ANALYSIS OF INDETERMINATE STRUCTURES

SUB CODE: 15CV52

2019-2020(ODD SEM)

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Vision of BIET

To be a center of excellence recognized nationally and internationally, in distinctive areas of engineering education and research, based on a culture of innovation and invention.

Mission of BIET

BIET contributes to the growth and development of its students by imparting a broad based engineering education and empowering them to be successful in their chosen field by inculcating in them positive approach, leadership qualities and ethical values

VISION OF THE DEPARTMENT

To train the students to become Civil Engineers with leadership qualities, having ability to take up professional assignments and research with a focus on innovative approaches to cater to the needs of the society.

MISSION OF THE DEPARTMENT

1. To provide quality education through updated curriculum and conducive teaching learning environment for the students to excel in higher studies, competitive examinations and professional career.
2. To impart soft skills, leadership qualities and professional ethics among the graduates to handle the projects independently with confidence.
3. To deal with the contemporary issues and to cater to the socio-economic needs.
4. To build industry-institute interaction and to establish good rapport with alumni.

PROGRAM EDUCATIONAL OBJECTIVES (PEOs)

***PEO 1: Core Competence*:** Graduates will be able to plan, analyse, design and construct sustainable Civil Engineering Infrastructure.

***PEO 2: Professional Skills*:** Graduates will be professional engineers with a sense of ethics, creativity, leadership, self-confidence and independent thinking to cater to the needs of the society.

***PEO 3: Societal Needs*:** Graduates will be able to contribute effectively for the development of industry and professional bodies.

***PEO 4: Cognitive Intelligence*:** Graduates will be able to take up competitive examinations, higher studies and involve in research and entrepreneurship activities.

PROGRAM SPECIFIC OUTCOMES (PSOs)

Students after the completion of the Program will be able to

1. Apply the fundamental concepts, software and codal provisions in the analysis, design and construction of sustainable civil engineering infrastructure.
2. Inculcate professional and leadership qualities, sense of ethics and confidence related to civil engineering.

Faculty will be able to

3. Contribute to the overall development of civil engineering community through the professional bodies and offer services to the society.

B. E. CIVIL ENGINEERING
Choice Based Credit System (CBCS) and Outcome Based Education (OBE)
SEMESTER - V

ANALYSIS OF INDETERMINATE STRUCTURES

Course Code	18CV52	CIE Marks	40
Teaching Hours/Week(L:T:P)	(3:2:0)	SEE Marks	60
Credits	04	Exam Hours	03

Course Learning Objectives: This course will enable students to

1. Apply knowledge of mathematics and engineering in calculating slope, deflection, bending moment and shear force using slope deflection, moment distribution method and Kani's method.
2. Identify, formulate and solve problems in structural analysis.
3. Analyze structural system and interpret data.
4. use the techniques, such as stiffness and flexibility methods to solve engineering problems
5. communicate effectively in design of structural elements

Module-1

Slope Deflection Method: Introduction, sign convention, development of slope deflection equation, analysis of continuous beams including settlements, Analysis of orthogonal rigid plane frames including sway frames with kinematic indeterminacy≤3.

Module-2

Moment Distribution Method: Introduction, Definition of terms, Development of method, Analysis of continuous beams with support yielding, Analysis of orthogonal rigid plane frames including sway frames with kinematic indeterminacy ≤3.

Module-3

Kani's Method: Introduction, Concept, Relationships between bending momentand deformations, Analysis of continuous beams with and without settlements, Analysis of frames with and without sway.

Module-4

Matrix Method of Analysis (Flexibility Method) : Introduction, Axes and coordinates, Flexibility matrix, Analysis of continuous beams and plane trusses using system approach, Analysis of simple orthogonal rigid frames using system approach with static indeterminacy ≤3.

Module-5

Matrix Method of Analysis (Stiffness Method): Introduction, Stiffness matrix, Analysis of continuous beams and plane trusses using system approach, Analysis of simple orthogonal rigid frames using system approach with kinematic indeterminacy ≤3.

Course Outcomes: After studying this course, students will be able to:

1. Determine the moment in indeterminate beams and frames having variable moment of inertia and subsidence using slope defection method
2. Determine the moment in indeterminate beams and frames of no sway and sway using moment distribution method.
3. Construct the bending moment diagram for beams and frames by Kani's method.
4. Construct the bending moment diagram for beams and frames using flexibility method
5. Analyze the beams and indeterminate frames by system stiffness method.

Question paper pattern:

- The question paper will have ten full questions carrying equal marks.
- Each full question will be for 20 marks.
- There will be two full questions (with a maximum of four sub- questions) from each module.
- Each full question will have sub- question covering all the topics under a module.
- The students will have to answer five full questions, selecting one full question from each module.

Textbooks:

1. Hibbeler R C, " **Structural Analysis**", Pearson Publication
2. L S Negi and R S Jangid, **"Structural Analysis"**, Tata *McGraw-Hill* Publishing Company Ltd.
3. D S PrakashRao, **"Structural Analysis: A Unified Approach"** , Universities Press
4. K.U. Muthu, H. Narendraetal, **"Indeterminate Structural Analysis"**, IK International Publishing Pvt. Ltd.

Reference Books:

1. Reddy C S, **"Basic Structural Analysis"**,Tata McGraw-Hill Publishing Company Ltd.
2. Gupta S P, G S Pundit and R Gupta, **"Theory of Structures"**, Vol II, Tata McGraw Hill Publications company Ltd.
3. V N Vazirani and M MRatwani, **"Analysis Of Structures "**, Vol. 2, Khanna Publishers
4. Wang C K, **"Intermediate Structural Analysis"**, McGraw Hill, International Students Edition.
5. S.Rajasekaran and G. Sankarasubramanian, **"Computational Structural Mechanics"**, PHI Learning Pvt. Ltd.

Title & Code	**Analysis of Indeterminate Structures (15CV52)**
CO	**Statement**
15CV52.1	**Analyse** the beams and frames by slope deflection method.
15CV52.2	**Analyse** the beams and frames by moment distribution method.
15CV52.3	**Analyse** the beams and frames by Kani's rotation contribution method
15CV52.4	**Analyse** the beams and frames by flexibility method (System Approach)
15CV52.5	**Analyse** the beams and frames by stiffness method (System Approach)
15CV52.6	**Analyse** trusses by flexibility and stiffness matrix methods (System Approach)

Course Title		**Analysis of Indeterminate Structures**										
CO	**PO1**	**PO2**	**PO3**	**PO4**	**PO5**	**PO6**	**PO7**	**PO8**	**PO9**	**PO10**	**PO11**	**PO12**
15CV52.1	**2**	**2**		**1**								**2**
15CV52.2	**2**	**2**		**1**								**2**
15CV52.3	**2**	**2**		**1**								**2**
15CV52.4	**2**	**2**		**1**								**2**
15CV52.5	**2**	**2**		**1**								**2**
15CV52.6	**2**	**2**		**1**								**2**
Average	**2**	**2**		**1**								**2**

CO	**PSO1**	**PSO2**
15CV52.1	**2**	**2**
15CV52.2	**2**	**2**
15CV52.3	**2**	**2**
15CV52.4	**2**	**2**
15CV52.5	**2**	**2**
15CV52.6	**2**	**2**
Average	**2**	**2**

Analysis of Indeterminate Structures

Introduction:

Anything built by man by using mat and machines are called structures.

Ex: Building, bridges, Dam, Tunnel, Highway, ways, Harbour etc.,

Calculation of unknowns is called analy There are two types of unknowns:

a) Reactive unknowns
b) Displacement unknowns

Reactive unknowns are R_V, R_H, BM, SF, RS, T, σ, E, P_b, τ etc., and

Displacement unknowns are Slope (θ), D -tion (Δ), Settlement, sway etc.,

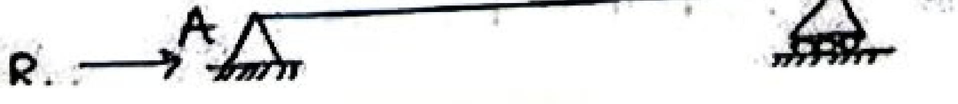
Equilibrium Equations:

(i) $\Sigma M = 0$; Body doesnot rotate in any direc
(ii) $\Sigma H = 0$; Body doesnot move in Horizontal
(iii) $\Sigma V = 0$; Body doesnot move in vertical di

Note: All structures are in equilibrium cond

If the unknowns can be calcula making use of three equilibrium equations -ed determinate structure.

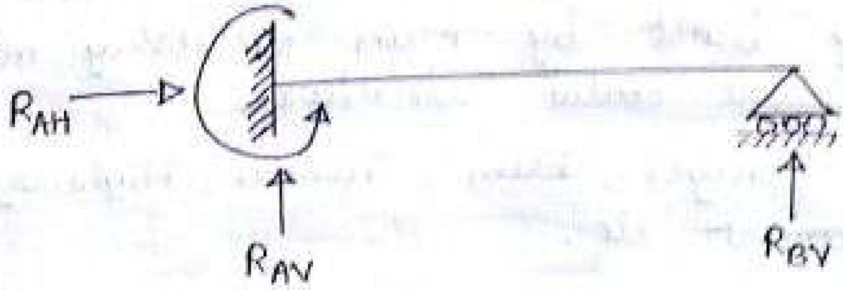
Ex:



If the unknowns cannot be calculated by using the equilibrium equation is called indeterminate structure.

We need extra equation to calculate unknow

Ex:

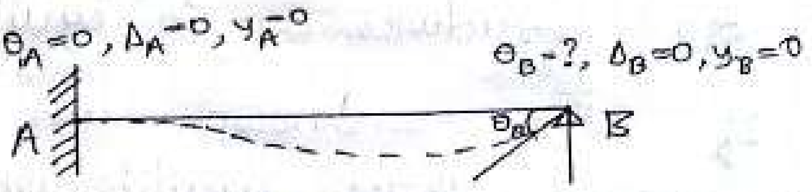


No. of unknowns = 4
Equilibrium eq^{ns} = 3
4 - 3 = 1

Number of extra equations required to calculate unknown reactive components is called static Indeterminancy. It is also called as degree of redundancy (DOR).

Number of extra equations required to calculate unknown displacement components is called Kinematic Indeterminancy. It is also called as Degree of freedom (DOF).

Degree of freedom is defined as 'the possible movement of a structure at the support @ at the joints.'



DOF = θ_B

∴ DOF = 1

There are various methods to analyse statically Indeterminate structures:

1. Slope - Deflection method
2. Moment Distribution method

3. Kani's rotation method
4. Stiffness matrix method
5. Flexibility matrix method

Sign Convention :

*) ↑ +ve ↓ -ve

*) → +ve ← -ve

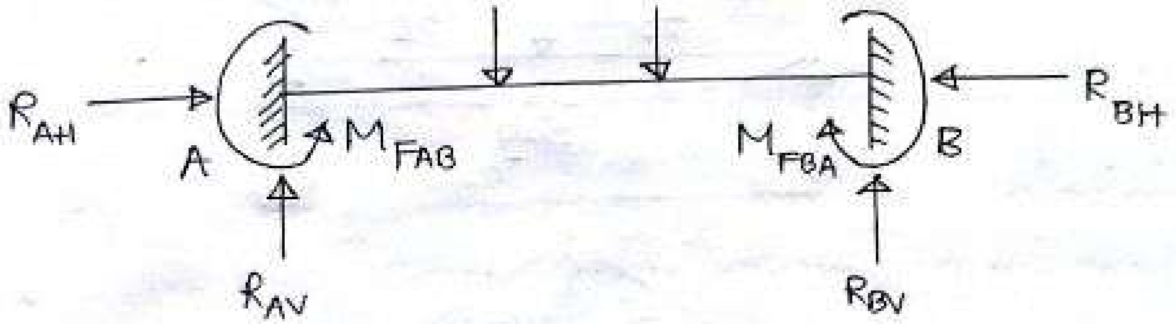
*) ↻ +ve ↺ -ve

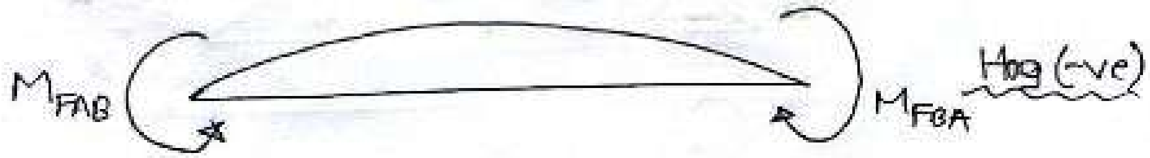
Imp *)

The moments which are developed at the support due to fixity end is called fixed end moment (FEM). The effect of fixed end moment is to 'Hog' the beam.

The moment developed at the centre called sagging bending moment (or) free bending moment (FBM).

Ex :





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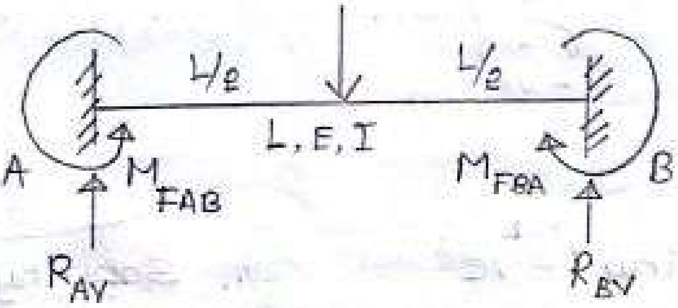
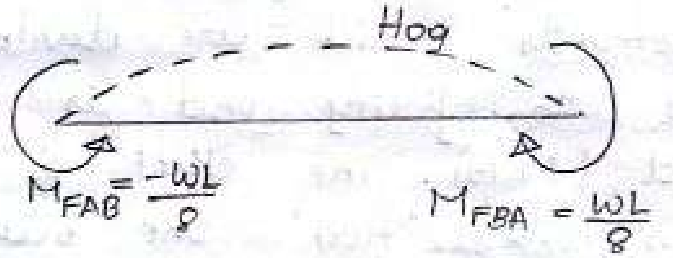
If the cross-section of the beam is unifo throughout is called "prismatic beam".

If the beam is made of same material throughout the length called "homogeneous", if elastic properties are same (E, G, K) called 'isotropic'.

Fixed end moments and free bending moments

1.





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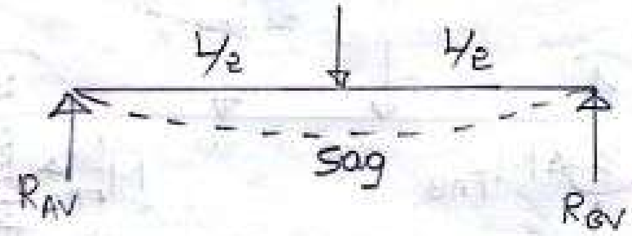

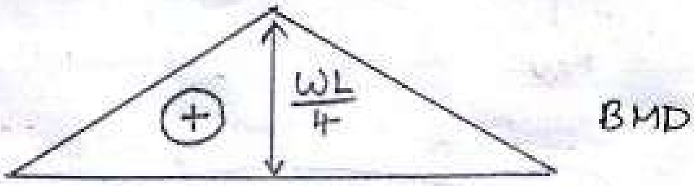

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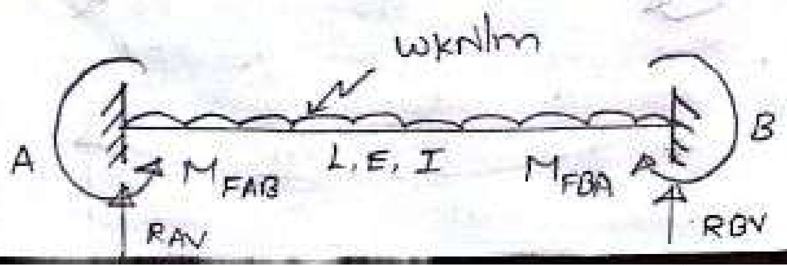
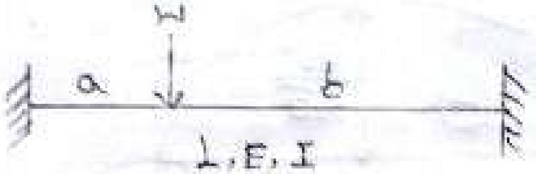
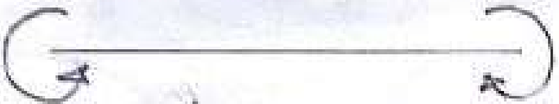


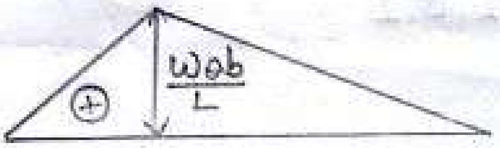
$M_{FAB} = \frac{-wL^2}{12}$ $M_{FBA} = \frac{wL^2}{12}$

3.





$M_{FAB} = \frac{-Wab^2}{L^2}$ $M_{FBA} = \frac{Wa^2b}{L^2}$



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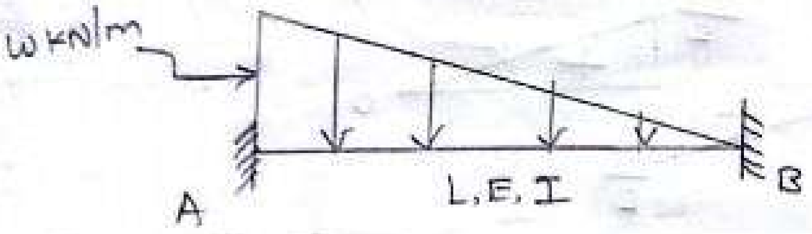
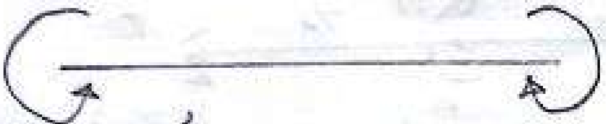
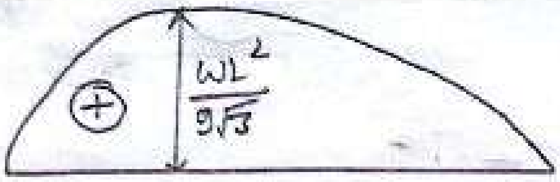




$M_{FAB} = \frac{-wL^2}{20}$ $M_{FBA} = \frac{wL^2}{30}$



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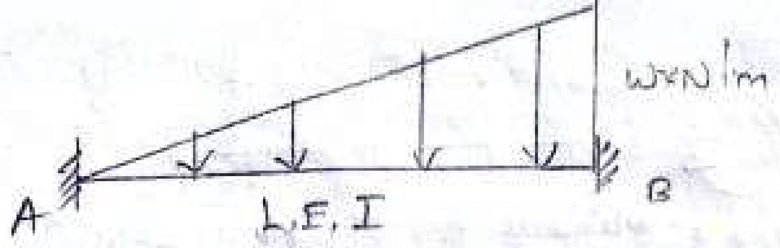
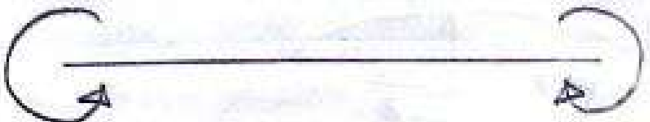
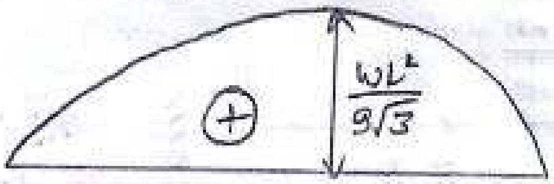
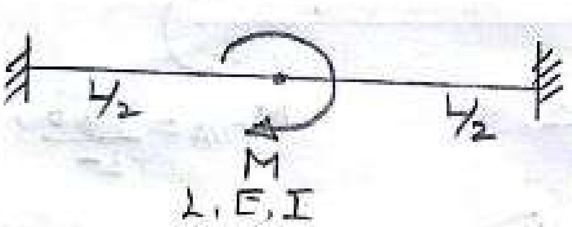




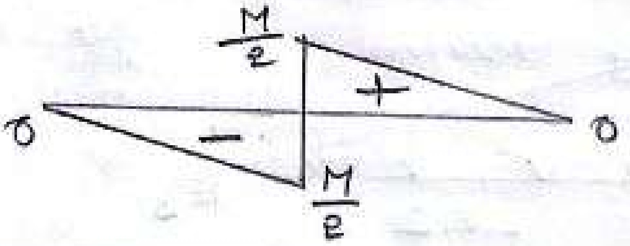
$M_{FAB} = \frac{-wL^2}{30}$ $F_{FBA} = \frac{wL^2}{20}$



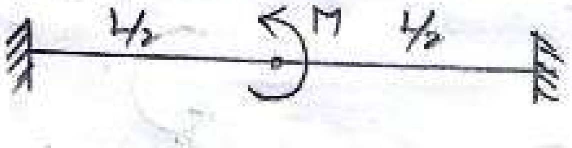
6.

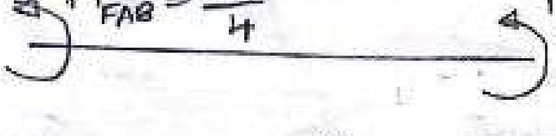


$M_{FAB} = \frac{M}{4}$ $M_{FBA} = -\frac{M}{4}$

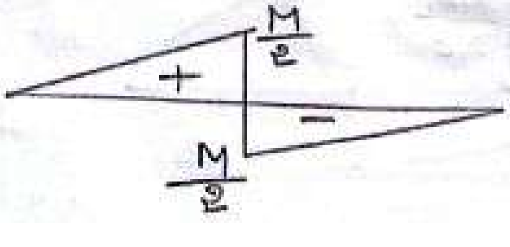


7.



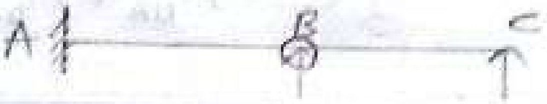


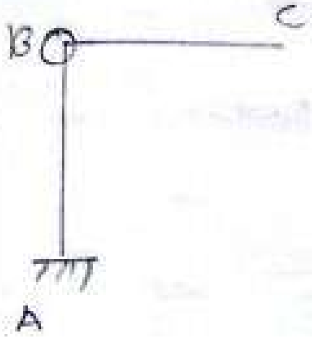
$M_{FAB} = \frac{-M}{4}$ $M_{FBA} = \frac{-M}{4}$

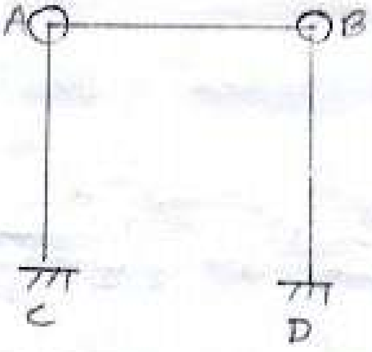


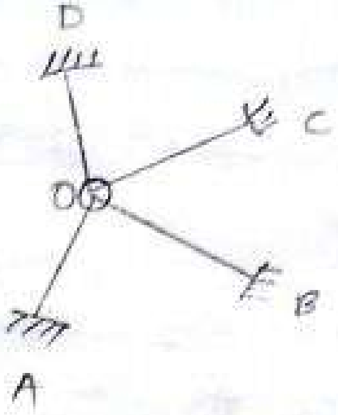
Joint :

Joint is a point where two @ more than two members meet.



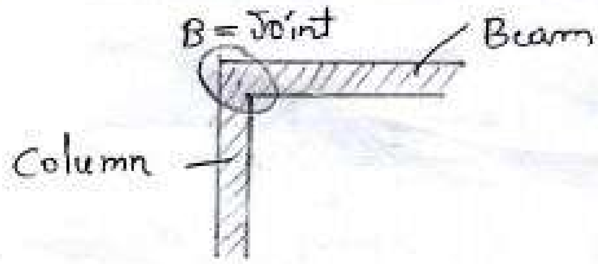




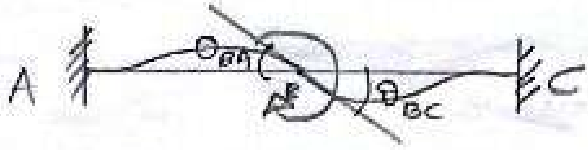


Characteristics of a joint :

1. Joint is monolithic and rigid.



2. Rotation at the joint is constant.



$$\theta_{BA} = \theta_{BC} = \theta_B$$

3. Joints can be replaced by two fixed points.

$$M_{BC} = -M_{BA}$$

$$\therefore M_{BA} + M_{BC} = 0$$

4. Sum of the moments @ the joint is zero.

$$\Sigma M_B = 0$$

i.e., $M_{BA} + M_{BC} = 0$

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Module - 1

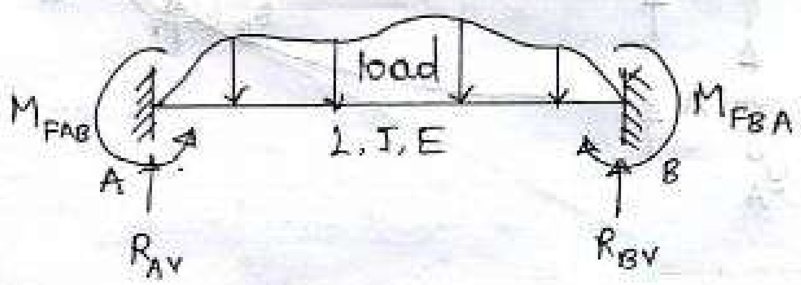
Slope - Deflection Method

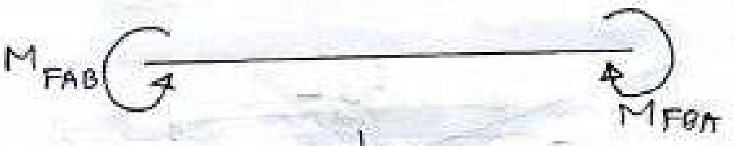
Slope - deflection method is one of the methods to analyse Indeterminate structures.

It is first introduced by an investigator G.A. Maney, hence known as Maney's method.

Slope deflection Equations:

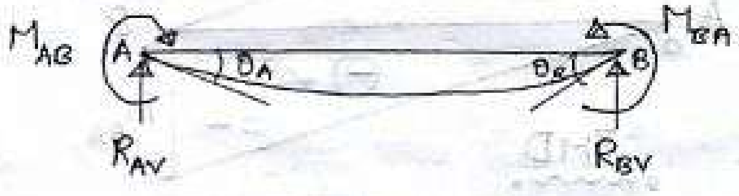
Consider a fixed beam of length 'L', moment of Inertia 'I' and Young's modulus 'E' for general loading condition. as shown in figure.





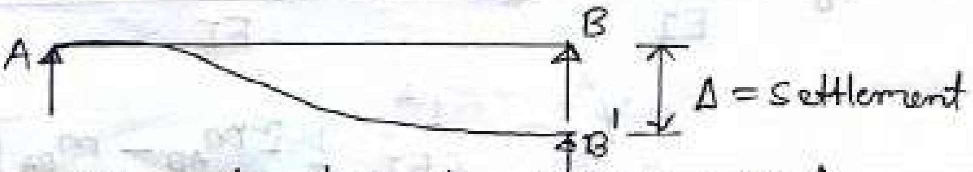
Moment due to Fixity (M_F)

+



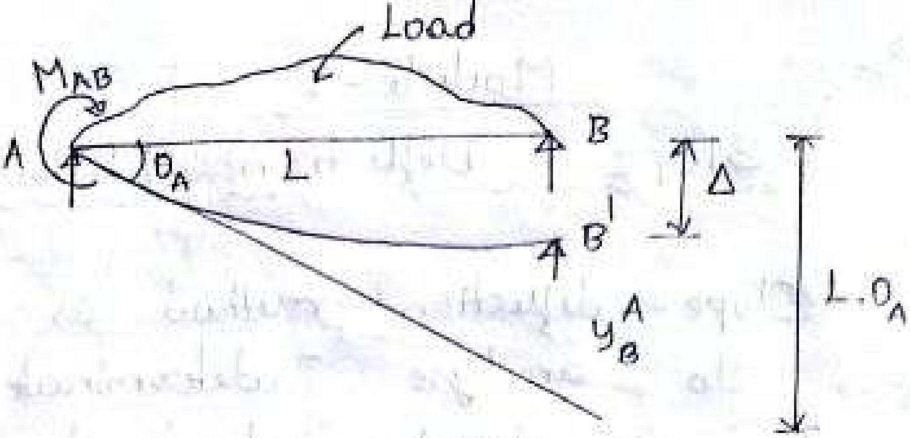
Moment due to rotation (θ)

+



Moment due to settlement (Δ)

$$\therefore \left.\begin{matrix}\text{Final Moment}\\(M)\end{matrix}\right\} = \left\{\begin{matrix}\text{Moment due}\\\text{to fixity}\end{matrix}\right\} + \left\{\begin{matrix}\text{Moment due}\\\text{to rotation}\end{matrix}\right\} + \left\{\begin{matrix}\text{moment}\\\text{to settlement}\end{matrix}\right\}$$



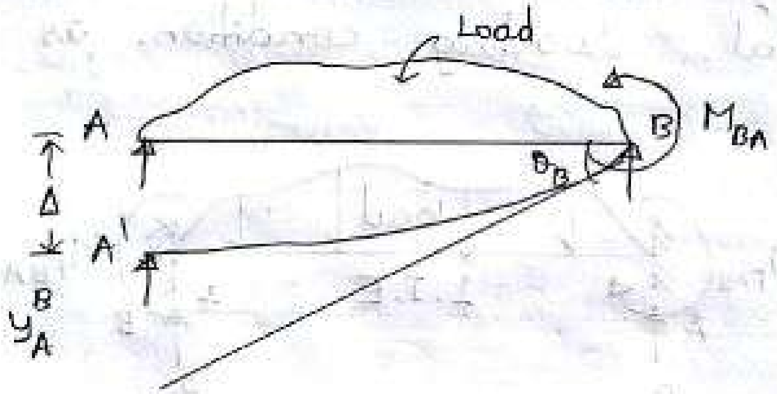
Deviation at 'B' when the moment is at 'A is i.e.,

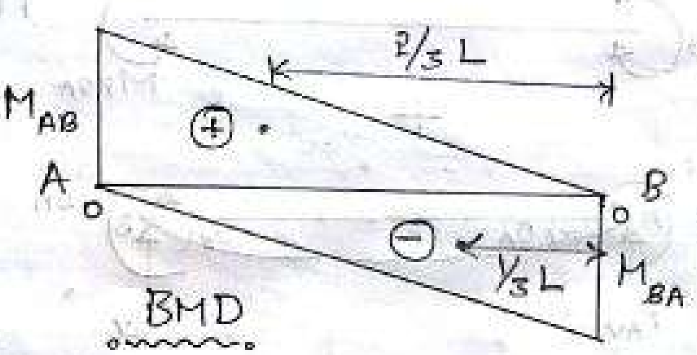
$$Y_B^A = (L.\theta_A - \Delta)$$

Similarly deviation at 'A' when the moment is at 'B' is

$$y_A^B = (L\theta_B - \Delta)$$

i.e.,





$$Y_B^A = \frac{A\bar{X}}{EI} = \frac{(^1/_2 \times m_{AB} \times L)\,^2/_3 L - (^1/_2 \times m_{BA} \times L)\,^1/_3 L}{EI}$$

$$Y_B^A = \frac{L^2}{6EI}\left[2m_{AB} - m_{BA}\right]$$

$$\frac{L^2}{6EI}\left[2m_{AB} - m_{BA}\right] = L\theta_A - \Delta$$

$$2m_{AB} - m_{BA} = \frac{6EI}{L^2}(L\theta_A - \Delta)$$

$$2m_{AB} - m_{BA} = \frac{6EI}{L}\left(\theta_A - \frac{\Delta}{L}\right) \longrightarrow \text{①}$$

Similarly,

$$2m_{BA} - m_{AB} = \frac{6EI}{L}\left(\theta_B - \frac{\Delta}{L}\right) \longrightarrow \text{②}$$

Solving the equations ① and ②, we get

$$2m_{AB} - m_{BA} = \frac{6EI}{L}\left(\theta_A - \Delta/L\right) \quad \times 2$$

$$2m_{BA} - m_{AB} = \frac{6EI}{L}\left(\theta_B - \Delta/L\right)$$

$$3m_{AB} = \frac{12EI}{L}\left(\theta_A - \Delta/L\right) + \frac{6EI}{L}\left(\theta_B - \Delta/L\right)$$

$$3m_{AB} = \frac{6EI}{L}\left\{\left(2\theta_A - \frac{2\Delta}{L}\right) + \theta_B - \Delta/L\right\}$$

$$m_{AB} = \frac{2EI}{L}\left(2\theta_A + \theta_B - \frac{3\Delta}{L}\right)$$

$$m_{BA} = \frac{2EI}{L}\left(2\theta_B + \theta_A - \frac{3\Delta}{L}\right)$$

∴ Final moment (M),

$$\boxed{\begin{aligned} M_{AB} &= M_{FAB} + \frac{2EI}{L}\left(2\theta_A + \theta_B - \frac{3\Delta}{L}\right) \rightarrow \text{③} \\ M_{BA} &= M_{FBA} + \frac{2EI}{L}\left(2\theta_B + \theta_A - \frac{3\Delta}{L}\right) \rightarrow \text{④} \end{aligned}}$$

Eq^{ns} ③ and ④ are the slope - deflection equations.

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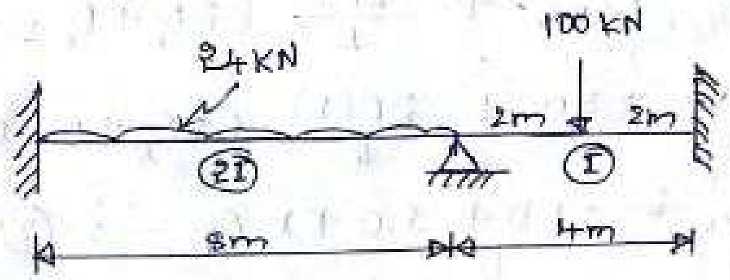
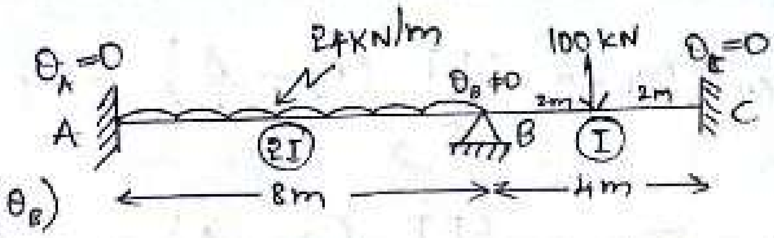
Problems:

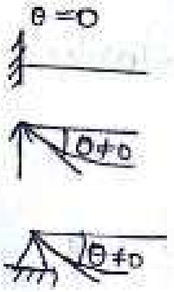
1. A continuous beam is loaded as shown below. Draw the BMD and elastic curve. by slope - deflection method.

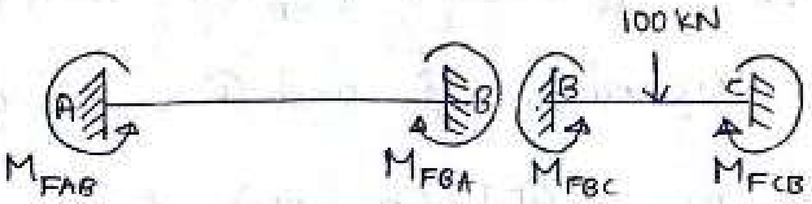


→ ①



DOF = 1(θB)





Step 1: Fixed End moments:

$$M_{FAB} = \frac{-WL^2}{12} = \frac{-24\times8^2}{12} = -128 \text{ kN-m}$$

$$M_{FBA} = \frac{WL^2}{12} = \frac{24\times8^2}{12} = 128 \text{ kN-m}$$

$$M_{FBC} = \frac{-WL}{8} = \frac{-100\times4}{8} = -50 \text{ kN-m}$$

$$M_{FCB} = \frac{WL}{8} = \frac{100\times4}{8} = 50 \text{ kN-m}$$

Step 2: Slope-deflection Equations:

$$M_{AB} = M_{FAB} + \frac{2EI}{L}\left(\cancel{2\theta_A}^{0} + \theta_B - \cancel{\frac{3\Delta}{L}}^{0}\right)$$

$$M_{AB} = -128 + \frac{2E(2I)}{8}(\theta_B)$$

$$M_{AB} = -128 + 0.5\ EI\ \theta_B \rightarrow ①$$

$$M_{BA} = M_{FBA} + \frac{2EI}{L}\left(2\theta_B + \theta_A - \frac{3\Delta}{L}\right)$$

$$= 128 + \frac{2E(2I)}{8}(2\theta_B + 0 - 0)$$

$$M_{BA} = 128 + 1.0\ EI\ \theta_B \longrightarrow ②$$

$$M_{BC} = M_{FBC} + \frac{2EI}{L}\left(2\theta_B + \theta_C - \frac{3\Delta}{L}\right)$$

$$= -50 + \frac{2E(I)}{4}(2\theta_B + 0 - 0)$$

$$M_{BC} = -50 + 1.0\ EI\ \theta_B \longrightarrow ③$$

$$M_{CB} = M_{FCB} + \frac{2EI}{L}\left(2\theta_c + \theta_B - \frac{3\Delta}{L}\right)$$

$$= 50 + \frac{2E(I)}{4}\left(0 + \theta_B - 0\right)$$

$$\therefore \quad M_{CB} = 50 + 0.5\ EI\ \theta_B \rightarrow ④$$

Step 3: Joint - Equilibrium Equations:

$$\Sigma M_B = 0; \quad M_{BA} + M_{BC} = 0$$

$$128 + 1.0\ EI\ \theta_B - 50 + 1.0\ EI\theta_B = 0$$

$$2\ EI\ \theta_B + 78 = 0$$

$$2\ EI\ \theta_B = -78$$

$$\therefore \boxed{\theta_B = \frac{-39}{EI}}$$

Step 4: Final Moments:

① $\Rightarrow$ $$M_{AB} = -128 + 0.5\ EI\left(\frac{-39}{EI}\right)$$

$$= -128 - 19.5$$

$$\therefore \underline{M_{AB} = -147.5\ \text{kN-m}}$$

② $\Rightarrow$ $$M_{BA} = 128 + 1.0\ EI\left(\frac{-39}{EI}\right)$$

$$= 128 - 39$$

$$\underline{M_{BA} = 89\ \text{kN-m}}$$

③ $\Rightarrow$ $$M_{BC} = -50 + 1.0\ EI\left(\frac{-39}{EI}\right)$$

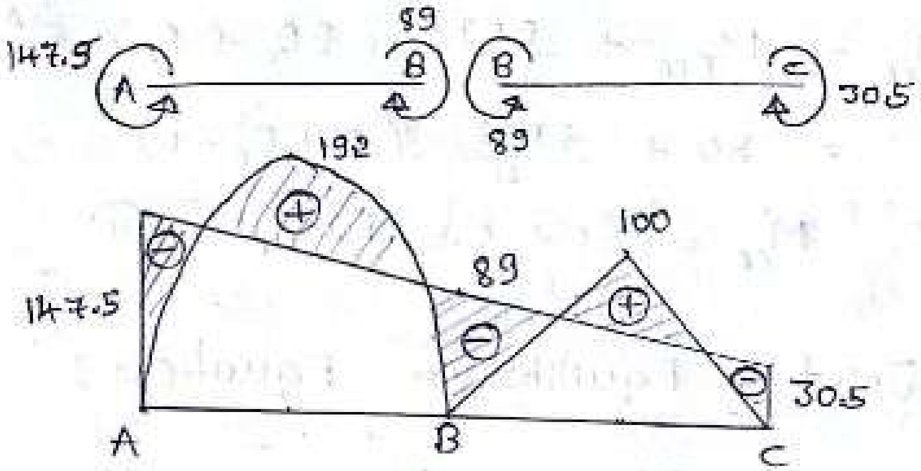
$$= -50 - 39$$

$$\underline{M_{BC} = -89\ \text{kN-m}}$$

④ $\Rightarrow$ $$M_{CB} = 50 + 0.5\ EI\left(\frac{-39}{EI}\right)$$

$$= 50 - 19.5$$

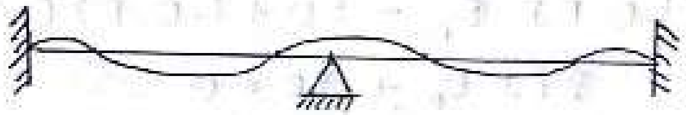
$$\underline{M_{CB} = 30.5\ \text{kN-m}}$$



$\frac{wL^2}{8}$; $\frac{wL}{4}$

=192 ; =100

BMD



Elastic Curve

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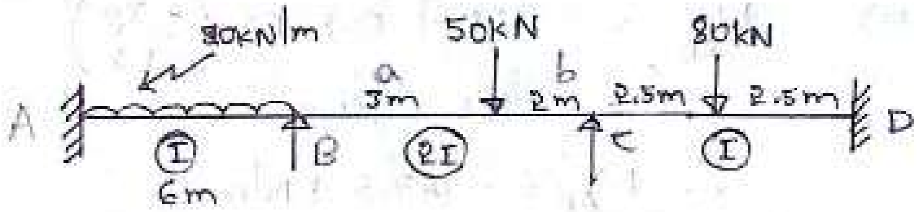
2. Analyse the continuous beam shown in the figure by slope deflection method. Draw BMD, elastic curve and SF diagram.



→ DOF = 2 (θ_B, θ_C)

Step 1: Fixed end moments :-

$$M_{FAB} = \frac{-wL^2}{12} = -\frac{20 \times 6^2}{12} = -60 \text{ kNm}$$

$$M_{FBA} = \frac{wL^2}{12} = \frac{20 \times 6^2}{12} = 60 \text{ kN-m}$$

$$M_{FBC} = \frac{-Wab^2}{L^2} = \frac{-50 \times 3 \times 2^2}{5^2} = -24 \text{ kN-m}$$

$$M_{FCB} = \frac{Wa^2b}{L^2} = \frac{50 \times 3^2 \times 2}{5^2} = 36 \text{ kN-m}$$

$$M_{FCD} = \frac{-WL}{8} = \frac{-80 \times 5}{8} = -50 \text{ kN-m}$$

$$F_{FDC} = \frac{WL}{8} = 50 \text{ kN-m}$$

Step 2 : Slope - Deflection equations :

$$M_{AB} = M_{FAB} + \frac{2EI}{L}\left(2\theta_A + \theta_B - \frac{3\Delta}{L}\right)$$

$$= -60 + \frac{2E(I)}{6}(0 + \theta_B - 0)$$

$$\therefore M_{AB} = -60 + 0.3333\ EI\ \theta_B \longrightarrow ①$$

$$M_{BA} = M_{FBA} + \frac{2EI}{L}\left(2\theta_B + \theta_A - \frac{3\Delta}{L}\right)$$

$$= 60 + \frac{2E(I)}{6}(2\theta_B + 0 - 0)$$

$$\therefore M_{BA} = 60 + 0.6667\ EI\ \theta_B \longrightarrow ②$$

$$M_{BC} = M_{FBC} + \frac{2EI}{L}\left(2\theta_B + \theta_C - \frac{3\Delta}{L}\right)$$

$$= -24 + \frac{2E(2I)}{5}(2\theta_B + \theta_C - 0)$$

$$\therefore M_{BC} = -24 + 1.6\ EI\ \theta_B + 0.8\ EI\ \theta_C \rightarrow ③$$

$$M_{CB} = M_{FCB} + \frac{2EI}{L}\left(2\theta_C + \theta_B - \frac{3\Delta}{L}\right)$$

$$= +36 + \frac{2E(2I)}{5}(2\theta_C + \theta_B - 0)$$

$$\therefore M_{CB} = 36 + 1.6\ EI\ \theta_C + 0.8\ EI\ \theta_B \rightarrow ④$$

$$M_{CD} = M_{FCD} + \frac{2EI}{L}\left(2\theta_C + \theta_D - \frac{3\Delta}{L}\right)$$

$$= -50 + \frac{2EI}{5}(2\theta_C + 0 - 0)$$

$$\therefore M_{CD} = -50 + 0.8\ EI\ \theta_C \longrightarrow ⑤$$

$$M_{DC} = M_{FDC} + \frac{2EI}{L}\left(2\theta_D + \theta_C - \frac{3\Delta}{L}\right)$$

$$= 50 + \frac{2EI}{L}(0 + \theta_C - 0)$$

$$\therefore M_{DC} = 50 + 0.4\ EI\ \theta_C \rightarrow ⑥$$

Step 3 : Joint Equilibrium equations :

$$\Sigma M_B = 0 ; \quad M_{BA} + M_{BC} = 0$$

$$60 + 0.6666\ EI\ \theta_B - 24 + 1.6\ EI\ \theta_B + 0.8\ EI\ \theta_C = 0$$

$$2.2666\,EI\,\theta_B + 0.8\,EI\,\theta_C = -36 \longrightarrow Ⓐ$$

$$\Sigma M_c = 0; \qquad M_{CB} + M_{CD} = 0$$

$$36 + 1.6\,EI\,\theta_C + 0.8\,EI\,\theta_B - 50 + 0.8\,EI\,\theta_C = 0$$

$$0.8\,EI\,\theta_B + 2.4\,EI\,\theta_C = 14 \longrightarrow Ⓑ$$

Now,

$$EI.\,\theta_B = \frac{\begin{vmatrix} -36 & 0.8 \\ 14 & 2.4 \end{vmatrix}}{\begin{array}{cc} \theta_B & \theta_C \\ \hline 2.2666 & 0.8 \\ 0.8 & 2.4 \end{array}} = \frac{-36 \times 2.4 - 14 \times 0.8}{2.667 \times 2.4 - 0.8 \times 0.8}$$

{ Cramer's rule

$$\theta_B = \frac{-20.93}{EI}$$

(or)

Multiplication method :

$$6.798\,EI\,\theta_B + 2.4\,EI\,\theta_C = -108$$

$$0.8\,EI\,\theta_B + 2.4\,EI\,\theta_C = 14$$

$$5.97\,EI\,\theta_B = -122$$

$$\therefore \theta_B = \frac{-20.34}{EI}$$

(or) Using calculator (Eq^n) :

$$\therefore \theta_B = -\frac{20.33}{EI}, \quad \theta_C = \frac{12.61}{EI}$$

Step 4 : Final moments :

① ⇒ $M_{AB} = -60 + 0.3333\,EI\left(\frac{-20.33}{EI}\right)$

$\therefore M_{AB} = -66.8$ KN-m

② ⇒ $M_{BA} = 46.4$ KN-m

③ ⇒ $M_{BC} = -24 + 1.6\,EI\left(\frac{-20.33}{EI}\right) + 0.8\,EI\left(\frac{12.61}{EI}\right)$

$M_{BC} = -46.4$ kN-m

④ ⇒ $M_{CB} = 39.9$ kN-m

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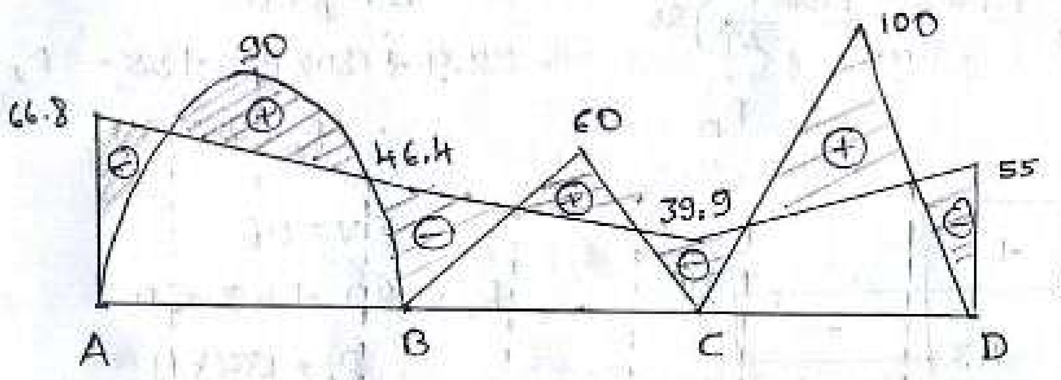
⑤ ⟹ $M_{CD} = -50 + 0.8EI\left(\frac{12.61}{EI}\right)$

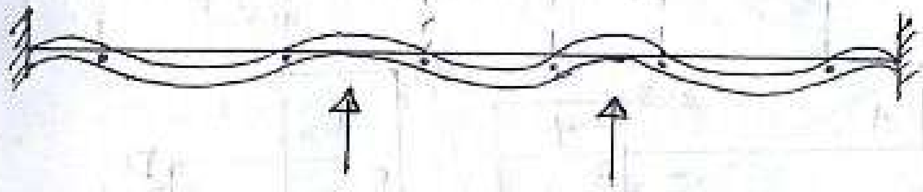
$M_{CD} = -39.9$ kN-m

⑥ ⟹ $M_{DC} = 50 + 0.4EI\left(\frac{12.61}{EI}\right)$

$\therefore M_{DC} = 55$ kN-m



BMD.

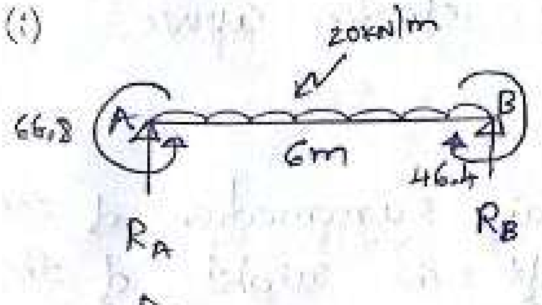


Elastic curve.

Note: The bentform shape of the beam due to external bending is known as 'elastic curve'.

To draw SF diagram:

(i)



$\Sigma M_A = 0$;

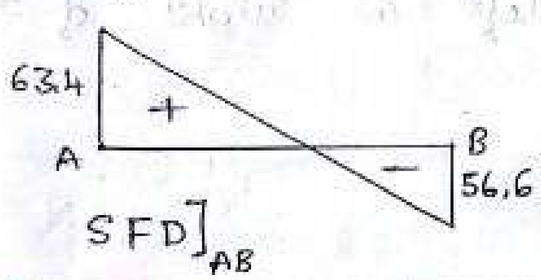
$-66.8 + (20\times6)3 - R_B\times6 + 46.4 = 0$

$\therefore R_B = 56.6$ kN

$(\overset{\uparrow}{+ve})\ \Sigma V = 0$;

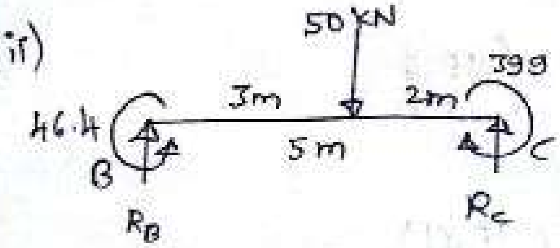
$R_A - (20\times6) + 56.6 = 0$

$\therefore R_A = 63.4$ kN



SFD]$_{AB}$

(ii)



$\Sigma M_B = 0;$

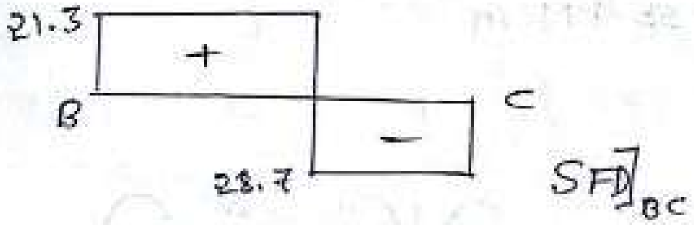
$-46.4 + 50\times3 - R_c\times5 + 39.9 = 0$

$\therefore R_c = 28.7$ kN

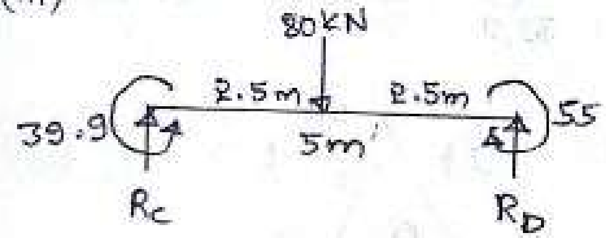
$\Sigma V = 0;$

$+R_B - 50 + 28.7 = 0$

$\therefore R_B = 21.3$ kN



(iii)



$\Sigma M_B = 0;$

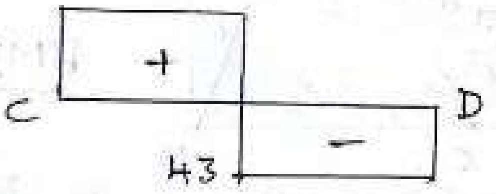
$-39.9 + 80\times2.5 + 55 - R_D\times5 = 0$

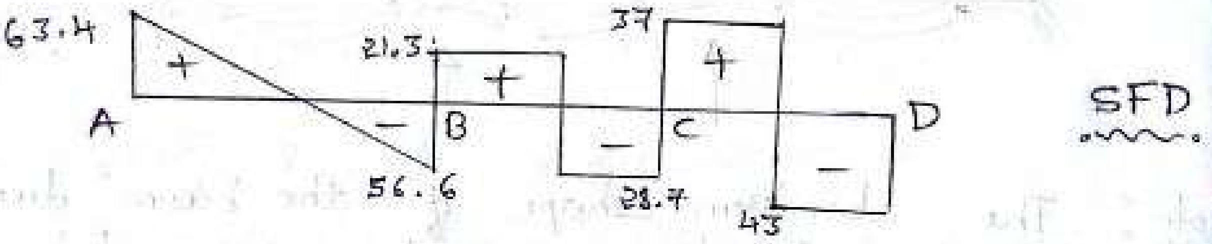
$\therefore R_D = 43$ KN

$\Sigma V = 0;$

$R_c - 80 + 43 = 0$

$\therefore R_c = 37$ kN





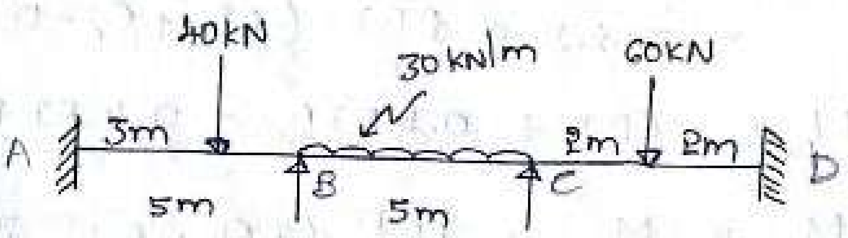
Elastic Curve:

Bekentum shape of the beam due to external load is called elastic curve.

Shear Force:

It is the algebraic summation of the vertical forces either left @ right of the section.

3. Analyse the continuous beam by slope-deflection method. Draw BMD, SFD & elastic curve.



→ Here moment of inertia is not given, hence we assume it as ⓘ (continuous beam).

DOF = 2 (θ_B, θ_C)

Step 1: Fixed end moments :

$$M_{FAB} = \frac{-wab^2}{L^2} = \frac{-40\times3\times2^2}{2} = -19.2 \text{ kN-m}$$

$$M_{FBA} = \frac{wa^2b}{L^2} = \frac{40\times3^2\times2}{2} = 28.8 \text{ kN-m}$$

$$M_{FBC} = \frac{-wL^2}{12} = \frac{-30\times5^2}{12} = -62.5 \text{ kN-m}$$

$$M_{FCB} = \frac{wL^2}{12} = \frac{30\times5^2}{12} = 62.5 \text{ kN-m}$$

$$M_{FCD} = \frac{-wL}{8} = \frac{-60\times4}{8} = -30 \text{ kN-m}$$

$$M_{FDC} = \frac{wL}{8} = \frac{60\times4}{8} = 30 \text{ kN-m}$$

Step 2: Slope - Deflection Equation:

$$M_{AB} = M_{FAB} + \frac{2EI}{L}\left(2\theta_A + \theta_B - \frac{3\Delta}{L}\right)$$

$$= -19.2 + \frac{2EI}{5}(0 + \theta_B - 0)$$

$$\therefore M_{AB} = -19.2 + 0.4\ EI\ \theta_B \rightarrow ①$$

$$M_{BA} = M_{FBA} + \frac{2EI}{L}\left(2\theta_B + \theta_A - 3\Delta/L\right)$$

$$= 28.8 + \frac{2EI}{5}(2\theta_B + 0 - 0)$$

$\therefore M_{BA} = 28.8 + 0.8\ EI\ \theta_B \rightarrow$ ②

Now, $M_{BC} = M_{FBC} + \frac{2EI}{L}\left(2\theta_B + \theta_C - \frac{3\Delta}{L}\right)$

$= -62.5 + \frac{2EI}{5}\left(2\theta_B + \theta_C - 0\right)$

$\therefore M_{BC} = -62.5 + 0.8 EI\theta_B + 0.4\ EI\ \theta_C \rightarrow$ ③

Now, $M_{CB} = M_{FCB} + \frac{2EI}{L}\left(2\theta_C + \theta_B - \frac{3\Delta}{L}\right)$

$= 62.5 + 0.4\ EI\ (2\theta_C + \theta_B - 0)$

$\therefore M_{CB} = 62.5 + 0.8\ EI\ \theta_C + 0.4\ EI\ \theta_B \rightarrow$ ④

Now, $M_{CD} = M_{FCD} + \frac{2EI}{L}\left(2\theta_C + \theta_D - \frac{3\Delta}{L}\right)$

$= -30 + \frac{2EI}{4}\left(2\theta_C + 0 - 0\right)$

$\therefore M_{CD} = -30 + EI.\ \theta_C \rightarrow$ ⑤

Now, $M_{DC} = M_{FDC} + \frac{2EI}{L}\left(2\theta_D + \theta_C - 3\Delta/L\right)$

$\therefore M_{DC} = 30 + 0.5.\ EI.\ \theta_C \rightarrow$ ⑥

Step 4: Joint Equilibrium Equations:

$\Sigma M_B = 0;\quad M_{BA} + M_{BC} = 0$

$28.8 + 0.8\ EI\ \theta_B - 62.5 + 0.8\ EI\ \theta_B + 0.4\ EI\ \theta_C = 0$

$1.6\ EI\ \theta_B + 0.4\ EI\ \theta_C = 33.7 \rightarrow$ Ⓐ

Now, $\Sigma M_C = 0;\ M_{CB} + M_{CD} = 0$

$62.5 + 0.8\ EI\ \theta_C + 0.4\ EI\theta_B - 30 + EI.\ \theta_C = 0$

$0.4\ EI\ \theta_B + 1.8\ EI\ \theta_C = -32.5 \rightarrow$ Ⓑ

$\therefore$ From eq^{ns} Ⓐ and Ⓑ,

$\theta_B = \frac{27.08}{EI},\quad \theta_C = -\frac{24.10}{EI}$

Step 4: Final Moments:

$\therefore M_{AB} = -19.2 + 0.4\ EI\left(\frac{27.08}{EI}\right) = -8.37$ KN-m

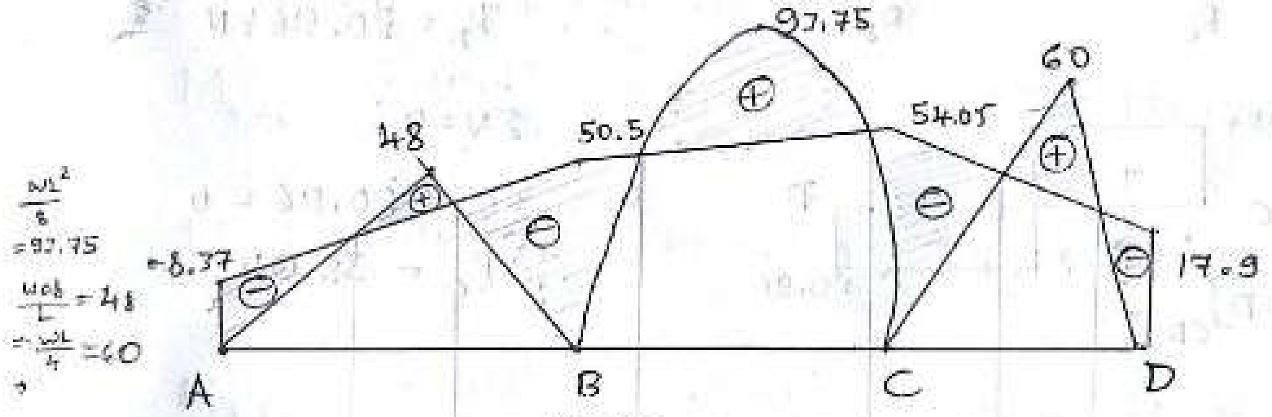
Also, $M_{BA} = 28.8 + 0.8\,EI\left(\frac{+27.08}{EI}\right) = 50.46$ kN-m

$M_{BC} = -62.5 + 21.664 - 9.64 = -50.47$ KN-m

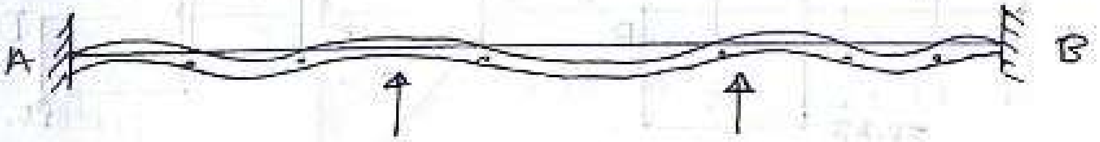
$M_{CB} = 62.5 + (-19.28) + 10.83 = 54.05$ KN-m

Also, $M_{CD} = -30 - 24.10 = -54.1$ KN-m

$M_{DC} = 30 - 12.05 = 17.9$ KN-m



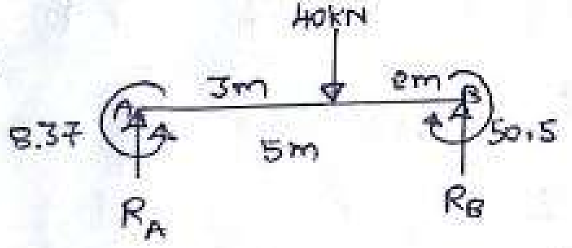
BMD



Elastic curve

Step 5: To draw SF diagram:

(i)



$\Sigma M_A = 0;$

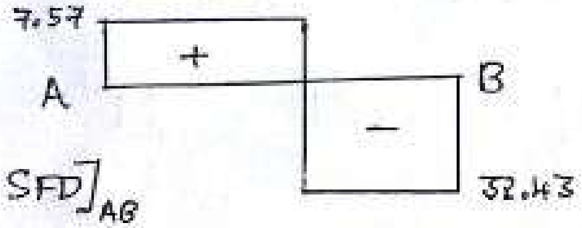
$-8.37 + (40 \times 3) - (R_B \times 5) + 50.5 =$

$\therefore R_B = 32.43$ KN

$(\overset{\uparrow}{+ve})\ \Sigma V = 0;$

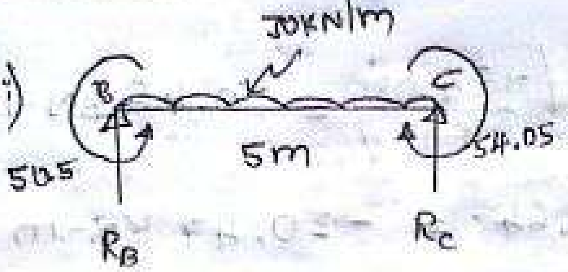
$R_A - 40 + 32.43 = 0$

$\therefore R_A = 7.57$ KN



$SFD]_{AB}$

(ii)



$\Sigma M_A = 0;$

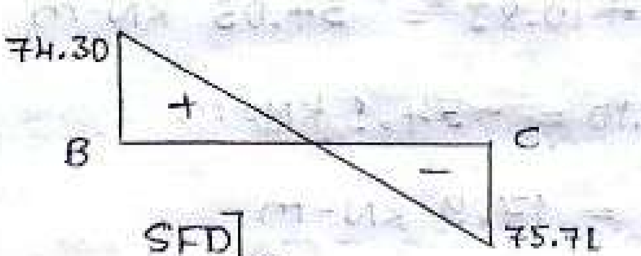
$-50.5 + (30 \times 5)2.5 + 54.05 - R_c \times 5 =$

$\therefore R_c = 75.71$ KN

$\Sigma V = 0;$

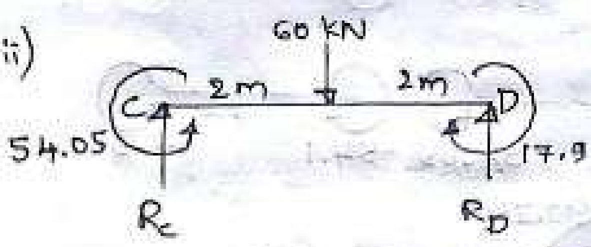
$R_B - 150 + R_c(75.71) = 0$

$\therefore R_B = 74.30$ KN



$SFD]_{BC}$

(iii)



$\Sigma M_A = 0;$

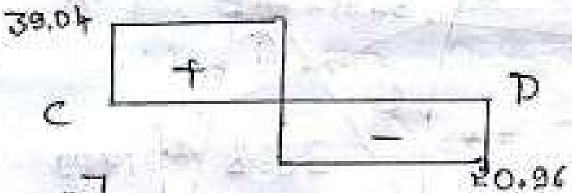
$-54.05 + (60 \times 2) + 17.9 - R_D \times 4 = 0$

$\therefore \quad R_D = 20.96$ KN

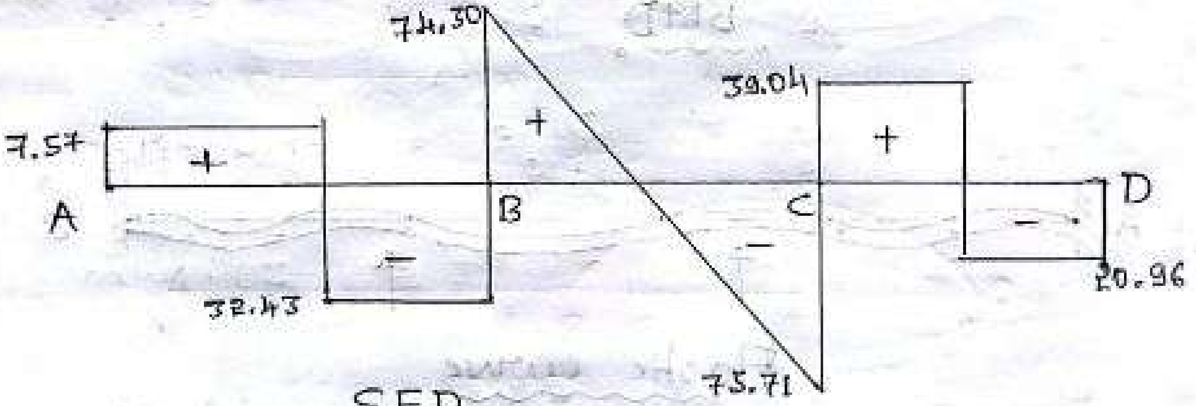
$\Sigma V = 0;$

$R_c - 60 + 20.96 = 0$

$\therefore R_c = 39.04$



$SFD]_{CD}$



SFD

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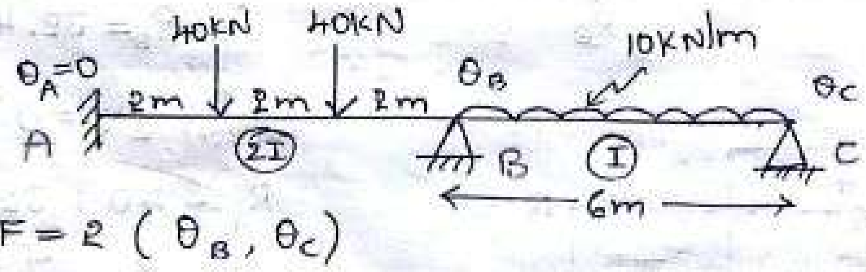
4. Analyse the continuous beam loaded shown in the figure by slope-deflection method. Draw BMD and SFD.



⟶ DOF = 2 (θ_B, θ_C)

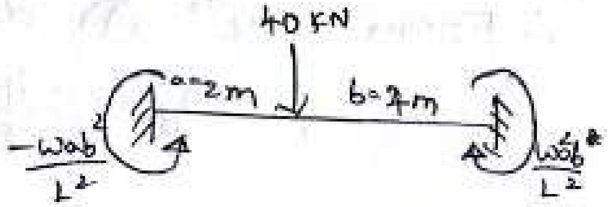
20/08/18

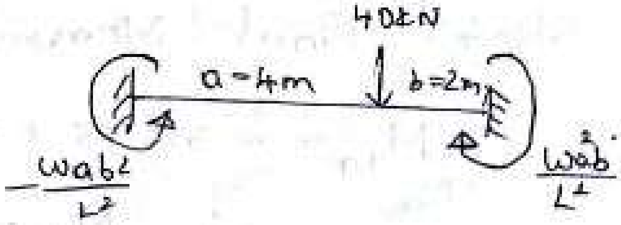
Step 1 : Fixed End Moments :

$$M_{FAB} = \frac{-wab^2}{L^2} - \frac{wab^2}{L^2}$$

$$= \frac{-40\times2\times4^2}{6^2} - \frac{40\times4\times2^2}{6^2}$$

$$= -53.33 \text{ KNm}$$





$$M_{FBA} = \frac{wa^2b}{L^2} + \frac{wa^2b}{L^2}$$

$$= \frac{40\times2^2\times4}{6^2} + \frac{40\times4^2\times2}{6^2}$$

$$= 53.33 \text{ KNm}$$

$$M_{FBC} = \frac{-wL^2}{12} = \frac{-10\times6^2}{12} = -30 \text{ KNm}$$

$$M_{FCB} = \frac{wL^2}{12} = \frac{10\times6^2}{12} = 30 \text{ KNm}$$

Step 2 : Slope - Deflection Equations :

$$M_{AB} = -53.33 + \frac{2E(2I)}{6}\left(2\overset{0}{\cancel{\theta_A}} + \theta_B - \overset{0}{\cancel{\frac{3\Delta}{L}}}\right)$$

$$= -53.33 + 0.667\ EI\ \theta_B \longrightarrow ①$$

$$M_{BA} = 53.33 + \frac{2E(2I)}{6}\left(2\theta_B + \overset{0}{\cancel{\theta_A}} - \overset{0}{\cancel{\frac{3\Delta}{L}}}\right)$$

$$= 53.33 + 1.333\ EI\ \theta_B \rightarrow ②$$

$$M_{BC} = -30 + \frac{2E(I)}{6}\left(2\theta_B + \theta_C - \overset{0}{\cancel{\frac{3\Delta}{L}}}\right)$$

$$= -30 + 0.667\ EI\ \theta_B + 0.333\ EI\ \theta_C \rightarrow ③$$

$$M_{CB} = 30 + \frac{2E(I)}{6}\left(2\theta_C + \theta_B - \overset{0}{\cancel{\frac{3\Delta}{L}}}\right)$$

$$= 30 + 0.667\ EI\ \theta_C + 0.333\ EI\ \theta_B \rightarrow ④$$

Step 3 : Joint Equilibrium Equations :

$$\Sigma M_B = 0; \quad M_{BA} + M_{BC} = 0$$

$$53.33 + 1.333\ EI\ \theta_B - 30 + 0.667\ EI\ \theta_B + 0.333\ EI\ \theta_C = 0$$

$$2.0\ EI\ \theta_B + 0.333\ EI\ \theta_C = -23.33 \rightarrow Ⓐ$$

$M_{CB}=0$;

$$0.333\,EI\theta_B + 0.667\,\theta_C = -30 \rightarrow \text{Ⓑ}$$

∴ From eqns Ⓐ and Ⓑ,

$$\theta_B = \frac{-4.55}{EI}, \quad \theta_C = \frac{-42.70}{EI}$$

Step 4: Final Moments:

$$M_{AB} = -53.33 + 0.667\,\cancel{EI}\left(\frac{-4.55}{\cancel{EI}}\right)$$

$$\therefore \quad M_{AB} = -56.4 \text{ KN-m}$$

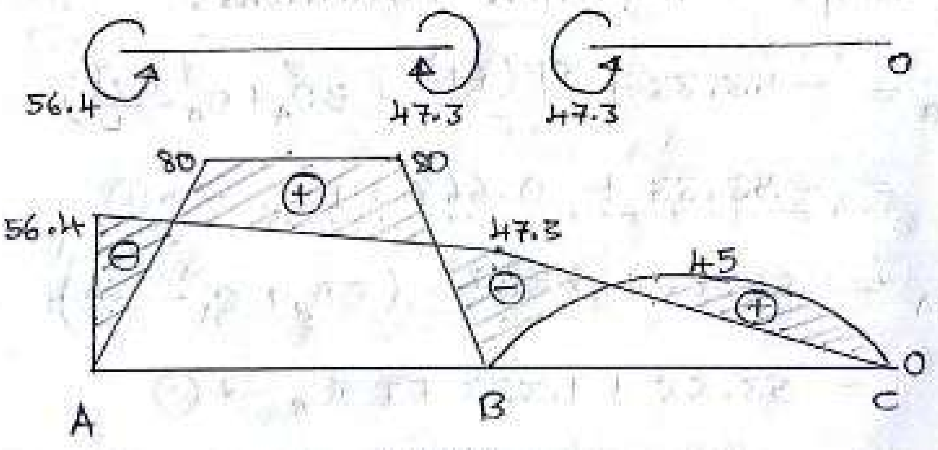
$$M_{BA} = +47.33 \text{ KNm}$$

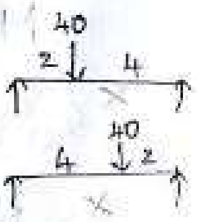
$$M_{BC} = -30 + 0.667\,EI\left(\frac{-4.55}{EI}\right) + 0.333\,EI\left(\frac{-42.70}{EI}\right)$$

$$M_{BC} = -47.3 \text{ KNm}$$

$$M_{CB} = 30 + 0.667\,EI\left(\frac{-42.7}{EI}\right) + 0.333\,EI\left(\frac{-4.55}{EI}\right)$$

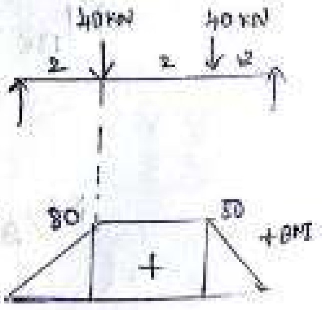
$$M_{CB} = 0 \text{ KN-m}$$





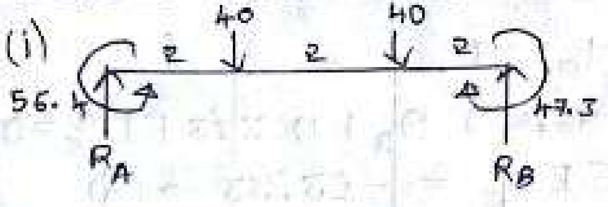
BMD

Elastic Curve



Step 5: To draw SFD:

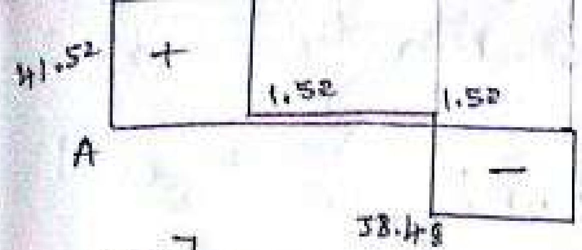
(i)



$\Sigma M_A = 0$;

$$-56.4 + 40\times2 + 40\times4 + 47.3 - R_B\times6 = 0$$

$$\therefore \quad R_B = 38.48 \text{ KN}$$



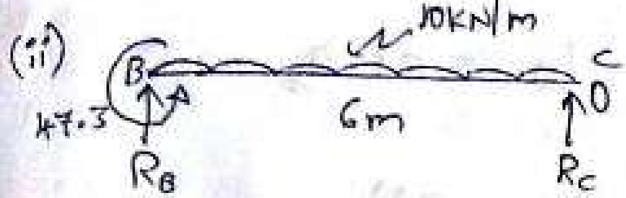
$\Sigma V = 0;$

$R_A - 40 - 40 + 38.48 = 0$

$R_A = 41.52$ kN

SFD]$_{AB}$

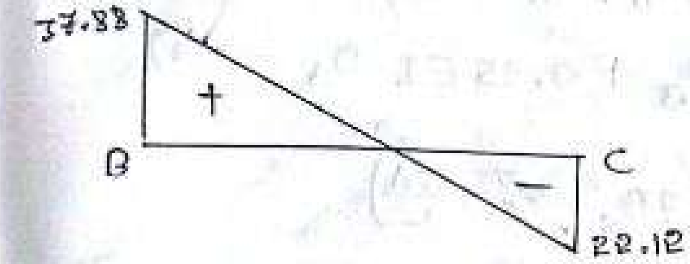
(ii)



$\Sigma M_B = 0;$

$47.3 + 60 \times 3 - R_c \times 6 = 0$

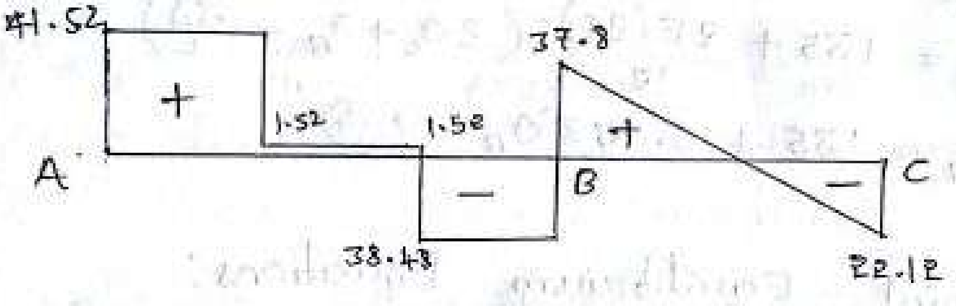
$\therefore R_c = 22.12$ kN



$\Sigma V = 0;$

$R_B - 60 + 22.12 = 0$

$R_B = 37.88$ kN



SFD

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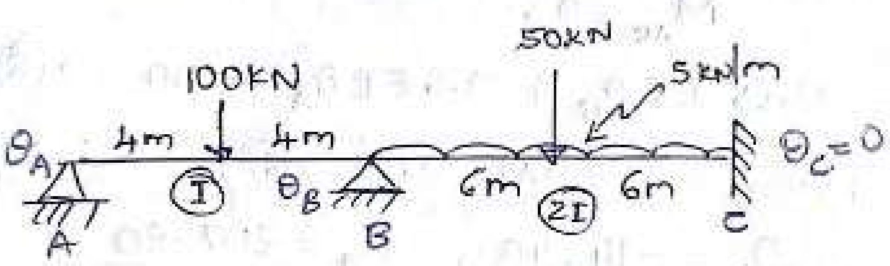
5. Analyse the continuous beam by slope deflection method. Draw BMD and SFD.



→ DOF = 2 (θ_A, θ_B)

Step 1: Fixed End Moments:

$$M_{FAB} = \frac{-WL}{8} = -\frac{100 \times 8}{8} = -100 \text{ kN-m}$$

$$M_{FBA} = \frac{WL}{8} = \frac{100 \times 8}{8} = 100 \text{ kN-m}$$

$$M_{FBC} = \frac{-WL}{8} - \frac{wL^2}{12} = \frac{-50 \times 12}{8} - \frac{5 \times 12^2}{12} = -135 \text{kN-m}$$

$$M_{FCB} = \frac{wL}{8} + \frac{wL^2}{12} = 135 \text{ kN-m}$$

Step 2: Slope - Deflection Equations:

$$M_{AB} = -100 + \frac{2E(I)}{8}\left(2\theta_A + \theta_B - \overset{0}{\cancel{\frac{3\Delta}{L}}}\right)$$

$$\therefore M_{AB} = -100 + 0.5\, EI\, \theta_A + 0.25\, EI\, \theta_B \rightarrow ①$$

$$M_{BA} = 100 + \frac{2E(I)}{8}\left(2\theta_B + \theta_A - \frac{3\Delta}{L}\right)$$

$$\therefore M_{BA} = 100 + 0.5\, EI\, \theta_B + 0.25\, EI\, \theta_A \rightarrow ②$$

$$M_{BC} = -135 + \frac{2E(2I)}{12}\left(2\theta_B + \overset{0}{\cancel{\theta_C}} - \overset{0}{\cancel{\frac{3\Delta}{L}}}\right)$$

$$\therefore M_{BC} = -135 + 0.667\overset{EI}{}\theta_B \rightarrow ③$$

$$M_{CB} = 135 + \frac{2E(2I)}{12}\left(2\theta_C + \theta_B - \frac{3\Delta}{L}\right)$$

$$\therefore M_{CB} = 135 + 0.333\overset{EI}{}\theta_B \rightarrow ④$$

Step 3: Joint - Equilibrium Equations;

$$\Sigma M_B = 0; \quad M_{BA} + M_{BC} = 0$$

$$100 + 0.5\, EI\, \theta_B + 0.25\, EI\, \theta_A - 135 + 0.667\, \theta_B = 0$$

$$1.167\, \theta_B + 0.25\, EI\, \theta_A = 35 \rightarrow Ⓐ$$

$$M_{AB} = 0;$$

$$0.25\, EI\, \theta_B + 0.5\, EI\, \theta_A = 100 \rightarrow Ⓑ$$

∴ From eq^{ns} Ⓐ and Ⓑ,

$$\theta_B = \frac{-14.40}{EI}, \quad \theta_A = \frac{207.20}{EI}$$

Step 4: Final moments:

$$M_{AB} = -100 + 0.5\, EI\left(\frac{207.20}{EI}\right) + 0.25\, EI\left(\frac{-14.40}{EI}\right)$$

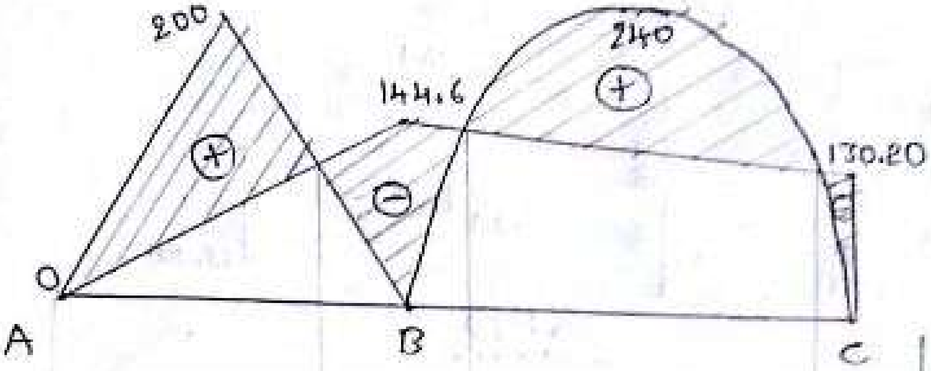
$$\therefore M_{AB} = 0.0 \text{ kN-m}$$

$M_{BA} = 144.6 \text{ KN-m}$

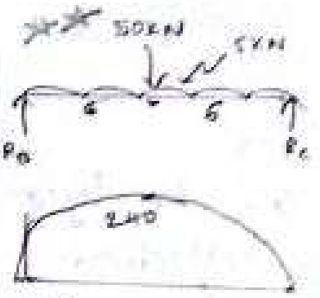
$M_{BC} = -135 + 0.667 (EI) \left(\frac{-14.40}{EI}\right)$

$M_{BC} = -144.6 \text{KN-m}$

$M_{CB} = 130.2 \text{ KN-m}$



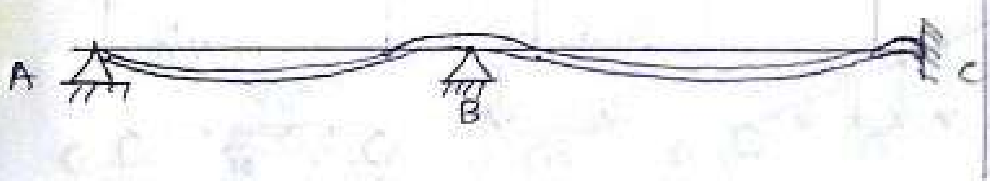
$\frac{wL}{4}$
$= \frac{100 \times 8}{4}$



BM]$_{\text{mid}}$ = 55×6 − (20×6×…

=

BMD.



