MODULE – 3: Automatic Generation Control
Mathematical Model of Automatic Load Frequency Control

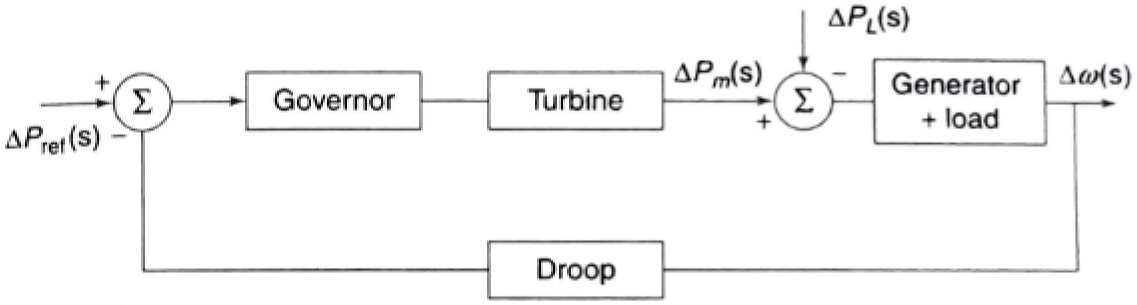

Fig.1 Functional block diagram of ALFC

The mathematical model for the ALFC can be done by developing the transfer function models for each element in the ALFC. The inputs to the ALFC element are the output power set point (ΔP_{ref}) and the change in active power load (ΔP_L). The functional block diagram is shown in fig.1.

*__Governor Model__:

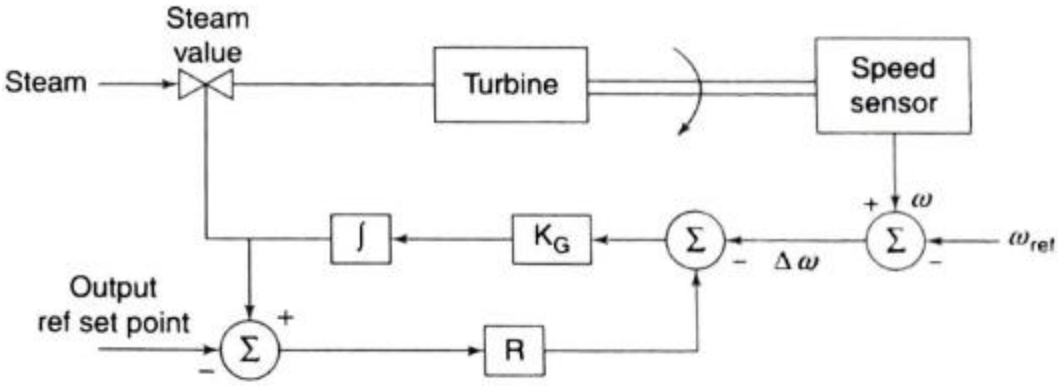

Fig.2 Block diagram of governor with droop

The block diagram of the governor with speed-droop mechanism is shown in fig.2. The output of speed sensor ω is compared with the reference speed ω_{ref} to produce the speed error Δω. By adjusting reference set point on a unit, its output can be varied while holding the system frequency close to the standard frequency. The error due to difference between actual power output and reference set point is fed back through the speed regulation R. The transfer function equivalent of fig. 2 is shown in fig.3.

An increase in governor output command ΔP_g results from an increase in ΔP_{ref} and a decrease in Δω. Then for small increments

