}, \quad L = 6 \text{ m}$$

For radiation from pipe :

$$\text{Surface area of pipe} = \pi D \cdot L$$
$$= \pi \times 0.06 \times 6.0 = 1.131 \text{ m}^2$$

The rate of heat loss by radiation from the pipe :

$$Q_r = e \sigma A \left(T_1^4 - T_2^4\right)$$
$$= 0.85 \times 5.67 \times 10^{-8} \times 1.131 \left(443^4 - 290^4\right)$$
$$= 1714 \text{ W}$$

The rate of heat loss by convection from the pipe :

$$Q_c = h_c A (T_1 - T_2)$$
$$= 1.64 (\Delta T)^{0.25} \times A (T_1 - T_2)$$
$$= 1.64 (153)^{0.25} \times 1.131 (443 - 290)$$
$$= 998 \text{ W}$$

$\therefore$ $$\text{Total heat loss} = Q_r + Q_c = 1714 + 998$$
$$= 2712 \text{ W}$$

... **Ans.**

Exchange of Energy between Two Parallel Plates / Planes of Different Emissivities :

Multiple Reflection Method :

When two non-black bodies are situated a small distance apart, part of the energy emitted by one body will be reflected back to it by the second body and will then be partly reabsorbed and partly reflected again. Thus the heat undergoes a series of internal reflections and absorptions.

Consider two large gray planes/surfaces that are maintained at absolute temperatures T_1 and T_2 respectively, a small distance apart and exchanging radiation. Let e_1 and e_2 be the emissivities of the surfaces.

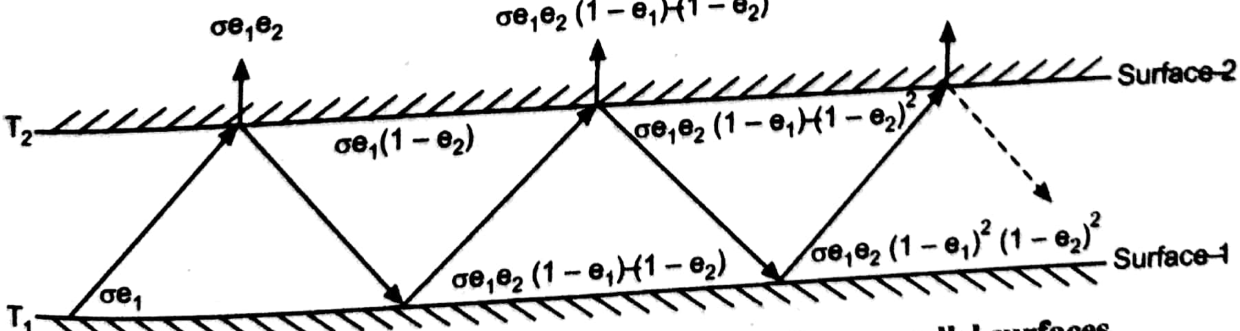


Fig. 4.3 : Radiant heat exchanger between infinite parallel surfaces (energy originating at surface-1 absorbed by surface-2)

Consider the energy radiated/emitted from the surface-1. Then, for per unit area per unit time, we have

- energy radiated from surface-1 $= \sigma \cdot e_1 T_1^4$ ... (a)
- of this, energy absorbed by surface-2 $= \sigma e_1 T_1^4 \; e_2$ ... (b)
- and energy reflected by surface-2 $= e_1 T_1^4 \; (1 - e_2)$ ... (c)
- of this, energy re-absorbed by surface-1 $= \sigma e_1 T_1^4 (1 - e_2) \; e_1$ ... (d)
- and energy re-reflected by surface-1 $= \sigma e_1 \; T_1^4 \; (1 - e_2)(1 - e_1)$ ... (e)
- and of this, energy absorbed by surface-2 $= \sigma e_1 T_1^4 (1 - e_2)(1 - e_1) \; e_2$ ... (f)

Hence, as a result of each complete cycle of internal reflection, it is clear by comparing (b) and (f) that the absorption is reduced by a factor $(1 - e_1)(1 - e_2)$. As the energy suffers an infinite number of reflections, we can write

Total transfer of energy from surface-1 to surface-2 per unit area per unit time is

$$= \sigma \cdot e_1 e_2 T_1^4 \; [1 + (1 - e_1)(1 - e_2) + (1 - e_1)^2 (1 - e_2)^2 \ldots \text{ to } \infty]$$

$$= \sigma \cdot e_1 e_2 T_1^4 \; \frac{1}{1 - (1 - e_1)(1 - e_2)}$$

$$= \frac{\sigma \cdot e_1 e_2}{e_1 + e_2 - e_1 e_2} \; T_1^4$$

In a similar manner, considering the radiation emitted by the surface 2, it can be shown that the total transfer of energy from surface 2 to surface 1 per unit area per unit time (i.e., energy emitted by the surface 2 and absorbed by the surface 1)

$$= \frac{e_1 e_2 \sigma}{e_1 + e_2 - e_1 e_2} \; T_2^4$$

Thus, the net energy transferred per unit area per unit time is

$$\left(\frac{Q}{A}\right)_{12} = \frac{e_1 e_2 \sigma}{e_1 + e_2 - e_1 e_2} \; (T_1^4 - T_2^4)$$

$$\left(\frac{Q}{A}\right)_{12} = \frac{\sigma (T_1^4 - T_2^4)}{\frac{1}{e_1} + \frac{1}{e_2} - 1} \qquad \ldots (4.26)$$

$$\left(\frac{Q}{A}\right)_{12} = \sigma \cdot F_{12} \; (T_1^4 - T_2^4) \qquad \ldots (4.27)$$

where,

$$F_{12} = \frac{1}{\frac{1}{e_1} + \frac{1}{e_2} - 1} \qquad \ldots (4.28)$$

(F_{12} is called overall interchange factor and is function of e_1 and e_2.)

Spheres or cylinders with spherical or cylindrical enclosures : The net exchange of radiative heat or radiant energy between inner and outer spheres is given by

$$Q = \frac{\sigma A_1}{\frac{1}{e_1} + \left(\frac{r_1}{r_2}\right)^2 \left(\frac{1}{e_2} - 1\right)} \; (T_1^4 - T_2^4) \qquad \ldots (4.29)$$

$$= \frac{\sigma A_1}{\frac{1}{e_1} + \frac{A_1}{A_2} \left(\frac{1}{e_2} - 1\right)} \; (T_1^4 - T_2^4) \qquad \ldots (4.30)$$

The net exchange of radiant energy between infinitely large concentric cylinders is given by

$$Q = \frac{\sigma \cdot A_1 (T_1^4 - T_2^4)}{\frac{1}{e_1} + \frac{A_1}{A_2}\left(\frac{1}{e_2} - 1\right)} = \frac{\sigma \cdot A_1 (T_1^4 - T_2^4)}{\frac{1}{e_1} + \frac{r_1}{r_2}\left(\frac{1}{e_2} - 1\right)} \quad \text{... (4.31)}$$

where A_1 and A_2 are the areas of the inner and outer cylinders/spheres respectively, e_1 and e_2 are the emissivities of the inner and outer cylindrical/spherical surfaces. T_1 and T_2 are the respective temperatures.

$$Q = \sigma A_1 F_{12} (T_1^4 - T_2^4) \quad \text{... (4.32)}$$

where $$F_{12} = \frac{1}{\frac{1}{e_1} + \frac{A_1}{A_2}\left(\frac{1}{e_2} - 1\right)} \quad \text{... (4.33)}$$

Example 4.6 : *Calculate the loss of heat by radiation from a steel tube of diameter 70 mm and 3 m long at a temperature of 500 K (227°C), if the tube is located in a square brick conduit 0.3 m side at 300 K (27°C). Assume 'e' for steel as 0.79 and for brick conduit as 0.93.*

Solution : The rate of heat loss by radiation is given by

$$Q = \frac{\sigma \cdot A_1 (T_1^4 - T_2^4)}{\frac{1}{e_1} + \frac{A_1}{A_2}\left(\frac{1}{e_2} - 1\right)}$$

where, $\sigma = 5.67 \times 10^{-8}$ W/(m²·K⁴)

e_1 = e of steel = 0.79, e_2 = e of brick = 0.93

$T_1 = 500$ K

$T_2 = 300$ K

A_1 = area of tube $= \pi \times \frac{70}{1000} \times 3 = 0.659$ m²

A_2 = area of square conduit = $4\,(0.3 \times 3) = 3.6$ m²

$$\therefore \quad Q = \frac{5.67 \times 10^{-8} \times 0.659 \times [\overline{500}^4 - \overline{300}^4]}{\frac{1}{0.79} + \frac{0.659}{3.6}\left(\frac{1}{0.93} - 1\right)}$$

$$Q = \mathbf{1588.5\ W} \quad \text{... Ans.}$$

Example 4.7 : *Calculate the net radiant heat exchange per square meter for very large planes at temperatures of 703 K (430°C) and 513 K (260°C) respectively. Assume that the emissivity of the hot and cold planes are 0.85 and 0.75 respectively.*

Solution : The net radiant heat exchange per 1 m² area between two planes is given by

$$\left(\frac{Q}{A}\right)_r = \frac{\sigma (T_1^4 - T_2^4)}{\frac{1}{e_1} + \frac{1}{e_2} - 1}$$

where $\sigma = 5.67 \times 10^{-8}$ W/(m²·K⁴)

$T_1 = 703$ K

$T_2 = 513$ K

$e_1 = 0.85$ and $e_2 = 0.75$

$$\left(\frac{Q}{A}\right)_r = \frac{5.67 \times 10^{-8}[(703)^4 - (513)^4]}{\frac{1}{0.85} + \frac{1}{0.75} - 1}$$

$$= \mathbf{6571\ W/m^2}$$ ... **Ans.**

Example 4.8 : *Determine the net radiant heat exchange between two parallel oxidised iron plates, placed at a distance of 25 mm having sides 3 × 3 m. The surface temperatures of two plates are 373 K (100°C) and 313 K (40°C) respectively. Emissivities of the plates are equal. Given : $e_1 = e_2 = 0.736$.*

Solution : The interchange factor is given by

$$F_{12} = \frac{1}{\frac{1}{e_1} + \frac{1}{e_2} - 1}$$

$$= \frac{1}{\frac{1}{0.736} + \frac{1}{0.736} - 1} = 0.5823$$

The radiant heat exchange between two parallel planes is given by

$$Q = \sigma A F_{12} \left(T_1^4 - T_2^4\right)$$

where
$$F_{12} = 0.5823$$
$$A = 3 \times 3 = 9\ m^2$$
$$\sigma = 5.67 \times 10^{-8}\ W/(m^2 \cdot K^4)$$
$$T_1 = 373\ K$$
$$T_2 = 313\ K$$

$\therefore$
$$Q = 5.67 \times 10^{-8} \times 9 \times 0.5823 \times [(373)^4 - (313)^4]$$
$$= \mathbf{2900\ W}$$ ... **Ans.**

The net radiant interchange between two parallel oxidised iron plates is 2900 W.