$\Sigma M_B = 0$

$300 - (60 \times 6) - R_C \times 12 = 0$

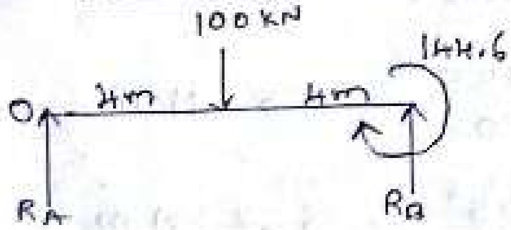
$\therefore R_C = 55 \text{KN}$

$\Sigma V = 0, \; R_B = 55 \text{ KN}$

Elastic Curve.

Step 5: To draw SFD:

(1)



$\Sigma M_A = 0;$

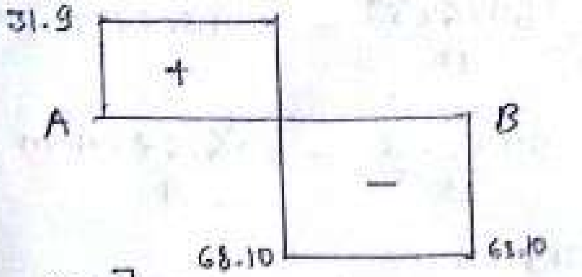
$100 \times 4 + 144.6 - R_B \times 8 = 0$

$R_B = 68.10 \text{ KN}$

$\Sigma V = 0;$

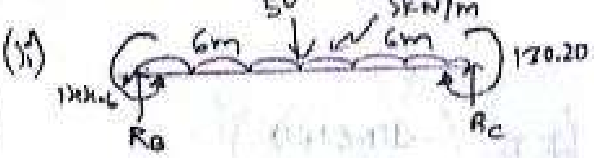
$R_A - 100 + 68.10 = 0$

$\therefore R_A = 31.9 \text{ KN}$



SFD]$_{AB}$

(ii)



$\Sigma M_A = 0;$

$-144.6 + (60\times6) + (50\times6) + 130.20 -$

$- R_C \times 12 = 0$

$\therefore R_C = 53.90\ kN$

$\Sigma V = 0;$

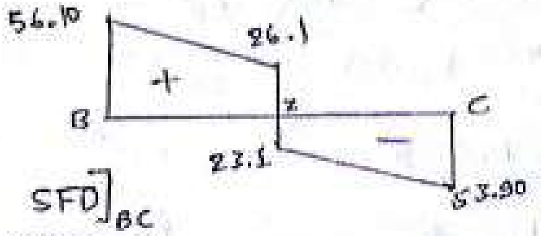
$R_B - 50 - 60 + 53.90 = 0$

$R_B = 56.10\ KN$

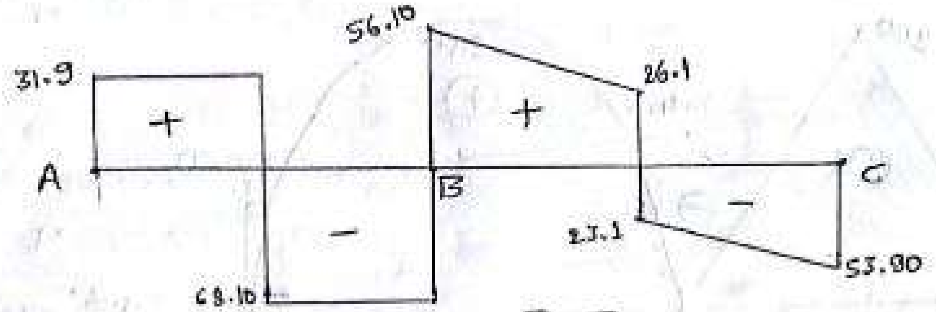
**

$\therefore SF]_Z = 56.10 - (5\times6) = +26.1$

$SF]_{PZ} = 56.10 - (5\times6) - 50 = -23.9$



$SFD]_{BC}$



SFD

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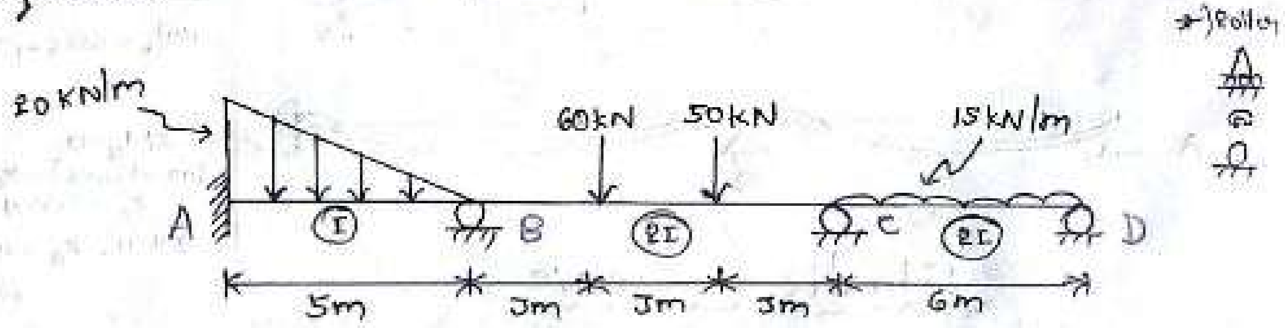
6. Analyse the continuous beam by slope deflection method. Draw BMD and SFD.



→ DOF = 3 $(\theta_B, \theta_C, \theta_D)$

Step 1: Fixed End moments:

$$M_{FAB} = -\frac{wL^2}{20} = \frac{-20\times5^2}{20} = -25\ KN\text{-}m$$

$$M_{FBA} = \frac{wL^2}{30} = \frac{20\times5^2}{30} = 16.67\ KN\text{-}m$$

$$M_{FBC} = \frac{-60\times3\times6^2}{9^2} - \frac{50\times6\times3^2}{9^2} = -113.33 KN\text{-}m$$

$$M_{FCB} = \frac{60\times3\times6^2}{9^2} + \frac{50\times6^2\times3}{9^2} = 106.67 KN\text{-}m$$

$$M_{FCD} = -\frac{15\times6^2}{12} = -45\ KN\text{-}m$$

$M_{FDC} = 45 \text{ kN-m}$

Step 2 : Slope - Deflection Equations :

$$M_{AB} = -25 + \frac{2E(I)}{5}(\theta_B)$$

$$\therefore M_{AB} = -25 + 0.4\,EI\,\theta_B \rightarrow ①$$

Now, $M_{BA} = 16.67 + \frac{2E(I)}{5}(2\theta_B)$

$$\therefore M_{BA} = 16.67 + 0.8\,EI\,\theta_B \rightarrow ②$$

$$M_{BC} = -113.33 + \frac{2E(2I)}{9}(2\theta_B + \theta_C)$$

$$\therefore M_{BC} = -113.33 + 0.88\,EI\,\theta_B + 0.44\,EI\,\theta_C \rightarrow ③$$

$$M_{CB} = 106.67 + \frac{2E(2I)}{9}(2\theta_C + \theta_B)$$

$$\therefore M_{CB} = 106.67 + 0.88\,EI\,\theta_C + 0.44\,EI\,\theta_B \rightarrow ④$$

$$M_{CD} = -45 + \frac{2E(2I)}{6}(2\theta_C + \theta_D)$$

$$\therefore M_{CD} = -45 + 1.33EI\,\theta_C + 0.667\,EI\,\theta_D \rightarrow ⑤$$

$$M_{DC} = 45 + \frac{2E(2I)}{6}(2\theta_D + \theta_C)$$

$$\therefore M_{DC} = 45 + 1.33\,EI\,\theta_D + 0.667\,EI\,\theta_C \rightarrow ⑥$$

Step 3 : Joint - Equilibrium Equations :

$\Sigma M_B = 0 ; \quad M_{BA} + M_{BC} = 0$

$$16.67 + 0.8\,EI\,\theta_B - 113.33 + 0.88\,EI\,\theta_B + 0.44\,EI\,\theta_C = 0$$

$$1.68\,EI\,\theta_B + 0.44\,EI\theta_C = 96.66 \rightarrow Ⓐ$$

$\Sigma M_C = 0 ; \quad M_{CB} + M_{CD} = 0$

$$106.67 + 0.88\,EI\theta_C + 0.44\,EI\theta_B - 45 + 1.33EI\,\theta_C + 0.67EI\theta_D = 0$$

$$0.44\,EI\theta_B + 2.21\,EI\,\theta_C + 0.667\,EI\theta_D = -61.67 \rightarrow Ⓑ$$

$\Sigma M_{DC} = 0 ;$

$$45 \ast 0.667\,EI\,\theta_C + 1.33\,EI\,\theta_D = -45 \rightarrow Ⓒ$$

$$\therefore \quad \theta_B = \frac{67.1}{EI} \quad ; \quad \theta_C = \frac{-36.58}{EI} \quad ; \quad \theta_D = \frac{-15.49}{EI}$$

Step 4 : Final moments :

$M_{AB} = 1.84$ kN-m

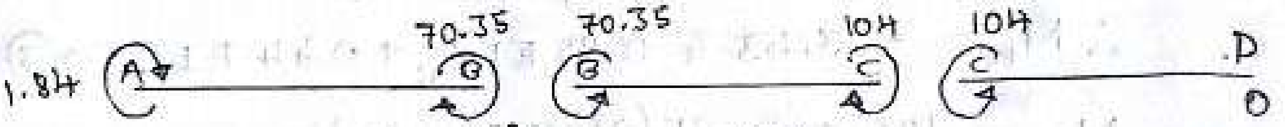
$M_{BA} = 70.35$ kN-m

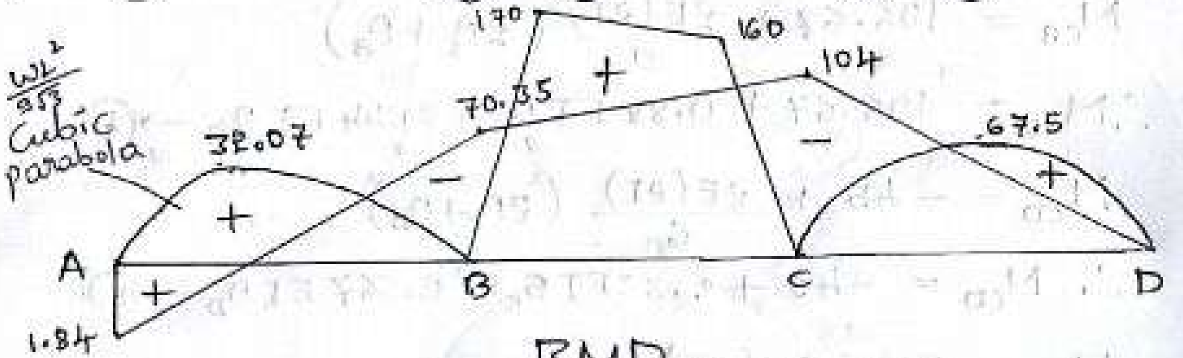
$M_{BC} = -70.35$ kN-m

$M_{CB} = 104$ kN-m

$M_{CD} = -104$ kN-m

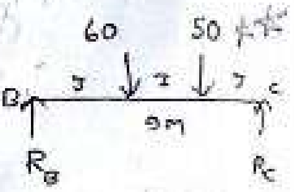
$M_{DC} = 0.0$ kN-m





BMD

Elastic Curve



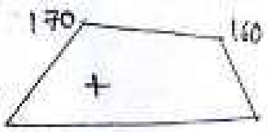
$\Sigma M_B = 0;$

$60\times3 + 50\times3 - R_c\times3$

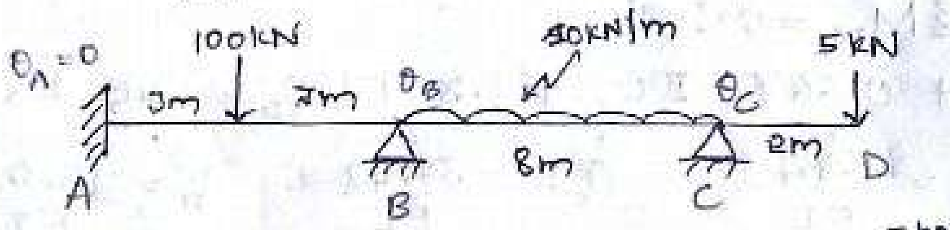
$R_c = 53.33$

$\Sigma V = 0;$

$R_B = 56.67$



7. Analyse the continuous beam loaded shown in the figure. Draw BMD, SFD & elastic cu



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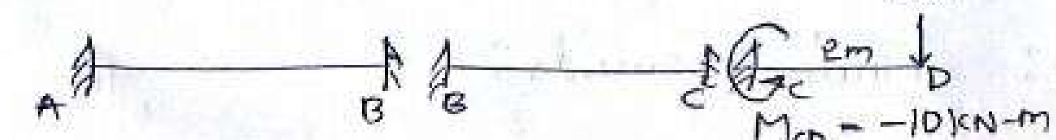


$M_{CD} = -10$ kN-m

**) Here CD is a cantilever which is a determinate one, hence $M_{CD} = -5 \times 2 = -10$ kN-m

No

Step 1: Fixed End moments:

$$M_{FAB} = -\frac{WL}{8} = \frac{-100 \times 6}{8} = -75 \text{ kN-m}$$

$$M_{FBA} = \frac{WL}{8} = \frac{100 \times 6}{8} = 75 \text{ kN-m}$$

$$M_{FBC} = \frac{-WL^2}{12} = \frac{-10 \times 8^2}{12} = -\overset{53.33}{\cancel{266.67}} \text{ kN-m}$$

$$M_{FCB} = \frac{WL^2}{12} = +53.33 \text{ kN-m}$$

Step 2: Slope-Deflection Equation:

$$M_{AB} = -75 + \frac{2E(I)}{6}\left(2\theta_A + \theta_B - \frac{3\Delta}{L}\right)$$

$$\therefore M_{AB} = -75 + 0.33 \text{ EI } \theta_B \rightarrow ①$$

$$M_{BA} = 75 + \frac{2E(I)}{6}\left(2\theta_B + \theta_A - \frac{3\Delta}{L}\right)$$

$$\therefore M_{BA} = 75 + 0.667 \text{ EI } \theta_B \rightarrow ②$$

$$M_{BC} = -53.33 + \frac{2EI}{8}(2\theta_B + \theta_C)$$

$$\therefore M_{BC} = -53.33 + 0.5 \text{ EI}\theta_B + 0.25 \text{ EI}\theta_C \rightarrow ③$$

$$M_{CB} = 53.33 + \frac{2EI}{8}(2\theta_C + \theta_B)$$

$$\therefore M_{CB} = 53.33 + 0.5 \text{ EI } \theta_C + 0.25 \text{ EI } \theta_B \rightarrow ④$$

Step 3: Joint Equilibrium Equations:

$$\Sigma M_B = 0; \quad M_{BA} + M_{BC} = 0;$$

$$75 + 0.667 \text{ EI}\theta_B - 53.33 + 0.5 \text{ EI}\theta_B + 0.25 \text{ EI}\theta_C = 0$$

$$1.166 \text{ EI}\theta_B + 0.25 \text{ EI}\theta_C = -21.67 \rightarrow Ⓐ$$

$$\Sigma M_C = 0; \quad M_{CB} + M_{CD} = 0;$$

$$53.33 + 0.5 \text{ EI}\theta_C + 0.25 \text{ EI}\theta_B - 10 = 0$$

$$0.25 \text{ EI } \theta_B + 0.5 \text{ EI}\theta_C = -43.33 \rightarrow Ⓑ$$

∴ From Ⓐ & Ⓑ,

$$\theta_D = \frac{-4.88 \times 10}{EI}, \quad \theta_B = \frac{-86.66}{EI}$$

Step 4: Final Moments :

$$\therefore M_{AB} = -75 + 0.33\,EI \left(\frac{-4.88\times10^{-3}}{EI}\right)$$

$$\therefore \quad M_{AB} = -75 \text{ kN-m}$$

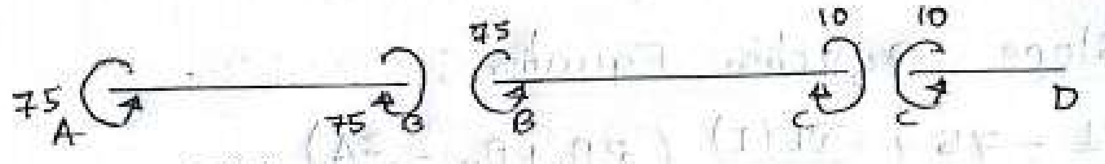
$$M_{BA} = 75 \text{ kN-m}$$

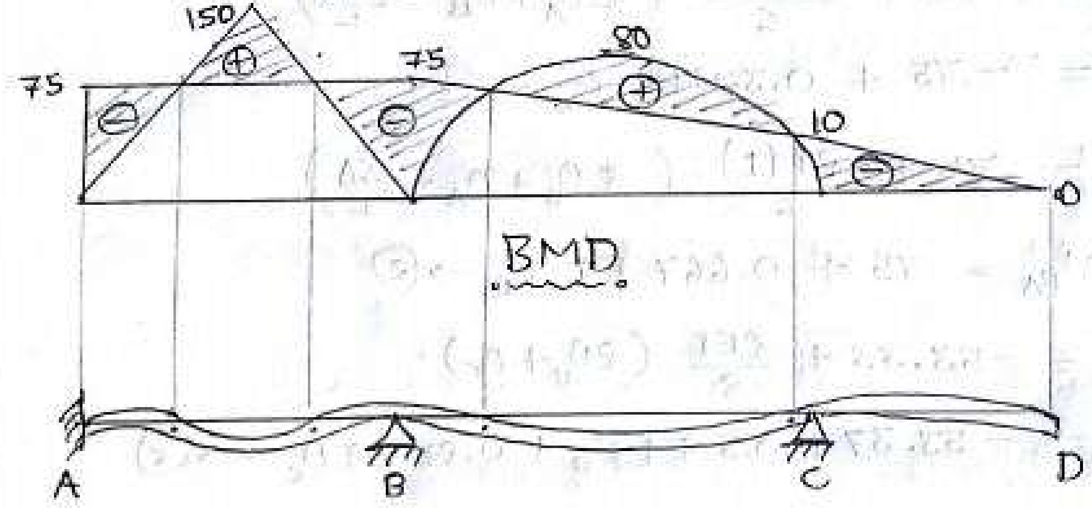
$$M_{BC} = -75 \text{ kN-m}$$

$$M_{CB} = 53.33 + 0.5\,EI \left(\frac{-86.66}{EI}\right)$$

$$\therefore \quad M_{CB} = +10 \text{ kN-m}$$

$$M_{CD} = -10 \text{ kN-m}$$

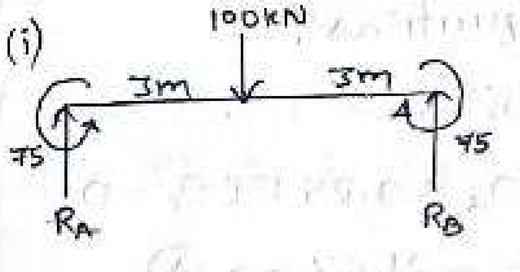




Elastic Curve

Step 5: To draw SFD :

(i)



$\Sigma M_A = 0;$

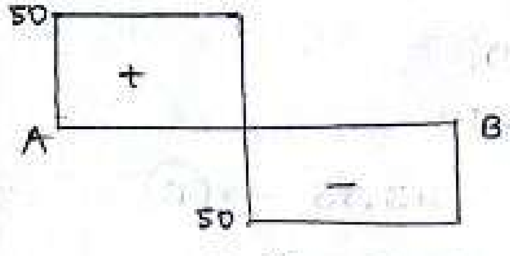
$-75 + 300 + 75 - R_B \times 6 = 0$

$\therefore R_B = 50 \text{ kN}$

$\Sigma V = 0;$

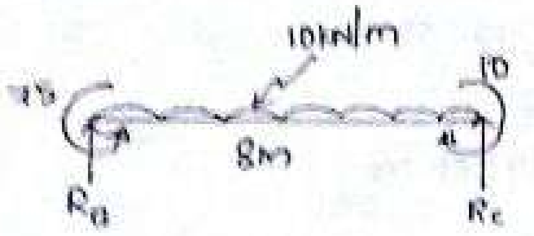
$R_A - 100 + 50 = 0$

$\therefore R_A = 50 \text{ kN}$



$SFD]_{AB}$

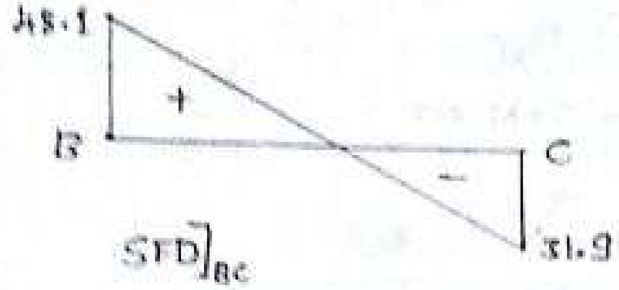
(ii)



$\Sigma M_A = 0:$

$-75 + 80 \times 4 + 10 - R_c \times 8 = 0$

$R_c = 31.90$ KN

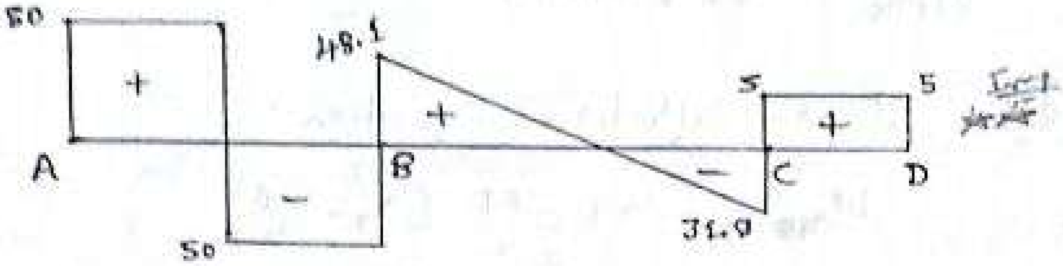


SFD$]_{BC}$

$\Sigma V = 0:$

$R_B - 80 + 31.90 = 0$

$\therefore R_B = 48.1$ KN



SFD

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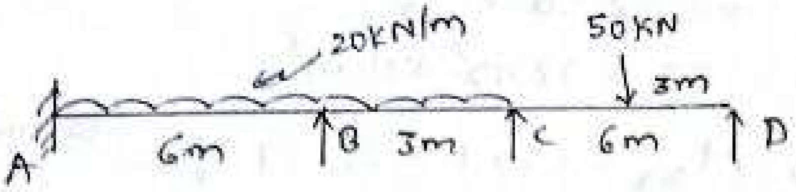
2. Analyse the continuous beam loaded as shown in the figure by slope deflection method. The support 'B' sinks by 10mm. $E = 2 \times 10^5$ N/mm^2, $I = 16 \times 10^7$ mm^4. Sketch the EMD.



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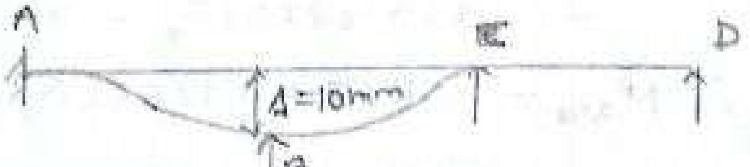
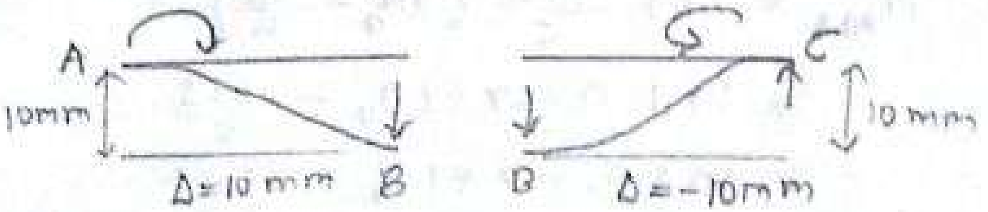




DOF = 3 ($\theta_B, \theta_C, \theta_D$)

Step 1: Fixed End Moments:

$$M_{FAB} = -\frac{20\times6^2}{12} = -60 kN\text{-}m$$

$$M_{FBA} = 60 kN\text{-}m$$

$$M_{FBC} = -\frac{20\times3^2}{12} = -15 kN\text{-}m$$

$$M_{FCB} = +15 kN\text{-}m$$

$$M_{FCD} = -\frac{W\times L}{8} = -\frac{50\times6}{8} = -37.5 kN\text{-}m$$

$$M_{FDC} = 37.5 \ kN\text{-}m$$

Step 2: Slope-Deflection Equation:

$$M_{FAB} = -60 + \frac{2EI}{6}\left(\theta_B - \frac{3\Delta}{L}\right)$$

$$M_{FAB} = -60 + 0.333 EI\theta_B - \frac{EI.\Delta}{6} \rightarrow ④$$

But, $EI = \left(\frac{1}{1000}\right) kN \left(\frac{1}{1000}\right)^2 m^2 \begin{cases} 1000 N = 1kN \\ 1 N = 1/1000 kN \end{cases}$

$$= \frac{1}{10^3}\times\frac{1}{10^6} kNm^2$$

$$\therefore EI = 1/10^9 \ kN\text{-}m^2$$

$$\therefore EI = (2\times10^5 \times 16\times10^7)\times 1/10^9$$

$$= 32\times10^{12}\times\frac{1}{10^9}$$

$$\therefore EI = 32\times10^3 \ kN\text{-}m^2$$

Now, ④ $\Rightarrow M_{FAB} = -60 + 0.333 EI\theta_B - \frac{\left(32\times10^3\times\frac{1}{100}\right)}{6}$

$$= -60 + 0.333 EI\theta_B - 53.33$$

$$\therefore M_{FAB} = -113.33 + 0.333 EI\theta_B \rightarrow ①$$

Now, $M_{FBA} = 60 + \frac{2EI}{6}\left(2\theta_B - \frac{3\Delta}{L}\right)$

$$= 60 + 0.667 EI\theta_B - \frac{EI\Delta}{6}$$

$$= 60 + 0.667 EI\theta_B - 53.33$$

$$\therefore M_{FBA} = 6.67 + 0.667 EI\theta_B \rightarrow ②$$

Now, $M_{FBC} = -15 + \frac{2EI}{3}\left(2\theta_B + \theta_C - \frac{3\Delta}{3}\right)$

$$M_{FBC} = -15 + 1.33\,EI\theta_B + 0.667\,EI\theta_C - 0.667\,EI\overset{-(1/100)}{\Delta}$$

$$\therefore M_{FBC} = -15 + 1.33\,EI\theta_B + 0.667\,EI\theta_C + 213.44$$

$$\therefore M_{FBC} = 198.144 + 1.33\,EI\theta_B + 0.667\,EI\theta_C \rightarrow ③$$

Now, $M_{FCB} = 15 + \frac{2EI}{3}\left(2\theta_C + \theta_B - \frac{3\Delta}{3}\right)$

$$M_{FCB} = 15 + 1.333\,EI\theta_C + 0.667\,EI\theta_B + 213.44$$

$$\therefore M_{FCB} = 228.44 + 1.333\,EI\theta_C + 0.667\,EI\theta_B \rightarrow ④$$

Now, $M_{FCD} = -37.5 + \frac{2EI}{6}\left(2\theta_C + \theta_D\right)$

$$\therefore M_{CD} = -37.5 + 0.667\,EI\theta_C + 0.333\,EI\theta_D \rightarrow ⑤$$

$$M_{DC} = 37.5 + 0.667\,EI\theta_D + 0.333\,EI\theta_C \rightarrow ⑥$$

Step 3: Joint Equilibrium Equations:

$\Sigma M_B = 0$; $M_{BA} + M_{BC} = 0$;

$$6.67 + 0.667\,EI\theta_B + 198.144 + 1.33\,EI\theta_B + 0.667\,EI\theta_C = 0$$

$$2.0\,EI\theta_B + 0.667\,EI\theta_C = -205.11 \rightarrow Ⓐ$$

Now, $\Sigma M_C = 0$; $M_{CB} + M_{CD} = 0$;

$$228.44 + 1.333\,EI\theta_C + 0.667\,EI\theta_B - 37.5 + 0.667\,EI\theta_C + 0.333\,EI\theta_D = 0\ ;$$

$$0.667\,EI\theta_B + 2.0\,EI\theta_C + 0.333\,EI\theta_D = \cancel{10} - 190.94 \quad Ⓑ$$

$M_{DC} = 0$;

$$0.333\,EI\theta_C + 0.667\,EI\theta_D = -37.5 \rightarrow Ⓒ$$

$$\theta_B = \frac{-81.07}{EI} \qquad \theta_C = \frac{-64.43}{EI} \qquad \theta_D = \frac{-24.05}{EI}$$

Step 4: Final moments:

$M_{AB} = -140.33$ KN-m

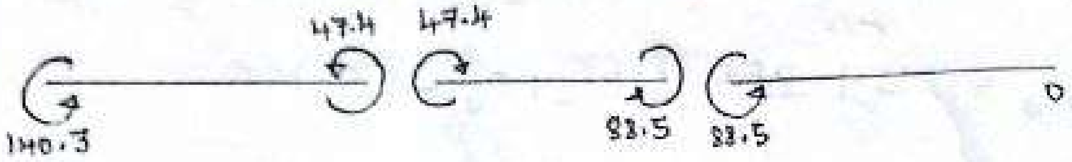
$M_{BA} = -47.40$ KN-m

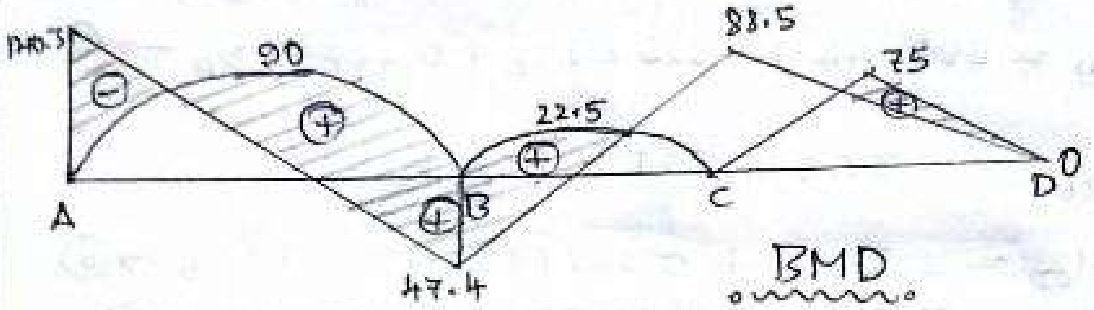
$M_{BC} = 47.40$ KN-m

$M_{CB} = 88.48$ KN-m

$$M_{CB} = -88.48 \text{ kN-m}$$

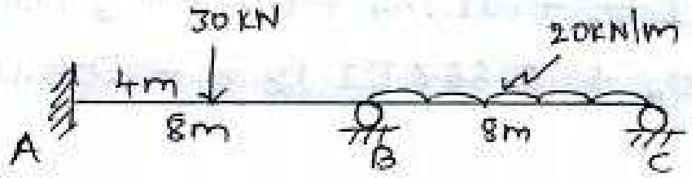
$$M_{DC} = 0.0 \text{ kN-m}$$





BMD

9. Analyse the continuous beam by slope-deflection method if the support 'B' sinks by 5mm. Draw BMD. Assume EI = 4000 kN m²



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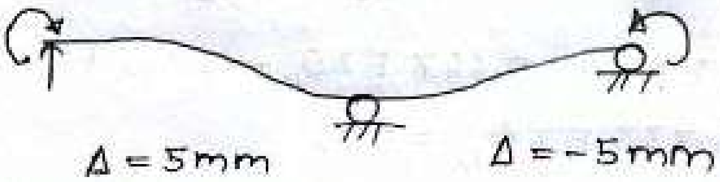


Δ = 5 mm

1000 mm = 1 m

$1 \text{mm} = \frac{1}{1000} \text{m}$

$\therefore 5 \text{mm} = \left(\frac{5}{1000}\right) \text{m}$

$\text{DoF} = 2 \; (\theta_B, \theta_C)$

<u>Step 1:</u> <u>Fixed End Moments:</u>

$$M_{FAB} = \frac{-WL}{8} = -\frac{30 \times 8}{8} = -30 \text{ kN-m}$$

$$M_{FBA} = \frac{WL}{8} = \frac{30 \times 8}{8} = 30 \text{ kN-m}$$

$$M_{FBC} = \frac{-WL^2}{12} = -\frac{20 \times 8^2}{12} = -106.67 \text{ kN-m}$$

$$M_{FCB} = \frac{WL^2}{12} = \frac{20 \times 8^2}{12} = 106.67 \text{ kN-m}$$

Step 2: Slope-Deflection Equations:

$$M_{AB} = -30 + \frac{2EI}{8}\left(\cancel{2\theta_A}^{0} + \theta_B - \frac{3\Delta}{8}\right)$$

$M_{AB} = -30 + 0.25 EI\theta_B - 0.25 \times 0.375 \times 4000 \times \left(\frac{5}{1000}\right)$

$M_{AB} = -31.87 + 0.25 EI\theta_B \rightarrow$ ①

Now, $M_{BA} = 30 + \frac{2EI}{8}\left(2\theta_B + \cancel{\theta_A}^{0} - \frac{3\Delta}{L}\right)$

$= 30 + 0.5 EI\theta_B - 1.87$

$\therefore M_{BA} = 28.13 + 0.5 EI\theta_B \rightarrow$ ②

Now, $M_{BC} = -106.67 + \frac{2EI}{8}\left(2\theta_B + \theta_C - \frac{3\Delta}{L}\right)$

$= -106.67 + 0.25 EI\left(2\theta_B + \theta_C - 0.375\left(\frac{5}{1000}\right)\right)$

$= -106.67 + 0.50 EI\theta_B + 0.25 EI\theta_C - 1.875$

$\therefore M_{BC} = -108.54 + 0.50 EI\theta_B + 0.25 EI\theta_C \rightarrow$ ③

Now, $M_{CB} = 106.67 + \frac{2EI}{8}\left(2\theta_C + \theta_B - \frac{3\Delta}{L}\right)$

$M_{CB} = 104.67 + 0.25 EI\theta_B + 0.50 EI\theta_C \rightarrow$ ④

Step 3: Joint Equilibrium Equations:

$\Sigma M_B = 0; \quad M_{BA} + M_{BC} = 0$

$28.13 + 0.5 EI\theta_B - 108.54 + 0.5 EI\theta_B + 0.25 EI\theta_C = 0$

$1.0 EI\theta_B + 0.25 EI\theta_C = 80.41 \rightarrow$ Ⓐ

$M_{CB} = 0;$

$0.25 EI\theta_B + 0.50 EI\theta_C = -104.67 \rightarrow$ Ⓑ

$\therefore \theta_B = \frac{151.71}{EI}; \quad \theta_C = \frac{-285.2}{EI}$

Step 4: Final Moments:

$M_{AB} = -31.87 + 0.25 EI\left(\frac{151.71}{EI}\right)$

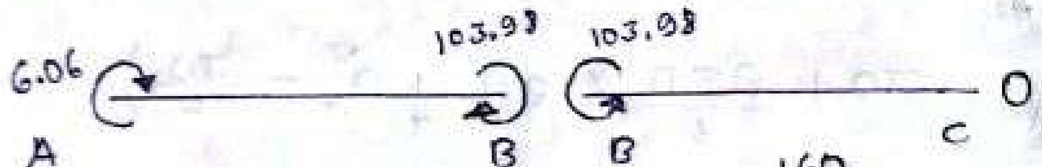
$\therefore M_{AB} = 6.06$ kN-m ;

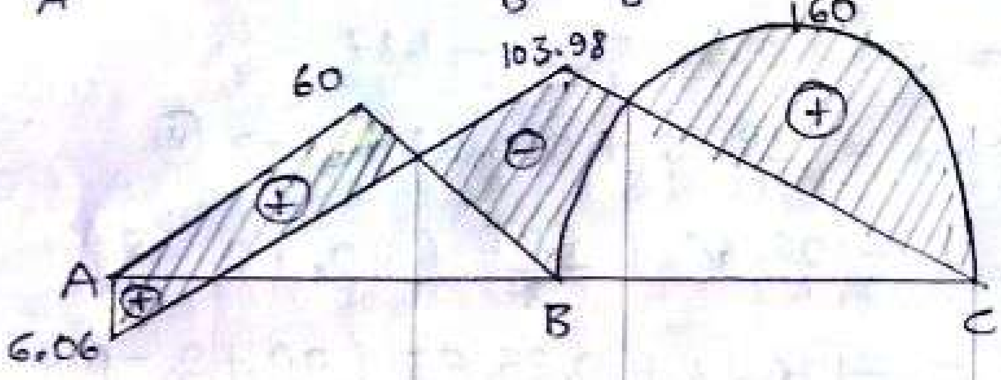
$M_{BA} = 28.13 + 0.5 EI\left(\frac{151.71}{EI}\right)$

$\therefore M_{BA} = 103.98$ kN-m ;

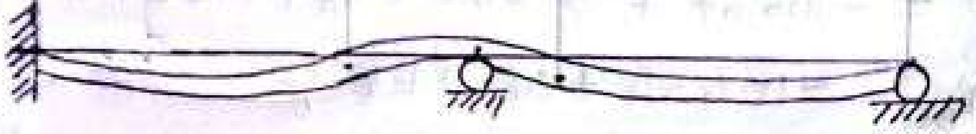
III^{ly}, $M_{BC} = -103.98$ kN-m ;

111^{ly}. $M_{CB} = 0.0$ kN-m





BMD

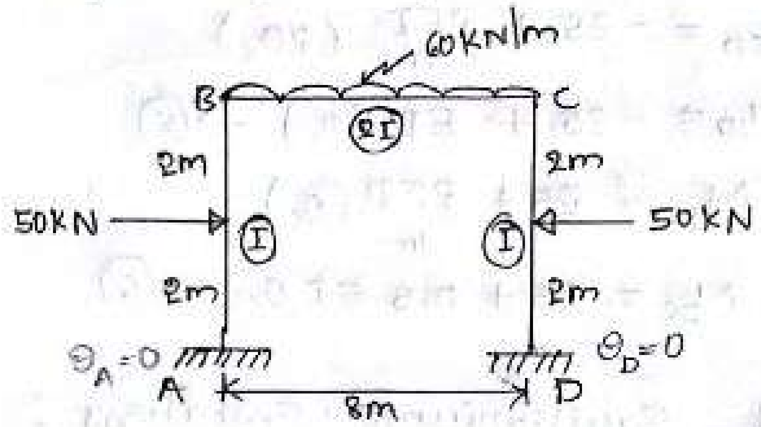


Elastic Curve

25/08/18

Rigid Plane Frames :

1. Analyse the portal frame shown in the figure by Slope - deflection method.



→ Here DOF = 2 (θ_B, θ_C)

Step 1 : Fixed End Moments :

$$M_{FAB} = \frac{-wL}{8} = \frac{-50\times4}{8} = -25 \text{ kNm}$$

$$M_{FBA} = \frac{wL}{8} = 25 \text{ kNm}$$

$$M_{FBC} = -\frac{wL^2}{12} = \frac{-60\times8^2}{12} = -320 \text{ kNm}$$

$$M_{FCB} = \frac{wL^2}{12} = \frac{60\times8^2}{12} = 320 \text{ kNm}$$

$$M_{FCD} = \frac{-wL}{8} = \frac{-50\times4}{8} = -25 \text{ kNm}$$

$$M_{FDC} = \frac{wL}{8} = 25 \text{ kNm}$$

Step 2 : Slope - Deflection Equations :

$$M_{AB} = M_{FAB} + \frac{2EI}{L}\left(2\theta_A + \theta_B - \frac{3\Delta}{L}\right)$$

$$= -25 + \frac{2EI}{4}(\theta_B)$$

$$\therefore M_{AB} = -25 + 0.5\,EI\,\theta_B \rightarrow ①$$

Now, $M_{BA} = 25 + \frac{2EI}{4}(2\theta_B)$

$$\therefore M_{BA} = 25 + EI(\theta_B) \rightarrow ②$$

Now, $M_{BC} = -320 + \frac{2E(2I)}{8}(2\theta_B + \theta_C)$

$\therefore \quad M_{BC} = -320 + EI\ \theta_B + 0.5\ EI\ \theta_C \rightarrow ③$

Now, $M_{CB} = 320 + \frac{2E(2I)}{8}\ (2\theta_C + \theta_B)$

$\therefore \quad M_{CB} = 320 + EI(\theta_C) + 0.5\ EI\ \theta_B \rightarrow ④$

Now, $M_{CD} = -25 + \frac{2EI}{4}\ (2\theta_C)$

$\therefore \quad M_{CD} = -25 + EI\ (\theta_C) \rightarrow ⑤$

Now, $M_{DC} = 25 + \frac{2EI}{4}(\theta_C)$

$\therefore \quad M_{DC} = 25 + 0.5\ EI\ \theta_C \rightarrow ⑥$

Step 3: Joint Equilibrium Equations:

$\Sigma M_B = 0;\quad M_{BA} + M_{BC} = 0$

$25 + EI\ \theta_B - 320 + EI\ \theta_B + 0.5\ EI\ \theta_C = 0$

$2EI\ \theta_B + 0.5\ EI\ \theta_C = 295 \rightarrow Ⓐ$

Now, $\Sigma M_C = 0;\quad M_{CB} + M_{CD} = 0$

$320 + EI\ \theta_C + 0.5\ EI\ \theta_B - 25 + EI\ \theta_C = 0$

$0.5\ EI\ \theta_B + 2\ EI\ \theta_C = -295 \rightarrow Ⓑ$

$\therefore \quad \theta_B = \frac{196.67}{EI} \qquad \theta_C = \frac{-196.67}{EI}$

Step 4: Final Moments:

$① \Rightarrow \quad M_{AB} = -25 + 0.5\ EI\left(\frac{196.7}{EI}\right)$

$= 73.35$ KN-m

$② \Rightarrow \quad M_{BA} = 25 + EI\left(\frac{196.7}{EI}\right)$

$= 221.7$ KN-m

$③ \Rightarrow \quad M_{BC} = -320 + EI\left(\frac{196.7}{EI}\right) + 0.5\ EI\left(-\frac{196.7}{EI}\right)$

$\therefore \quad M_{BC} = -221.7$ KN-m

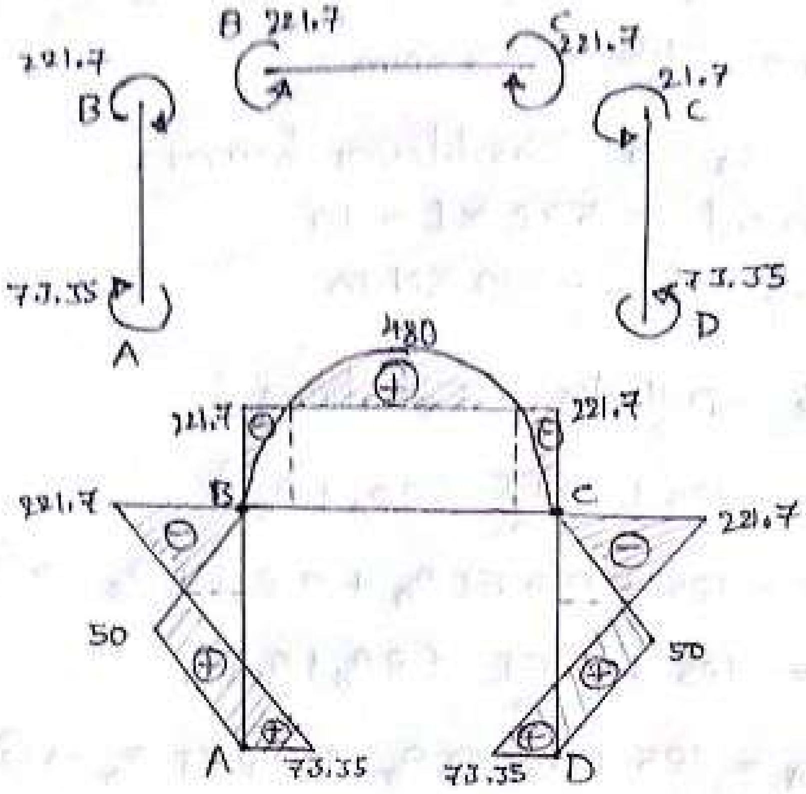
$④ \Rightarrow \quad M_{CB} = 320 + EI\left(\frac{-196.7}{EI}\right) + 0.5\ EI\left(\frac{196.7}{EI}\right)$

$= 221.7$ KN-m

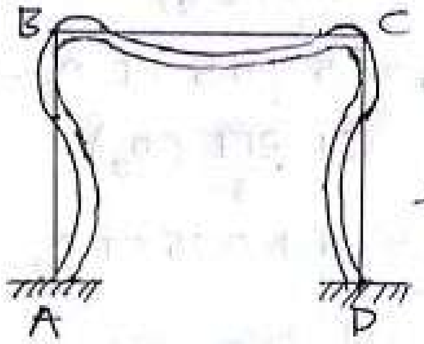
$⑤ \Rightarrow \quad M_{CD} = -221.7$ KNm

⑥ $\Rightarrow$ $M_{DC} = 25 + 0.5\,EI\left(\frac{-196.7}{EI}\right)$

$= -73.35$ KN-m



BMD



Imp **

Elastic Curve

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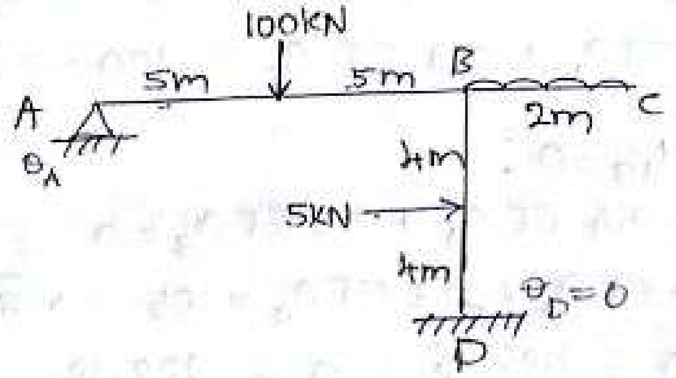
2. Analyse the frame shown in the figure by slope-deflection method. Draw BMD. ** V.Imp

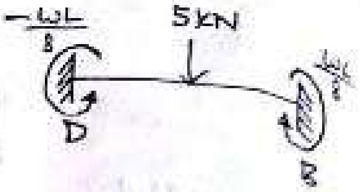


$\rightarrow$ Here DOF = 2 (θ_A, θ_B)

Step 1: Fixed End Moments:

$M_{FAB} = -\frac{WL}{8} = \frac{-100 \times 10}{8} = -125$ KNm

$$M_{FBA} = \frac{wL}{8} = 125 \text{ kNm}$$



$$M_{FBD} = \frac{wL}{8} = \frac{5 \times 8}{8} = 5 \text{ kNm}$$

$$M_{FDB} = \frac{-wL}{8} = -5 \text{ kNm}$$

★ Note: BC is a cantilever beam,
moment $= 5 \times 2 \times 1 = 10$
$\therefore \quad M_{BC} = -10$ KN-m

Step 2: Slope - Deflection Equations:

$$M_{AB} = -125 + \frac{2EI}{10}(2\theta_A + \theta_B)$$

$$\therefore M_{AB} = -125 + 0.4\, EI\, \theta_A + 0.2\, EI\, \theta_B \rightarrow ①$$

$$\text{Now, } M_{BA} = 125 + \frac{2EI}{10}(2\theta_B + \theta_A)$$

$$\therefore \quad M_{BA} = 125 + 0.4\, EI\, \theta_B + 0.2\, EI\, \theta_A \rightarrow ②$$

$$\text{Now, } M_{BD} = 5 + \frac{2EI}{8}(2\theta_B)$$

$$\therefore \quad M_{BD} = 5 + 0.5\, EI\, \theta_B \rightarrow ③$$

$$\text{Now, } \quad M_{DB} = -5 + \frac{2EI}{8}(\theta_B)$$

$$\therefore M_{DB} = -5 + 0.25\, EI\, \theta_B \rightarrow ④$$

Step 3: Joint Equilibrium Equations:

$$\Sigma M_B = 0; \quad M_{BA} + M_{BC} + M_{BD} = 0$$

$$125 + 0.4\, EI\, \theta_B + 0.2\, EI\, \theta_A - 10 + 5 + 0.5\, EI\, \theta_B = 0$$

$$\therefore \quad 0.2\, EI\theta_A + 0.9\, EI\, \theta_B = -120 \rightarrow Ⓐ$$

$$M_{AB} = 0;$$

$$-125 + 0.4\, EI\, \theta_A + 0.2\, EI\, \theta_B = 0$$

$$0.4\, EI\, \theta_A + 0.2\, EI\, \theta_B = 125 \rightarrow Ⓑ$$

$$\therefore \quad \theta_A = \frac{426.56}{EI}; \quad \theta_B = \frac{-228.12}{EI}$$

Step 4: Final Moments:

$$M_{AB} = -125 + 0.4\, EI\left(\frac{426.56}{EI}\right) + 0.2\, EI\left(\frac{-228.12}{EI}\right)$$

$$\therefore M_{AB} = 0.0 \text{ kNm}$$

Now, $M_{BA} = 125 + 0.4 EI \left(\frac{-228.12}{EI}\right) + 0.2 EI \left(\frac{426.56}{EI}\right)$

$$\therefore M_{BA} = 119 \text{ kNm}$$

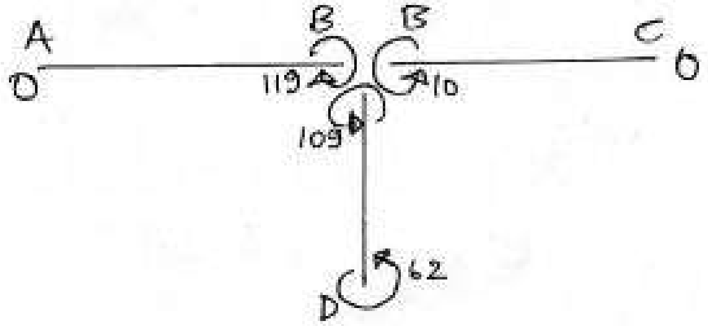
$$M_{BC} = -10 \text{ kNm}$$

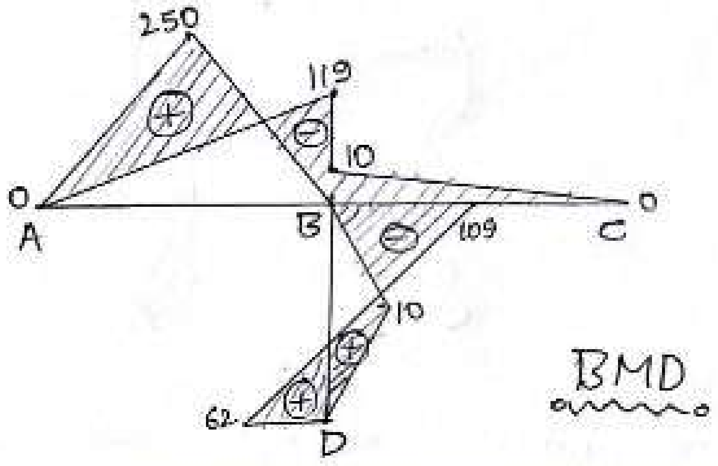
Now, $M_{BD} = 5 + 0.5 EI \left(\frac{-228.12}{EI}\right)$

$$= -109 \text{ kNm}$$

Now, $M_{DB} = -5 + 0.25 \left(\frac{-228.12}{EI}\right) EI$

$$\therefore M_{DB} = -62 \text{ kNm}$$





BMD

(i) $\frac{wL}{4} = \frac{1000\times10}{4}$

$= 250$

(ii) $\frac{wL}{4} = \frac{5\times8}{4}$

$= 10$

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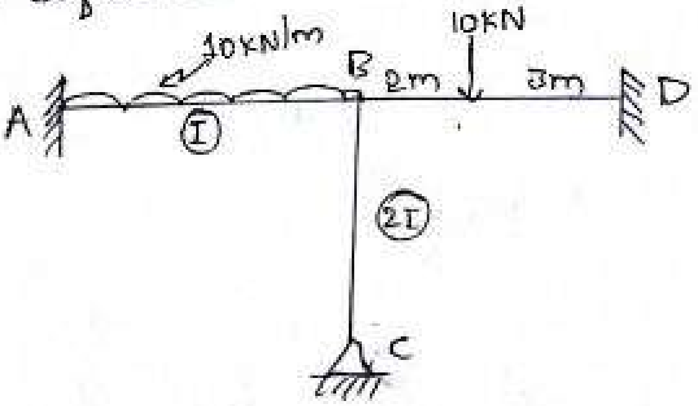
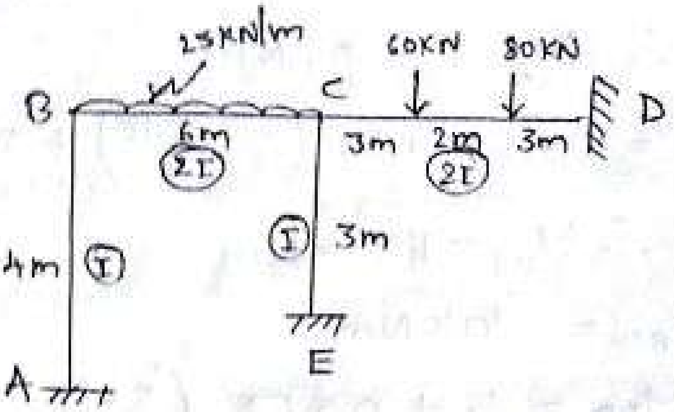
3. Analyse the frame shown in the figure by slope - deflection method.



→ DOF = 2 (θ_B, θ_C)

27/08/18

4.



→ Here DOF = 2 (θ_B, θ_C)

Step 1: Fixed End Moments:

$$M_{FAB} = \frac{-WL}{8} = \frac{-0\times4}{8} = 0 \; ; \; M_{FBA} = 0$$

$$M_{FBC} = \frac{-wL^2}{12} = \frac{-25\times6^2}{12} = -75 \text{ kNm}$$

$$M_{FCB} = 75 \text{ KN-m}$$

$$M_{FCD} = \frac{-Wab^2}{L^2} - \frac{Wab^2}{L^2}$$

$$= \frac{-60\times3\times5^2}{8^2} - \frac{80\times5\times3^2}{8^2}$$

$$\therefore M_{FCD} = -126.56 \text{ KNm}$$

$$M_{FDC} = \frac{Wa^2b}{L^2} + \frac{Wa^2b}{L^2} = \frac{60\times3^2\times5}{8^2} + \frac{80\times5^2\times3}{8^2}$$

$$\therefore M_{FDC} = 135.94 \text{ KNm}$$

$$M_{FCE} = 0 \; ; \; M_{FEC} = 0$$

Step 2: Slope - Deflection Equations:

$$M_{AB} = 0 + \frac{2EI}{4}(\theta_B)$$

$$\therefore M_{AB} = 0.5\, EI\,\theta_B \rightarrow ①$$

$$M_{BA} = 0 + \frac{2EI}{4}(2\theta_B)$$

$$\therefore M_{BA} = 1.0\, EI\,\theta_B \rightarrow ②$$

$$M_{BC} = -75 + \frac{2E(2I)}{6}(2\theta_B + \theta_C)$$

$$\therefore M_{BC} = -75 + 1.33\, EI\,\theta_B + 0.67\, EI\,\theta_C \rightarrow ③$$

Now, $M_{CB} = 75 + \frac{2E(2I)}{6}(2\theta_C + \theta_B)$

$$\therefore M_{CB} = 75 + 1.33\, EI\,\theta_C + 0.67\, EI\,\theta_B \rightarrow ④$$

$$M_{CD} = -126.56 + \frac{2E(2I)}{8}(2\theta_c)$$

$$\therefore M_{CD} = -126.56 + 1.0\,EI\,\theta_c \longrightarrow ⑤$$

$$M_{DC} = 135.94 + \frac{2E(2I)}{8}(\theta_c)$$

$$\therefore M_{DC} = 135.94 + 0.5\,EI\,\theta_c \longrightarrow ⑥$$

$$M_{CE} = 0 + \frac{2EI}{3}(2\theta_c)$$

$$M_{CE} = 1.33\,EI\,\theta_c \rightarrow ⑦$$

Now, $M_{EC} = 0 + \frac{2EI}{3}(\theta_c)$

$$\therefore M_{EC} = 0.67\,EI\,\theta_c \rightarrow ⑧$$

Step3: Joint Equilibrium Equations:

$$\Sigma M_B = 0; \quad M_{BA} + M_{BC} = 0$$

$$1.0\,EI\,\theta_B + (-75) + 1.33\,EI\theta_B + 0.67\,EI\,\theta_c \rightarrow A$$

$$2.33\,EI\,\theta_B + 0.67\,EI\,\theta_c = 75 \longrightarrow Ⓐ$$

$$\Sigma M_c = 0; \quad M_{CB} + M_{CD} + M_{CE} = 0$$

$$75 + 1.33\,EI\theta_C + 0.67\,EI\,\theta_B - 126.56 + 1.0\,EI\,\theta_C + 1.33\,EI\theta_c = 0$$

$$0.67\,EI\theta_B + 3.33\,EI\,\theta_c = 51.56 \longrightarrow Ⓑ$$

$$\therefore \quad \theta_B = \frac{29.7}{EI} \quad ; \quad \theta_c = \frac{8.65}{EI}$$

Step4: Final Moments:

$$M_{AB} = 0.5\,EI\left(\frac{29.7}{EI}\right) = 14.8 \text{ KNm}$$

$$M_{BA} = 1.0\,EI\left(\frac{29.7}{EI}\right) = 29.7 \text{ KNm}$$

$$M_{BC} = -75 + 1.33\,EI\left(\frac{29.7}{EI}\right) + 0.67\,EI\left(\frac{8.65}{EI}\right) = -29.7 \text{ kNm}$$

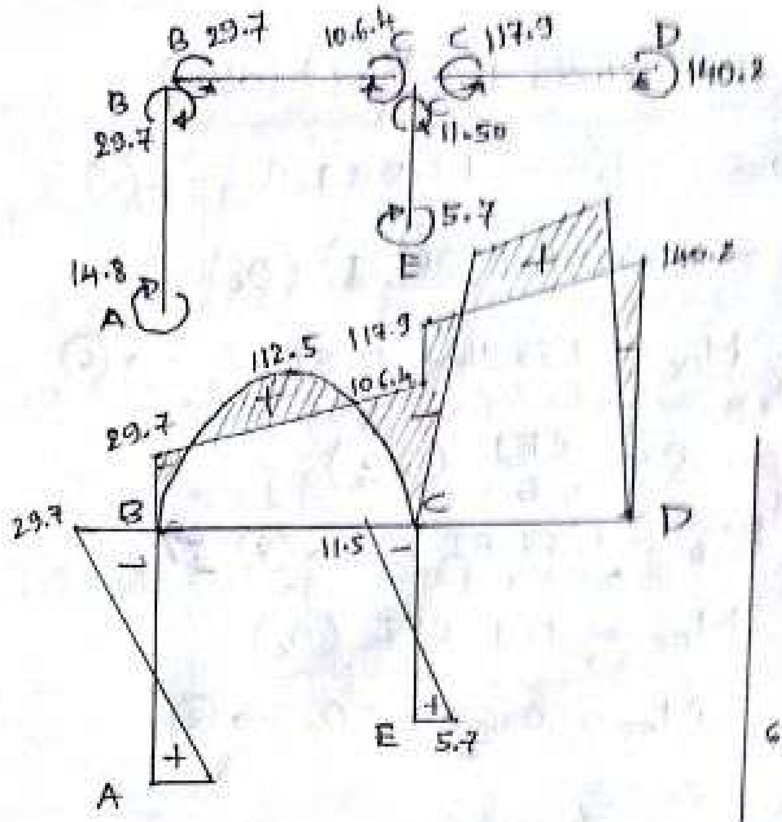
$$M_{CB} = 75 + 1.33\,EI\left(\frac{8.65}{EI}\right) + 0.67\,EI\left(\frac{29.7}{EI}\right) = 106.4 \text{ kNm}$$

$$M_{CD} = -117.9 \text{ kNm}$$

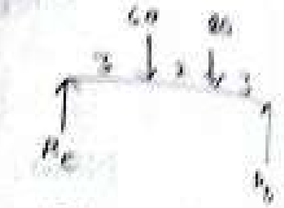
$$M_{DC} = 140.2 \text{ kNm}$$

$$M_{CE} = 11.50 \text{ KNm}$$

$$M_{EC} = 5.7 \text{ KNm}$$



BMD



$\Sigma M_c = 0;$

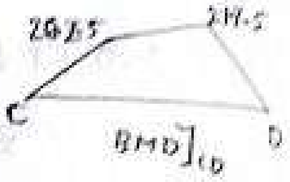
$60 \times 3 + 80 \times 3 - R_D \times 3 = ($

$R_D = 72.5 \text{ kN}$

$\Sigma V = 0;$

$R_c - 60 - 80 + 72.5 =$

$R_c = 67.5$



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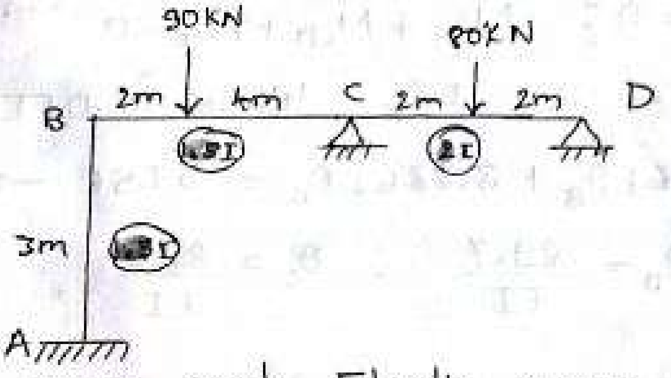


Draw BMD and Elastic curve:

→ DOF = 3 $(\theta_B, \theta_C, \theta_D)$

1: $M_{FAB} = 0$; $M_{FBA} = 0$

$$M_{FBC} = \frac{-wab^2}{L^2} = \frac{-90 \times 2 \times 4^2}{6^2} = -80 \text{ KNm}$$

$$M_{FCB} = \frac{+wa^2b}{L^2} = \frac{+90 \times 2^2 \times 4}{6^2} = 40 \text{ KNm}$$

$$M_{FCD} = \frac{-wL}{8} = \frac{-80 \times 4}{8} = -40 \text{ KNm}$$

$$M_{FDC} = \frac{wL}{8} = \frac{8 \times 4}{8} = +40 \text{ KNm}$$

Step 2: Slope - Deflection Equations:

$$M_{AB} = 0 + \frac{2EI}{3}\left(\cancel{2\theta_A}^{0} + \theta_B - 0\right)$$

$$\therefore \quad M_{AB} = 0.67\,EI\,\theta_A \rightarrow ①$$

$$M_{BA} = 0 + \frac{2EI}{3}\left(2\theta_B + 0 - 0\right)$$

$$\therefore \quad M_{BA} = 1.34\,EI\,\theta_B \rightarrow ②$$

$$M_{BC} = -80 + \frac{2EI}{6}\left(2\theta_B + \theta_C - 0\right)$$

$$M_{BC} = -80 + 0.67\,EI\theta_B + 0.33\,EI\,\theta_C \rightarrow ③$$

$$M_{CB} = 40 + \frac{2EI}{6}\left(2\theta_C + \theta_B - 0\right)$$

$$\therefore M_{CB} = 40 + 0.67\,EI\,\theta_C + 0.33\,EI\,\theta_B \rightarrow ④$$

$$M_{CD} = -40 + \frac{2EI}{4}\left(2\theta_C + \theta_D - 0\right)$$

$$\therefore \quad M_{CD} = -40 + 1.0\,EI\theta_C + 0.5\,EI\,\theta_D \rightarrow ⑤$$

$$M_{DC} = 40 + \frac{2EI}{4}\left(2\theta_D + \theta_C - 0\right)$$

$$\therefore \quad M_{DC} = 40 + 1.0\,EI\,\theta_D + 0.5\,EI\,\theta_C \rightarrow ⑥$$

Step 3: Joint Equilibrium Equations:

$$\Sigma M_B = 0\;; \quad M_{BA} + M_{BC} = 0$$

$$1.34\,EI\,\theta_B - 80 + 0.67\,EI\theta_B + 0.33\,EI\,\theta_C = 0$$

$$2.01\;EI\;\theta_B + 0.33\,EI\,\theta_C = 80 \rightarrow Ⓐ$$

$$\Sigma M_C = 0\;; \quad M_{CB} + M_{CD} = 0$$

$$40 + 0.67\,EI\theta_C + 0.33\,EI\,\theta_B - 40 + 1\,EI\,\theta_C + 0.5\,EI\,\theta_D = 0$$

$$0.33\,EI\theta_B + 1.67\,EI\,\theta_C + 0.5\,EI\,\theta_D = 0 \rightarrow Ⓑ$$

$$M_{DC} = 0\;;$$

$$\cancel{40}\;\; 0.5\,EI\,\theta_C + 1.0\,EI\,\theta_D = -40 \rightarrow Ⓒ$$

$$\therefore \quad \theta_B = \frac{38.97}{EI} \;;\quad \theta_C = \frac{5.03}{EI} \;;\quad \theta_D = \frac{-42.5}{EI}$$

Step 4: Final Moments:

$$M_{AB} = 0.67\,EI\left(\frac{38.97}{EI}\right) = 26.11 \text{ KNm}$$

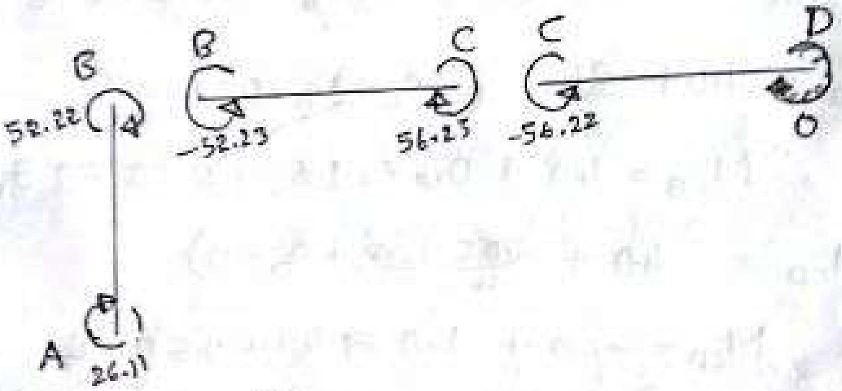
$$M_{BA} = 1.34\,EI\left(\frac{38.97}{EI}\right) = 52.22 \text{ KNm}$$

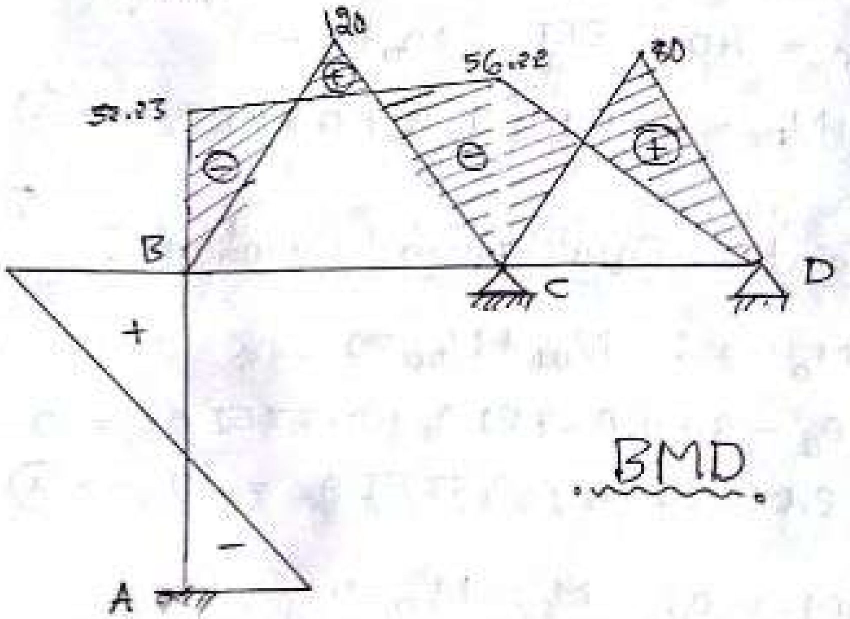
$$M_{BC} = -80 + 0.67\,EI\left(\frac{38.97}{EI}\right) + 0.33\,EI\left(\frac{5.03}{EI}\right) = -52.23 \text{ KN}$$

$$M_{CB} = 40 + 0.67\,EI\left(\frac{5.03}{EI}\right) + 0.33\,EI\left(\frac{38.97}{EI}\right) = 56.23 \text{ KN}$$

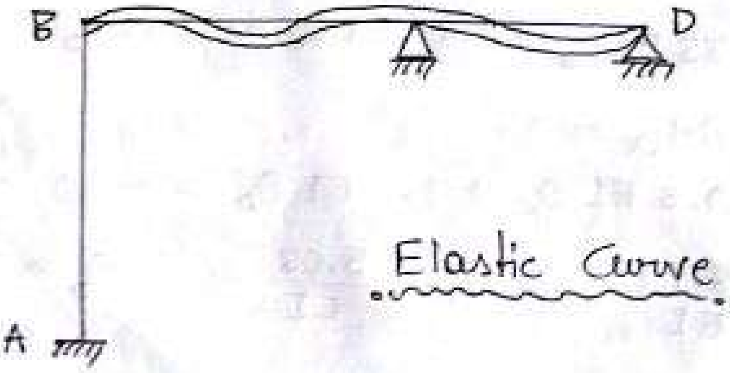
$$M_{CD} = -40 + EI\left(\frac{5.03}{EI}\right) + 0.5\,EI\left(\frac{-42.5}{EI}\right) = -56.22 \text{ KN}$$

$$M_{DC} = 40 + EI\left(\frac{-42.5}{EI}\right) + 0.5\,EI\left(\frac{5.03}{EI}\right) = 0 \text{ KNm}$$



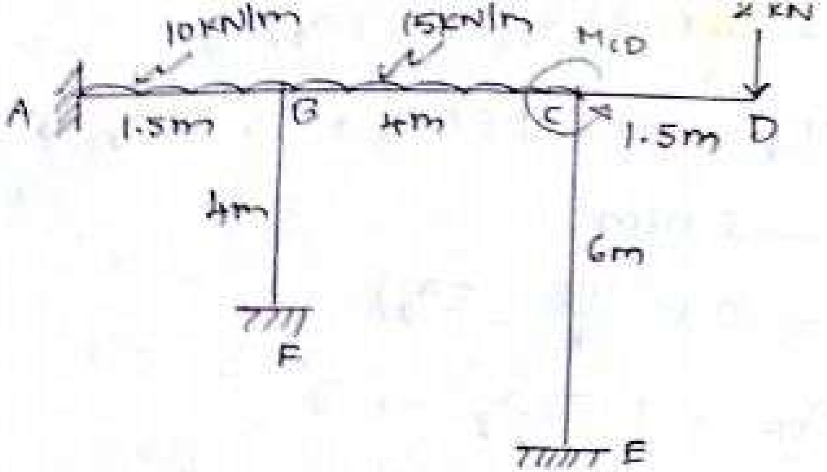


BMD.



Elastic Curve.

6.



→ DOF = 2 (θ_B, θ_C)

Here $M_{CD} = -2 \times 1.5 = -3$ KNm

Step 1 : Fixed End Moments :

$$M_{FAB} = \frac{-wL^2}{12} = \frac{-10\times(1.5)^2}{12} = -1.875 \text{ KNm}$$

$$M_{FBA} = \frac{wL^2}{12} = \frac{10\times(1.5)^2}{12} = 1.875 \text{ KNm}$$

$$M_{FBC} = \frac{-wL^2}{12} = \frac{-15\times4^2}{12} = -20 \text{ KNm}$$

$$M_{FCB} = \frac{wL^2}{12} = \frac{15\times4^2}{12} = 20 \text{KNm}$$

$M_{FBF} = 0$

$M_{FFB} = 0$

$M_{FCE} = 0$

$M_{FEC} = 0$

Step 2 : Slope - deflection Equations :

$$M_{AB} = -1.87 + \frac{2EI}{1.5}(0 + 2\theta_B - 0)$$

$M_{AB} = -1.87 + 1.33 EI \theta_B$ → ①

$$M_{BA} = 1.87 + \frac{2EI}{1.5}(2\theta_B - 0 - 0)$$

$M_{BA} = 1.87 + 2.67 \theta_B EI$ → ②

$$M_{BC} = -20 + \frac{2EI}{4}(2\theta_B + \theta_C)$$

$\therefore M_{BC} = -20 + 1 EI \theta_B + 0.5 EI \theta_C$ → ③

$M_{CB} = 20 + \frac{2EI}{4}(2\theta_c + \theta_B)$

$\therefore M_{CB} = 20 + 1 EI \theta_c + 0.5 EI \theta_B \rightarrow$ ④

$M_{CD} = -3$ KNm

$M_{BF} = 0 + \frac{2EI}{4}(2\theta_B)$

$M_{BF} = 1 . EI\theta_B \rightarrow$ ⑤

$M_{FB} = 0 + \frac{2EI}{4}(\theta_B)$

$\therefore M_{FB} = 0.5 EI \theta_B \rightarrow$ ⑥

$M_{CE} = 0 + \frac{2EI}{6}(2\theta_c)$

$M_{CE} = 0.67 EI \theta_c \rightarrow$ ⑦

$|||^{ly}$, $M_{EC} = 0 + \frac{2EI}{6}(\theta_c)$

$M_{EC} = 0.33 EI \theta_c \rightarrow$ ⑧

Step 3 : Joint Equilibrium Equations :

$\Sigma M_B = 0$; $M_{BA} + M_{BC} + M_{BF} = 0$

$1.87 + 2.67 EI\theta_B - 20 + EI\theta_B + 0.5 EI \theta_c + EI \theta_B = 0$

$4.67 EI \theta_B + 0.5 EI \theta_c = 18.43 \rightarrow$ Ⓐ

Now, $\Sigma M_C = 0$; $M_{CB} + M_{CD} + M_{CE} = 0$

$20 + EI\theta_c + 0.5 EI \theta_B - 3 + 0.67 EI \theta_c = 0$

$0.5 EI\theta_B + 1.67 EI \theta_c = -17 \rightarrow$ Ⓑ

∴ From Ⓐ and Ⓑ,

$\therefore \theta_B = \frac{5.20}{EI}$; $\theta_c = \frac{-11.74}{EI}$

Step 4 : Final Moments :

$\therefore M_{AB} = -1.87 + 1.33 EI \left(\frac{5.20}{EI}\right) = 5.05$ KNm

$M_{BA} = 1.87 + 2.67 EI \left(\frac{5.20}{EI}\right) = 15.75$ KNm

$M_{BC} = -20 + EI\left(\frac{5.2}{EI}\right) + 0.5 EI\left(\frac{-11.74}{EI}\right) = -20.7$ KN

$M_{CB} = 20 + EI\left(\frac{-11.74}{EI}\right) + 0.5\,EI\left(\frac{5.2}{EI}\right) = 10.9$ KNm

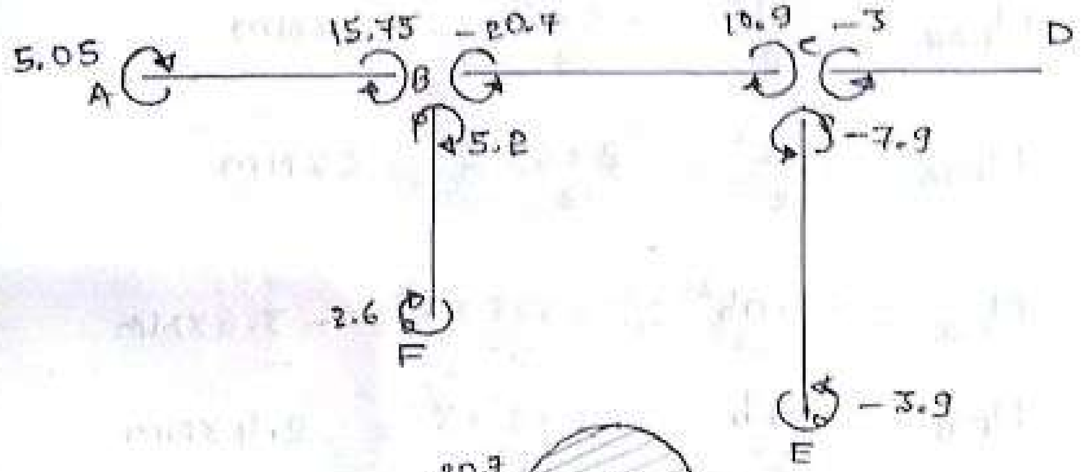
$M_{CD} = -3$ KNm

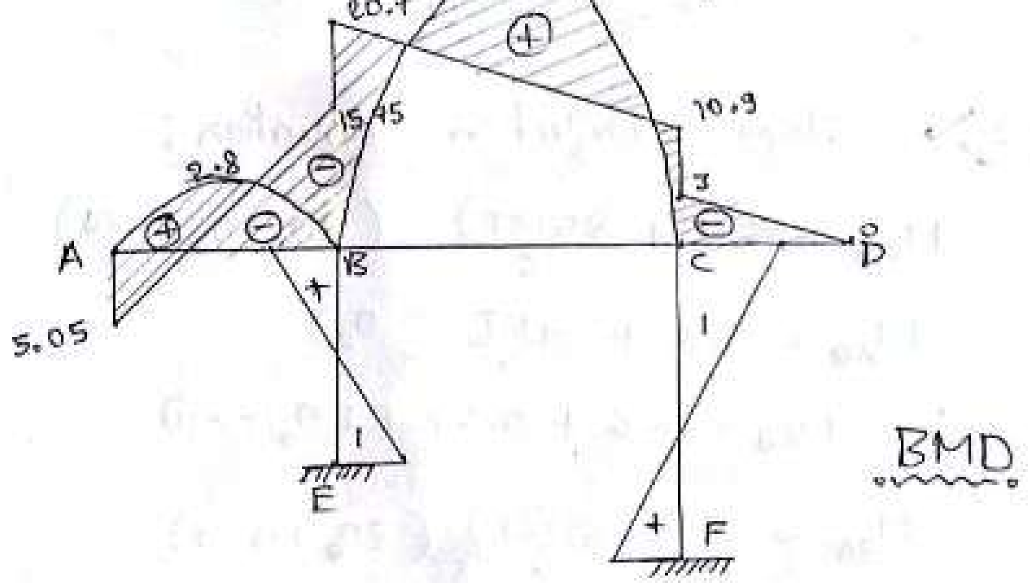
$M_{BF} = EI\left(\frac{5.20}{EI}\right) = 5.20$ KNm

$M_{FB} = 0.5\,EI\left(\frac{5.20}{EI}\right) = 2.6$ KNm

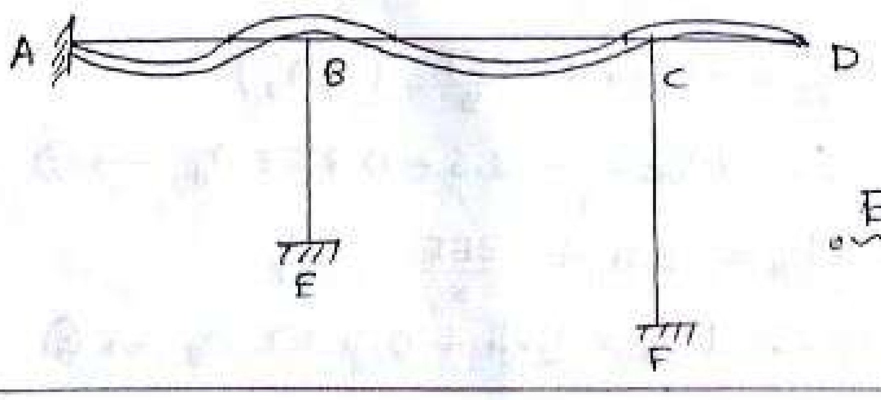
$M_{CE} = 0.67\,EI\left(\frac{-11.74}{EI}\right) = -7.9$ KNm

$M_{EC} = 0.33\,EI\left(\frac{-11.74}{EI}\right) = -3.9$ KNm



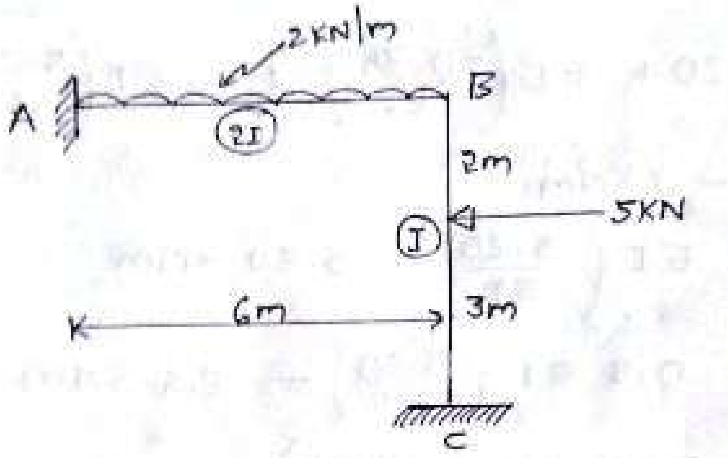


BMD



Elastic Curve

7.