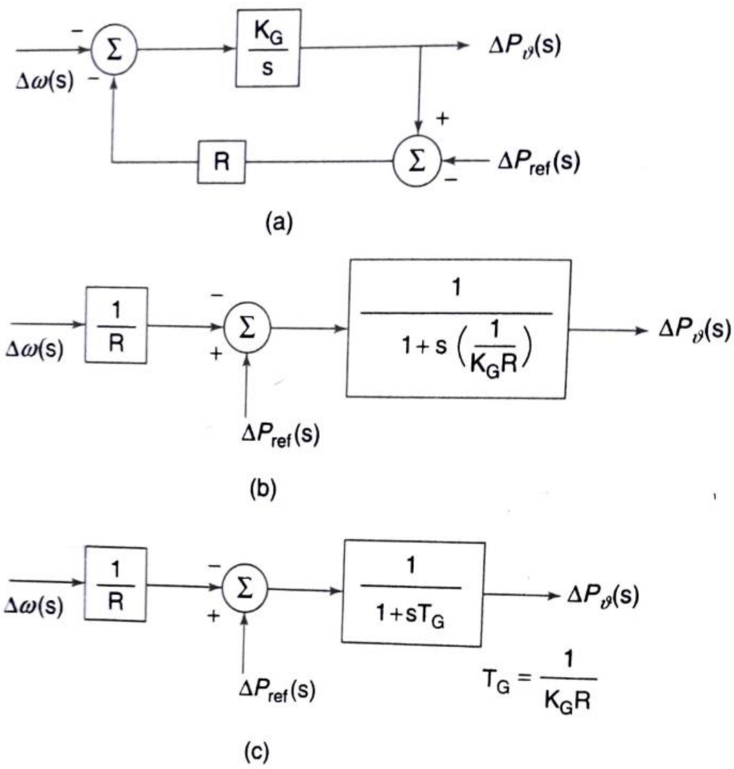

Fig.3 Model of governor

$$\Delta P_g = \Delta P_{ref} - \frac{1}{R}\,\Delta\omega \qquad MW ----\rightarrow (1) \quad \text{where R = speed regulation or droop in Hz/MW}$$

Laplace transformation of equation (1) gives, $\Delta P_g(s) = \Delta P_{ref}(s) - \frac{1}{R}\,\Delta\omega\,(s) \quad ----\rightarrow (2)$

The input position Δx of the valve actuator increases as a result of an increased command ΔP_g but decreases due to increased valve output, ΔP_V. Equal increase in both ΔP_g and ΔP_V should result in $\Delta x = 0$. Then, $\Delta x = \Delta P_g - \Delta P_v \qquad MW ----\rightarrow (3)$

For small changes in Δx, the oil flow into the hydraulic motor is proportional to position Δx of the pilot valve. Thus for the position of the main piston, $\Delta P_v = K_G \int \Delta x \; dt \quad ----\rightarrow (4)$

Taking Laplace transformation of equations (3) and (4), and elimination of Δx gives the transfer function of the hydraulic valve actuator

$$G_H(s) = \frac{\Delta P_v(s)}{\Delta P_g(s)} = \frac{1}{1 + s\,T_G} \quad ---\rightarrow (5)$$

Where the hydraulic time constant, $T_G = \frac{1}{K_G R}$ (typically assumes values around 0.1 sec.)

T_G is the governor time constant. It depends on the speed regulation R and the hydraulic amplifier gain, K_G.

***Turbine Model:** In normal steady state, the turbine power P_m keeps balance with the electromechanical air-gap power P_G resulting in zero acceleration and a constant speed or frequency. Perturbations ΔP_m and ΔP_G will upset the above balance. If the difference power, (ΔP_m - ΔP_G), is positive, the turbine generator unit will accelerate, if negative, it will decelerate.

The turbine power increment ΔP_m depends entirely upon the valve power increment ΔP_V and the response characteristics of the turbine. In general, the turbine response is slow with response times measured in several seconds. From the turbine dynamics, a so called non-reheat turbine has the simplest transfer function consisting of a single time constant (T_{TR}), i.e,

$$G_T = \frac{\Delta P_m(s)}{\Delta P_V(s)} = \frac{1}{1 + s\,T_{TR}} \quad ---\rightarrow (6)$$

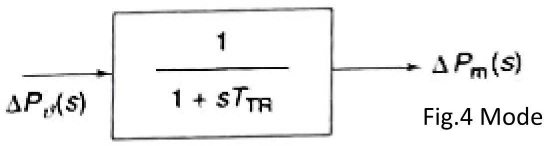

Fig.4 Model of turbine

***Generator Model:**

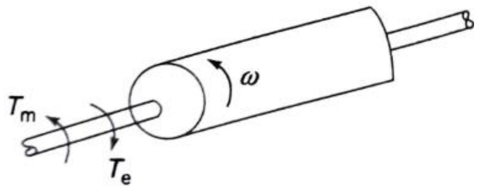

Fig.5 Torques acting on generator

There are two torques acting on generator: the shaft torque (due to the prime mover) and the electromagnetic torque, neglecting losses. The shaft torque tend to accelerate the generator in the positive direction of rotation and the electromagnetic torque in the negative direction.