Example 4.9 : *Calculate the rate of heat loss from a thermoflask if the polished silvered surfaces have emissivities of 0.05. The liquid in the flask is at 368 K (95°C) and the casing is at 293 K (20°C). Calculate the loss if both the surfaces were black.*

Steafan-Boltzmann constant (σ) = 5.67×10^{-8} $W/(m^2 \cdot K^4)$

Solution : The interchange factor is given by

$$F_{12} = \frac{1}{\frac{1}{e_1} + \frac{A_1}{A_2}\left(\frac{1}{e_2} - 1\right)}$$

Given :
$$A_1 = A_2 \text{ and } e_1 = e_2 = 0.05$$

$$F_{12} = \frac{1}{\frac{1}{0.05} + \left(\frac{1}{0.05} - 1\right)} = 0.0256$$

We have :
$$T_1 = 368\ K$$
$$T_2 = 293\ K$$

The heat loss by thermal radiation per unit area of the silvered surface to the surroundings is

$$\frac{Q}{A} = \sigma F_{12} \left(T_1^4 - T_2^4\right)$$

$$= 5.67 \times 10^{-8} \times 0.0256 \, [(368)^4 - (293)^4]$$

$$= \mathbf{15.92 \ W/m^2}$$

... **Ans.**

When the two surfaces are black :

$$e_1 = e_2 = 1. \text{ Therefore,}$$

$$F_{12} = \frac{1}{\frac{1}{1} + \left(\frac{1}{1} - 1\right)} = 1$$

The heat loss by radiation in this case is

$$\frac{Q}{A} = 5.67 \times 10^{-8} \times 1 \times [(368)^4 - (293)^4] = \mathbf{622 \ W/m^2}$$

... **Ans.**

Example 4.10 : *The inner sphere of a Diwar flask is 30 cm in diameter and outer sphere is 36 cm diameter. Both spheres are coated with a material for which emissivity is 0.05. Find the rate at which liquid oxygen (latent heat = 21.44 kJ/kg) would evaporate at 90 K (– 183°C) when the outer sphere temperature is 293 K (20°C). Assume that the other modes of heat transfer are absent.*

Solution : Let us calculate the interchange factor. Denote the inner sphere by 1 and the outer by 2. The ratio of their areas is

$$\frac{A_1}{A_2} = \frac{d_1^2}{d_2^2} = \frac{(30)^2}{(36)^2} = 0.6944$$

Given : $e_1 = e_2 = 0.05$

We have :

$$F_{12} = \frac{1}{\frac{1}{e_1} + \frac{A_1}{A_2}\left(\frac{1}{e_2} - 1\right)} = \frac{1}{\frac{1}{0.05} + 0.6944\left(\frac{1}{0.05} - 1\right)} = 0.03$$

The radiation heat transfer through the walls into the flask is given by

$$\frac{Q}{A} = \sigma \cdot F_{12} \left(T_1^4 - T_2^4\right)$$

$$= 5.67 \times 10^{-8} \times 0.03 \, [\overline{293}^4 - \overline{90}^4] = 12.42 \ W/m^2$$

$$\therefore \quad Q = 12.42 \times A$$

$$= 12.42 \times \pi \, (0.3)^2 = 3.51 \ W \equiv 12.64 \ kJ/h$$

Latent heat of vaporisation liquid oxygen = 21.44 kJ/kg

Amount of oxygen evaporated = $\dot{m} = \frac{12.64}{21.44} = \mathbf{0.59 \ kg/h}$ ... **Ans.**

Example 4.11 : *Liquid oxygen at atmospheric pressure (boiling point = 90 K (– 183°C) is stored in a spherical vessel of 300 mm outside diameter. The system is insulated by enclosing the container inside another concentric sphere of 500 mm inside diameter with the space between them evacuated. Both the sphere surfaces are made of aluminium for which emissivity may be taken as 0.3. The temperature of the outer sphere is 313 K (40°C).*

Calculate the rate of heat flow by radiation.

What will be the reduction in heat flow if a polished aluminium with an emissivity of 0.5 is used for the container walls ?

Solution : Case-I : The rate of heat flow by radiation is given by

$$Q = \frac{\sigma \cdot A_1 \left(T_1^4 - T_2^4\right)}{\frac{1}{e_1} + \frac{A_1}{A_2}\left(\frac{1}{e_2} - 1\right)}$$

where

$$\begin{aligned}
\sigma &= 5.67 \times 10^{-8}\ \text{W/(m}^2\cdot\text{K}^4\text{)} \\
e_1 &= e_2 = 0.3 \\
T_1 &= 90\ \text{K} \\
T_2 &= 313\ \text{K} \\
A_1 &= \pi D_1^2 = \pi \times (0.3)^2 = 0.283\ \text{m}^2 \\
A_2 &= \pi D_2^2 = \pi \times (0.5)^2 = 0.785\ \text{m}^2
\end{aligned}$$

$$Q = \frac{5.67 \times 10^{-8} \times 0.283\ [\overline{90}^4 - \overline{313}^4]}{\frac{1}{0.3} + \frac{0.283}{0.785}\left[\frac{1}{0.3} - 1\right]} = -36.64\ \text{W}$$

$$\text{Rate of heat flow} = 36.64\ \text{W}$$

Case-II : Let us calculate Q using the polished aluminium for the inner pipe.

$$\begin{bmatrix}\text{Emissivity of} \\ \text{polished aluminium}\end{bmatrix} = e_1 = 0.05$$

$$\begin{bmatrix}\text{Emissivity of} \\ \text{aluminium}\end{bmatrix} = e_2 = 0.5$$

The rate of heat flow by radiation is

$$Q = \frac{\sigma A_1 \left(T_1^4 - T_2^4\right)}{\frac{1}{e_1} + \frac{A_1}{A_2}\left(\frac{1}{e_2} - 1\right)}$$

$$Q = \frac{5.67 \times 10^{-8} \times 0.283\ [(90)^4 - (313)^4]}{\frac{1}{0.05} + \frac{0.283}{0.785}\left(\frac{1}{0.30} - 1\right)} = -7.34\ \text{W}$$

$$\text{Rate of heat flow} = -7.34\ \text{W}$$

$$\therefore \quad \begin{bmatrix}\text{Reduction in} \\ \text{the heat flow}\end{bmatrix} = \frac{36.64 - 7.34}{36.64} \times 100 = \mathbf{79.97\ \%}$$

... **Ans.**

Example 4.12 : *Liquid nitrogen boiling at 77 K (– 196°C) is stored in a 15 litre spherical container of diameter 32 cm. The container is surrounded by a concentric spherical shell of diameter 36 cm at a temperature of 303 K (30°C) and the space between the two spheres is evacuated. The surfaces of the spheres facing each other are silvered and have an emissivity of 0.03. Take the latent heat of vaporisation for liquid nitrogen as 201 kJ/kg. Find the rate at which the nitrogen evaporates.*

Solution : The radiant heat exchange rate between the inner and outer shell or the radiation heat transfer through the walls into the container is given by

$$Q = \frac{\sigma A_1 \left(T_1^4 - T_2^4\right)}{\frac{1}{e_1} + \frac{A_1}{A_2}\left(\frac{1}{e_2} - 1\right)}$$

where, T_1 and T_2 are temperatures of the inner and outer surfaces, respectively.

$$\sigma = 5.67 \times 10^{-8}\ W/(m^2 \cdot K^4)$$
$$T_1 = 77\ K$$
$$T_2 = 303\ K$$
$$D_1 = 32\ cm = 0.32\ m, \quad D_2 = 36\ cm = 0.36\ m$$
$$A_1 = \pi D_1^2 = \pi (0.32)^2 = 0.3217\ m^2$$
$$A_2 = \pi D_2^2 = \pi (0.36)^2 = 0.407\ m^2$$
$$e_1 = e_2 = 0.3$$

$$Q = \frac{5.67 \times 10^{-8} \times 0.3217\ [\overline{77}^4 - \overline{303}^4]}{\frac{1}{0.03} + \frac{0.3217}{0.407}\left(\frac{1}{0.03} - 1\right)}$$

$$Q = -2.63\ W \equiv -9.5\ kJ/h$$

Rate of radiant heat transfer = 2.63 W ≡ 9.5 kJ/h

Latent heat of vaporisation of liquid nitrogen = 201 kJ/kg

where $\dot{m}$ is the rate of vaporisation of liquid nitrogen.

Rate of evaporation rate $= \dot{m} = \frac{Q}{\lambda} = \frac{9.5}{201} = 0.047$ kg/h ... **Ans.**

Example 4.13 : *A space between the two concentric spherical vessels is completely evacuated. The inner sphere contains air at 76 K (– 197°C). The ambient temperature is 300 K (27°C). The surfaces of the spheres are highly polished (e = 0.04). Find the rate of evaporation of liquid air per hour.*

Diameter of inner sphere = 250 mm

Diameter of outer sphere = 350 mm

Latent heat of vaporisation of air = 200 kJ/kg

Solution : The rate of radiation heat transfer the walls into the vessel is given by the relation :

$$Q = \frac{\sigma A_1 \left(T_1^4 - T_2^4\right)}{\frac{1}{e_1} + \frac{A_1}{A_2}\left(\frac{1}{e_2} - 1\right)} = \frac{\sigma \pi D_1^2 \left(T_1^4 - T_2^4\right)}{\frac{1}{e_1} + \left(\frac{D_1^2}{D_2^2}\right)\left(\frac{1}{e_2} - 1\right)}$$

where,

$$\sigma = 5.67 \times 10^{-8}\ W/(m^2 \cdot K^4)$$
$$A_1 = \pi D_1^2, \qquad A_2 = \pi D_2^2$$
$$D_1 = 250\ mm = 0.25\ m, \quad D_2 = 350\ mm = 0.35\ m$$
$$T_1 = 76\ K$$
$$T_2 = 300\ K$$

$$Q = \frac{5.67 \times 10^{-8} \times \pi (0.25)^2\ [\overline{76}^4 - \overline{300}^4]}{\frac{1}{0.04} + \left(\frac{0.25}{0.35}\right)^2 \left[\frac{1}{0.04} - 1\right]} = -2.45\ W$$

Rate of radiant heat transfer or heat flow by radiation

= 2.45 W ≡ 2.45 J/s ≡ 8.82 kJ/h

Rate of evaporation of liquid air $= \dot{m} = \frac{Q}{\lambda} = \frac{8.82}{200} = \mathbf{0.0441\ kg/h}$ ... **Ans.**

Radiation Shields :

In order to reduce the heat transfer by radiation between two surfaces, a third surface is introduced in between them. This surface is known as a radiation shield.

A shield or wall of material interposed between a source of radiation and a radiation sensitive body to protect the body.

Radiation shields increase the surface resistance without removing any heat from the overall system. Thin sheets of plastic that are coated, on both sides, with highly reflecting metallic films act as very effective radiation shields. They are used for the insulation of cryogenic storage tanks.

Suppose two infinite and parallel planes (each of area A) at temperatures T_1 and T_3 are separated by a third plane that is opaque to direct radiation between the two and which is extremely thin (radiation shield) as shown in Fig. 4.4. The net heat exchange between two initial parallel planes (i.e., without a radiation shield) is given by

$$Q = \frac{\sigma A}{\frac{1}{e_1} + \frac{1}{e_3} - 1} (T_1^4 - T_3^4)$$

where e_1 and e_3 are the emissivities of the planes.

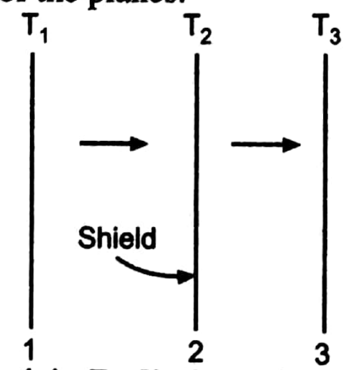


Fig. 4.4 : Radiation with shield

Let e_2 be the emissivity of the radiation shield.

With shield, the net heat exchange from 1 to 3 is given by

$$Q_1 = \frac{\sigma A}{\frac{1}{e_1} + \frac{1}{e_2} - 1} (T_1^4 - T_2^4) = \frac{\sigma A}{\frac{1}{e_2} + \frac{1}{e_3} - 1} (T_2^4 - T_3^4)$$

[$Q_1 = Q_{1-2} = Q_{2-3}$ as the shield does deliver or remove heat from the system.]