→ $DOF = 1 (\theta_B)$

Step 1: Fixed End Moments:

$$M_{FAB} = \frac{-wL^2}{12} = \frac{-2 \times 6^2}{12} = -6 \text{ KNm}$$

$$M_{FBA} = \frac{wL^2}{12} = \frac{2 \times 6^2}{12} = 6 \text{ KNm}$$

$$M_{FBC} = \frac{-wab^2}{L^2} = \frac{-5 \times 2 \times 3^2}{5^2} = -3.6 \text{ KNm}$$

$$M_{FCB} = \frac{wa^2b}{L^2} = \frac{5 \times 2^2 \times 3}{5^2} = 2.4 \text{ KNm}$$

Step 2: Slope - Deflection Equation:

$$M_{AB} = -6 + \frac{2E(2I)}{6}\left(2\theta_A + \theta_B - \frac{3\Delta}{L}\right)$$

$$M_{AB} = -6 + \frac{4EI}{6}(\theta_B)$$

$$\therefore \quad M_{AB} = -6 + 0.67\, EI\, \theta_B \rightarrow ①$$

$$M_{BA} = 6 + \frac{2E(2I)}{6}(2\theta_B + 0 - 0)$$

$$\therefore \quad M_{BA} = 6 + 1.34\, EI\, \theta_B \rightarrow ②$$

$$M_{BC} = -3.6 + \frac{2EI}{5}(2\theta_B)$$

$$\therefore \quad M_{BC} = -3.6 + 0.8\, EI\, \theta_B \rightarrow ③$$

$$M_{CB} = 2.4 + \frac{2EI}{5}(\theta_B)$$

$$\therefore \quad M_{CB} = 2.4 + 0.4\, EI\, \theta_B \rightarrow ④$$

Step 3 : Joint Equilibrium Equation :

$$\Sigma M_B = 0 ; \quad M_{BA} + M_{BC} = 0$$

$$6 + 1.34\ EI\theta_B - 3.6 + 0.8\ EI\ \theta_B = 0$$

$$2.4 + 2.14\ EI\theta_B = 0$$

$$2.14\ EI\ \theta_B = -2.4$$

$$\therefore \quad \theta_B = \frac{-1.12}{EI}$$

Step 4 : Final Moments :

$$M_{AB} = -6 + 0.67\ EI \left(\frac{-1.12}{EI}\right)$$

$$\therefore \quad M_{AB} = -6.7 \text{ KNm}$$

$$M_{BA} = 6 + 1.34\ EI \left(\frac{-1.12}{EI}\right)$$

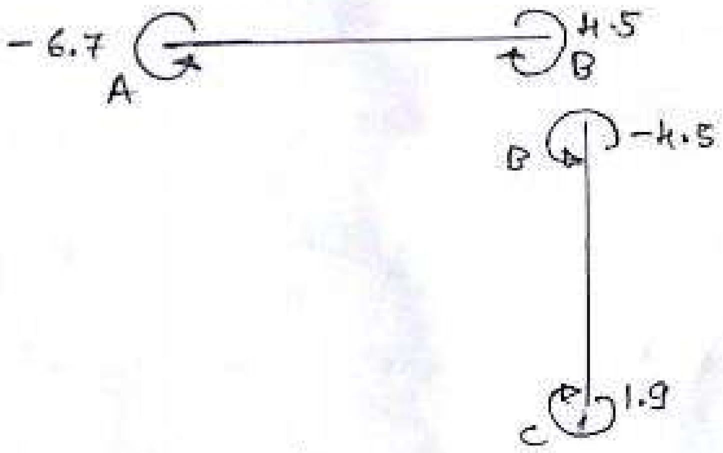
$$\therefore \quad M_{BA} = 4.5 \text{ kNm}$$

$$M_{BC} = -3.6 + 0.8\ EI \left(\frac{-1.12}{EI}\right)$$

$$\therefore \quad M_{BC} = -4.5 \text{ kNm}$$

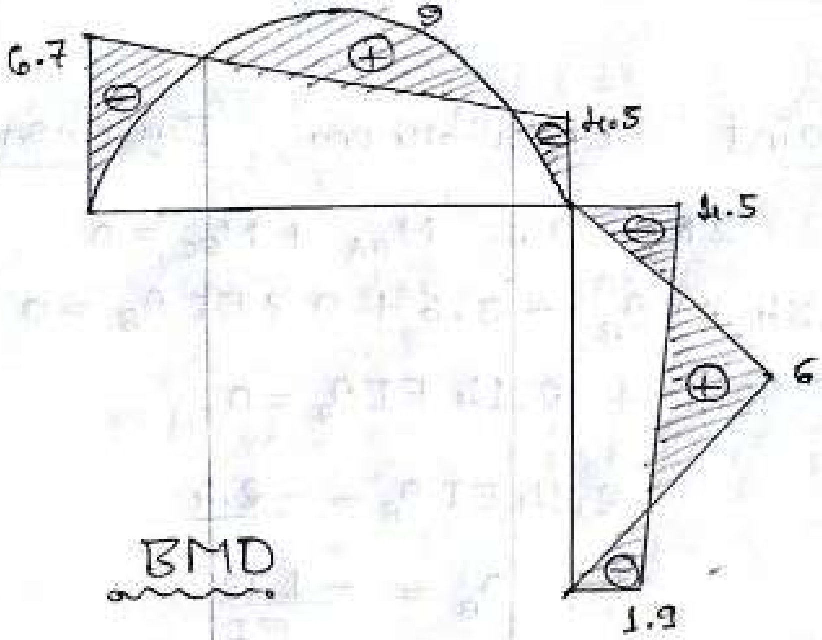
$$M_{CB} = 2.4 + 0.4\ EI \left(\frac{-1.12}{EI}\right)$$

$$\therefore \quad M_{CB} = 1.9 \text{ KNm}$$



Final Moments

BMD → pto



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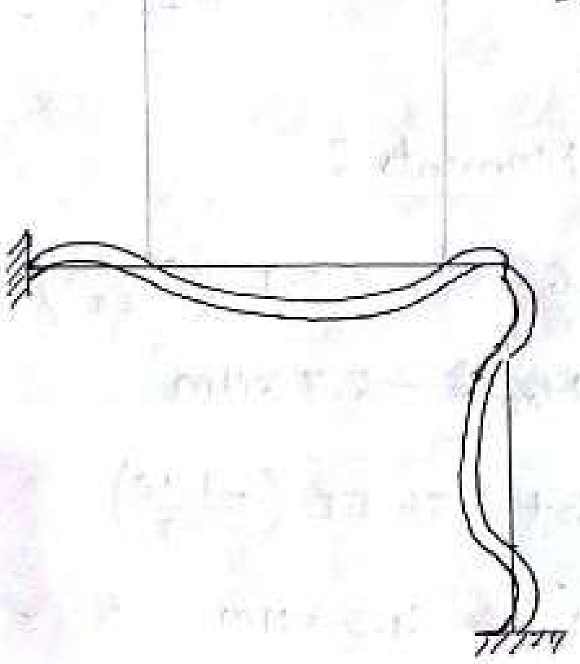


Elastic Curve

* * *

28/08/18

Module - 2

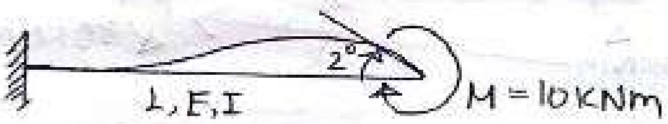
SUJITH.N.S

Moment Distribution Method

It is an another method to analyse statically Indeterminate structure introduced by Prof. Hardy, hence the method is also known as Hardy cross method.

Stiffness : (K) :-

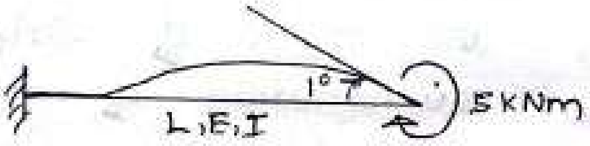
The moment required to produce 1° rotation is called stiffness. Stiffness is also defined as the ratio of moment of Inertia to length.



$2° \longrightarrow 10kNm$

For $1° \longrightarrow \frac{10}{2} = 5 KN\text{-}m$

$\therefore \quad K = \left(\frac{M}{\theta}\right)$

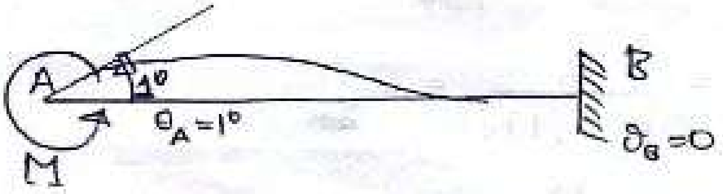


$K = \frac{M}{\theta} = \frac{5}{1°} KNm$

$\boxed{OR}$

$K = \left(\frac{I}{L}\right) \qquad \left[\because \begin{matrix} K \propto I \\ K \propto 1/L \end{matrix}\right]$

Consider a beam of length 'L', Young's modulus 'E', MoI 'I' subjected to a moment 'M' to produce 1° rotation as shown in the fig.



Now, $\quad M_{AB} = 0 + \frac{2EI}{L}(2\theta_A + 0 - 0)$

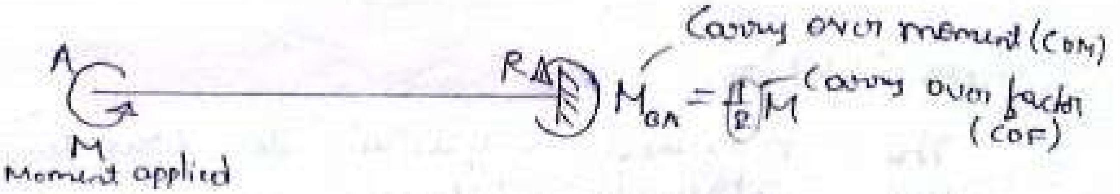
$\therefore \quad M = \frac{4EI}{}$

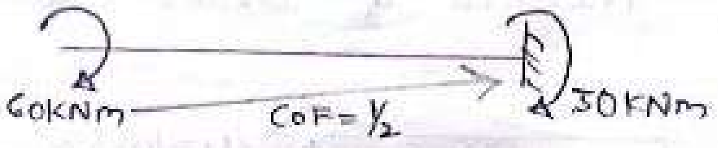
Now, $M_{BA} = 0 + \frac{2EI}{L}(2\times0+1-0)$

$\therefore M_{BA} = \frac{2EI}{L}$

Consider. $\frac{M_{BA}}{M_{AB}} = \frac{M_{BA}}{M} = \frac{2EI/L}{4EI/L}$

$\frac{M_{BA}}{M_{AB}} = \frac{1}{2}$ @ $\boxed{M_{BA} = \frac{1}{2} M}$

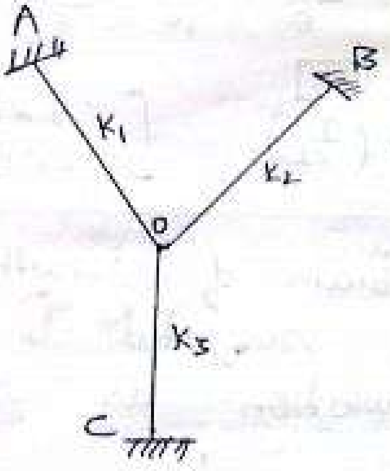




Note:

Distribution factor (DF) @ $\sqrt{\ }$:

It is the ratio of stiffness of one member to the summation of all the members meeting at the joint.



$\sqrt{\ }_{OA} = \frac{K_1}{K_1+K_2+K_3} = \frac{K_1}{\Sigma K}$

$\sqrt{\ }_{OB} = \frac{K_2}{\Sigma K}$

$\sqrt{\ }_{OC} = \frac{K_3}{\Sigma K}$

$$\therefore \sqrt{}_{OA} + \sqrt{}_{OB} + \sqrt{}_{OC} = \frac{K_1}{\Sigma K} + \frac{K_2}{\Sigma K} + \frac{K_3}{\Sigma K}$$

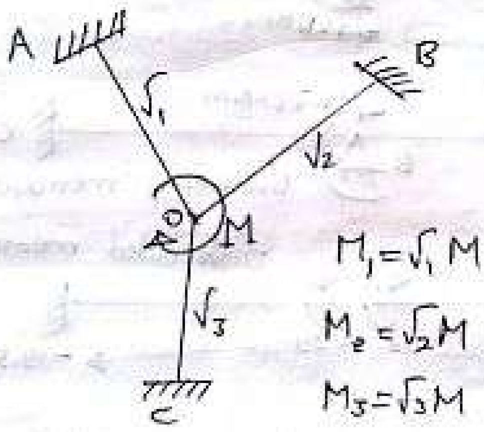
$$= \frac{K_1 + K_2 + K_3}{\Sigma K}$$

$$= \frac{\Sigma K}{\Sigma K} = 1$$

$$\boxed{\sqrt{}_1 + \sqrt{}_2 + \sqrt{}_3 = 1}$$

Sum of the distribution factor = 1

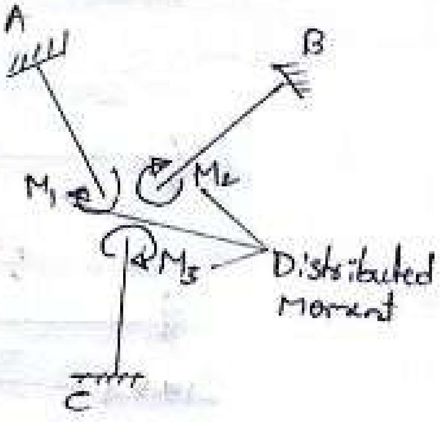
If a moment is applied at a joint, momn will be get distributed among the members based on the distribution factor.



$M_1 = \sqrt{}_1 M$

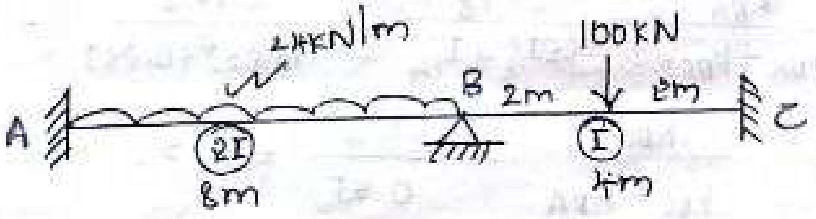
$M_2 = \sqrt{}_2 M$

$M_3 = \sqrt{}_3 M$



Problems:

1. Analyse the continuous beam by Moment-Distribution method. Draw BMD & SFD.



*Note: Distribution factors are to be calculated only at the joints

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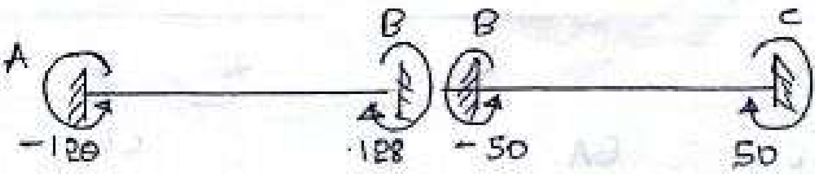


30/08/18

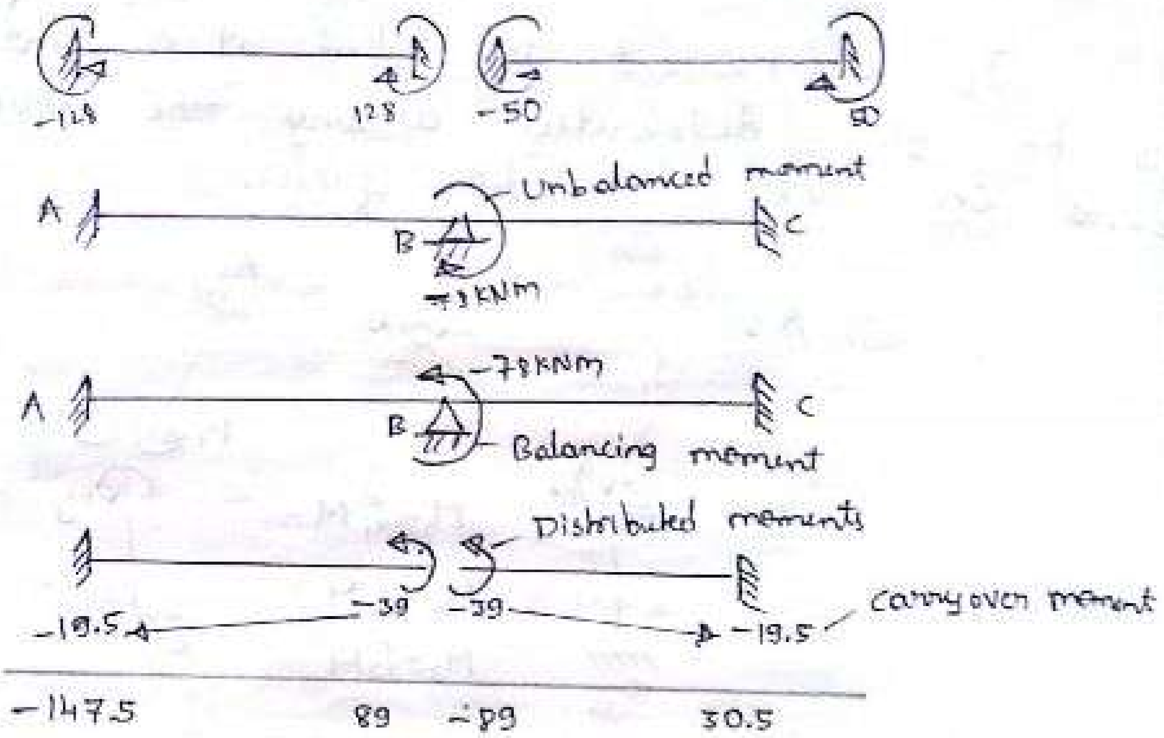
Step 1 : Fixed End Moments ;

$$M_{FAB} = \frac{-wl^2}{12} = -\frac{24\times8^2}{12} = -128 \text{ KNm}$$

$$M_{FBA} = \frac{wL^2}{12} = \frac{24\times8^2}{12} = 128 \text{ KNm}$$

$$M_{FBC} = \frac{-100\times4}{8} = -50 \text{ KNm}$$

$$M_{FCB} = 50 \text{KNm}$$



Step 2 : Distribution Factors ;

@ Joint 'B',

$$\delta_{BA} = \frac{K_{BA}}{K_{BA}+K_{BC}} = \frac{2I/8}{2I/8 + I/4} = \frac{0.25I}{0.25I+0.25I} = 0.5$$

$$\delta_{BC} = \frac{K_{BC}}{K_{BC}+K_{BA}} = \frac{I/4}{0.5I} = 0.5$$

Step 3 ; Moment - Distribution table ;

Joint	A	B		C
Members	AB	BA	BC	CB
DF	–	0.5	0.5	–

$\delta_{AB} = \frac{k_{BA}}{3}$

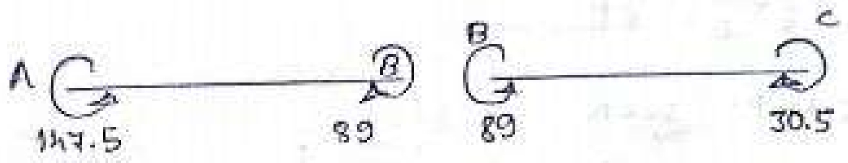
FEM	−128	128	−50 $\boxed{-78}$	50
	−19.5	−39	−39	−19.5
Final m-oments KNm	−147.5	+89	− 89	30.5

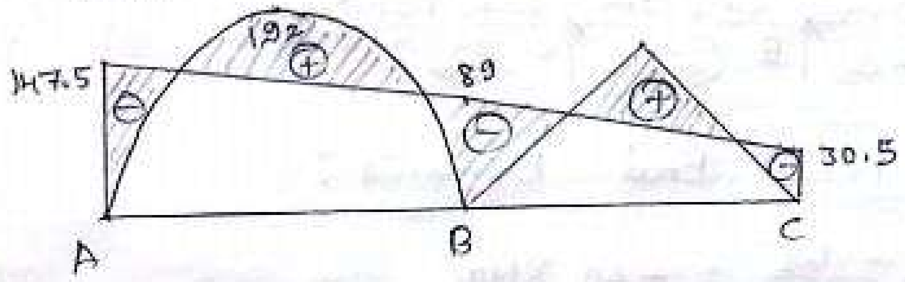
Balancing moment

Distributed moment

Carry over moment

1 cycle

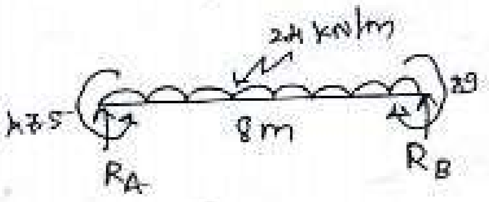




BMD

Elastic curve

To find SF:



$\Sigma M_A = 0$;

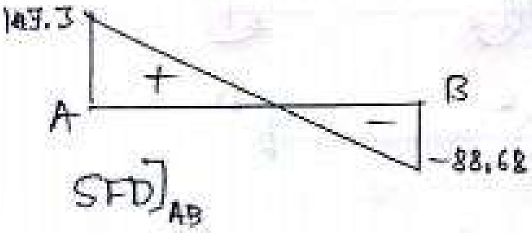
$-147.5 + (24 \times 8)4 - R_B \times 8 + 89 = 0$

$\therefore R_B = 88.68$ KN

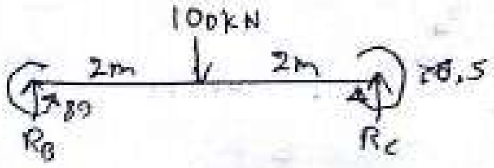
$\Sigma V = 0$;

$R_A - (24 \times 8) + 88.68 = 0$

$R_A = 103.3$ KN



SFD$]_{AB}$



$\Sigma M_B = 0$;

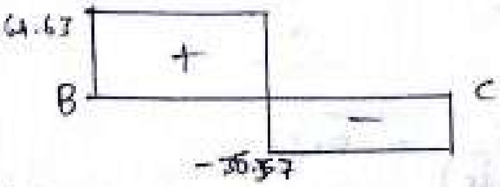
$-89 + 100 \times 2 + 30.5 - R_c \times 4 = 0$

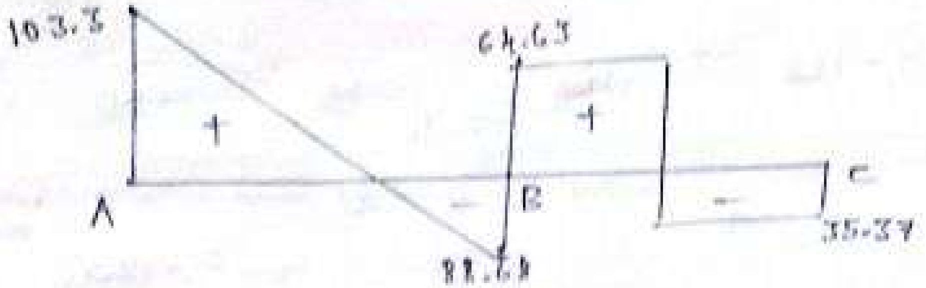
$R_c = 35.37$ KN

$\Sigma V = 0$;

$R_B - 100 + 35.37 = 0$

$R_B = 64.63$ KN

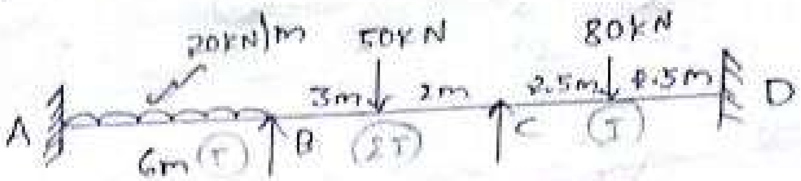




SFD

P. Draw BMD by moment distribution method.

$2 I_{AB} = I_{BC} = 2 I_{CD} = 2I$



→ Step 1 : Fixed End Moments :

$$M_{FAB} = \frac{-wL^2}{12} = -60 \text{ kNm}$$

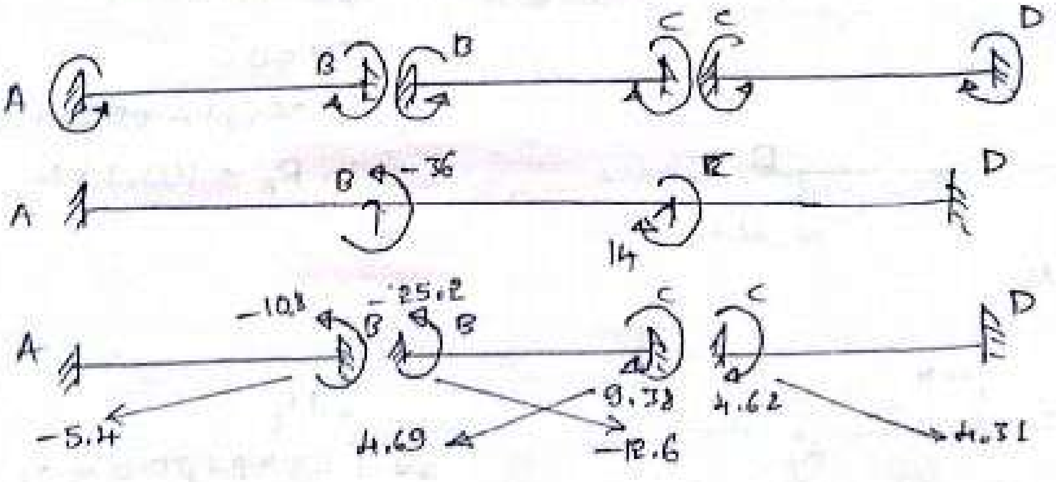
$$M_{FBA} = \frac{wL^2}{12} = 60 \text{ kNm}$$

$$M_{FBC} = \frac{-wab^2}{L^2} = -24 \text{ kNm}$$

$$M_{FCB} = \frac{wa^2b}{L^2} = 36 \text{ kNm}$$

$$M_{FCD} = \frac{-WL}{8} = -150 \text{ kNm}$$

$$M_{FDC} = \frac{WL}{8} = 50 \text{ kNm}$$



Step 2 : DF's :

ⓐ Joint 'B',

$$\delta_{BA} = \frac{K_{BA}}{K_{BA} + K_{BC}} = \frac{I/6}{I/6 + 2I/5} = \frac{(1/6)}{(1/6 + 2/5)} = 0.3$$

$$\delta_{BC} = \frac{K_{BC}}{K_{BA}+K_{BC}} = \frac{2I/5}{I/6+2I/5} = 0.7$$

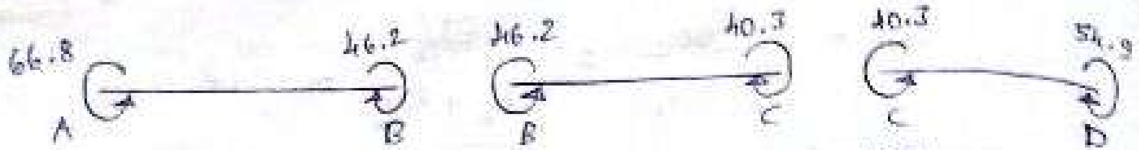
@ Joint 'C',

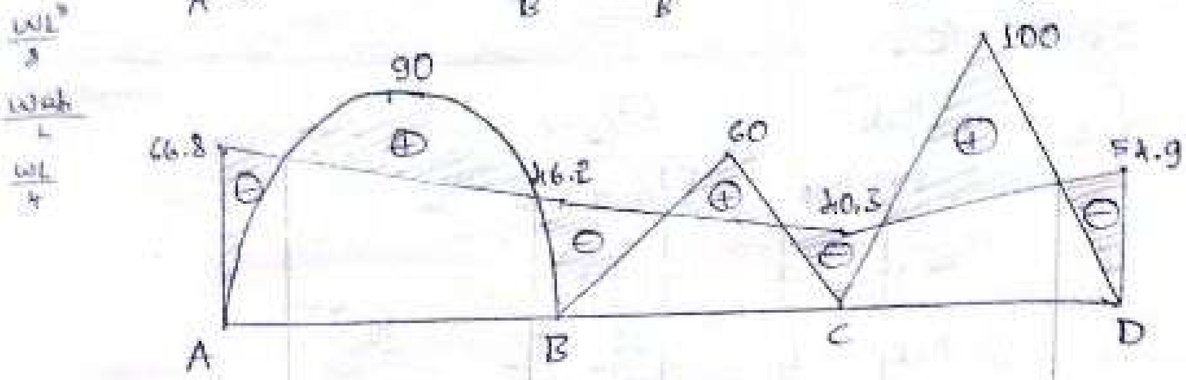
$$\delta_{CB} = \frac{K_{CB}}{K_{CB}+K_{CD}} = \frac{2I/5}{2I/5+I/5} = 0.67$$

$$\delta_{CD} = \frac{K_{CD}}{K_{CB}+K_{CD}} = \frac{I/5}{2I/5+I/5} = 0.33$$

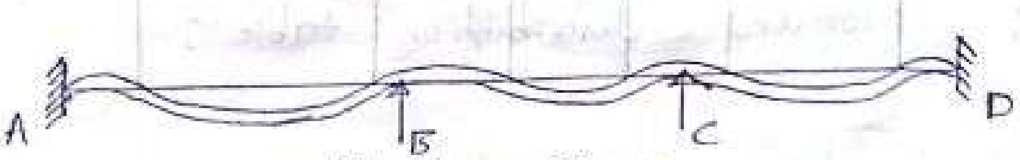
Step 3 : Moment distribution table :

Joint	A	B		C		D	
Members	AB	BA	BC	CB	CD	DC	
DF	–	0.3	0.7	0.67	0.33	–	
FEM	-60	60 $\boxed{-36}$	-24	36	$\boxed{14}$ -50	50	
	-5.4	-10.8 $\boxed{-4.69}$	-25.2 4.69	9.38 -12.6	4.62 $\boxed{12.6}$	2.31	1^{th} cycle
	-0.7	-1.407 $\boxed{-4.22}$	-3.283 4.22	8.44 -1.64	4.15 $\boxed{1.64}$	2.07	2^{th} cycle
	-0.63	-1.26 $\boxed{-0.54}$	-2.95 0.54	1.09 -1.47	0.54 $\boxed{1.47}$	0.19	3^{th} cycle
	-0.08	-0.162 $\boxed{-0.49}$	-0.372 0.49	0.985 -0.189	0.485 $\boxed{0.189}$	0.24	4^{th} cycle
	-0.073	-0.147	-0.34 +0.063	0.126 -0.17	0.06	0.03	5^{th} cycle
Final Moments	-66.8	46.2	-46.2	40.3	-40.3	54.94	





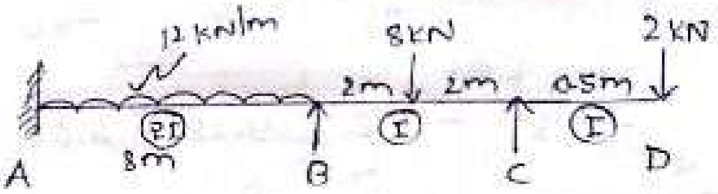
BMD



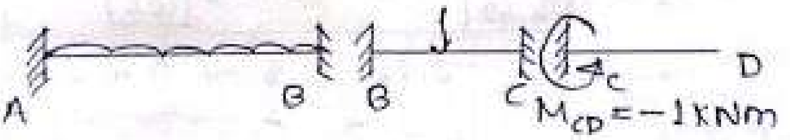
Elastic Curve

03/09/18

3. Analyse the continuous beam loaded as shown in the figure. Draw BMD and SFD



→



Step 1: Fixed End Moments:

$$M_{FAB} = \frac{-12 \times 8^2}{12} = -64 \text{ KNm}$$

$$M_{FBA} = 64 \text{ KNm}$$

$$M_{FBC} = \frac{-WL}{8} = \frac{-8 \times 4}{8} = -4 \text{ KNm}$$

$$M_{FCB} = 4 \text{ KNm}$$

Step 2: Distribution factor:

@ Joint B, $D_{BA} = \frac{K_{BA}}{K_{BA} + K_{BC}} = \frac{2I/8}{2I/8 + I/4} = 0.5$

$$D_{BC} = \frac{K_{BC}}{K_{BA} + K_{BC}} = \frac{I/4}{2I/8 + I/4} = 0.5$$

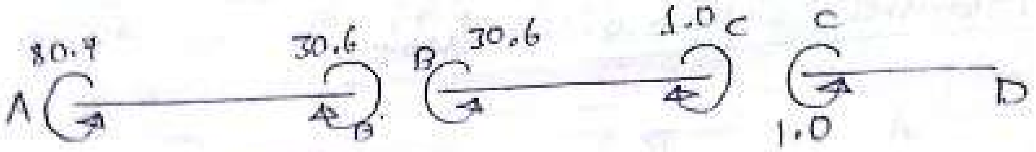
@ Joint C. $\int_{CB} = \frac{K_{CB}}{K_{CB}+K_{CD}} = \frac{I/4}{I/4+0} = 1.0$

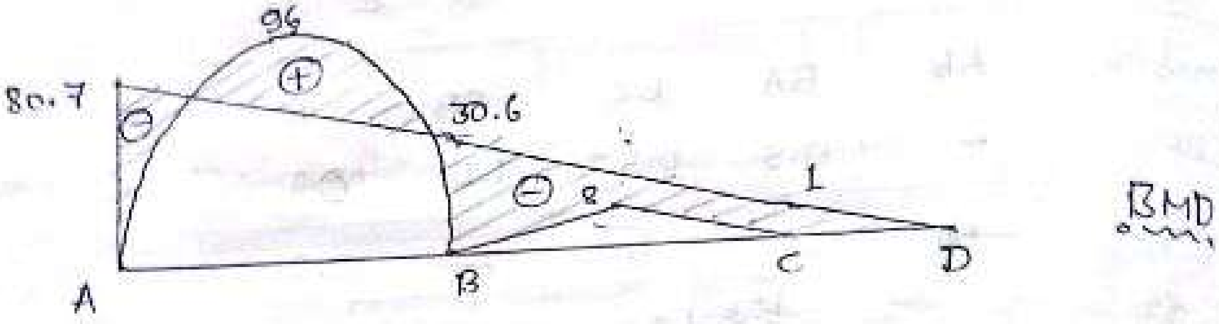
$\int_{CD} = \frac{K_{CD}}{K_{CB}+K_{CD}} = \frac{0}{K_{CB}+K_{D}} = 0.0$

Step 3: Moment distribution table:

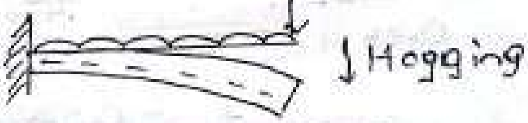
Joint	A	B		C	
Members	AB	BA	BC	CB	CD
DF	–	0.5	0.5	1.0	+0.0
FEM	-64	64 $\boxed{-60}$	-4	4 $\boxed{-3.0}$	-1.0
	~~-30~~ -15	-30 $\boxed{1.5}$	-30 -1.5	-3.0 -15 $\boxed{15}$	0.0
	0.375	0.75 $\boxed{-7.5}$	0.75 7.5	15.0 0.375 $\boxed{-0.375}$	0.0
	-1.875	-3.15 $\boxed{0.1875}$	-3.75 -0.1875	-0.375 -1.875 $\boxed{1.875}$	0.0
	0.045	0.09 $\boxed{-0.9}$	-0.09 0.9	1.875 0.045 $\boxed{-0.045}$	0.0
	-0.225	-0.45 $\boxed{0.022}$	-0.45 -0.0225	-0.045 -0.225 $\boxed{0.225}$	0.0
	0.003	0.011 $\boxed{-0.112}$	0.011 0.112	0.225 0.005 $\boxed{-0.005}$	0.0 0.0
	-0.0551 -0.022	-0.0551	-0.0551 -0.002	-0.005 -0.022	0.0

Final Moment	-80.7	30.6	- 30.6	1.0	-1.0



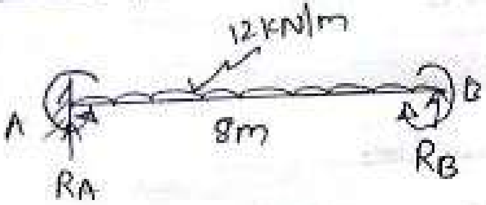


Note: There is no sagging BM↑ in the cantilever. Only Hogging BM occurs.



To draw SFD:

(i)



$\Sigma M_A = 0$;

$-80.7 - (96 \times 4) + 30.6 - R_B \times 8 =$

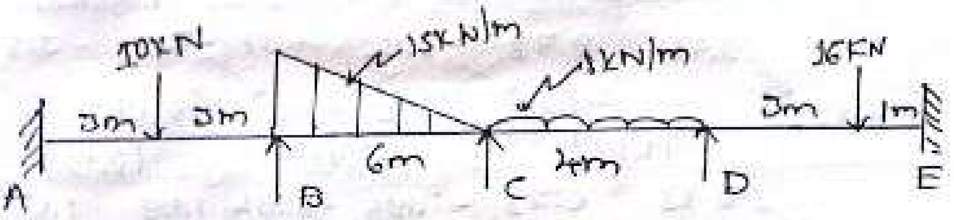
$R_B = -51.26$ kN

$\Sigma M = 0$;

$R_A - 96 - 51.26 = 0$

$\therefore R_A = 147.26$ kN

4.



→ Step 1: Fixed End moments:

$$M_{FAB} = -7.5\ KNm$$

$$M_{FBA} = 7.5\ KNm$$

$$M_{FBC} = \frac{-15 \times 6^2}{20} = -27\ KNm$$

$$M_{FCB} = \frac{15 \times 6^2}{30} = 18\ KNm$$

$$M_{FCD} = \frac{-8 \times 4^2}{12} = -10.67\ KNm$$

$$M_{FDC} = 10.67\ KNm$$

$$M_{FDE} = \frac{-16 \times 3 \times 1^2}{4^2} = -3.0\ KNm$$

$$M_{FED} = \frac{16 \times 3^2 \times 1}{4^2} = 9.0\ KNm$$

Step 2: DF: @ Joint 'B'.

$$\int_{BA} = \frac{I/6}{I/6 + I/6} = 0.5 \qquad \int_{BC} = 0.5$$

@ Joint 'C',

$$\surd_{CB} = \frac{I/6}{I/6 + I/4} = 0.4$$

$$\surd_{CD} = \frac{I/4}{I/6 + I/4} = 0.6$$

@ Joint 'D',

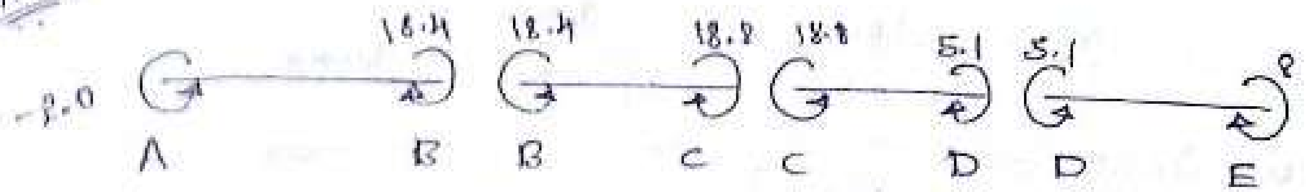
$$\surd_{DC} = \frac{K_{DC}}{K_{DC} + K_{DE}} = \frac{I/4}{I/4 + I/4} = 0.5$$

$$\surd_{DE} = 0.5$$

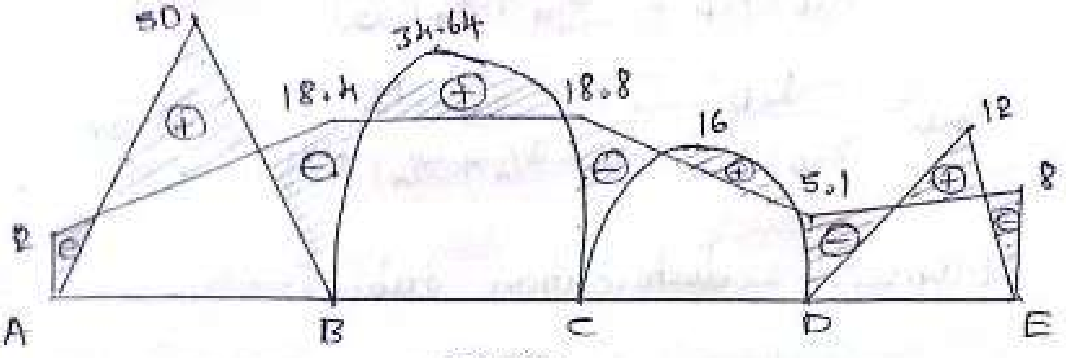
Step 3 : Distribution table :

Joint	A	B		C		D		E
Members	AB	BA	BC	CB	CD	DC	DE	ED
DF	—	0.5	0.5	0.4	0.6	0.5	0.5	—
FEM	−7.5	7.5 $\boxed{19.5}$	−27	18 $\boxed{-7.3}$	−10.67	10.67 $\boxed{-7.63}$	−3.0	9.0
	4.89	9.78 $\boxed{1.46}$	9.78 −1.46	−2.92 4.89 $\boxed{-2.99}$	−4.38 −1.9	−3.8 −2.19 $\boxed{2.19}$	−3.8	−1.9
	0.365	0.73 $\boxed{0.598}$	0.73 −0.598	−1.196 0.365 $\boxed{-0.91}$	−1.794 0.545	1.09 −0.898 $\boxed{0.895}$	1.09	0.545
	0.145	0.29 $\boxed{0.18}$	0.29 −0.18	−0.36 0.145 $\boxed{-0.368}$	−0.54 0.223	0.447 −0.27 $\boxed{0.27}$	0.447	0.223
	0.045	0.09 $\boxed{0.07}$	0.09 −0.07	−0.14 0.045 $\boxed{-0.112}$	−0.21 0.067	0.136 −0.105 $\boxed{0.105}$	0.136	0.067
	0.0175	0.035	0.035 −0.02	−0.04 0.0175	−0.06 −0.025	0.05 −0.02	0.05	0.025
Final Moments	−2.07	18.4	−18.4	18.8	−18.8	5.1	−5.1	7.96≈8

09/09/18



Fixed Moments



BMD

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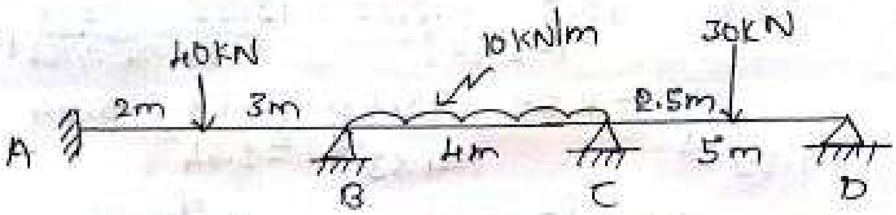
5. Analyse the continuous beam by moment-distribution method. Draw BMD and SFD. EI is constant.



→ Step 1: Fixed End Moments:

$$M_{FAB} = \frac{-40\times2\times3^2}{5^2} = -28.8 \text{ KNm}$$

$$M_{FBA} = \frac{40\times2^2\times3}{5^2} = 19.2 \text{ KNm}$$

$$M_{FBC} = \frac{-WL^2}{12} = \frac{-10\times4^2}{12} = -13.33 \text{ KNm}$$

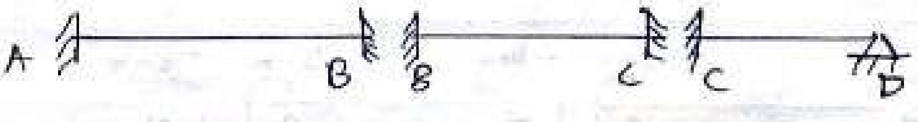
$$M_{FCB} = 13.33 \text{ KNm}$$

$$M_{FCD} = \frac{-WL}{8} = \frac{-30\times5}{8} = -18.7 \text{ KNm}$$

$$M_{FDC} = 18.7 \text{ KNm}$$

Step 2: Distribution factors:

@ Joint 'B', $D_{BA} = \frac{K_{BA}}{K_{BA}+K_{BC}} = \frac{I/5}{I/5+I/4} = 0.44$



$$\sqrt{\ }_{BC} = \frac{k_{BC}}{k_{BA}+k_{BC}} = \frac{I/4}{I/5 + I/4} = 0.56$$

@ Joint 'C',

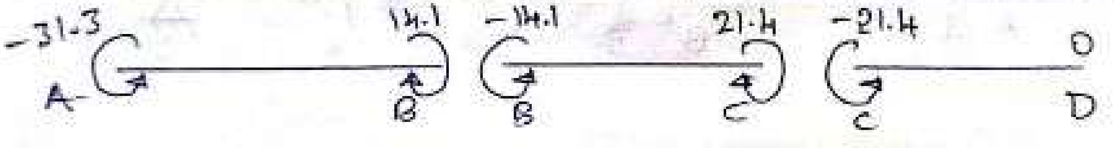
$$\sqrt{\ }_{CB} = \frac{k_{CB}}{k_{CB}+k_{CD}} = \frac{I/4}{I/4 + 3/4(I/5)} = 0.63$$

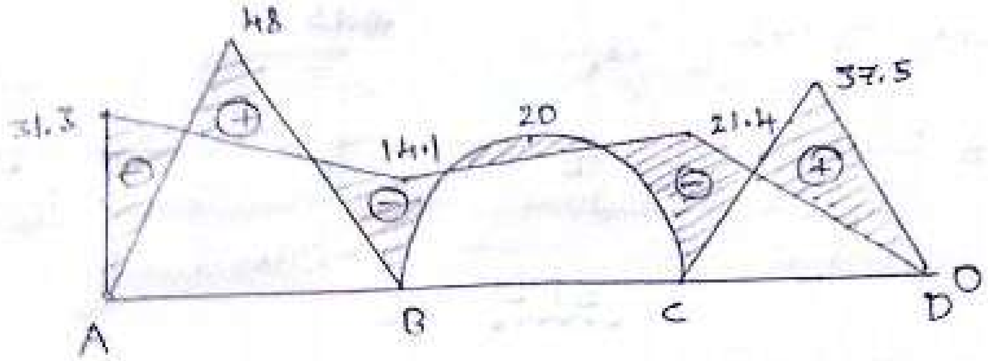
$$\sqrt{\ }_{BCD} = \frac{k_{CD}}{k_{CB}+k_{CD}} = \frac{3/4(I/5)}{I/4 + 3/4(I/5)} = 0.37$$

Step 3: <u>Moment distribution table:</u>

Joint	A	B		C		D
Member	AB	BA	BC	CB	CD	DC
DF	–	0.44	0.56	0.63	0.37	–
FEM	−28.8	19.2 $\boxed{-5.87}$	−13.33	13.33 $\boxed{+14.72}$	−18.7 $\boxed{-9.35}$	18.7 $\boxed{-18.7}$
	−1.29	−2.58 $\boxed{-4.63}$	−3.28 4.63	9.27 −1.64 $\boxed{1.64}$	5.44	
	−1.02	−2.03 $\boxed{-0.51}$	−2.59 0.51	1.03 −1.29 $\boxed{1.29}$	0.60	
	−0.11	−0.22 $\boxed{-0.405}$	−0.28 0.405	0.81 −0.14 $\boxed{0.14}$	0.47	
	0.08	−0.179 $\boxed{-0.04}$	−0.22 0.04	0.08 −0.11 $\boxed{0.11}$	0.051	
	−0.008	−0.017	−0.022 0.035	0.067 −0.011	0.04	
Final Moments	−31.38	14.1	−14.1	21.40	−21.4	0.0

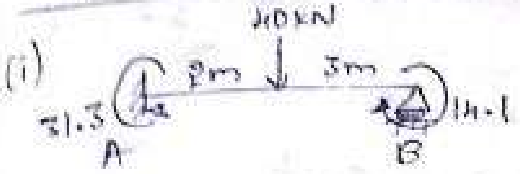
for V.I.





To draw SFD:

(i)



$\Sigma M_A = 0;$

$-31.3 + 80 + 14.1 - R_B \times 5 = 0$

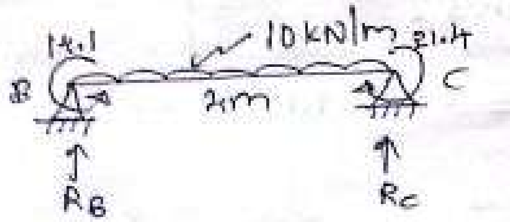
$R_B = 12.56\ kN$

$\Sigma V = 0;$

$R_A - 40 + R_B = 0$

$R_A = 27.44\ kN$

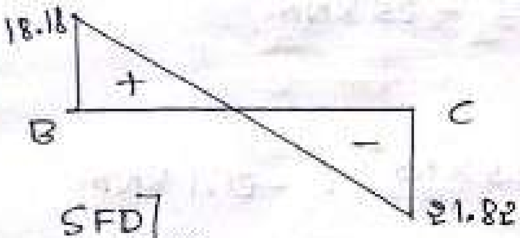
(ii)



$\Sigma M_B = 0;$

$-14.1 + (10 \times 4)2 + 21.4 - R_c \times 4 = 0$

$R_c = 21.82\ kN$



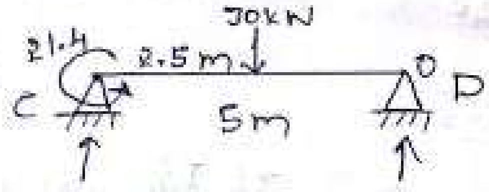
$SFD]_{BC}$

$\Sigma V = 0;$

$R_B - 40 + 21.82 = 0$

$\therefore R_B = 18.18\ kN$

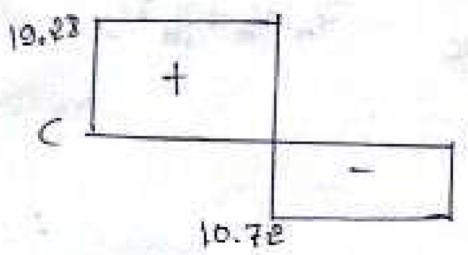
(ii)



$\Sigma M_c = 0;$

$-21.4 + (30 \times 2.5) - R_D \times 5 = 0$

$R_D = 10.72\ kN$



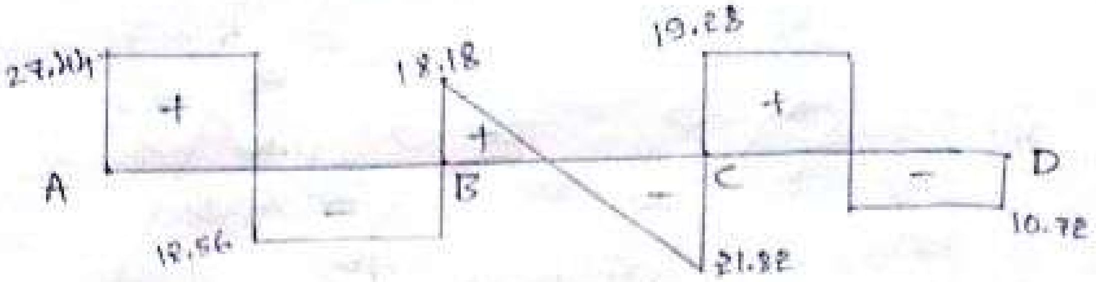
$SFD]_{CD}$

$\Sigma V = 0;$

$R_c - (30) + 10.72 = 0$

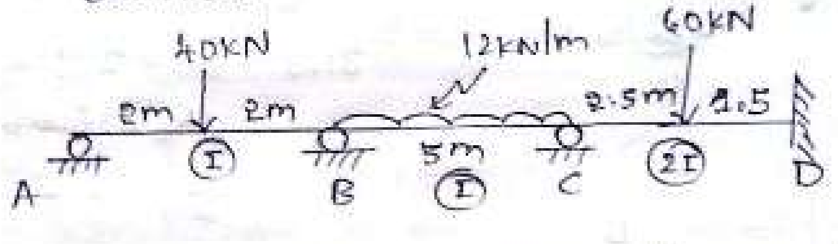
$R_c = 19.28\ kN$

∴ Shear force is given by,



SFD

6. Analyse the fixed beam shown in the fig by moment distribution.



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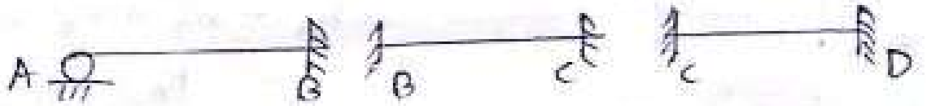


Step I : Fixed End Moments :

$$M_{FAB} = -\frac{WL}{8} = -\frac{40\times4}{8} = -20\text{KNm}$$

$$M_{FBA} = +20\text{ KNm}$$

$$M_{FBC} = \frac{-WL^2}{12} = \frac{-12\times5^2}{12} = -25\text{ KNm}$$

$$M_{FCB} = 25\text{ KNm}$$

$$M_{FCD} = -\frac{wab^2}{L^2} = \frac{-60\times2.5\times1.5^2}{4^2} = -21.1\text{ KNm}$$

$$M_{FDC} = \frac{wa^2b}{L^2} = \frac{60\times2.5^2\times1.5}{4^2} = 35.15\text{ KNm}$$

Step 2 : Distribution factor :

@ Joint B, $\delta_{BA} = \frac{K_{BA}}{K_{BA}+K_{BC}} = \frac{3/4\ I/4}{3/4\ I/4 + \frac{3}{4}\ I/5} = 0.48$

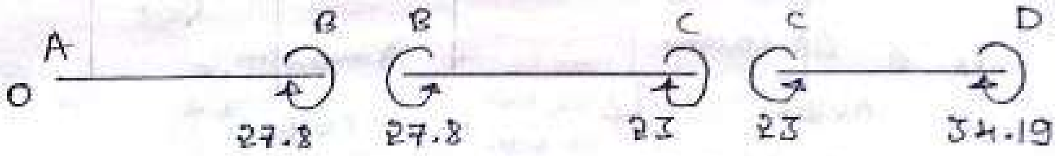
$$\delta_{BC} = \frac{K_{AC}}{K_{BA}+K_{BC}} = 0.52$$

@ 'C', $\delta_{CB} = \frac{K_{CB}}{K_{CB}+K_{CD}} = 0.3$

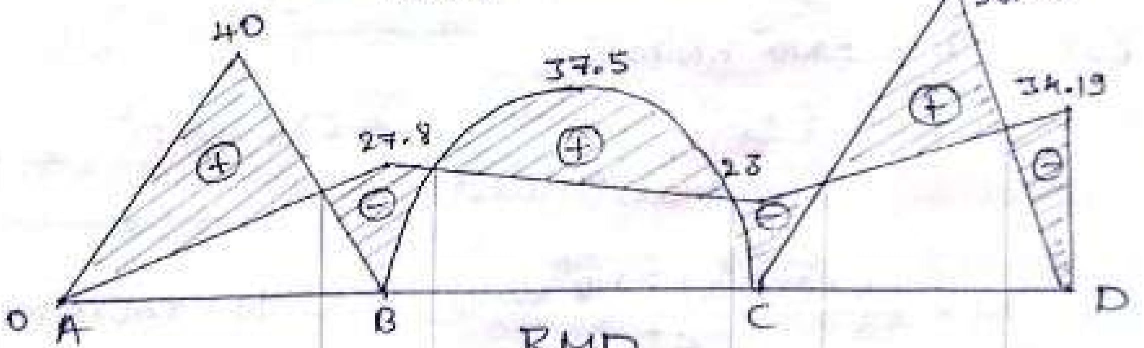
III^{ly}, $\delta_{CD} = 0.7$

Step 3 : Moment Distribution table :

Joint	A	B		C		D
Member	AB	BA	BC	CB	CD	DC
DF	—	0.48	0.52	0.3	0.7	—
FEM	-20 +20 →	20 10	-25 $\boxed{-5}$	25 $\boxed{-3.91}$	-21.09	35.15
		-2.4 $\boxed{0.58}$	-2.6 -0.58	-1.17 -1.3 $\boxed{1.3}$	-2.73	-1.36
		0.28 $\boxed{-0.195}$	0.30 0.195	0.39 0.15 $\boxed{-0.15}$	0.91	0.455
		-0.094	-0.10 -0.0225	-0.045 -0.05	-0.105	-0.052
Final Moments	0.0	27.80	-27.8	23	-23	34.19



Fixed Moments



$\frac{wL}{4}$

$\frac{wL^2}{8}$

$\frac{wab}{L}$

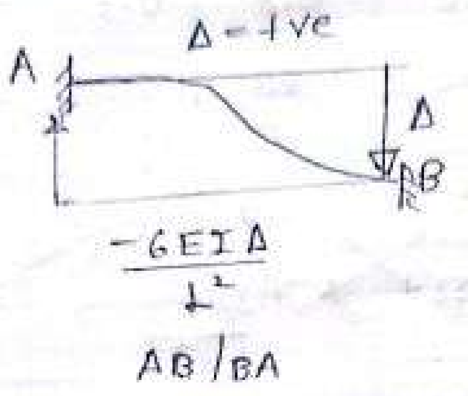
BMD

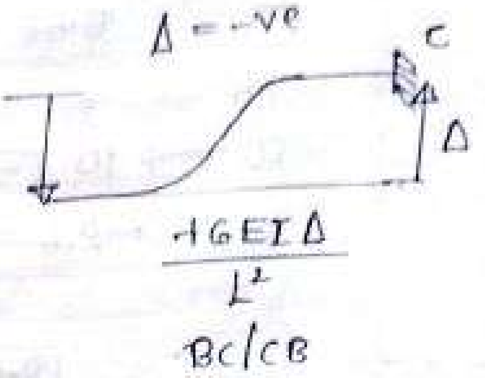
Elastic Curve

05/09/18

Settelement (Δ) :

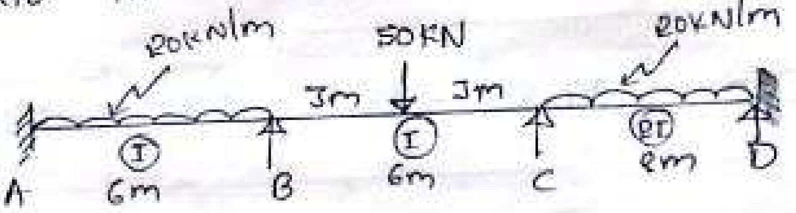
Settlement 'Δ' is converted to equivalen moment. that moment is added to fixe end moment and adopt distribution proces it is.



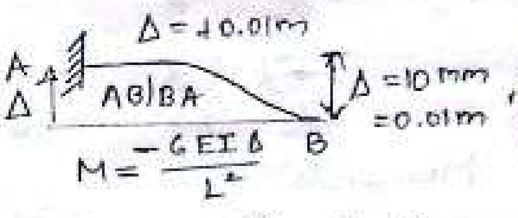


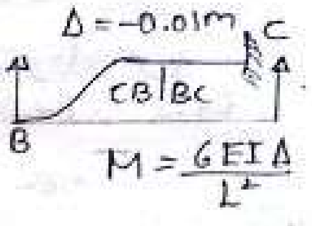
Problems :

1. Analyse the continuous beam loaded shown in the fig. by moment distribution method support 'B' sinks by 10 mm. E is 200 kN/mm²
$I = 1.2 \times 10^{-4}$ m⁴



→





But $E = 2 \times 10^5$ kN/mm²

$$E = \frac{2 \times 10^5 \left(\frac{1}{1000}\right) kN}{\left(\frac{1}{1000}\right) m \left(\frac{1}{1000}\right) m} = 2 \times 10^8 \text{ kN/m}^2$$

$$\therefore M = \frac{-6 \times 2 \times 10^8 \times 1.2 \times 10^{-4} \times 0.01}{6^2} \qquad \therefore M = +40 \text{kNm}$$

$$\therefore M = -40 \text{ kNm}$$

Step 1 : Fixed End Moments :

$$M_{FAB} = -\frac{WL^2}{12} = \frac{-20 \times 6^2}{12} - 40 = -60 - 40 = -100 \text{ kNm}$$

$$M_{FBA} = \frac{20\times6^2}{12} - 40 = 20\,\text{KNm}$$

$$M_{FBC} = \frac{-WL}{8} = \frac{-50\times3}{8} + 40 = 2.5\,\text{KNm}$$

$$M_{FCB} = \frac{WL}{8} = \frac{50\times3}{8} + 40 = 77.5\,\text{KNm}$$

$$M_{FCD} = \frac{-WL^2}{12} = \frac{-20\times6^2}{12} = -106.67\,\text{KNm}$$

$$M_{FDC} = 106.67\,\text{KNm}$$

Step 2: Distribution factors:

@ Joint 'B',

$$\delta_{BA} = \frac{K_{BA}}{K_{BA}+K_{BC}} = \frac{I/6}{I/6+I/6} = 0.5$$

$$\delta_{BC} = \frac{I/6}{I/6+I/6} = 0.5$$

@ Joint 'C',

$$\delta_{CB} = \frac{I/6}{\frac{I}{6}+3/4\left(\frac{2I}{8}\right)} = 0.47$$

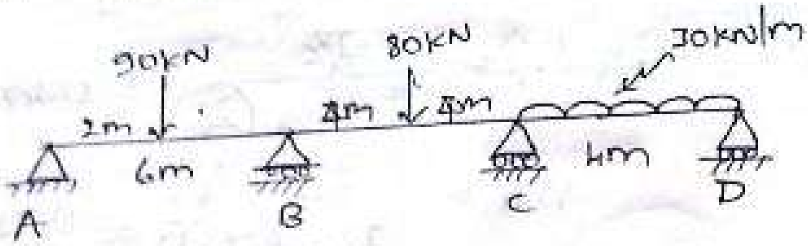
$$\delta_{CD} = \frac{3/4\left(2I/8\right)}{I/6+3/4\left(\frac{2I}{8}\right)} = 0.53$$

Step 3: Moment distribution table:

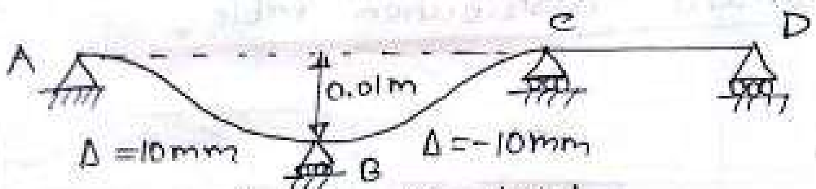
Joint	A	B		C		D
Members	AB	BA	BC	CB	CD	DC
DF	—	0.5	0.5	0.47	0.53	—
FEM	-100	20 $\boxed{-22.5}$	2.5	77.5 $\boxed{82.7}$	-106.67 -53.33	106.67 -106.67
	-5.625	-11.25 $\boxed{-19.38}$	-11.25 19.38	38.77 -5.625 $\boxed{5.625}$	43.72	
	-4.84	-9.69 $\boxed{-1.32}$	-9.69 1.32	2.64 -4.84 $\boxed{4.84}$	2.98	
	-0.33	-0.66 $\boxed{-1.135}$	-0.66 1.135	2.27 -0.33 $\boxed{0.33}$	2.56	

18/10/18

2. Draw the BMD & SFD for the continuous b loaded shown in the figure. Support 'B' yield by 10 mm below the level of ACD. $I = 132 \times 10^6 mm$
$E = 200 GPa$



→ Note: Yielding means settlement



$M = \frac{-6EI\Delta}{L^2}$ AB/BA

$M = \frac{6EI\Delta}{L^2}$ BC/CB

Step1: Fixed End Moments:

$$M_{FAB} = \frac{-90 \times 2 \times 4^2}{6^2} - \frac{6EI\Delta}{L^2}$$

$$\therefore \quad EI = 200 \times 10^6 \times 132 \times 10^{-6}$$

$$EI = 26400 \cancel{\times} KNm^2$$

$$\therefore \quad M_{FAB} = -\overset{80}{\cancel{180}} - \frac{6(26,400)0.01}{6^2}$$

$$= -180 - 44$$

$$\therefore \quad M_{FAB} = -124 \text{ KNm}$$

$$M_{FBA} = \frac{90 \times 2^2 \times 4}{6^2} - 44 = -4 \text{ kNm}$$

$E = 200 GPa$

$E = 200 \times 10^9 N/m^2$

$E = 200 \times 10^9 (\frac{1}{1000})$

$E = 200 \times 10^6 KN/m$

$I = 132 \times 10^6 (\frac{1}{10^3})^4$

$I = 132 \times 10^{-6} m^4$

$$M_{FBC} = -\frac{80\times8}{8} + 24.75$$

$$= -55.25\ \text{kNm}$$

$$M_{FCD} = \frac{80\times8}{8} + 24.75$$

$$\therefore\ M_{FCB} = 104.75\ \text{kNm}$$

$$M_{FCD} = -\frac{30\times4^2}{12} = -40\text{kNm}$$

$$M_{FDC} = \frac{30\times4^2}{12} = 40\text{kNm}$$

Step 2: Distribution factor:

@ Joint 'B'.

$$\sqrt{}_{BA} = \frac{K_{BA}}{K_{BA}+K_{BC}} = \frac{3/4(I/6)}{3/4(I/6)+I/8} = 0.5$$

$$\sqrt{}_{BC} = \frac{K_{BC}}{K_{BA}+K_{BC}} = 0.5$$

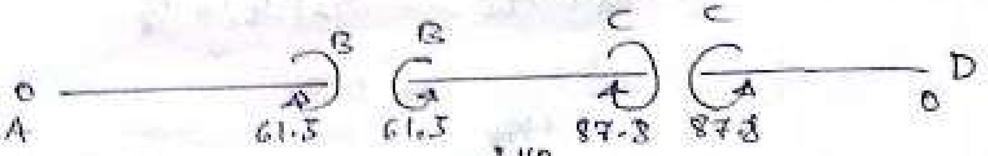
@ Joint 'C', $\sqrt{}_{CB} = \frac{K_{CB}}{K_{CB}+K_{CD}} = \frac{I/8}{I/8+3/4\,(I/4)} = 0.4$

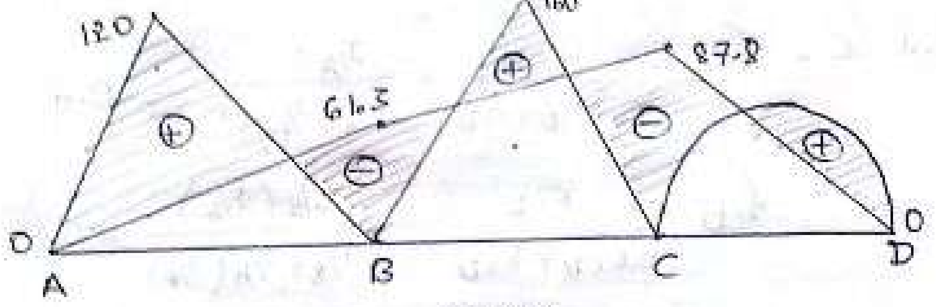
$$\sqrt{}_{CD} = \frac{K_{CD}}{K_{CB}+K_{CD}} = \frac{3/4(I/4)}{I/8+3/4(I/4)} = 0.6$$

Step 3: Moment distribution table:

Joint	A	B		C		D
Members	AB	BA	BC	CB	CD	DC
DF	–	0.5	0.5	0.4	0.6	–
FEM	−124 +124	−4 62	−55.25 $\boxed{-2.75}$	104.75 $\boxed{-44.75}$	40 −20	40 −40
		−1.375 $\boxed{8.95}$	−1.375 −8.95	−17.9 −0.68	−26.85 $\boxed{0.68}$	
		4.475 $\boxed{-0.13}$	4.475 0.13	0.27 2.23	0.41 $\boxed{-2.23}$	

		-0.065	-0.065	-0.892	-1.338	
			-0.446	-0.0325		
		$\boxed{0.446}$			$\boxed{0.0325}$	
		0.223	0.223	0.013	0.02	
			0.006	0.1115		
		$\boxed{-0.006}$			$\boxed{-0.1115}$	
		-0.003	-0.003	-0.045	-0.07	
			-0.0225	-0.0015		
Final moments	0.0	61.3	-61.3	87.8	-87.8	0.0

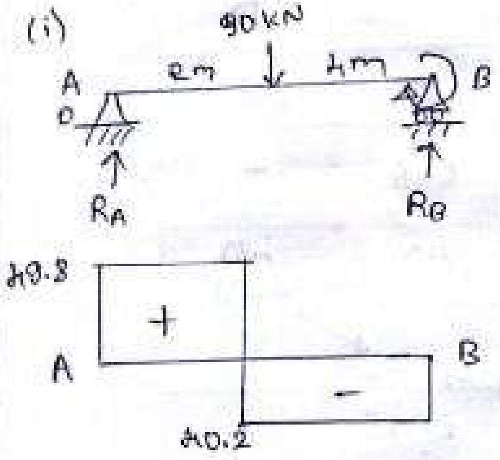




BMD

To find SF :

(i)



$\Sigma M_A = 0$;

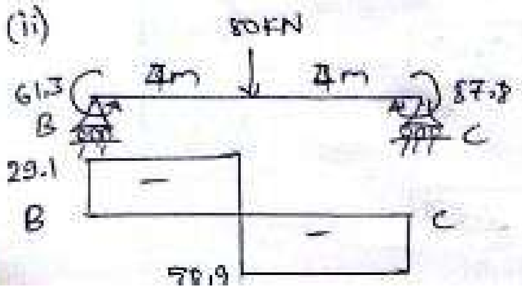
$90 \times 2 - R_B \times 6 + 61.3 = 0$

$R_B = 40.2$

$\Sigma V = 0$; $R_A - 90 + 40.2 = 0$

$R_A = 49.8$

(ii)



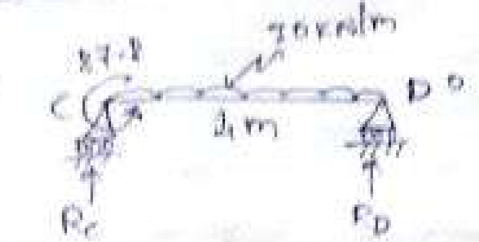
$\Sigma M_B = 0$;

$80 \times 4 + 87.8 - R_C \times 8 - 61.3 = 0$

$\therefore R_C = 50.9$

$\Sigma V = 0$; $R_B = 29.1$

(iii)



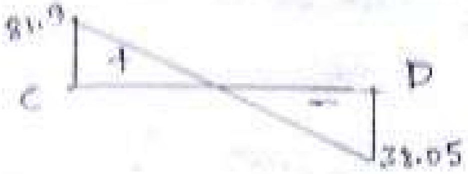
$\Sigma M_c = 0;$

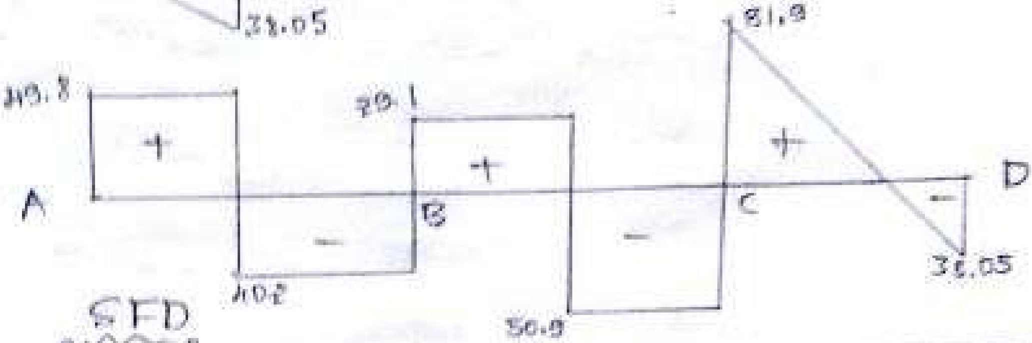
$-87.8 + (30 \times 4) \times 2 - R_D \times 4 = 0$

$\therefore R_D = 38.05$ kN

$\Sigma V = 0;\ R_c - (30 \times 4) + 38.05 = 0$

$R_c = 81.9$ kN





SFD

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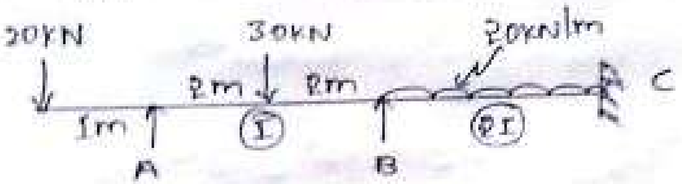
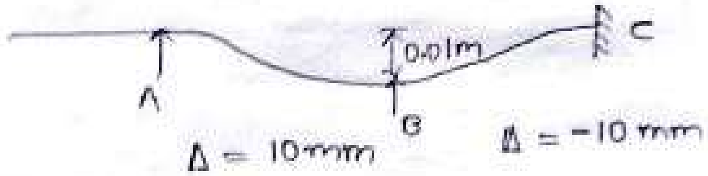
3. Analyse the continuous beam by moment-distribution method. 'B' sinks by 10mm. EI = 4000 kNm²

Draw BMD and SFD.



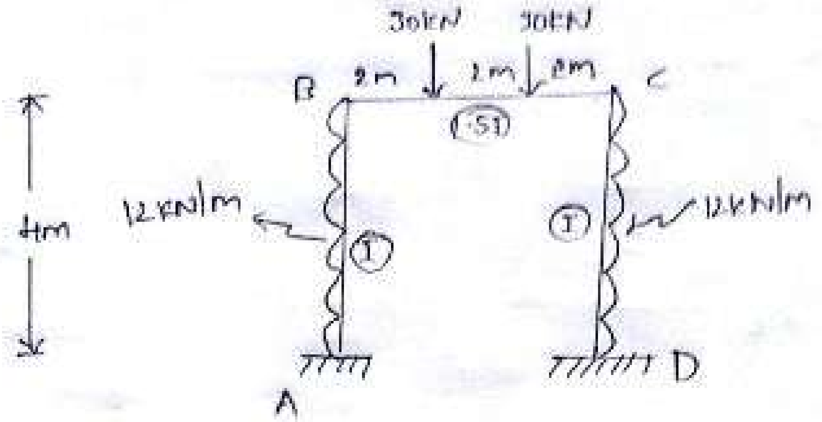
→



07/09/18

Frames:

1. Analyse the portal frame by moment-distribution method. Draw BMD.



→ **Step 1: Fixed End Moments:**

$$M_{FAB} = -\frac{wL^2}{12} = -16\,KNm$$

$$M_{FBA} = \frac{wL^2}{12} = 16\,KNm$$

$$M_{FBC} = \frac{-30\times 2\times 4^2}{6^2} - \frac{30\times 4\times 2^2}{6^2}$$

$$\therefore M_{FBC} = -40\,KNm$$

$$M_{FCB} = \frac{30\times 2^2\times 4}{6^2} + \frac{30\times 4^2\times 2}{6^2}$$

$$\therefore M_{FCB} = 40\,KNm$$

$$M_{FCD} = \frac{-wL^2}{12} = -16\,KNm$$

$$M_{FDC} = 16\,KNm$$

Step 2: DF:

@ Joint 'B', $\delta_{BA} = \frac{K_{BA}}{K_{BA}+K_{BC}} = \frac{I/4}{I/4 + \frac{1.5I}{6}} = 0.5$

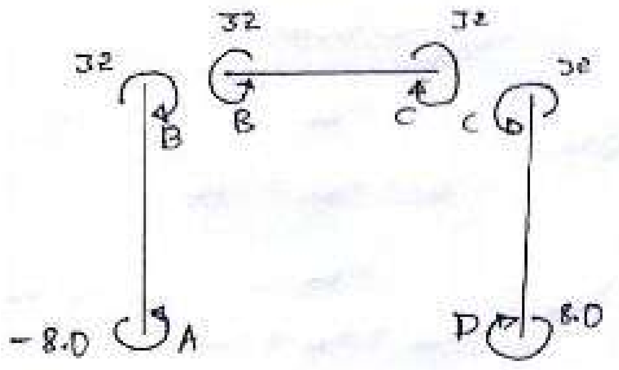
$$\delta_{BC} = \frac{K_{BC}}{K_{BA}+K_{BC}} = 0.5$$

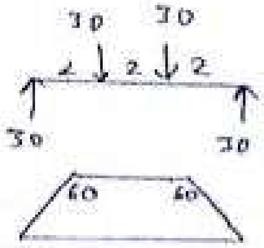
@ Joint 'C', $\delta_{CB} = \frac{K_{CB}}{K_{CB}+K_{CD}} = \frac{1.5I/6}{1.5I/6 + I/4} = 0.5$

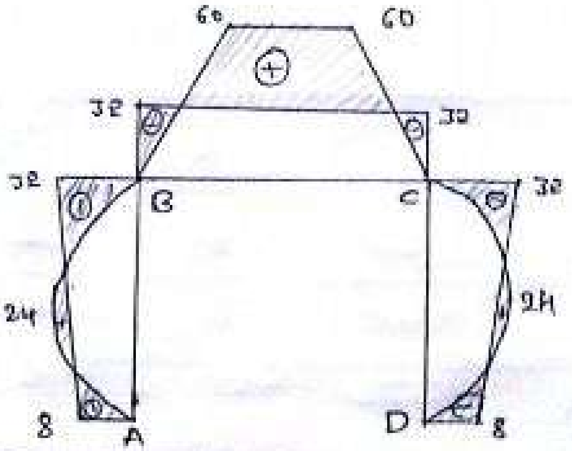
$$\delta_{CD} = \frac{K_{CD}}{K_{CB}+K_{CD}} = 0.5$$

Step 3 : Moment Distribution table :

Joint	A	B		C		D
Members	AB	BA	BC	CB	CD	DC
DF	–	0.5	0.5	0.5	0.5	–
FEM	−16.0	16.0	−40	40	−16	16
		$\boxed{24.0}$		$\boxed{-24}$		
		12.0	12.0	−12.0	12.0	
	6.0		−6.0	6.0		−6.0
		$\boxed{6.0}$		$\boxed{-6.0}$		
		3.0	3.0	−3.0	−3.0	
	1.5		−1.5	1.5		−1.5
		$\boxed{1.5}$		$\boxed{-1.5}$		
		0.75	0.75	−0.75	−0.75	
	−0.375		−0.375	0.375		−0.375
		$\boxed{0.375}$		$\boxed{-0.375}$		
		0.187	0.187	−0.187	−0.187	
	0.09		−0.09	0.09		−0.09
Final Moments	−8.03	32.0	−32.0	32.0	−32.0	8.0

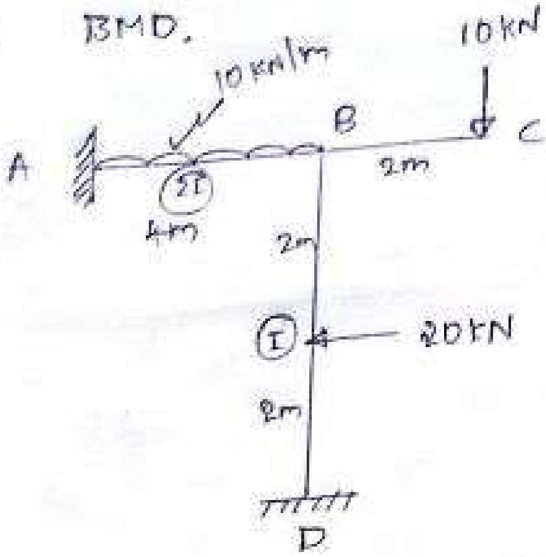






BMD

3. Analyse the frame by moment - distribution -od. Draw BMD.



→ Step 1: Fixed End moments:

$M_{FAB} = \frac{-wL^2}{12} = -13.33 \text{ kN}$; $M_{BC} = -20 \text{ kNm}$

$M_{FBA} = 13.33 \text{ kN}$

$M_{FBD} = \frac{-wL}{8} = -10 \text{ kNm}$

$M_{FDB} = 10 \text{ kNm}$

Step 2: Distribution factor:

@ Joint B, $\sqrt{BA} = \frac{K_{BA}}{K_{BA} + K_{BC} + K_{BD}} = \frac{4I/4}{2I/4 + 0 + I/4}$

$\therefore \sqrt{BA} = 0.67$

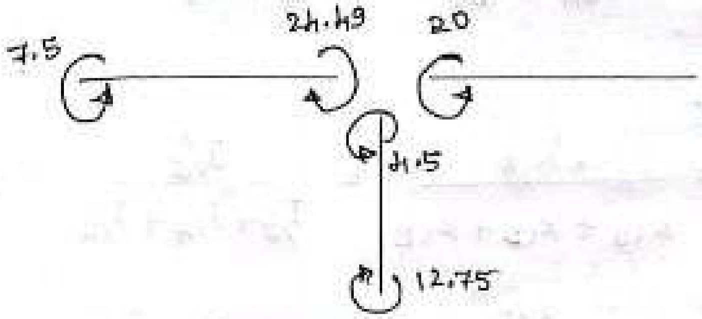
$\sqrt{BC} = \frac{K_{BC}}{K_{BA} + K_{BC} + K_{BD}} = \frac{0}{(\ )} = 0$

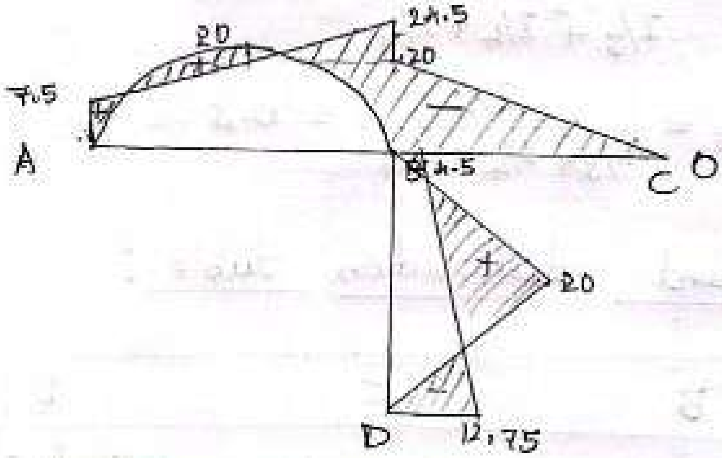
$\sqrt{BD} = \frac{K_{BD}}{K_{BA} + K_{BC} + K_{BD}} = 0.33$

Step 3: MDT:

Joint	A	B			D
Members	AB	BA	BC	BD	DB
DF	—	0.67	0	0.33	—
FEM	−13.33	13.33	−20 $\boxed{16.67}$	−10	10

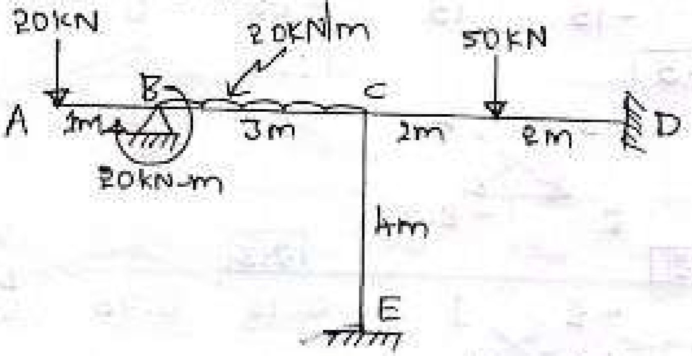
	5.58	11.16	0.0	5.5	2.75
Final moments	-7.5	24.49	-20	-4.5	12.75





BMD

4. Analyse the portal frame by moment distributio method. Draw BMD.



Step 1: Fixed End Moments:

$$M_{FBC} = \frac{-WL^2}{12} = \frac{-20 \times 3^2}{12} = -15\ KN\text{-}m$$

$$M_{FCB} = \frac{WL^2}{12} = 15\ KN\text{-}m$$

$$M_{FCD} = \frac{-WL}{8} = \frac{-50 \times 4}{8} = -25\ KN\text{-}m$$

$$M_{FDC} = \frac{WL}{8} = 25\ KN\text{-}m$$

$M_{FCE} = 0$; $M_{FEC} = 0$; $M_{BA} = 20 KN\text{-}m$

12/09/18

Step 2 : Distribution factor :

@ Joint 'B',

$$D_{BA} = \frac{K_{BA}}{K_{BA} + K_{BC}} = \frac{0}{0 + I/3} = 0$$

$$D_{BC} = \frac{K_{BC}}{K_{BA} + K_{BC}} = \frac{I/3}{0 + I/3} = 1$$

@ Joint 'C',

$$D_{CB} = \frac{K_{CB}}{K_{CB} + K_{CD} + K_{CE}} = \frac{I/3}{I/3 + I/4 + I/4} = 0.4$$

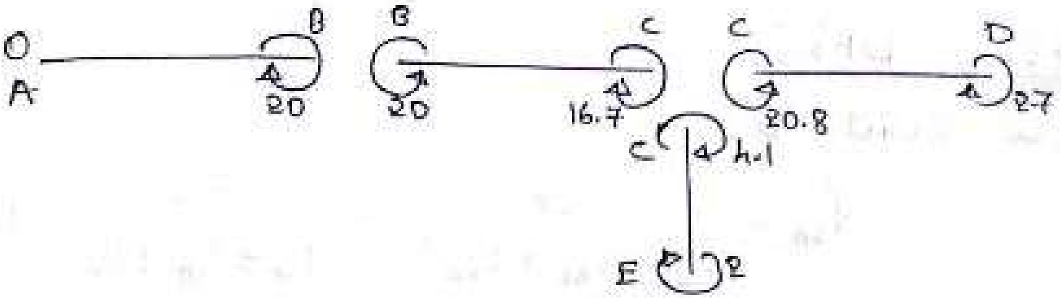
$$D_{CD} = \frac{K_{CD}}{I/3 + I/4 + I/4} = 0.3$$

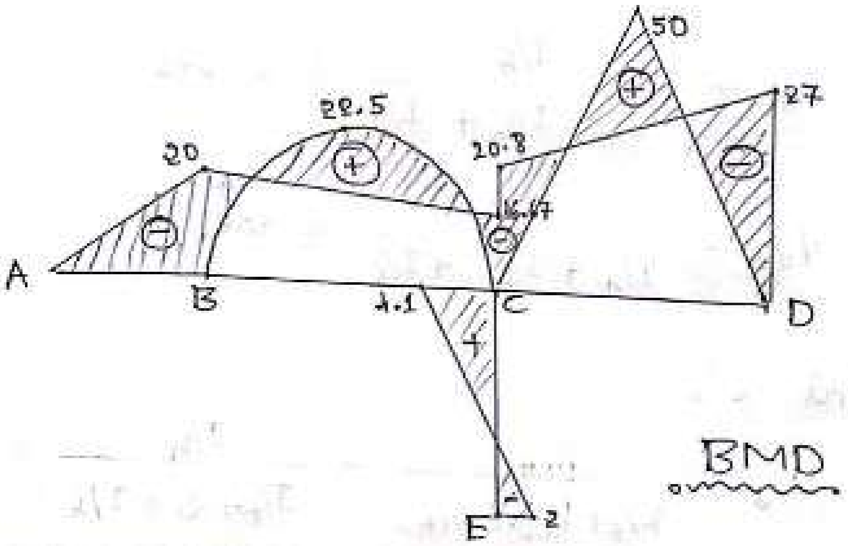
$$D_{CE} = \frac{I/4}{I/3 + I/4 + I/4} = 0.3$$

Step 3 : Moment distribution table :

Joint	B		C			D	E
Members	BA	BC	CB	CD	CE	DE	EC
DF	—	1	0.4	0.3	0.3	—	—
FEM	20	−15 $\boxed{-5}$	15	−25 $\boxed{10}$	0	25	0
	0	−5 2 $\boxed{-2}$	4 −2.5	3 $\boxed{2.5}$	3	1.5	1.5
	0	−2 0.5 $\boxed{-0.5}$	1 −1	0.75 $\boxed{1}$	0.75	0.375	0.375
	0	−0.5 0.2 $\boxed{-0.2}$	0.4 −0.025	0.3 $\boxed{0.025}$	0.3	0.15	0.15
	0	−0.2 −0.05 $\boxed{0.005}$	−0.1 −0.1	0.075 $\boxed{0.1}$	0.075	0.0375	0.0375

	0.0	0.05 0.02	0.04 0.025	0.03	0.03	0.015	0.015
Final Moments	20	−20	16.7	−20.8	4.1	27	2.0





BMD

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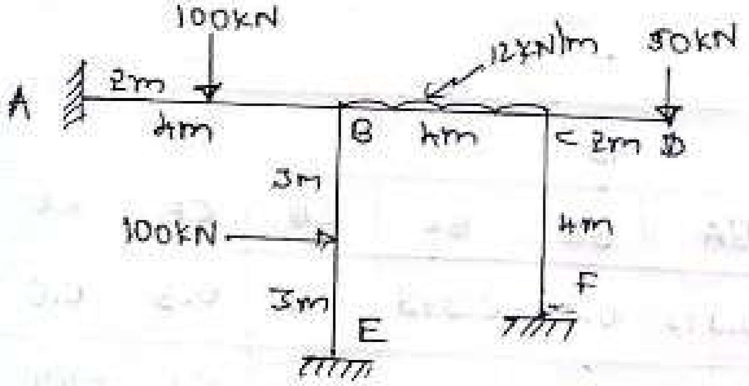
5. Analyse the frame shown in the figure by moment distribution method. Draw BMD.