Total accelerating torque is given by $T_a = T_m - T_e --\rightarrow (1)$

From Newton's law of motion, $J\,\alpha = T \;\; --\rightarrow (2)$

Where J = moment of inertia, α = angular acceleration and T = net torque.

Equation (1) can be written as, $J\,\frac{d^2\theta_m}{dt^2} = T_m - T_e --\rightarrow (3)$

Where θ_m is the rotor angle and is converted into an angle measured with respect to a synchronously rotating reference axis such that $\delta_m = \theta_m - \omega_{sm} t \;\; --\rightarrow (4)$

Where ω_{sm} is the synchronous speed in rad/sec. and δ_m is the angular displacement in rad.
From equation (4), $\frac{d^2\delta_m}{dt^2} = \frac{d^2\theta_m}{dt^2} \quad ---\rightarrow (5)$
Substituting into equation (3), we get

$$J\,\frac{d^2\delta_m}{dt^2} = T_m - T_e \;--\rightarrow (6)$$

Multiplying both sides by the angular velocity, ω_m, we get

$\omega_m\, J\, \frac{d^2\delta_m}{dt^2} = \omega_m(T_m - T_e) \;--\rightarrow (7)$

where $\omega_m\, J = M$ = angular momentum or inertia constant.
$\omega_m\, T_m = P_m$= mechanical power input at the shaft minus rotational losses.
$\omega_m\, T_e = P_e$ = electrical power output minus losses.

$$\therefore\; M\,\frac{d^2\delta_m}{dt^2} = P_m - P_e = P_a \quad in\ W \;--\rightarrow (8)$$

M depends on the speed ω_m. However, since the deviation in speed is limited, M can be assumed to be a constant. The value of M varies over a wide range depending on the rating and type of the generator. Hence, another constant H is used to specify the energy stored in the machine.

$$H = \frac{\text{Stored kinetic energy in MJ at synchronous speed}}{\text{Machine rating in MVA}}\ \text{MJ/MVA} \;--\rightarrow (9)$$

It is also called inertia constant. It lies in a narrow range for different machines.
M and H are related as follows

$$M = \frac{2\,G\,H}{\omega_{sm}}\ \ MJ - sec/mech.rad \;\;--\rightarrow (10)$$

Where G = MVA rating of machine. In pu, M = 2H.

Equation (8) can be written as

$$\frac{2\,H}{\omega_{sm}}\,\frac{d^2\delta_m}{dt^2} = \frac{P_m - P_e}{G} \quad ---\rightarrow (11)$$

We can express both δ_m *and* ω_m in terms of electrical radians to get

$\frac{2\,H}{\omega_s}\,\frac{d^2\delta_m}{dt^2} = P_m - P_e = P_a \quad in\ \ pu \quad ---\rightarrow (12)$ -> Swing equation.

Where $P_m = \frac{P_m \text{ in MW}}{G}$ = per unit mechanical power and P_a = acceleration power.
$\omega_s = \frac{P}{2}\omega_{sm}$ = synchronous speed in electrical rad/sec.
Linearize the equation (12), we get $\frac{2\,H}{\omega_s}\,\frac{d^2\Delta\delta}{dt^2} = \Delta P_m - \Delta P_e \quad ---\rightarrow (13)$

$$2\,H\,\frac{d}{dt}\left(\frac{\Delta\omega}{\omega_s}\right) = \Delta P_m - \Delta P_e$$

With speed expressed in pu, $\frac{d\,\Delta\omega}{dt} = \frac{\Delta P_m - \Delta P_e}{2\,H} \qquad \because \frac{\Delta\omega}{\omega_s} = \Delta\omega \quad \text{in pu}$

Taking Laplace transform, $\Delta\omega(s) = \frac{1}{2\,H\,s}(\Delta P_m(s) - \Delta P_e(s)) \;---\rightarrow (14)$

Equation (14) can be written a block diagram form as,

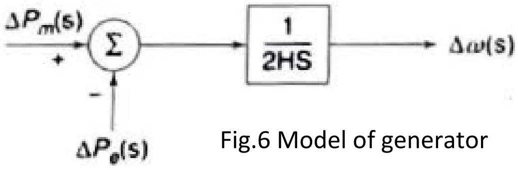

Fig.6 Model of generator

***Load Model:**

In general, power system loads are a composite of a variety of electrical devices. For resistive loads, such as lighting and heating loads, the electrical power is independent of frequency. In the case of motor loads, such as fans and pumps, the electrical power changes with frequency due to changes in motor speed. The overall frequency-dependent characteristic of a composite load may be expressed as

$$\Delta P_e = \Delta P_L + D\,\Delta\omega \quad --\rightarrow (15)$$

Where, ΔP_L = non-frequency-sensitive load change.