If $e_1 = e_3$, then

$$T_2^4 = \frac{1}{2} (T_1^4 - T_3^4)$$

Then,

$$Q_1 = \frac{\sigma A}{\frac{1}{e_1} + \frac{1}{e_2} - 1} \frac{1}{2} (T_1^4 - T_3^4)$$

If $e_1 = e_2 = e$, then

$$Q_1 = \frac{\sigma \cdot A}{\frac{2}{e} - 1} \times \frac{1}{2} (T_1^4 - T_3^4)$$

for

$$e_1 = e_3 = e$$

$$Q = \frac{\sigma A}{\frac{2}{e} - 1}\left(T_1^4 - T_3^4\right)$$

$$\therefore \quad Q_1 = \frac{1}{2} Q$$

It shows that due to insertion of one shield between two parallel planes/surfaces, the radiation heat transfer rate between them reduces to half of the initial value (i.e., the radiation heat transfer rate is halved).

For the simple case when 'n' shields are employed, each having the same emissivities as the initial planes,

$$Q_n = \frac{1}{n+1} Q$$

$$Q \text{ with 'n' shields} = \frac{1}{n+1} Q \text{ without shield}$$

where Q is the net heat exchange if the initial planes were not separated by shields.

Example 4.14 : *A double walled flask may be idealised to be equivalent to two infinite parallel plates. The emissivities of walls are 0.3 and 0.7 respectively. The space between them is evacuated. A shield of polished aluminium of e = 0.05 is inserted between them. Find the reduction in heat transfer due to insertion of the radiation shield.*

Solution :

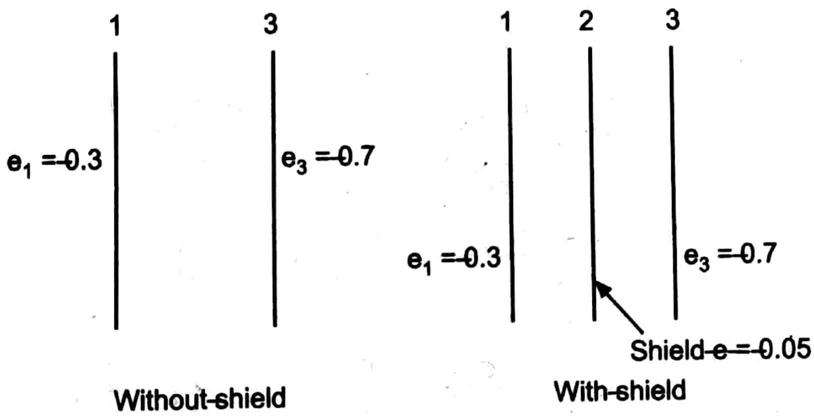


Fig. E 4.14

In the absence of shield, the radiant heat transfer rate is given by

$$Q = \frac{\sigma A\left(T_1^4 - T_3^4\right)}{\frac{1}{e_1} + \frac{1}{e_3} - 1} \quad \dots (1)$$

$$= \frac{\sigma A\left(T_1^4 - T_3^4\right)}{\frac{1}{0.3} + \frac{1}{0.7} - 1} = \frac{\sigma A\left(T_1^4 - T_3^4\right)}{3.76} = \frac{Z}{3.76}$$

[assuming $\sigma A\left(T_1^4 - T_3^4\right) = Z$]

With shield, the radiant heat transfer rate is given by

$$Q_1 = \frac{\sigma A}{\frac{1}{e_1}+\frac{1}{e_2}-1}(T_1^4 - T_2^4) = \frac{\sigma A}{\frac{1}{e_2}+\frac{1}{e_3}-1}(T_2^4 - T_3^4) \quad \dots (2)$$

$$[Q_1 = Q_{1-2} = Q_{2-3}]$$

Therefore,

$$\frac{\sigma A}{\frac{1}{0.3}+\frac{1}{0.05}-1}(T_1^4 - T_2^4) = \frac{\sigma A}{\frac{1}{0.05}+\frac{1}{0.7}-1}(T_2^4 - T_3^4)$$

$$\frac{T_1^4 - T_2^4}{22.33} = \frac{T_2^4 - T_3^4}{20.42}$$

$$T_1^4 - T_2^4 = 1.093\,(T_2^4 - T_3^4)$$

$$T_1^4 + 1.093\,T_3^4 = 2.093\,T_2^4$$

$$\frac{T_1^4 + 1.093\,T_3^4}{2.093} = T_2^4$$

Substituting the value of T_2^4, Equation (2) becomes

$$Q_1 = \frac{\sigma A}{\frac{1}{e_1}+\frac{1}{e_2}-1}(T_1^4 - T_2^4)$$

$$= \frac{\sigma A}{\frac{1}{e_1}+\frac{1}{e_2}-1}\left[T_1^4 - \frac{(T_1^4 + 1.093 + T_3^4)}{2.093}\right]$$

$$= \frac{\sigma A}{\frac{1}{e_1}+\frac{1}{e_2}-1}\left[\frac{1.093}{2.093}\right][T_1^4 - T_3^4]$$

$$= \frac{\sigma A \times 1.093}{\left(\frac{1}{0.3}\times\frac{1}{0.05}-1\right)(2.093)} \times (T_1^4 - T_3^4)$$

$$= 0.02338\,\sigma \cdot A\,(T_1^4 - T_3^4) = \frac{\sigma \cdot A\,(T_1^4 - T_3^4)}{42.766} = \frac{Z}{42.766}$$

$$\begin{bmatrix}\text{Reduction in}\\ \text{heat transfer using shield}\end{bmatrix} = \frac{Q - Q_1}{Q} \times 100 = \frac{\frac{Z}{3.76} - \frac{Z}{42.766}}{\frac{Z}{3.76}} \times 100$$

$$= \mathbf{91.2\ \%} \quad \dots \text{Ans.}$$

Example 4.15 : *Find the heat transfer rate per unit area due to radiation between two infinitely long parallel planes. The first plane has an emissivity of 0.4 and is maintained at 473 K (200°C). The emissivity of the second plane is 0.2 and is maintained at 300 K (30°C). If a radiation shield having e = 0.5 is interposed between the given planes, find the percentage reduction in heat transfer rate and the steady-state temperature attained by the shield.*

Solution : The radiant heat transfer without the shield is

$$\frac{Q}{A} = \frac{\sigma\left(T_1^4 - T_3^4\right)}{\frac{1}{e_1} + \frac{1}{e_3} - 1}$$

where,

$$\sigma = 5.67 \times 10^{-8}\ \text{W/(m}^2\cdot\text{K}^4)$$
$$e_1 = 0.4, \qquad e_3 = 0.2$$
$$T_1 = 473\ \text{K}$$
$$T_2 = 303\ \text{K}$$

$$\frac{Q}{A} = \frac{5.67 \times 10^{-8}\left[\overline{473}^4 - \overline{303}^4\right]}{\frac{1}{0.4} + \frac{1}{0.2} - 1}$$

$$= \mathbf{363.1\ W/m^2}$$ ... **Ans.**

When the shield is introduced, the rate of heat transfer is

$$\frac{Q_1}{A} = \frac{\sigma\left(T_1^4 - T_2^4\right)}{\frac{1}{e_1} + \frac{1}{e_2} - 1} = \frac{\sigma A\left(T_2^4 - T_3^4\right)}{\frac{1}{e_2} + \frac{1}{e_3} - 1}$$

where $e_2 = 0.5$ and T_2 is the temperature of the shield in K.

The temperature of the shield can be obtained by equating the heat exchange between each plate and the shield.

$$\frac{T_1^4 - T_2^4}{\frac{1}{e_1} + \frac{1}{e_2} - 1} = \frac{T_2^4 - T_3^4}{\frac{1}{e_2} + \frac{1}{e_3} - 1}$$

$$\frac{\overline{473}^4 - T_2^4}{\frac{1}{0.4} + \frac{1}{0.5} - 1} = \frac{T_2^4 - \overline{303}^4}{\frac{1}{0.5} + \frac{1}{0.2} - 1}$$

$$\overline{473}^4 - T_2^4 = \frac{3.5}{6}\left[T_2^4 - \overline{303}^4\right]$$

Solving, we get $T_2 = 431.66$ K

Temperature of the shield $= 431.66$ K ($158.66°$C)

$$\frac{Q_1}{A} \text{ with shield} = \frac{\sigma\left(T_1^4 - T_2^4\right)}{\frac{1}{e_1} + \frac{1}{e_2} - 1}$$

$$= \frac{5.67 \times 10^{-8}\left[\overline{473}^4 - \overline{431.66}^4\right]}{\frac{1}{0.4} + \frac{1}{0.5} - 1} = 248.44\ \text{W/m}^2$$

$$\left[\begin{array}{l}\text{\% Reduction in} \\ \text{heat loss}\end{array}\right] = \frac{363.1 - 248.44}{363.1} \times 100$$

$$= \mathbf{31.58}$$ ... **Ans.**

Example 4.16 : *Two long planes A and B are maintained at 600 K (327°C) and 300 K (27°C) and their surface emissivities are 0.8 and 0.5, respectively. Two thin radiation shields C and D having emissivities 0.5 and 0.4 are introduced between the given planes. The given planes are in order A, C, D and B. Assuming all the planes to be infinitely long, find the rate of heat exchange per unit area and steady-state temperatures attained by the planes C and D.*

Solution : Since the shield does not deliver or remove heat from the system, the heat exchange between C-D is equal to that between D-B.

In steady state, thus we can write

$$Q_{CD} = Q_{DB}$$

$$\frac{\sigma\left(T_C^4 - T_D^4\right)}{\frac{1}{e_C}+\frac{1}{e_D}-1} = \frac{\sigma\left(T_D^4 - T_B^4\right)}{\frac{1}{e_D}+\frac{1}{e_B}-1}$$

$$e_C = 0.5,\ e_D = 0.4,\quad e_B = 0.5$$

$$\frac{T_C^4 - T_D^4}{\frac{1}{0.5}+\frac{1}{0.4}-1} = \frac{T_D^4 - T_B^4}{\frac{1}{0.4}+\frac{1}{0.5}-1}$$

$$T_C^4 - T_D^4 = T_D^4 - T_B^4$$

$$T_B = 300\text{ K}$$

$$T_D^4 = \frac{1}{2}\left(T_C^4 - T_B^4\right)$$

$$T_D^4 = \frac{1}{2}\left[T_C^4 - 300^4\right] \qquad \ldots (1)$$

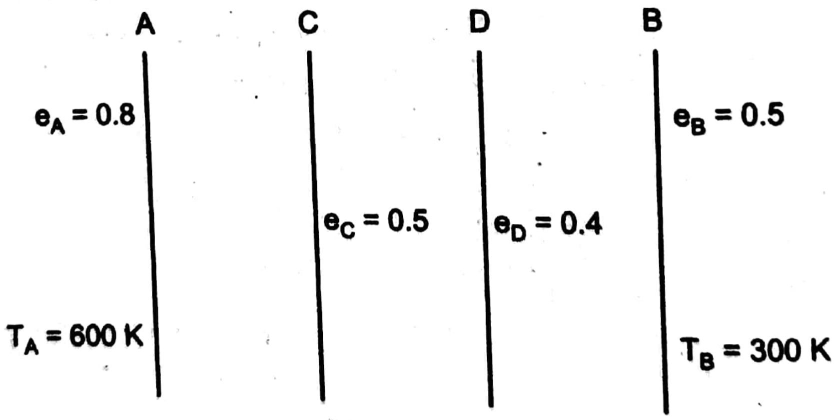


Fig. E 4.16

Similarly,

$$Q_{AC} = Q_{CD}$$

$$\frac{\sigma\left(T_A^4 - T_C^4\right)}{\frac{1}{e_A}+\frac{1}{e_C}-1} = \frac{\sigma\left(T_C^4 - T_D^4\right)}{\frac{1}{e_C}+\frac{1}{e_D}-1}$$

$$T_A = 600\text{ K}$$

$$e_A = 0.8,\qquad e_C = 0.5,\qquad e_D = 0.4$$

$$\frac{\sigma\left[\overline{600}^4 - T_C^4\right]}{\frac{1}{0.8}+\frac{1}{0.5}-1} = \frac{\sigma\left(T_C^4 - T_D^4\right)}{\frac{1}{0.5}+\frac{1}{0.4}-1}$$

$$\frac{\overline{600}^4 - T_C^4}{2.25} = \frac{T_C^4 - T_D^4}{3.5}$$

$$1.56\left[\overline{600}^4 - T_C^4\right] = T_C^4 - T_D^4 \qquad \ldots (2)$$

Substituting the value of T_D in terms of T_C from Equation (1) in Equation (2), we get

$$1.56\left[\overline{600}^4 - T_C^4\right] = T_C^4 - \frac{1}{2}\left[T_C^4 - \overline{300}^4\right]$$