→ Here $M_{CD} = -50 \times 2 = -100 KNm$

Step 1: $M_{FAB} = \frac{-WL}{8} = -50 KN m$

$M_{FBA} = \frac{WL}{8} = 50 KNm$

$M_{FBC} = \frac{-WL^2}{12} = -16 KNm$

$M_{FCB} = \frac{WL^2}{12} = 16 KNm$

$$M_{FBE} = \frac{100 \times 6}{8} = 75 \text{ KNm}$$

$$M_{FEB} = -75 \text{ KNm}$$

$$M_{FCF} = 0 \ ; \ M_{FFC} = 0$$

Step 2: DF's:

@ Joint 'B',

$$D_{BA} = \frac{K_{BA}}{K_{BA} + K_{BC} + K_{BE}} = \frac{I/4}{I/4 + I/4 + I/6} = 0.375$$

$$D_{BC} = \frac{I/4}{I/4 + I/4 + I/6} = 0.375$$

$$D_{BE} = \frac{I/6}{I/4 + I/4 + I/6} = 0.25$$

@ Joint 'C',

$$D_{CB} = \frac{K_{CB}}{K_{CB} + K_{CD} + K_{CF}} = \frac{I/4}{I/4 + 0 + I/4} = 0.5$$

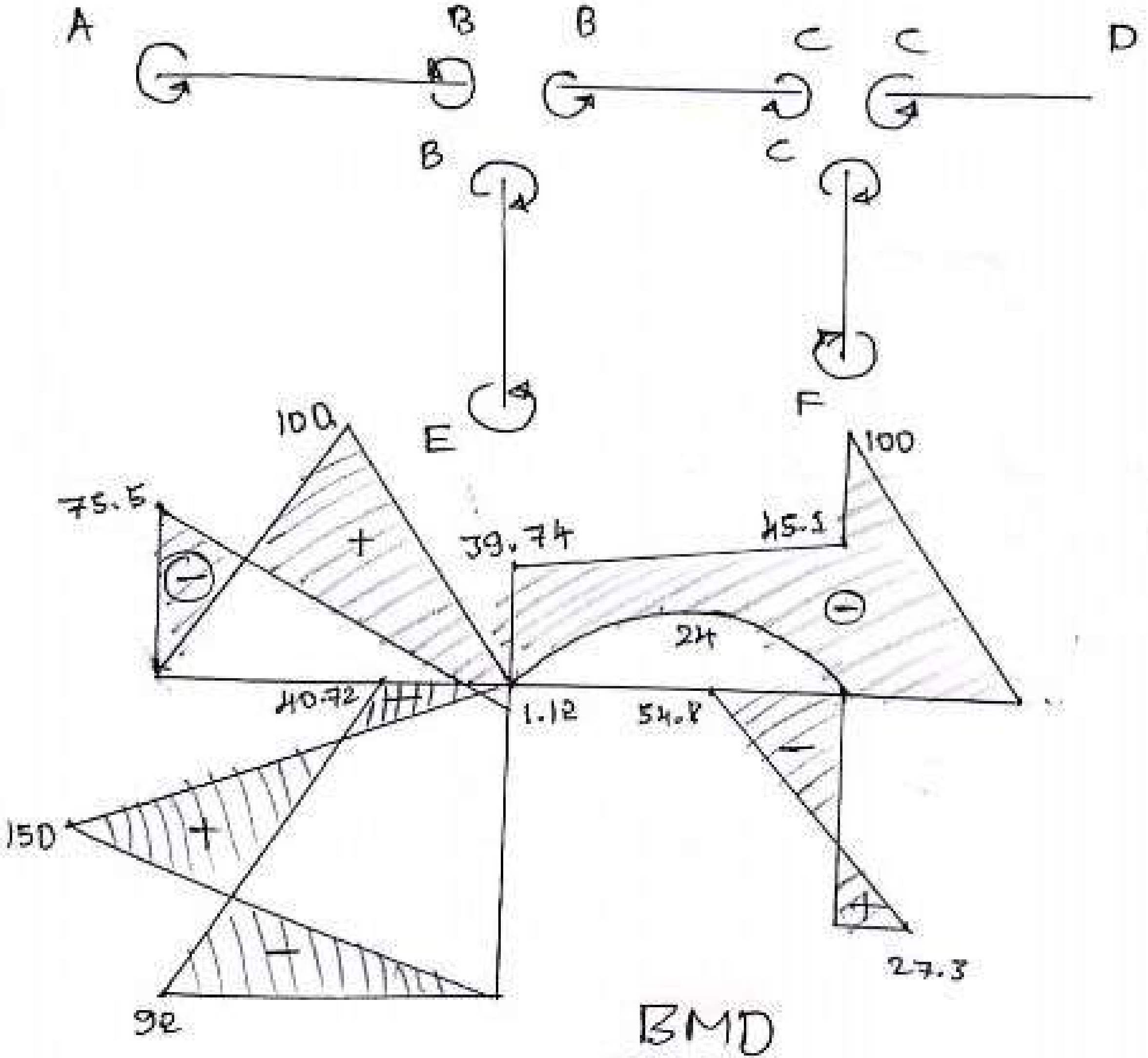
$$D_{CD} = 0$$

$$D_{CF} = 0.5$$

Step 3: MDT:

Joint	A	B			C			E	F
Members	AB	BA	BE	BC	CB	CF	CD	EB	FC
DF	—	0.375	0.25	0.375	0.5	0.5	0.0	—	—
FEM	−50	50	75	−16	16	0.0	−100	−75	0.0
			$\boxed{-109}$			$\boxed{84}$			
		−40.87	−27.25	−40.87	42	42	0		
	−20.47			21	−20.47			−13.625	21
			$\boxed{-21}$			$\boxed{21.43}$			
		−7.87	−5.25	−7.87	10.21	10.21	0.0		
	−3.93			5.105	−3.93			−2.625	5.105
			$\boxed{-5.105}$			$\boxed{3.93}$			

		−1.914	−1.276	−1.914	1.967	1.967	0.0		
	−0.957			0.983	−0.957			−0.638	0.983
			$\boxed{-0.983}$			$\boxed{0.957}$			
		−0.368	−0.245	−0.368	0.48	0.48	0.0		
	−0.184			0.24	−0.184			−0.1225	0.24
			$\boxed{-0.24}$			$\boxed{0.184}$			
		−0.09	−0.06	−0.09	0.092	0.09	0.0		
	−0.045			0.046	−0.045			−0.03	0.045
Final Moments	−75.5	−1.112	40.72	−39.73	45.2	54.8	−100	−92.04	27.37



BMD

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Module-3

SUJITH.N.S
V 'A'

Kani's Method

It is one of the methods to analyse statically indeterminate structure.

It was first introduced by an investigato Grasper Kani, hence known as Kani's rotation method. It is an improved version of momn distribution method. It is easy, faster & quickly compared to other two methods. It is the best suited method for 3 storey building analysis.

A complete one cycle in moment-distributio method is incorporated in a single factor called as rotation factor (RF).

$$RF = -\frac{1}{2}\left[\frac{K}{\Sigma K}\right]$$

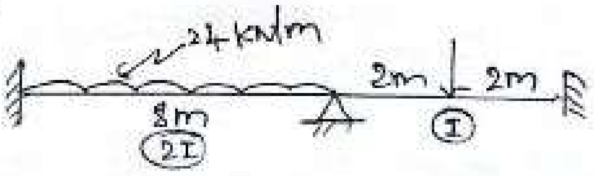
The summation of all RF at a joint = -0.5

Final moment, $M_F - M_{FAB} + 2$ (moment of near end) + 1 (moment of far end)

i.e., $M_F = M_{FAB} + 2\,m_{AB} + m_{BA}$

Problems:

1. Analyse the continuous beam loaded shown in the fig, by Kani's rotation method. Draw BMD and SFD.



Note: Steps in kanis rotation method

Step 1: FEM

Step 2: Calculation of rotation factors

Step 3: Kani's table

Step 4: Final moments

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Step 1: Fixed End moments:

$$M_{FAB} = \frac{-WL^2}{12} = -128 \text{ KN-m}$$

$$M_{FBA} = \frac{WL^2}{12} = 128 \text{ KN-m}$$

$$M_{FBC} = \frac{-WL}{8} = -50 \text{ KN-m}$$

$$M_{FCB} = \frac{WL}{8} = 50 \text{ KN-m}$$

Step 2: Calculation of rotation factors:

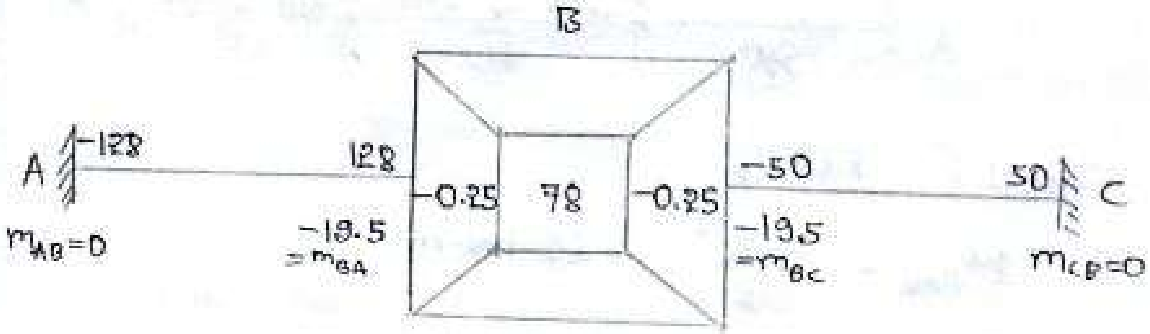
@ Joint 'B'.

$$RF]_{BA} = -\frac{1}{2}\left[\frac{K_{BA}}{K_{BA}+K_{BC}}\right] = -\frac{1}{2}\left[\frac{2I/8}{2I/8 + I/4}\right]$$

$$RF]_{BA} = -0.25$$

$$RF]_{BC} = -\frac{1}{2}\left[\frac{K_{BC}}{K_{BA}+K_{BC}}\right] = -0.25$$

Step 3: Kani's table:



Step 4: Final moments:

$$M_{AB} = M_{FAB} + 2\,m_{AB} + m_{BA}$$

$$= -128 + 0 - 19.5$$

$$\therefore \quad M_{AB} = -147.5 \text{ KN-m}$$

$$M_{BA} = M_{FBA} + 2\,m_{BA} + m_{AB}$$

$$= +128 + 2(-19.5) + 0$$

$$\therefore \quad M_{BA} = 89 \text{ kN-m}$$

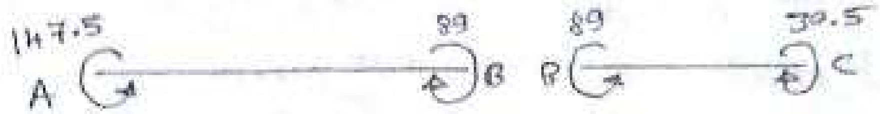
IIIly,

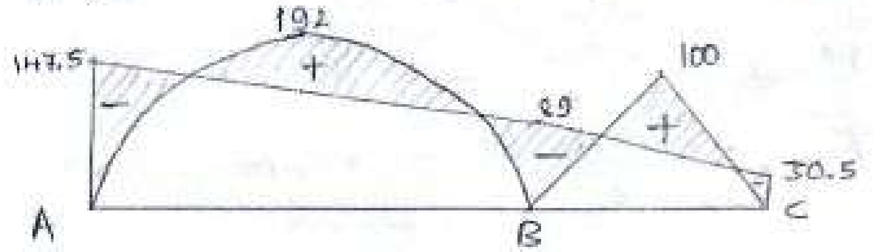
$$M_{BC} = -50 + 2\,(-19.5) + 0$$

$$\therefore \quad M_{BC} = -89 \text{ kN-m}$$

$$M_{CB} = 50 + 2(0) + (-19.5)$$

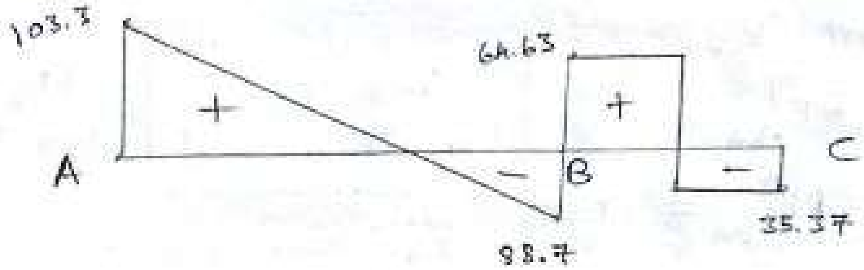
$$\therefore \; M_{CD} = 30.5 \text{ KN-m}$$





BMD

To draw SFD:



SFD

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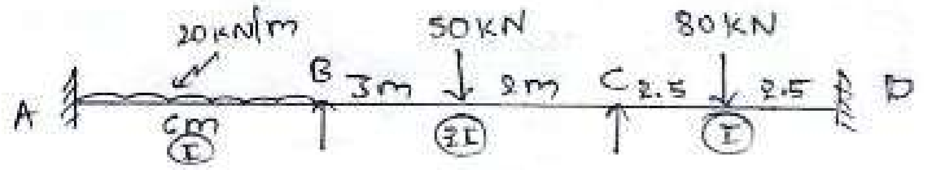
2. Analyse by Kani's rotation method. Draw BMD and SFD and elastic curve.



→ Step 1: FEM:

$$M_{FAB} = \frac{-wL^2}{12} = -60 \text{ KN-m}$$

$$M_{FBA} = \frac{wL^2}{12} = 60 \text{ KN-m}$$

$$M_{FBC} = \frac{-wab^2}{L^2} = -24 \text{ KN-m}$$

$$M_{FCB} = \frac{wa^2b}{L^2} = 36 \text{ KN-m}$$

$$M_{FCD} = \frac{-wL}{8} = -50 \text{ KN-m}$$

$$M_{FDC} = \frac{wL}{8} = 50 \text{ KN-m}$$

Step 2: Calculation of RF:

@ Joint 'B', $RF\Big]_{BA} = -\frac{1}{2}\left[\frac{K_{BA}}{K_{BA}+K_{BC}}\right] = -\frac{1}{2}\left[\frac{I/6}{I/6 + 2I/5}\right.$

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$\therefore\ RF]_{BA} = -0.15$

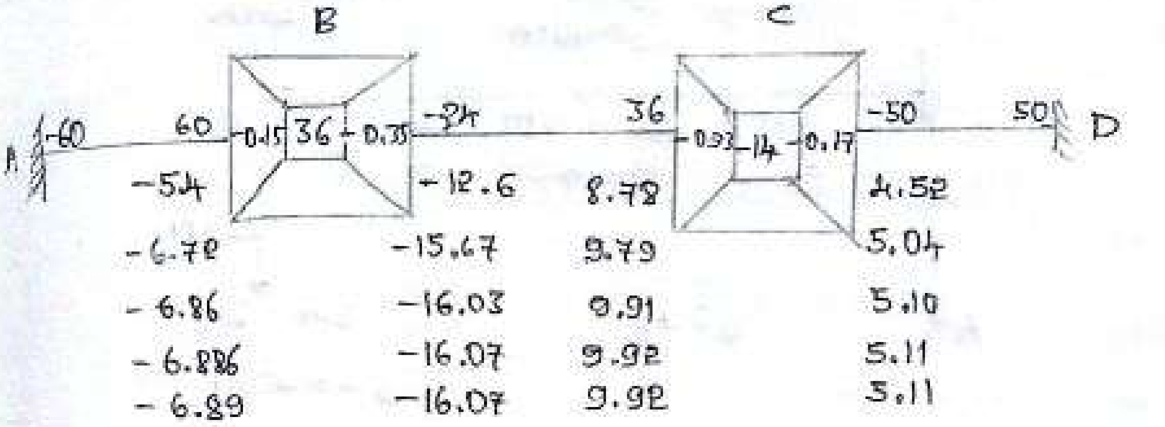
Now, $RF]_{BC} = -\frac{1}{2}\left[\frac{2I/5}{I/6 + 2I/5}\right] = -0.35$

@ Joint 'C',

$$RF]_{CB} = -\frac{1}{2}\left[\frac{2I/5}{2I/5 + I/5}\right] = -0.33$$

$$RF]_{CD} = -\frac{1}{2}\left[\frac{I/5}{2I/5 + I/5}\right] = -0.17$$

Step-3: Kani's table:



$m_{AB}=0$ $m_{BA}=$ $m_{BC}=$ $m_{CB}=$ $m_{CD}=$ $m_{DC}=0$

Step 4: Final Moments:

$$M_{AB} = M_{FAB} + 2(m_{AB}) + m_{BA}$$
$$= -60 - 6.89 = -66.89 \text{ KN-m}$$

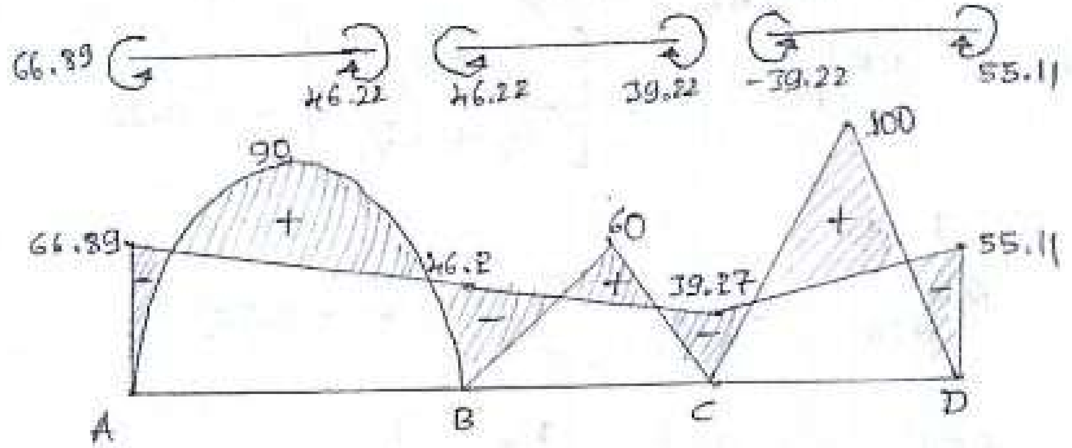
$$M_{BA} = 60 + 2(-6.80) + 0$$
$$= 46.22 \text{ KN-m}$$

$$M_{BC} = -24 + 2(-16.07) + 9.92$$
$$= -46.22 \text{ KN-m}$$

$$M_{CB} = 36 + 2(9.92) - 16.07$$
$$= 39.77 \text{ KN-m}$$

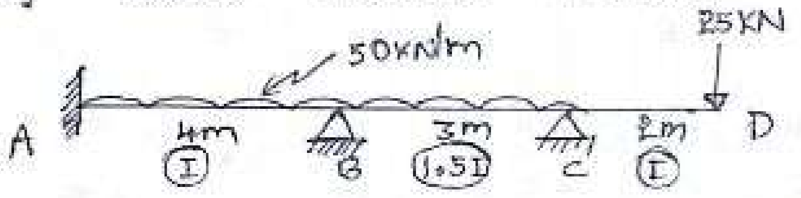
$$M_{CD} = -50 + 2(5.11) + 0$$
$$= -39.77 \text{ KN-m}$$

$$M_{DC} = 50 + 2(0) + 5.11$$
$$= 55.11 \text{ KN-m}$$

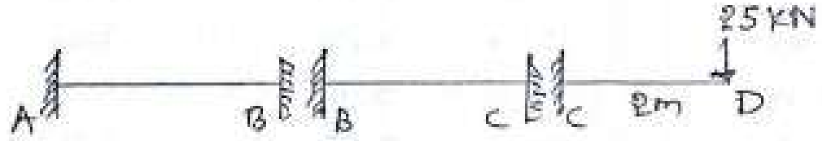


BMD.

3. Analyse the continuous beam shown in the figure by Kani's rotation method.



→



$M_{CD} = -50 KN\text{-}m$

Step 1: Fixed End Moment's:

$$M_{FAB} = -\frac{wL^2}{12} = -66.67 \text{ KN-m}$$

$$M_{FBA} = +66.67 \text{ KN-m}$$

$$M_{FBC} = -\frac{wL^2}{12} = -\frac{50\times3^2}{12} = -37.5 \text{ KN-m}$$

$$M_{FCB} = +37.5 \text{ KN-m}$$

$$M_{CD} = -50 \text{ KN-m}$$

Step 2: Calculation of Rotation factor:

@ Joint 'B',

$$RF]_{BA} = -\frac{1}{2}\left[\frac{K_{BA}}{K_{BA}+K_{BC}}\right] = -\frac{1}{2}\left[\frac{I/4}{I/4+1.5I/3}\right] = -0.167$$

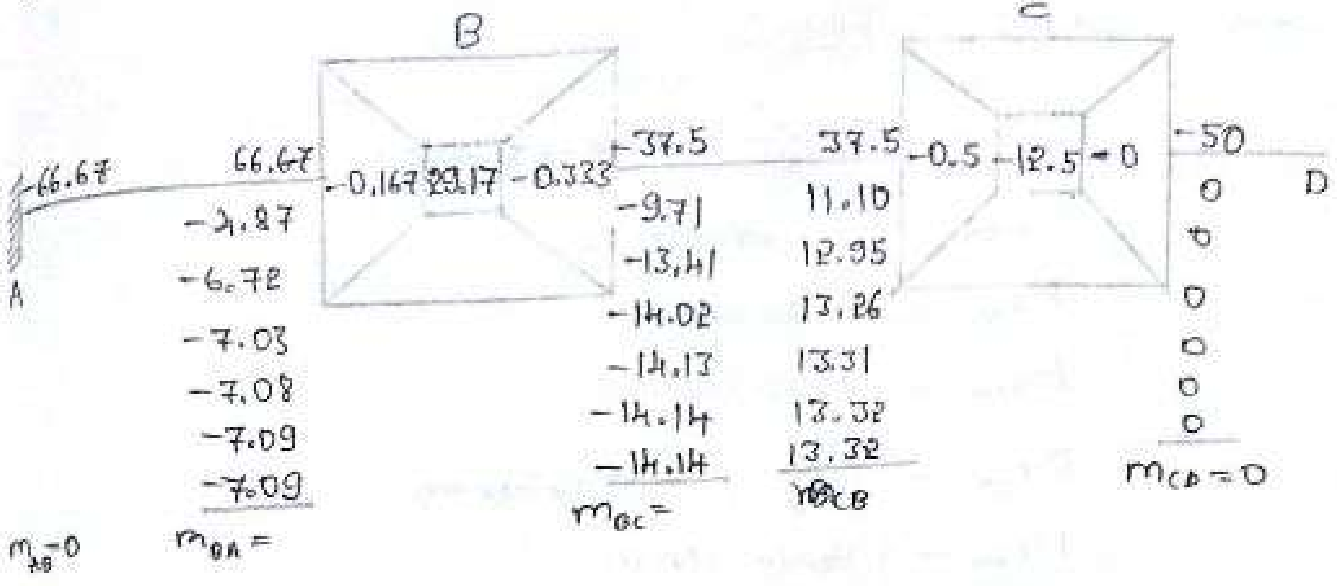
$$RF]_{BC} = -\frac{1}{2}\left[\frac{I/2}{I/4+1.5I/3}\right] = -0.333$$

@ Joint 'C',

$$RF]_{CB} = -1/2\left[\frac{1.5I/3}{1.5I/3+0}\right] = -0.5$$

$RF]_{CD} = 0$

Step 3: Kani's table:



Step 4: Final Moments:

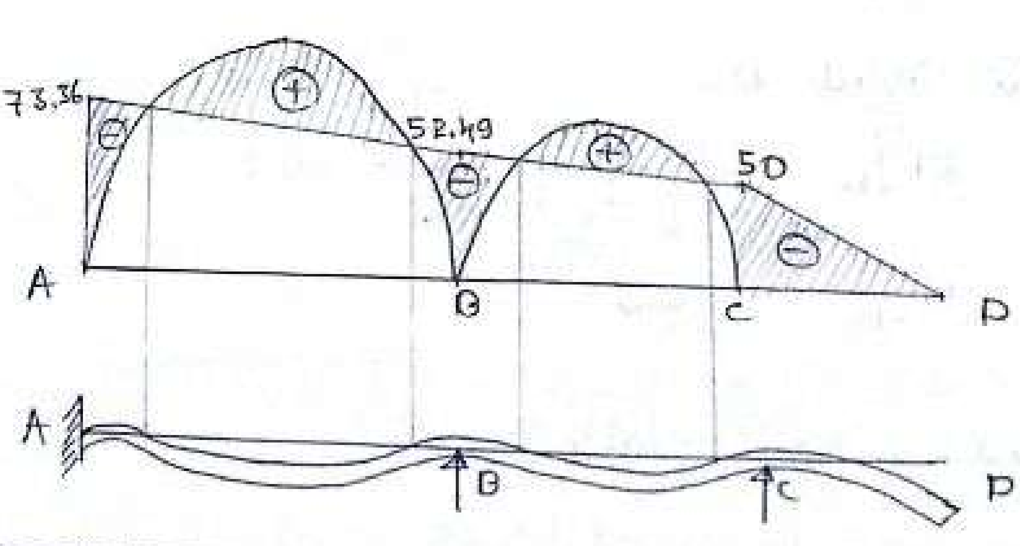
$$M_{AB} = -66.67 + 2(0) - 7.09$$
$$= -73.76 \text{ kN-m}$$

$$M_{BA} = 66.67 + 2(-7.09) + 0$$
$$= 52.49 \text{ kN-m}$$

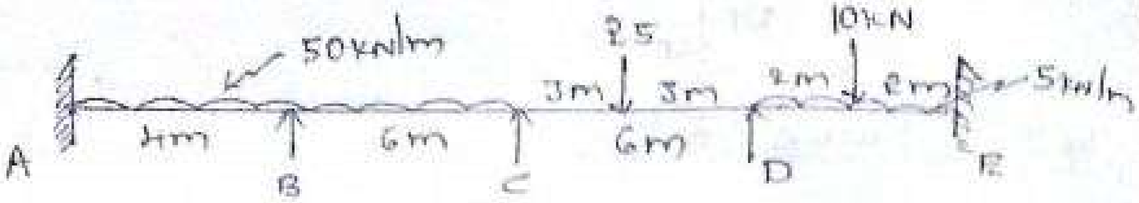
$$M_{BC} = -37.5 + 2(-14.14) + 13.32$$
$$= -52.5 \text{ kN-m}$$

$$M_{CB} = 37.5 + 2(13.32) - 14.14$$
$$= 50 \text{ kN-m}$$

$$M_{CD} = -50 \text{ kN-m}$$



4.



→ **Step 1 : FEM :**

$$M_{FAB} = \frac{-wL^2}{12} = -66.67 \text{ kNm}$$

$$M_{FBA} = +66.67 \text{ kN-m}$$

$$M_{FBC} = -150 \text{ kN-m}$$

$$M_{FCB} = +150 \text{ kN-m}$$

$$M_{FCD} = \frac{-WL}{8} = -18.75 \text{ kN-m}$$

$$M_{FDC} = +18.75 \text{ kN-m}$$

$$M_{FDE} = \frac{-wL^2}{12} - \frac{WL}{8} = \frac{-5 \times 4^2}{12} - \frac{10 \times 4}{8} = -11.67$$

$$M_{FED} = +11.67 \text{ kN-m}$$

Step 2 : RF's :

@ Joint 'B',

$$RF]_{BA} = -\frac{1}{2}\left[\frac{K_{BA}}{K_{BA}+K_{BC}}\right] = -\frac{1}{2}\left[\frac{I/4}{I/4+I/6}\right] = -0.3$$

$$RF]_{BC} = -0.2$$

@ Joint 'C',

$$RF]_{CB} = -\frac{1}{2}\left[\frac{I/6}{I/6+I/6}\right] = -0.25$$

$$RF]_{CD} = -0.25$$

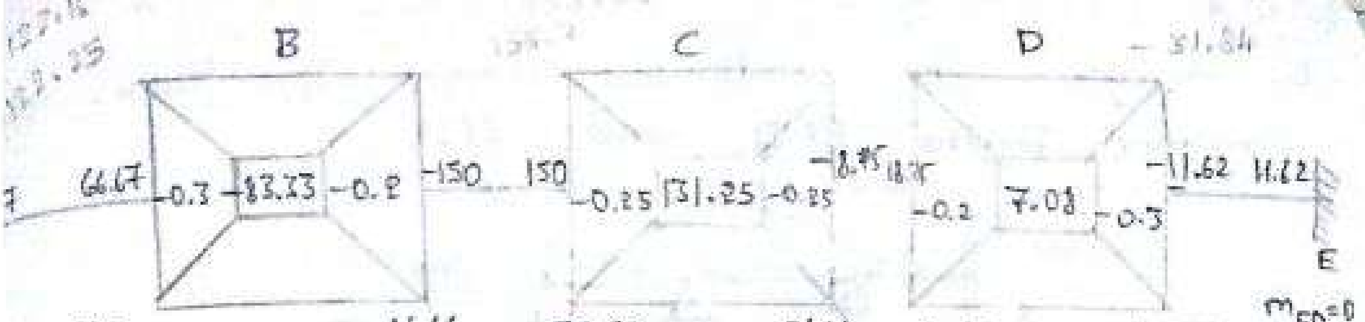
@ Joint 'D',

$$RF]_{DC} = -\frac{1}{2}\left[\frac{I/6}{I/6+I/4}\right] = -0.2$$

$$RF]_{DE} = -0.3$$

Step 3 : Kani's table :

P.T.O →



0	25	16.66	-36.98	-36.98	5.98	8.97
	36.09	24.06	-38.83	-38.83	6.35	9.52
	36.65	24.43	-38.92	-38.92	6.37	9.55
	36.67	24.45	-38.92	-38.92	6.37	9.55
	36.67	24.45	-38.92	-38.92	6.37	9.55
	m_{BA}	m_{BC}	m_{CB}	m_{CD}	m_{DC}	m_{DE}

Step 4: Final Moments:

$$M_{AB} = -66.7 + 2(0) + 36.67 = -30.03 \text{ KNm}$$

$$M_{BA} = 66.7 + 2(36.67) + 0 = 140.04 \text{ KNm}$$

$$M_{BC} = -150 + 2(24.45) - 38.92 = -140.02 \text{ KNm}$$

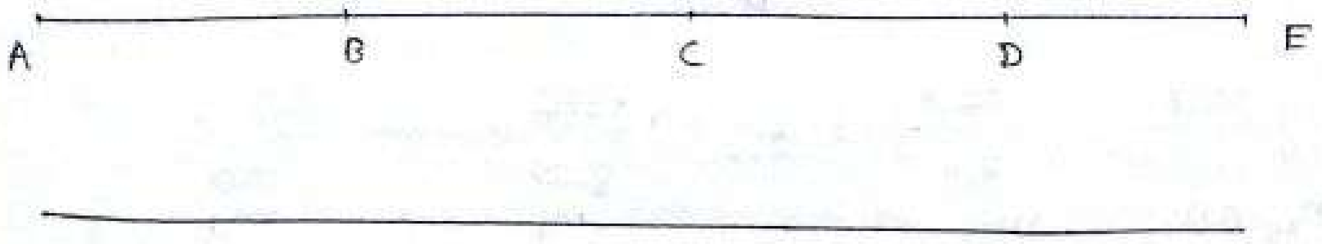
$$M_{CB} = 150 + 2(-38.92) + 24.45 = 96.61 \text{ KNm}$$

$$M_{CD} = -18.75 + 2(-38.92) + 6.37 = -90.22 \text{ KNm}$$

$$M_{DE} = 18.75 + 2(6.37) - 38.92 = -7.43 \text{ KNm}$$

$$M_{DE} = -11.67 + 2(9.55) + 0 = 7.43 \text{ KNm}$$

$$M_{ED} = 11.67 + 2(0) + 9.55 = 21.22 \text{ KNm}$$



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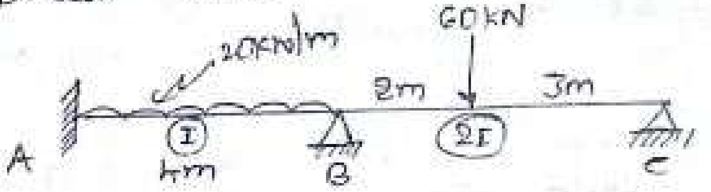
5. Analyse the continuous beam by kani's rotation method. Draw BMD and SFD.



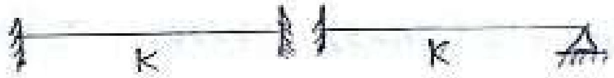
→ Step 1 : FEM's :

$$M_{FAB} = \frac{-WL^2}{12} = -26.67 \text{ kNm}$$

$$M_{FBA} = 26.67 \text{ KNm}$$

$$M_{FBC} = \frac{-wab^2}{L^2} = \frac{-60 \times 2 \times 3^2}{5^2} = -43.2 \text{ kNm}$$

$$M_{FCB} = \frac{60 \times 2^2 \times 3}{5^2} = 28.8 \text{ KNm}$$



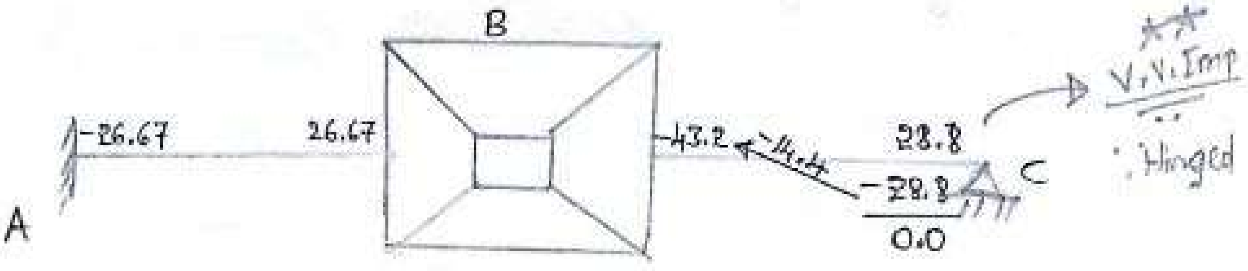
Step 2 : RF's :

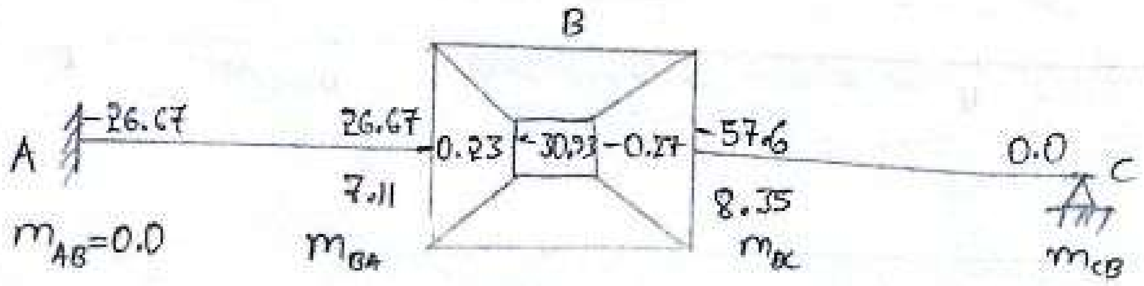
@ Joint 'B'.

$$RF]_{BA} = -\frac{1}{2}\left[\frac{k_{BA}}{k_{BA}+k_{BC}}\right] = -\frac{1}{2}\left[\frac{I/4}{I/4 + 3/4\,(2I/5)}\right] = -0.23$$

$$RF]_{BC} = -\frac{1}{2}\left[\frac{3/4\left(\frac{2I}{5}\right)}{I/4 + \frac{3}{4}\left(\frac{2I}{5}\right)}\right] = -0.27$$

Step 3 : Kani's table :





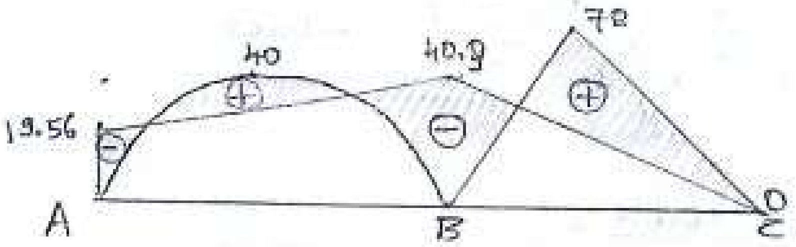
Step 4: Final Moments:

$M_{AB} = -26.67 + 2(0) + 7.11 = -19.56$ KNm

$M_{BA} = 26.67 + 2(7.11) + 0 = 40.89$ KNm

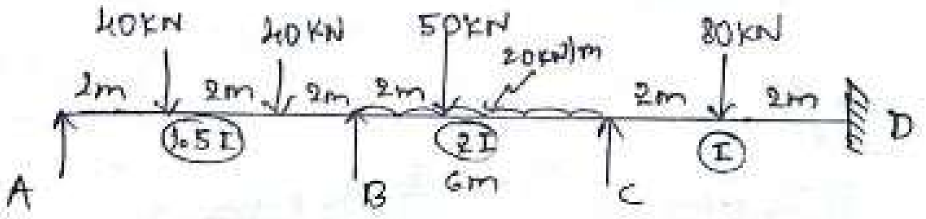
$M_{BC} = -$ ~~4~~ $57.6 + 2(8.35) + 0 = -40.9$ KNm

$M_{CB} = 0.0$ KNm



BMD

6. Analyse the beam by Kani's rotation method.



→ Step 1: FEM's:

$$M_{FAB} = \frac{-40\times2\times4^2}{6^2} - \frac{40\times4\times2^2}{6^2} = -53.33 \text{ kNm}$$

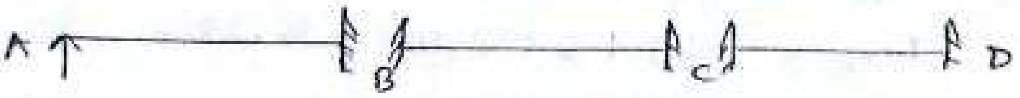
$$M_{FBA} = \frac{+40\times2\times4^2}{6^2} + \frac{40\times4\times2^2}{6^2} = 53.33 \text{ kNm}$$

$$M_{FBC} = \frac{-wL^2}{12} - \frac{Wab^2}{L^2} = \frac{-20\times6^2}{12} - \frac{50\times2\times4^2}{6^2} = -104.4$$

$$M_{FCB} = \frac{20\times6^2}{12} + \frac{50\times2^2\times4}{6^2} = 82.22 \text{ kNm}$$

$$M_{FCD} = \frac{-80\times4}{8} = -40 \text{ KNm}$$

$$M_{FDC} = 40 \text{ KNm}$$



Step 2: RF's:

@ Joint 'B',

$$RF]_{BA} = -\frac{1}{2}\left[\frac{3/4\left(\frac{1.5I}{6}\right)}{\frac{3}{4}\left(\frac{1.5I}{6}\right)+\frac{2I}{6}}\right] = -0.18$$

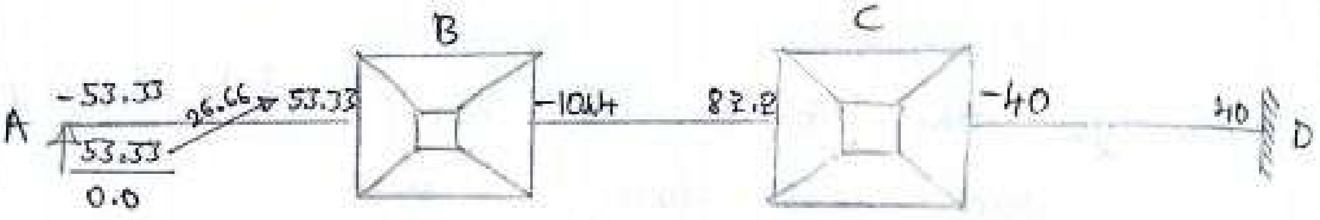
$$RF]_{BC} = -\frac{1}{2}\left[\frac{2I/6}{\frac{3}{4}\left(\frac{1.5I}{6}\right)+\frac{2I}{6}}\right] = -0.32$$

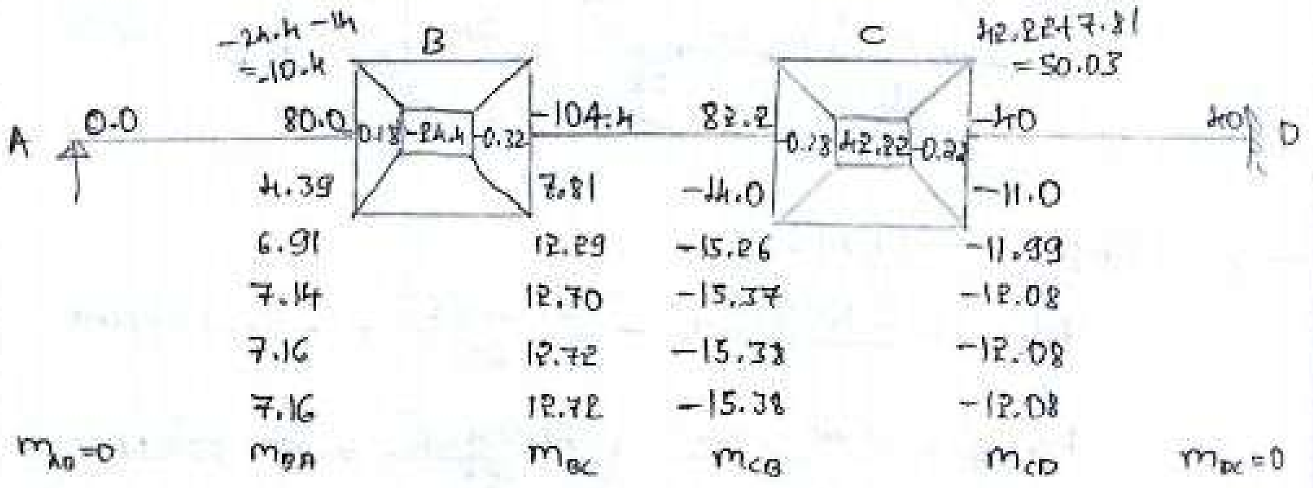
@ Joint 'C',

$$RF]_{CB} = -\frac{1}{2}\left[\frac{(2I/6)}{\frac{2I}{6}+I/4}\right] = -0.28$$

$$RF]_{CD} = -\frac{1}{2}\left[\frac{I/4}{2I/6+I/4}\right] = -0.22$$

Step 3: Kani's table:





Step 4: Final Moments:

$M_{AB} = 0$

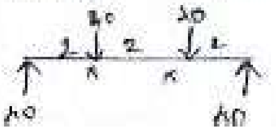
$M_{BA} = 80 + 2(7.16) + 0 = 94.3$ KNm

$M_{BC} = -104.4 + 2(12.72) - 15.38 = -94.3$ KNm

$M_{CB} = 82.22 + 2(-15.38) + 1(12.70) = 64.18$ KNm

$M_{CD} = -40 + 2(12.08) - 15.38 = -64.18$ KNm

$M_{DC} = 27.92$ KNm

BM ⇒ 

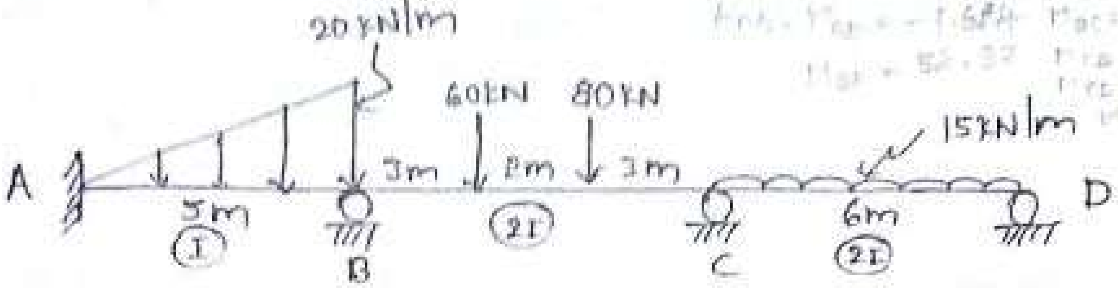
$BM]_x = 80$; 

$\frac{wl^2}{8} + \frac{wab}{l}$

BMD

Ans. $M_{AB} = -1.684$ $M_{BC} = -52.37$
$M_{BA} = 52.37$ $M_{CB} = 31$
$M_{CD} = -31$
$M_{DC} = 0$

7.



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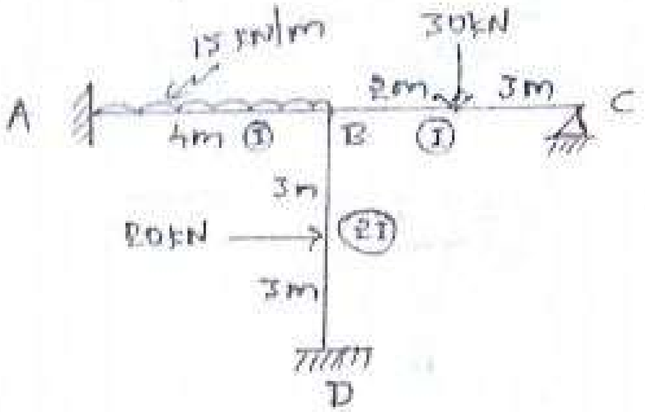
Frames :

1. Analyse the frame by Kani's rotation method.



→ Step I : FEM's :

$$M_{FAB} = \frac{-wl^2}{12} = -20 \text{ KNm}$$

$$M_{FBA} = \frac{wl^2}{12} = 20 \text{ KNm}$$

$$M_{FBC} = \frac{-30 \times 2 \times 3^2}{5^2} = -21.6 \text{ kNm}$$

$$M_{FCB} = \frac{30 \times 2^2 \times 3}{5^2} = 14.4 \text{ KNm}$$

$$M_{FBD} = \frac{20 \times 6}{8} = 15 \text{ kNm}$$

$$M_{FDB} = -15 \text{ KNm}$$

Step 2 : RF's :

@ Joint B,

$$RF\rfloor_{BA} = -\frac{1}{2}\left[\frac{I/4}{I/4 + 3/4(I/5) + 2I/6}\right]$$

$$RF\rfloor_{BA} = -0.17$$

$$RF\rfloor_{BC} = -\frac{1}{2}\left[\frac{3/4(I/5)}{I/4 + 3/4(I/5) + 2I/6}\right]$$

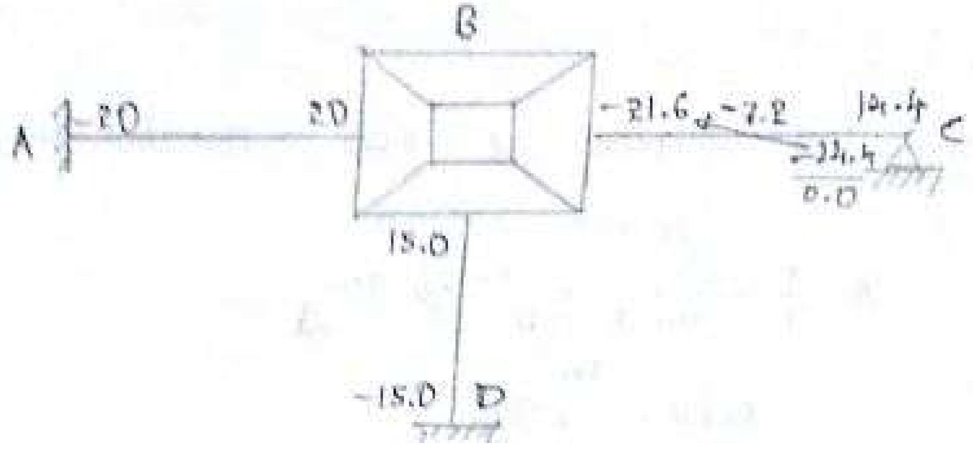
$$\therefore RF\rfloor_{BC} = -0.10$$

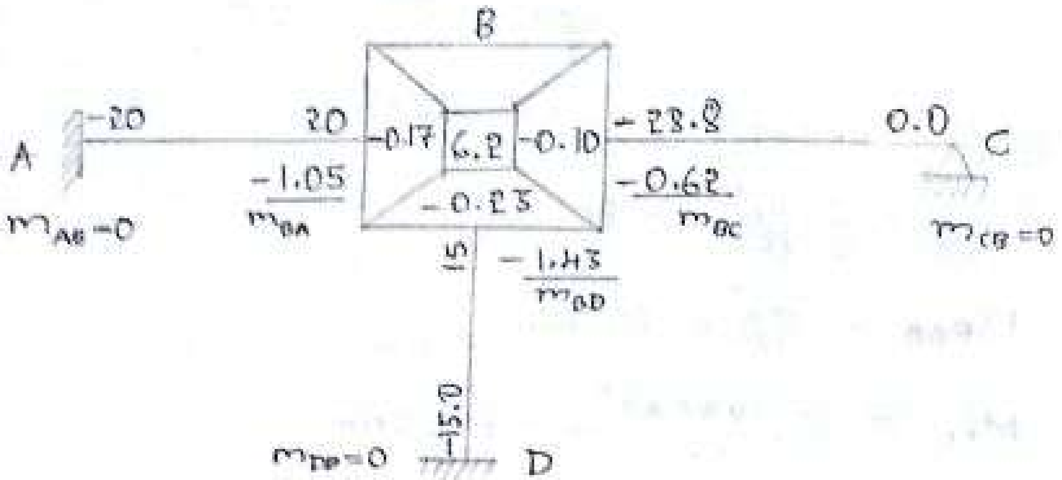
$$RF\rfloor_{BD} = -\frac{1}{2}\left[\frac{2I/6}{I/4 + 3/4(I/5) + 2I/6}\right]$$

$$RF\rfloor_{BD} = -0.23$$

Step 3 : Kani's table :

P.T.O →





Step 4: Final Moments:

$M_{AB} = -20+2(0)+(-1.05) = -21.05$ kNm

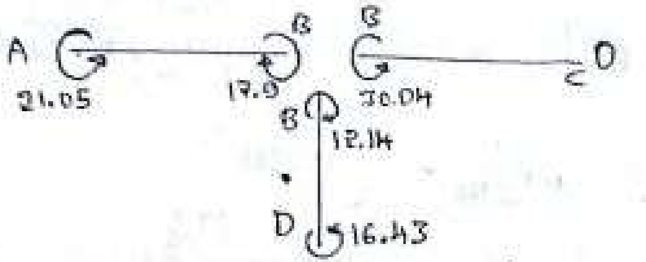
$M_{BA} = 20+2(-1.05)+0 = 17.0$ kN-m

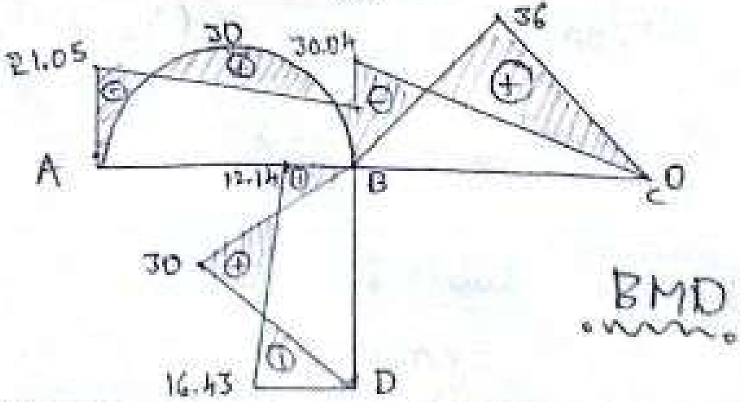
$M_{BC} = -28.8+2(-0.62)+0 = -30.04$ kNm

$M_{CB} = 0$

$M_{BD} = 15+2(-1.43)+0 = 12.14$ kN-m

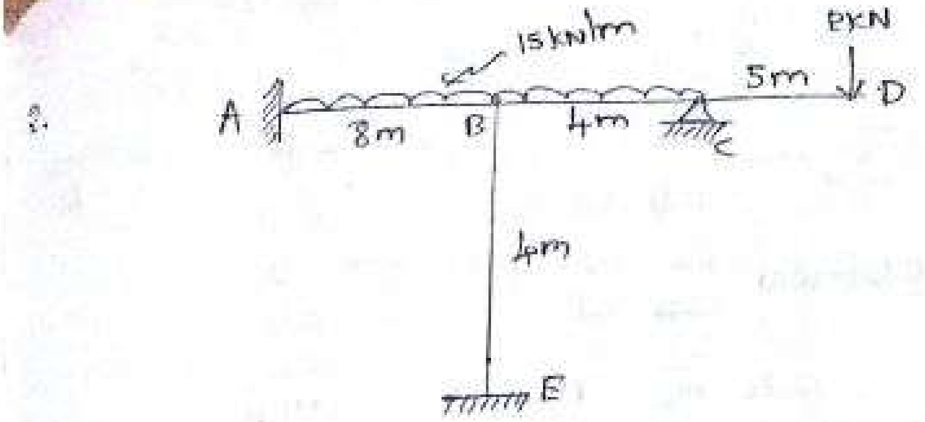
$M_{DB} = -15+2(0)-1.43 = -16.43$ kNm





BMD

2.



Here $\cdot$ $M_{CD} = -10\,KNm$

Step 1 : FEM's :

$$M_{FAB} = -\frac{WL^2}{12} = -80\,KNm$$

$$M_{FBA} = +80\,KNm$$

$$M_{FBC} = \frac{-WL^2}{12} = -20\,KNm$$

$$M_{FCB} = 20\,KNm$$

$$M_{CD} = -10\,KNm$$

$$M_{FBE} = 0 \;;\quad M_{FEB} = 0$$

Step 2 : RF's :

@ 'B', $RF\big]_{BA} = -\frac{1}{2}\left[\frac{I/8}{I/8 + I/4 + I/4}\right] = -0.1$

$$RF\big]_{BC} = -\frac{1}{2}\left[\frac{I/4}{I/8 + I/4 + I/4}\right] = -0.2$$

$$RF\big]_{BE} = -\frac{1}{2}\left[\frac{I/4}{I/8 + I/4 + I/4}\right] = -0.2$$

@ Joint 'C',

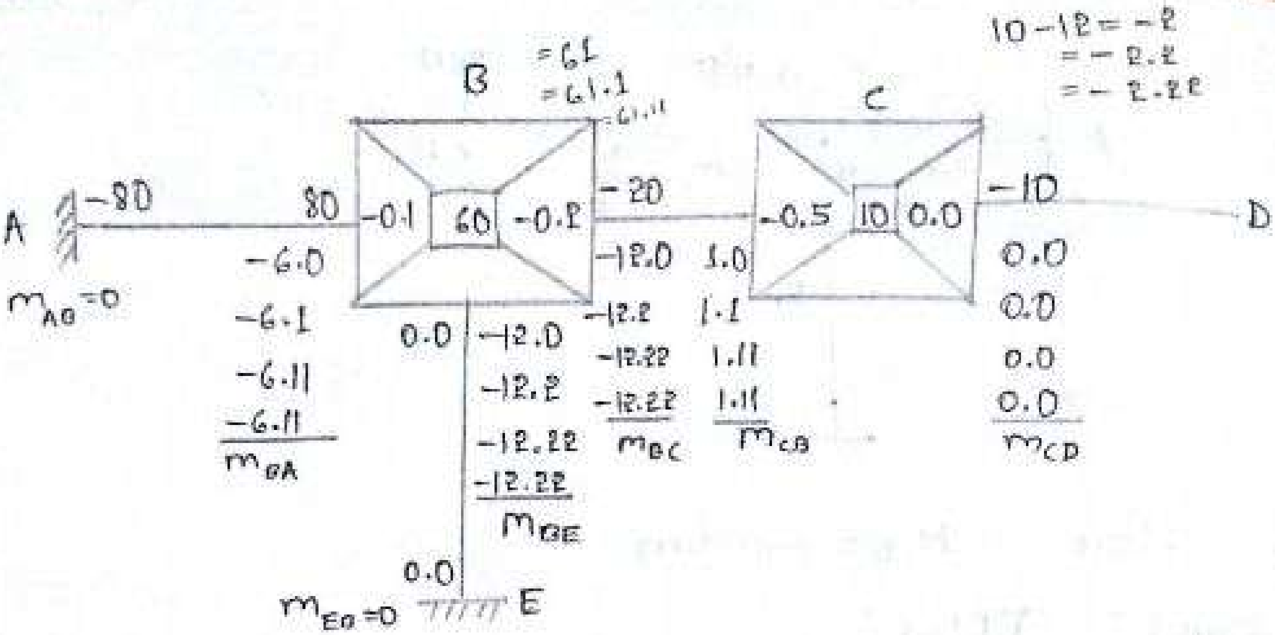
$$RF\big]_{CB} = -\frac{1}{2}\left[\frac{I/4}{I/4 + 0}\right] = -0.25$$

$$RF\big]_{CD} = 0$$

Step 3 :

Kani's rotation table :

P.T.O →



Step 4 : Final Moments :

$$M_{AB} = -80 + 2(0) + (-6.11) = -86.11 \text{ KNm}$$

$$M_{BA} = 80 + 2(-6.11) + 0 = 67.78 \text{ KNm}$$

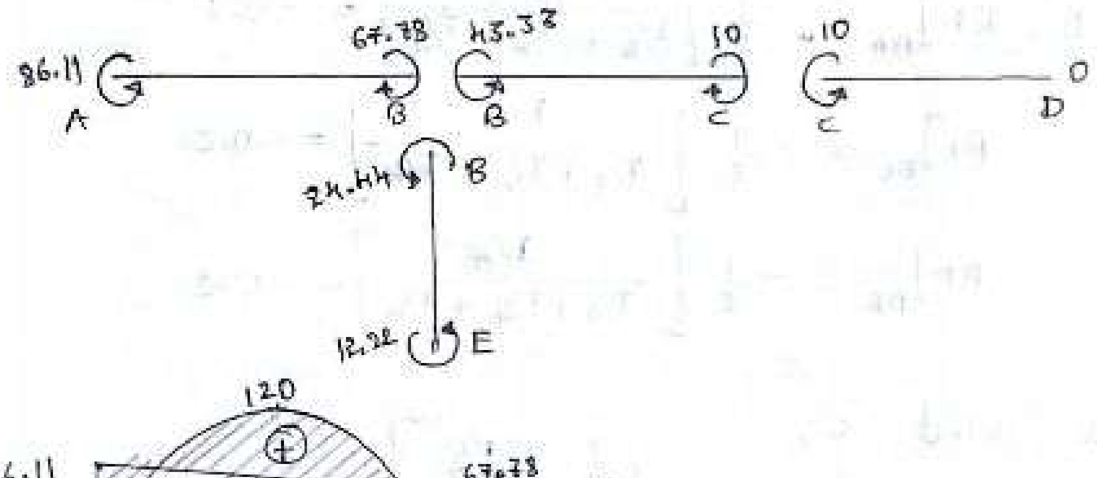
$$M_{BC} = -20 + 2(-12.22) + 1.11 = -43.33 \text{ KNm}$$

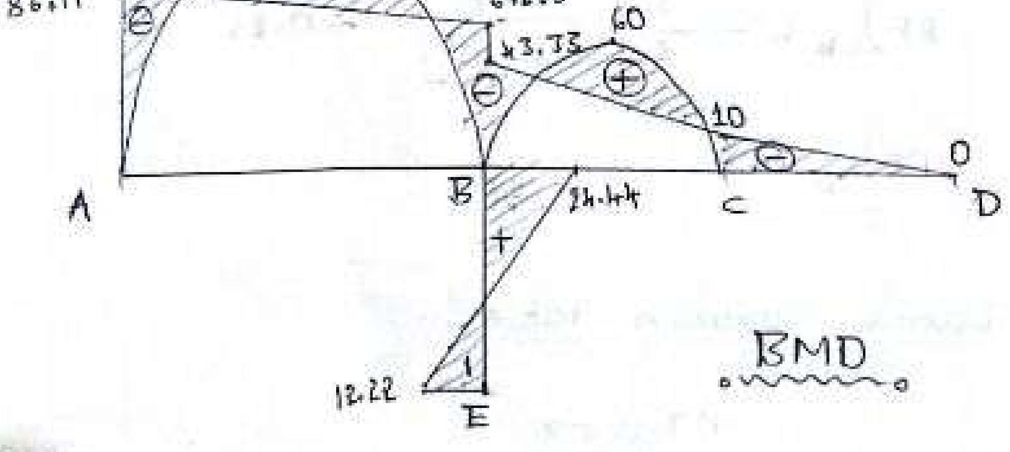
$$M_{CB} = 20 + 2(1.11) - 12.22 = 10 \text{ KNm}$$

$$M_{CD} = -10 \text{ KNm}$$

$$M_{BE} = 0.0 + 2(-12.22) + 0 = -24.44 \text{ KNm}$$

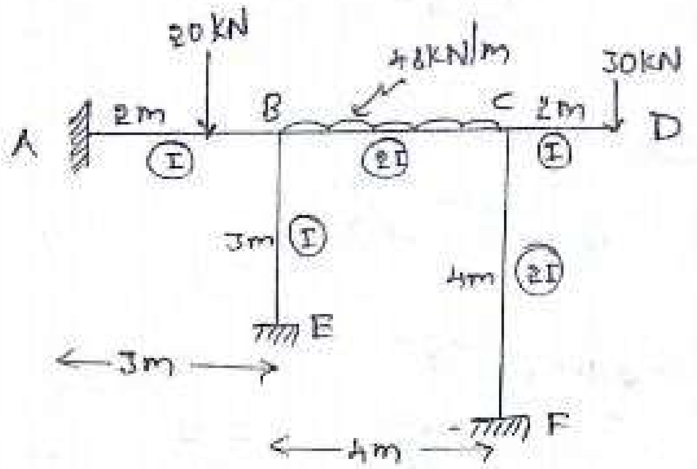
$$M_{EB} = 0 + 2(0) - 12.22 = -12.22 \text{ KNm}$$





04/10/18

3. Analyse the rigid frame by Kani's rotation method. Draw BMD.



→ Step 1 : FEM :

$$M_{FAB} = \frac{-wab^2}{L^2} = \frac{-20 \times 2 \times 1^2}{9} = -4.44 \text{ KNm}$$

$$M_{FBA} = \frac{wa^2b}{L^2} = \frac{20 \times 2^2 \times 1}{3^2} = +8.89 \text{ KNm}$$

$$M_{FBC} = \frac{-wL^2}{12} = \frac{-48 \times 4^2}{12} = -64 \text{ KNm}$$

$$M_{FCB} = +64 \text{ KNm}$$

$$M_{CD} = -60 \text{ KNm}$$

$M_{FBE} = 0$; $M_{FEB} = 0$

$M_{FCF} = 0$; $M_{FFC} = 0$

Step 2 : Calculation of RF's :

@ Joint 'B',

$$RF]_{BA} = -\frac{1}{2}\left[\frac{I/3}{I/3 + 2I/4 + I/3}\right] = -0.143$$

$$RF]_{BC} = -\frac{1}{2}\left[\frac{2I/4}{I/3 + 2I/4 + I/3}\right] = -0.214$$

$$RF]_{BE} = -\frac{1}{2}\left[\frac{I/3}{I/3 + 2I/4 + I/4}\right] = -0.143$$

@ Joint 'c',

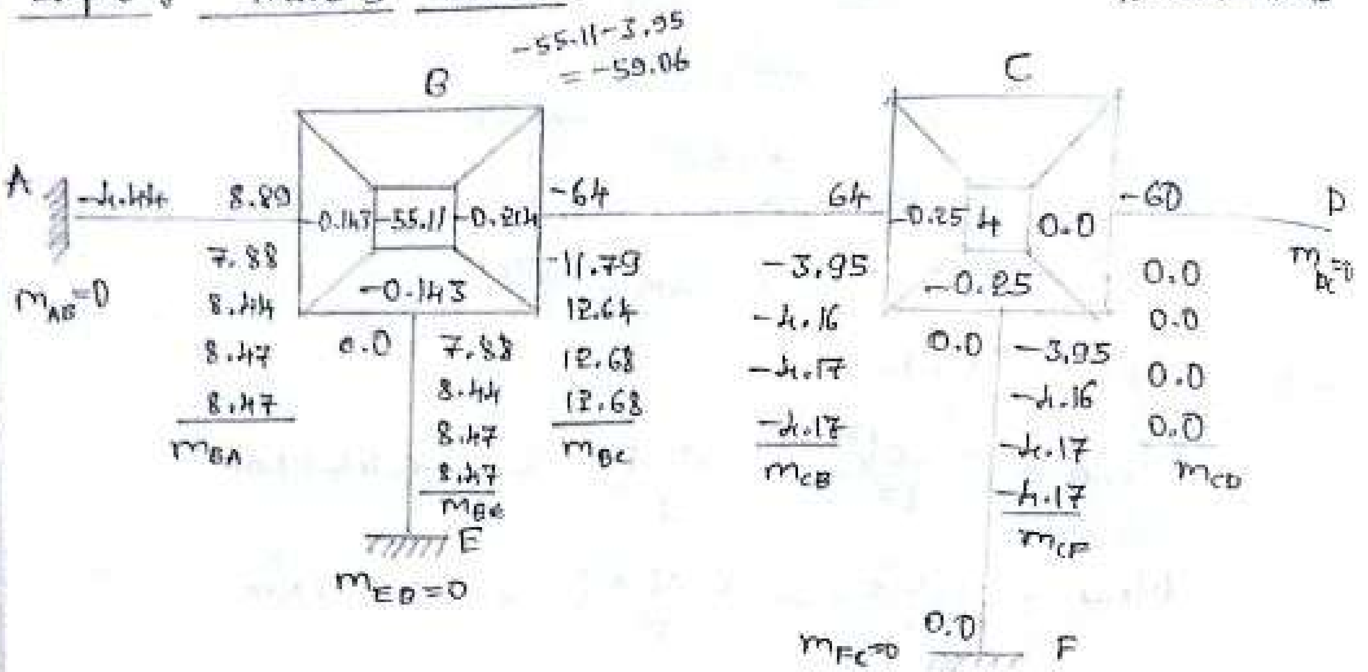
$$RF]_{CB} = -\frac{1}{2}\left[\frac{2I/4}{2I/4 + 0 + 2I/4}\right] = -0.25$$

$$RF]_{CD} = 0$$

$$RF]_{CF} = -\frac{1}{2}\left[\frac{2I/4}{2I/4 + 0 + 2I/4}\right] = -0.25$$

Step 3 : Kani's table :

11.79+4 = 15.79



Step 4 : Final Moments :

$M_{AB} = 4.03$ KNm

$M_{BA} = 8.89 + 2(8.47) + 0 = 25.83$ KNm

$M_{BC} = -64 + 2(12.68) + (-4.17) = -42.81$ KNm

$M_{CB} = 64 + 2(-4.17) + 12.68 = 68.34$ KNm

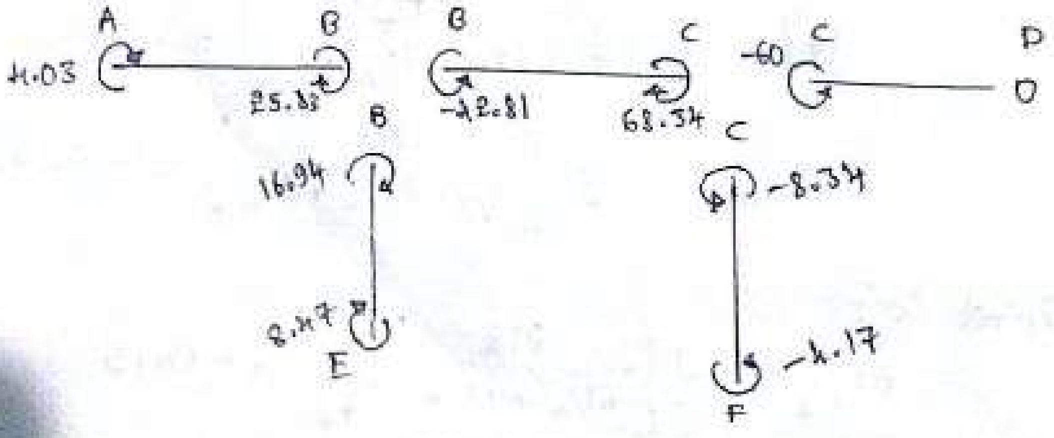
$M_{CD} = -60$ KNm

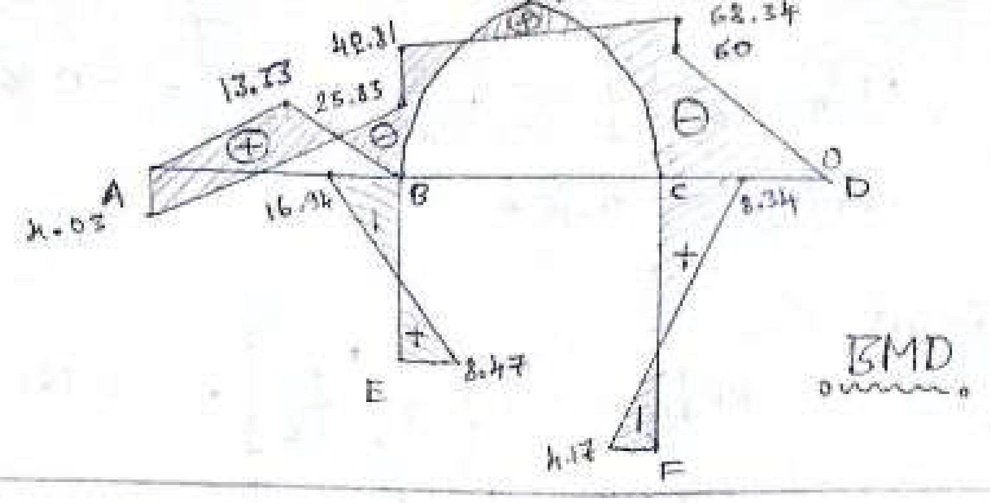
$M_{BE} = 16.94$ KNm

$M_{EB} = 0 + 2(0) + 8.47 = 8.47$ KNm

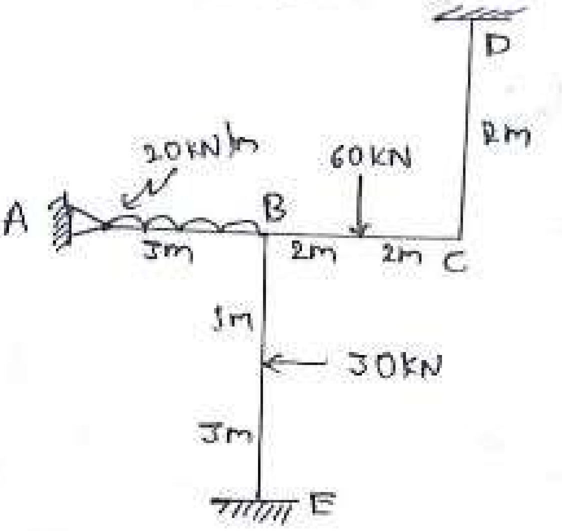
$M_{CF} = 0 + 2(-4.17) + 0 = -8.34$ KNm

$M_{FC} = 0 + 2(0) - 4.17 = -4.17$ KNm





4. Analyse the rigid frame.



→ Step 1 : FEM's :

$$M_{FAB} = \frac{-wL^2}{12} = -15 \text{ kNm}$$

$$M_{FBA} = \frac{wL^2}{12} = 15 \text{ kNm}$$

$$M_{FBC} = \frac{-wL}{8} = -30 \text{ kNm}$$

$$M_{FCB} = 30 \text{ KNm}$$

$$M_{FCD} = M_{FDC} = 0 \text{ kNm}$$

$$M_{FBE} = \frac{-wab^2}{L^2} = \frac{-30 \times 1 \times 3^2}{4^2} = -16.87 \text{ kNm}$$

$$M_{FEB} = \frac{30 \times 1^2 \times 3}{4^2} = 5.62 \text{ KNM}$$

Step 2 : RF's !

@ Joint B, $$RF\Big]_{BA} = -\frac{1}{2}\left[\frac{3/4\,(I/3)}{3/4\,(I/3) + I/4 + I/4}\right]$$

$$RF\Big]_{BA} = -0.167$$

$$RF]_{BC} = -\frac{1}{2}\left[\frac{I/4}{3/4(I/3)+I/4+I/4}\right] = -0.167$$

$$RF]_{BE} = -0.167$$

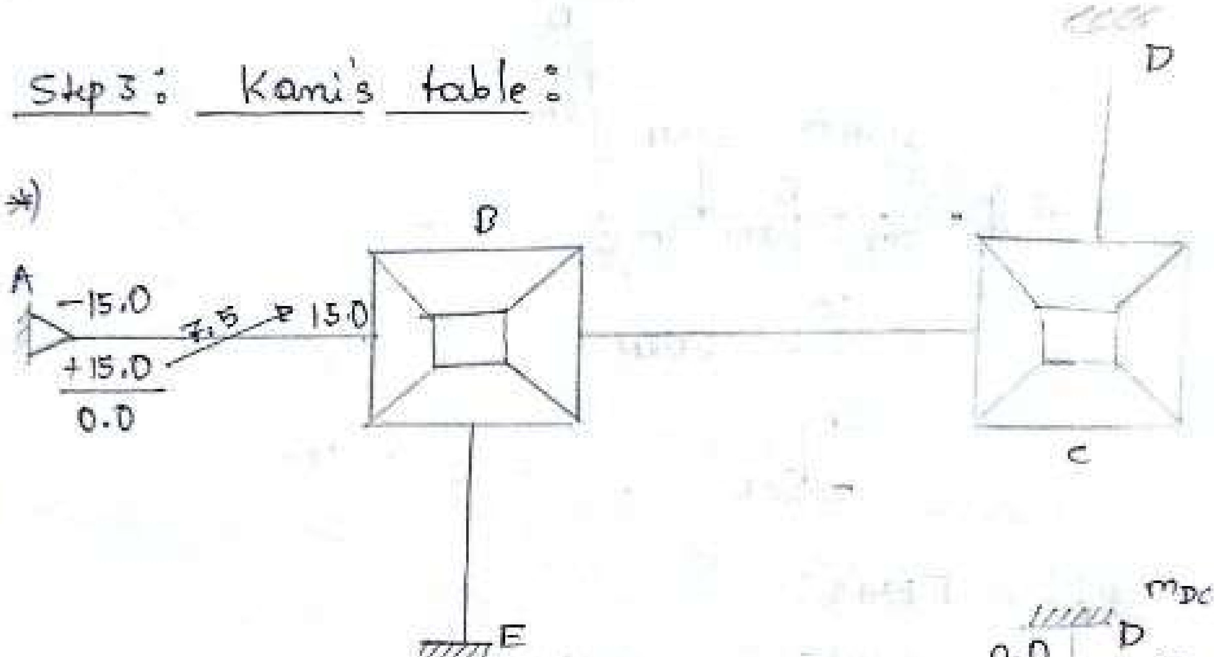
@ Joint 'C',

$$RF]_{CB} = -\frac{1}{2}\left[\frac{I/4}{I/4+I/2}\right] = -0.167$$

$$RF]_{CD} = -\left[\frac{I/2}{I/4+I/2}\right] = -0.333$$

Step 3: Kani's table:

*)



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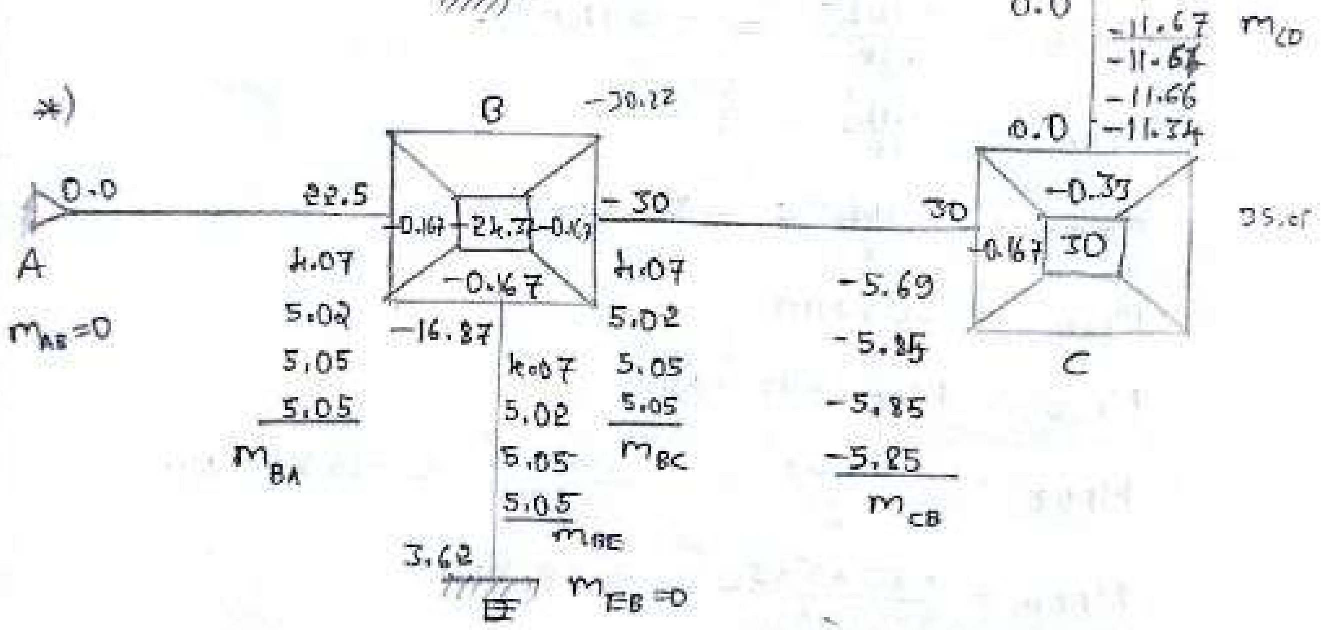


Step 4: Final Moments:

$M_{AB} = 0.0$ KNm

$M_{BA} = 22.5 + 2(5.05) + 0 = 32.6$ KNm

$M_{BC} = -30 + 2(5.05) - 5.85 = -25.75$ KNm

$$M_{CB} = 30 + 2(-5.85) + 5.05 = 23.35 \text{ kNm}$$

$$M_{CD} = 0.0 + 2(-11.67) + 0 = -23.34 \text{ kNm}$$

$$M_{DC} = 0.0 + 2(0) - 11.67 = -11.67 \text{ kNm}$$

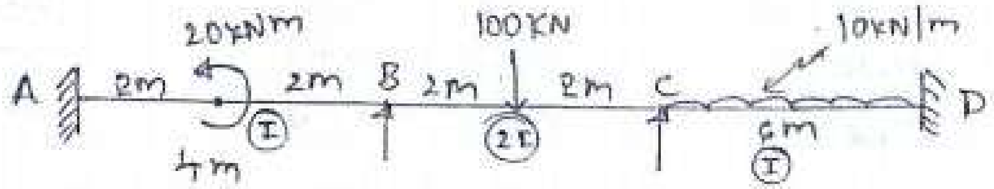
$$M_{BE} = -16.87 + 2(5.05) + 0 = -6.77 \text{ kNm}$$

$$M_{EB} = 3.62 + 2(0) + 5.05 = 8.67 \text{ kNm}$$

Imp

5.