$D\,\Delta\omega$ = frequency-sensitive load change

D = load-damping constant

The damping constant is expressed as a percent change in load for one percent change in frequency. Typical values of D are 1 to 2%. A value of D = 2 means that a 1% change in frequency would cause a 2% change in load. The system block diagram including the effect of the load damping is shown in fig.(7).

The transfer function $\frac{1}{2\,H\,s + D}$ can be written in the form of a standard first-order transfer function, $\frac{K_{PS}}{1 + s\,T_{PS}}$.

Where, $K_{PS} = \frac{1}{D}$ = power system gain and $T_{PS} = \frac{2\,H}{D}$ = power system time constant.

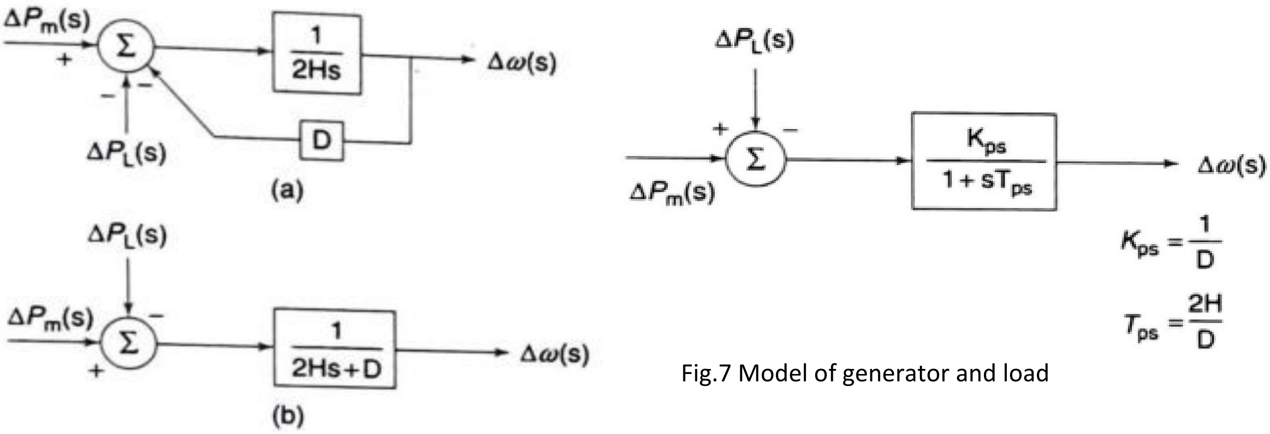

Fig.7 Model of generator and load

***Composite Model:**

The collective performance of all the generators connected to the system is considered to build the ALFC model. Generators connected to a system are assumed to swing coherently so that we can represent them as a single equivalent generator, with an inertia constant $M_{eq} = M_1 + M_2 + \ldots$, driven by the combined mechanical outputs of the turbines connected to the generators. The block diagram is shown in fig.(8) for a system with n generators.

The effects of the frequency dependency of the loads are lumped into a single damping constant D. The speed of the equivalent system determines the frequency.

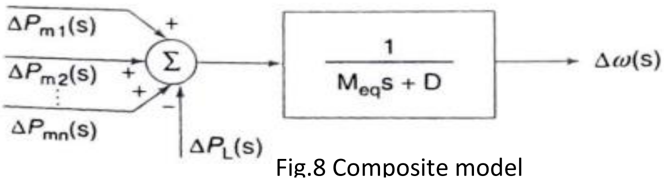

Fig.8 Composite model

The power output-frequency characteristics will depend on the effect of the droops of all generator speed governors and on the frequency characteristics of all the loads. For a system with n generators, the steady state frequency deviation after a load change ΔP_L is given by,

$$\Delta f_{SS} = \frac{-\Delta P_L}{\left(\frac{1}{R_1}+\frac{1}{R_1}+\cdots+\frac{1}{R_n}\right)+D} = \frac{-\Delta P_L}{\frac{1}{R_{eq}}+D} \quad --\rightarrow (16)$$

Where, $R_{eq} = \frac{1}{\frac{1}{R_1}+\frac{1}{R_1}+\cdots+\frac{1}{R_n}}$ and D = composite load damping constant.