Solving, we get $\quad T_C = \mathbf{560.94\ K\ (287.94°C)}$ ... **Ans.**

We have :

$$T_D^4 = \frac{1}{2}\left[T_C^4 - \overline{300}^4\right]$$

$$= \frac{1}{2}\left[\overline{560.94}^4 - \overline{300}^4\right]$$

$$T_D = 461.73\ K \quad (188.73°C)$$

The rate of heat exchange per unit area is

$$\frac{Q}{A} = \frac{\sigma\left(T_A^4 - T_C^4\right)}{\frac{1}{e_A}+\frac{1}{e_C}-1} = \frac{5.67\times10^{-8}\left[\overline{600}^4 - \overline{560.94}^4\right]}{\frac{1}{0.8}+\frac{1}{0.5}-1}$$

$$= \mathbf{770.94\ W/m^2}$$ ... **Ans.**

Example 4.17 : *A 200 mm outside diameter pipe carrying steam runs in a large room and is exposed to air at a temperature of 303 K (30°C). The temperature of the pipe surface is 673 K (400°C). Calculate the loss of heat to the surroundings per meter length of the pipe due to thermal radiation. The emissivity of the pipe material is 0.8.*

What would be the loss of heat due to radiation if the pipe is enclosed in a 400 mm diameter brick conduit of emissivity 0.91 ?

Solution : Case - I : Pipe to surroundings

$$Q_1 = e\cdot A_1\sigma\left(T_1^4 - T_2^4\right)$$

where $\quad e = 0.8, \quad \sigma = 5.67\times10^{-8}\ W/(m^2\cdot K^4)$

$T_1 = 674\ K, \quad T_2 = 303\ K$

$D_o = 200\ mm = 0.2\ m$

Consider 1 m length of the pipe.

$$L = 1\ m$$

$$A_1 = \pi D\cdot L = \pi\times0.2\times1.0 = 0.628\ m^2/m$$

The loss of heat per 1 m length of the pipe is

$$Q_1 = 0.8\times0.628\times5.67\times10^{-8}\left[\overline{674}^4 - \overline{303}^4\right]$$

$$= \mathbf{5.6\times10^3\ W/m}$$ ... **Ans.**

Case - II : Concentric cylinders

$$Q_2 = \frac{\sigma A_1 \left(T_1^4 - T_2^4\right)}{\frac{1}{e_1} + \frac{A_1}{A_2}\left(\frac{1}{e_2} - 1\right)} = \frac{\sigma A_1 \left(T_1^4 - T_2^4\right)}{\frac{1}{e_1} + \frac{D_1}{D_2}\left(\frac{1}{e_2} - 1\right)}$$

where
$$\sigma = 5.67 \times 10^{-8}\ \text{W/(m}^2\cdot\text{K}^4)$$
$$e_1 = 0.8, \quad e_2 = 0.91$$
$$D_1 = 0.2\ \text{m}, \quad D_2 = 0.40\ \text{m}$$
$$A_1 = 0.628\ \text{m}^2\text{/m length}$$

$$Q_2 = \frac{5.67 \times 10^{-8} \times 0.628 \left(\overline{673}^4 - \overline{303}^4\right)}{\frac{1}{0.8} + \frac{0.20}{0.40}\left(\frac{1}{0.91} - 1\right)}$$

$$= \mathbf{5.39 \times 10^3\ W/m} \qquad \text{... Ans.}$$

$\therefore$ Reduction in heat loss $= 5.6 \times 10^3 - 5.39 \times 10^3$

$= \mathbf{210\ W/m}$ ... **Ans.**

Example 4.18 : *Two concentric tubes A and B with diameters 30 cm and 25 cm respectively are maintained at temperatures of 813 K (540°C) and 473 K (200°C). The emissivity of tube A is 0.87 and that of B is 0.26. Determine the net heat transfer by radiation between the surfaces of tubes expressed in watts for each square meter of B.*

Solution : For concentric cylinders, we have

$$Q/A_1 = \frac{\sigma \left(T_1^4 - T_2^4\right)}{\frac{1}{e_1} + \frac{D_1}{D_2}\left(\frac{1}{e_2} - 1\right)}$$

where
$$\sigma = 5.67 \times 10^{-8}\ \text{W/(m}^2\cdot\text{K}^4)$$
$$T_1 = 813\ \text{K}, \quad T_2 = 473\ \text{K}$$
$$e_1 = 0.87, \quad e_2 = 0.26$$
$$D_1 = 25\ \text{cm} = 0.25\ \text{m}$$
$$D_2 = 30\ \text{cm} = 0.30\ \text{m}$$

$$Q/A_1 = \frac{5.67 \times 10^{-8} \left(\overline{813}^4 - \overline{473}^4\right)}{\frac{1}{0.87} + \frac{0.25}{0.30}\left[\frac{1}{0.26} - 1\right]}$$

$$= \mathbf{6228\ W/m^2} \qquad \text{... Ans.}$$

Radiation Shape Factor (F') :

The concept of radiation shape factor is useful in the analysis of radiant heat transfer between two surfaces. It is also called as view factor, geometric configuration factor or simply configuration factor. The shape factor depends upon the shape and size of surfaces, the orientation of the surfaces w.r.t. one another and the distance between them. The shape factor is defined as *the fraction of the radiant energy that is emitted from one surface and intercepted by the other surface directly without intervening reflections*. It is represented by the symbol F_{mn}

which means the shape factor from a surface A_m to another surface A_n. (F'_{mn} represents fraction of energy leaving the surface 'm' which reaches the surface 'n').

Consider two black surfaces A_1 and A_2 at uniform temperatures T_1 and T_2 respectively between which there is a net interchange/exchange of thermal radiation.

The energy leaving surface-1 and arriving at surface-2, is $E_{b_1} A_1 F'_{12}$ and that between 2 and 1 is $E_{b_2} A_2 F'_{21}$.

The net energy exchange between A_1 and A_2 is

$$Q_{12} = Q = E_{b_1} A_1 F'_{12} - E_{b_2} A_2 F'_{21} \quad \text{... (4.34)}$$

If both the surfaces are at the same temperature, then $Q_{12} = 0$ and $E_{b_1} = E_{b_2}$

$$\therefore \quad A_1 F'_{12} = A_2 F'_{21} \quad \text{... (4.35)}$$

The above equation is called as the *reciprocity theorem.*

In general, for exchange of heat between any two surfaces, we can write

$$A_m F'_{mn} = A_n F'_{nm}$$

The net heat exchange is

$$Q = Q_{12} = A_1 F'_{12} (E_{b_1} - E_{b_2}) = A_2 F'_{21} (E_{b_1} - E_{b_2})$$

$$= A_1 F'_{12} \sigma\left(T_1^4 - T_2^4\right) = A_2 F'_{21} \sigma\left(T_1^4 - T_2^4\right)$$

Mathematically, F'_{12} is given as

$$F'_{12} = Q_2/Q_1 \quad \text{... (4.36)}$$

where Q_1 is the energy emitted by surface-1 and Q_2 is the energy intercepted directly by the surface-2 out of Q_1.

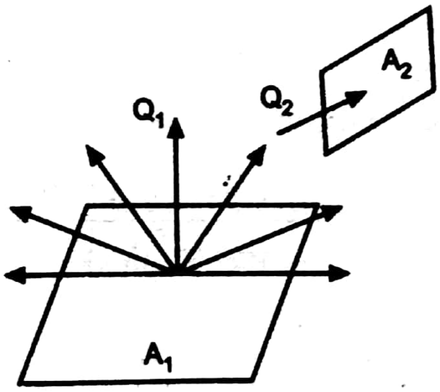


Fig. 4.5

If surface A_1 can see only surface A_2, then the shape factor F'_{12} is unity.

Consider a case of enclosure in which one surface is exchanging radiant energy with all other surfaces in the enclosure including itself, if the surface is concave. This is due to the fact that it can view another part of it. On contrary, a convex or flat surface cannot see any other part of it. The shape factor of a convex surface with its enclosure is always unity since all the heat radiated from a convex surface is intercepted by its enclosure and not vice versa.

If n surfaces form an enclosure then the energy radiated/emitted from one surface is always intercepted by the other (n – 1) surfaces as well as by the surface itself if the surface is concave-.

$$F'_{11} + F'_{12} + F'_{13} + \dots + F'_{1n} = 1.0 \qquad \dots (4.37)$$

$$F'_{21} + F'_{22} + F'_{23} + \dots + F'_{2n} = 1.0 \qquad \dots (4.38)$$

$$F'_{n1} + F'_{n2} + F'_{n3} + \dots + F'_{nn} = 1.0 \qquad \dots (4.39)$$

F'_{11}, F'_{22}, F'_{nn} are the shape factors with respect to surface itself.

Shape factor with respect to itself is *the fraction of incident energy emitted by the surface that gets intercepted by itself.*

When the surface is concave, it has a shape factor with respect to itself. But for plane or convex surface, the shape factor with respect to itself is zero.

$$\therefore \qquad F_{nn} = 0 \text{ for a convex or a flat surface}$$

$$\text{and} \qquad F'_{nn} \neq 0 \text{ for a concave surface}$$

Example 4.19 : *Two parallel black plates 0.5 by 1.0 m are spaced 0.5 m apart, plate 1 is maintained at 1273 K (1000°C) and plate 2 is maintained at 773 K (500°C). What is the net radiant heat exchange between the plates ?*

Take $F'_{12} = 0.285$

Solution : The net radiant heat exchange between the plates is

$$Q = A_1 F'_{12} (E_{b_1} - E_{b_2}) = \sigma A_1 F'_{12} \left(T_1^4 - T_2^4\right)$$

where

$$\sigma = 5.67 \times 10^{-8} \text{ W/(m}^2 \cdot \text{K}^4)$$

$$A_1 = 0.5 \times 1 = 0.5 \text{ m}^2$$

$$F'_{12} = 0.285$$

$$T_1 = 1273 \text{ K and } T_2 = 773 \text{ K}$$

$$Q = 5.67 \times 10^{-8} \times 0.5 \times 0.285 \left(\overline{1273}^{\,4} - \overline{773}^{\,4}\right)$$

$$= \mathbf{18333 \text{ W}}$$

Ans.