*)



→ **Step1: FEM's:**

$$M_{FAB} = -\frac{M}{4} = \frac{-20}{4} = -5 \text{ kNm}$$

$$M_{FBA} = -\frac{M}{4} = \frac{-20}{4} = -5 \text{ kNm}$$

$$M_{FBC} = \frac{-WL}{8} = -\frac{100\times4}{8} = -50 \text{ kNm}$$

$$M_{FCB} = \frac{WL}{8} = \frac{100\times4}{8} = 50 \text{ kNm}$$

$$M_{FCD} = \frac{-WL^2}{12} = \frac{-10\times6^2}{12} = -30 \text{ kNm}$$

$$M_{FDC} = 30 \text{ kNm}$$

Step2: Rotation factor:

@ Joint 'B', $RF]_{BA} = -\frac{1}{2}\left[\frac{I/4}{I/4 + 2I/4}\right] = -0.167$

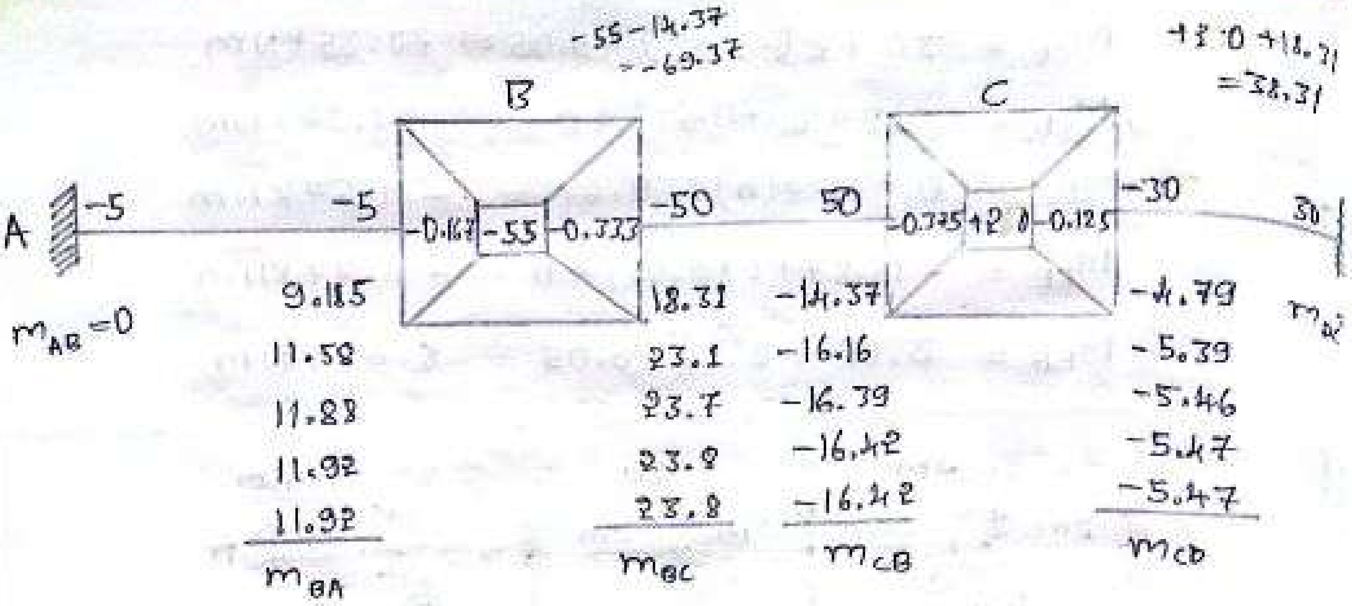
$RF]_{BC} = -0.333$

@ Joint 'C', $RF]_{CB} = -\frac{1}{2}\left[\frac{2I/4}{2I/4 + I/6}\right] = -0.375$

$\therefore\ RF]_{CD} = -0.125$

i.e., $RF]_{CD} = -\frac{1}{2}\left[\frac{I/6}{2I/4 + I/6}\right] = -0.125$

Step 3: Kani's table:



Step 4: Final Moments:

$$M_{AB} = -5 + 2(0) - 11.92 = -16.92 \text{ KNm}$$

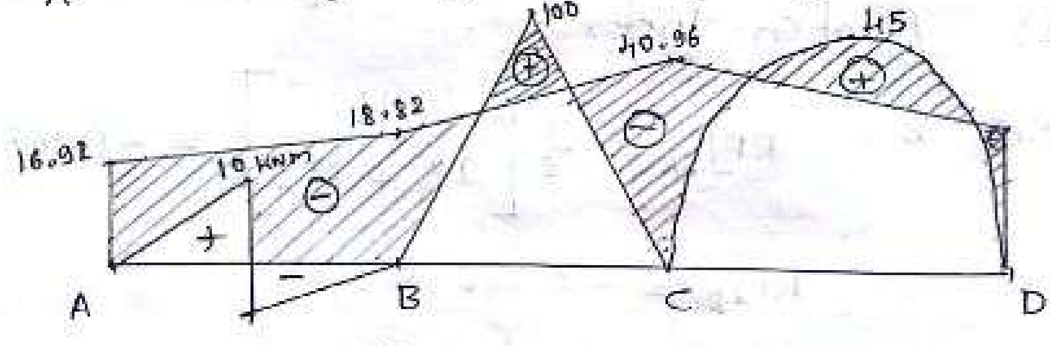
$$M_{BA} = -5 + 2(+11.92) + 0 = +18.82 \text{ KNm}$$

$$M_{BC} = -50 + 2(23.8) - 16.42 = -18.82 \text{ KNm}$$

$$M_{CB} = 50 + 2(-16.42) + 23.8 = 40.96 \text{ KNm}$$

$$M_{CD} = -30 + 2(-5.47) + 0 = -40.94 \text{ KNm}$$

$$M_{DC} = 30 + 2(0) - 5.47 = 24.53 \text{ KNm}$$



BMD

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6. Analyse the total frame by Kani's rotation method. Draw BMD.

→ **Step 1: FEM's:**

$$M_{FAB} = \frac{-WL}{8} = -37.5 \text{ KNm}$$

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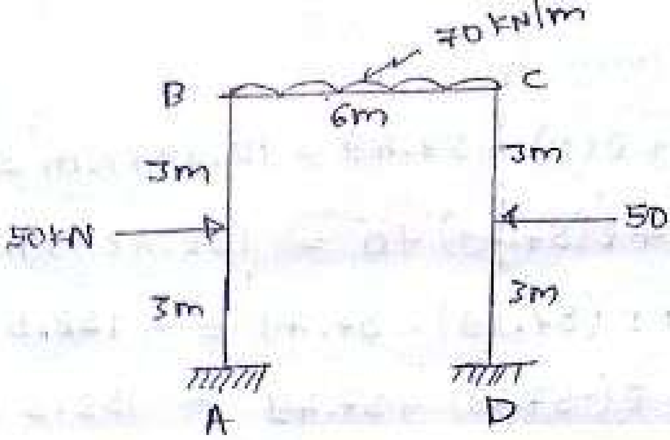


$M_{FBA} = 37.5 \text{ kNm}$

$M_{FBC} = \frac{-wL^2}{12} = -210 \text{kNm}$

$M_{FCB} = 210 \text{ kNm}$

$M_{FCD} = \frac{-WL}{8} = -37.5 \text{ kNm}$

$M_{FDC} = 37.5 \text{ kNm}$

Step 2: RF's:

@ Joint 'B',

$RF]_{BA} = -\frac{1}{2}\left[\frac{I/6}{I/6 + I/6}\right] = -0.25$

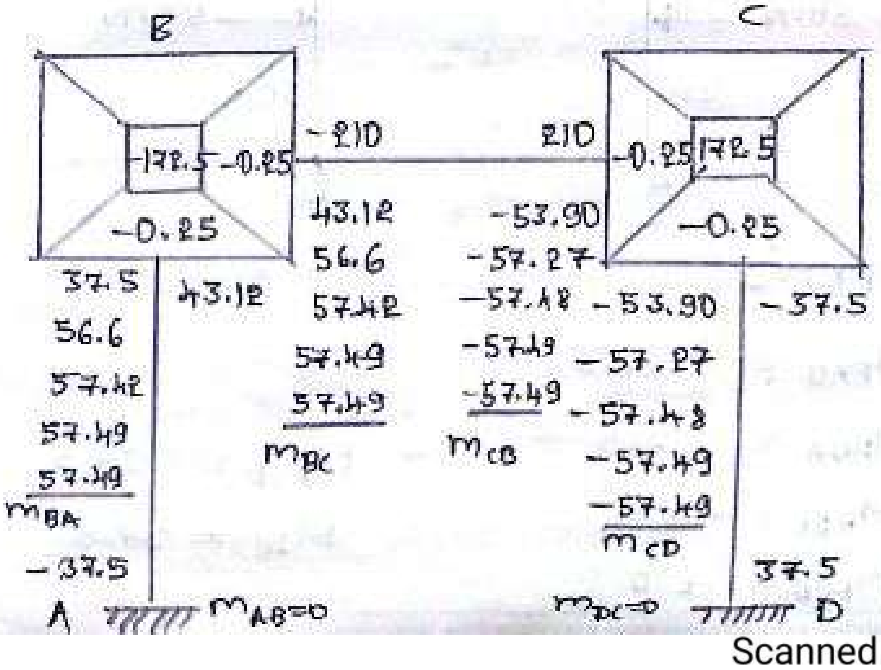
$RF]_{BC} = -0.25$

@ Joint 'C',

$RF]_{CB} = -\frac{1}{2}\left[\frac{I/6}{I/6+I/6}\right] = -0.25$

$RF]_{CD} = -0.25$

Step 3: Kani's table:



Step 4: Final Moments:

$M_{AB} = -37.5 + 2(0) + 57.49 = 19.99$ kNm $\simeq$ 20 kNm

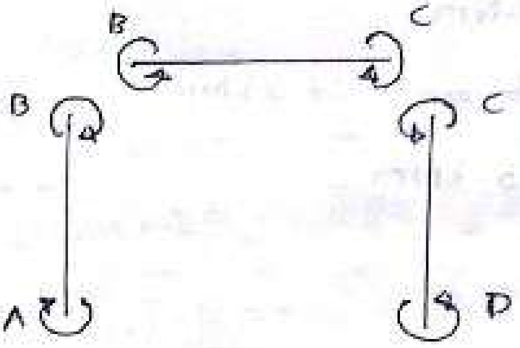
$M_{BA} = 437.12 + 2(57.49) + 0 = 152.48$ kNm

$M_{BC} = -210 + 2(57.49) - 57.49 = -152.5$ kNm

$M_{CB} = 210 + 2(-57.49) + 57.49 = 152.5$ kNm

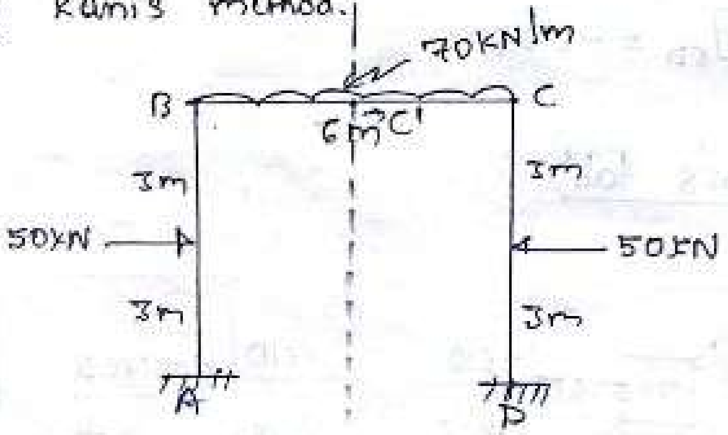
$M_{CD} = -37.5 + 2(-57.49) + 0 = -152.5$ kNm

$M_{DC} = 37.5 + 2(0) - 57.49 = -20$ kNm



(OR)

**) If the load and frame are symmetrical no need to analyse the entire frame. Analyse the half portion taking the axis of symmetry, of either left @ right side. For the cut member take stiffness as half. This is the advantage of Kani's method.



Step 1: FEM's:

$M_{FAB} = -37.5$

$M_{FBA} = 37.5$

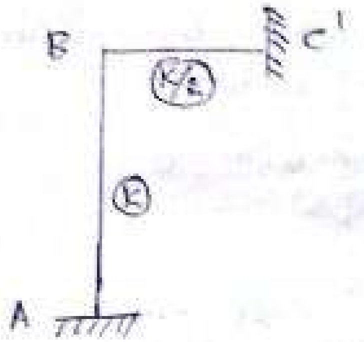
$M_{FBC} = -210$

$M_{FCB} = 210$

$M_{FCD} = -37.5$

$M_{DC} = 37.5$

Step 2 :



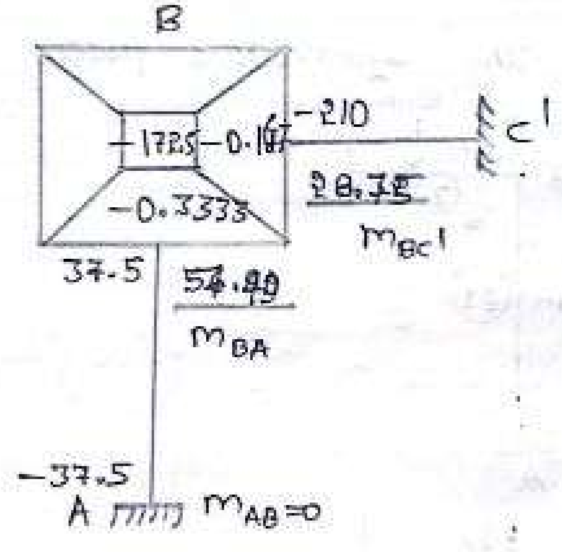
$$RF\Big]_{BA} = -\frac{1}{2}\left[\frac{K_{BA}}{K_{BA}+K_{BC}}\right] = -\frac{1}{2}\left[\frac{K}{k+K/2}\right] =$$

$$= -\frac{1}{2}\left[\frac{I/6}{I/6+\frac{1}{2}(I/6)}\right]$$

$$RF\Big]_{BA} = -0.3333$$

$$RF\Big]_{BC'} = -\frac{1}{2}\left[\frac{K/2}{k+K/2}\right] = -\frac{1}{2}\left[\frac{1/2(I/6)}{I/6+\frac{1}{2}(I/6)}\right] = -0.167$$

Step 3 : Kani's rotation table :



Step 4 : Final Moments :

$$M_{AB} = -37.5 + 2(0) + 57.49 = 19.99 \text{ KNm} \simeq 20 \text{ KNm}$$

$$M_{BA} = 37.5 + 2(57.49) + 0 = 152.5 \text{ KNm}$$

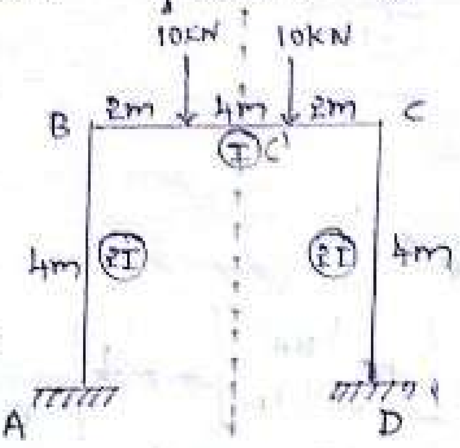
$$M_{BC'} = -210 + 2(28.75) + 0 = -152.5 \text{ KNm}$$

$\therefore$ $M_{AB} = M_{DC} = -20$ KNm

$M_{BA} = M_{CD} = -152.5$ KNm

$M_{BC} = M_{CB} = 152.5$ KNm

7. Analyse the frame by Kani's rotation method



→ <u>Step 1:</u> <u>FEM's:</u>

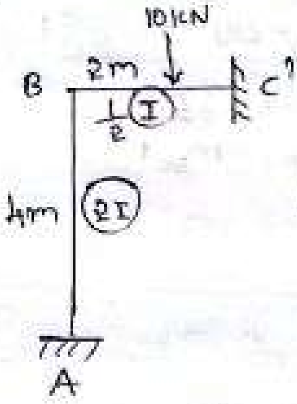
$M_{FAB} = M_{FBA} = 0$ KNm

$M_{FCD} = M_{DC} = 0$ KNm

$$M_{FBC} = \frac{-wab^2}{L^2} - \frac{wab^2}{L^2} = \frac{-10\times 2\times 6^2}{8^2} - \frac{10\times 6\times 2^2}{8^2}$$

$M_{FBC} = -15$ KNm

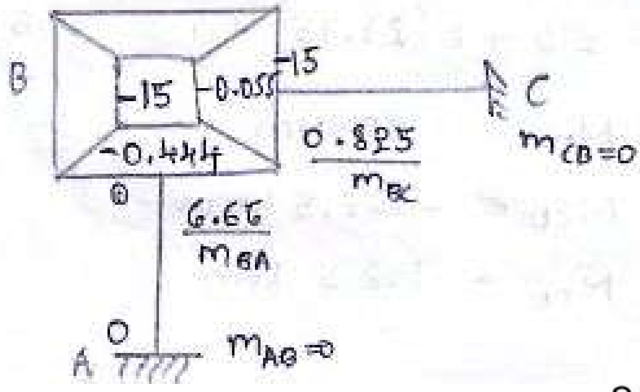
<u>Step 2:</u> Consider the symmetric part,



$$RF]_{BA} = -\frac{1}{2}\left[\frac{2I/4}{2I/4 + \frac{1}{2}(I/8)}\right] = -0.444$$

$$RF]_{BC} = -\frac{1}{2}\left[\frac{1/2\,(I/8)}{2I/4 + \frac{1}{2}(I/8)}\right] = -0.055$$

<u>Step 3:</u>



Step 4 : **Final Moments :**

$M_{AB} = 0 + 2(0) + 6.66 = 6.66$ KNm

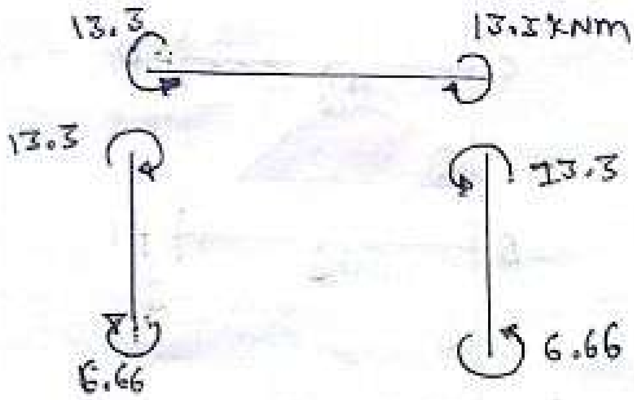
$M_{BA} = 0 + 2(6.66) + 0 = 13.35$ KNm

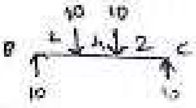
$M_{BC} = -15 + 2(0.825) + 0 = -13.35$ KNm

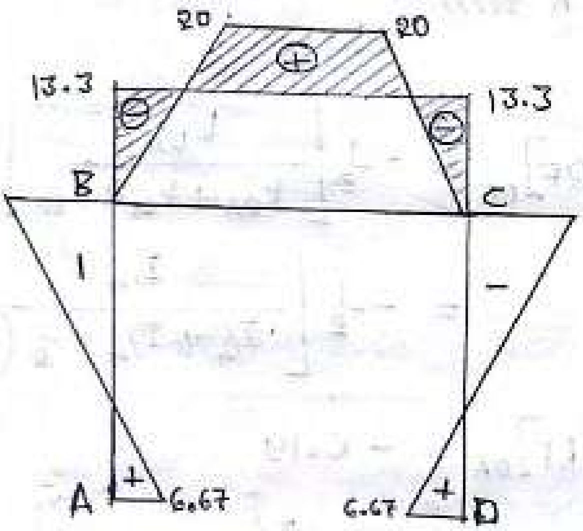
$\therefore$ $M_{AB} = M_{DC} = -6.66$ KNm

$M_{BA} = M_{CD} = -13.3$ KNm

$M_{BC} = M_{CB} = +13.3$ KNm







BMD

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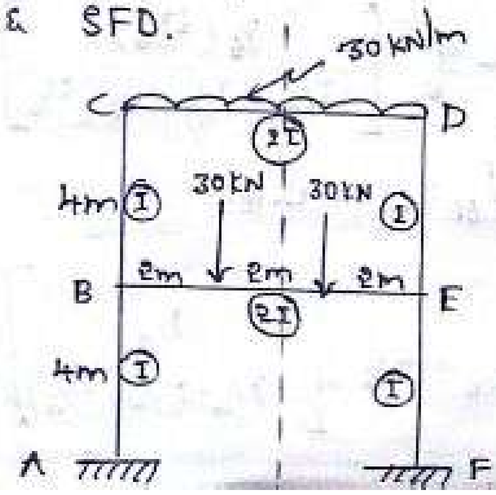
6. Analyse symmetrical two storey frame by kani's rotation method taking the advantage of symmetry. Draw BMD & SFD.



09/10/18

→ 1 : $M_{FAB} = M_{FBA} = M_{FFE} = M_{FEF} = 0$

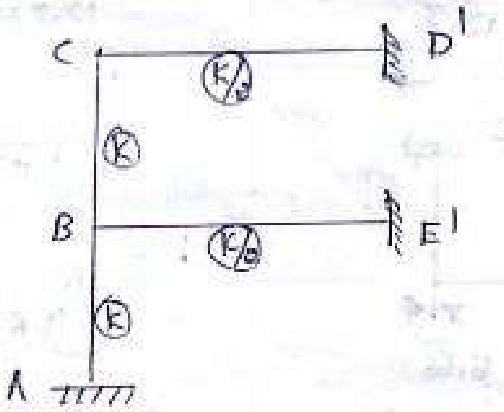
$$M_{FCD} = \frac{-30 \times 6^2}{12} = -90\,kNm$$

$$M_{FDC} = 90\,kNm$$

$$M_{FBE} = -\frac{30 \times 2 \times 4^2}{6^2} - \frac{30 \times 4 \times 2^2}{6^2} = -40\,kNm$$

$$M_{FEB} = 40\,kNm$$

$$M_{FCB} = M_{FBC} = M_{FDE} = M_{FED} = 0$$



Step 2 :

@ Joint 'B',
$$RF]_{BA} = -\frac{1}{2}\left[\frac{K_{BA}}{K_{BA}+K_{BC}+K_{BE'}}\right]$$

$$= -\frac{1}{2}\left[\frac{I/4}{I/4 + I/4 + \frac{1}{2}\left(2I/6\right)}\right]$$

$$RF]_{BA} = -0.19$$

$$RF]_{BC} = -\frac{1}{2}\left[\frac{I/4}{I/4 + I/4 + 1/2\left(2I/6\right)}\right]$$

$$RF]_{BC} = -0.19$$

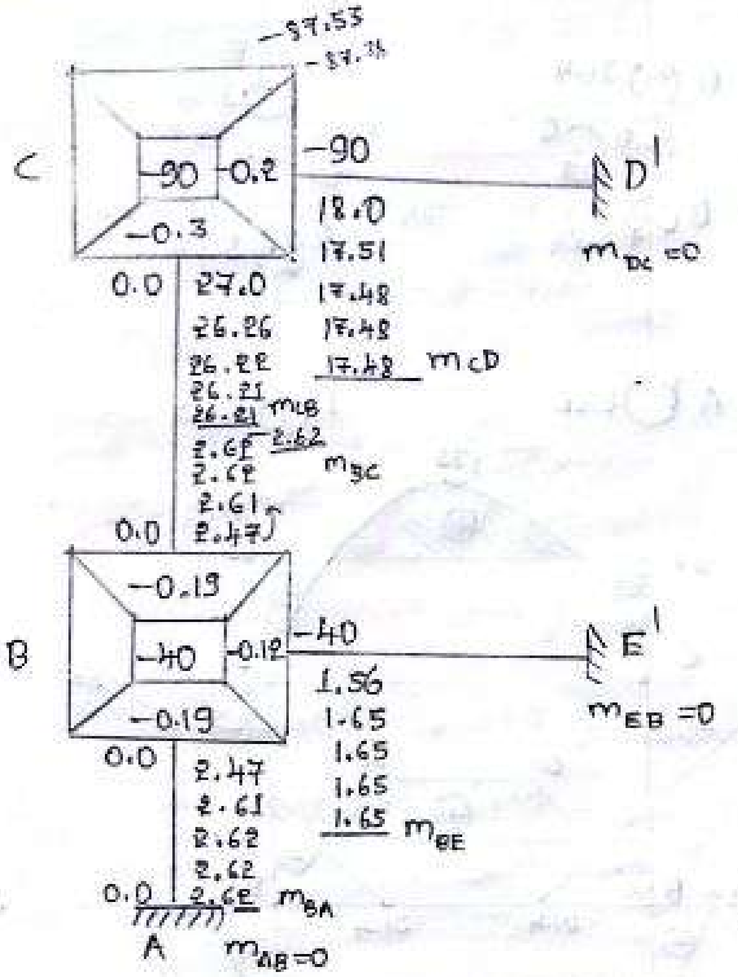
$$RF]_{BE'} = -\frac{1}{2}\left[\frac{1/2\left(2I/6\right)}{I/4 + I/4 + 1/2\left(2I/6\right)}\right]$$

$$RF]_{BE'} = -0.12$$

@ Joint 'C',
$$RF]_{CB} = -\frac{1}{2}\left[\frac{I/4}{I/4 + 1/2\left(2I/6\right)}\right] = -0.3$$

$$RF]_{CD'} = -\frac{1}{2}\left[\frac{1/2\left(2I/6\right)}{I/4 + 1/2\left(\frac{2I}{6}\right)}\right] = -0.2$$

Step 3: Kani's rotation table:



Step 4: Final Moments:

$M_{AB} = 0 + 2(0) + 2.62 = 2.62$ KNm

$M_{BA} = 0 + 2(2.62) + 0 = 5.24$ KNm

$M_{BC} = 0 + 2(2.62) + 26.21 = 31.4$ KNm

$M_{CB} = 0 + 2(26.21) + 2.62 = 55.0$ kNm

$M_{BE} = -40 + 2(1.65) + 0 = -36.7$ kNm

$M_{CD} = -90 + 2(17.41) + 0 = -55.0$ KNm

$\therefore$ $M_{FE} = -2.62$ kNm

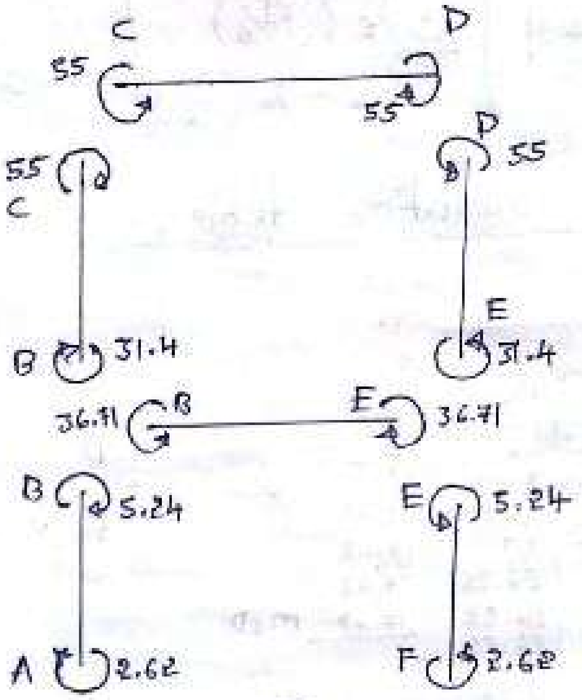
$M_{EF} = -5.24$ kNm

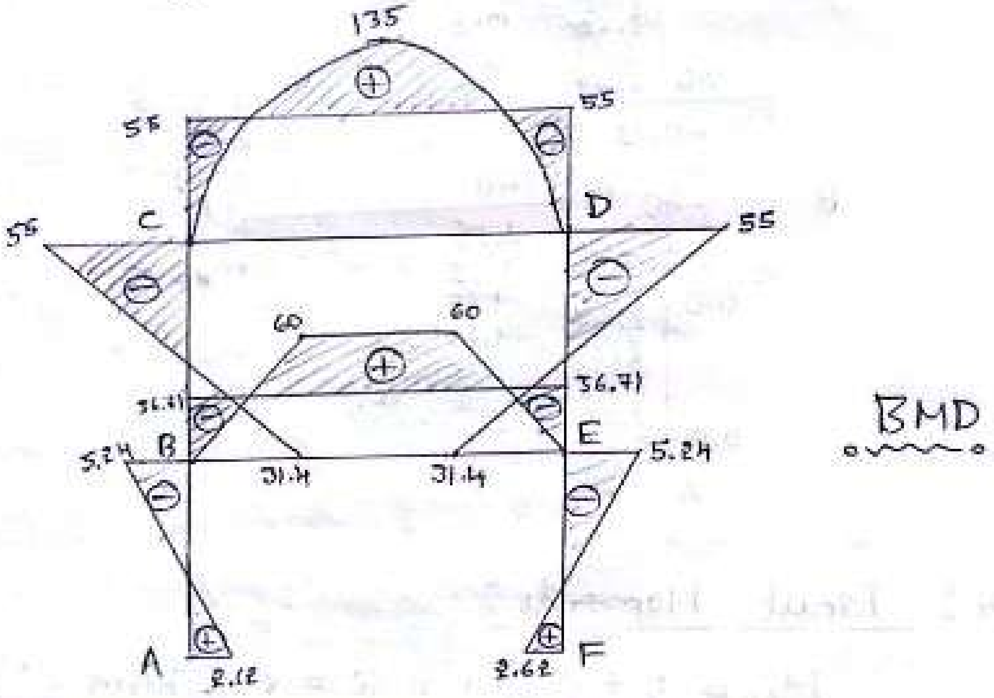
$M_{ED} = -31.4$ KNm

$M_{DE} = -55.0$ KNm

$M_{DC} = 55.0$ KNm

$M_{EB} = 36.7$ KNm





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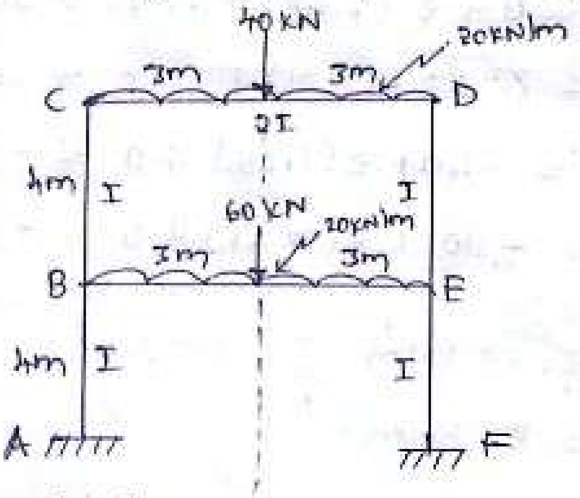
9. Analyse the frame by taking advantage of symmetry.



→ Step 1 : FEM's :

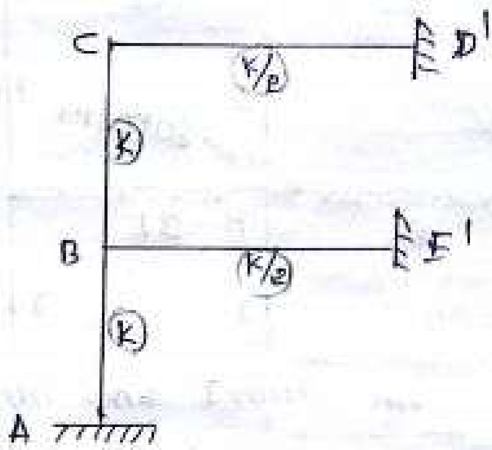
$M_{FAB} = M_{FBA} = M_{FBC} = M_{FCB} = M_{FDE} = M_{FED} = M_{FEF} = M_{FFE} = 0$

$M_{FCD} = -\frac{20 \times 6^2}{12} - \frac{40 \times 6}{8} = -90\ kNm$

$M_{FDC} = 90 \text{ KNm}$

$M_{FBE} = -\frac{20 \times 6^2}{12} - \frac{60 \times 6}{8} = -105 \text{ KNm}$

$M_{FEB} = 105 \text{ KNm}$



Step2 : RF's :

@ Joint 'B', $RF\rfloor_{BA} = -\frac{1}{2}\left[\frac{I/4}{I/4 + I/4 + \frac{1}{2}(I/6)}\right] = -0.21$

$RF\rfloor_{BC} = -0.21$

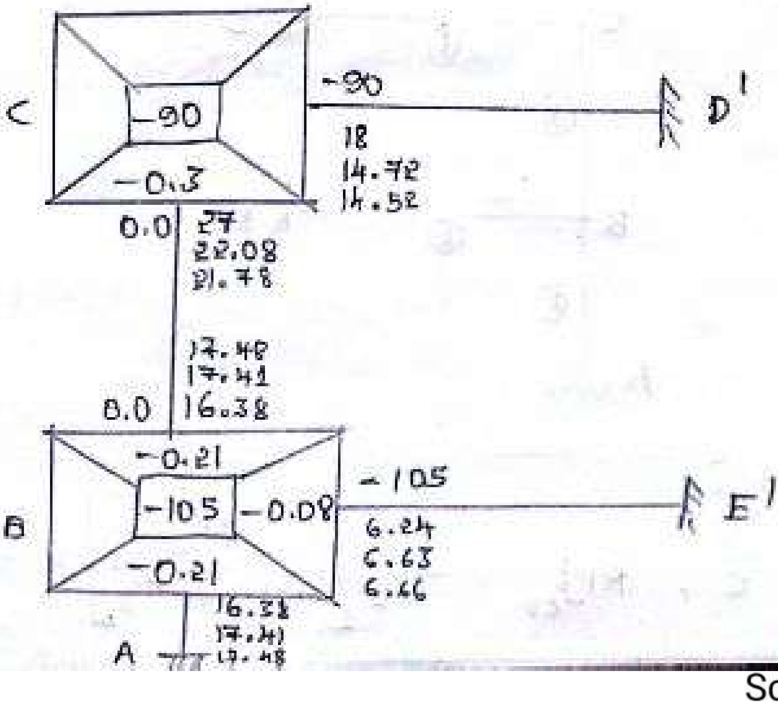
$RF\rfloor_{BE} = -\frac{1}{2}\left[\frac{1/2\,(I/6)}{I/4 + I/4 + 1/2\,(I/6)}\right] = -0.08$

@ Joint 'C',

$RF\rfloor_{CD} = -\frac{1}{2}\left[\frac{1/2\,(2I/6)}{1/2\,(2I/6) + I/4}\right] = -0.2$

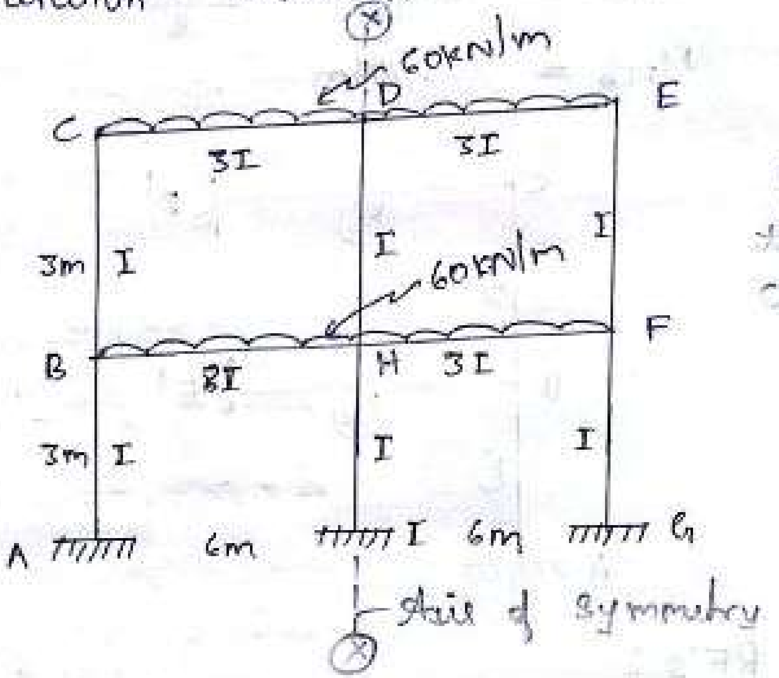
$RF\rfloor_{CB} = -0.3$

Step3 :



10/10/18

10. Analyse the multistorey frame / building by Kani's rotation method.



Moment in the inner columns is zero.

→ **Step 1 : FEM's :**

$M_{FAB} = M_{FBA} = M_{FCB} = M_{FBC} = 0$

$M_{FDH} = M_{FHD} = M_{FHI} = M_{FIH} = 0$

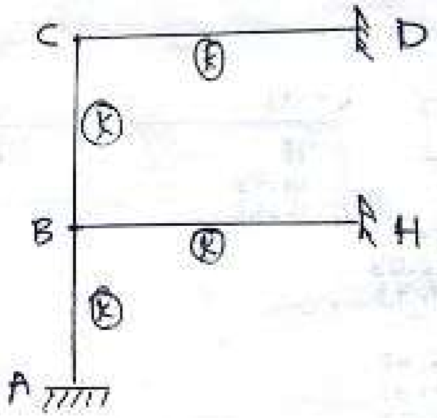
$M_{FCD} = -\frac{WL^2}{12} = -180\,KNm$

$M_{FDC} = 180\,KNm$

$M_{FBH} = -\frac{60\times6^2}{12} = -180\,KNm$

$M_{FHB} = 180\,KNm$

$M_{FEF} = M_{FFE} = M_{FGF} = M_{FFG} = 0$



Step 2 : RF's :

ⓐ Joint 'C', $RF]_{CD} = -\frac{1}{2}\left[\frac{3I/6}{3I/6 + I/3}\right] = -0.3$

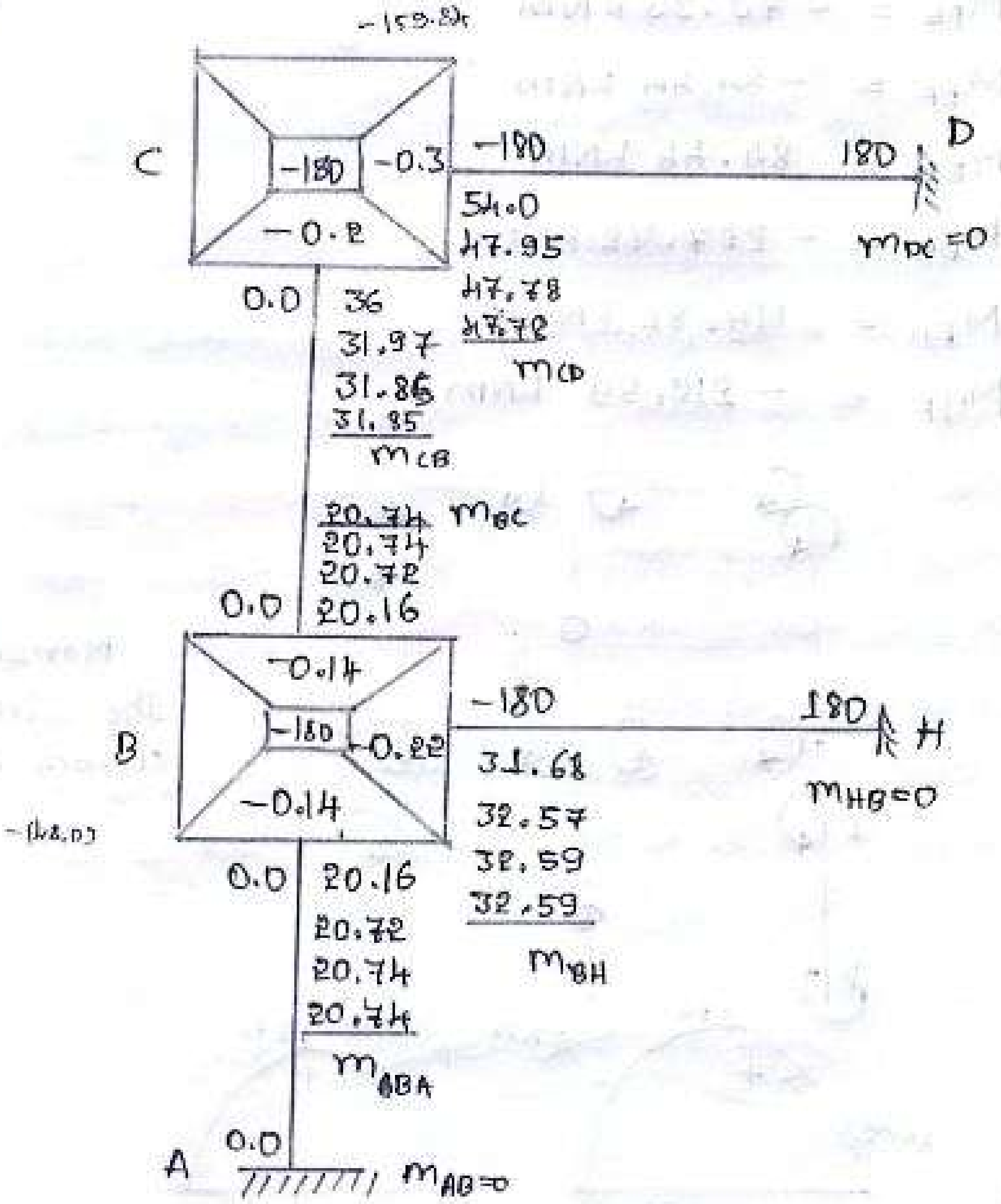
$$RF\Big]_{CB} = -\frac{1}{2}\left[\frac{I/3}{3I/3 + I/3}\right] = -0.2$$

@ 'B', $$RF\Big]_{BC} = -\frac{1}{2}\left[\frac{I/3}{I/3 + 3I/6 + I/3}\right] = -0.14$$

$$RF\Big]_{BA} = -0.14$$

$$RF\Big]_{BH} = -\frac{1}{2}\left[\frac{3I/6}{I/3 + 3I/6 + I/3}\right] = -0.22$$

Step 3 :



Step 4 : Final Moments :

$M_{AB} = 0 + 2(0) + 20.74 = 20.74$ KNm

$M_{BA} = 0 + 2(20.74) + 0 = 41.48$ KNm

$M_{BC} = 0 + 2(20.74) + 31.85 = 73.33$ KNm

$M_{CB} = 0 + 2(31.85) + 20.74 = 84.44$ KNm

$M_{CD} = -180+2(47.78)+0 = -84.44$ KNm

$M_{DC} = 180+2(0)+47.78 = 227.78$ KNm

$M_{BH} = -180+2(32.59)+0 = -114.82$ KNm

$M_{HB} = 180+2(0)+32.59 = 212.59$ KNm

∴ By symmetry,

$M_{GH} = -20.74$ KNm

$M_{FG} = -41.48$ KNm

$M_{FE} = -73.33$ KNm

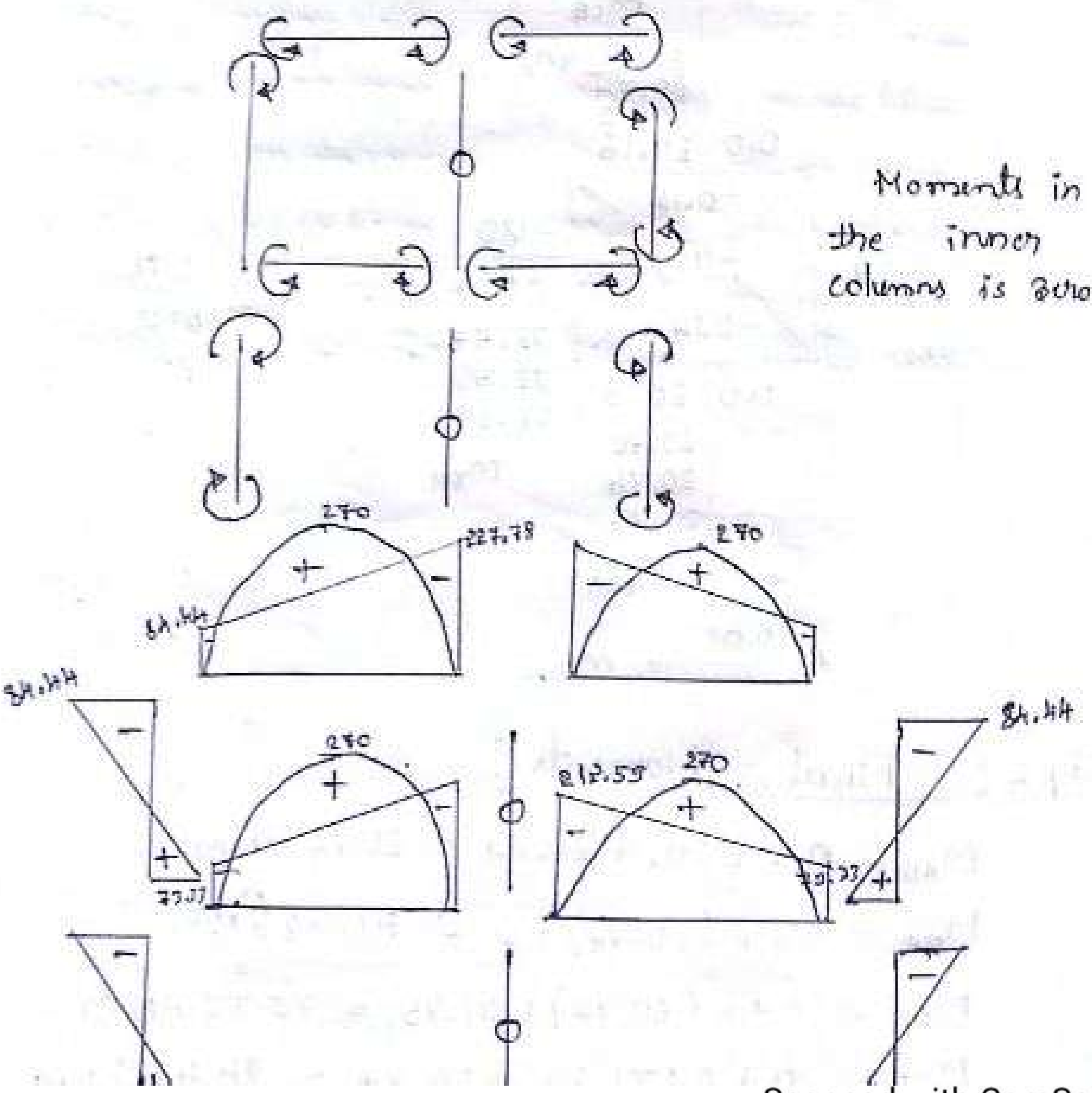
$M_{EF} = -84.44$ KNm

$M_{ED} = 84.44$ KNm

$M_{DE} = -227.78$ KNm

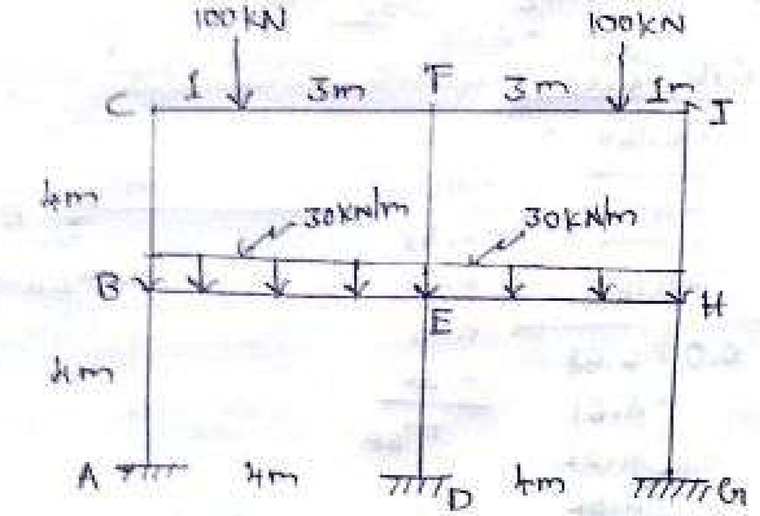
$M_{FH} = 114.82$ KNm

$M_{HF} = -212.59$ KNm



Moments in the inner columns is zero.

11. Analyse the frame shown in figure. Draw BMD.



→ Step I : FEM's :

$M_{FAB} = M_{FBA} = M_{FBC} = M_{FCB} = 0$

$M_{FCF} = \frac{-100\times1\times3^2}{4^2} = -56.25$ KNm

$M_{FFC} = \frac{100\times1^2\times3}{4^2} = 18.75$ KNm

$M_{FBE} = -40$KNm ; $M_{FEB} = 40$KNm

Step 2 : RF's :

@ Joint 'C', $RF]_{CB} = -\frac{1}{2}\left[\frac{3/4}{3/4+3/4}\right] = -0.25$

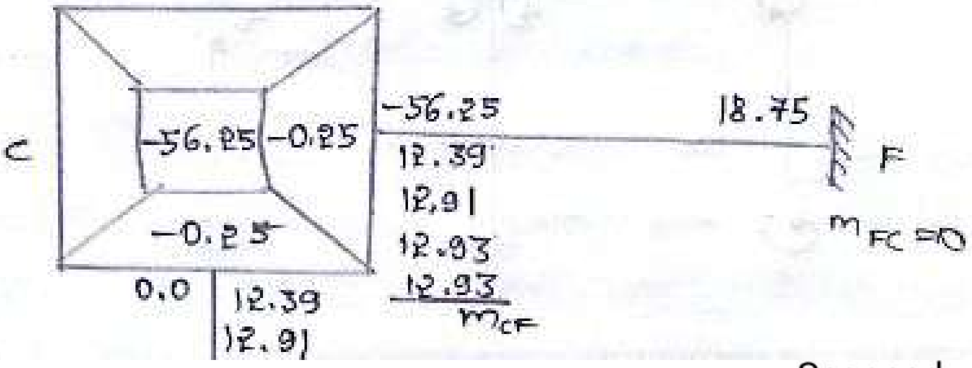
$RF]_{CF} = -0.25$

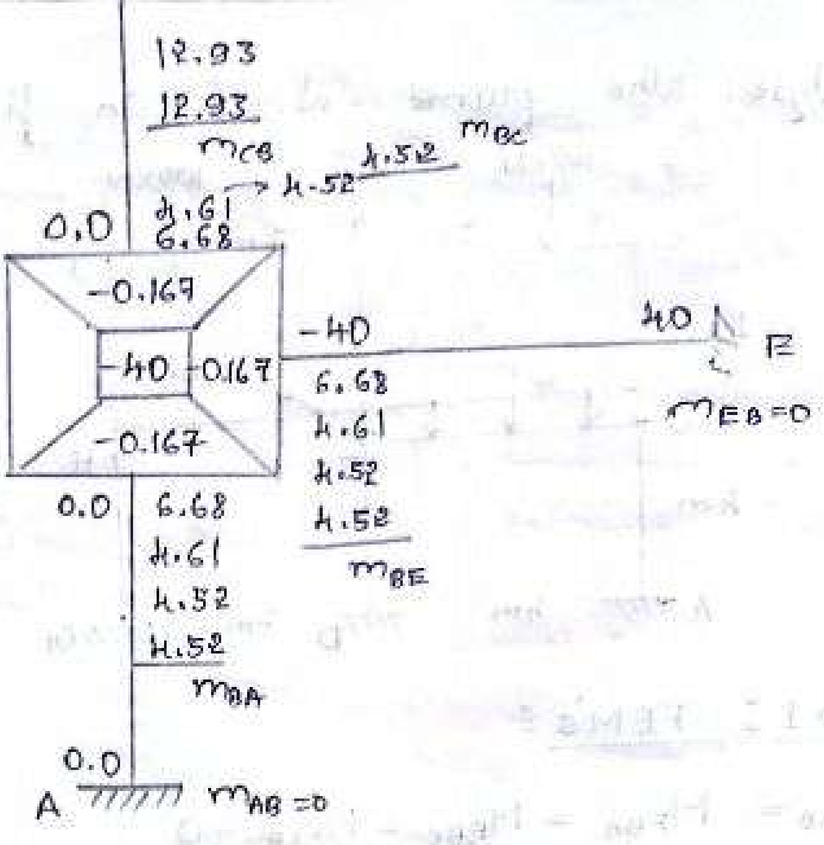
@ Joint 'B', $RF]_{BC} = -0.167$

$RF]_{BA} = -0.167$

$RF]_{BE} = -0.167$

Step 3 : Kani's table :





Step 4: Final Moments:

$M_{AB} = 4.52$ KNm

$M_{BA} = 9.04$ KNm

$M_{BC} = 21.97$ KNm

$M_{CB} = 30.38$ KNm

$M_{CF} = -30.39$ KNm

$M_{FC} = 31.68$ KNm

$M_{BE} = -30.96$ KNm

$M_{EB} = 44.52$ KNm

$M_{GH} = -4.52$ KNm

$M_{HG} = -9.04$ KNm

$M_{HE} = -21.97$ KNm

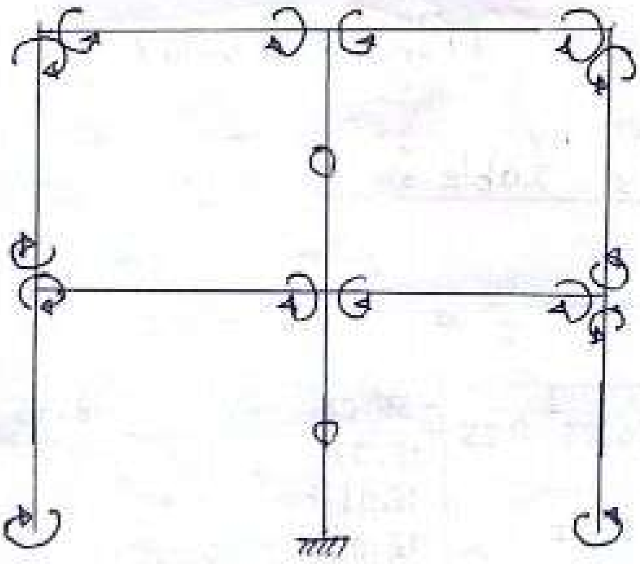
$M_{IH} = -30.38$ KNm

$M_{IF} = 30.39$ KNm

$M_{FI} = -31.68$ KNm

$M_{HE} = 30.96$ KNm

$M_{EH} = -44.52$ KNm



12/11/18

SUJITH.N.S

Module - 4

Flexibility Matrix Method

(system approach)(Force Method, Displacement method)

The deflection produced due to unit load (or) rotation developed due to unit moment is called flexibility denoted by 'δ'.

Flexibility, $[\delta] = \frac{[\Delta]}{[P]}$ mm/KN

(or)

$$[\delta] = \frac{[\theta]}{[M]} \ °/KN\text{-}m$$

$$[\delta][M] = [\theta]$$

where, $[\theta]$ → Slope due to external load
$[M]$ → Unknown moment
$[\delta]$ → Flexibility matrix

**) The reaction of conjugate beam gives slope 'θ'.

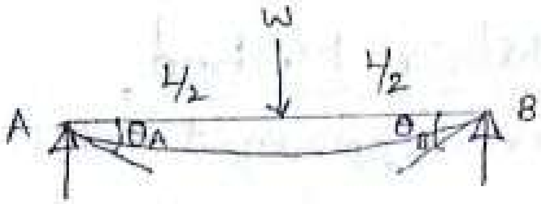
The general equation of flexibility matrix is

$$[\delta][M] = [\theta_s] - [\theta_L]$$

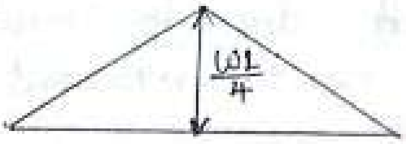
where, $[\theta_s]$ → Rotation @ system co-ordinate due to extra loads at supports.
$[\theta_L]$ → Rotation @ system co-ordinate due to external loads.

Conjugate beam:

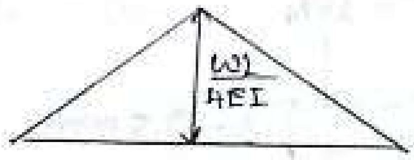
1.



Load @ centre (Symmetrical)

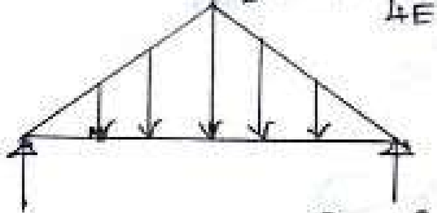


'M' diagram



'$\frac{M}{EI}$' diagram

$$\frac{1}{2} \times L \times \frac{WL}{4EI} = \frac{WL^2}{8EI}$$

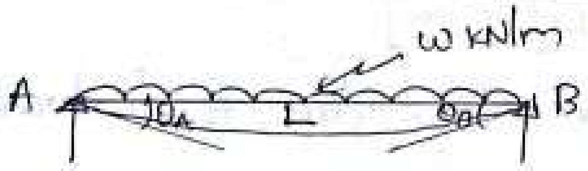


'Conjugate beam'

$R_A = \theta_A$ $\quad$ $R_B = \theta_B$

i.e., $R_A = \frac{WL^2}{16EI}$ $\quad$ $R_B = \frac{WL^2}{16EI}$

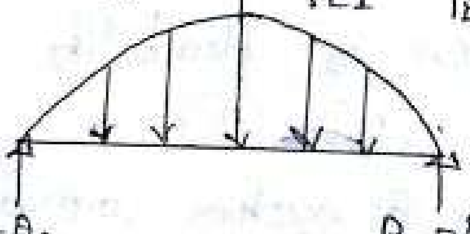
2.



Load (UDL)

'$\frac{M}{EI}$' diagram

$$\frac{2}{3} \times L \times \frac{WL^2}{8EI} = \frac{WL^3}{12EI}$$

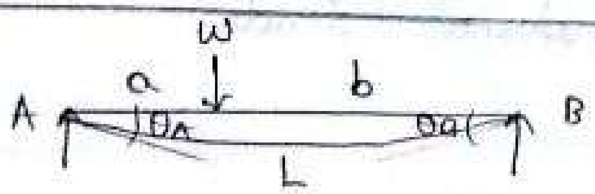


Conjugate beam dia

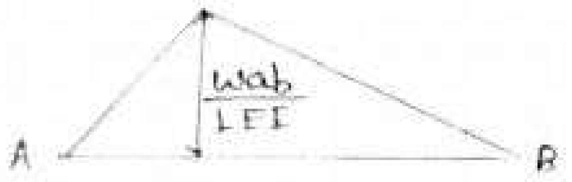
$R_A = \theta_A$ $\quad$ $R_B = \theta_B$

i.e., $R_A = \frac{WL^3}{24EI}$ $\quad$ $R_B = \frac{WL^3}{24EI}$

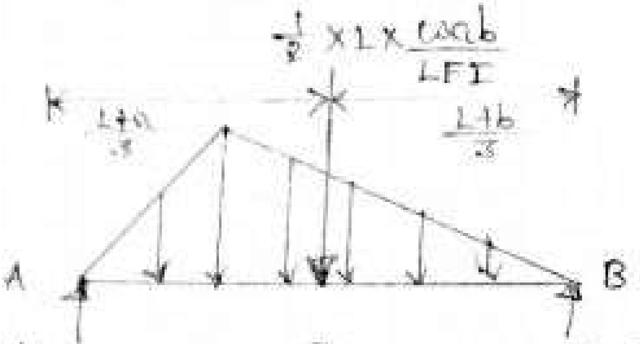
3.



Load not @ centre (Unsymmetrical)



$\frac{M}{EI}$ diagram



Conjugate beam dia.

$R_A = \frac{wab}{EI}\left[\frac{1}{2} - \left(\frac{L+a}{6L}\right)\right]$ $R_B = \frac{wab}{6EIL}[L+a]$

∵ w.k.t $\Sigma M_A = 0$;

$$\frac{wab}{2EI}\left(\frac{L+a}{3}\right) - R_B \times L = 0$$

$$\therefore R_B = \frac{wab(L+a)}{6EIL}$$

Also, w.k.t $\Sigma V = 0$;

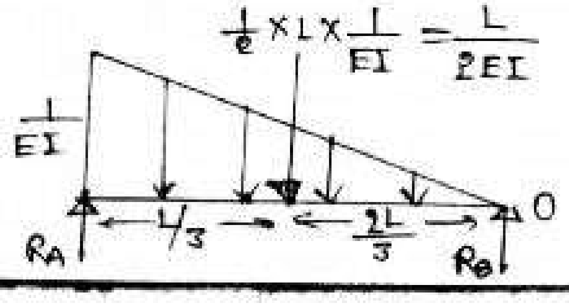
$$R_A - \frac{wab}{2EI} + \frac{wab(L+a)}{6EIL} = 0$$

$$R_A = \frac{wab}{2EI} - \frac{wab(L+a)}{6EIL}$$

$$\therefore R_A = \frac{wab}{EI}\left[\frac{1}{2} - \left(\frac{L+a}{6L}\right)\right]$$

Flexibility :

1.



Here $\Sigma M_A = 0$;

$$\frac{L}{2EI}\left(\frac{L}{3}\right) - R_B \times L = 0$$

$$\frac{L^2}{6EI} = R_B L$$

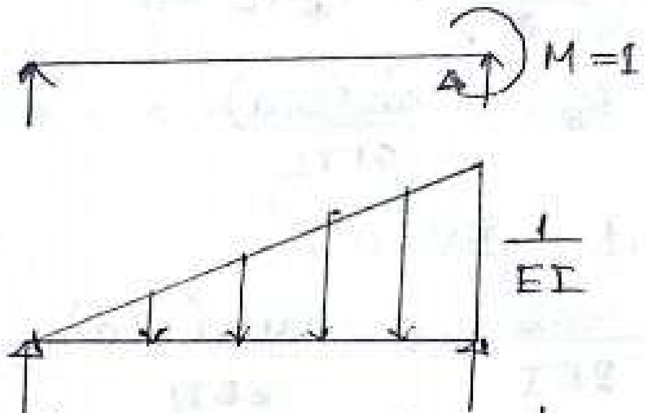
$$\therefore R_B = \frac{L}{6EI}$$

$|||^{ly}$, $\Sigma V = 0$;

$$R_A - \frac{L}{2EI} + \frac{L}{6EI} = 0$$

$$\therefore R_A = \frac{L}{3EI}$$

2.



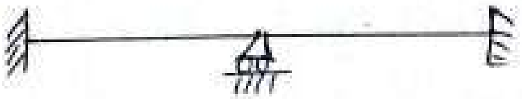
$\theta_A = \frac{L}{6EI}$ $\theta_B = \frac{L}{3EI}$

<u>Note</u> : In flexibility matrix, take degree of redunde-cy (DOR) i.e., moments as unknown.

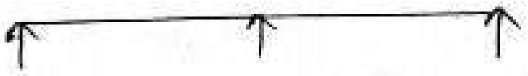
(i)

$5 - 2 = 3$ (DOR)

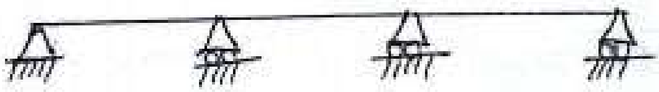
(ii)



(iii)



(iv)



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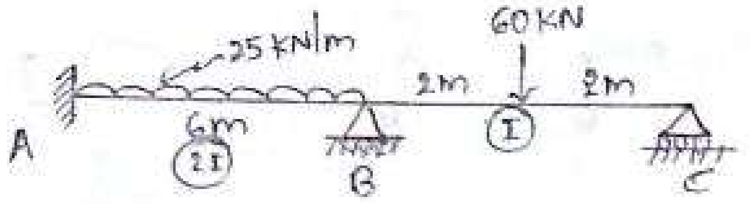
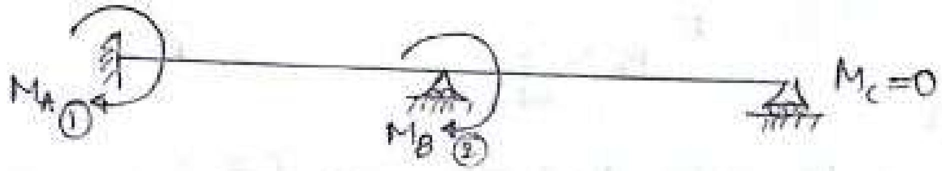
Problems :

1. Analyse the continuous beam by flexibility matrix method. Draw BMD, SFD.



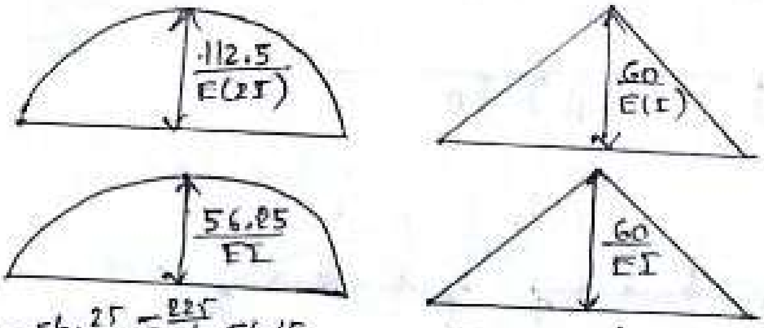
→ $DOR = 4 - 2 = 2$



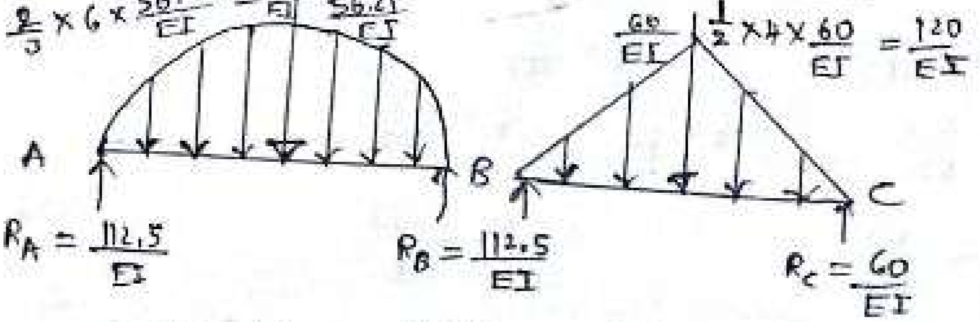
$$\text{w.k.t}\quad [\delta][M] = [\theta_s] - [\theta_L] \quad ***$$

$$M = \begin{bmatrix} M_A \\ M_B \end{bmatrix} ; \quad [\theta_s] = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

But to find $[\theta_L]$.



'$\frac{M}{EI}$' diagram



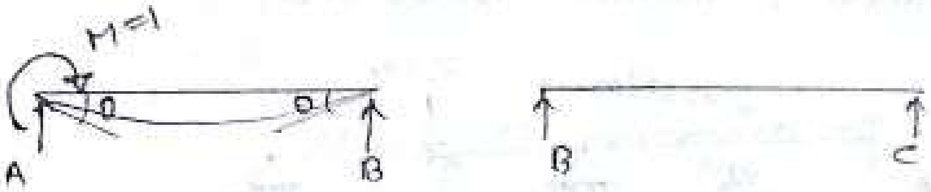
Conjugate beam

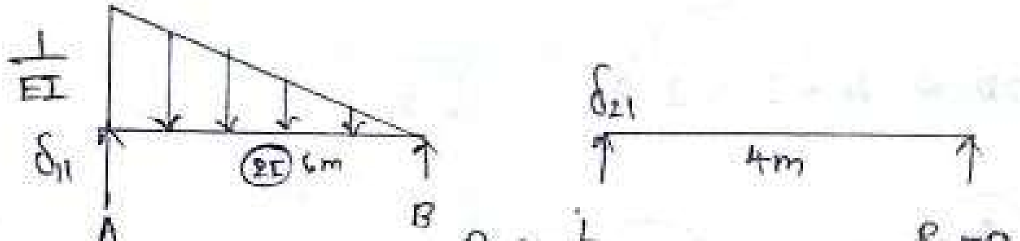
$$\theta_{1L} = \frac{112.5}{EI}$$

$$\theta_{2L} = \frac{112.5}{EI} + \frac{60}{EI} = \frac{172.5}{EI}$$

$$\therefore \quad \theta_L = \begin{bmatrix} 112.5/EI \\ 172.5/EI \end{bmatrix}$$

(i)





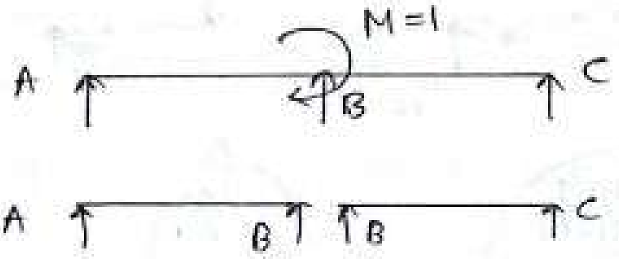
$R_B = \frac{L}{6EI}$ $\quad$ $R_c = 0$

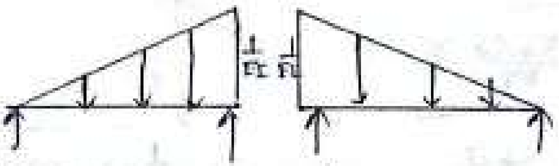
$R_A = \frac{L}{3EI}$

$R_A = \frac{6}{3E(2I)} = \frac{10}{EI}$ $\quad$ $R_B = \frac{6}{6EI(2I)} = \frac{0.5}{EI}$

$\delta_{11} = \frac{1.0}{EI}$ $\quad$ $\delta_{21} = \frac{0.5}{EI}$

(ii) Apply a unit moment at system co-ordinate ②.





$R_A = \frac{L}{6EI}$ $\quad$ $R_B = \frac{L}{3EI}$

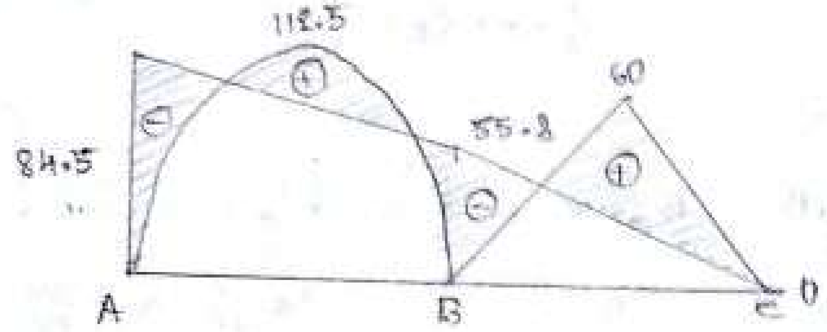
$R_A = \frac{0.5}{EI}$ $\quad$ $R_B = \frac{1}{EI}$ $\quad$ $R_B = \frac{1.33}{EI}$

$\delta_{12} = \frac{0.5}{EI}$ $\quad$ $\delta_{22} = \frac{2.33}{EI}$

$$\therefore \quad \frac{1}{EI}\begin{bmatrix} 1 & 0.5 \\ 0.5 & 2.33 \end{bmatrix}\begin{bmatrix} M_A \\ M_B \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} - \begin{bmatrix} 112.5/EI \\ 172.5/EI \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0.5 \\ 0.5 & 2.33 \end{bmatrix} \begin{bmatrix} M_A \\ M_B \end{bmatrix} = \begin{bmatrix} -112.5 \\ -172.5 \end{bmatrix}$$

$$\begin{bmatrix} M_A \\ M_B \end{bmatrix} = \begin{bmatrix} -84.5 \\ -55.8 \end{bmatrix} \text{ kNm}$$



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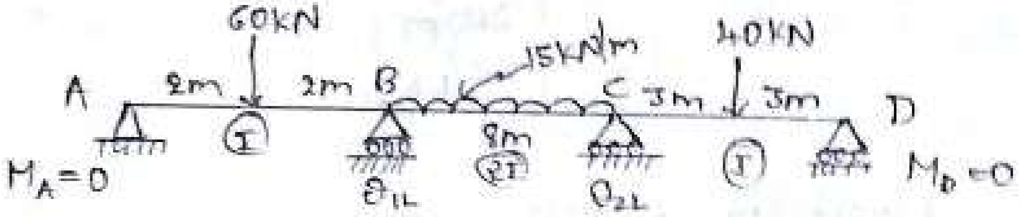
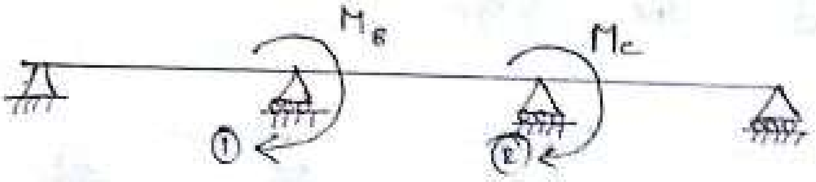
2. Analyse the continuous beam by flexibility matrix method . and draw BMD.



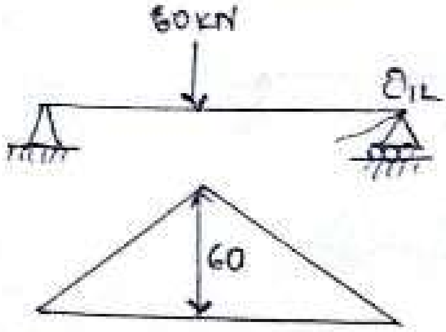
→ DOR = 4 − 2 = 2 (M_B, M_C)

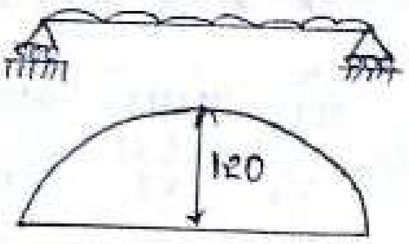


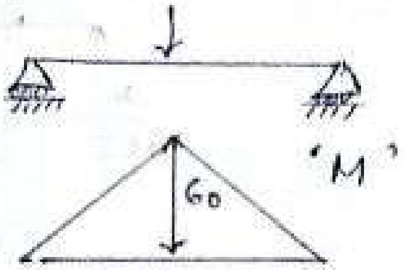
$$[\delta][M] = [\theta_s]-[\theta_L]$$

$$[M] = \begin{bmatrix} M_B \\ M_C \end{bmatrix} \qquad [\theta_s] = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

θ_L is the slope developed at System co-ordinate due to external load.







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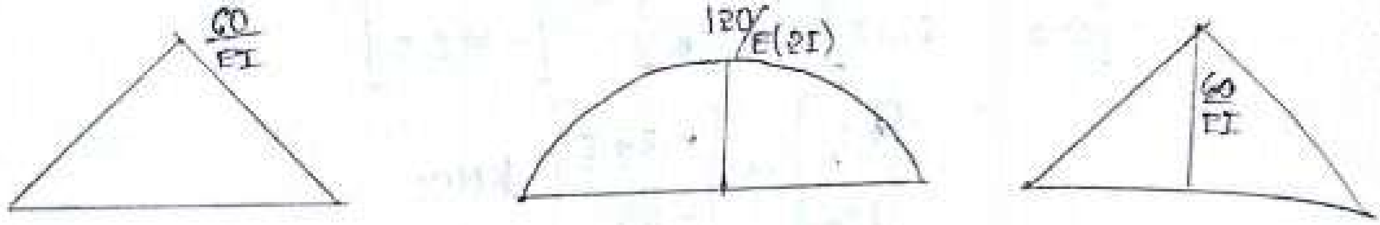
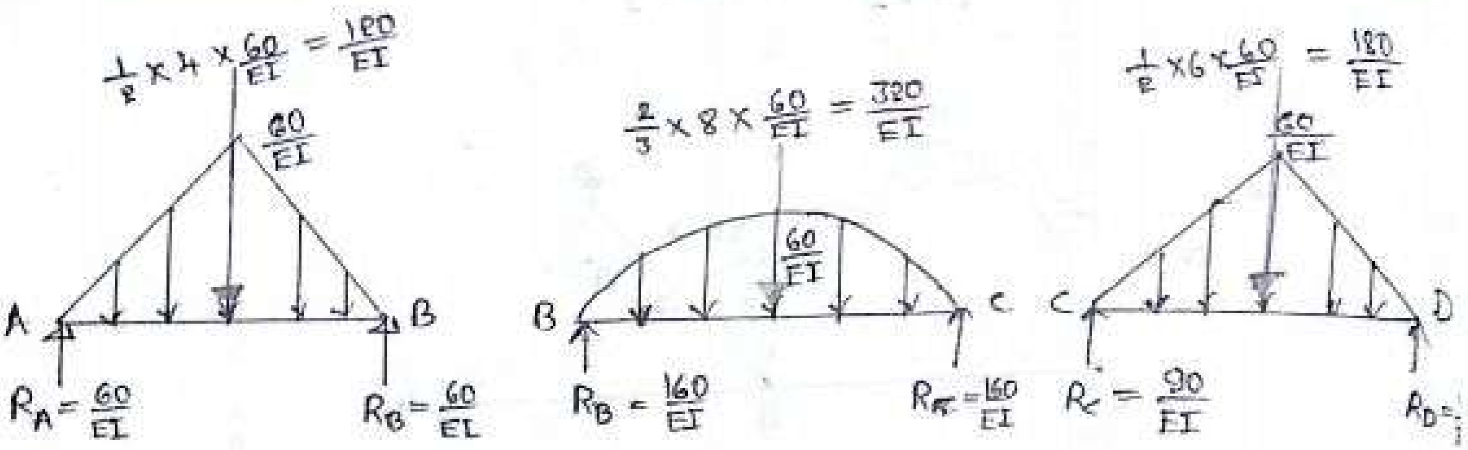




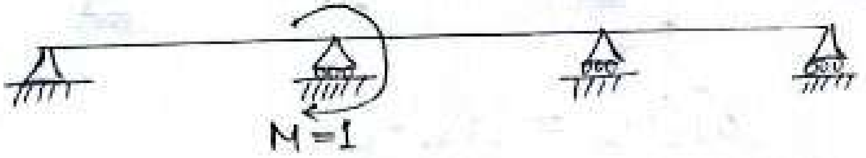
$$\theta_{1L} = \frac{60}{EI} + \frac{160}{EI} \qquad \theta_{2L} = \frac{160}{EI} + \frac{90}{EI}$$

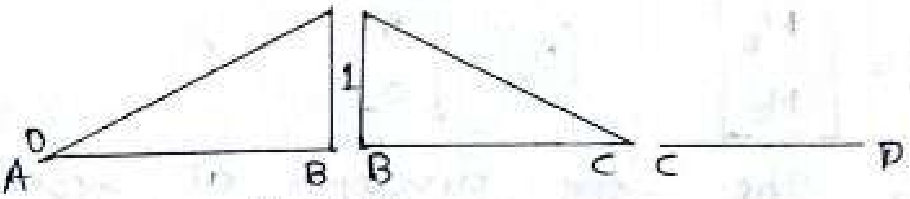
$$\therefore \theta_{1L} = \frac{220}{EI} \qquad \therefore \theta_{2L} = \frac{250}{EI}$$

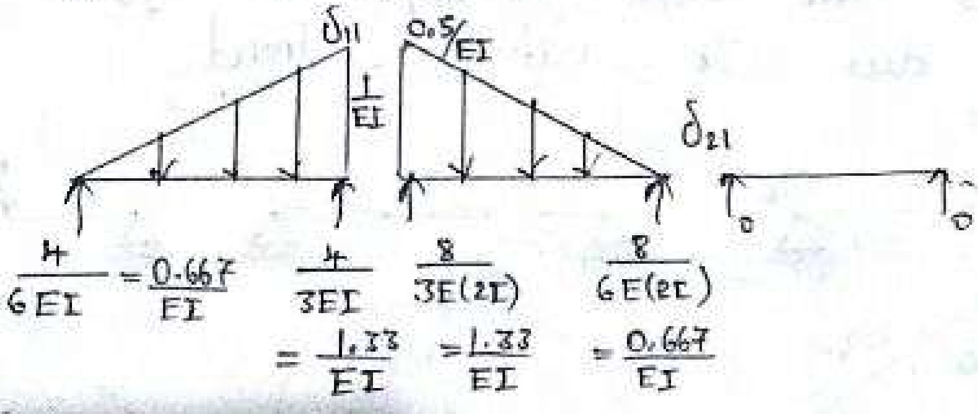
$$\therefore \theta_L = \begin{bmatrix} 220/EI \\ 250/EI \end{bmatrix}$$

To find Flexibility matrix:

(i) Apply a unit moment in system co-ordinate ① and find the slope.