The composite frequency response of the system is given by

$$\beta = \frac{-\Delta P_L}{\Delta f_{SS}} = \frac{1}{R_{eq}} + D \quad --\rightarrow (17)$$

β is referred to as stiffness of the system or frequency bias factor. The unit of β is MW / Hz. The composite regulating characteristic is 1/β. If there is an increase of load by ΔP_L at nominal frequency, then we have an increase of ΔP_G in generation and a reduction of load of ΔP_D due to frequency dependence of the load (note that an increase in load will cause a drop in frequency which causes the frequency dependent loads to reduce). Therefore,

$$\Delta P_L = \Delta P_G - \Delta P_D$$

$$\Delta P_G = -\frac{\Delta f}{R} \quad \text{and} \quad \Delta P_D = D\,\Delta f$$

$$\therefore \Delta P_L = -\frac{\Delta f}{R} - D\,\Delta f = \Delta f\left[-\frac{1}{R} - D\right]$$

$$\text{or} \qquad \Delta f = \frac{-\Delta P_L}{\frac{1}{R}+D} ---\rightarrow (18)$$

(We have used $\Delta P_L = -\frac{\Delta f}{R}$, because when Δf is negative, it implies that the load has increased and P_G has to increase).

***Complete ALFC Model:**

Now connect the block diagrams of governor, turbine, generator and load models to obtain the block diagram of the complete ALFC, shown in fig.(9).

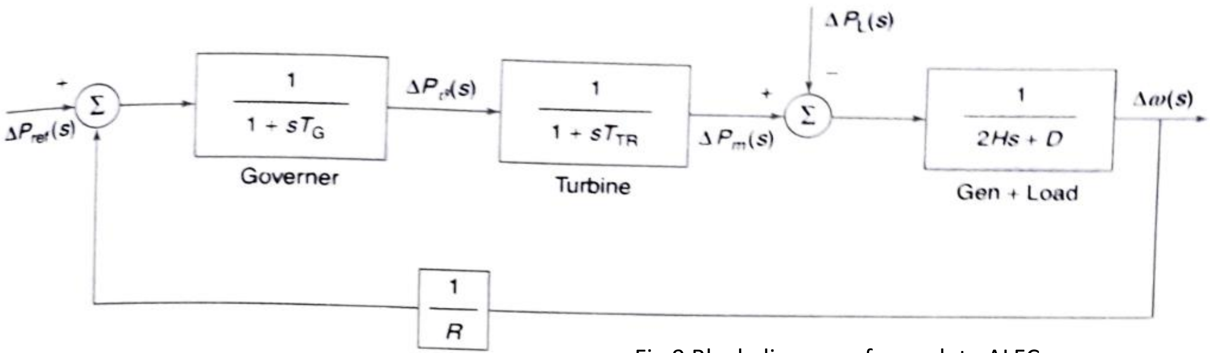

Fig.9 Block diagram of complete ALFC

The primary ALFC loop has one output Δω and two important incremental inputs: ΔP_{ref} = the change in speed changer setting (reference set point) and ΔP_L= the change in load demand.

Now, consider a situation in which the speed changer has a fixed setting so that $\Delta P_{ref} = 0$ and the load demand changes. This is known as a free governor operation.

From the block diagram we obtain the transfer function G(s) as,

$$G(s) = \frac{\Delta\omega(s)}{-\Delta P_L(s)} = \frac{\frac{1}{2\,Hs\, +\, D}}{1+\frac{1}{R}\left(\frac{1}{2\,Hs\,+\,D}\right)\left(\frac{1}{1+s\,T_G}\right)\left(\frac{1}{1+s\,T_{TR}}\right)} \quad --\rightarrow (19)$$

$$G(s) = \frac{\Delta\omega(s)}{-\Delta P_L(s)} = \frac{(1+s\,T_G)(1+s\,T_{TR})}{(2\,Hs\,+\,D)(1+s\,T_G)(1+s\,T_{TR})+\frac{1}{R}} \quad --\rightarrow (20)$$

$$G(s)=\frac{\Delta\omega(s)}{-\Delta P_L(s)}=\frac{K_{PS}\,(1+s\,T_G)(1+s\,T_{TR})}{(1+s\;T_{PS})(1+s\,T_G)(1+s\,T_{TR})+\frac{K_{PS}}{R}}\;--\rightarrow(21)\qquad\because\frac{1}{2\,H\,s\,+\,D}=\frac{K_{PS}}{1\,+\,s\,T_{PS}}$$