Electrical Network Analogy for Thermal Radiation Systems :

The electrical network analogy is an alternate approach for analysing thermal radiation between gray or black surfaces in which case a radiation problem is transferred to an equivalent electrical problem. The terms used in the electrical analogy approach are irradiation and radiosity. This method gives a simpler formula for estimating the rate of flow by comparison with Ohm's law.

$$Q = \frac{\text{Equivalent potential difference between surfaces}}{\text{Equivalent thermal resistance of the system}} \quad \ldots (4.40)$$

For black surfaces, the radiation transfer can be estimated from the relation

$$Q = A_1 \sigma F'_{12} \left(T_1^4 - T_2^4\right) = \frac{\sigma\left(T_1^4 - T_2^4\right)}{1/\left(A_1 \cdot F'_{12}\right)} \quad \ldots (4.41)$$

$\therefore$ Equivalent potential difference $= \sigma\left(T_1^4 - T_2^4\right)$

Equivalent thermal resistance $= 1/\left(A_1 \cdot F'_{12}\right)$

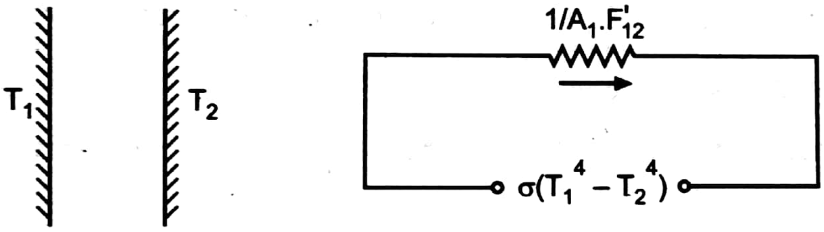


Fig. 4.6 : Equivalent electrical circuit for black surfaces

Radiosity-Irradiation Approach :

Radiosity (J) : It is the *total radiation leaving a surface (i.e., the total amount of energy leaving a surface) per unit time per unit area.* It is the sum of the energy emitted and energy reflected by the surface.

Irradiation (G) : It is the *total amount of radiation incident upon a surface per unit time per unit area.*

Let us consider an elementary grey surface A_1 at T_1 having an emissivity of e_1.

Let E_b be the emissive power of the surface.

Let G be the total radiation incident upon the surface.

Let J be the radiosity which is the sum of the energy emitted and energy reflected when no energy is transmitted.

Then, the net energy leaving a surface is the difference between the radiosity and the irradiation of the surface.

$$\therefore \quad Q/A = J - G \quad \ldots (4.42)$$

$$J = e E_b + rG \quad \ldots (4.43)$$

As the transmissivity, $\tau = 0$, we have

$$a + r + \tau = 1 \quad \text{but} \quad \tau = 0 \quad \therefore \; r = 1 - a = 1 - e$$

$$\therefore \quad J = e\,E_b + (1 - e)\,G \qquad \dots (4.44)$$

$$J - e\,E_b = (1 - e)\,G$$

$$G = J - e\,E_b/(1 - e) \qquad \dots (4.45)$$

$$\therefore \quad \frac{Q}{A} = J - \frac{J - e\,E_b}{(1 - e)} \qquad \dots (4.46)$$

$$= \frac{J\,(1 - e) - J + e\,E_b}{(1 - e)}$$

$$\frac{Q}{A} = \frac{(E_b - J)}{(1 - e)}$$

$$Q = \frac{e\,A\,(E_b - J)}{(1 - e)} = \frac{E_b - J}{(1 - e)/eA} \qquad \dots (4.47)$$

Comparison of the above equation with Ohm's law ($I = \Delta V/R$), i.e., by considering the net flow rate of heat as the current and ($E_b - T$) as the potential difference gives

$$\text{Equivalent resistance} = (1 - e)/eA \qquad \dots (4.48)$$

As this resistance depends upon the property of a surface (emissivity), it is known as a **surface resistance.**

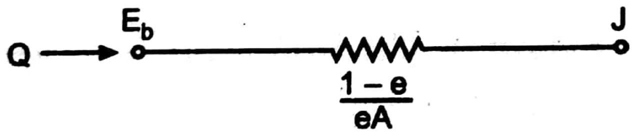


Fig. 4.7 : Surface resistance in radiation network method

Now, consider the exchange of radiant energy between two surfaces A_1 and A_2. In this case, there is a restriction on the free flow of energy between two surfaces owing to their orientation (shape) and this restriction may be referred as a shape resistance. The shape resistance can be obtained as follows.

Of the total radiant energy that leaves surface-1, the energy that reaches surface-2 is $A_1 F'_{12} J_1$ and of the total radiant energy leaving surface-2, an energy that reaches surface-1 is $A_2 F'_{21} J_2$

The net interchange or exchange of energy from surface-1 to surface-2 is

$$Q_{12} = A_1 F'_{12} J_1 - A_2 F'_{21} J_2 \qquad \dots (4.49)$$

But $$A_1 F'_{12} = A_2 F'_{21}$$

$$Q_{12} = A_1 F'_{12} (J_1 - J_2) = \frac{(J_1 - J_2)}{1/A_1 F'_{12}} \qquad \dots (4.50)$$

Comparing with Ohm's law, we get

$$\text{Shape resistance} = 1/A_1 F'_{12}$$

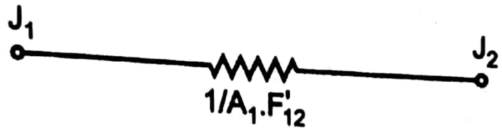


Fig. 4.8 : Element representing shape resistance in radiation network method

Hence, two surfaces which exchange heat may each be considered as having a surface resistance of $(1 - e)/eA$ and a shape resistance of $1/A_1\ F'_{12}$ between their radiosity potential. To construct a network for the radiation heat transfer between the surfaces, we only need to connect a surface resistance $[1 - 1/eA]$ to each surface and a shape resistance $1/A_m\ F'_{mn}$ between the radiosity potentials. For example, in case of two surfaces which exchange heat with each other, the radiation network is as shown in Fig. 4.9.

For this case, the net heat transfer/exchange is the ratio of the overall potential difference to the sum of resistances, as given by

$$Q_{net} = \frac{E_{b_1} - E_{b_2}}{(1 - e_1)/e_1 A_1 + 1/A_1 F'_{12} + (1 - e_2)/e_2 A_2} \quad \ldots (4.51)$$

$$= \frac{\sigma\left(T_1^4 - T_2^4\right)}{(1 - e_1)/e_1 A_1 + 1/A_1 F'_{12} + (1 - e_2)/e_2 A_2} \quad \ldots (4.52)$$

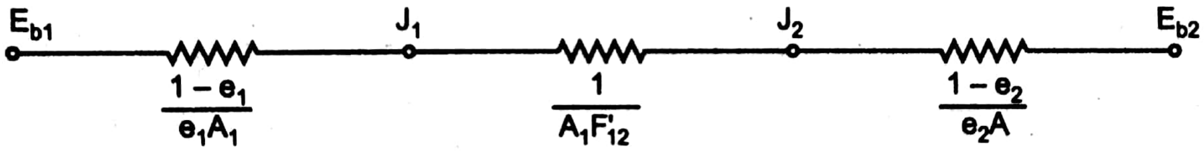


Fig. 4.9 : Radiation network for two surfaces which see each other and nothing else

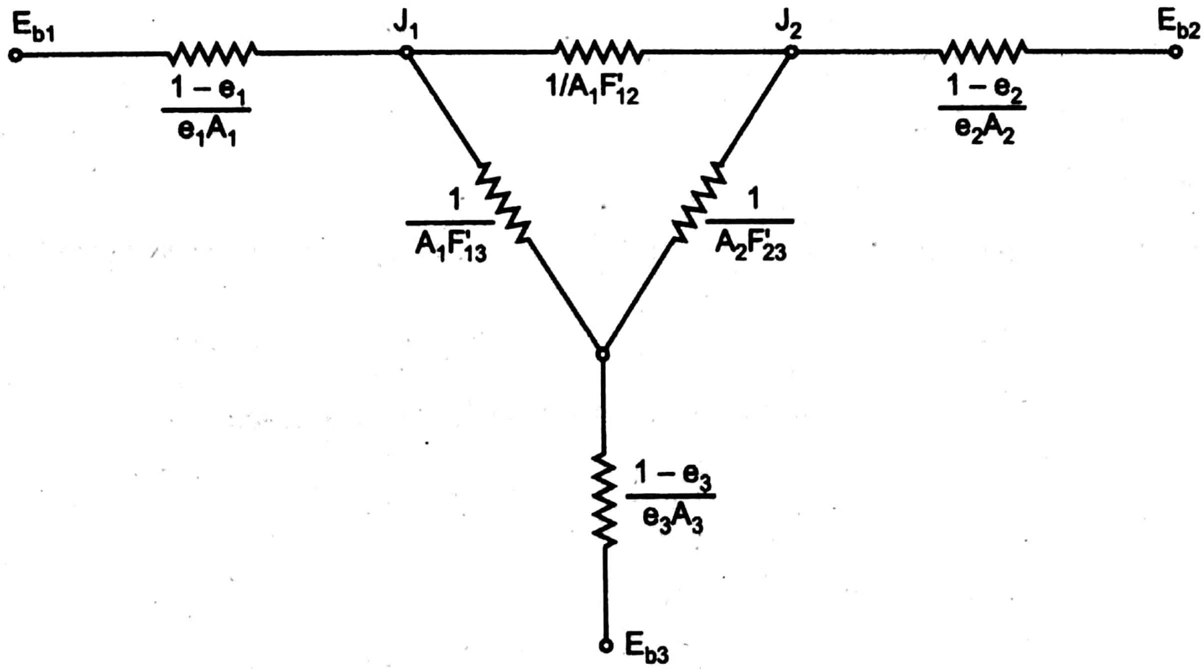


Fig. 4.10 : Radiation network for three surfaces which see each other and nothing else

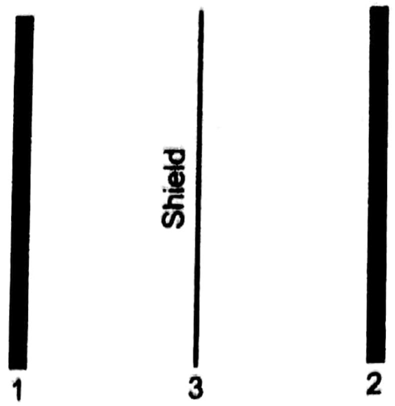


Fig. 4.11 : Radiation between parallel infinite planes with a radiation shield

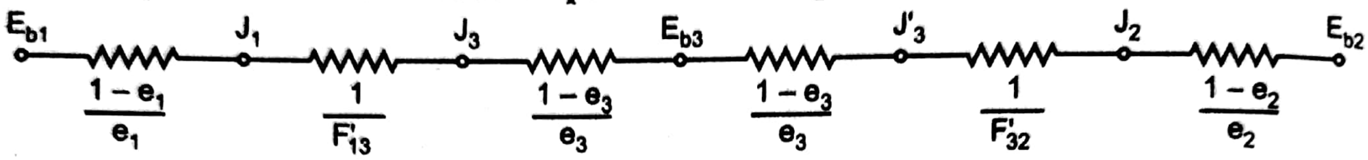


Fig. 4.12 : Radiation network for two parallel planes/surfaces separated by one radiation shield

(i) Consider radiation between two infinite surfaces-1 and -2 maintained at temperatures T_1 and T_2. Using the electrical analogy, we have

$$Q_{12} = Q = \frac{\sigma\left(T_1^4 - T_2^4\right)}{\frac{1-e_1}{A_1 e_1} + \frac{1}{A_1 F'_{12}} + \frac{1-e_2}{e_2 A_2}} \quad \ldots (4.53)$$

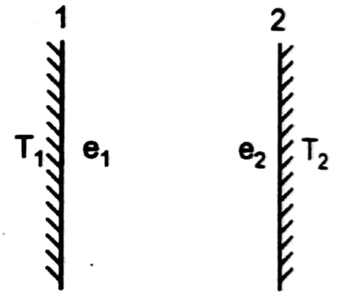


Fig. 4.13

As the surfaces are infinite, $A_1 = A_2 = A$

and $F'_{12} = 1.0$

$\therefore$ Equation (4.53) becomes

$$Q = \frac{\sigma\left(T_1^4 - T_2^4\right)}{\frac{1-e_1}{A e_1} + \frac{1}{A} + \frac{1-e_2}{e_2 A}}$$

$$Q = \frac{A\sigma\left(T_1^4 - T_2^4\right)}{\frac{1-e_1}{e_1} + 1 + \frac{1-e_2}{e_2}}$$

$$= \frac{A\sigma\left(T_1^4 - T_2^4\right)}{\frac{1}{e_1} - 1 + 1 + \frac{1}{e_2} - 1} = \frac{A\sigma\left(T_1^4 - T_2^4\right)}{\frac{1}{e_1} + \frac{1}{e_2} - 1} \quad \ldots (4.54)$$

$$Q = A\sigma F_{12}\left(T_1^4 - T_2^4\right) \quad \ldots (4.55)$$

F_{12} – Overall interchange factor $= \dfrac{1}{\frac{1}{e_1} + \frac{1}{e_2} - 1}$

(ii) For two concentric infinitely long cylinders at T_1 and T_2 by electrical analogy, we have

$$Q_{12} = Q = \frac{\sigma\left(T_1^4 - T_2^4\right)}{\dfrac{1-e_1}{A_1 e_1} + \dfrac{1}{A_1 F_{12}^{'}} + \dfrac{1-e_2}{A_2 e_2}}$$

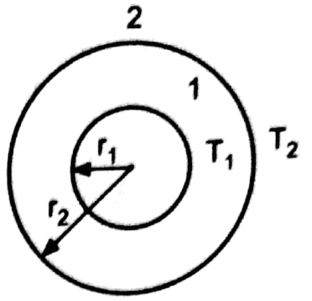


Fig. 4.14

[The network diagram is the same as given in Fig. 4.14.]