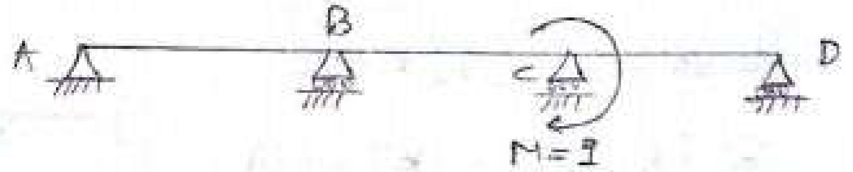


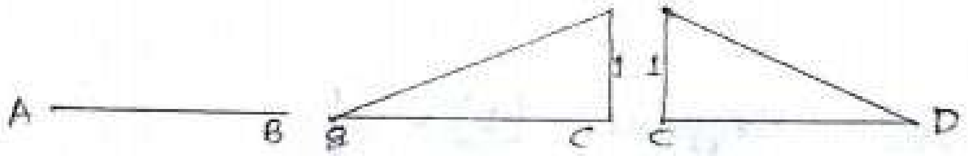


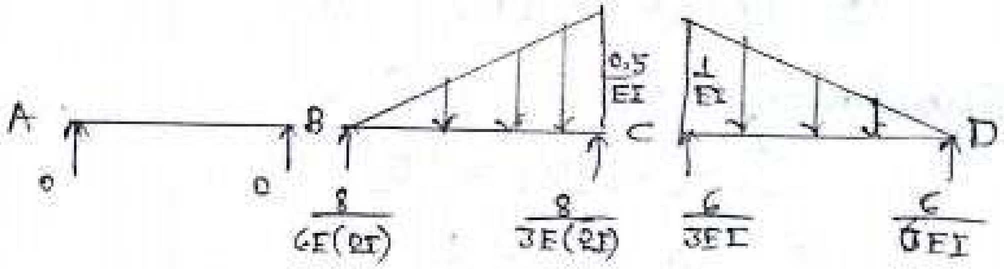
$$\delta_{11} = \frac{1.33}{EI} + \frac{1.33}{EI} + = \frac{2.66}{EI}$$

$$\delta_{21} = \frac{0.667}{EI} + 0 = \frac{0.667}{EI}$$

(ii) Apply a unit moment in the system co-ordinate ② and find the slope.







$R_B = \frac{0.667}{EI}$ $R_c = \frac{1.33}{EI}$ $= 2/EI$ $= \frac{1}{EI}$

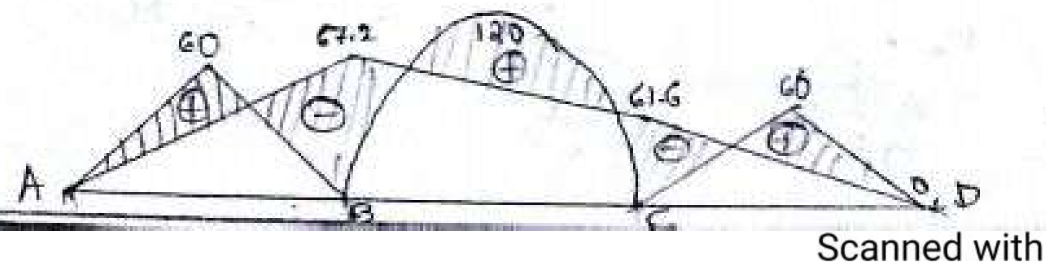
$$\delta_{12} = 0 + \frac{0.667}{EI} = \frac{0.667}{EI}$$

$$\delta_{22} = \frac{1.33}{EI} + \frac{2}{EI} = \frac{3.33}{EI}$$

$$\therefore [\delta] = \frac{1}{EI}\begin{bmatrix} 2.66 & 0.667 \\ 0.667 & 3.33 \end{bmatrix}$$

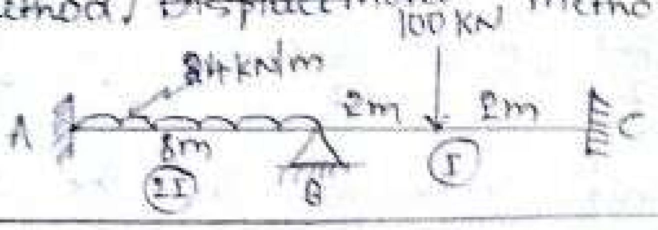
$$\therefore \frac{1}{EI}\begin{bmatrix} 2.66 & 0.667 \\ 0.667 & 3.33 \end{bmatrix}\begin{bmatrix} M_B \\ M_C \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} - \begin{bmatrix} 220/EI \\ 250/EI \end{bmatrix}$$

$$\therefore M_B = -67.26 \text{ KNm}$$
$$M_c = -61.6 \text{ KNm}$$



BMD.

3. Analyse the continuous beam by flexibility matrix method / ~~Displacement~~ Force method.



Relation b/w Flexibility matrix and Stiffness matrix

w.r.t flexibility, $[\delta] = \frac{[\theta]}{[M]}$

(or) $[\delta][M] = [\theta] \rightarrow ①$

Also, Stiffness, $[k] = \frac{[M]}{[\theta]}$

(or) $[k][\theta] = [M] \longrightarrow ②$

Substituting eqⁿ ② in ①,

$$[\delta][k][\theta] = [\theta]$$

$$[\delta][k] = 1$$

$$[\delta] = \frac{1}{[k]} = [k]^{-1}$$

(or) $$[k] = \frac{1}{[\delta]} = [\delta]^{-1}$$

- → Flexibility is inverse of stiffness (or)
- → Stiffness is inverse of flexibility (or)
- → Flexibility and Stiffness are inverse to each other
- → product of flexibility and stiffness yields identity matrix.

$$\boxed{[k][\delta] = [I]}$$

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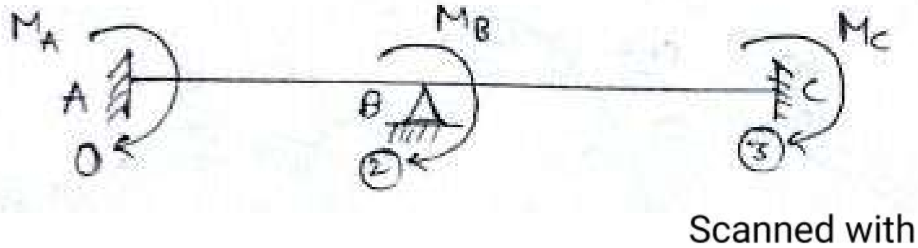
3. → DOR = 5 − 2 = 3 (M_A, M_B, M_C)



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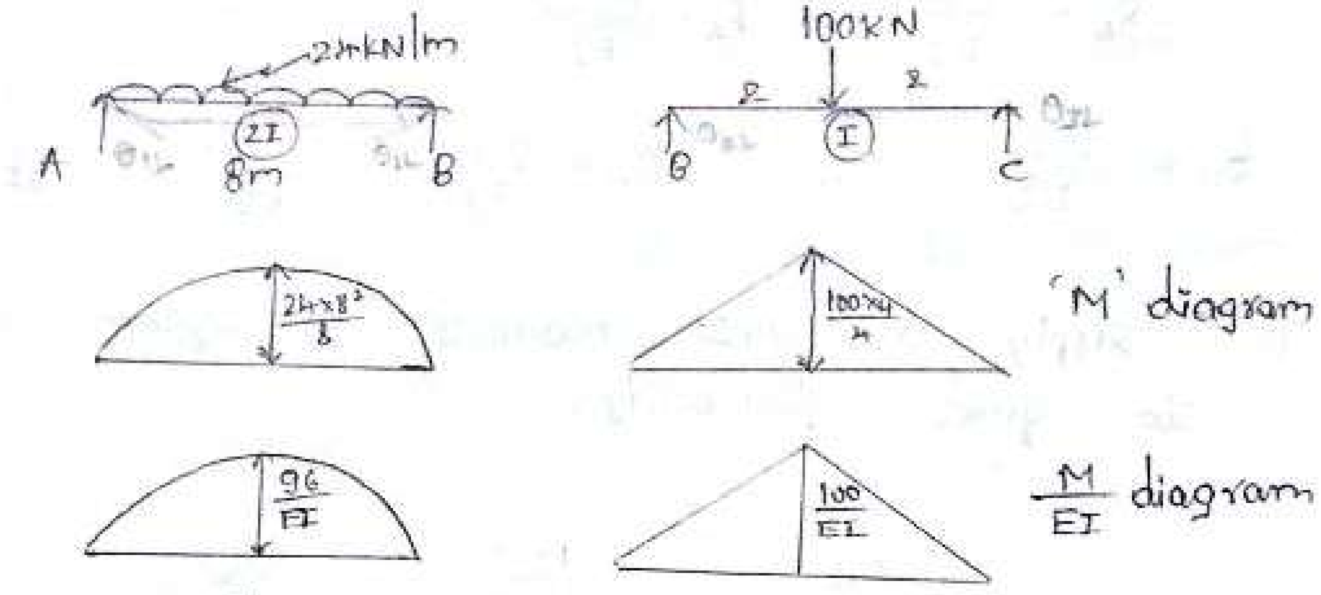
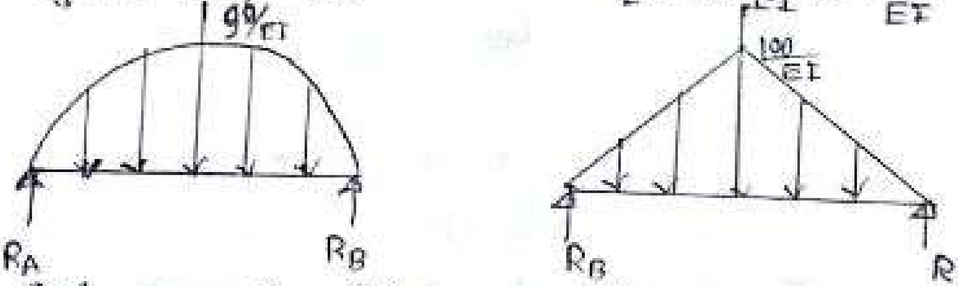
w.k.t $[\delta][M] = [\theta_s] - [\theta_L]$

$$[M] = \begin{bmatrix} M_A \\ M_B \\ M_C \end{bmatrix} ; \; [\theta_s] = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

But to find $[\theta_L]$,



$\frac{2}{3} \times 8 \times \frac{96}{EI} = \frac{512}{EI}$ $\qquad$ $\frac{1}{2} \times 4 \times \frac{100}{EI} = \frac{200}{EI}$



$\therefore R_A = \frac{256}{EI}$ $\quad R_B = \frac{256}{EI}$ $\quad R_B = \frac{100}{EI}$ $\quad R_C = \frac{100}{EI}$

$$\theta_{1L} = \frac{256}{EI} \; ; \; \theta_{2L} = \frac{256}{EI} + \frac{100}{EI} = \frac{356}{EI} \; ; \; \theta_{3L} = \frac{100}{EI}$$

$$\therefore [\theta_L] = \begin{bmatrix} 256/EI \\ 356/EI \\ 100/EI \end{bmatrix}$$

Element flexibility matrix:

(i) Apply a unit moment in system co-ordinate ① to find flexibility matrix:

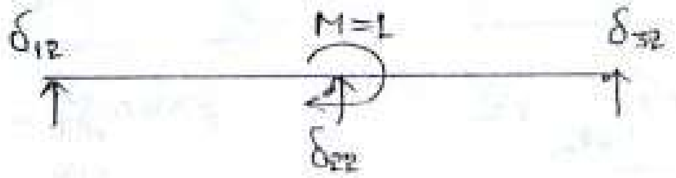
Scanned with CamScanner

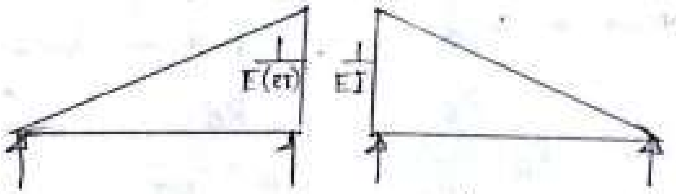
$R_A = \frac{L}{3E(2I)}$ $R_B = \frac{L}{6E(2I)}$ $R_B = 0$ $R_C = 0$

$R_A = \frac{1.33}{EI}$ $R_B = \frac{0.667}{EI}$

$\delta_{11} = \frac{1.33}{EI}$; $\delta_{21} = \frac{0.667}{EI} + 0 = \frac{0.667}{EI}$; $\delta_{31} = 0$

(ii) Apply a unit moment in system co-ordinate ② to find flexibility.



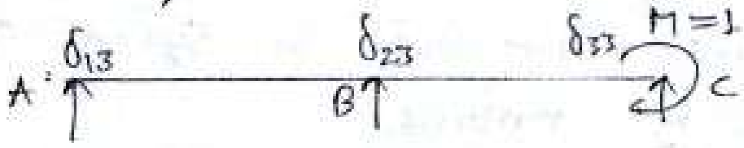


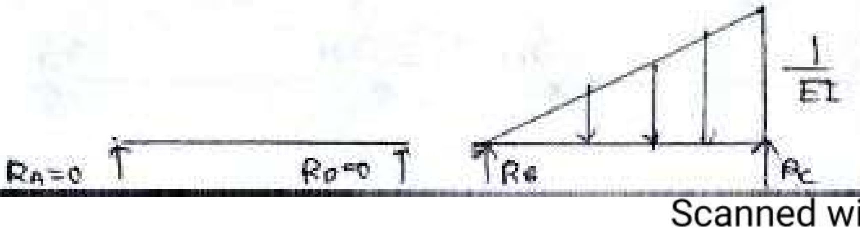
$R_A = \frac{8}{6E(2I)}$ $R_B = \frac{8}{3E(2I)}$ $R_B = \frac{4}{3E(I)}$ $R_C = \frac{4}{6E(I)}$

$R_A = \frac{0.667}{EI}$ $R_B = \frac{1.33}{EI}$ $R_B = \frac{1.33}{EI}$ $R_C = \frac{0.667}{EI}$

$\delta_{12} = \frac{0.667}{EI}$; $\delta_{22} = \frac{1.33}{EI} + \frac{1.33}{EI} = \frac{2.66}{EI}$; $\delta_{32} = \frac{0.667}{EI}$

(iii) Apply a unit moment in system co-ordinate ③ to find flexibility.





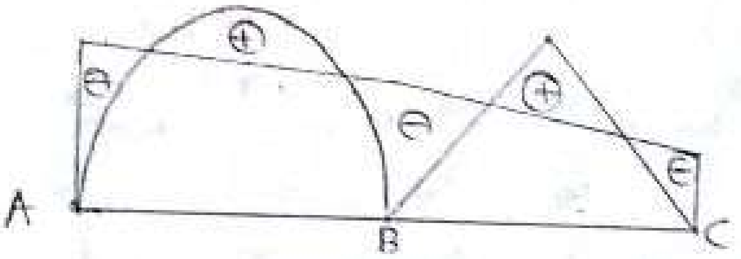
$$R_B = \frac{0.667}{EI} \quad ; \quad R_C = \frac{1.33}{EI}$$

$$\delta_{13} = 0 \quad ; \quad \delta_{22} = 0 + \frac{0.667}{EI} = \frac{0.667}{EI} \quad ; \quad \delta_{33} = \frac{1.33}{EI}$$

$$\therefore \; [\delta][M] = [\theta_s] - [\theta_L]$$

$$\frac{1}{EI}\begin{bmatrix} 1.33 & 0.667 & 0 \\ 0.667 & 0.66 & 0.667 \\ 0 & 0.667 & 1.33 \end{bmatrix}\begin{bmatrix} M_A \\ M_B \\ M_C \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} - \begin{bmatrix} 256 \\ 356 \\ 100 \end{bmatrix} EI$$

$M_A = -147.78$ KNm ; $M_B = -89.13$ KNm ; $M_C = -30.48$ KNm



BMD

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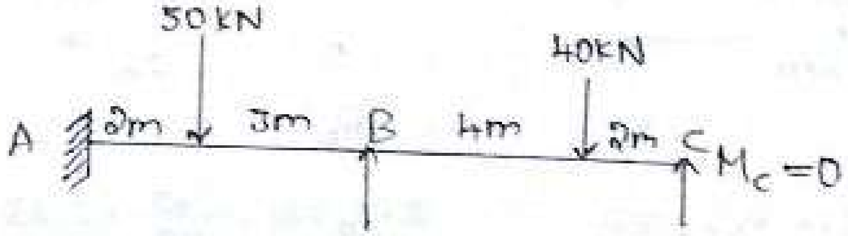
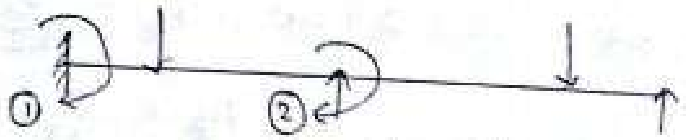
4. Analyse the beam by flexibility matrix method. Draw BMD. EI is constant.



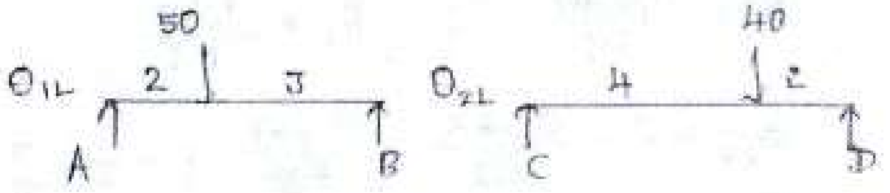
→ DOR = 4 − 2 = 2 (M_A, M_B)



w.k.t $[\delta][M] = [\theta_s] - [\theta_L]$

$$[M] = \begin{bmatrix} M_A \\ M_B \end{bmatrix} \quad ; \quad \theta_s = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

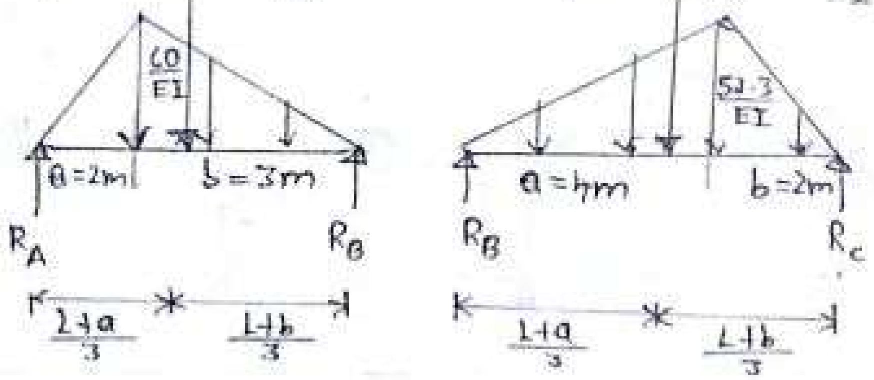
But to calculate $[\theta_L]$:



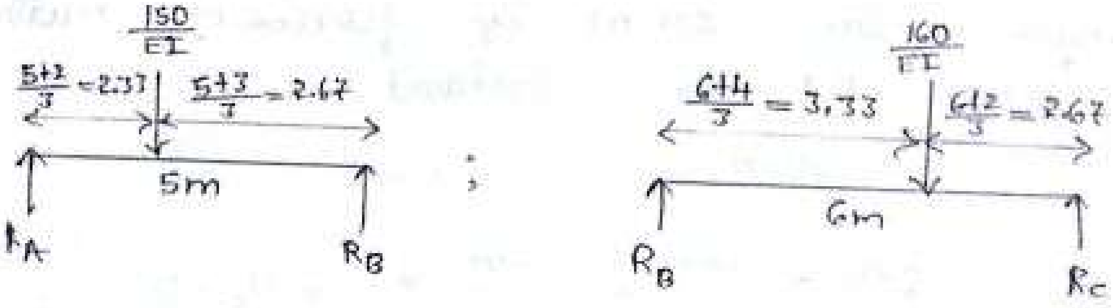
'M' diagram

$\frac{'M'}{EI}$

$\frac{1}{2}\times5\times\frac{60}{EI}=\frac{150}{EI}$ $\frac{1}{2}\times6\times\frac{53.3}{EI}=\frac{160}{EI}$



Conjugate b



$\Sigma M_A=0\,;\ \frac{150}{EI}\times2.33-R_B\times5=0$

$R_B=\frac{69.99}{EI}\Rightarrow\frac{70}{EI}$

$\Sigma V=0\,;\ R_A-\frac{150}{EI}+\frac{70}{EI}=0$

$\therefore\ R_A=\frac{80}{EI}$

$\Sigma M_B=0\,;\ \frac{160}{EI}\times3.33-R_c\times6=0$

$\therefore\ R_c=\frac{88.8}{EI}$

$\Sigma V=0\,;\ R_B-\frac{160}{EI}+\frac{88.8}{EI}=0$

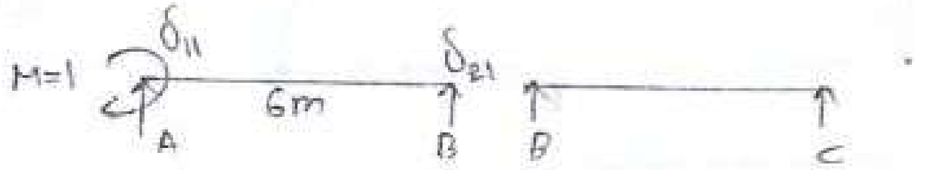
$\therefore\ R_B=\frac{71.2}{EI}$

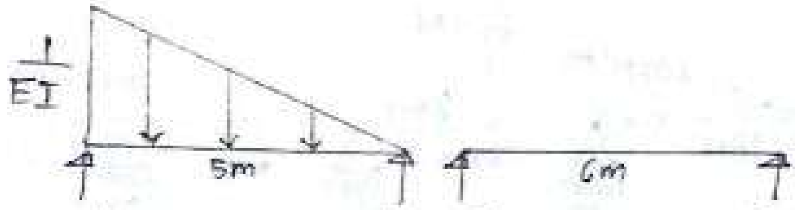
$\theta_{1L}=R_A=\frac{80}{EI}\ ;\ \theta_{2L}=\frac{70.0}{EI}+\frac{71.2}{EI}=\frac{141.2}{EI}$

$$\therefore\ [\theta_L]=\begin{bmatrix}80/EI\\141.2/EI\end{bmatrix}$$

Flexibility matrix :

(i) Apply a unit moment in ① to find flexibility.



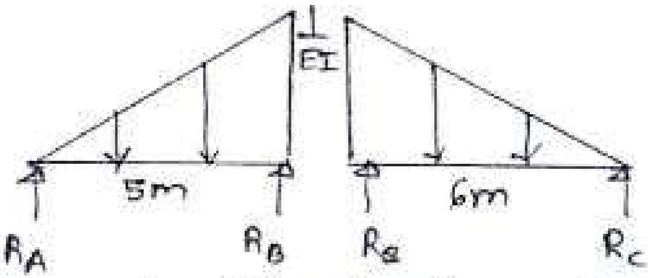


$R_A = \frac{L}{3EI}$ $\quad R_B = \frac{L}{6EI}$ $\quad R_B = 0$ $\quad R_C = 0$

$R_A = \frac{1.67}{EI}$ $\quad R_B = \frac{0.83}{EI}$

$$\therefore \quad \delta_{11} = \frac{1.67}{EI} \quad ; \quad \delta_{21} = \frac{0.83}{EI} + 0 = \frac{0.83}{EI}$$

(ii) Apply a unit moment in ② to find flexibility.



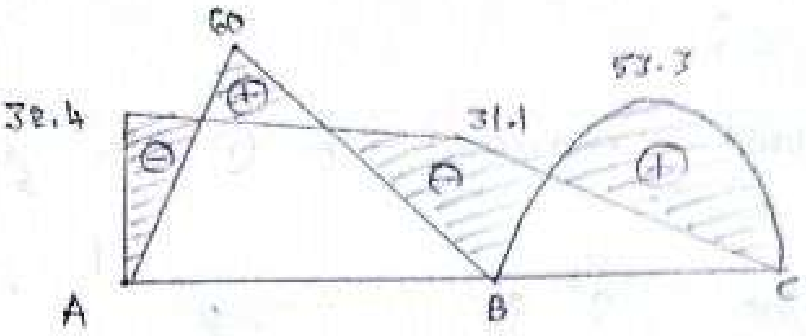
$R_A = \frac{0.83}{EI}$ $\quad R_B = \frac{1.67}{EI}$ $\quad R_B = \frac{2}{EI}$ $\quad R_C = \frac{1.0}{EI}$

$$\delta_{12} = \frac{0.83}{EI} \quad ; \quad \delta_{22} = \frac{1.67}{EI} + \frac{2}{EI} = \frac{3.67}{EI}$$

$$\therefore \quad \delta = \frac{1}{EI}\begin{bmatrix} 1.67 & 0.83 \\ 0.83 & 3.67 \end{bmatrix}$$

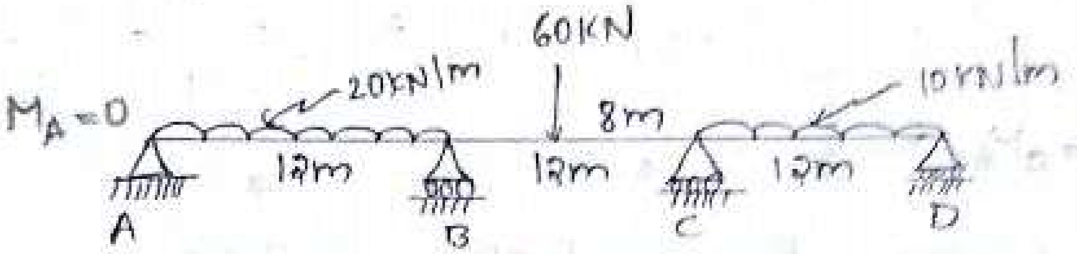
$$\therefore \quad \frac{1}{EI}\begin{bmatrix} 1.67 & 0.83 \\ 0.83 & 3.67 \end{bmatrix}\begin{bmatrix} M_A \\ M_B \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} - \begin{bmatrix} 80 \\ 141.2 \end{bmatrix}\frac{1}{EI}$$

$M_A = -32.4$ KNm ; $M_B = -31.1$ KNm , $M_C = 0$



BMD

5.



→ DOR = 4 − 2 = 2 (M_B, M_C)

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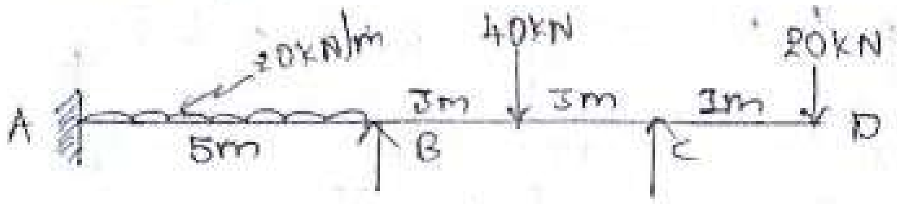
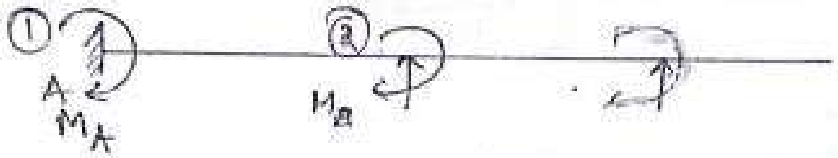
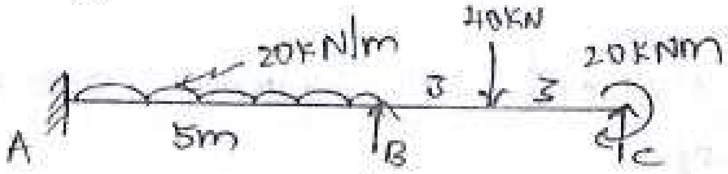
6. Analyse the continuous beam by flexibility matrix method.



→ DOR = 4 − 2 = 2 (

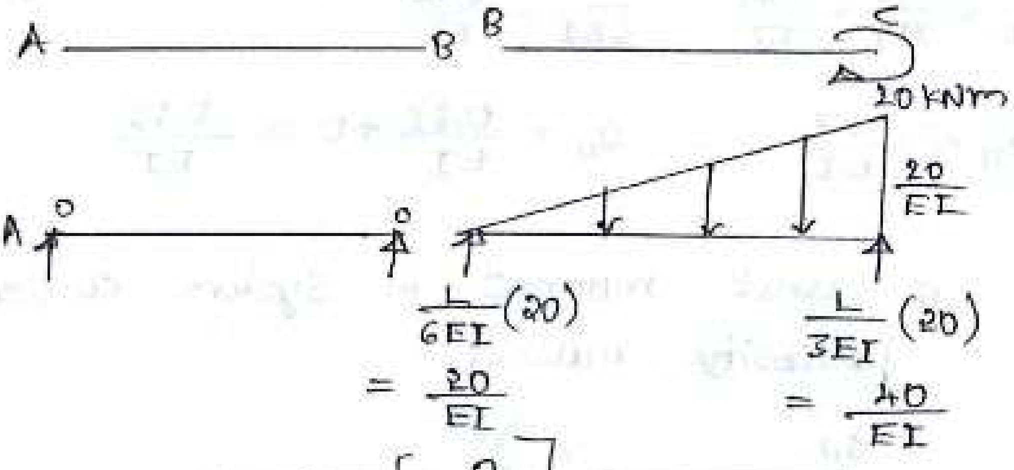




Take the moment 20 KNm as the load at the support.

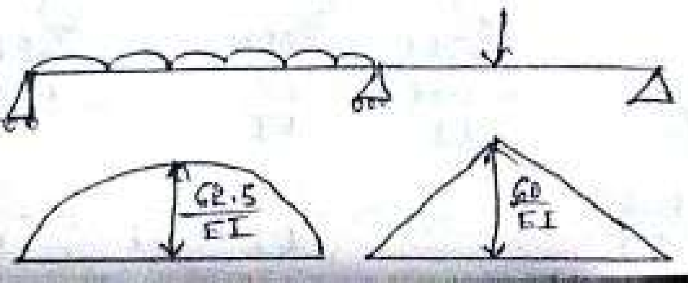
w.k.t $[\delta][M] = [\theta_s] - [\theta_L]$

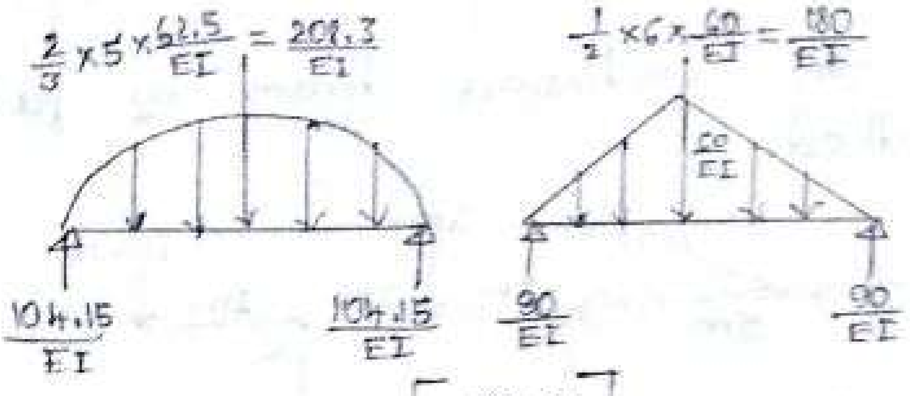
But $[M] = \begin{bmatrix} M_A \\ M_B \end{bmatrix}$; $[\theta_s] = \begin{bmatrix} 0 \\ \theta_{s2} \end{bmatrix}$



$\therefore [\theta_s] = \begin{bmatrix} 0 \\ 20/EI \end{bmatrix}$

To find $[\theta_L]$:

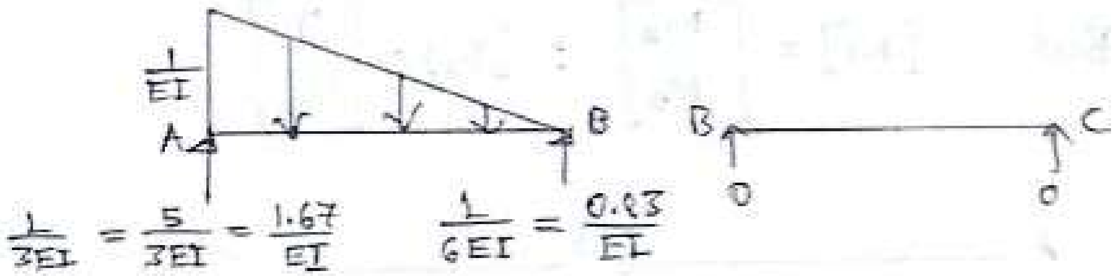




$$\therefore [\theta_L] = \begin{bmatrix} \frac{104.15}{EI} \\ \frac{194.15}{EI} \end{bmatrix}$$

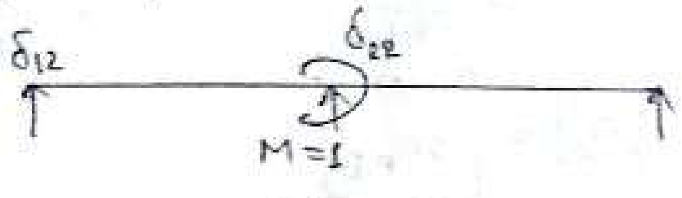
To find [δ] :

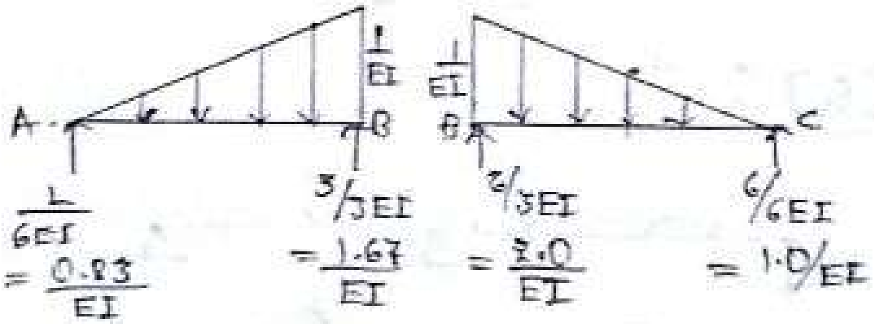
(i) Apply unit moment in system co-ordinate ①
find flexibility.



$$\therefore \delta_{11} = \frac{1.67}{EI} \quad ; \quad \delta_{21} = \frac{0.83}{EI} + 0 = \frac{0.83}{EI}$$

(ii) Apply a unit moment in system co-ordinate ②
find flexibility matrix.



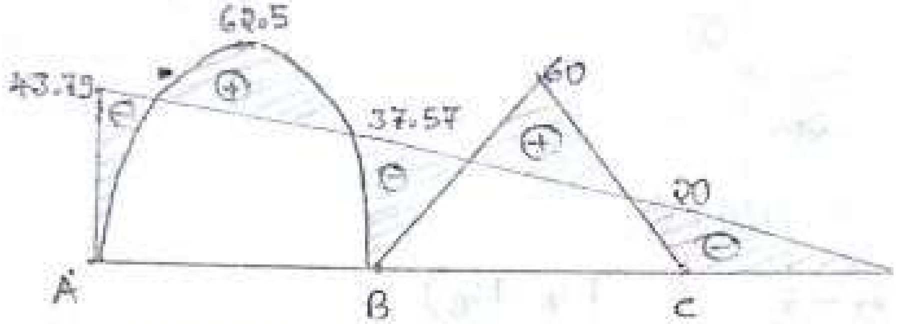


$$\therefore \delta_{12} = \frac{0.83}{EI} \quad ; \quad \delta_{22} = \frac{1.67}{EI} + \frac{2}{EI} = \frac{3.67}{EI}$$

$$\therefore [\delta] = \frac{1}{EI}\begin{bmatrix} 1.67 & 0.83 \\ 0.83 & 3.67 \end{bmatrix}$$

$$\therefore \frac{1}{EI}\begin{bmatrix} 10.67 & 0.83 \\ 0.83 & 3.67 \end{bmatrix}\begin{bmatrix} M_A \\ M_B \end{bmatrix} = \frac{1}{EI}\begin{bmatrix} 0 \\ 20 \end{bmatrix} - \frac{1}{EI}\begin{bmatrix} 104.15 \\ 194.15 \end{bmatrix}$$

$\therefore M_A = -43.70$ KNm ; $M_B = -37.57$ KNm

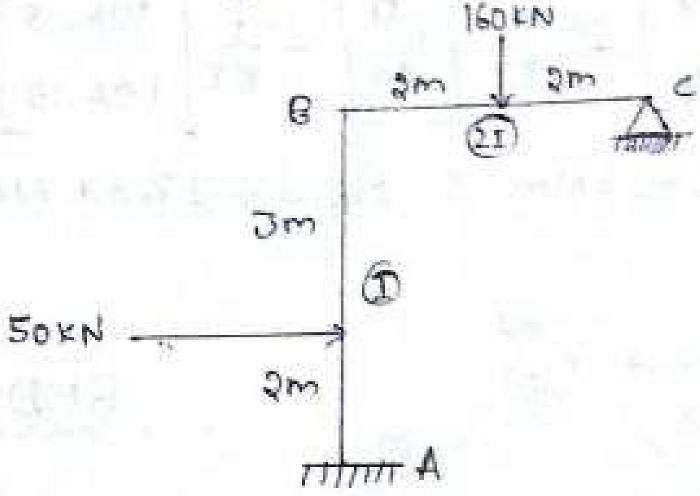


BMD

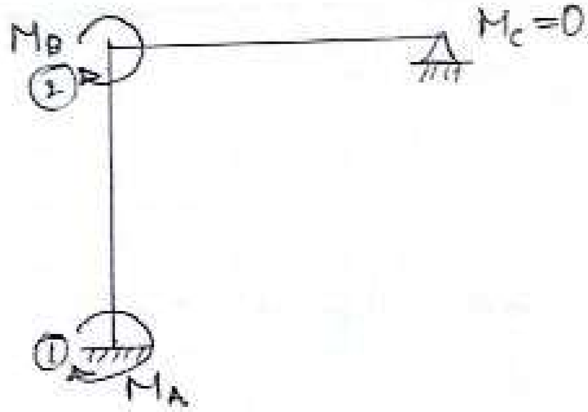
16/11/18

Frames:

1. Analyse L-frame by flexibility matrix method.



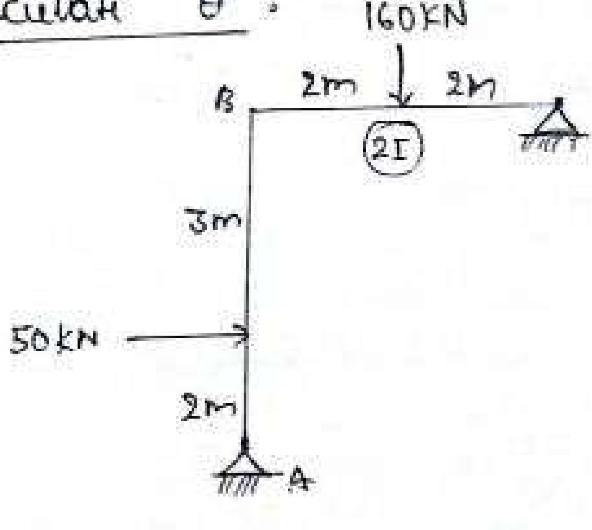
→ DOR = 4−2 = 2 (M_A, M_B)

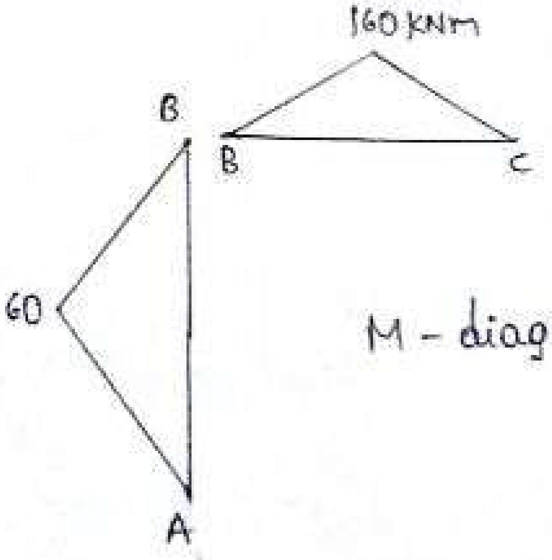


w.k.t $[\delta][M] = [\theta_S]-[\theta_L]$

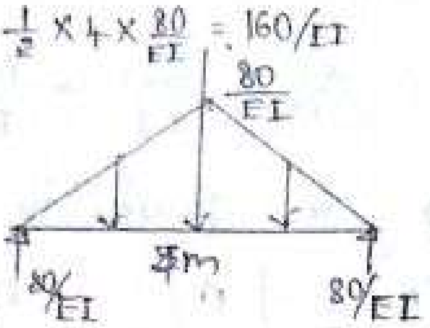
$$[M] = \begin{bmatrix} M_A \\ M_B \end{bmatrix} ; \quad [\theta_s] = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

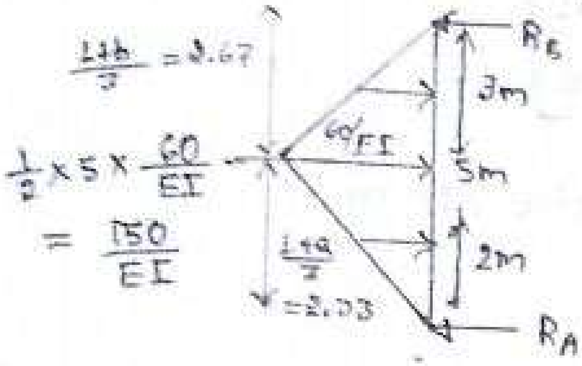
To calculate 'θ':





M - diagram





$$\Sigma M_A = 0;\ \frac{150}{EI}\times2.33 - R_B\times5 = 0$$

$$\therefore R_B = \frac{70}{EI}$$

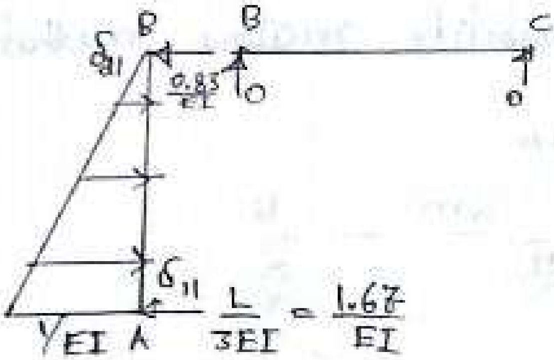
$$\Sigma V = 0;\ R_A - \frac{150}{EI} + \frac{70}{EI} = 0$$

$$\therefore R_A = \frac{80}{EI}$$

$$\therefore [\theta_L] = \begin{bmatrix} 80/EI \\ 150/EI \end{bmatrix}$$

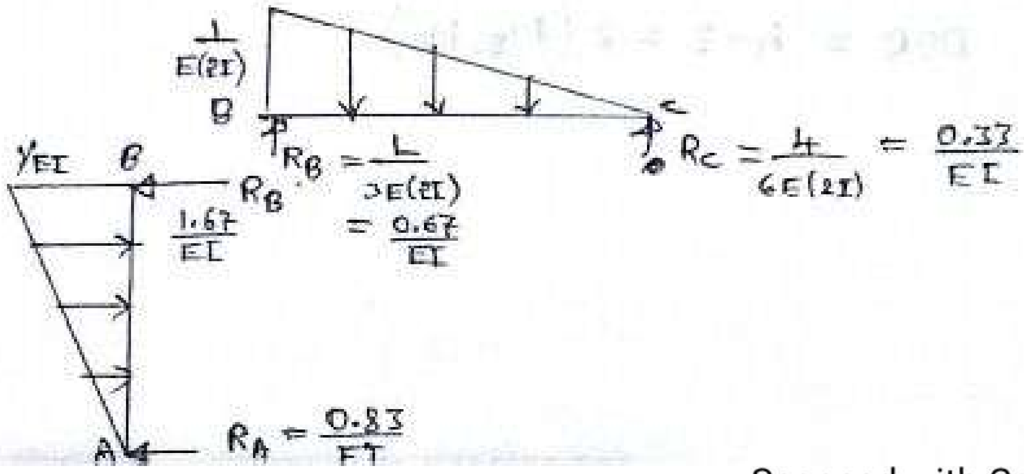
To find flexibility:

(i) Apply a unit moment in ① to find δ,



$$\therefore \delta_{11} = \frac{1.67}{EI};\ \delta_{21} = \frac{0.83}{EI} + 0 = \frac{0.83}{EI}$$

(ii) Apply a unit moment in ② to find δ,

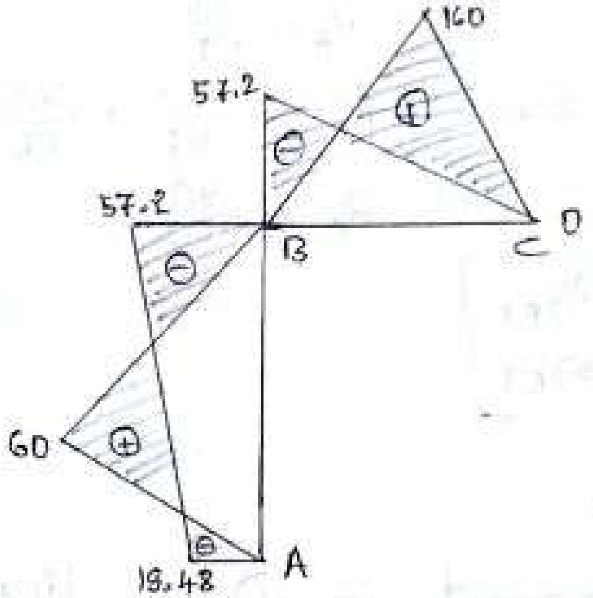


$$\therefore \delta_{12} = \frac{0.83}{EI} \; ; \; \delta_{22} = \frac{1.67}{EI} + \frac{0.67}{EI} = \frac{2.34}{EI}$$

Substituting in flexibility eq^n,

$$\frac{1}{EI}\begin{bmatrix} 1.67 & 0.83 \\ 0.83 & 2.34 \end{bmatrix}\begin{bmatrix} M_A \\ M_B \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} - \begin{bmatrix} -80 \\ -150 \end{bmatrix}$$

$$\therefore M_A = -19.48 \text{ kNm} \; ; \; M_B = -57.2 \text{ kN-m}$$



BMD

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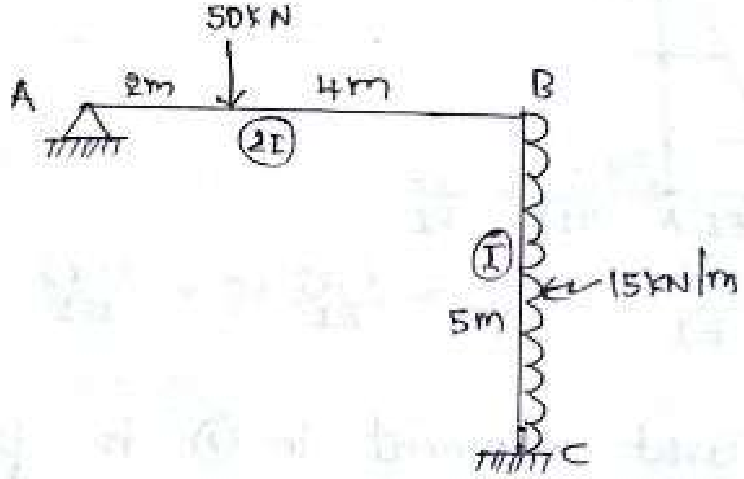
2. Analyse by flexibility matrix method.



→ $DOR = 4 - 2 = 2 \; (M_B, M_C)$

Nholis

SUJITH.N.S
V.'A'

Module-5

Stiffness Matrix method.
(System approach)

Stiffness:

The load required to produce unit deflection @ the moment required to produce unit rotation is called stiffness denoted by the letter 'k'

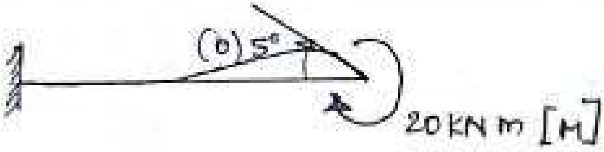
Fig(a):

5 mm → 10 KN

1 mm → $\left(\frac{10}{5}\right)$ KN/mm

$[K]=\left[\frac{P}{\Delta}\right]$ in KN/mm

Fig(b):



For 5° → 20KNm

1° → $\left(\frac{20}{5}\right)$ KN-m/rad

$[K]=\left[\frac{M}{\theta}\right]$

Stiffness is also defined as the ratio of moment of inertia to length.

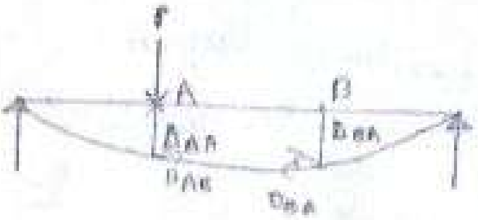
i.e., $k = I/L$

$k \propto I$ & $k \propto 1/L$

According to Maxwell's reciprocal theorem,

$\Delta_{AB} = \Delta_{BA}$

@ $\theta_{AB} = \theta_{BA}$



In general, $K_{12} = K_{21}$

Position of def^n/slope

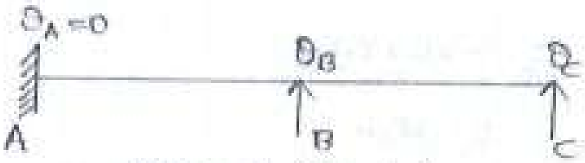
Position of ~~slope~~ load

System co-ordinate:

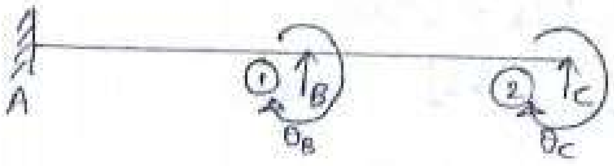
Giving / Assigning the numbers to the unknown displacement component called system co-ordinate, written inside the circle.

No. of system co-ordinates = No. of DOF's

Eg:



DOF = 2 (θ_B, θ_C)



System co-ordinate

Basic equation of stiffness matrix method is

$$[K][\theta] = [P_S] - [P_L]$$

where, $[K]$ = Stiffness at the system co-ordinate

$[\theta]$ = Unknown rotation

$[P_S]$ = Extra moment / load acting in system co-ordinate

$[P_L]$ = Sum of the fixed end moments acting at the system co-ordinate

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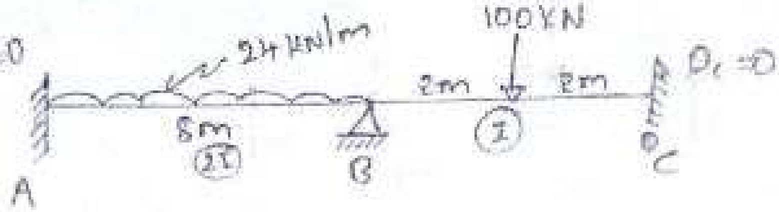
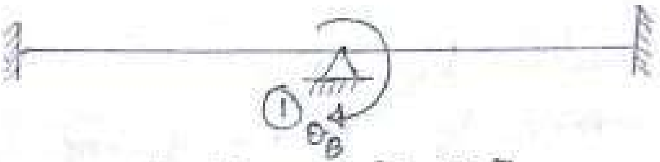
Problems:

1. Analyse the given beam by Stiffness matrix method. Draw BMD.

15/10/18



→ DOF = 1 (θ_B)



$$[k][\theta] = [P_s] - [P_L]$$

Here $[\theta] = [\theta_B]$

Also, $[P_s] = 0$

But $M_{FAB} = \frac{-24\times8^2}{12} = -128$ kNm

$M_{FBA} = 128$ kNm

$M_{FBC} = -50$ kNm

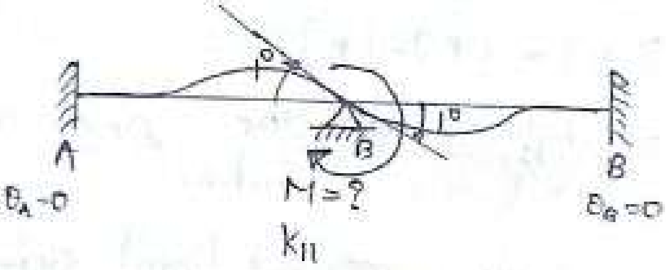
$M_{FCB} = 50$ kNm

$\therefore [P_L] = [M_{FBA} + M_{FBC}]$

$= 128 + (-50)$

$\therefore [P_L] = [78]$

To find the stiffness :



$$K_{11} = M_{BA} + M_{BC}$$

$$= \frac{2EI}{L}(2\theta_B+\theta_A) + \frac{2EI}{L}(2\theta_B+\theta_C]$$

$$= \frac{2E(2I)}{8}(2\times1+0) + \frac{2EI}{4}(2\times1)$$

$$= 1.0\,EI + 1.0\,EI$$

$$K_{11} = 2.0\,EI$$

Now, $[K][\theta] = [P_s]-[P_L]$

$$[2.0EI][\theta_B] = [0]-[78]$$

$$[\theta_B] = -\frac{78}{2.0EI}$$

$$[\theta_B] = -39/EI$$

Final moments:

$$M_{AB} = -128 + \frac{2E(2I)}{8}\left(2\times0+\left(\frac{-39}{EI}\right)\right) - \cancel{\frac{3\Delta}{L}}^{\to 0}$$

$$= -128 + \frac{4EI}{8}\left(\frac{-39}{EI}\right)$$

$\therefore$ $M_{AB} = -147.5$ KNm

$$M_{BA} = 128 + \frac{2E(2I)}{8}\left(2\left(\frac{-39}{EI}\right)+0\right) - \cancel{\frac{3\Delta}{L}}^{\to 0}$$

$\therefore$ $M_{BA} = 89$ KNm

$$M_{BC} = -50 + \frac{2EI}{4}\left(2\left(\frac{-39}{EI}\right)+0\right) - \cancel{\frac{3\Delta}{L}}^{\to 0}$$

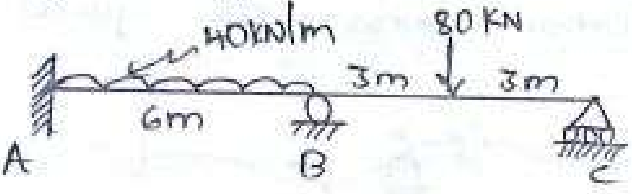
$$M_{BC} = -89 \text{ KNm}$$

$$M_{CB} = 50 + \frac{2EI}{4}\left(2\times0 - \frac{39}{EI} - \cancel{\frac{3\Delta}{L}}^{0}\right)$$

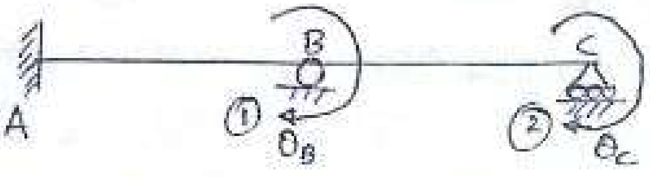
$$= 30.5 \text{ KNm}$$

---

2. Analyse the continuous beam shown in the figure by stiffness matrix method.



→ DOF = 2 (θ_B & θ_C)



$$[K][\theta] = [P_s]-[P_L]$$

$$[\theta] = \begin{bmatrix}\theta_B\\ \theta_C\end{bmatrix}$$

But $$[P_S] = \begin{bmatrix} P_{S_1} \\ P_{S_2} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$[P_L] = \begin{bmatrix} P_{L_1} \\ P_{L_2} \end{bmatrix}$$

Fixed End Moments :

$$M_{FAB} = \frac{-40\times6^2}{8} = -120 \text{ KNm}$$

$$M_{FBA} = 120 \text{ KNm}$$

$$M_{FBC} = -60 \text{ KNm}$$

$$M_{FCB} = 60 \text{ kNm}$$

Now, $P_{L_1} = M_{FBA} + M_{FBC}$

$= 120 + (-60)$

$= 60$

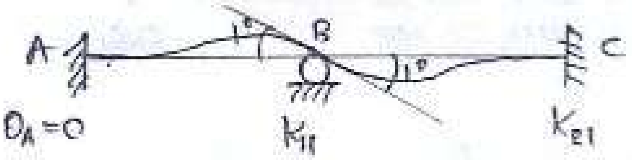
$P_{L_2} = M_{FCB}$

$= 60$

$$\therefore \quad P_L = \begin{bmatrix} 60 \\ 60 \end{bmatrix}$$

To find the stiffnees :

Apply the moment 'M' to produce 1° rotation keeping all other ends as fixed.



$K_{11} = M_{BA} + M_{BC}$

$= \frac{2EI}{6}(2\times1+0) + \frac{2EI}{6}(2\times1)$

$= 1.33 \text{ EI}$

$K_{21} = M_{CB}$

$= \frac{2EI}{6}(2\times0+1)$

$= 0.333 \text{ EI}$

Apply the moment 'M' in system co-ordinate ① to produce 1° rotation keeping all other ends as fixed.

$K_{12} = M_{BA} + M_{BC}$

$= \frac{2EI}{6}(2\times0+0) + \frac{2EI}{6}(2\times0+1)$

$K_{12} = 0.333\ EI$

$K_{22} = M_{CB}$

$= \frac{2EI}{6}(2\times1+0)$

$= 0.67\ EI$

$$\therefore\ K = \begin{array}{c|cc} & ① & ② \\ \hline ① & 1.33\,EI & 0.333\,EI \\ ② & 0.333\,EI & 0.67\,EI \end{array}$$

Now, $[K][\theta] = [P_s] - [P_L]$

$$\begin{bmatrix} 1.33EI & 0.333EI \\ 0.333EI & 0.67EI \end{bmatrix}\begin{bmatrix} \theta_B \\ \theta_C \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} - \begin{bmatrix} 60 \\ 60 \end{bmatrix}$$

$$EI\begin{bmatrix} 1.33 & 0.333 \\ 0.333 & 0.67 \end{bmatrix}\begin{bmatrix} \theta_B \\ \theta_C \end{bmatrix} = \begin{bmatrix} -60 \\ -60 \end{bmatrix}$$

$1.33\overset{EI}{\theta_D} + 0.333\overset{EI}{\theta_C} = -60$

$0.333\overset{EI}{\theta_B} + 0.67\overset{EI}{\theta_C} = -60$

$\therefore\ [\theta_B] = \frac{-25.92}{EI}\ ;\ [\theta_C] = \frac{-76.67}{EI}$

Final moments:

$\therefore\ M_{AB} = -120 + \frac{2EI}{6}\left(2\times0 + \left(\frac{-25.92}{EI}\right) - \overset{\to 0}{\frac{3\Delta}{L}}\right)$

$M_{AB} = -128.64\ KNm$

$M_{BA} = 120 + \frac{2EI}{6}\left(2\left(\frac{-25.92}{EI}\right) + 0 - \overset{0}{\frac{3\Delta}{L}}\right)$

$\therefore\ M_{BA} = 102.7\ KNm$

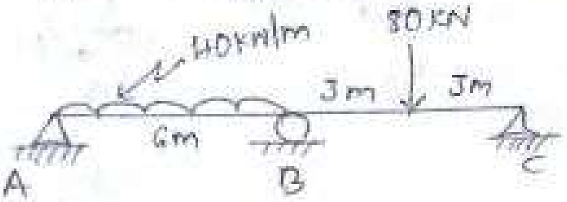
$M_{BC} = -60 + \frac{2EI}{6}\left(2\left(\frac{-25.92}{EI}\right) + \left(-\frac{76.67}{EI}\right) - \overset{0}{\frac{3\Delta}{L}}\right)$

$M_{BC} = -102.7\ KNm$

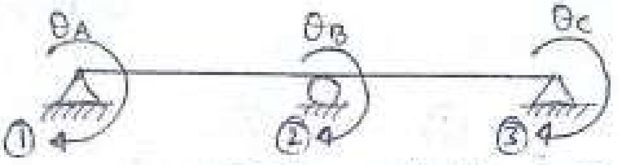
$M_{CB} = 60 + \frac{2EI}{6}\left(2\left(-\frac{76.67}{EI}\right) + \left(-\frac{25.92}{EI}\right) - 0\right)$

$\therefore\ M_{CB} = 0.0\ KNm$

3. Analyse by Stiffness matrix method.



→ DOF = 3 $(\theta_A, \theta_B, \theta_C)$



$$[K][\theta] = [P_S] - [P_L]$$

Here $[\theta] = \begin{bmatrix} \theta_A \\ \theta_B \\ \theta_C \end{bmatrix}$; $[P_S] = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$; $[P_L] = \begin{bmatrix} P_{L_1} \\ P_{L_2} \\ P_{L_3} \end{bmatrix}$

FEM's :

$$M_{FAB} = \frac{-WL^2}{12} = \frac{-40 \times 6^2}{12} = -120 \text{ kNm}$$

$$M_{FBA} = 120 \text{KNm}$$

$$M_{FBC} = \frac{-WL}{8} = \frac{-80 \times 6}{8} = -60 \text{ kNm}$$

$$M_{FCB} = 60 \text{KNm}$$

∴ $P_{L_1} = M_{FAB} = -120$ KNm

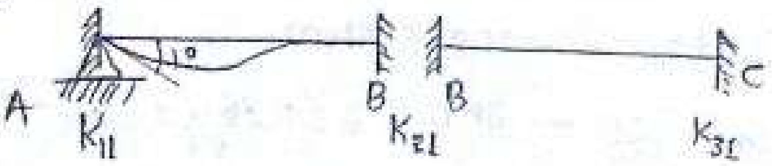
$P_{L_2} = M_{FBA} + M_{FBC} = 120 - 60 = 60$ kNm

$P_{L_3} = M_{FCB} = 60$ kNm

$$\therefore P_L = \begin{bmatrix} -120 \\ 60 \\ 60 \end{bmatrix}$$

To find the stiffness :

Apply the moment in the system co-ordinate ① to produce 1° rotation keeping all other ends as fixed.



$$K_1 = M_{AB}$$

$$K_{11} = \frac{2EI}{6}(2\times1+0) = 0.667\ EI$$

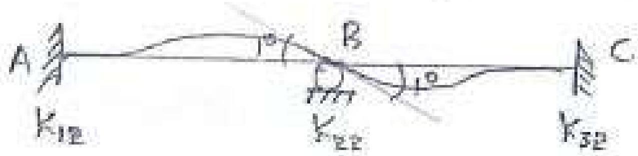
$$K_{21} = M_{BA} + M_{BC}$$

$$K_{21} = \frac{2EI}{6}(2\times0+1) + \frac{2EI}{6}(2\times0+0) = 0.333\ EI$$ ~~KN~~

$$K_{31} = M_{CB}$$

$$K_{31} = \frac{2EI}{6}(2\times0+0) = 0\ \text{KNm}$$

Now, apply the moment 'M' in the system co-ordination ② to produce 1° rotation keeping all other ends as fixed.

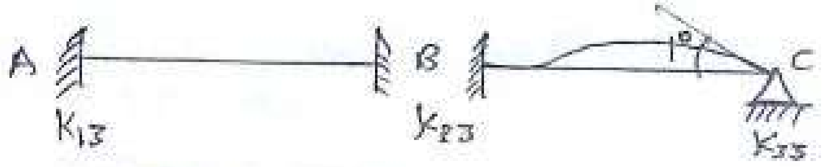


$$K_{12} = M_{AB} = \frac{2EI}{6}(2\times0+1) = 0.333\ EI$$

$$K_{22} = \frac{2EI}{6}(2\times1+0) + \frac{2EI}{6}(2\times1+0) = 1.334\ EI$$

$$K_{32} = \frac{2EI}{6}(2\times0+1) = 0.333\ EI$$

Now, apply the moment 'M' in the system co-ordination ③ to produce 1° rotation keeping all other ends as fixed.



$$K_{13} = \frac{2EI}{6}(2\times0+0) = 0\ \text{KNm}$$

$$K_{23} = \frac{2EI}{6}(2\times0+0) + \frac{2EI}{6}(2\times0+1) = 0.333\ EI$$

$$K_{33} = \frac{2EI}{6}(2\times1+0) = 0.667\ EI$$

$$\therefore\ K = EI \begin{array}{c|ccc|} & ① & ② & ③ \\ ① & 0.667 & 0.333 & 0 \\ ② & 0.333 & 1.334 & 0.333 \\ ③ & 0 & 0.333 & 0.667 \end{array}$$

$$\therefore EI \begin{bmatrix} 0.667 & 0.333 & 0 \\ 0.333 & 1.334 & 0.333 \\ 0 & 0.333 & 0.667 \end{bmatrix} \begin{bmatrix} \theta_A \\ \theta_B \\ \theta_C \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} - \begin{bmatrix} -120 \\ 60 \\ 60 \end{bmatrix}$$

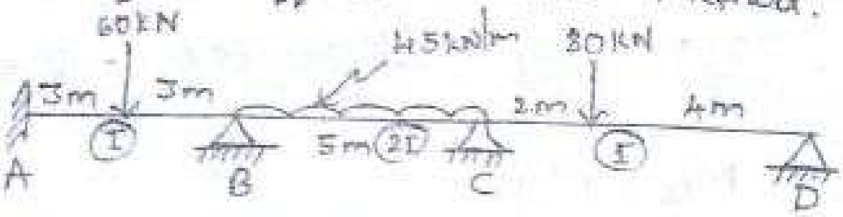
$$0.667 EI\,\theta_A + 0.333\,EI\,\theta_B + 0.\theta_C = +120$$

$$0.333 EI\,\theta_A + 1.334 EI\,\theta_B + 0.333\,\theta_C = -60$$

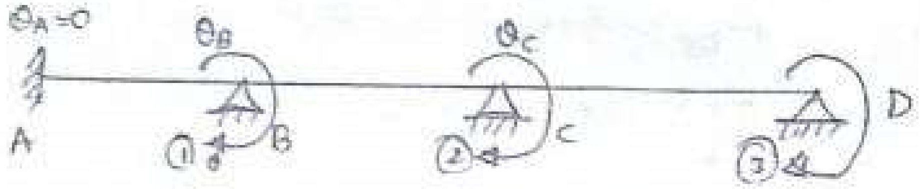
$$0.\theta_A + 0.333\,EI\,\theta_B + 0.667\,\theta_C = -60$$

$$\theta_A = \frac{224.75}{EI} \;;\quad \theta_B = \frac{-89.82}{EI} \;;\quad \theta_C = \frac{-45.1}{EI}$$

4. Analyse by Stiffness matrix method.



→ DOF = 3 $(\theta_B, \theta_C, \theta_D)$



$$[K][\theta] = [P_s] - [P_L]$$

$$[\theta] = \begin{bmatrix} \theta_B \\ \theta_C \\ \theta_D \end{bmatrix} ; \quad [P_s] = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} ; \quad [P_L] = \begin{bmatrix} P_{L1} \\ P_{L2} \\ P_{L3} \end{bmatrix}$$

FEM's :

$M_{FAB} = -45$ KNm

$M_{FBA} = 45$ KNm

$M_{FBC} = -93.75$ KNm

$M_{FCB} = +93.75$ KNm

$M_{FCD} = -71.11$ KNm

$M_{FDC} = 35.55$ KNm

Now, $P_{L1} = M_{FBA} + M_{BC} = -48.75$ KNm

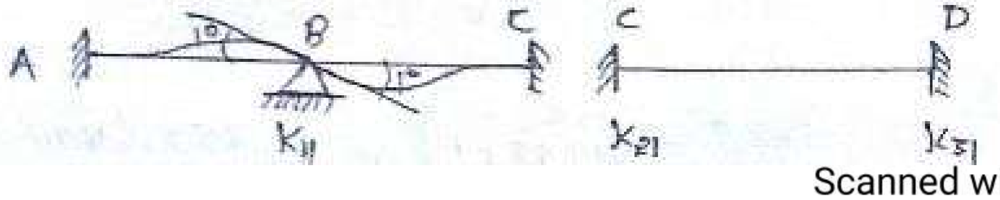
$P_{L2} = M_{FBCB} + M_{FCD} = 22.64$ KNm

$P_{L3} = M_{FDC} = 35.55$ KNm

$$\therefore \quad [P_L] = \begin{bmatrix} -48.75 \\ 22.64 \\ 35.55 \end{bmatrix}$$

To find the stiffness:

Apply the moment in the system co-ordinate ① to produce 1° rotation keeping all other ends as fixed.



$$K_{11} = M_{BA} + M_{BC}$$

$$K_{11} = \frac{2EI}{6}(2\times1) + \frac{2E(2I)}{5}(2\times1) = 2.26\ EI$$

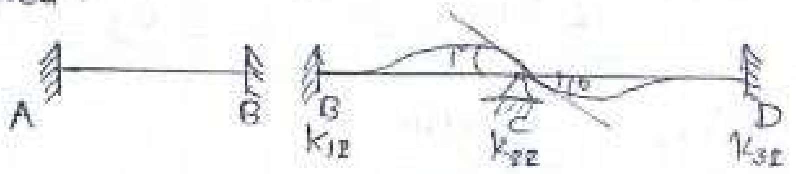
$$K_{21} = M_{CB} + M_{CD}$$

$$= \frac{2E(2I)}{5}(1) + \frac{2EI}{6}(0+0) = 0.8\ EI$$

$$K_{31} = M_{DC}$$

$$= \frac{2EI}{6}(0+0) = 0$$

Now, apply the M in the system co-ordinate ② to produce 1° rotation keeping all other ends as fixed.



$$K_{12} = M_{BA} + M_{BC}$$

$$= \frac{2EI}{6}(0+0) + \frac{2E(2I)}{5}(1) = 0.8EI$$

$$K_{22} = M_{CB} + M_{CD}$$

$$= \frac{2E(2I)}{5}(2\times1) + \frac{2EI}{6}(2\times1) = 2.26EI$$

$$K_{32} = M_{DC}$$

$$= \frac{2EI}{6}(0+1) = 0.333\ EI$$

Now, apply the 'M' in the system co-ordinate ③ to produce 1° rotation keeping all other ends as fixed.

$$K_{13} = M_{BA} + M_{BC}$$

$$= \frac{2EI}{6}(0+0) + \frac{2E(2I)}{5}(0+0) = 0$$

$$K_{23} = M_{CB} + M_{CD}$$

$$= \frac{2E(2I)}{5}(0+0) + \frac{2EI}{6}(0+1) = 0.333EI$$

$K_{33} = M_{DC} = \frac{2EI}{6}(2\times1+0) = 0.667\,EI$

$$\therefore \quad K = EI \begin{array}{c} \\ ① \\ ② \\ ③ \end{array} \begin{array}{ccc} ① & ② & ③ \\ \end{array}$$

$$\therefore \quad K = EI \begin{array}{c|ccc|} & ① & ② & ③ \\ ① & 2.26 & 0.8 & 0 \\ ② & 0.8 & 2.26 & 0.333 \\ ③ & 0 & 0.333 & 0.667 \end{array}$$

$$\therefore \quad EI\begin{bmatrix} 2.26 & 0.8 & 0 \\ 0.8 & 2.26 & 0.333 \\ 0 & 0.333 & 0.667 \end{bmatrix} \begin{bmatrix} \theta_B \\ \theta_C \\ \theta_D \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} - \begin{bmatrix} -48.75 \\ 22.64 \\ 35.55 \end{bmatrix}$$

$2.26\,\theta_B + 0.8\,\theta_C + 0.\theta_D = 48.75$

$0.8\,\theta_B + 2.26\,\theta_C + 0.333\,\theta_D = -22.64$

$0.\theta_B + 0.333\,\theta_C + 0.667\,\theta_D = -35.55$

$\therefore \quad \theta_B = \frac{25.9}{EI} \;;\quad \theta_C = \frac{-12.23}{EI} \;;\quad \theta_D = \frac{-47.2}{EI}$

Final moments :

$M_{AB} = -45 + \frac{2EI}{6}\left(0 + \frac{25.9}{EI}\right)$

$= -36.7$ KNm

$M_{BA} = 45 + \frac{2EI}{6}\left(2\left(\frac{25.9}{EI}\right) + 0\right)$

$= 62.26$ KNm

$M_{BC} = -93.75 + \frac{2E(2I)}{5}\left(2\left(\frac{25.9}{EI}\right) - \frac{12.23}{EI}\right)$

$= -62.3$ KNm

$M_{CB} = 93.75 + \frac{2E(2I)}{5}\left(2\left(\frac{-12.23}{EI}\right) + \left(\frac{25.9}{EI}\right)\right)$

$= 94.9$ KNm

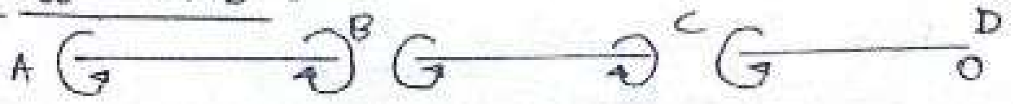
$M_{CD} = -71.11 + \frac{2EI}{6}\left(2\left(\frac{-12.23}{EI}\right) + \left(\frac{-47.2}{EI}\right)\right)$

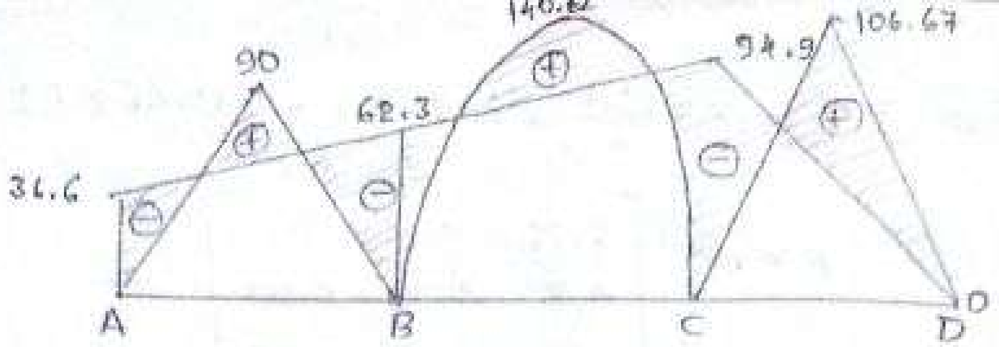
$= -94.9$ KNm

$M_{DC} = 35.55 + \frac{2EI}{6}\left(2\left(\frac{-47.2}{EI}\right) + \left(\frac{-12.23}{EI}\right)\right)$

$= 0.0$ KNm

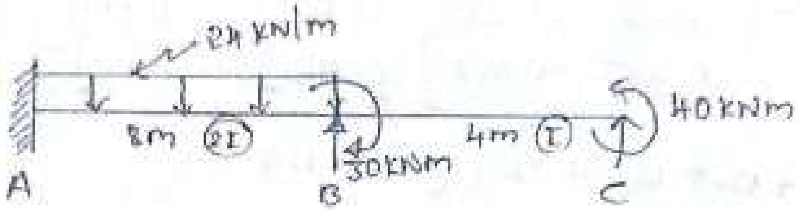
To draw BMD :



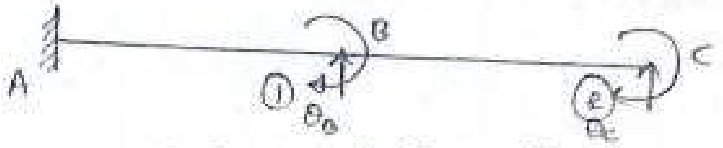


BMD

5.



→ DOF = 2 (θ_B, θ_C)



$$[K][\theta] = [P_S] - [P_L]$$

$$[\theta] = \begin{bmatrix} \theta_B \\ \theta_C \end{bmatrix} \quad ; \quad [P_S] = \begin{bmatrix} 30 \\ -40 \end{bmatrix} \quad ; \quad P_L = \begin{bmatrix} P_{L_1} \\ P_{L_2} \end{bmatrix}$$

FEM's:

$M_{FAB} = -128$ KNm

$M_{FBA} = 128$ KNm

$M_{FBC} = 0$ KNm

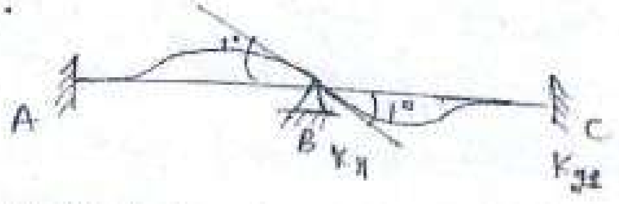
$M_{FCB} = 0$ KNm

$\therefore \quad P_{L_1} = M_{FBA} + M_{FBC} = 128$ KNm

$P_{L_2} = M_{FCB} = 0$ KNm

To find stiffness matrix:

Apply the moment 'M' in the system co-ordinat ① to produce 1° rotation keeping all othe as fixed.



$$K_{11} = \frac{2E(2I)}{8}(2\times1) + \frac{2EI}{4}(2\times1) = 2.0\,EI$$

$$K_{21} = \frac{2EI}{4}(2\times0+1) = 0.5\,EI$$

Now, apply the moment in the system co-ordinate ② to produce 1° rotation.

$$K_{12} = \frac{2EI}{4}(2\times0+1) = 0.5\,EI$$

$$K_{22} = \frac{2EI}{4}(2\times1+0) = 1.0\,EI$$

$$\therefore \text{ Stiffness matrix, } [K] = EI\begin{array}{c|cc} & ① & ② \\ \hline ① & 2.0 & 0.5 \\ ② & 0.5 & 1.0 \end{array}$$

$$\therefore \quad EI\begin{bmatrix} 2.0 & 0.5 \\ 0.5 & 1.0 \end{bmatrix}\begin{Bmatrix} \theta_B \\ \theta_C \end{Bmatrix} = \begin{bmatrix} 30 \\ -40 \end{bmatrix} - \begin{bmatrix} 128 \\ 0 \end{bmatrix}$$

$$2.0\,EI\,\theta_B + 0.5\,EI\,\theta_C = -98$$

$$0.5\,EI\,\theta_B + 1.0\,EI\,\theta_C = -40$$

$$\therefore \theta_B = \frac{-44.57}{EI} \; ; \; \theta_C = \frac{-17.71}{EI}$$

Final moments :

$$M_{AB} = -128 + \frac{2(2EI)}{8}\left(2\times0 - \frac{44.57}{EI}\right) = -150.3\text{ kNm}$$

$$M_{BA} = 128 + \frac{2E(2I)}{8}\left(2\left(\frac{-44.57}{EI}\right) - 0\right) = 83.43\text{ kNm}$$

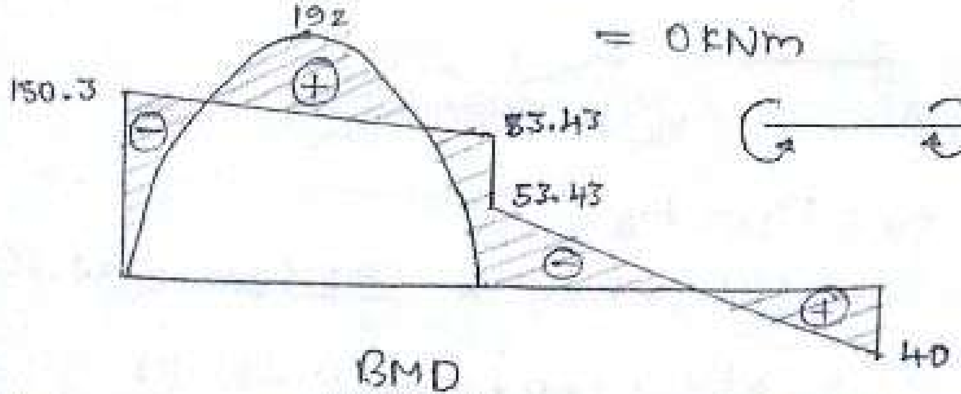
$$M_{BC} = 0 + \frac{2EI}{4}\left(2\left(\frac{-44.57}{EI}\right) - \frac{17.71}{EI}\right) = -53.43\text{ kNm}$$

$$-30 = -83.43$$

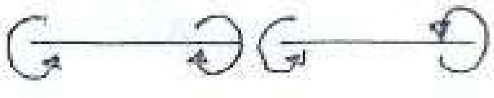
$$M_{CB} = 0 + \frac{2EI}{4}\left(\left(\frac{-17.71}{EI}\right)2 - \frac{44.57}{EI}\right) = -40\text{ kNm}$$

$$+40\text{kNm}$$

$$= 0\text{kNm}$$



BMD



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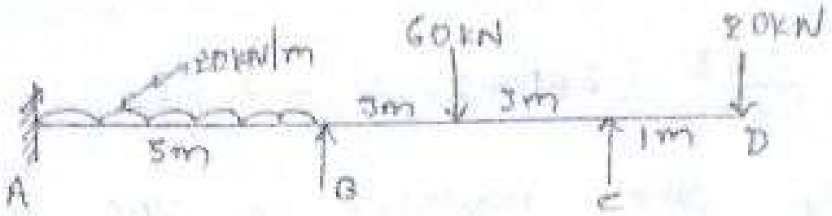
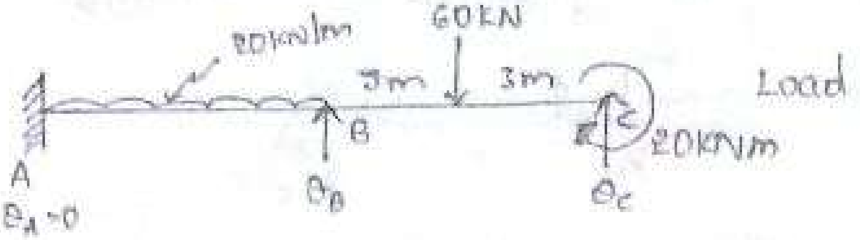
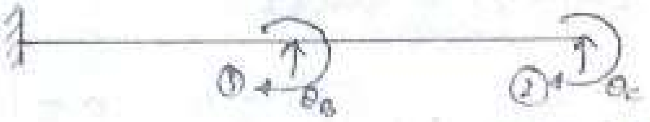
6.