***Steady State Analysis:**

From equation (21), $G(s)=\frac{\Delta\omega(s)}{-\Delta P_L(s)}\;-\rightarrow\;\Delta\omega(s)=-\Delta P_L(s)\,G(s)$

For a step load change of constant magnitude, $\Delta P_L = M$, we have $\Delta P_L(s)=\frac{M}{s}$

Using the final value theorem, we can obtain the steady state (static) frequency deviation

$$\Delta\omega_{ss}=\lim_{s\to 0}[s\,\Delta\omega(s)]=-\,s\;\times\frac{M}{s}\times\frac{K_{PS}}{1+\left(\frac{K_{PS}}{R}\right)}\;=-\;\frac{M}{\left(D+\frac{1}{R}\right)}\;\;\text{Hz}---\rightarrow(22)$$

$\because\;K_{PS}=\frac{1}{D}$ or $D=\frac{1}{K_{PS}}$, area frequency response characteristic, $\beta=D+\frac{1}{R}$ $\quad$ pu MW/Hz $--\rightarrow(23)$

and then, $\qquad\Delta\omega_{ss}=-\,\frac{M}{\beta}\;\text{Hz}---\rightarrow(24)$

If there is no frequency dependant load D = 0, then, $\Delta\omega_{ss}=-R\,M---(25)$

Automatic Generation Controller

We have seen with primary speed control, the amount of a steady-state frequency deviation for a change in the system load depends on the governor droop characteristics and frequency sensitivity of the load. All the generating units will change their response to the load change, irrespective of the location of the load. Restoration of the system frequency to the scheduled value requires supplementary control to load reference set point. This secondary control, called AGC, becomes the basic means of controlling prime mover power to match the variations of the system load. The controller should satisfy the following performance specifications:

- Stable closed loop control operation
- Keep frequency deviation to a minimum
- Limit the integral of the frequency error
- Divide the load economically

In an isolated system, there is no interchange power to be considered between the power system areas. The function of the AGC is purely to maintain the frequency at the scheduled value. This is achieved by adding an integral controller in the feedback path to change the load reference setting depending on the frequency deviation. From control theory it is known that the steady-state error of an integral controller is zero.

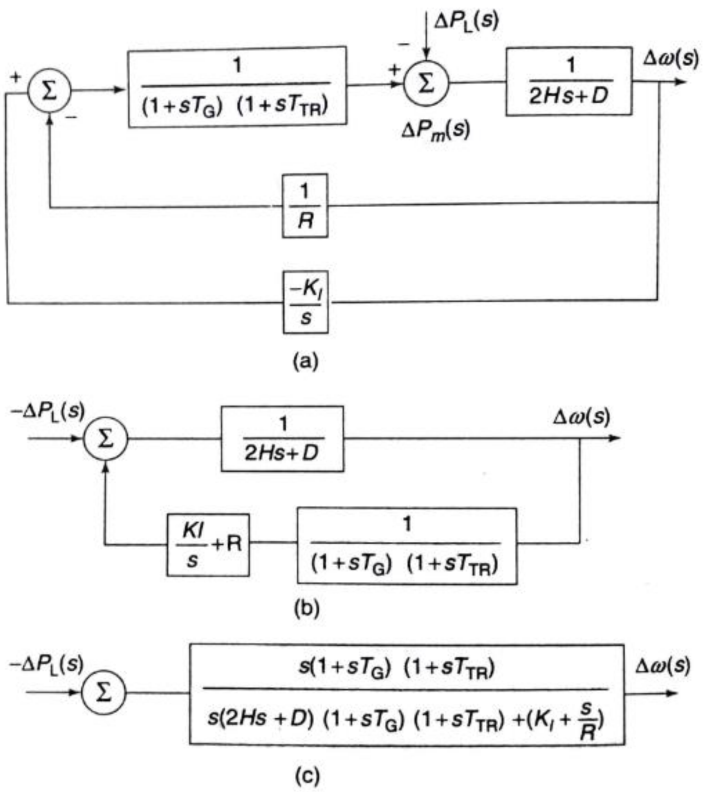

Fig.10 AGC with integral controller

Integral Controller:

By using the control strategy, that is integral control added to the primary ALFC loop shown in fig.(10), we obtain an overall system that will meet performance specifications.