We have, $$F_{11}^{'} + F_{12}^{'} = 1.0$$

$$F_{12}^{'} = 1 - F_{11}^{'}$$

For convex surface, $$F_{11} = 0$$

$\therefore$ $$F_{12}^{'} = 1.0$$

$\therefore$ $$Q = \frac{\left(T_1^4 - T_2^4\right)}{\dfrac{1-e_1}{A_1 e_1} + \dfrac{1}{A_1} + \dfrac{1-e_2}{A_2 e_2}}$$

$$= \frac{A_1 \sigma\left(T_1^4 - T_2^4\right)}{\dfrac{1-e_1}{e_1} + 1 + \dfrac{A_1}{A_2}\left(\dfrac{1-e_2}{e_2}\right)}$$

$$= \frac{A_1 \sigma\left(T_1^4 - T_2^4\right)}{\dfrac{1}{e_1} - 1 + 1 + \dfrac{A_1}{A_2}\left(\dfrac{1}{e_2} - 1\right)}$$

$$= \frac{A_1 \sigma\left(T_1^4 - T_2^4\right)}{\dfrac{1}{e_1} + \dfrac{A_1}{A_2}\left(\dfrac{1}{e_2} - 1\right)} \qquad \dots (4.56)$$

$$Q = A_1 \sigma F_{12}\left(T_1^4 - T_2^4\right) \qquad \dots (4.57)$$

where F_{12} is the overall interchange factor.

$$F_{12} = \frac{1}{\dfrac{1}{e_1} + \dfrac{A_1}{A_2}\left(\dfrac{1}{e_2} - 1\right)}$$

Please note : F_{12} is the interchange factor and F'_{12} is the shape factor.

(iii) For one radiation shield of emissivity e_3 between two parallel infinite planes of emissivities e_1 and e_2 by electrical analogy, we have

$$(Q/A)_s = Q/A \text{ with shield} = \frac{\sigma\left(T_1^4 - T_2^4\right)}{\frac{1-e_1}{e_1} + \frac{1}{F'_{13}} + \frac{1-e_3}{e_3} + \frac{1-e_3}{e_3} + \frac{1}{F'_{32}} + \frac{1-e_2}{e_2}}$$

$$\sum R = \frac{(1-e_1)}{e_1} + \frac{1}{F'_{13}} + \frac{(1-e_3)}{e_3} + \frac{(1-e_3)}{e_3} + \frac{1}{F'_{32}} + \frac{(1-e_2)}{e_2}$$

But $\quad \frac{1}{F'_{13}} = 1, \quad \frac{1}{F'_{32}} = 1$

The above equation reduces to

$$(Q/A)_s = \frac{\sigma\left(T_1^4 - T_2^4\right)}{\frac{1}{e_1} - 1 + 1 + \frac{1}{e_3} - 1 + \frac{1}{e_3} - 1 + 1 + \frac{1}{e_2} - 1}$$

$$= \frac{\sigma\left(T_1^4 - T_2^4\right)}{\frac{1}{e_1} + \frac{1}{e_2} + \frac{2}{e_3} - 2} \quad \ldots (4.58)$$

where e_3 is the emissivity of the shield.

This equation can be generalized for two parallel plates separated by n shields of emissivities $e_{s_1}, e_{s_2}, e_{s_n}$ as

$$(Q/A)_s = \frac{\sigma\left(T_1^4 - T_2^4\right)}{\frac{1}{e_1} + \frac{1}{e_2} + 2\sum_{i=1}^{n} \frac{e}{e_{s_i}} - (n+1)} \quad \ldots (4.59)$$

For n = 1,

$$(Q/A)_s = \frac{\sigma\left(T_1^4 - T_2^4\right)}{\frac{1}{e_1} + \frac{1}{e_2} + \frac{1}{e_{s_1}} - 2} \quad \ldots (4.60)$$

$$e_{s_1} = e_3$$

For n = 2,

$$(Q/A)_s = \frac{\sigma\left(T_1^4 - T_2^4\right)}{\frac{1}{e_1} + \frac{1}{e_2} + \frac{2}{e_{s_1}} + \frac{2}{e_{s_2}} - 3} \quad \ldots (4.61)$$

e_{s_1} and e_{s_2} are the emissivities of radiation shields.

(iv) For cylindrical radiation shields used in cylindrical thermal systems [Fig. 4.15 (a)] the radiation network diagram is as given below :

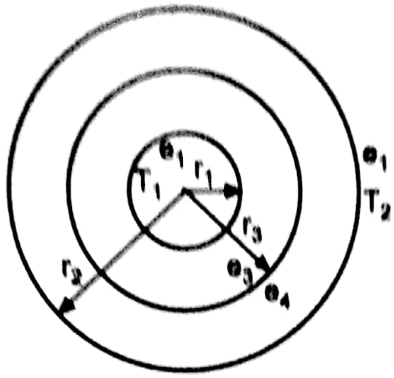


Fig. 4.15 (a) : Cylindrical radiation shield

where e_1 and e_2 are the emissivities of concentric cylindrical surfaces, and e_3 and e_4 are the surface emissivities (inner and outer) of the cylindrical radiation shield.

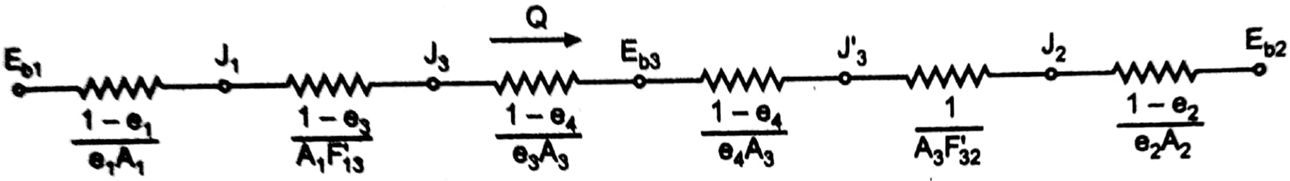


Fig. 4.15 (b) : Radiation network for cylindrical radiation shield

$$(Q)_s = Q \text{ with shield} = \frac{\sigma\left(T_1^4 - T_2^4\right)}{\sum R} \qquad \dots (4.62)$$

$$\sum R = (1 - e_1/e_1 A_1) + 1/A_1 F'_{13} + (1 - e_3/e_3 A_3) + (1 - e_4/e_4 A_3) + 1/A_3 F'_{32} + (1 - e_2/e_2 A_2)$$

$$(Q)_s = \frac{\sigma\left(T_1^4 - T_2^4\right)}{\frac{1-e_1}{e_1 A_1} + \frac{1}{A_1 F'_{13}} + \frac{1-e_3}{e_3 A_3} + \frac{1-e_4}{e_4 A_3} + \frac{1}{A_3 F'_{32}} + \frac{1-e_2}{e_2 A_2}} \qquad \dots (4.63)$$

$$\frac{1}{F'_{13}} = \frac{1}{F'_{32}} = 1$$

$$(Q)_s = \frac{\sigma\left(T_1^4 - T_2^4\right)}{\frac{1-e_1}{A_1 e_1} + \frac{1}{A_1} + \frac{1-e_3}{e_3 A_3} + \frac{1-e_4}{e_4 A_3} + \frac{1}{A_2} + \frac{1-e_2}{e_2 A_2}}$$

$$(Q)_s = \frac{\sigma A_1\left(T_1^4 - T_2^4\right)}{(1/e_1) - 1 + 1 + \frac{A_1}{A_3}(1/e_3 - 1) + \frac{A_1}{A_3}(1/e_4 - 1) + \frac{A_1}{A_3} + \frac{A_1}{A_2}(1/e_2 - 1)}$$

$$= \frac{A_1 \sigma\left(T_1^4 - T_2^4\right)}{\frac{1}{e_1} + \frac{A_1}{A_2}\left(\frac{1}{e_2} - 1\right) + \frac{A_1}{A_3}\left(\frac{1}{e_3} + \frac{1}{e_4} - 1\right)} \qquad \dots (4.64)$$

With A = πDL or 2πrL, we get

$$(Q)_S = \frac{2\pi r_1 L\,\sigma\left(T_1^4 - T_2^4\right)}{\frac{1}{e_1} + \frac{r_1}{r_2}\left(\frac{1}{e_2} - 1\right) + \frac{r_1}{r_3}\left(\frac{e}{e_3} + \frac{1}{e_4} - 1\right)} \qquad \cdots (4.65)$$

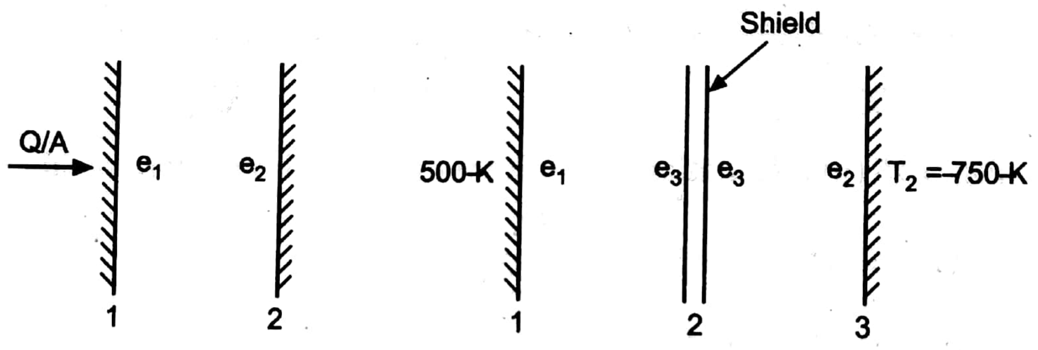
[You can write $(Q)_S$ or Q with shield, or case-II with shield and then use Q.]

Example 4.20 : *Consider two large parallel plates, one at T_1 = 750 K (477°C) of emissivity e_1 = 0.75 and other at 500 K (227°C) of emissivity e_2 = 0.50. An aluminium radiation shield having an emissivity (on both sides) of e_2 = 0.05 is placed between the plates.*

(i) Sketch a radiation network for the system with or without the radiation shield.

(ii) Calculate the percent reduction in heat transfer rate due to incorporation of the radiation shield.