→



DOF = 2 (θ_B, θ_C)



$$[K][\theta] = [P_s] - [P_L]$$

$$[\theta] = \begin{bmatrix} \theta_B \\ \theta_C \end{bmatrix} ; \; [P_s] = \begin{bmatrix} 0 \\ 20 \end{bmatrix}$$

FEM's:

$$M_{FAB} = -\frac{wL^2}{12} = -41.67 \text{ KNm}$$

$$M_{FBA} = 41.67 \text{ KNm}$$

$$M_{FBC} = -45 \text{ KNm}$$

$$M_{FCB} = 45 \text{ KNm}$$

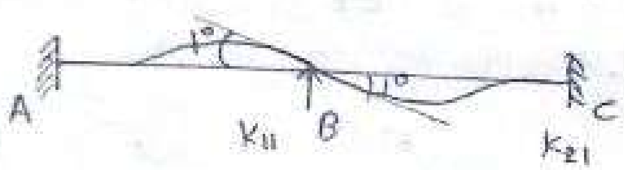
$$\therefore \; P_{L_1} = M_{FBA} + M_{FBC} = -3.33 \text{ KNm}$$

$$P_{L_2} = M_{FCB} = 45 \text{ KNm}$$

$$\therefore \; [P_L] = \begin{bmatrix} -3.33 \\ 45 \end{bmatrix}$$

Stiffness:

Apply a moment in system co-ordinate ① to produce 1" rotation keeping all other ends fix

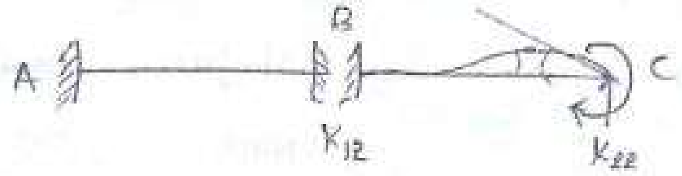


$$K_{11} = M_{BA} + M_{BC}$$

$$= \frac{2EI}{5}(2\times1+0) = \frac{2EI}{6}(2\times1+0) = 1.467 EI$$

$$K_{21} = \frac{2EI}{6}(2\times0+1) = 0.333 \text{ EI}$$

② Apply the moment in the system co-ordinate to produce 1° rotation keeping other ends fixed.



$$K_{12} = M_{BA} + M_{BC}$$

$$= \frac{2EI}{5}(2\times0+0) + \frac{2EI}{6}(2\times0+1) = 0.333EI$$

$$K_{22} = M_{CB} = \frac{2EI}{6}(2\times1)+0) = 0.667EI$$

$$\therefore \text{Element stiffness matrix, } k = EI \begin{array}{c|cc} & ① & ② \\ ① & 1.467 & 0.333 \\ ② & 0.333 & 0.667 \end{array}$$

$$\therefore \quad EI \begin{bmatrix} 1.467 & 0.333 \\ 0.333 & 0.667 \end{bmatrix} \begin{bmatrix} \theta_B \\ \theta_C \end{bmatrix} = \begin{bmatrix} 0 \\ 20 \end{bmatrix} - \begin{bmatrix} -3.33 \\ 45 \end{bmatrix}$$

$$1.467\,EI\theta_B + 0.333\,\theta_C = 3.33$$

$$0.333\,EI\theta_B + 0.667\,EI\theta_C = -25$$

$$\therefore \quad \theta_B = \frac{12.15}{EI} \; ; \; \theta_C = \frac{-43.54}{EI}$$

Final moments:

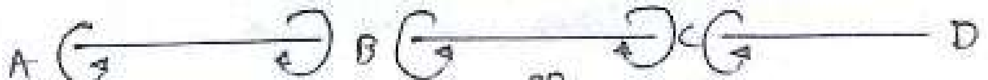
$$M_{AB} = -41.67 + \frac{2EI}{5}\left(2\times0 + \left(\frac{12.15}{EI}\right)\right) = -36.8\text{ KNm}$$

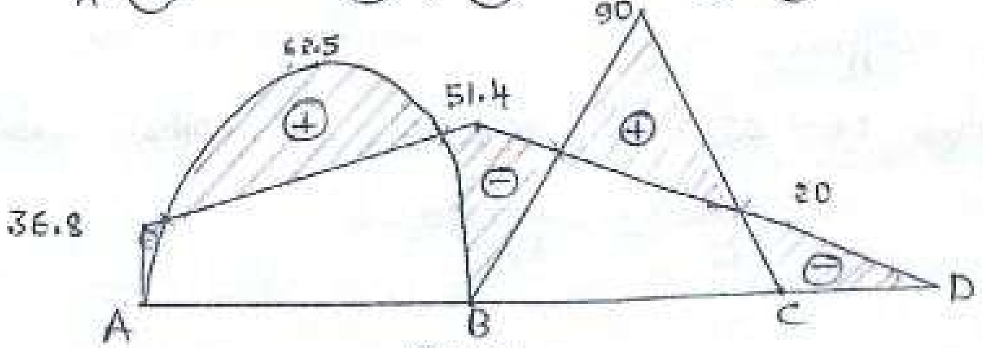
$$M_{BA} = 41.67 + \frac{2EI}{5}\left(2\left(\frac{12.15}{EI}\right)+0\right) = 51.4\text{ KNm}$$

$$M_{BC} = -45 + \frac{2EI}{6}\left(2\left(\frac{12.15}{EI}\right) - \frac{43.54}{EI}\right) = -51.4\text{ KNm}$$

$$M_{CB} = 45 + \frac{2EI}{6}\left(2\left(\frac{-43.54}{EI}\right) + \frac{12.15}{EI}\right) = 20.0\text{ KNm}$$

$$\therefore \quad M_{CD} = -20\text{ KNm}$$





BMD

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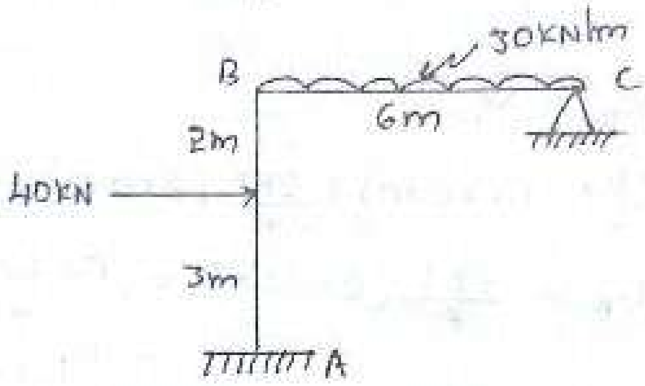
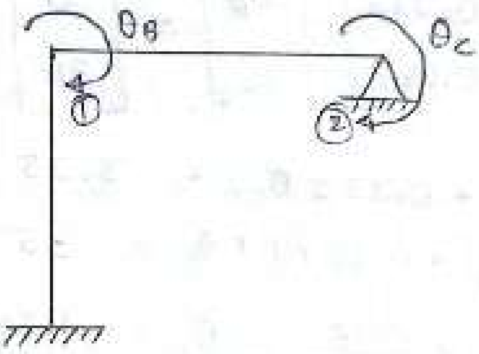
Frames :

1. Analyse the frame by stiffness matrix method



→ DOF = 2(θ_B, θ_C)



$$[\theta] = \begin{bmatrix} \theta_B \\ \theta_C \end{bmatrix} ; [P_S] = \begin{bmatrix} 0 \\ 0 \end{bmatrix} ; [P_L] = \begin{bmatrix} P_{L1} \\ P_{L2} \end{bmatrix}$$

FEM's :

$$M_{FAB} = -\frac{wab^2}{L^2} = -19.2 \text{ KNm}$$

$$M_{FBA} = 28.8 \text{ KNm}$$

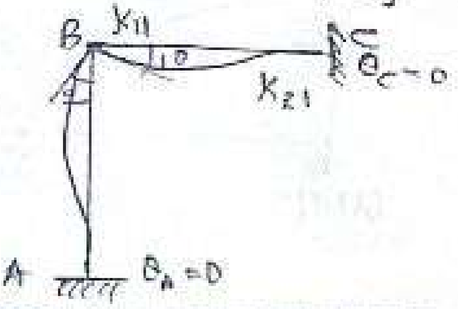
$$M_{FBC} = -\frac{wL^2}{12} = -90 \text{ KNm}$$

$$M_{FCB} = 90 \text{ KNm}$$

$$P_{L} = \begin{bmatrix} M_{FBA} + M_{FBC} \\ M_{FCB} \end{bmatrix} = \begin{bmatrix} -61.2 \\ 90 \end{bmatrix}$$

To find stiffness :

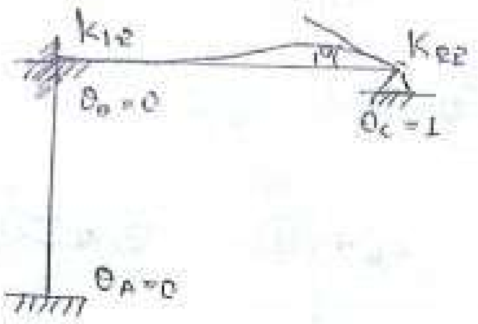
Apply M at ① keeping all other ends fixed.



$$K_{11} = \frac{2EI}{5}(2\times1+0) + \frac{2EI}{6}(2\times1+0) = 1.467\ EI$$

$$K_{21} = M_{CB} = \frac{2EI}{6}(2\times0+1) = 0.333\ EI$$

Now, apply the 'M' in ② keeping all other ends fixed to produce 1° rotation.



$$K_{12} = \frac{2EI}{5}(0) + \frac{2EI}{6}(2\times0+1) = 0.333\ EI$$

$$K_{22} = \frac{2EI}{6}(2\times1+0) = 0.667\ EI$$

$$\therefore \quad K = EI \begin{array}{c} \\ ① \\ ② \end{array}\begin{array}{cc} ① & ② \\ \end{array}$$

$$\therefore \quad K = EI\;\begin{array}{c|cc} & ① & ② \\ \hline ① & 1.467 & 0.333 \\ ② & 0.333 & 0.667 \end{array}$$

$$\therefore \quad EI\begin{bmatrix} 1.467 & 0.333 \\ 0.333 & 0.667 \end{bmatrix}\begin{bmatrix} \theta_B \\ \theta_C \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} - \begin{bmatrix} -61.2 \\ 90 \end{bmatrix}$$

$$1.467\ EI\theta_B + 0.333\ EI\theta_C = 61.2$$

$$0.333\ EI\theta_B + 0.667\ EI\theta_C = -90$$

$$\therefore \quad \theta_B = \frac{81.6}{EI} \;;\; \theta_C = \frac{-175.66}{EI}$$

Final moments :

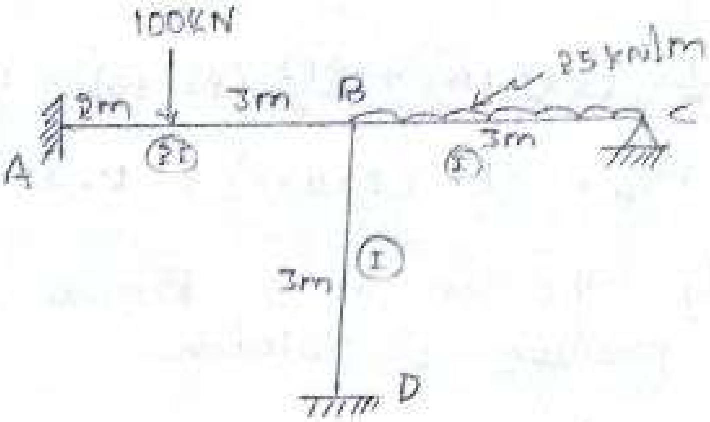
$$M_{AB} = -19.2 + \frac{2EI}{5}\left(2\times0 + \frac{81.6}{EI}\right) = 13.44\ KNm$$

$$M_{BA} = 28.8 + \frac{2EI}{5}\left(2\left(\frac{81.6}{EI}\right) + 0\right) = 94.1\ KNm$$

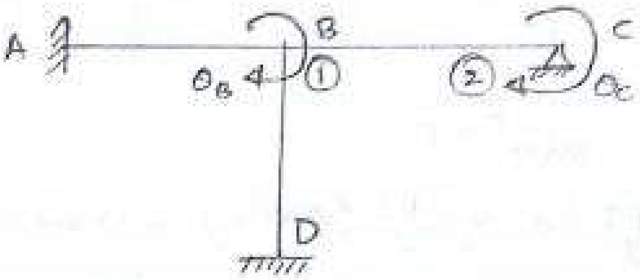
$$M_{BC} = -90 + \frac{2EI}{6}\left(2\left(\frac{81.6}{EI}\right) - \frac{175.66}{EI}\right) = -94.1\ KNm$$

$$M_{CB} = 90 + \frac{2EI}{6}\left(2\left(-\frac{175.66}{EI}\right) + \frac{81.6}{EI}\right) = 0.0\ KNm$$

Analyse the frame by stiffness matrix method. Draw BMD.



→ DOF = 2 (θ_B, θ_C)



"System approach"

$$[K][\theta] = [P_S] - [P_L]$$

$$[\theta] = \begin{bmatrix} \theta_B \\ \theta_C \end{bmatrix} ; [P_S] = \begin{bmatrix} 0 \\ 0 \end{bmatrix} ; [P_L] = \begin{bmatrix} P_{L1} \\ P_{L2} \end{bmatrix}$$

FEM's :

$$M_{FAB} = \frac{-Wab^2}{L^2} = -72 \text{ KNm}$$

$$M_{FBA} = 48.04 \text{ KNm}$$

$$M_{FBC} = -18.75 \text{ KNm}$$

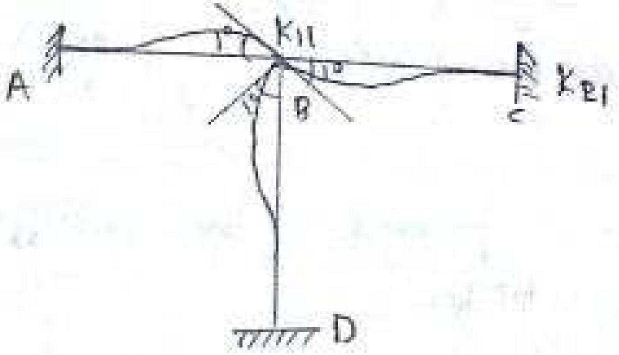
$$M_{FCB} = 18.75 \text{ KNm}$$

$$M_{FBD} = M_{FDB} = 0$$

$$[P_L] = \begin{bmatrix} M_{FBA} + M_{FBC} + M_{FBD} \\ M_{FCB} \end{bmatrix} = \begin{bmatrix} 29.25 \\ 18.75 \end{bmatrix}$$

To find the stiffness :

Apply the moment 'M' in ① keeping ends fixed.

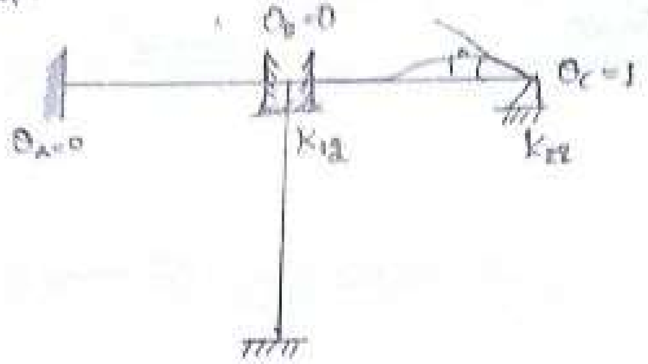


$$K_{11} = M_{BA} + M_{BC} + M_{BD}$$

$$K_{11} = \frac{2E(2I)}{5}(2\times 1+0) + \frac{2EI}{3}(2\times 1+0) + \frac{2EI}{3}(2\times 1+0) = 4.267\ EI$$

$$K_{21} = M_{CB} = \frac{2EI}{3}(2\times 0+1) = 0.667\ EI$$

Now, apply the moment in the ② keeping all other fixed.



$$K_{12} = \frac{2E(2I)}{5}(2\times 0+0) + \frac{2EI}{3}(2\times 0+1) + \frac{2EI}{3}(2\times 0+0)$$

$$\therefore K_{11} = 0.667\ EI$$

$$K_{22} = M_{CB} = 1.333\ EI$$

$$\therefore K = EI\begin{bmatrix} 4.267 & 0.667 \\ 0.667 & 1.333 \end{bmatrix}$$

$$\therefore EI\begin{bmatrix} 4.267 & 0.667 \\ 0.667 & 1.333 \end{bmatrix}\begin{bmatrix} \theta_B \\ \theta_C \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} - \begin{bmatrix} 29.25 \\ 18.75 \end{bmatrix}$$

$$\therefore \theta_B = \frac{-5.05}{EI} \ ; \ \theta_C = \frac{-11.54}{EI}$$

Final moments:

$$M_{AB} = -72 + \frac{2E(2I)}{5}\left(2\times 0 - \frac{5.05}{EI}\right) = -76.04\ kNm$$

$$M_{BA} = 48.04 + \frac{2E(2I)}{5}\left(2\left(-\frac{5.05}{EI}\right) + 0\right) = 40\ KNm$$

$$M_{BC} = -18.75 + \frac{2EI}{3}\left(2\left(\frac{-5.05}{EI}\right) - \frac{11.54}{EI}\right) = -33.2\ kNm$$

$$M_{CB} = 0\ kN\text{-}m$$

$$M_{BD} = 0 + \frac{2EI}{3}\left(2\left(-\frac{5.05}{EI}\right) + 0\right) = -6.73\ KNm$$

$$M_{DB} = 0 + \frac{2EI}{3}\left(2(0) - \frac{5.05}{EI}\right) = -3.37\ KNm$$

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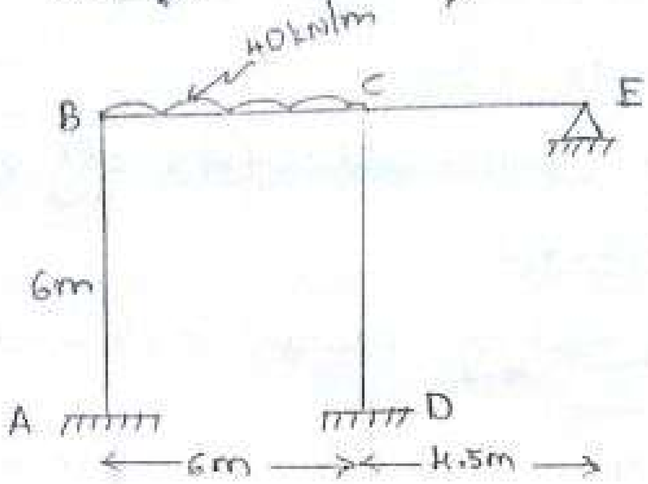
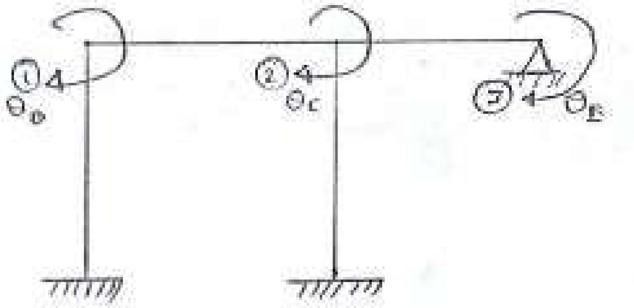
4. Using stiffness matrix method and system approach analyse the frame. Draw BMD. EI const.



→ DOF = 3 $(\theta_B, \theta_C, \theta_E)$



$$[k][\theta] = [P_S] - [P_L] \longrightarrow ①$$

But $[\theta] = \begin{bmatrix} \theta_B \\ \theta_C \\ \theta_E \end{bmatrix}$; $[P_S] = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

FEM's :

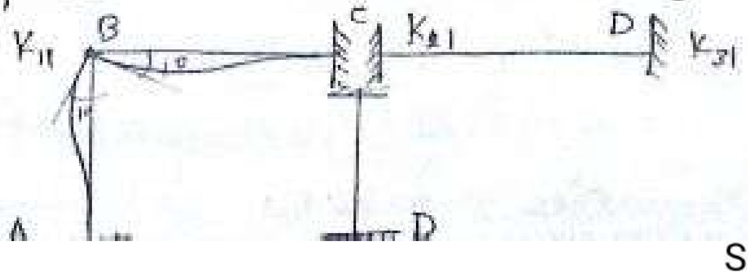
$$M_{FBC} = \frac{-40 \times 6^2}{120} = -120 \text{ KNm}$$

$$M_{FCB} = 120 \text{ KNm}$$

$$[P_L] = \begin{bmatrix} -120 \\ 120 \\ 0 \end{bmatrix}$$

To find stiffness :

Apply the moment 'M' in system co-ordinate ① to produce 1 rotation keeping all other ends fixed.



Now, $K_{11} = M_{BA} + M_{BC}$

$$K_{11} = \frac{2EI}{6}(2\times1) + \frac{2EI}{6}(2\times1) = 1.33\ EI$$

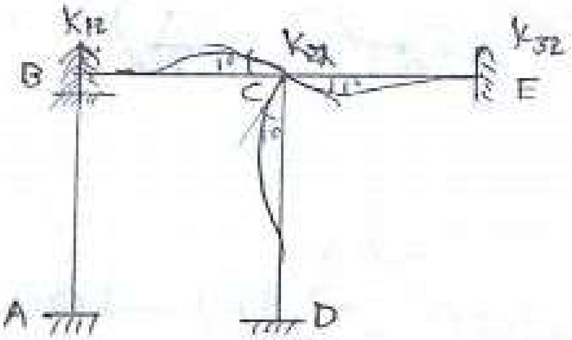
$$K_{21} = M_{CB} + M_{CD} + M_{CE}$$

$$= \frac{2EI}{6}(0+1) + \frac{2EI}{6}(0+0) + \frac{2EI}{4.5}(0+0)$$

$$K_{21} = 0.333\ EI$$

$$K_{31} = M_{EC} = \frac{2EI}{4.5}(2\times0+0) = 0.0\ \text{kNm}$$

Now, apply the moment in system co-ordinat to produce 1° rotation keeping all other end as fixed.



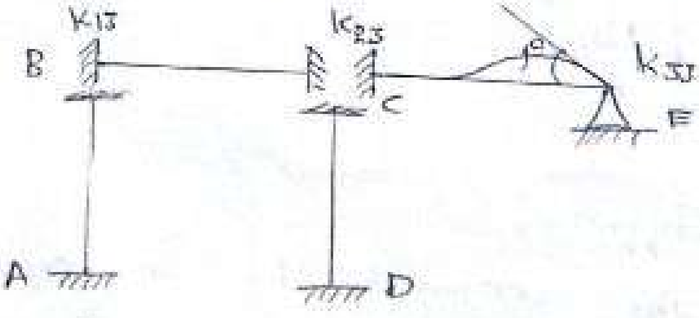
$$K_{12} = M_{BA} + M_{BC} = 0.33EI$$

$$K_{22} = M_{CB} + M_{CD} + M_{CE} = \frac{2EI}{6}(2\times1) + \frac{2EI}{6}(2\times1) + \frac{2EI}{4.5}$$

$$\therefore K_{22} = 2.2\ EI$$

$$K_{32} = \frac{2EI}{4.5}(2\times0+1) = 0.44\ EI$$

Now, apply the moment in ③ to produce rotation keeping all other ends as fixed.



$$K_{13} = 0+0 = 0.0\ \text{kNm}$$

$$K_{23} = M_{CB} + M_{CD} + M_{CE}$$

$$= 0+0+\frac{2EI}{4.5}(2\times0+1) = 0.44\ EI$$

$$K_{33} = M_{EC} = 0.88\ EI$$

Then $K = EI \begin{bmatrix} 1.33 & 0.33 & 0.0 \\ 0.33 & 2.22 & 0.44 \\ 0.0 & 0.44 & 0.88 \end{bmatrix}$

Now, eq^n ① becomes.

$$EI \begin{bmatrix} 1.33 & 0.33 & 0.0 \\ 0.33 & 2.22 & 0.44 \\ 0.0 & 0.44 & 0.88 \end{bmatrix} \begin{bmatrix} \theta_B \\ \theta_C \\ \theta_D \end{bmatrix} = \begin{bmatrix} 120 \\ -120 \\ 0 \end{bmatrix}$$

$\therefore \quad \theta_B = \frac{109.56}{EI} \; ; \; \theta_C = \frac{-78.14}{EI} \; ; \; \theta_D = \frac{39.12}{EI}$

Final moments:

$$M_{AB} = 0 + \frac{2EI}{6}\left(2\times0 + \frac{108.56}{EI} - 0\right) = 360.59 \text{ KNm}$$

$$M_{BA} = 0 + \frac{2EI}{6}\left(2\left(\frac{109.56}{EI}\right) - 0 - 0\right) = 73.04 \text{ KNm}$$

$$M_{BC} = -120 + \frac{2EI}{6}\left(2\left(\frac{109.56}{EI}\right) - \frac{78.14}{EI} - 0\right) = -73.01 \text{ KNm}$$

$$M_{CB} = 120 + \frac{2EI}{6}\left(2\left(\frac{-78.14}{EI}\right) + \left(\frac{109.56}{EI}\right)\right) = 104.4 \text{ KNm}$$

$$M_{CE} = 0 + \frac{2EI}{4.5}\left(2\left(\frac{-78.14}{EI}\right) + \frac{39.12}{EI}\right) = -59.4 \text{ KNm}$$

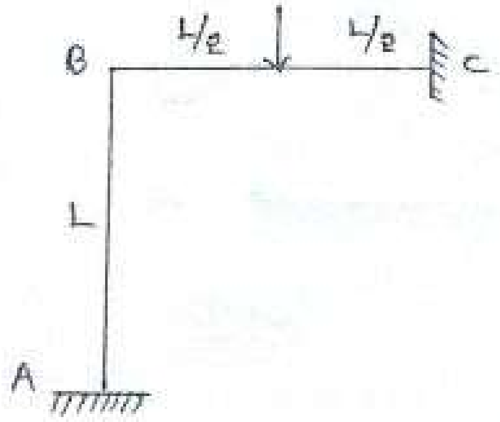
$$M_{EC} = 0 + \frac{2EI}{4.5}\left(2\left(\frac{39.12}{EI}\right) - \frac{78.14}{EI}\right) = -52.2 \text{ KNm}$$

$$M_{CD} = 0 + \frac{2EI}{6}\left(2\left(\frac{-78.14}{EI}\right) + 0\right) = -56.$$

$$M_{DC} = 0 + \frac{2EI}{6}\left(2(0) - \frac{78.14}{EI}\right) = -26.05$$

---

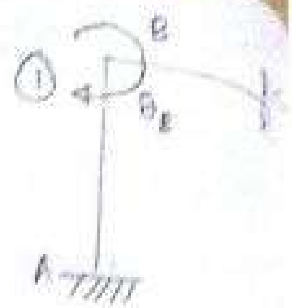
5. Analyse the frame by stiffness matrix method.



→ DOF = 1 (θ_B)

$$[K][\theta] = [P_E] - [P_L]$$

$$[K][\theta_B] = [0] - \left[\frac{-WL}{8}\right]$$

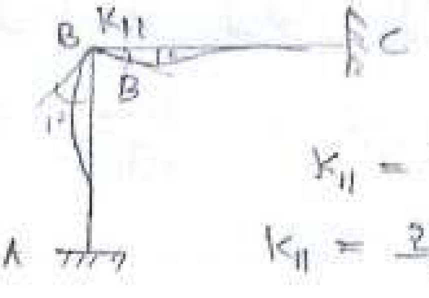


$$\because \quad [P_L] = M_{FBA} + M_{FBC}$$

$$= 0 + \left(-\frac{WL}{8}\right) = \frac{-WL}{8}$$

Stiffness:

Apply the moment at ① keeping ends fixed



$$K_{11} = M_{BA} + M_{BC}$$

$$K_{11} = \frac{2EI}{L}(2\times1) + \frac{2EI}{L}(2\times1)$$

$$K_{11} = \left(\frac{8EI}{L}\right)$$

$$\therefore \quad \left[\frac{8EI}{L}\right][\theta_B] = [0] - \left[\frac{-WL}{8}\right]$$

$$\therefore \quad \theta_B = \frac{WL^2}{64EI}$$

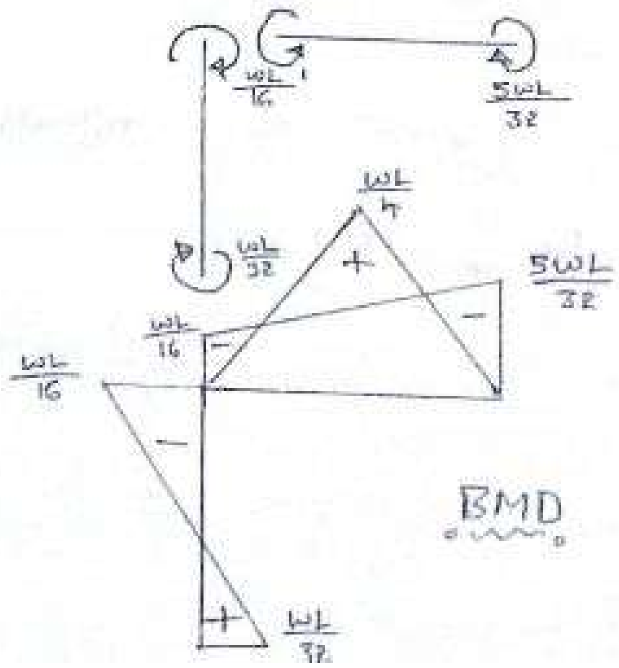
Final Moments:

$$M_{AB} = 0.0 + \frac{2EI}{L}\left(2\times0 + \frac{WL^2}{64EI}\right) = \frac{WL}{32}$$

$$M_{BA} = 0 + \frac{2EI}{L}\left(2\left(\frac{WL^2}{64EI}\right) + 0\right) = \frac{WL}{16}$$

$$M_{BC} = \frac{-WL}{8} + \frac{2EI}{L}\left(2\left(\frac{WL^2}{64EI}\right) + 0\right) = -\frac{WL}{16}$$

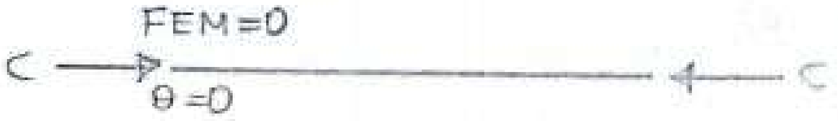
$$M_{CB} = \frac{+WL}{8} + \frac{2EI}{L}\left(2(0) + \frac{WL^2}{64EI}\right) = \frac{5WL}{32}$$



BMD

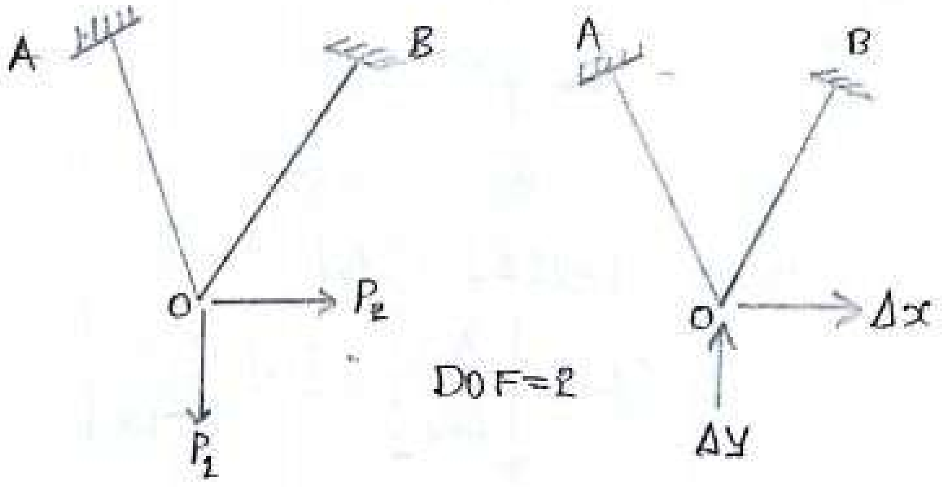
Analysis of truss by Stiffness matrix method :

Member of a truss is subjected to either tension @ compression. NO bending is allowed. Therefore rotation 'θ' is zero and external load on the member of a truss is zero.



$$[K][\theta] = [P_s] - [P_L]$$

$[P_L] = 0$; θ = unknown displacement (Δ)



$\therefore$ $[K][\Delta] = [P_s]$ $\quad \because [P_L] = 0$

$$K_{11} = \sum \frac{AE}{L} \cos^2\theta$$

$$K_{12} = \sum \frac{AE}{L} \sin\theta\cos\theta$$

$$K_{22} = \sum \frac{AE}{L} \sin^2\theta$$

$$\therefore \quad F = -\frac{AE}{L}(\Delta_x \cos\theta + \Delta_y \sin\theta)$$

Problems :

1. Find the forces in the member of a joint by stiffness matrix method.

Scanned with CamScanner

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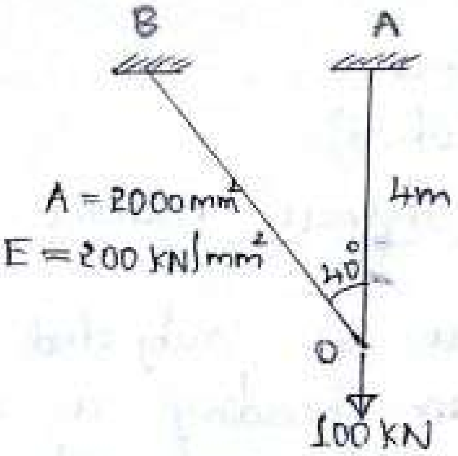
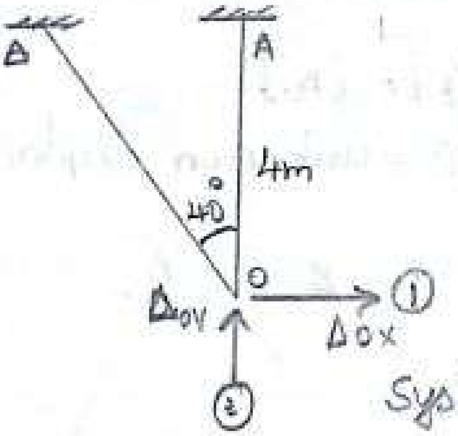


→ Let Δ_{ox} and Δ_{oy} are unknown displacement components (DOF = 2)



System co-ordinates

$\cos 40° = \frac{OA}{OB}$

$\therefore OB = 5.22m$

w.r.t $[K][\Delta] = [P_s] \longrightarrow$ ①

$$[\Delta] = \begin{bmatrix} \Delta_{ox} \\ \Delta_{oy} \end{bmatrix} ; [P_s] = \begin{bmatrix} 0 \\ -100 \end{bmatrix}$$

Members	$\frac{AE}{L}$	θ	cosθ	Sinθ	$\frac{AE}{L}\cos^2\theta$	$\frac{AE}{L}\sin^2\theta$	$\frac{AE\sin\theta}{L}$
OA	$\frac{2000 \times 200}{400}$ = 100 kN/mm	90°	0	1	0	100	0
OB	$\frac{2000 \times 200}{5220}$ = 76.63 kN/mm	130°	−0.64	0.766	31.39	44.96	−37.57
					$K_{11} = 31.39$	$K_{22} = 144.96$	$K_{12} = K_{21}$

$$\therefore [K] = \begin{bmatrix} 31.39 & -37.57 \\ -37.57 & 144.96 \end{bmatrix}$$

$$\therefore Eq^n ① \Rightarrow \begin{bmatrix} 31.39 & -37.57 \\ -37.57 & 144.96 \end{bmatrix} \begin{bmatrix} \Delta_{ox} \\ \Delta_{oy} \end{bmatrix} = \begin{bmatrix} 0 \\ -100 \end{bmatrix}$$

$\therefore \Delta_{OX} = -1.197 \quad ; \Delta_{OY} = -1.0$

$$\therefore F_{OA} = \frac{-AE}{L}\left[\Delta_{OX}\cos\theta + \Delta_{OY}\sin\theta\right]$$

$$= -100\left[(-1.197\times 0) + (-1.00\times 1)\right]$$

$$\therefore \boxed{F_{OA} = 100 \text{ kN (T)}}$$

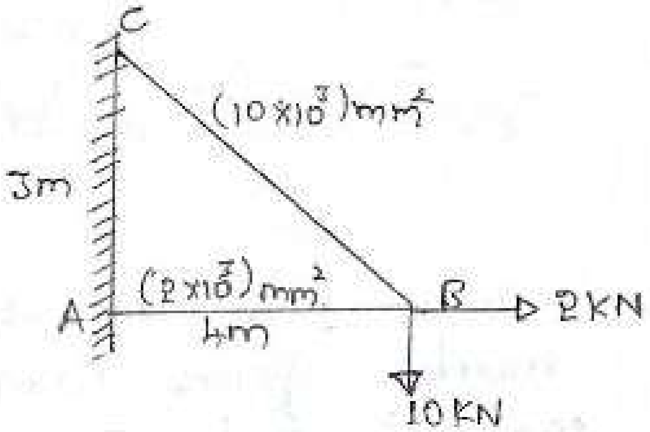
$|||^{ly}$, $$F_{OB} = \frac{-AE}{L}\left[\Delta_{OX}\cos\theta + \Delta_{OY}\sin\theta\right]$$

$$= -76.63\left[(-1.19\times -0.64) + (-1.00\times 0.766)\right]$$

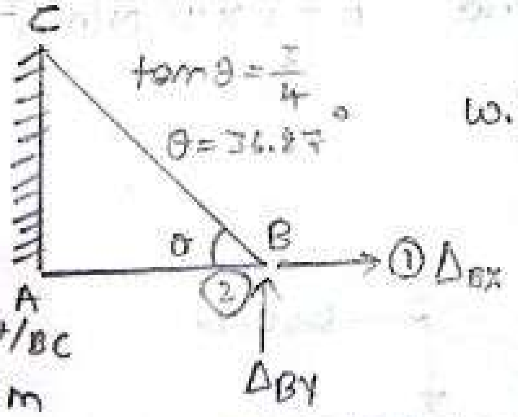
$$\boxed{F_{OB} = 0.0 \text{ KN}}$$

---

2. Using stiffness method find the forces in the member, take $E = 2\times10^5$ N/mm^2, Area of the cross-section of each member is given in paranthesis.



→



w.k.t $[K][\Delta] = [P_s] \rightarrow$ ①

$$[\Delta] = \begin{bmatrix}\Delta_{BX}\\ \Delta_{BY}\end{bmatrix} ; \quad [P_s] = \begin{bmatrix}2\\ -10\end{bmatrix}$$

$\therefore \cos\theta = 4/BC$

$\therefore BC = 5$m

Members	$\frac{AE}{L}$	θ	$\cos\theta$	$\sin\theta$	$\frac{AE}{L}\cos^2\theta$	$\frac{AE}{L}\sin^2\theta$	$\frac{AE}{L}\sin\theta\cos\theta$
BA	$\frac{2\times10^3\times2\times10^2}{4000}$ $= 100$ kN/mm	180°	−1	0	100	0	0
BC	$\frac{10\times10^3\times2\times10^2}{5000}$ $= 400$ kN/mm	143.13°	−0.8	0.6	256.36	144	−191.76
					$K_{11} = 355.36$	$K = 144$	$K = K = -191$

$$\therefore K = \begin{bmatrix} 355.36 & -191.76 \\ -191.76 & 144 \end{bmatrix}$$

$$\therefore ① \Rightarrow \begin{bmatrix} 355.36 & -191.76 \\ -191.76 & 144 \end{bmatrix} \begin{bmatrix} \Delta_{BX} \\ \Delta_{BY} \end{bmatrix} = \begin{bmatrix} 2 \\ -10 \end{bmatrix}$$

$$\therefore \Delta_{BX} = -0.113 \; ; \; \Delta_{BY} = -0.22$$

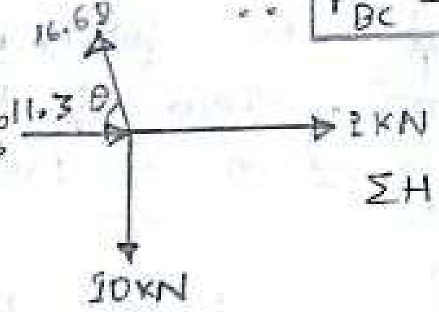
$$\therefore F_{BA} = -\frac{AE}{L}\left[\Delta_{BX}\cos\theta + \Delta_{BY}\sin\theta\right]$$

$$\therefore \boxed{F_{BA} = -11.3 \text{ kN} (C)}$$

Also, $F_{BC} = -400\left[(-0.113 \times -0.8) + (-0.22)(0.6)\right]$

$$\therefore \boxed{F_{BC} = 16.68 \text{ kN} (T)}$$

Not required
Check

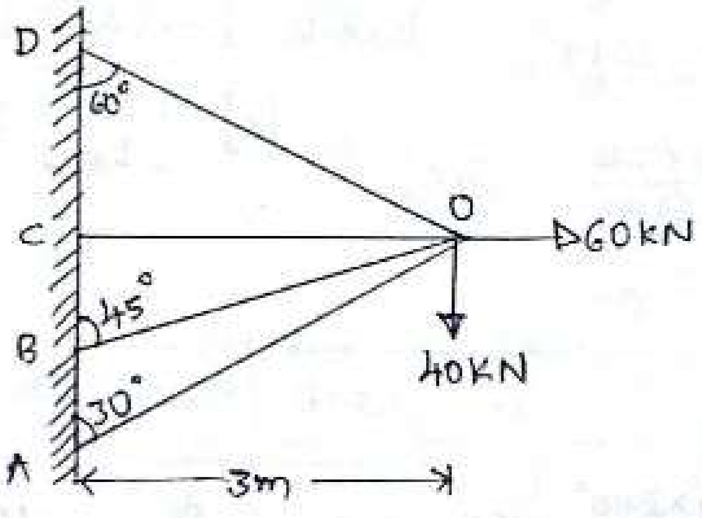


$\Sigma H = 0 ; -16.68 \cos 36.87 + 11.3 + 2 = 0$
$0 = 0$ ✓

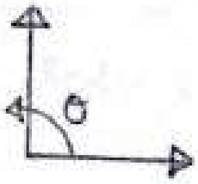
$\Sigma V = 0 : 16.68 \sin 36.87 - 10 = 0$
$0 = 0$ ✓

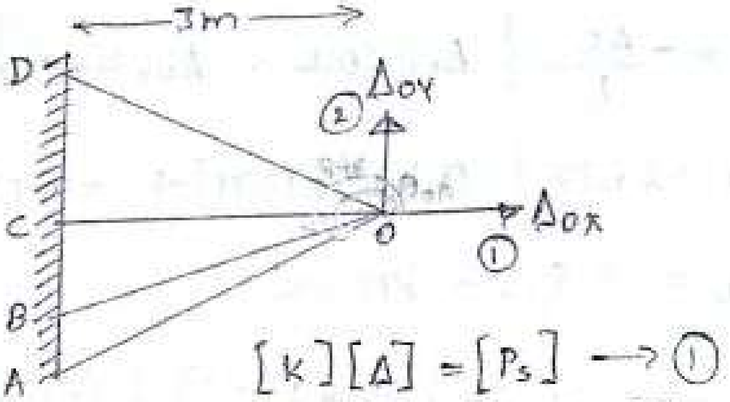
---

3. Analyse the truss joint by system approach and tabulate the member forces. Cross section of all the members is 1000 mm^2 and $E = 2 \times 10^5$ N/mm^2.



→ DOF = 2 (Δ_{ox}, Δ_{oy})

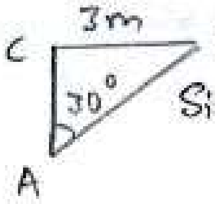




$$[K][\Delta] = [P_s] \longrightarrow ①$$

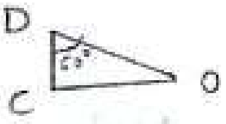
But $\Delta = \begin{bmatrix} \Delta_{ox} \\ \Delta_{oy} \end{bmatrix}$; $[P_s] = \begin{bmatrix} 60 \\ -40 \end{bmatrix}$

Members	$\frac{AE}{L}$; kN/mm²	θ	Cosθ	Sinθ	$\frac{AE}{L}\cos^2\theta$	$\frac{AE}{L}\sin^2\theta$	$\frac{AE}{L}\sin\theta\cos\theta$
OA	$\frac{1000\times200}{6000}$ = 33.33	240	−0.5	−0.87	8.33	25.23	14.5
OB	47.17	225°	−0.71	−0.71	23.78	23.78	23.78
OC	66.67	180°	−1	0	66.67	0	0
OD	57.8	150°	−0.87	0.5	43.75	14.45	−25.14
					K_{11} = 142.53	K_{21} = 63.46	K_{12} = K_{21} = 13.14



$\sin 30° = \frac{3}{OA}$ ∴ $\underline{OA = 6m}$* ;

$\sin 45° = \frac{3}{OB}$ $\underline{= 4.24m}$*



$\sin 60° = \frac{3}{OD}$ $\underline{= 3.46m}$*

$$\therefore [K] = \begin{bmatrix} 142.53 & 13.14 \\ 13.14 & 63.46 \end{bmatrix}$$

$$Eq^n\ ① \Rightarrow \begin{bmatrix} 142.53 & 13.14 \\ 13.14 & 63.46 \end{bmatrix} \begin{bmatrix} \Delta_{ox} \\ \Delta_{oy} \end{bmatrix} = \begin{bmatrix} 60 \\ -40 \end{bmatrix}$$

$$\Delta_{ox} = 0.49 \ ; \ \Delta_{oy} = -0.73$$

∴ w.k.t $F_{OA} = \frac{-AE}{L}\left[\Delta_{ox}\cos\theta + \Delta_{oy}\sin\theta\right]$

$= -33.33\left[0.49(-0.5) + (-0.73)(-0.87)\right]$

$F_{OA} = -13$ KN (C)

$$F_{OB} = -\frac{AE}{L}\left[\Delta_{Ox}\cos\theta + \Delta_{Oy}\sin\theta\right]$$

$$= -47.17\left[0.49(-0.71) + (-0.73 \times -0.71)\right]$$

$$F_{OB} = -8.04 \text{ kN (C)}$$

$$F_{OC} = -66.67\left[0.49(-1) + (-0.73 \times 0)\right]$$

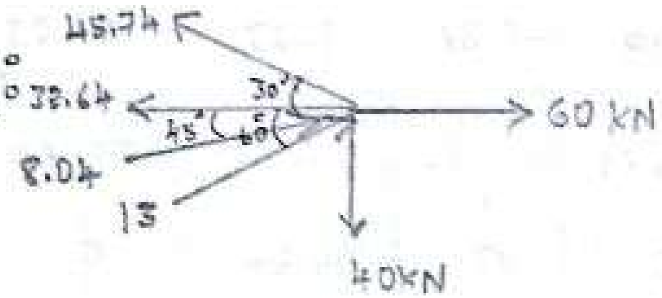
$$F_{OC} = 32.67 \text{ kN (T)}$$

$$F_{OD} = -57.8\left[0.49(-0.87) + (-0.73 \times 0.5)\right]$$

$$F_{OD} = 45.74 \text{ kN (T)}$$

(Not required)

Checking :



$$\Sigma V = 0;\quad 45.74 \sin 30° + 8.04 \sin 45° + 13 \sin 60° - 40 = 0$$

$$0 = 0$$

$$\Sigma H = 0;\quad -45.74 \cos 30° + 8.04 \cos 45° - 32.64 \cos 0° + 60 = 0$$

$$0 = 0$$

Imp

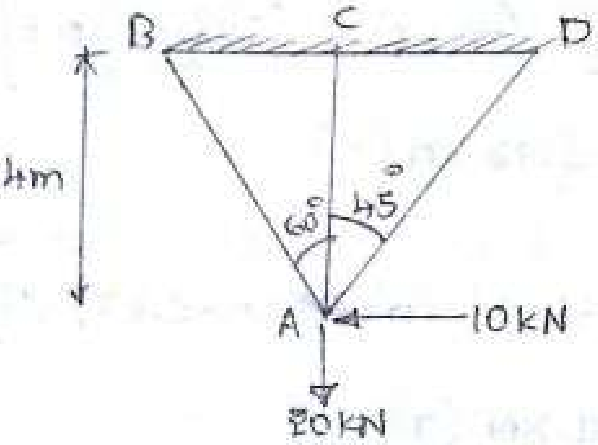
Tabulation :

Members	Forces (KN)	Nature
OA	·13.0	Compression
OB	8.04	Compression
OC	32.6	Tension
OD	45.74	Tension

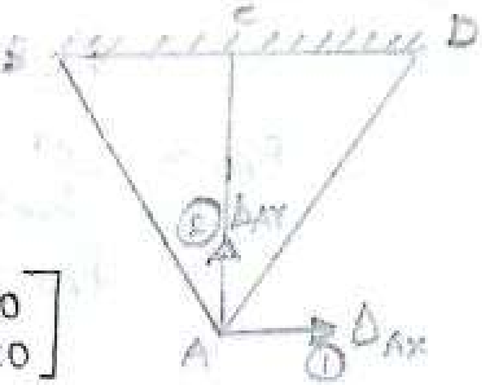
---

4. Find the forces in the member by Stiffness matrix method. Take AE as constant.

→ Note : Here AE is called axial rigidity and is constant for all members.



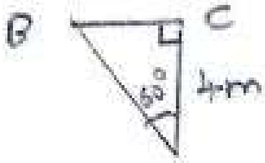
$\rightarrow$ $DOF = 2\ (\Delta_{ox}, \Delta_{oy})$



$$[K][\Delta] = [P_s] \longrightarrow ①$$

But $\Delta = \begin{bmatrix} \Delta_{AX} \\ \Delta_{AY} \end{bmatrix}$; $P_s = \begin{bmatrix} -10 \\ -20 \end{bmatrix}$

						K_{11}	K_{22}	$K_{12} = K_{21}$
Members	L	$\frac{AE}{L}$	θ	cosθ	Sinθ	$\frac{AE}{L}\cos^2\theta$	$\frac{AE}{L}\sin^2\theta$	$\frac{AE}{L}\sin\theta\cos\theta$
AB	8m	$\frac{AE}{8}$	150°	−0.87	0.5	AE(0.09)	AE(0.03)	−AE(0.05)
AC	4m	$\frac{AE}{4}$	90°	0	1	AE(0)	AE(0.25)	0
AD	5.65m	$\frac{AE}{5.65}$	45°	0.71	0.71	AE(0.09)	AE(0.09)	AE(0.09)
						AE(0.18)	AE(0.37)	AE(0.04)



$\cos 60^\circ = \frac{4}{OB}$;

$\cos 45^\circ = \frac{4}{AD}$

$$\therefore \quad [K] = \begin{bmatrix} 0.18 & 0.04 \\ 0.04 & 0.37 \end{bmatrix} AE$$

$$\therefore \quad ① \Rightarrow AE \begin{bmatrix} 0.18 & 0.04 \\ 0.04 & 0.37 \end{bmatrix} \begin{bmatrix} \Delta_{AX} \\ \Delta_{AY} \end{bmatrix} = \begin{bmatrix} -10 \\ -20 \end{bmatrix}$$

$$\therefore \Delta_{AX} = \frac{-44.61}{AE} \ ; \quad \Delta_{AY} = \frac{-49.23}{AE}$$

$$\therefore \quad F_{AB} = -\frac{AE}{L}\left[\Delta_{AX}\cos\theta + \Delta_{AY}\sin\theta\right]$$

$$F_{AB} = -\frac{AE}{8}\left[\frac{-44.61}{AE}(-0.87) + \left(\frac{-49.23}{AE}\right)0.5\right]$$

$$\therefore F_{AB} = -1.77 \text{ kN } (C)$$

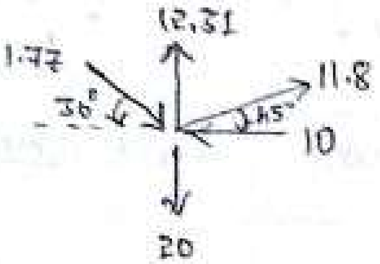
$$F_{AC} = -\frac{AE}{4}\left[(-44.61)(0) + (-49.23)(1)\right]\frac{1}{AE}$$

$$F_{AC} = 12.31 \text{ kN } (T)$$

$$F_{AD} = \frac{-AE}{5.65}\left[(-44.61)(0.71) + (-49.23)(0.71)\right]\frac{1}{AE}$$

$$F_{AD} = 11.8 \text{ kN } (T)$$

Checking:



$$\Sigma H = 0;\ -10 + 11.8 \cos 45^\circ + 1.77 \cos 30^\circ = 0$$

$$0 = 0$$

CBCS SCHEME

USN 4 B D 1 8 C V 0 7 2

18CV52

Fifth Semester B.E. Degree Examination, Jan./Feb. 2021
Analysis of Indeterminate Structures

Time: 3 hrs. Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

1 Analyze the continuous beam shown in Fig.Q1 by slope deflection method. Draw BMD, SFD and elastic curve.

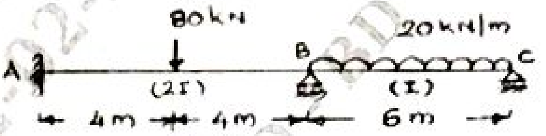


Fig.Q1

(20 Marks)

OR

2 Analyze the portal frame shown in Fig.Q2 by slope deflection method. Draw BMD and elastic curve.

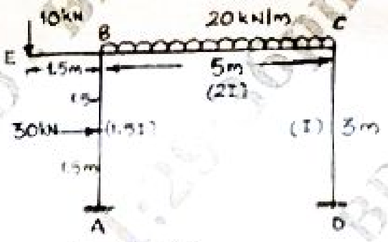


Fig.Q2

(20 Marks)

Module-2

3 Analyze the continuous beam shown in Fig.Q3 by using moment distribution method. Draw BMD SFD and elastic curve the support B sinks by 1 cm. Take EI = 500 kN-m^2.

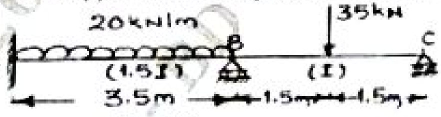


Fig.Q3

(20 Marks)

OR

4 Analyze the portal frame shown in Fig.Q4 by moment distribution method. Draw BMD, SFD and elastic curve.

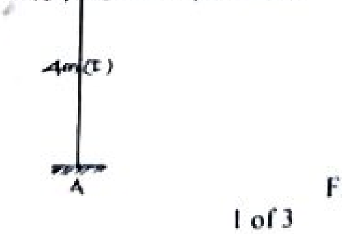


Fig.Q4

(20 Marks)

Module-3

5 Analyze the continuous beam shown in Fig.Q5 by using Kani's method. The support C sinks by 20 mm. Take E = 200 kN/mm^2, I = 170 × 10^6 mm^4. Draw BMD, SFD and EC.

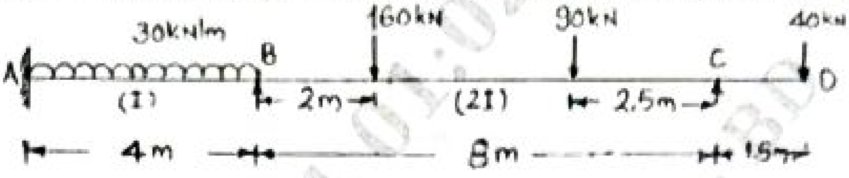


Fig.Q5

(20 Marks)

OR

6 Analyze the portal frame shown in Fig.Q6 by using Kani's method. Assume EI is constant throughout. Draw BMD and elastic curve.

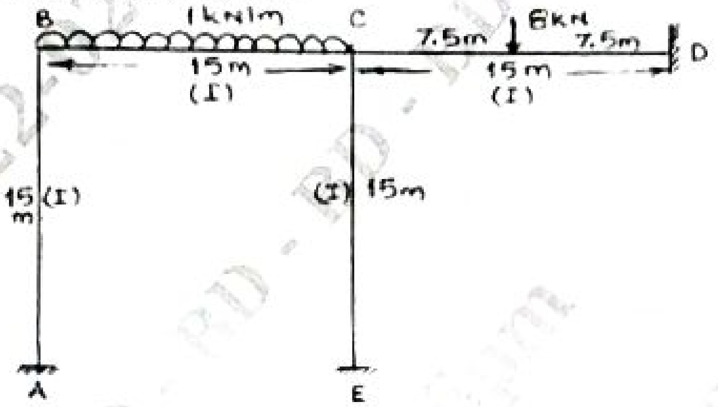


Fig.Q6

(20 Marks)

Module-4

7 Analyze the continuous beam by using flexibility matrix method. Draw BMD, SFD and elastic curve. Refer Fig.Q7.

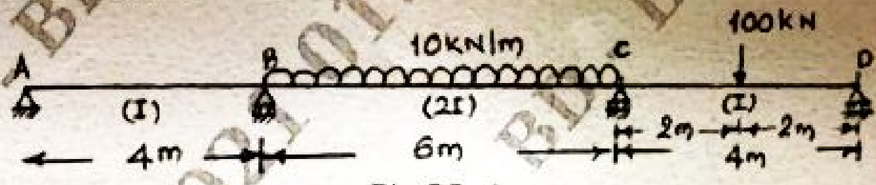


Fig.Q7

(20 Marks)

OR

8 Analyze the truss shown in Fig.Q8 by flexibility matrix method choosing force in the member AD as redundant. Assume constant EI for all the members.

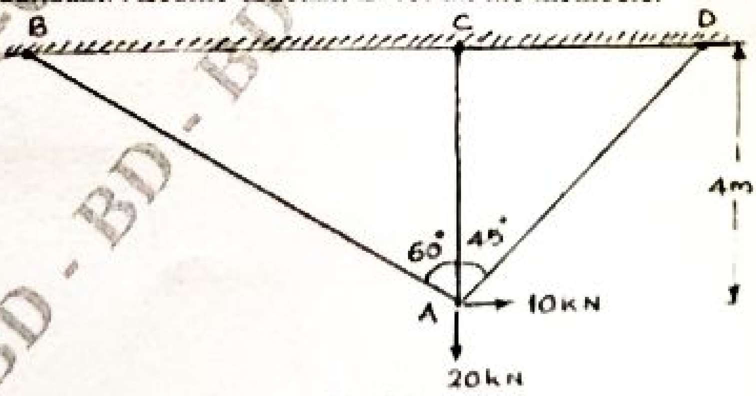


Fig.Q8

(20 Marks)

Module-5

9 Analyze the continuous beam shown in Fig.Q9 by using stiffness matrix method. Draw BMD, SFD and elastic curve.

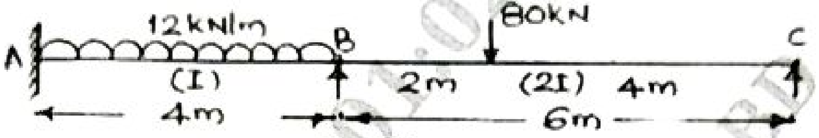


Fig.Q9

(20 Marks)

OR

10 Analyze the portal frame shown in Fig.Q10 by stiffness matrix method. Draw BMD and elastic curve.

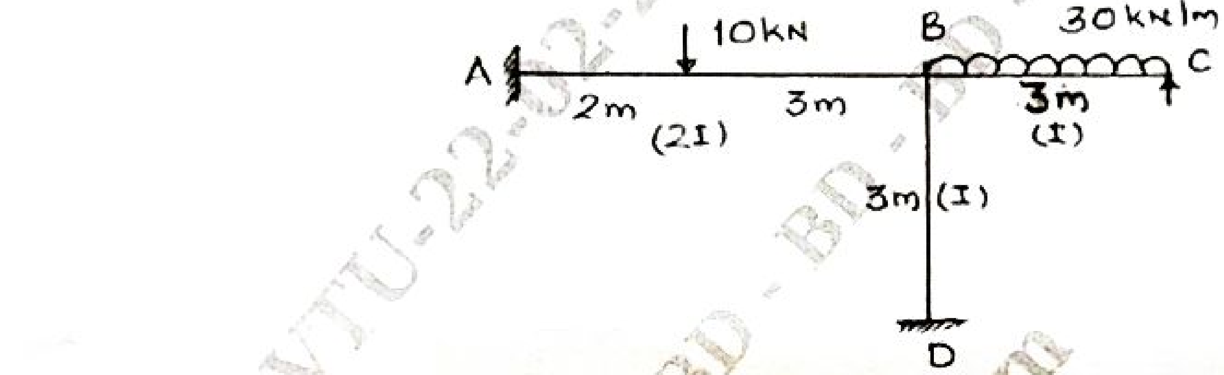


Fig.Q10

(20 Marks)

* * * * *

Modified

CBCS SCHEME

USN ☐☐☐☐☐☐☐☐☐☐

17CV52

Fifth Semester B.E. Degree Examination, Dec.2019/Jan.2020
Analysis of Indeterminate Structures

Time: 3 hrs. Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

Module-1

1 Analyse the beam completely by slope deflection method relative to support A support B sinks by 1mm and support C rises by 0.5 mm. Take EI = 30000 kN-m2. Refer Fig.Q1. Draw BMD, SFD and Elastic curve.

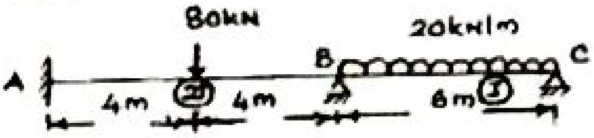


Fig.Q1 **(20 Marks)**

OR

2 Analyse the given frame by slope deflection method. Draw SFD, BMD and elastic curve. Refer Fig.Q2.

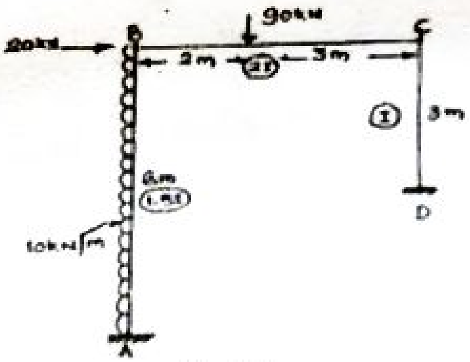


Fig.Q2 **(20 Marks)**

Module-2

3 Analyse the beam shown in Fig.Q3 by moment distribution method. Draw BMD, SFD and elastic curve.

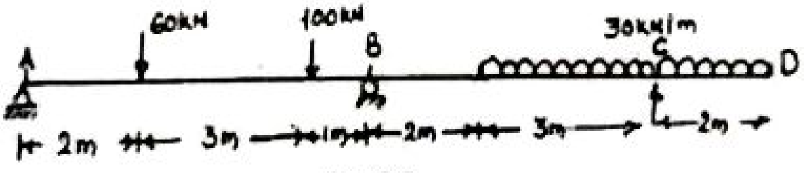


Fig.Q3 **(20 Marks)**

OR

4 Analyse the frame by moment distribution method. Draw BMD, SFD and elastic curve. Refer Fig.Q4.

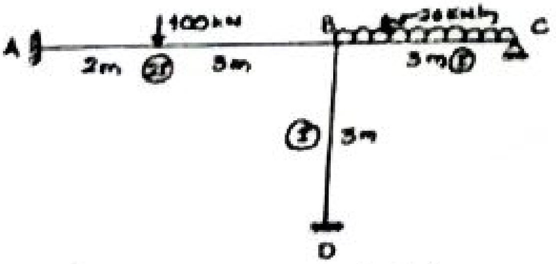


Fig.Q4

(20 Marks)

Module-3

5 Analyse the three span continuous beam shown in Fig.Q5 by using Kani's method. Draw BMD, SFD and elastic curve.

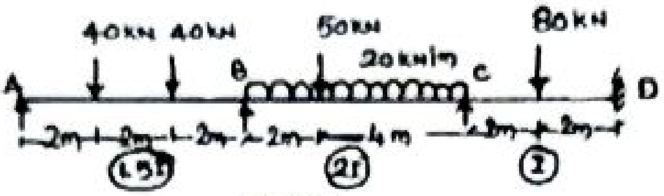


Fig.Q5

(20 Marks)

OR

6 Analyse the portal frames shown in Fig.Q6 by using Kani's method. Draw BMD, SFD and elastic curve.

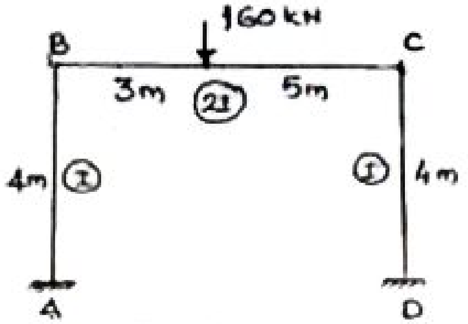


Fig.Q6

(20 Marks)

Module-4

7 Analyse the continuous beam shown in Fig.Q7 by flexibility method using system approach. Support B sinks by 5 mm sketch BMD, SFD and elastic curve. Take $EI = 15 \times 10^3$ kN-m^2.

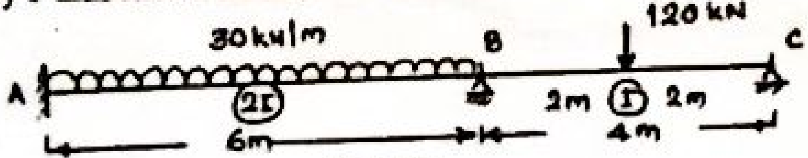


Fig.Q7

(20 Marks)

OR

8 Analyse the pin jointed plane truss shown in Fig.Q8 by using flexibility matrix method. Assume $\frac{L}{AE}$ for each member = 0.025 mm/kN. Tabulate the member forces.

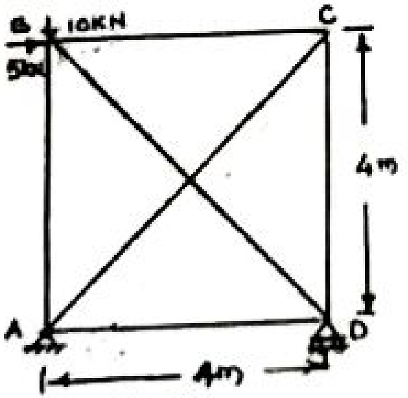


Fig.Q8

(20 Marks)

Module-5

9 Analyse the frame shown in Fig.Q9 by stiffness matrix method and draw BMD, SFD and Elastic curve. Assume EI is constant throughout.

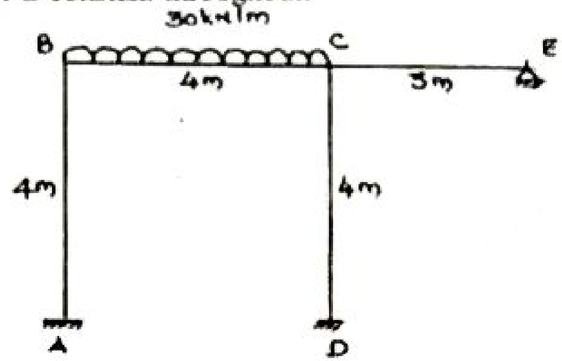


Fig.Q9

(20 Marks)

OR

10 Analyse the continuous beam shown in Fig.Q10 by using stiffness matrix method.

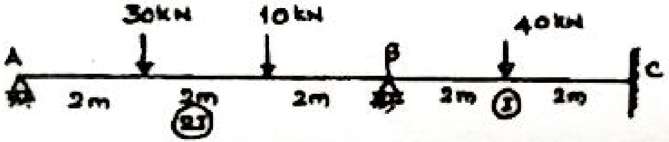


Fig.Q10

(20 Marks)

* * * * *

Visvesvaraya Technological University
Belagavi, Karnataka - 590 018

17CV52

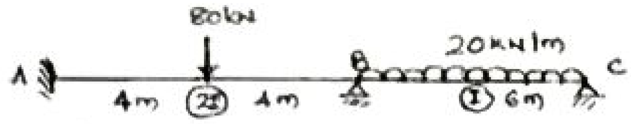
Signature of Scrutinizer

Scheme & Solution

Subject Title : Analysis of Indeterminate Structures **Subject Code :** 17CV52

Question Number	Solution	Marks Allocated

1



(a) F.E.M: $M_{FAB} = -\frac{80\times8}{8} = -80\text{kN.m}$ $M_{FBA} = +\frac{80\times8}{8} = 80\text{ kN.m}$ 04 Marks

$M_{FBC} = \frac{-20\times6^2}{12} = -60\text{kN.m}$ $M_{FCB} = \frac{20\times6^2}{12} = 60\text{ kN.m}$

(b) S.D. Equations:- 04 Marks

$M_{AB} = -80 + \frac{2E(2I)}{8}\left(\theta_B - \frac{3\times1}{8000}\right) = -80 + \frac{30000}{2}\left(\theta_B - \frac{3}{8000}\right)$

$= -80 + 15{,}000\theta_B - 5.625 \Rightarrow 15.000\theta_B - 85.625$ —(i)

$M_{BA} = +80 + \frac{30000}{2}\left(2\theta_B - \frac{3\times1}{8000}\right) \Rightarrow 30{,}000\theta_B + 74.375$ —(ii)

$M_{BC} = -60 + \frac{2}{6}\times30{,}000\left(2\theta_B + \theta_C - \frac{3\times(-1.5)}{6000}\right\} \Rightarrow 20{,}000\theta_B + 10000\theta_C - 52.5$ —(iii)

$M_{CB} = +60 + \frac{2}{6}(30000)\left\{\theta_B + 2\theta_C + \frac{3\times1.5}{6000}\right\} \Rightarrow 10{,}000\theta_B + 20{,}000\theta_C + 67.5$ —(iv)

(c) Equilibrium equations: 03 Marks

$M_{BA} + M_{BC} = 0 \Rightarrow 50{,}000\theta_B + 10{,}000\theta_C = -21.875$ —(I)

$M_{CB} = 0 \Rightarrow 10{,}000\theta_B + 20{,}000\theta_C = -67.5$ —(II)

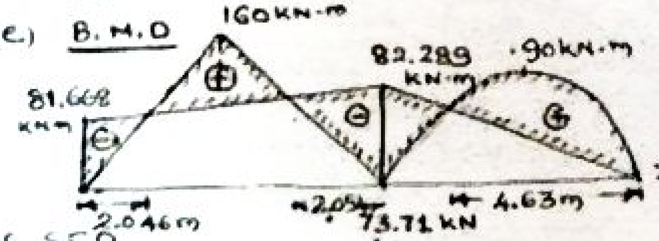
By solving $\theta_B = 2.638\times10^{-4}$ $\theta_C = -3.507\times10^{-3}$

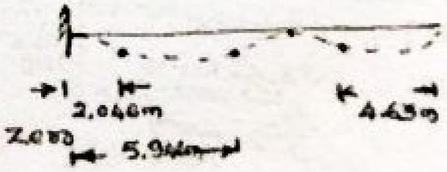
(d) Final Moments:

$M_{AB} = -81.668\text{ kN.m}$ $M_{BA} = 82.289\text{ kN.m}$ $M_{BC} = -82.289\text{kN}$ 04 Marks

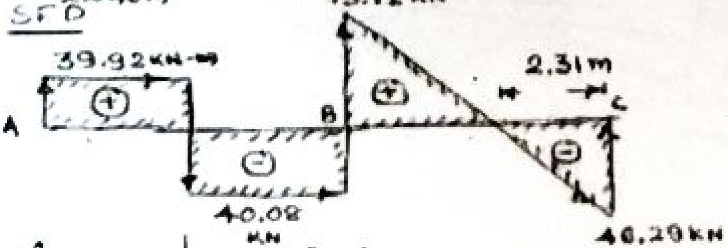
$M_{CB} = 0$

(e) B.M.D 02+01 Marks





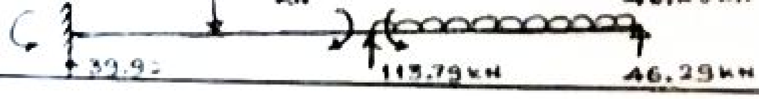
f SFD



SF at xx is zero

2.3145m from C 02 Marks

Maximum BM in BC = 53.56 kN.m

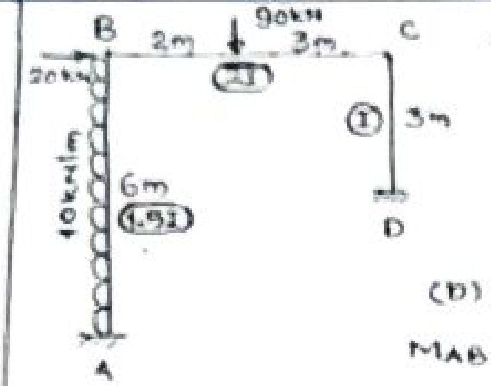


"APPROVED"

Registrar (Evaluation)
Visvesvaraya Technological University,
BELAGAVI - 590018

Question Number	Solution	Marks Allocated

2



(a) F.E.M :- $M_{FAB} = -\frac{10\times6^2}{12} = -30$ kN·m — 04 Marks

$M_{FBA} = +30$ kN·m

$M_{FBC} = -\frac{90\times2\times3^2}{5^2} = -64.8$ kN·m

$M_{FCB} = \frac{90\times2^2\times3}{5^2} = 43.2$ kN·m

(b) S.D. Equations — 03 Marks

$M_{AB} = +\frac{3EI}{6}\left(\theta_B - \frac{3\delta}{6}\right) - 30$

$= 0.5EI\theta_B - 0.25EI\Delta - 30$ —(i)

$M_{BA} = EI\theta_B - 0.25EI\Delta + 30$ —(ii)

$M_{BC} = -64.8 + 1.6EI\theta_B + 0.8EI\theta_C$ —(iii)

$M_{CB} = 43.2 + 0.8EI\theta_B + 1.6EI\theta_C$ —(iv)

$M_{CD} = 1.33EI\theta_C - 0.67EI\Delta$ —(v)

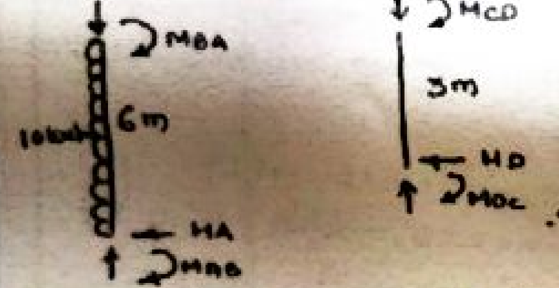
$M_{DC} = 0.67EI\theta_C - 0.67EI\Delta$ —(vi)

(c) Equilibrium equations — 02+02+01 Marks

$M_{BA} + M_{BC} = 0 \Rightarrow 2.6EI\theta_B + 0.8EI\theta_C - 0.25EI\Delta = 34.8$ —Ⓘ

$M_{CB} + M_{CD} = 0 \Rightarrow 0.8EI\theta_B + 2.93EI\theta_C - 0.67EI\Delta = -43.2$ —Ⓘⓘ

(d) column shear condition



$H_A + H_D - 20 - 10\times6 = 0$

$H_A = \frac{1}{6}\left[-M_{AB} - M_{BA} + 10\times6\times3\right]$

$H_D = \frac{1}{3}\left[-M_{CD} - M_{DC}\right]$

$\therefore \frac{-M_{AB} - M_{BA} + 180}{6} - \frac{(M_{CD} + M_{DC})}{3} - 80 = 0$

$\Rightarrow \frac{-[1.5EI\theta_B - 0.5EI\Delta] + 180}{6} - \frac{(2EI\theta_C - 1.33EI\Delta)}{3} - 80 = 0$

$\Rightarrow -0.25EI\theta_B - 0.67EI\theta_C + 0.5278EI\Delta = 50$ — Ⓘⓘⓘ

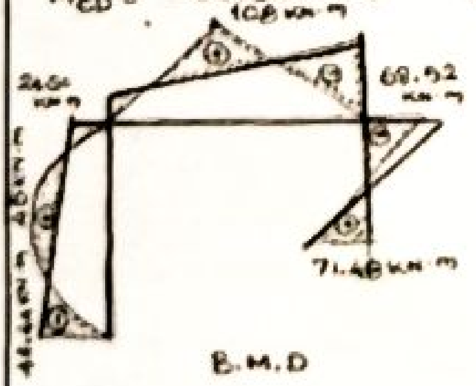
By solving $\theta_B = 22.6954/EI$ $\theta_C = 4.48/EI$ $\Delta = 111.17/EI$

(e) Final Moments — 03 Marks

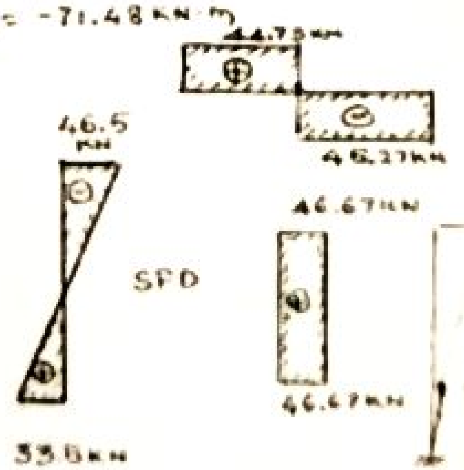
$M_{AB} = -46.44$ kN·m $M_{BA} = 24.90$ kN·m $M_{BC} = -24.90$ kN·m $M_{CB} = 68.52$ kN·m

$M_{CD} = -68.52$ kN·m $M_{DC} = -71.48$ kN·m

02+02M +01M



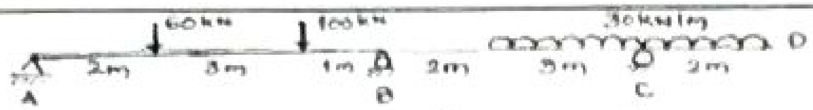
B.M.D



SFD

RC

Question Number	**Solution**	**Marks Allocated**
3		



(a) F.E.M: $M_{FAB} = -\frac{60\times2\times4^2}{6^2} - \frac{100\times5\times1^2}{6^2} = -67.22$ kN·m

$M_{FBA} = \frac{60\times2^2\times4}{6^2} + \frac{100\times5^2\times1}{6^2} = \cdot 96.11$ kN·m

$M_{FBC} = -\int_2^5 \frac{30dx(x)(5-x)^2}{5^2} = -\frac{30}{25}\left[\frac{25x^2}{2} - \frac{10x^3}{3} + \frac{x^4}{4}\right]_2^5 = -29.7$ kN·m

$M_{FCB} = \int_2^5 \frac{30dx}{5^2} x^2(5-x) = \frac{30}{25}\left[\frac{5x^3}{3} - \frac{x^4}{4}\right]_2^6 = 51.3$ kN·m

02+04=06

$M_{CD} = -30\times2\times1 = -60$ kN·m

01M

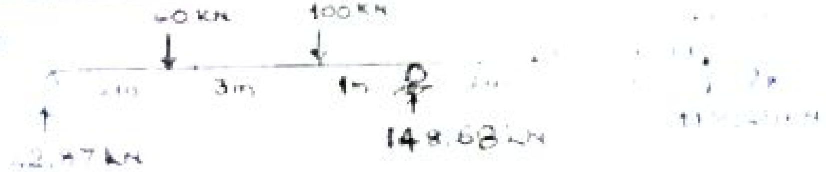
(b) Distribution Factor at the joint

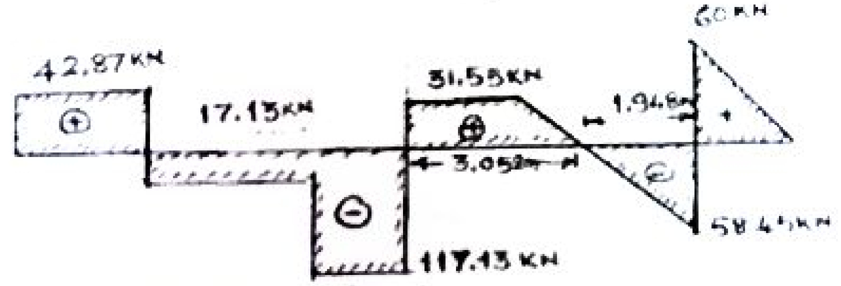
Joint	B		C	
Member	BA	BC	CB	CD
k	3/4 I/6	3/4 I/5	I/5	0
Σk	0.2751		I/5	
DF	0.45	0.55	1	0

04M

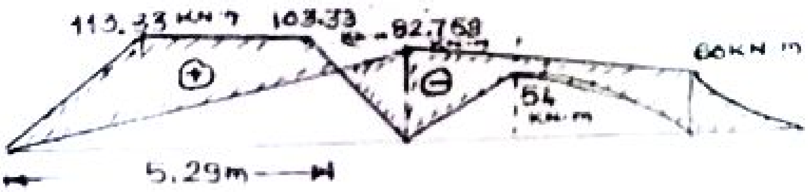
(c) M.D. Table

Joint	A	B		C	
Member	AB	BA	BC	CB	CD
DF	—	0.45	0.55	1.0	0
FEM	-67.22	96.11	-29.7	51.30	-60
Bal	+67.22	-29.88	-36.53	+8.70	0
CO		+33.61	4.35		
Bal		-17.082	-20.878		
Final moment	0	82.756	-82.758	[illegible]	





02 Marks



02 Marks

01 Mark

Question Number	Solution	Marks Allocated

4

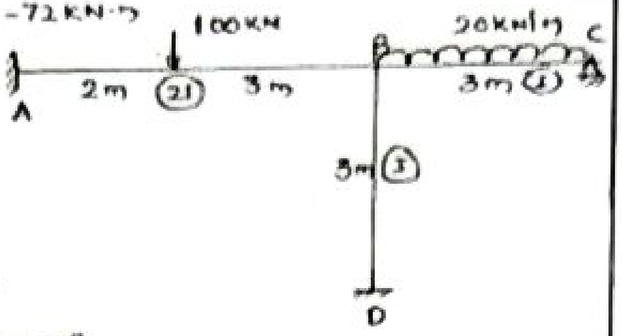
(a) F.E.M: $M_{FAB} = -\frac{100\times2\times3^2}{5^2} = -72\,KN\cdot m$

$M_{FBA} = +\frac{100\times2^2\times3}{5^2} = 48\,KN\cdot m$

$M_{FBC} = -\frac{20\times3^2}{12} = -15\,KN\cdot m$

$M_{FCB} = +15\,KN\cdot m$

$M_{FBD} = M_{FDB} = 0$



06 Marks

(b) D.F. at Joints

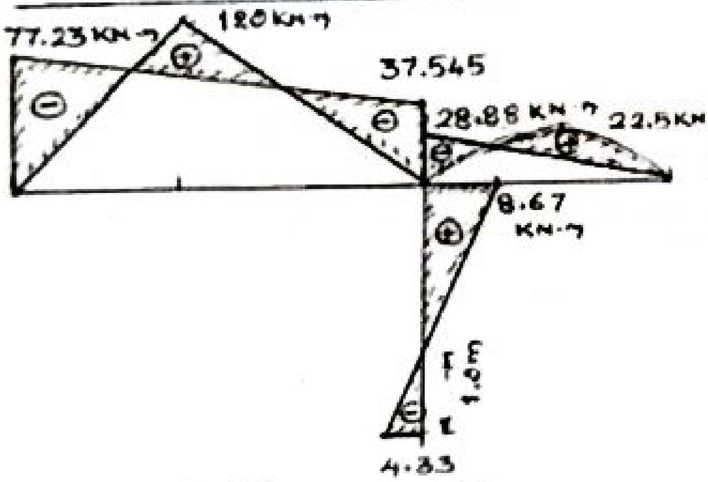
Joint	B		
Member	BA	BC	BD
K	2I/5	3/4 I/3	I/3
ΣK	0.981		
DF	0.407	0.25	0.34

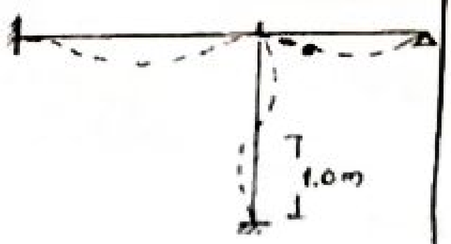
03 Marks

(c) M.D Table

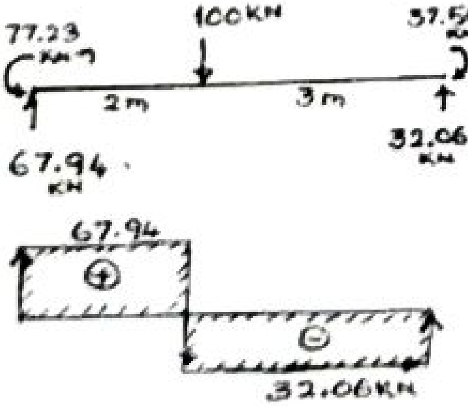
Joint	A	B			D	C
Member	AB	BA	BC	BD	DB	CB
DF	–	0.41	0.25	0.34	–	–
F.E.M	−72	48	−15	0		15
Balance		−13.53	−8.25	−11.22		−15
CO	−6.765		−7.50		−5.61	
Balance		+3.075	1.875	+2.55		
CO	1.54				+1.28	
Final Moments	−77.23	37.55	−28.88	−8.67	−4.33	0