Let the speed changer be commanded by a signal obtained by first amplifying and then integrating the frequency error, that is,

$$\Delta P_{ref}=\;-\,K_I\int\Delta\omega\;\,dt---\rightarrow(26)$$

Where K_I is the integral gain and its unit is per-unit MW per Hertz and second.

The integral controller increases the system type by 1, which forces the final frequency deviation to zero. The gain constant K_I must be adjusted for a satisfactory transient response. It controls the rate of integration and thus speed of response of the loop. The negative polarity of the integral controller must be chosen so as to cause a positive frequency error to give rise to negative or decrease command. (The signal generated by the integral controller must be opposite sign to $\Delta\omega(s)$. This means that for a decrease in $\Delta\omega(s)$, the generation must increase ($\Delta P_{ref}(s)$ must be positive)).

The signal fed into the integrator is referred to as area control error (ACE).

$$\mathrm{ACE} = \Delta\omega \text{ or } \Delta f - - - \rightarrow (27)$$

Integral control will give rise to zero static frequency error (steady state error) following a step load change for the physical reason: As long as an error remains, the integrator output will increase, causing the speed changer to move. The integrator output and thus the speed changer position, attains a constant value only when the frequency error has reduced to zero.

The secondary ALFC loop has one output $\Delta\omega$ or Δf and one input ΔP_L. From the block diagram, we obtain the closed loop transfer function of the ALFC as

$$G(s) = \frac{\Delta\omega(s)}{-\Delta P_L(s)} = \frac{s\,K_{PS}\,(1 + s\,T_G)(1 + s\,T_{TR})}{s\,(1 + s\,T_{PS})(1 + s\,T_G)(1 + s\,T_{TR}) + K_{PS}(K_I + \frac{s}{R})} - - \rightarrow (28)$$

or

$$G(s) = \frac{\Delta\omega(s)}{-\Delta P_L(s)} = \frac{K_{PS}\,(1 + s\,T_G)(1 + s\,T_{TR})}{(1 + s\,T_{PS})(1 + s\,T_G)(1 + s\,T_{TR}) + K_{PS}(\frac{K_I}{s} + \frac{1}{R})} - - \rightarrow (28)$$

For a step load change of constant magnitude, $\Delta P_L = M$, we have $\Delta P_L(s) = \frac{M}{s}$

Using the final value theorem, we can obtain the steady state (static) frequency deviation

$$\Delta\omega_{ss} = \lim_{s\to 0}[s\,\Delta\omega(s)] = -\,s\,\times\frac{M}{s}\times\frac{K_{PS}}{1 + K_{PS}\left(\frac{K_I}{0} + \left(\frac{1}{R}\right)\right)} = -\frac{1}{\infty} = 0$$

The steady-state deviation is driven to zero, irrespective of the choice of the integral gain and R. The integrator output is zero only when the speed deviation is zero. Under this condition, $\Delta P_{ref} = 0$.

This supplementary control is much slower than the primary speed control action. It comes into effect only after the primary control has stabilized the system frequency. The primary control acts on all units with speed regulation, whereas AGC adjusts the load reference setting in only a few selected units. The outputs of these units override the effect of the composite frequency regulation characteristics of the system and in the process force the generation of all other units not on AGC to scheduled values.

AGC in Interconnected Power Systems

Introduction: We studied the importance of having automatic generation control to keep down frequency deviations. When we have interconnected systems, the AGC has to take into account the tie-line flows. We have seen in the case of an isolated system, with which only the primary speed control, any change in the system load will lead to a steady-state frequency deviation, depending on the droop (R) of the governor droop characteristic and the frequency dependence of the load (D). All the generation units equipped with governor control will have a change in their generation levels, irrespective of where the load change takes place. To restore the frequency to the nominal value, we need to use a supplementary control which changes the load reference set point, which matches the prime mover power to variations in the load.