Solution :



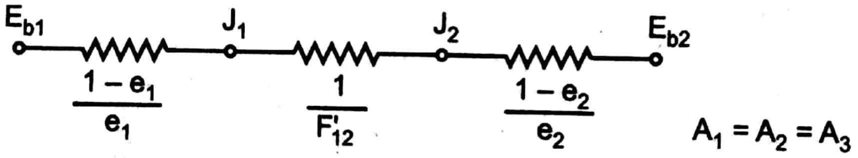


Fig. E 4.20 (A) : Radiation network without shield

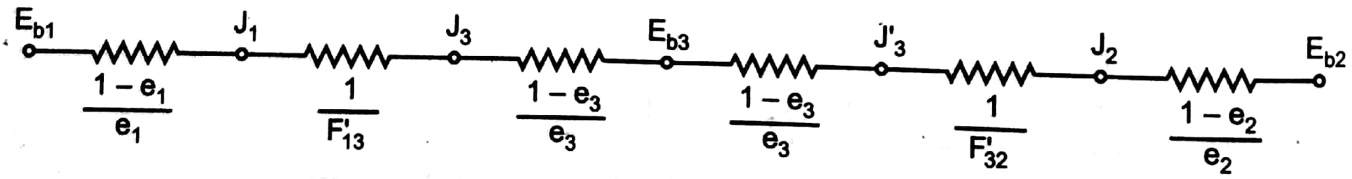


Fig. E 4.20 (B) : Radiation network for two parallel plates separated by one radiation shield

The heat transfer without shield is given by

$$Q/A = \frac{\sigma\left(T_1^4 - T_2^4\right)}{\frac{1}{e_1} + \frac{1}{e_2} - 1}$$

where $\sigma = 5.67 \times 10^{-8}\ W/(m^2 \cdot K^4)$

$T_1 = 750$ K, $T_2 = 500$ K, $e_1 = 0.75$ and $e_2 = 0.50$

$$Q/A = \frac{5.67 \times 10^{-8}\left(\overline{750}^4 - \overline{550}^4\right)}{\frac{1}{0.75} + \frac{1}{0.5} - 1} = 6170 \text{ W/m}^2$$

Heat transfer with shield is given by :

The radiation network shield is shown in Fig. E 4.20 (B).

Various resistances are calculated as follows per unit area.

(i) $(1 - e_1)/e_1 = (1 - 0.75)/0.75 = 0.333$

(ii) $1/F'_{13} = 1/1 = 1$

(iii) $(1 - e_3)/e_3 = (1 - 0.05)/0.05 = 19$

(iv) $(1 - e_3)/e_3 = (1 - 0.05)/0.05 = 19$

(v) $1/F'_{32} = 1.0$

(vi) $(1 - e_2)/e_2 = (1 - 0.5)/0.5 = 1.0$

$\therefore$ Total resistance $= 0.333 + 1.0 + 19 + 19 + 1.0 + 1.0$

$= 41.333$

$\therefore$ The net heat transfer with shield is :

$$(Q/A)_S = \frac{\sigma\left(T_1^4 - T_2^4\right)}{\sum R}$$

where $\sigma = 5.67 \times 10^{-8}$ W/(m²·K⁴)

$T_1 = 750$ K, $T_2 = 500$ K and $\sum R = 41.333$

$$(Q/A)_S = \frac{5.67 \times 10^{-8}\left(\overline{750}^4 - \overline{500}^4\right)}{41.333}$$

$= 348.30$ W/m²

$\therefore$ % reduction in heat transfer due to the shield $= \frac{6170 - 348.3}{6170} \times 1000 = \mathbf{94.34}$ ... **Ans.**

Example 4.21 : *Two very large parallel planes with emissivities 0.3 and 0.8 exchange heat. Calculate the percent reduction in heat transfer when a polished-aluminium radiation shield of emissivity 0.04 is placed between them.*

Solution : Let the planes be at temperatures T_1 and T_2 respectively.

The radiant heat transfer without the shield is

$$Q/A = \frac{\sigma\left(T_1^4 - T_2^4\right)}{\frac{1}{e_1} + \frac{1}{e_2} - 1}$$

where $e_1 = 0.3, \quad e_2 = 0.8$

$$Q/A = \frac{\sigma\left(T_1^4 - T_2^4\right)}{\frac{1}{0.3} + \frac{1}{0.8} - 1} = 0.2791\left[\sigma\left(T_1^4 - T_2^4\right)\right] = 0.2791\ z\ W/m^2$$

where $z = \sigma\left(T_1^4 - T_2^4\right)$

Heat transfer with the radiation shield of emissivity (e_3) of 0.04 :

The radiation network for this problem is the same as that shown in Fig. E 4.20 (B).

The resistances are :

(i) $(1 - e_1)/e_1 = (1 - 0.3)/0.3 = 2.333$

(ii) $1/F'_{13} = 1.0$

(iii) $(1 - e_3)/e_3 = (1 - 0.04)/0.04 = 24$

(iv) $(1 - e_3)/e_3 = (1 - 0.04)/0.04 = 24$

(v) $1/F'_{32} = 1.0$

(iv) $(1 - e_2)/e_2 = (1 - 0.8)/0.8 = 0.25$

Total resistance = $2.333 + 1.0 + 24 + 24 + 1 + 0.25 = 52.583$

The radiant heat transfer with the shield placed between the two planes is

$$(Q/A)_s = \frac{\sigma\left(T_1^4 - T_2^4\right)}{\sum R} = \frac{\sigma\left(T_1^4 - T_2^4\right)}{52.583}$$

$$= 0.019\,\sigma\left(T_1^4 - T_2^4\right) = 0.019\ z\ \ W/m^2$$

$$\text{\% reduction in heat transfer using shield} = \frac{(Q/A)_s - Q/A}{(Q/A)_s} \times 100$$

$$\therefore \quad \text{\% reduction in heat transfer} = \frac{0.2791\ z - 0.019\ z}{0.2791\ z} \times 100$$

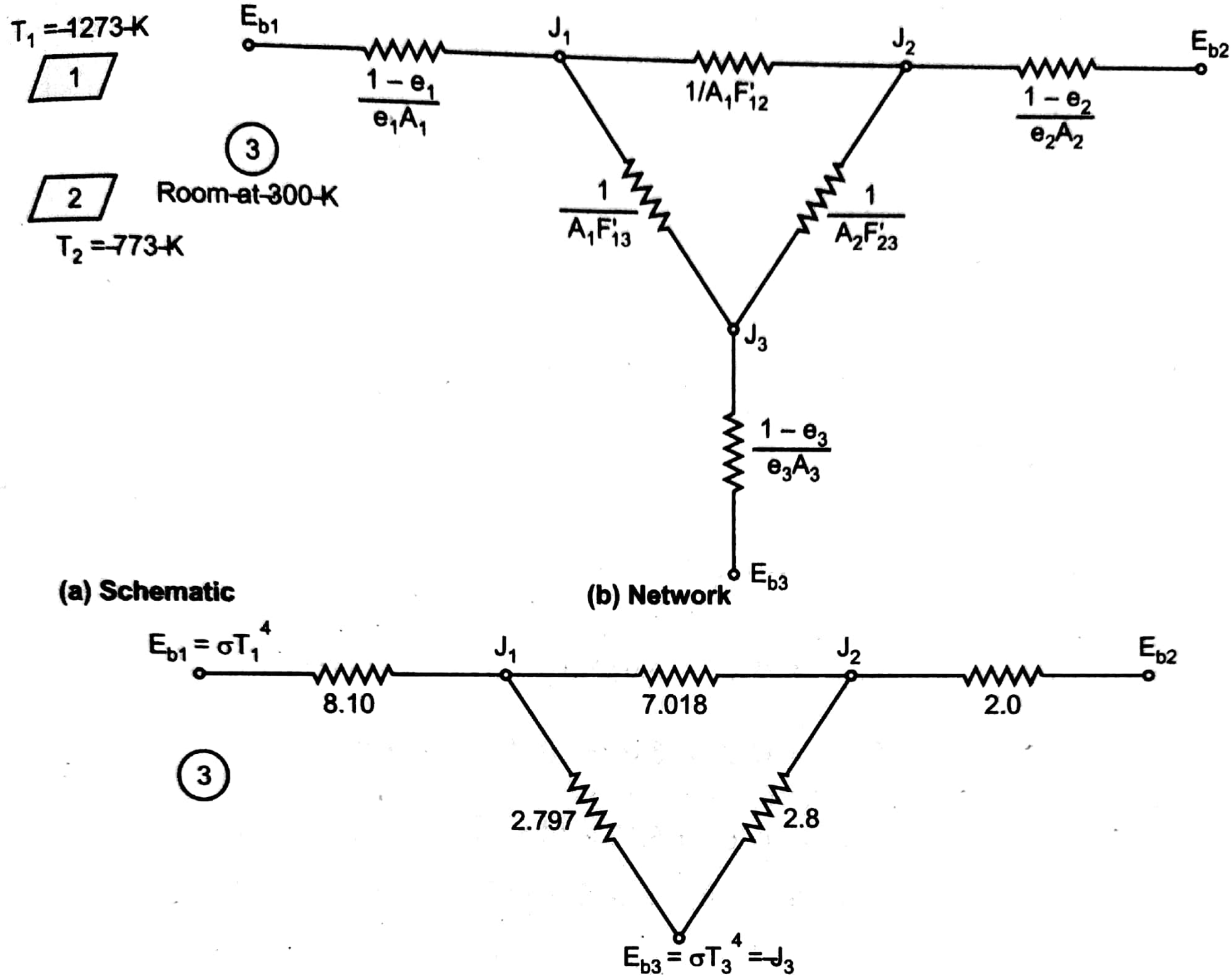
$$= \mathbf{93.2}$$ ... Ans.

The heat transfer is reduced by 93.2 percent by interposing the shield.

Example 4.22 : *The parallel plates 0.5 by 1.0 m are placed 0.5 m apart. One plate is maintained at 1273 K (100°C) and second plate at 773 K (500°C). The emissivities of these plates are 0.2 and 0.5 respectively. The plates are located in a very large room, the walls of which are maintained at 300 K (27°C). The plates exchange heat with each other and with surroundings but only the plate surfaces facing each other are to be considered in the analysis. Find the net heat transfer to each plate and to the room.*

Data : Radiation shape factors between first-second plate and between second-first plate are 0.285 each, between first plate and room is 0.715 and between second plate and room is 0.715. Surface resistance of the room may be taken as zero.