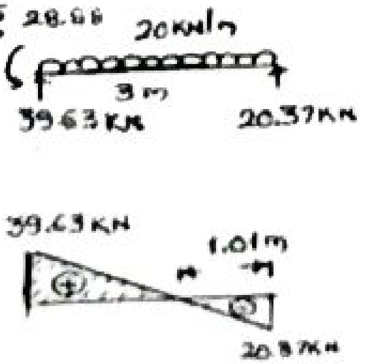
(05+01) Marks

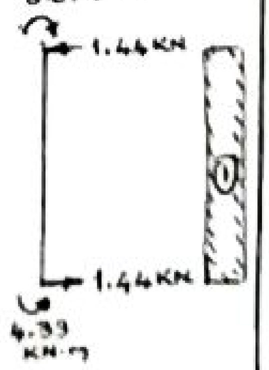




02+01 Marks



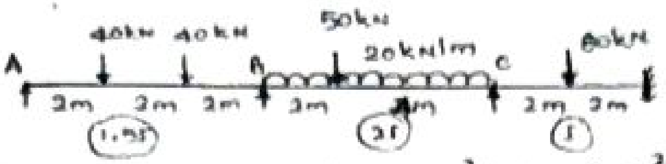




02 Marks

Question Number	Solution	Marks Allocated

5



(06 Marks)

(a) F.E.M :- $M_{FAB} = -\frac{40\times2\times4^2}{6^2} - \frac{40\times4\times2^2}{6^2} = -53.33\,kN\cdot m$

$M_{FBA} = +53.33\,kN\cdot m$

$M_{FBC} = \frac{-20\times6^2}{12} - \frac{50\times2\times4^2}{6^2} = -104.44\,kN\cdot m$

$M_{FCB} = \frac{20\times6^2}{12} + \frac{50\times2^2\times4}{6^2} = 82.22\,kN\cdot m$

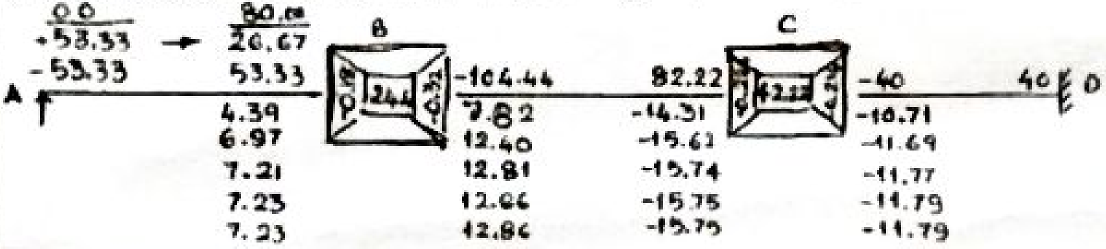
$M_{FCD} = -\frac{80\times4}{8} = -40\,kN\cdot m$ $M_{FDC} = +40\,kN\cdot m$

(b) Rotational Factor U

Joint	B		C	
Member	BA	BC	CB	CD
K	$\frac{3}{4}\cdot\frac{1.5I}{6}$	$\frac{2I}{6}=\frac{I}{3}$	$\frac{2I}{6}=I/3$	I/4
ΣK	0.521		0.583	
U	-0.18	-0.32	-0.286	-0.214

(02 Marks)

(c) Rotational Moments : $M'ab = U[\Sigma M_F + \Sigma M'ba]$



(04 Marks)

(d) Final moments

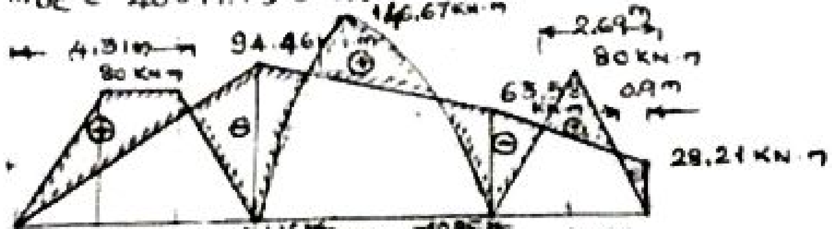
$M_{AB} = 0$, $M_{BA} = 80 + 2(7.23) = 94.46\,kN\cdot m$, $M_{BC} = -104.44 + 2(12.86) + (-15.75 = -94.47\,kN\cdot m$

$M_{CB} = 82.22 + 2(-15.75) + 12.86 = 63.58\,kN\cdot m$

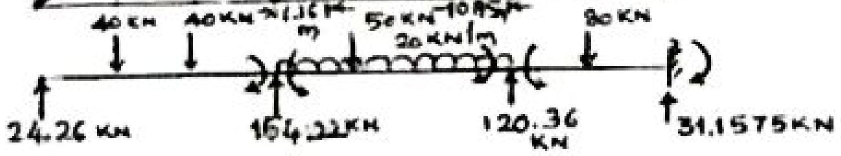
$M_{CD} = -40 + 2(-11.79) = -63.58\,kN\cdot m$

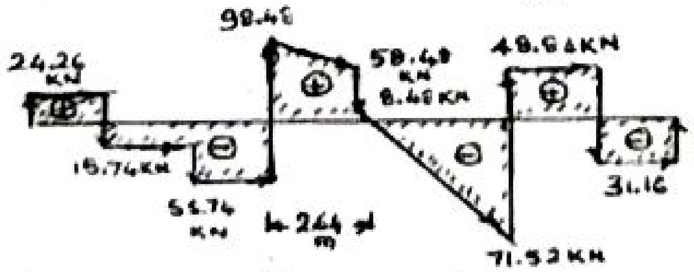
$M_{DC} = 40 - 11.79 = 28.21\,kN\cdot m$

(03 Marks)



(02 Marks)





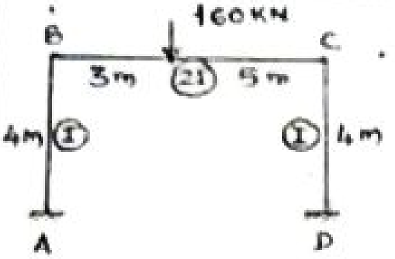
(02 Marks)

(01 Marks)

Subject Title : Analysis of Indeterminate Structure **Subject Code :** 17CV52

Question Number	Solution	Marks Allocated

6



(a) F.E.M :- $M_{FBC} = -\frac{160 \times 3 \times 5^2}{8^2} = -187.5$ KN-m

$M_{FCB} = \frac{160 \times 3^2 \times 5}{8} = 112.5$ KN-m

$M_{FAB} = M_{FBA} = 0$

$M_{FCD} = M_{FDC} = 0$

(03 Marks)

(b) Rotation Factor U

Joint	B		C	
Member	BA	BC	CB	CD
K	I/4	2I/8	2I/8	I/4
ΣK	I/2		I/2	
U	-1/4	-1/4	-1/4	-1/4

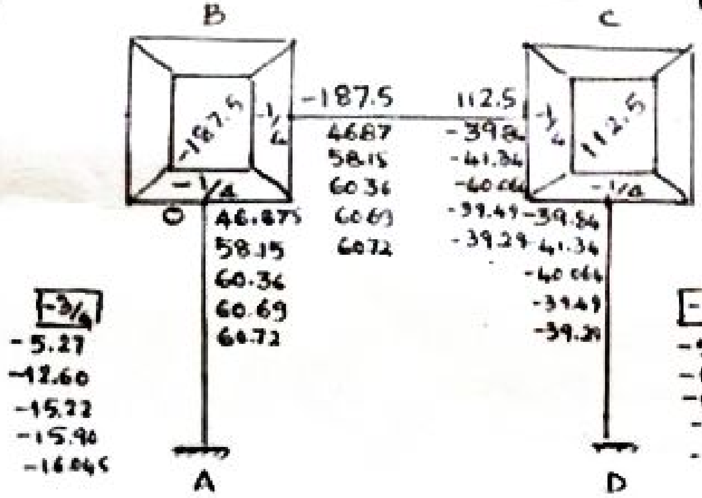
(02 Marks)

(c) Displacement Factor V

Member	AB	CD
k	I/4	I/4
Σk	I/2	
V	-3/4	-3/4

(02 Marks)

(d) Rotation & Displacement contribution



(05 Marks)

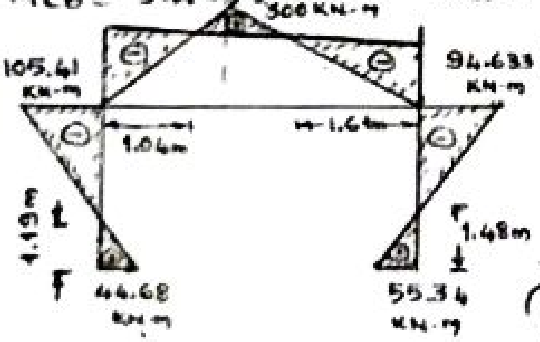
(e) Final Moments

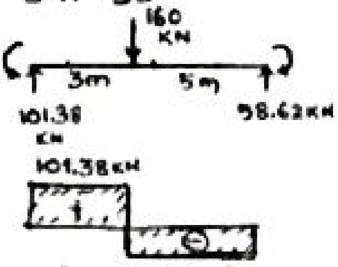
MAB = 60723 - 16.045 = 44.678 KN·m MBA = 2×60.723 - 16.045 = 105.406 KN·m

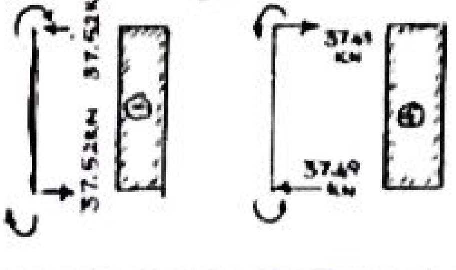
MBC = -187.5 + 2(60.723) - 39.294 = -105.35 KN·m

MCB = 94.635 KN·m MCD = - 94.633 KN·m MDC = - 55.339 KN·m

(03 Marks)







02 + 02 + 01 Marks

Question Number	Solution	Marks Allocated

7

(a) No of unknowns: M_A, R_A, R_B, M_B, R_C = 4 Nos

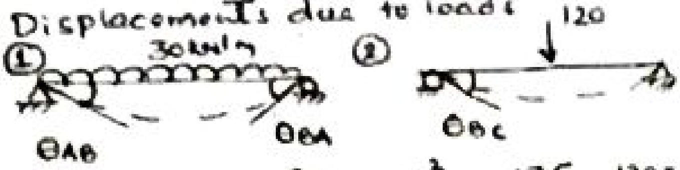
No of equilibrium equations } $\Sigma F_y = 0$, $\Sigma M_A = 0$, $\Sigma M_B = 0$ = 2 Nos

No of redundants = 2

M_A, M_B as redundants

(02 Marks)

(b) Displacements due to loads



$\Delta_{L1} = \frac{wl^3}{24EI} = \frac{30\times6^3}{24(2EI)}$

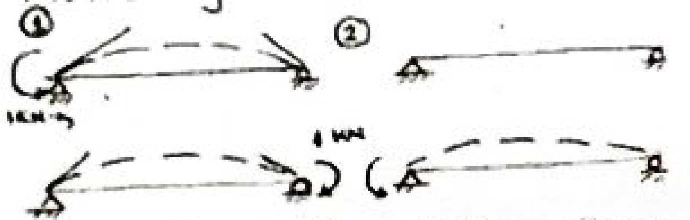
$= 135/EI$

$\Delta_{L2} = \frac{wl^3}{24EI}) + \frac{wl^2}{16EI} = \frac{135}{EI} + \frac{120\times4^2}{16EI} = \frac{255}{EI}$

$\Delta_L = \begin{bmatrix} 135/EI \\ 255/EI \end{bmatrix}$

(02 Marks)

(c) Flexibility Matrix Apply unit loads



$F_{11} = -\frac{1\times6}{3E(2I)} = -\frac{1}{EI}$

$F_{21} = -\frac{1\times6}{6E(2I)} = \frac{-0.5}{EI}$

$F_{12} = \frac{-1\times6}{6E(2I)} = \frac{-0.5}{EI}$

$F_{22} = -\frac{1\times6}{3E(2I)} - \frac{1\times4}{3EI} = -7/3EI$

(04 Marks)

$[F] = \frac{-1}{EI}\begin{bmatrix} 1 & 1/2 \\ 1/2 & 7/3 \end{bmatrix}$

WKT $\begin{bmatrix} M_A \\ M_B \end{bmatrix} = [F]^{-1}[\Delta - \Delta_L]$

Δ = Displacement due to sinking of supports

$\Delta_1 = -\frac{5}{6000} = -\frac{1}{1200}$ $\Delta_2 = \frac{5}{4000} + \frac{5}{6000} = \frac{1}{480}$

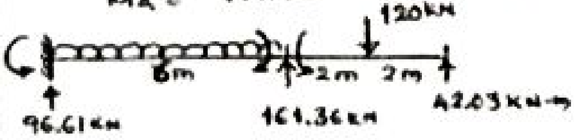
(02 Marks)

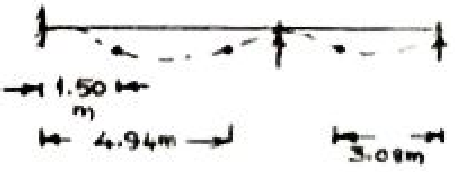
$\begin{bmatrix} M_A \\ M_B \end{bmatrix} = -\frac{15\times10^3}{(7/3 - 1/4)}\begin{bmatrix} 7/3 & -1/2 \\ -1/2 & 1 \end{bmatrix}\left[\begin{Bmatrix} -1/1200 \\ 1/480 \end{Bmatrix} - \begin{Bmatrix} 135/15\times10^3 \\ 255/15\times10^3 \end{Bmatrix}\right] = \frac{-15\times12\times10^3}{25}\begin{bmatrix} 7/3 & -1/2 \\ -1/2 & 1 \end{bmatrix}\begin{bmatrix} 9.8\times10^{-4} \\ -0.0149 \end{bmatrix}$

(03 Marks)

$M_A = 111.55$ kN-m $M_B = 71.88$ kN-m

(02 Marks)





02+02 +01 Marks

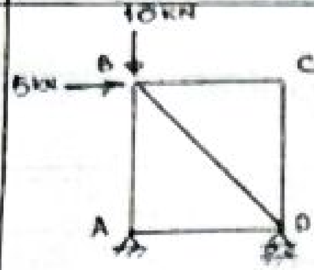
Maximum BM in AB

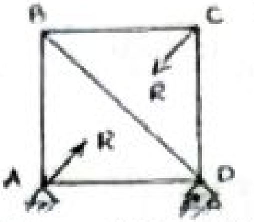
$96.61\times3.22 - 111.36 - 30\times\frac{3.22^2}{2}$

$= 44.1982$ kN.m

Question Number | **Solution** | **Marks Allocated**

8

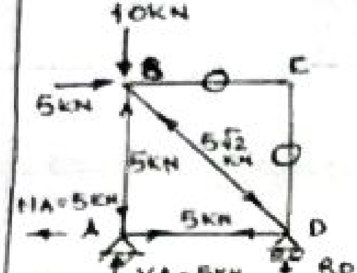


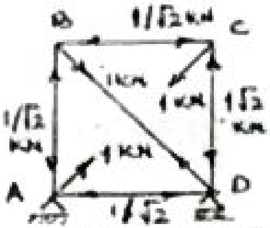


Released Structure along with Redundant

(a) no of reactions 2+1=3
no of members = 6
no of joints = 4
no of redundants
$R = m-(2j-3)$
$= 6-(2\times4-3)$
$= 1$
Assume Force in AC as redundant

(02Marks)





$\Sigma F_x = 0 \Rightarrow H_A = \overleftarrow{5KN}$
$\Sigma M_A = 0 \Rightarrow R_D = 5KN\uparrow$
$\Sigma F_y = 0 \Rightarrow R_A = 5KN\uparrow$

03+03 (Marks)

Forces in released structures for external loads and unit load

Displacement and flexibility coefficients

Member	F KN	f KN	l m	$\frac{Ffl}{AE}=\Delta L_1$	$\frac{f^2l}{AE}=\delta_{11}$	F+fR KN
AB	-5	$-1/\sqrt{2}$	4	$10\sqrt{2}$	2	-6.46
BC	0	$-1/\sqrt{2}$	4	0	2	-1.46
CD	0	$-1/\sqrt{2}$	4	0	2	-1.46
DA	5	$-1/\sqrt{2}$	4	$-10\sqrt{2}$	2	+3.54
DB	$-5\sqrt{2}$	1	$4\sqrt{2}$	-40.	$4\sqrt{2}$	-5.00
AC	–	1	$4\sqrt{2}$	–	$4\sqrt{2}$	+2.07
			Σ	-40	$8(1+\sqrt{2})$	

(03Marks)

Compatibility equation $\Delta_{L1} + \delta_{11}\times R = 0$

$$\Delta_{L1} = \frac{\Sigma Ffl}{AE} = \frac{-40}{AE} \qquad \delta_{11} = \frac{\Sigma f^2 l}{AE} = \frac{8(1+\sqrt{2})}{AE}$$

By above equation

$$-\frac{40}{AE} + \frac{8(1+\sqrt{2})}{AE}\times R = 0 \qquad \therefore R = 2.07KN$$

(03Marks)

Final forces

$F_{AB} = F+fR = -5-\frac{1}{\sqrt{2}}\times2.07 = -6.46KN$

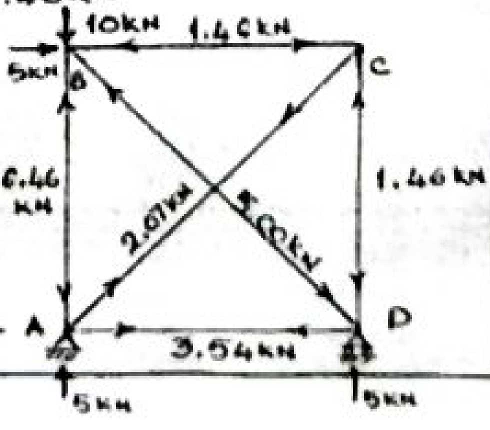
$F_{BC} = -\frac{1}{\sqrt{2}}\times2.07 = -1.46KN$

$F_{CD} = -1.46KN$

$F_{DA} = +5-\frac{1}{\sqrt{2}}\times2.07 = +3.54KN$

$F_{DB} = -5\sqrt{2}+2.07 = -5.00KN$

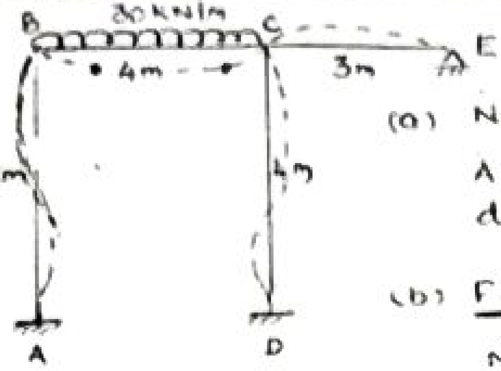
$F_{AC} = 0+2.07 = 2.07KN$



(06Marks)

Subject Title: Analysis of Indeterminate Structures Subject Code: 17CV52.

Question Number	Solution	Marks Allocated
9		



(a) No of Redundants 3, θ_B, θ_C, θ_E (01 Marks) + (01 Marks)

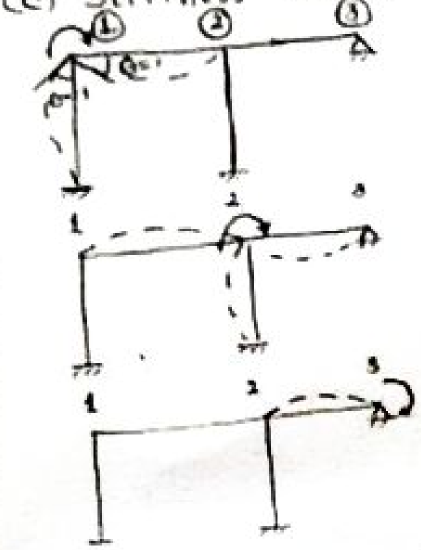
Assumed to act in clockwise direction

(b) **FEM** - $M_{FBC} = -\frac{30\times4^2}{12} = -40$ kN.m (01 Marks)

$M_{FCB} = +40$ kN.m

$P_1 = -40$ kN.m $P_2 = 40$ kN.m

(c) Stiffness Matrix - Apply unit moment at 1, 2, 3



$K_{11} = \frac{4EI}{4} + \frac{4EI}{4} = 2EI$

$K_{21} = \frac{2EI}{4} = 0.5EI$ $K_{31} = 0$

$K_{12} = \frac{2EI}{4} = 0.5EI$

$K_{22} = \frac{4EI}{4} + \frac{4EI}{4} + \frac{4EI}{3} = \frac{10EI}{3}$

$K_{32} = \frac{2EI}{3}$

$K_{33} = \frac{4EI}{3}$ $K_{13} = 0$ $K_{23} = \frac{2EI}{3}$ (07 Marks)

$$[k] = EI\begin{bmatrix} 2 & 0.5 & 0 \\ 0.5 & 10/3 & 2/3 \\ 0 & 2/3 & 4/3 \end{bmatrix} \qquad \begin{bmatrix}\Delta_1\\ \Delta_2\\ \Delta_3\end{bmatrix} = \frac{1}{EI}\begin{bmatrix} 2 & 0.5 & 0 \\ 0.5 & 10/3 & 2/3 \\ 0 & 2/3 & 4/3 \end{bmatrix}\begin{bmatrix}P-P_L\end{bmatrix}$$

$$[\Delta] = \frac{1}{EI\left[2\left(\frac{40}{9}-\frac{4}{9}\right)-0.5\left(\frac{2}{3}\right)\right]}\begin{bmatrix} 9 & -2/3 & 1/3 \\ -2/3 & +8/3 & -4/3 \\ 1/3 & -4/3 & 6.4167 \end{bmatrix}\begin{bmatrix}40\\ -40\\ 0\end{bmatrix}$$ 03 Marks

$\Delta_1 = 24.3478/EI$ $\Delta_2 = -17.3913/EI$ $\Delta_3 = 8.6956/EI$

(d) Final Moments

$M_{AB} = \frac{2EI}{4}(24.3478) = 12.17$ kN.m $M_{BA} = 24.35$ kN.m

$M_{BC} = -40 + \frac{2EI}{4}(2\times24.3478 - 17.3913) = -24.35$ "

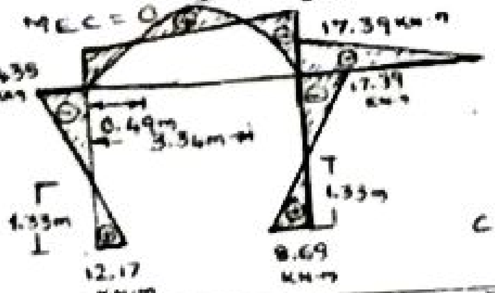
$M_{CB} = 40 + 2EI/4(-2\times17.3913 + 24.3478) = 34.7826$ "

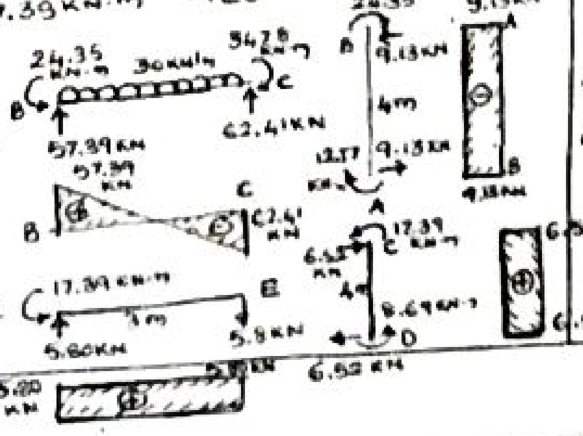
$M_{DC} = 2EI/4(-17.3913) = -8.69$

$M_{CE} = -17.3913$ kN.m (03 Marks)

$M_{CD} = \frac{2EI}{4}(-2\times17.3913) = -17.39$ kN.m

$M_{EC} = 0$



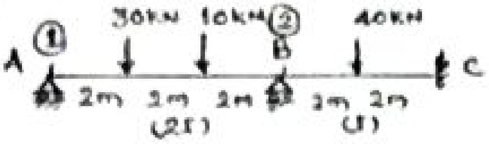


02 Marks + 02 Marks

Subject Title : Analysis of Indeterminate structures Subject Code : 17CV52

Question Number	Solution	Marks Allocated

10



(a) DOF :- 2. θA, θB as redundants — 01 Marks

(b) F.E.M :- $M_{FAB} = -\frac{30\times2\times4^2}{6^2} - \frac{10\times4\times2^2}{6^2} = -31.11$ KN·m — 04 Marks

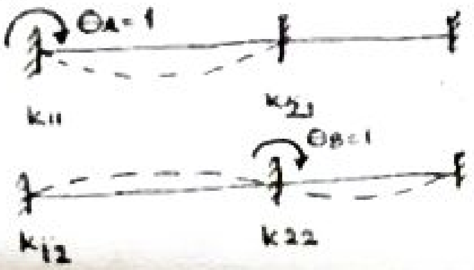
$M_{FBA} = +\frac{30\times2^2\times4}{6^2} + \frac{10\times4^2\times2}{6^2} = 22.22$ KN·m

$M_{BC} = -\frac{40\times4}{8} = -20$ KN·m $M_{FCB} = +20$ KN·m

(c) Force or moment due to applied load

$$[P_L] = \begin{bmatrix} P_{1L} \\ P_{2L} \end{bmatrix} = \begin{bmatrix} -31.11 \\ 22.22-20 \end{bmatrix} = \begin{bmatrix} -31.11 \\ 2.22 \end{bmatrix}$$ — 01 Marks

(d) Apply unit moments or Rotations at coordinates to get stiffness matrix k



$k_{11} = \frac{4E(2I)}{6} = \frac{4EI}{3}$

$k_{21} = \frac{2E(2I)}{6} = \frac{2EI}{3}$

$k_{12} = \frac{2E(2I)}{6} = \frac{2EI}{3}$

$k_{22} = \frac{4E(2I)}{6} + \frac{4EI}{4} = \frac{7EI}{3}$ — (02 Marks)

Stiffness Matrix[k] : $k = \begin{bmatrix} 4EI/3 & 2EI/3 \\ 2EI/3 & 7EI/3 \end{bmatrix}$ $[k]^{-1} = \frac{3}{EI(28-4)}\begin{bmatrix} 7 & -2 \\ -2 & 4 \end{bmatrix}$

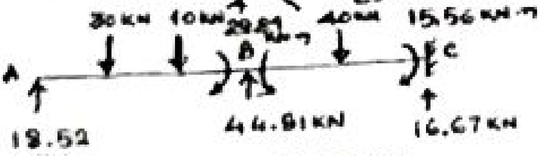
$\Delta = [K]^{-1}[P-P_L] \Rightarrow \begin{bmatrix} \theta_A \\ \theta_B \end{bmatrix} = \frac{1}{8EI}\begin{bmatrix} 7 & -2 \\ -2 & 4 \end{bmatrix}\begin{bmatrix} 0-(-31.11) \\ 0-2.22 \end{bmatrix}$ — (01 Marks)

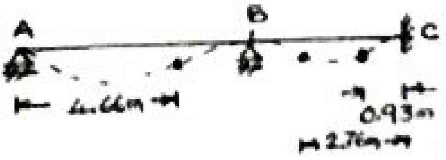
$\theta_A = 27.78/EI$ $\theta_B = -8.89/EI$ — 02 Marks

Final moments $M_{AB} = 0 \Rightarrow -31.11 + \frac{2E(2I)}{6}\left(2\times\frac{27.78}{EI} - \frac{8.89}{EI}\right) = 0$

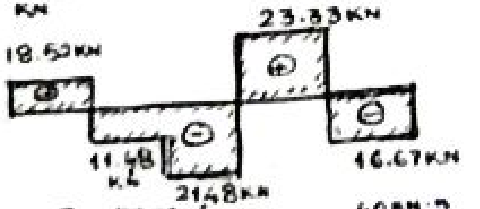
$M_{BA} = 22.22 + \frac{2E(2I)}{6}\left[-2\times\frac{8.89}{EI} + \frac{27.78}{EI}\right] = 28.89$ KN·m

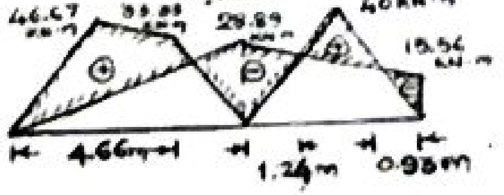
$M_{BC} = -20 + \frac{2EI}{4}\left(-2\times\frac{8.89}{EI}\right) = -28.89$ KN·m $M_{CB} = +20 + \frac{2EI}{4}\left(-\frac{8.89}{EI}\right) = 15.56$ KN·m — (04 Marks)





02+02+01





– END –

Gangadhar Hugar
Asst Prof, P.D.I.T
Hospete.

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Registrar (Evaluation)
Visvesvaraya Technological University
BELAGAVI - 590018

PCL XL Error

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CBCS SCHEME

USN ☐☐☐☐☐☐☐☐☐☐ **15CV52**

Fifth Semester B.E. Degree Examination, Dec.2019/Jan.2020
Analysis of Indeterminate Structures

Time: 3 hrs. Max. Marks: 80

Note: ***Answer any FIVE full questions, choosing ONE full question from each module.***

Module-1

1 Analyse the continuous beam shown in Fig Q1 by slope deflection method. Draw bending moment diagram and shear force diagram.

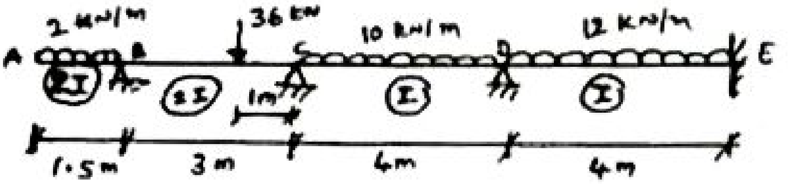


Fig Q1 **(16 Marks)**

OR

2 Analyse the portal frame shown in Fig Q2 by slope deflection method. Draw bending moment diagram.

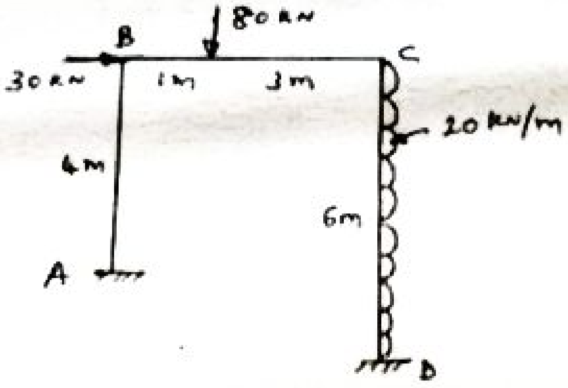


Fig Q2 **(16 Marks)**

Module-2

3 Analyse the continuous beam shown in Fig Q3 by moment distribution method. Draw bending moment diagram and shear force diagram. Support at B sinks by 10mm.

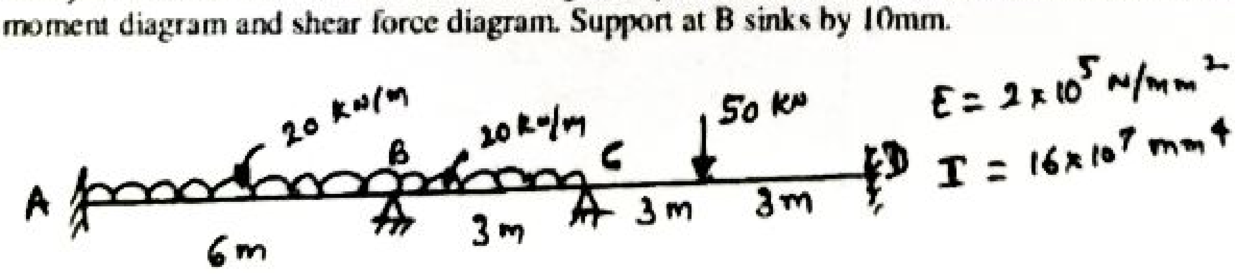


Fig Q3 **(16 Marks)**

OR

4 Analyse the frame shown in Fig Q4 by moment distribution method. Draw bending moment diagram.

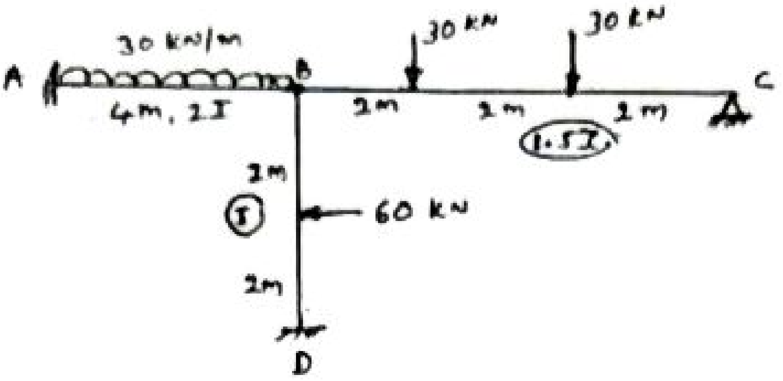


Fig Q4 (16 Marks)

Module-3

5 Analyse the continuous beam shown in Fig Q5 by rotation contribution method. Draw bending moment diagram and shear force diagram.

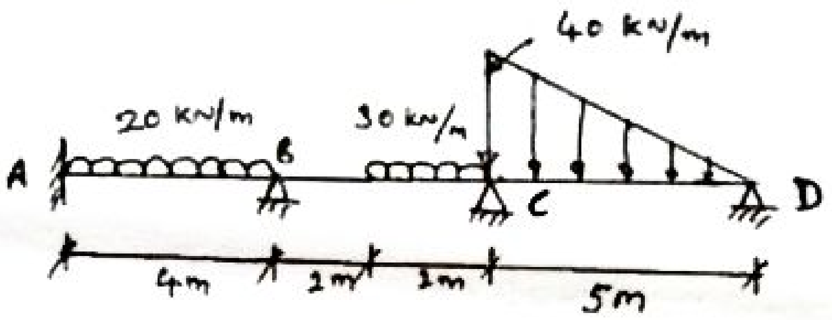


Fig Q5 (16 Marks)

OR

6 Analyse the frame shown in Fig Q6 by Kani's method. Draw bending moment diagram. Use axis of symmetry approach.

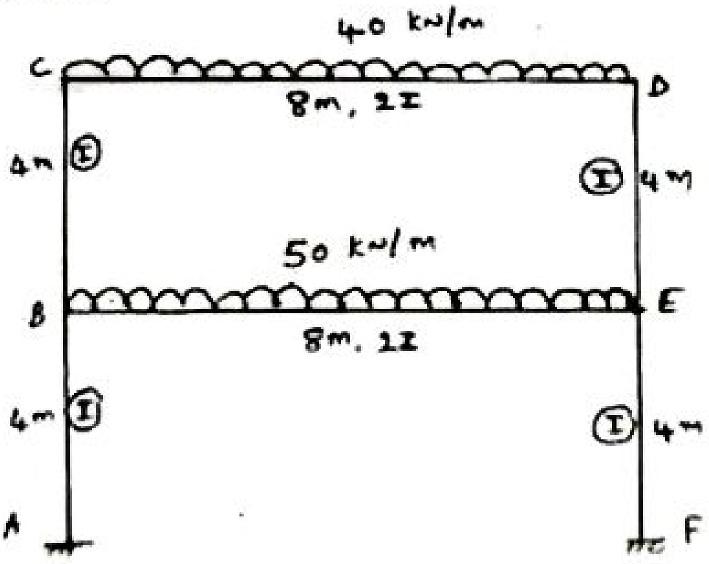


Fig Q6 (16 Marks)

Module-4

7 Analyse the continuous beam shown in Fig Q7 by flexibility matrix method. Draw BMD and SFD.

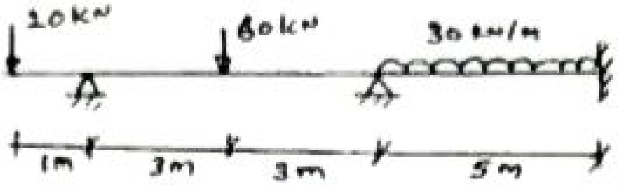


Fig Q7

(16 Marks)

OR

8 Analyse the pin jointed plane shown in Fig Q8 by flexibility matrix method to compute axial forces in the members. Assume $\frac{L}{AE}$ for each member is 0.025mm/kN.

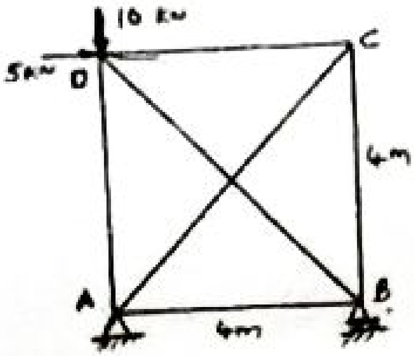


Fig Q8

(16 Marks)

Module-5

9 Analyse the continuous beam shown Fig Q9 by stiffness matrix method. Draw SFD and BMD.

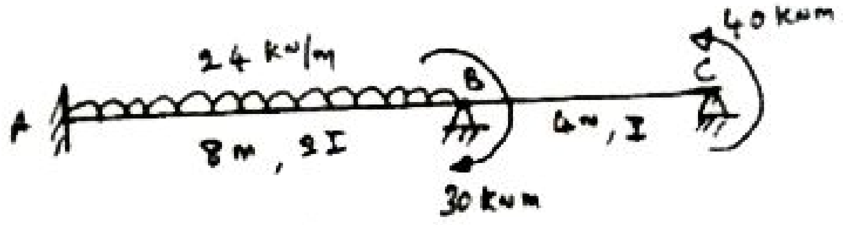


Fig Q9

(16 Marks)

OR

10 Analyse the portal frame shown in Fig Q10 by stiffness matrix method. Draw bending moment diagram.

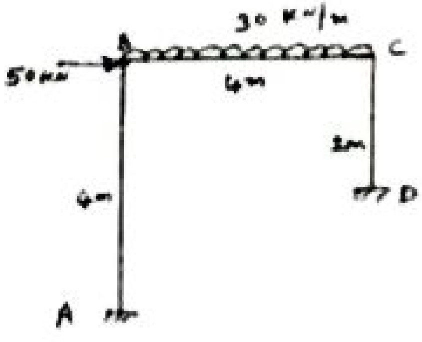


Fig Q10

(16 Marks)

Visvesvaraya Technological University
Belagavi, Karnataka - 590 018

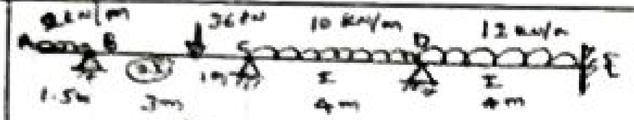
Signature of Scrutinizer

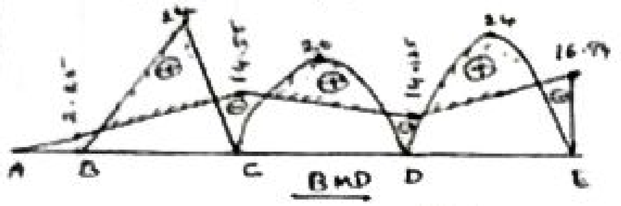
Scheme & Solution

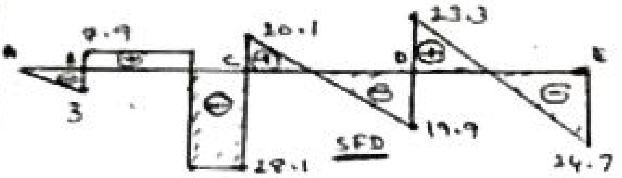
Subject Title: Analysis of Indeterminate Structures **Subject Code:** 15CV52

Question Number | **Solution** | **Marks Allocated**

1







$M_{BA} = +2 \times \frac{1.5^2}{2} = +2.25$ kNm

Fixed end moment

$\overline{M}_{AB} = -8$: $\overline{M}_{BA} = 16$: $\overline{M}_{BC} = -13.33$

$\overline{M}_{CB} = 13.33$: $\overline{M}_{CD} = -16 = -\overline{M}_{DC}$ — (02)

Slope deflection Eqn

$M_{BC} = -8 + \frac{8}{3} EI\theta_B + \frac{4}{3} EI\theta_C$

$M_{CB} = 16 + \frac{4}{3} EI\theta_B + \frac{8}{3} EI\theta_C$

$M_{CD} = -13.33 + EI\theta_C + 0.5EI\theta_D$

$M_{DC} = 13.33 + 0.5EI\theta_C + EI\theta_D$ — (03)

$M_{DE} = -16 + EI\theta_D$

$M_{ED} = 16 + 0.5EI\theta_D$

Equilibrium Eqns

$M_{BA} + M_{BC} = 0$: $8EI\theta_B + 4EI\theta_C = 17.25$

$M_{CB} + M_{CD} = 0$: $4EI\theta_B + 11EI\theta_C + 1.5EI\theta_D = -8.01$

$M_{DC} + M_{DE} = 0$: $0.5EI\theta_C + 3EI\theta_D = 2.67$ — (03)

$EI\theta_B = 3.2366$: $EI\theta_C = -2.1608$

$EI\theta_D = 1.8752$

Final moment : $M_{BA} = 2.25$: $M_{BC} = -2.25$

$M_{CB} = 14.55$: $M_{CD} = -14.55$ — (02)

$M_{DC} = 14.125$: $M_{DE} = -14.125$

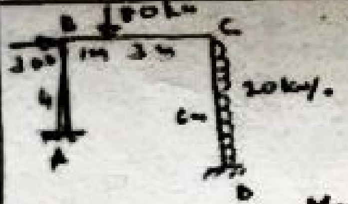
$M_{ED} = 16.94$

Reaction : $R_B = 10.9$: $R_C = 48$ — (03)

$R_D = 43.2$: $R_E = 24.7$ kN

SFD – BMD – $\frac{1}{2}$

2.



Fixed end Moment : $\overline{M}_{BC} = -45$: $\overline{M}_{CB} = 15$ — (02)

$\overline{M}_{CD} = -60 = -\overline{M}_{DC}$

Slope deflection Eqn

$M_{AB} = 0.5EI\theta_B - 0.375EI\delta$: $M_{BA} = EI\theta_B - \frac{3}{8}EI\delta$

$M_{BC} = -45 + EI\theta_B + 0.5EI\theta_C$: $M_{CB} = 15 + 0.5EI\theta_B + EI\theta_C$ — (05)

$M_{CD} = -60 + \frac{2}{3}EI\theta_C - \frac{EI\delta}{6}$: $M_{DC} = 60 + \frac{1}{3}EI\theta_C - \frac{EI\delta}{6}$

Equilibrium Eqn

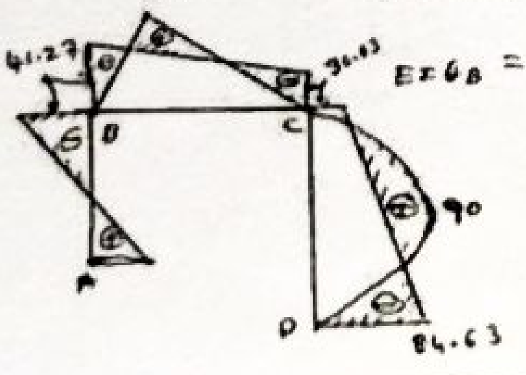
$M_{BA} + M_{BC} = 0$; $3EI\theta_B + 0.5EI\theta_C - \frac{3}{8}EI\delta = 45$

$M_{CB} + M_{CD} = 0$: $0.5EI\theta_B + 1.67EI\theta_C - 0.167EI\delta = 45$ — (05)

Shear Equation

$2.25EI\theta_B + EI\theta_C - 1.458EI\delta = 180$

$EI\theta_B = -2.14$: $EI\theta_C = 16.00$: $EI\delta = -115.78$



Final Moment

$M_{AB} = 42.34$ kNm

$M_{BA} = 41.27$

$M_{BC} = -41.27$ — (02)

$M_{CB} = 30.03$

$M_{CD} = -30.03$

$M_{DC} = 84.63$ kN-m

BMD – (02)

Subject Title: Analysis of Indeterminate Structures **Subject Code:** 15CV52

Question Number | **Solution** | **Marks Allocated**

③

EI = 32,000 kNm²

Fixed end Moments and Max sagging moment

$\overline{M}_{AB} = -113.33 : \overline{M}_{BA} = 6.67$

$\overline{M}_{BC} = 198.33 : \overline{M}_{CB} = 228.33$

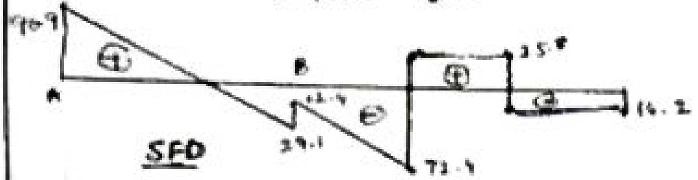
$\overline{M}_{CD} = -37.5 : M_{DC} = 37.5$

(02)

Distribution factor

Jt	Mem	R.S	T.S	D.F
B	BA	3I/6	0.5I	1/2
	BC	I/3		1/2
C	CB	I/3	0.5I	2/3
	CD	3I/6		1/3

(02)



Moment Distribution table:

Jt	[illegible]	B		C		D
Mem	AB	BC	CB	CB	CD	DC
D.F	–	1/2	1/2	2/3	1/3	–
FEM	-113.33	6.67	198.33	228.33	-37.5	37.5
Bal C.O Bal	-34.16	-68.32 31.20	-136.67 -63.61 45.40	-127.33 -68.33 45.36	-63.61 22.77	-31.80
Ultimate Final	-134.51	-46.12	45.62	81.13	-81.12	16.08

(06)

Reaction

$R_A = 90.9$

$R_B =$ [redacted] 16.7

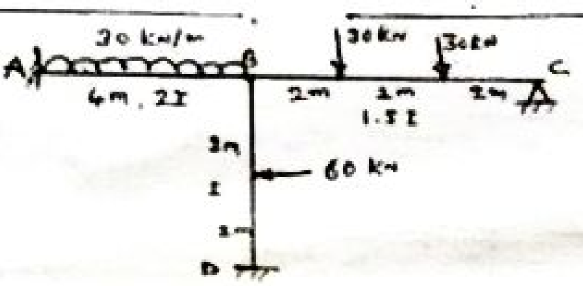
$R_C = 108.2$

$R_D = 16.2$

(02)

SFD - (02)
BMD - (02)

④



Fixed end Moment

$\overline{M}_{AB} = -40 : \overline{M}_{BA} = 40$

$\overline{M}_{BC} = -40 : M_{CB} = 40$

$\overline{M}_{BD} = -30 : \overline{M}_{DB} = 30$

(03)

Distribution factor

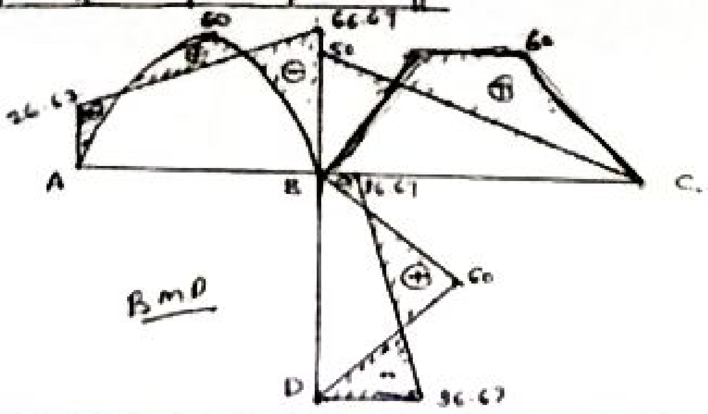
Jt Mem R.S T.S D.F
AB 2I/4 0.533
B BC (3/4)1.5I/6 0.20
BD I/4 0.267

(03)

Moment Distribution table

Jt	A	B			C	D
Mem		BA	BC	BD	CB	DB
FEM	-40	40	-40	-30	40	30
DF		0.533	0.2	0.267	●	
Rel. & C.O			[illegible] -20		-40	
	-40	40	-60	-30	0	30
Bal		26.66	10	13.33		
C.O	13.33					6.66
Final MF	-26.6	66.64	-50	-16.67	0	36.66

(07)



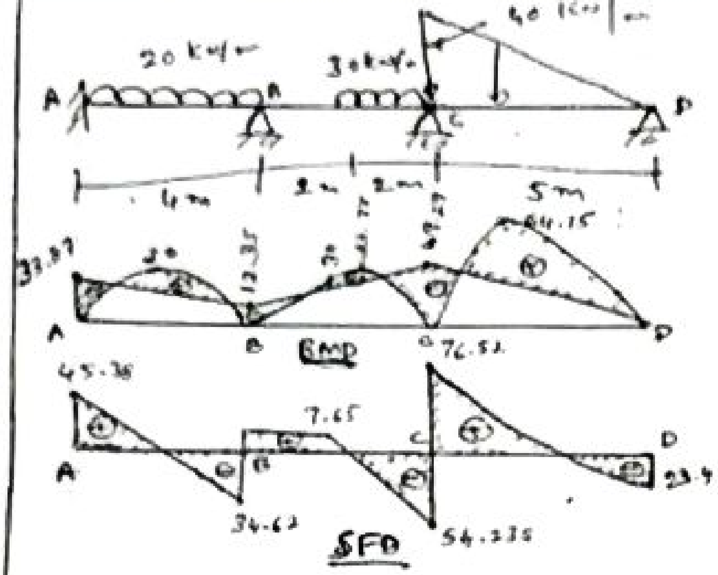
BMD

BMD - (03)

Subject Title: Analysis of Indeterminate Structure **Subject Code:** 15CV52

Question Number	Solution	Marks Allocated

5.



Fixed end moment

$\bar{M}_{AB} = -\frac{80}{3} = -\bar{M}_{BA}$

$\bar{M}_{BC} = -12.5$; $\bar{M}_{CB} = 27.5$

$\bar{M}_{CD} = -50$; $M_{DC} = 33.33$ — (02)

Rotation factors

Jt	Mem	R.S	T.S	R.F
B	BA	3/4	I/2	−1/4
	BC	I/4		−1/4
C	CB	I/4	$\frac{3I}{5}$	−5/16
	CD	$\frac{3}{4}$ I/5		−3/16

(02)

Rotation contribution

$M'_{AB} = 0$; $M'_{BA} = -7.16$; $M'_{BC} = -7.16$; $M'_{CB} = 14.4767$; $M'_{CD} = 8.686$ — (05)

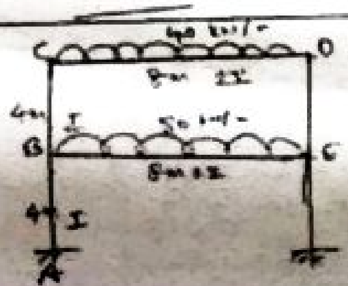
Final moment

$M_{AB} = -33.67$; $M_{BA} = 12.35$; $M_{BC} = -12.35$; $M_{CB} = 69.29 = M_{CB}$ — (02)

Reaction: $R_A = 45.38$; $R_B = 40.385$; $R_C = 130.75$; $R_D = 23.40$ — (02)

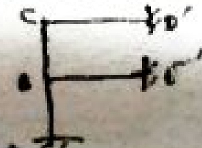
BMD - (02)
SFD (02)

6.



Fixed end moment $\bar{M}_{CD} = -213.33$; $\bar{M}_{BE} = -266.67$ — (02)

Modified Structure

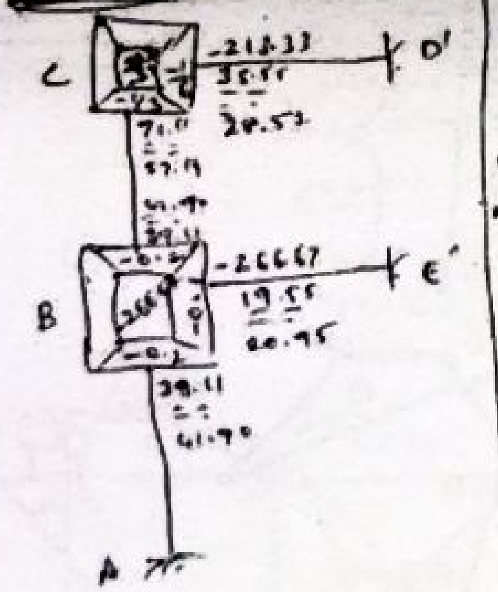


Rotation factors

Jt	mem	R.S	T.S	R.F
B	BA	I/4		−0.2
	BE'	$\frac{1}{2}$ 2I/8	$\frac{5I}{8}$	−0.1
	BC	I/4		−0.2
C	CB	I/4	$\frac{3I}{8}$	−1/3
	CD'	$\frac{1}{2}$ 2I/8		−1/6

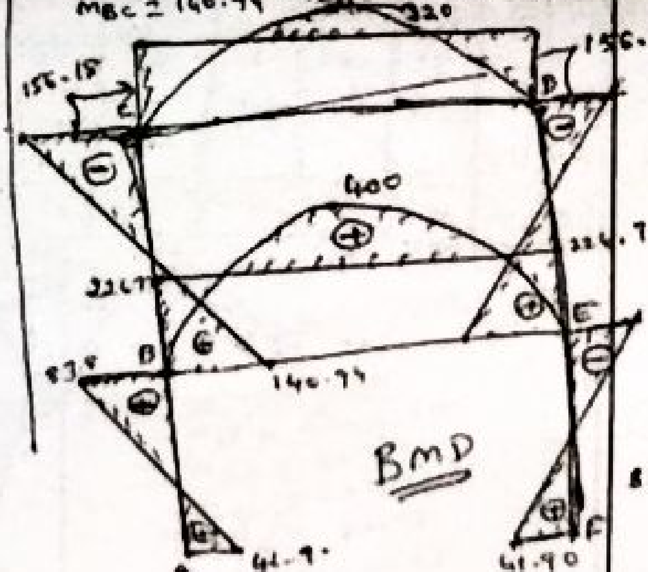
(02)

Rotation Contribution — (07)



Final moment

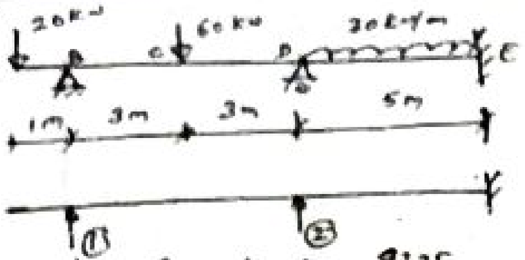
$M_{AB} = 41.9$; $M_{BA} = 83.8$; $M_{BE} = -224.71$
$M_{BC} = 140.94$ $M_{CB} = 156.18$; $M_{CD} = -156.19$ — (03)



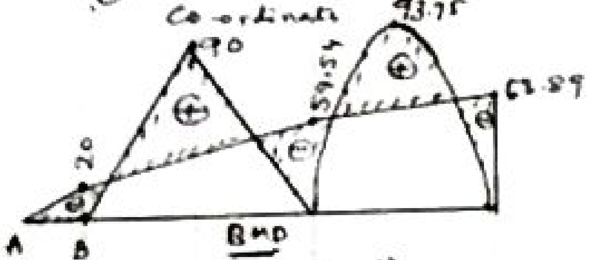
BMD (03)

Page 3/5

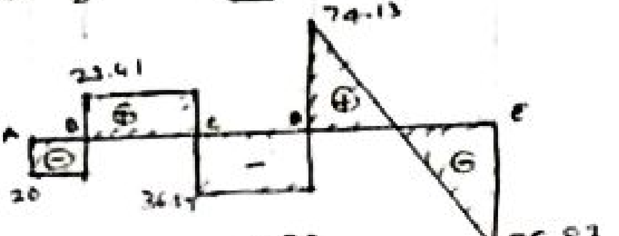
7.



Co-ordinate



BMD



SFD

$D_S = 2 < \begin{matrix} R_B \\ R_D \end{matrix}$ (02)

Displacement matrix

$$[D_R] = \frac{1}{EI}\begin{bmatrix} -32177 \\ -9677.08 \end{bmatrix}$$ (03)

Flexibility matrix

$$[F] = \frac{1}{EI}\begin{bmatrix} 443.66 & 116.66 \\ 116.66 & 41.66 \end{bmatrix}$$ (03)

Redundant

$$[R] = -[F]^{-1}[D_R]$$

$$\begin{bmatrix} R_B \\ R_D \end{bmatrix} = \begin{bmatrix} 43.41 \\ 110.72 \end{bmatrix}$$ (02)

Moment

$M_A = 0 : M_B = -20$ (02)

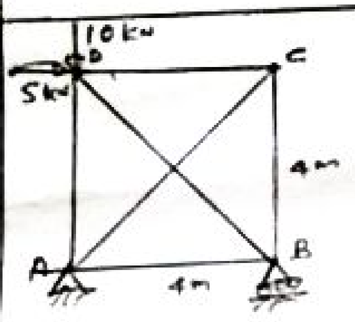
$M_D = -59.54$

$M_E = -63.89$

SFD (02)

BMD (02)

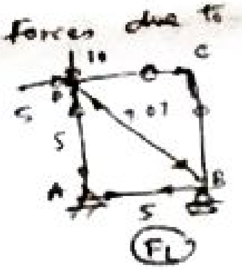
8



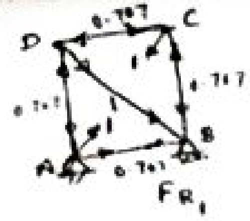
Degree of Redundancy = 1 (03)

Let F_{AC} be Redundant

Forces due to loads & unit action



F_L



F_{R_1}

(05)

Mem	F_L	F_R	F_LF_R	F_R^2
AB	5	−0.707	−3.535	0.5
BC	0	−0.707	0	0.5
CD	0	−0.707	0	0.5
DA	−5	−0.707	3.533	0.5
AC	0	1.0	0	1.0
BD	−7.07	1.0	−7.07	1.0
		Σ	−7.07	4.0

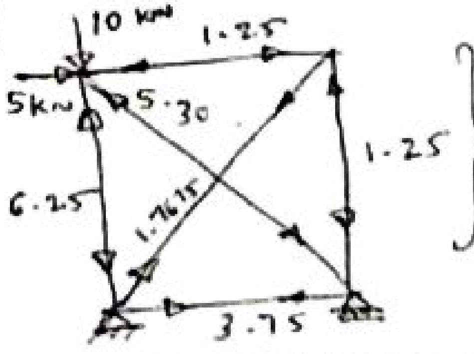
$$D_{R_1} = \Sigma \frac{F_L F_R L}{AE} = -0.17675$$

$$F_{11} = \Sigma \frac{F_R^2 L}{AE} = 0.1$$

$$[R] = -[F]^{-1}[D_R]$$

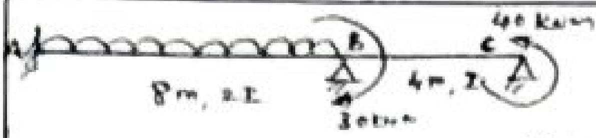
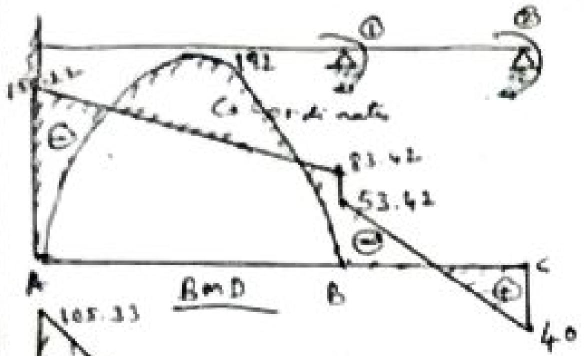
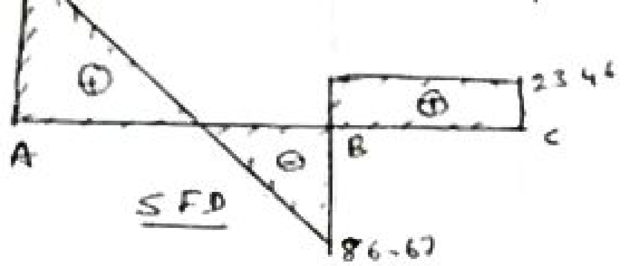
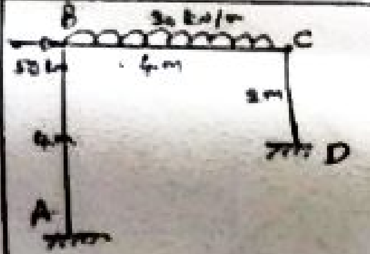
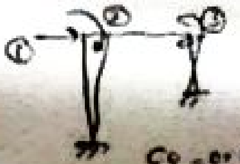
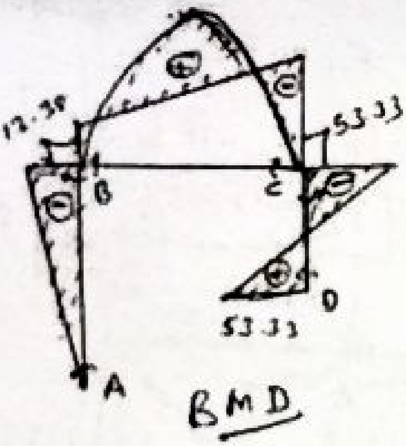
$$\boxed{F_{AC} = 1.7675 \text{ kN}}$$

(06)



(03)

Subject Title: Analysis of Indeterminate Structure **Subject Code:** 15CV52

Question Number	Solution	Marks Allocated
9	 $D_K = 2 < \begin{matrix}\theta_B \\ \theta_C\end{matrix}$	(02)
	 BMD **Action matrix** $[A] = \begin{bmatrix} -98 \\ -40 \end{bmatrix}$	(03)
	Stiffness matrix $S = EI \begin{bmatrix} 2 & 0.5 \\ 0.5 & 1 \end{bmatrix}$	(03)
	 SFD **Displacement matrix** $[D] = [S]^{-1}[A]$ $\begin{bmatrix} \theta_B \\ \theta_C \end{bmatrix} = \frac{1}{EI}\begin{bmatrix} -44.52 \\ -17.71 \end{bmatrix}$	(02)
	Final moment $M_{AB} = -150.27$ $M_{BA} = 83.43$ $M_{BC} = -52.42$ $M_{CB} = -40$	(02) SFD (02) BMD (02)
10	 Degree of Kinematic Redundancy = 3  Co-ordinate Unknown = $\delta, \theta_B, \theta_C$	(02)
	Load matrix: $A = \begin{bmatrix} 50 \\ 40 \\ -40 \end{bmatrix}$	(03)
	Stiffness matrix: ~~S=~~ $S = EI \begin{bmatrix} 1.6875 & -0.375 & -1.5 \\ -0.375 & 2 & 0.5 \\ -1.5 & 0.5 & 3 \end{bmatrix}$	(04)
	 BMD $D = S^{-1}A$ $\begin{bmatrix} \delta \\ \theta_B \\ \theta_C \end{bmatrix} = \frac{1}{EI}\begin{bmatrix} 35.56 \\ 26.62 \\ 0 \end{bmatrix}$	(02)
	Final Moment $M_{AB} = 0$; $M_{BA} = 13.285$ $M_{BC} = -13.38$; $M_{CB} = 53.3$ $M_{CD} = -53.3$; $M_{DC} = -53.3$	(02) BMD-(02)

CBCS SCHEME

USN ☐☐☐☐☐☐☐☐☐☐

17CV52

Fifth Semester B.E. Degree Examination, Dec.2019/Jan.2020

Analysis of Indeterminate Structures

Time: 3 hrs.

Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

1 Analyse the beam completely by slope deflection method relative to support A support B sinks by 1mm and support C rises by 0.5 mm. Take EI = 30000 kN-m2. Refer Fig.Q1. Draw BMD, SFD and Elastic curve.

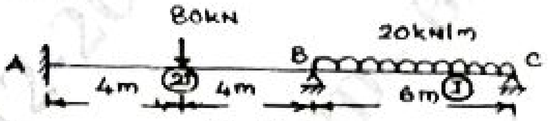


Fig.Q1

(20 Marks)

OR

2 Analyse the given frame by slope deflection method. Draw SFD, BMD and elastic curve. Refer Fig.Q2.

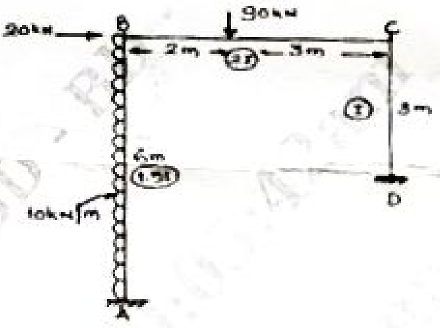


Fig.Q2

(20 Marks)

Module-2

3 Analyse the beam shown in Fig.Q3 by moment distribution method. Draw BMD, SFD and elastic curve.

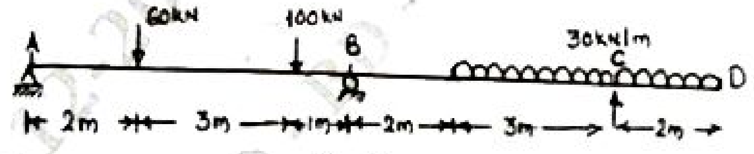


Fig.Q3

(20 Marks)

OR

4 Analyse the frame by moment distribution method. Draw BMD, SFD and elastic curve. Refer Fig.Q4.

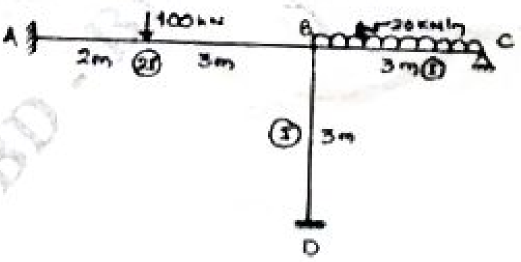


Fig.Q4

(20 Marks)

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg. 42+8 = 50, will be treated as malpractice.

Module-3

5 Analyse the three span continuous beam shown in Fig.Q5 by using Kani's method. Draw BMD, SFD and elastic curve.

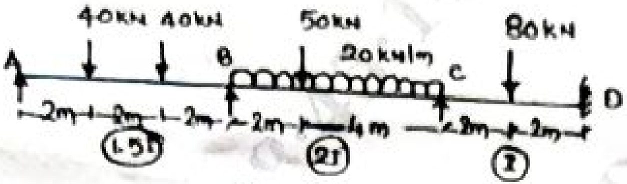


Fig.Q5

(20 Marks)

OR

6 Analyse the portal frames shown in Fig.Q6 by using Kani's method. Draw BMD, SFD and elastic curve.

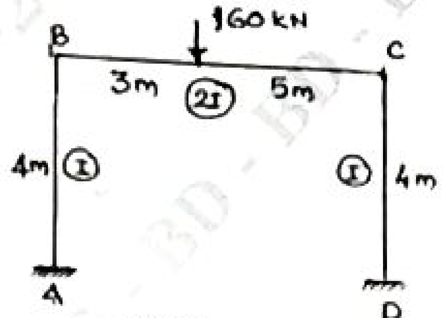


Fig.Q6

(20 Marks)

Module-4

7 Analyse the continuous beam shown in Fig.Q7 by flexibility method using system approach. Support B sinks by 5 mm sketch BMD, SFD and elastic curve. Take EI = 15×10^3 kN-m^2.

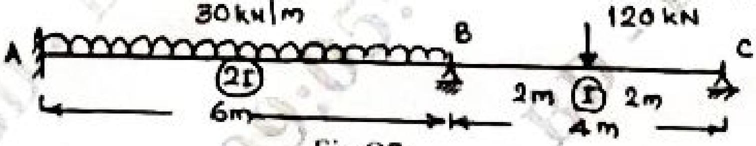


Fig.Q7

(20 Marks)

OR

8 Analyse the pin jointed plane truss shown in Fig.Q8 by using flexibility matrix method. Assume $\frac{L}{AE}$ for each member = 0.025 mm/kN. Tabulate the member forces.

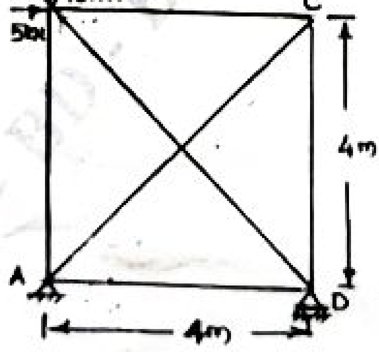


Fig.Q8

(20 Marks)

Module-5

9 Analyse the frame shown in Fig.Q9 by stiffness matrix method and draw BMD, SFD and Elastic curve. Assume EI is constant throughout.

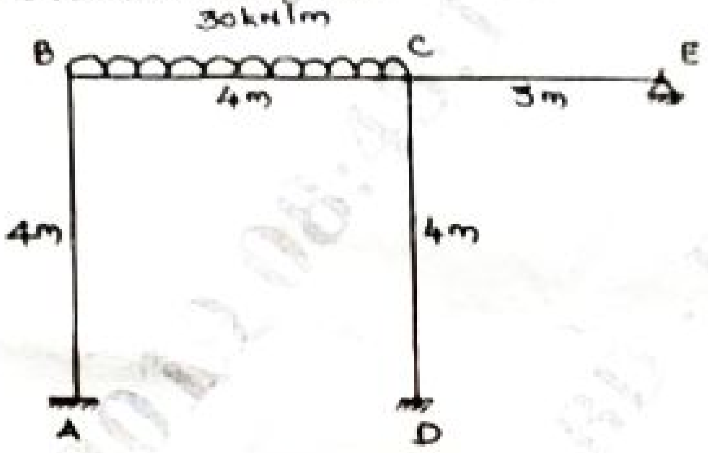


Fig.Q9

(20 Marks)

OR

10 Analyse the continuous beam shown in Fig.Q10 by using stiffness matrix method.

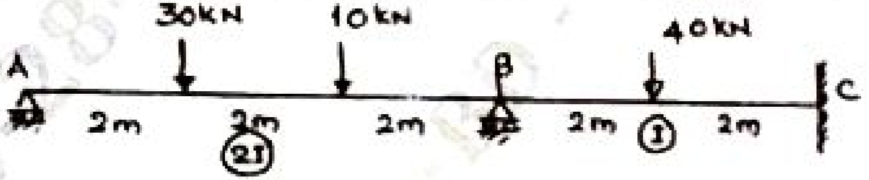


Fig.Q10

(20 Marks)

* * * * *

CBCS SCHEME

USN ☐☐☐☐☐☐☐☐☐☐ 15CV52

Fifth Semester B.E. Degree Examination, Dec.2019/Jan.2020

Analysis of Indeterminate Structures

Time: 3 hrs. Max. Marks: 80

Note: *Answer any FIVE full questions, choosing ONE full question from each module.*

Module-1

1 Analyse the continuous beam shown in Fig Q1 by slope deflection method. Draw bending moment diagram and shear force diagram.

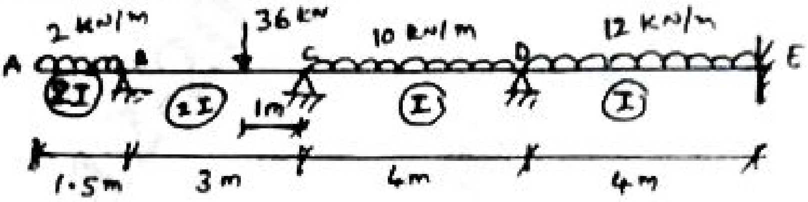


Fig Q1 (16 Marks)

OR

2 Analyse the portal frame shown in Fig Q2 by slope deflection method. Draw bending moment diagram.

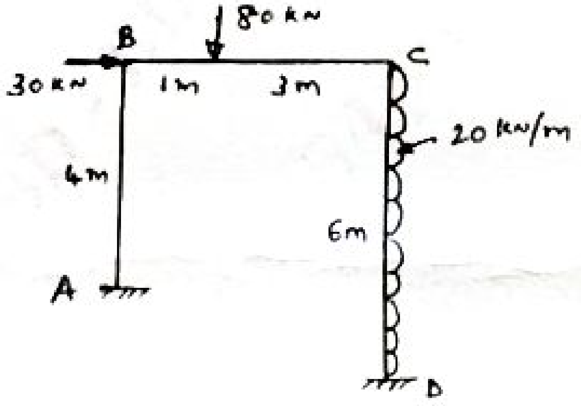
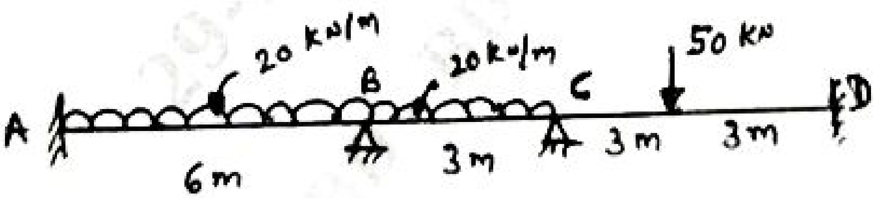


Fig Q2 (16 Marks)

Module-2

3 Analyse the continuous beam shown in Fig Q3 by moment distribution method. Draw bending moment diagram and shear force diagram. Support at B sinks by 10mm.



$E = 2\times10^{5}\ N/mm^{2}$

$I = 16\times10^{7}\ mm^{4}$

Fig Q3 (16 Marks)

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg. 42+8 = 50, will be treated as malpractice.

OR

4 Analyse the frame shown in Fig Q4 by moment distribution method. Draw bending moment diagram.

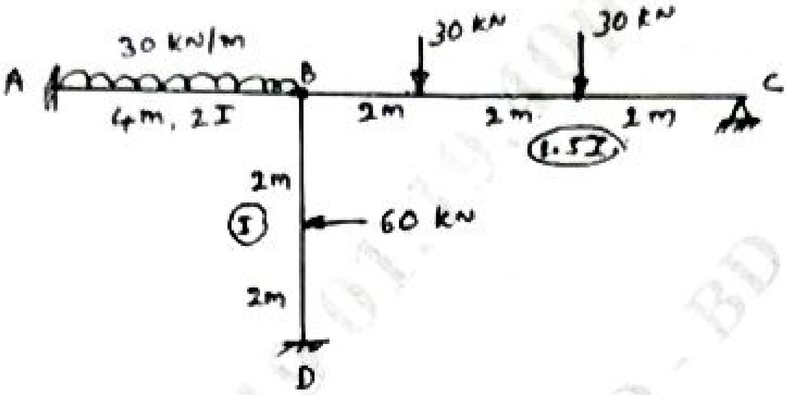


Fig Q4

(16 Marks)

Module-3

5 Analyse the continuous beam shown in Fig Q5 by rotation contribution method. Draw bending moment diagram and shear force diagram.

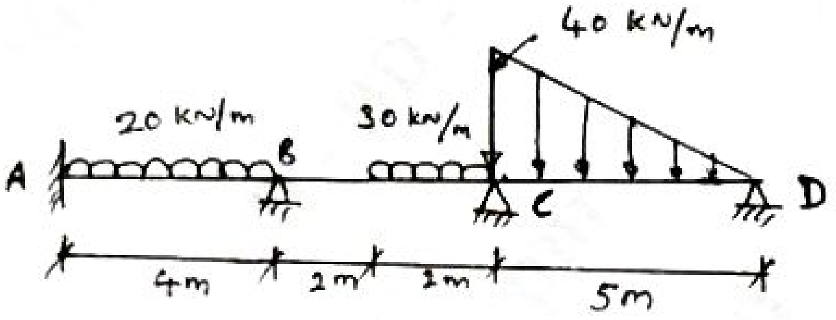


Fig Q5

(16 Marks)

OR

6 Analyse the frame shown in Fig Q6 by Kani's method. Draw bending moment diagram. Use axis of symmetry approach.

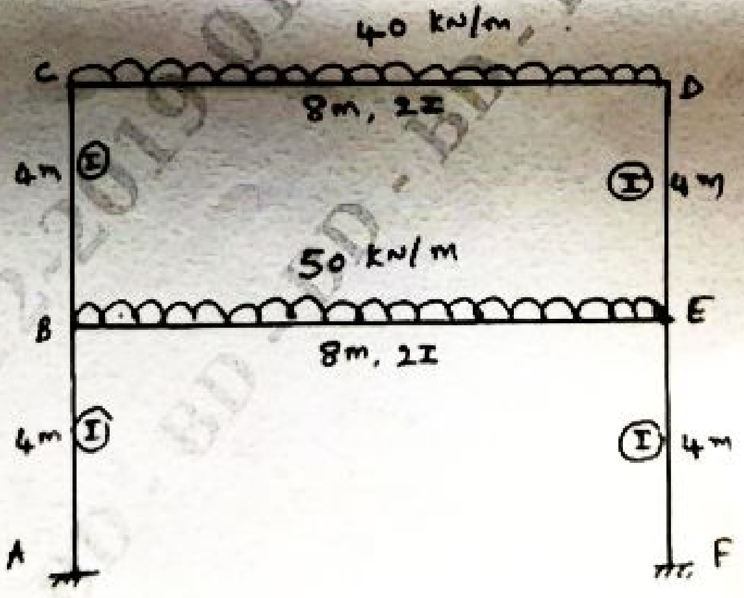


Fig Q6

(16 Marks)

Module-4

7 Analyse the continuous beam shown in Fig Q7 by flexibility matrix method. Draw BMD and SFD.

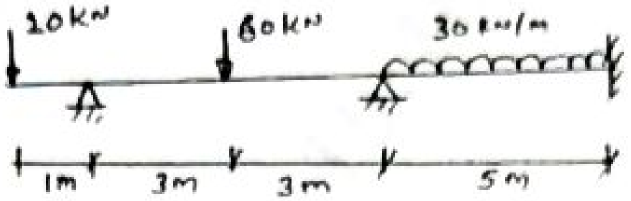


Fig Q7 (16 Marks)

OR

8 Analyse the pin jointed plane shown in Fig Q8 by flexibility matrix method to compute axial forces in the members. Assume $\frac{L}{AE}$ for each member is 0.025mm/kN.

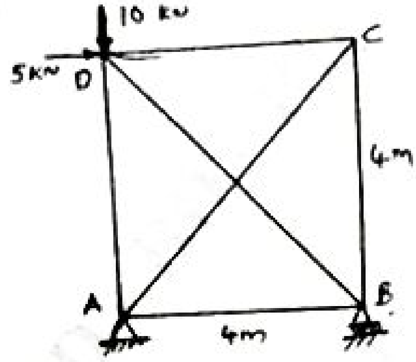


Fig Q8 (16 Marks)

Module-5

9 Analyse the continuous beam shown Fig Q9 by stiffness matrix method. Draw SFD and BMD.

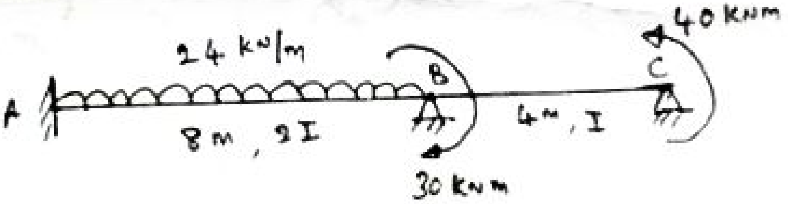


Fig Q9 (16 Marks)

OR

10 Analyse the portal frame shown in Fig Q10 by stiffness matrix method. Draw bending moment diagram.

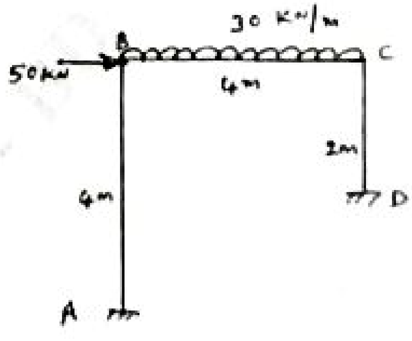


Fig Q10 (16 Marks)

Bapuji Educational Association ®
Bapuji Institute of Engineering and Technology, Davangere-577 004
Department of Civil Engineering

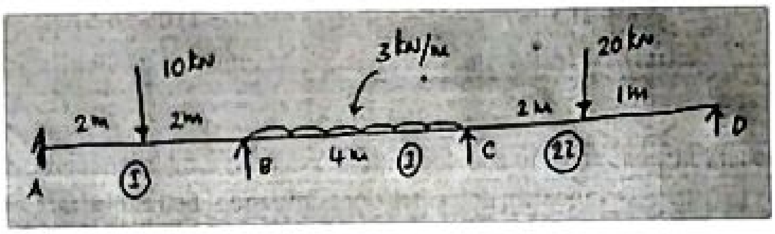
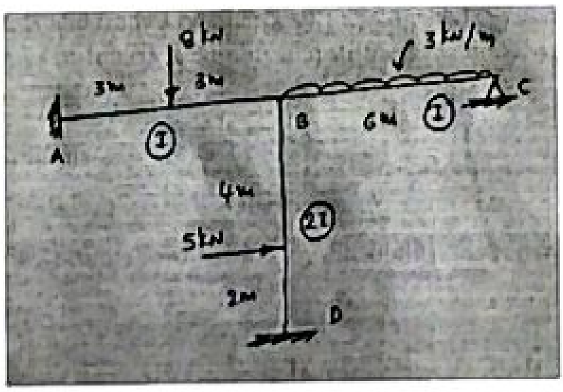
USN										

Course/Subject Title	Analysis of Indeterminate Structures	**Course/Subject Code**	18CV52
Semester	5^{th} A	**Scheme**	CBCS – 18
Date	12-01-2021	**CIE No.**	03
Time	8.00am-9.00am	**Max. Marks**	30

Course Outcome Statements : After the successful completion of the course, the students will be able to

CO1	To analyse the beams and frames by slope deflection method.
CO2	To analyse the beams and frames by Moment Distribution method.
CO3	To analyse the beams and frames by Kani's rotation contribution method.
CO4	To analyse the beams and frames by matrix flexibility method (System Approach).
CO5	To analyse the beams and frames by matrix stiffness method (System Approach).
CO6	To analyse the trusses by matrix flexibility and stiffness method (System Approach).

Note : Answer **BOTH** questions

Q. No.	**Questions**	**Marks**	**RBT Level**	**CO**
1	Analyse the continuous beam shown in Fig by Flexibility matrix method and also draw BMD & SFD.  	15	L4	5
2	Analyse the rigid jointed frames as shown in fig by Stiffness matrix method method .Draw BMD.  	15	L4	6

RBT (Revised Bloom's Taxonomy) Levels : Cognitive Domain		
L1 : Remembering	L2 : Understanding	L3 : Applying
L4 : Analysing	L5 : Evaluating	L6 : Creating

11/1/2021

Course Coordinator **Coordinator** **Program Coordinator**