Solution :



(a) Schematic (b) Network

(c) Network for problem

Fig. E 4.22

It is a three body problem, two plates and the room, therefore the radiation network is as shown in Fig. E 4.22.

$$T_1 = 1273 \text{ K}, \quad T_2 = 773 \text{ K} \quad \text{and} \quad T_3 = 300 \text{ K}$$

$$A_1 = A_2 = 0.5 \times 1 = 0.5 \text{ m}^2$$

$$e_1 = 0.2 \text{ and } e_2 = 0.5$$

$$F'_{12} = F'_{21} = 0.285$$

$$F'_{13} = F'_{12} = 1.0$$

$$\therefore \quad F'_{13} = 1 - F'_{12} = 1 - 0.285 = 0.715$$

$$F'_{23} = 1 - F'_{21} = 1 - 0.285 = 0.715$$

The resistances of the network are calculated as :

1. $1 - e_1/e_1 A_1 = 1 - 0.2/(0.2 \times 0.5) = 8.0$
2. $1 - e_2/e_2 A_2 = 1 - 0.5/(0.5 \times 0.5) = 2.0$
3. $1/A_1 F'_{12} = 1/(0.5 \times 0.285) = 7.018$
4. $1/A_1 F'_{13} = 1/0.5 \times 0.715 = 2.797$
5. $1/A_2 F'_{23} = 1/0.5 \times 0.715 = 2.797$
6. $1 - e_3/e_3 A_3 = 0$ (given)

For calculating the heat flow at each surface, we must determine the radiosities J_1 and J_2. This network is solved by setting the sum of heat currents entering nodes J_1 and J_2 to zero.

$$\text{Node } J_1 : \quad \frac{E_{b_1} - J_1}{8.0} + \frac{J_2 - J_1}{7.018} + \frac{E_{b_3} - J_1}{2.797} = 0 \qquad \dots (a)$$

$$\text{Node } J_2 : \quad \frac{J_1 - J_2}{7.018} + \frac{E_{b_3} - J_2}{2.797} + \frac{E_{b_2} - J_2}{2.0} = 0 \qquad \dots (b)$$

$$E_{b_1} = \sigma T_1^4 = 5.67 \times 10^{-8} \left(\overline{1273}^{\,4}\right) = 148900 \text{ W/m}^2$$

$$E_{b_2} = \sigma T_2^4 = 5.07 \times 10^{-8} \left(\overline{773}^{\,4}\right) = 20244 \text{ W/m}^2$$

$$E_{b_3} = \sigma T_3^4 = 5.67 \times 10^{-8} \left(\overline{300}^{\,4}\right) = 459 \text{ W/m}^2$$

Substituting the values of E_{b_1}, E_{b_2} and E_{b_3} in Equations (a) and (b) and solving simultaneously, we get

$$J_1 = 33515 \text{ W/m}^2 \text{ and } J_2 = 15048 \text{ W/m}^2$$

$$J_3 = E_{b_3} = 459 \text{ W/m}^2$$

The total heat lost by the plate-1 is

$$Q_1 = \frac{E_{b_1} - J_1}{(1 - e_1)/e_1 A_1} = \frac{148900 - 33515}{8.0} = \mathbf{14423 \text{ W/m}^2} \qquad \dots \text{Ans.}$$

The total heat lost by the plate-2 is

$$Q_2 = \frac{E_{b_2} - J_2}{(1 - e_2)/e_2 A_2} = \frac{20244 - 15048}{2.0} = \mathbf{2598 \text{ W/m}^2} \qquad \dots \text{Ans.}$$

The total heat received by the room is

$$Q_3 = \frac{J_1 - J_3}{J/A_1 F'_{13}} + \frac{J_2 - J_3}{J/A_2 F'_{23}}$$

$$= \frac{33515 - 459}{2.797} + \frac{15048 - 459}{2.797}$$

$$= \mathbf{17034 \text{ W/m}^2} \qquad \dots \text{Ans.}$$

Here, the overall heat balance must be satisfied :

$$Q_3 = Q_1 + Q_2$$

because the net energy lost by both plates must be absorbed by the room.

$$17034 \text{ W/m}^2 \approx 14423 + 2598 = \mathbf{17021 \text{ W/m}^2}$$ ... **Ans.**

Example 4.23 : *Two very large and parallel surfaces each having an emissivity of 0.7. With surface-1 is at 866.5 K (593.5°C) and surface-2 at 588.8 K (315°C) some distance apart. What is the net radiation loss of surface-1 ? To reduce this loss, two additional radiation shields also having an emissivity of 0.7 each are placed between the original surfaces. What is the new radiation loss ?*

Solution :

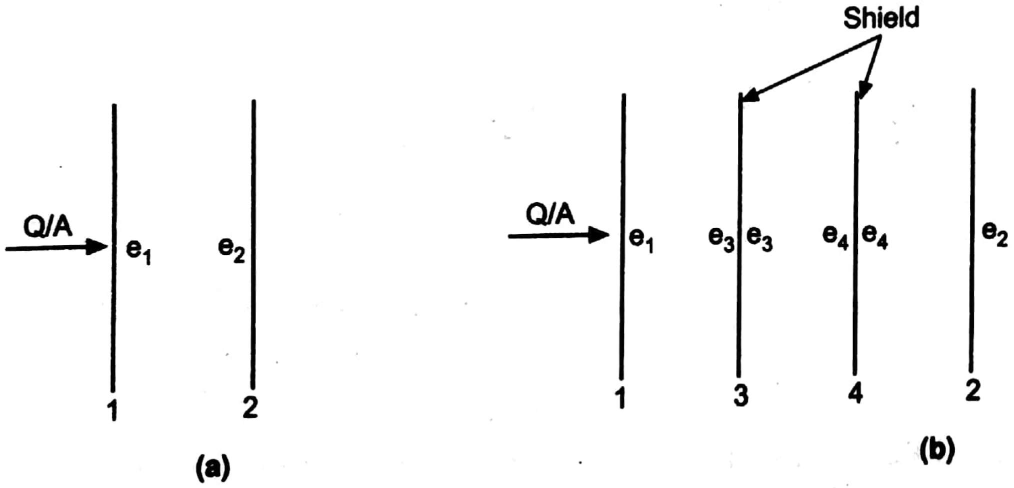


Fig. E 4.23 : Radiation between planes with and without shields

The net radiation heat loss without shields is

$$Q/A = \frac{\sigma\left(T_1^4 - T_2^4\right)}{1/e_1 + 1/e_2 - 1}$$

where

$$\sigma = 5.67 \times 10^{-8} \text{ W/(m}^2\cdot\text{K}^4)$$

$$e_1 = e_2 = 0.7$$

$$T_1 = 866.5 \text{ K and } T_2 = 588.8 \text{ K}$$

$$Q/A = \frac{5.67 \times 10^{-8}\left(\overline{866.5}^{\,4} - \overline{588.6}^{\,4}\right)}{1/0.7 + 1/0.7 - 1} = 13541.4 \text{ W/m}^2$$

For the simple case of $e_1 = e_2 = e_3 = e_4 = e = 0.7$, we have

$$Q \text{ with n shields} = \frac{1}{n+1} Q \text{ without shields}$$

where

$$n = \text{Number of shields} = 2$$

$$Q \text{ with shields} = \frac{1}{2+1} \times 13541.4$$

$$= \mathbf{4513.8 \text{ W/m}^2}$$ ... **Ans.**

Alternately,

We have with shields :

$$Q/A = \frac{\sigma\left(T_1^4 - T_2^4\right)}{(1/e_1) + (1/e_2) + 2\sum_{i=1}^{n}\frac{1}{e_{si}} - (n+1)}$$

For n = 2,

$$Q/A = \frac{\sigma\left(T_1^4 - T_2^4\right)}{(1/e_1) + (1/e_2) + 2\,[1/e_{s_1} + 1/e_{s_2}] - (2+1)}$$

$$= \frac{\sigma\left(T_1^4 - T_2^4\right)}{1/e_1 + 1/e_2 + \frac{2}{e_{s_1}} + \frac{2}{e_{s_2}} - 3}$$

$$= \frac{\sigma\left(T_1^4 - T_2^4\right)}{1/e_1 + 1/e_2 + 2/e_{s_1} + 2/e_{s_2} - 3}$$

Here $e_1 = e_2 = e_{s_1} = e_{s_2} = e = 0.7$

$$\therefore \quad Q/A = \frac{5.67 \times 10^{-8}\left(\overline{866.5}^{\,4} - \overline{588.8}^{\,4}\right)}{1/0.7 + 1/0.7 + 2/0.7 + 2/0.7 - 3}$$

$$= \mathbf{4513.9\ W/m^2}$$ ... **Ans.**

Example 4.24 : *A cryogenic fluid flows through a tube 30 mm diameter concentric with a tube of 90 mm diameter, the surface emissivities of inner and outer tube are 0.12 and 0.15 and are at temperatures of 100 K (–173°C) and 300 K (27°C) respectively. Determine (i) the heat gained by fluid per 1 m length of tube and (ii) percent reduction in heat gain, if the radiation shield with diameter 45 mm and emissivities 0.1 on the inner surface and 0.05 on the outer surface is interposed between the tubes.*

Solution : (I) Without shield :

$$Q = \frac{A_1\sigma\left(T_1^4 - T_2^4\right)}{1/e_1 + A_1/A_2\,(1/e_2 - 1)}$$

where $e_1 = 0.12, \quad e_2 = 0.15, \quad T_1 = 100\text{ K}, \quad T_2 = 300\text{ K}$

$A_1 = 2\pi\, r_1\, L$ and $A_2 = 2\pi\, r_2 L$ $\qquad r_1 = 15\text{ mm} = 0.015\text{ m}$

$r_2 = 45\text{ mm} = 0.045\text{ m}$

$$Q/L = \frac{2\pi\, r_1\,\sigma\left(T_1^4 - T_2^4\right)}{1/e_1 + r_1/r_2\ (1/e_2 - 1)} = \frac{2\pi \times 0.015 \times 5.67 \times 10^{-8}\left(\overline{100}^{\,4} - \overline{300}^{\,4}\right)}{\frac{1}{0.12} + \frac{0.015}{0.045}\ (1/0.15 - 1)}$$

$$= -4.182\ \text{W/m}$$

–ve sign indicates that the net heat flow is in the radial inward direction.

(II) With cylindrical radiation shield :

$$\frac{Q_s}{L} = \frac{2\pi\, r_1\, \sigma\left(T_1^4 - T_2^4\right)}{1/e_1 + r_1/r_2\,(1/e_2 - 1) + r_1/r_3\,(1/e_3 + 1/e_4 - 1)})$$

where $r_3 = 22.5 \text{ mm} = 0.0225 \text{ m}$

$e_3 = 0.10$ and $e_4 = 0.05$

$$\frac{Q_s}{L} = \frac{2\pi \times 0.015 \times 5.67 \times 10^{-8}\left(\overline{100}^{\,4} - \overline{300}^{\,4}\right)}{\frac{1}{0.12} + \frac{0.015}{0.045}\left(\frac{1}{0.15} - 1\right) + \frac{0.015}{0.0225}\left(\frac{1}{0.10} + \frac{1}{0.05} - 1\right)}$$

$$= -1.446 \text{ W/m}^2$$

% reduction in heat gain $= \dfrac{4.182 - 1.446}{4.182} \times 100 = \mathbf{65.42}$ ... **Ans.**

Radiation network work approach :

With shield : Take $L = 1$ m

$$(Q_s) = \frac{\sigma\left(T_1^4 - T_2^4\right)}{\frac{1-e_1}{e_1 A_1} + \frac{1}{A_1 F'_{13}} + \frac{1-e_3}{e_3 A_3} + \frac{1-e_4}{e_4 A_3} + \frac{1}{A_3 F_{32}} + \frac{1-e_2}{e_2 A_2}}$$

The resistances involved in the network are calculated as :

(i) $(1 - e_1)/e_1 A_1 = (1 - 0.12)/0.12 \times 2 \times \pi \times 0.015 \times 1.0 = 79.8$

(ii) $1/A_1 F_{13} = 1/2 \times \pi \times 0.015 \times 1.0 \times 1.0 = 10.6$

(iii) $(1 - e_3)/(e_3 A_3) = (1 - 0.10)/0.10 \times 2 \times \pi \times 0.0225 \times 1.0 = 63.66$

(iv) $(1 - e_4)/e_4 A_3 = (1 - 0.05)/0.05 \times 2 \times \pi \times 0.0225 \times 1.0 = 134.4$

(v) $1/A_3 F_{32} = 1/2 \times \pi \times 0.0225 \times 1.0 \times 1.0 = 7.08$

(vi) $(1 - e_2)/e_2 A_2 = (1 - 0.15)/0.15 \times 2 \times \pi \times 0.045 \times 1.0 = 20.04$

$$\sum R = 79.8 + 10.6 + 63.66 + 134.4 + 7.08 + 20.04 = 315.58$$

$$(Q)_s = \frac{5.67 \times 10^{-8}\left(\overline{100}^{\,4} - \overline{300}^{\,4}\right)}{315.58}$$

$$= \mathbf{-1.44 \text{ W/m}^2}$$ ... **Ans.**