A group of generators coupled closely will swing together and may be replaced by a single equivalent generator. The generator turbines also tend to have the same response characteristics. Such generators are said to be coherent and the system is called a control area. When different control areas are connected, the objectives of the AGC are as follows:

1. Hold the system frequency close to the nominal value of 50 Hz.
2. To maintain a correct value of power interchange between control areas.
3. Maintain generation of each unit at the most economic value.

Tie-Line Control with Primary Speed Control

The load frequency dynamics of multi-area system can be realised by studying first the dynamics of the two-area interconnected system. Let us consider a power system consisting of two control areas interconnected via a relatively weak tie line. The areas are generally of different size and characteristics.

Made the following assumptions:

- a control area is characterized by the same frequency throughout, so that the area network is rigid or strong.
- each area individually strong.
- interconnected with a weak tie-line, leads the frequency deviations in the two areas can be represented by two variables $\Delta\omega_1$ and $\Delta\omega_2$ respectively.

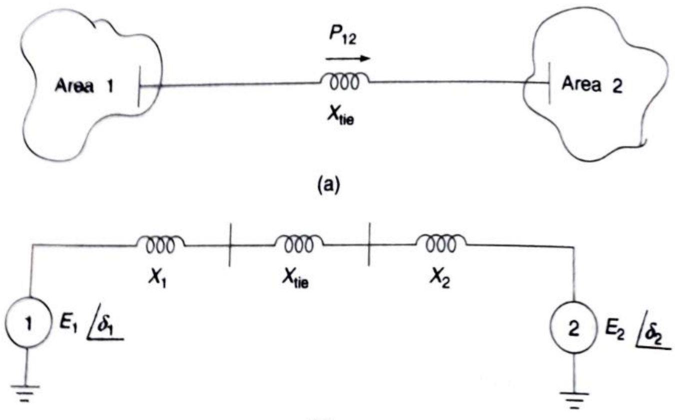

Fig.11 Two area system

In normal operation the positive power flow (P_{12}) on the tie-line from area-1 to area-2 is

$$P_{12} = \frac{|E_1||E_2|}{X_{12}} \sin(\delta_1 - \delta_2) \quad -----\rightarrow (1)$$

Where δ_1 and δ_2 are the angles of end voltages E_1 and E_2 respectively and $X_{12} = X_1 + X_{tie} + X_2$.
For small deviations in the angles δ_1 and δ_2 (that is, $\Delta\delta_{12} = \Delta\delta_1 - \Delta\delta_2$), the tie-line power changes with the amount

$$\Delta P_{12} \approx \left.\frac{dP_{12}}{d\delta_{12}}\right|_{\delta_{12}^0} \Delta\delta_{12} \approx \frac{|E_1||E_2|}{X_{12}} \cos(\delta_1 - \delta_2)\,(\Delta\delta_1 - \Delta\delta_2) \quad \text{MW} -----\rightarrow (2)$$

Define the electric stiffness or slope of the power angle curve at δ_{12}^0 or synchronising coefficient of a tie-line as

$$T = \left.\frac{dP_{12}}{d\delta_{12}}\right|_{\delta_{12}^0} = \frac{|E_1||E_2|}{X_{12}} \cos(\delta_1 - \delta_2) \quad \text{MW/rad} \quad -----\rightarrow (3)$$

Then, $\quad \Delta P_{12} = T\ \Delta\delta_{12} = T\,(\Delta\delta_1 - \Delta\delta_2) \quad \text{MW} -----\rightarrow (4)$

The frequency deviation $\Delta\omega$ is related to the reference angle $\Delta\delta$ by the equation,

$$\Delta\omega = \frac{d}{dt}(\delta^0 + \Delta\delta) = \frac{d}{dt}(\Delta\delta) \quad ---\rightarrow (5) \qquad \because\ \delta = \omega\, t \text{ and } \delta^0 \approx 0$$

or inversely $\Delta\delta = \int_0^t \Delta\omega\ dt \quad \text{rad} ---\rightarrow (6)$

By expressing tie-line deviation in terms of $\Delta\omega$ rather than $\Delta\delta$, the equation (4),

$$\Delta P_{12} = T\left(\int_0^t \Delta\omega_1\ dt - \int_0^t \Delta\omega_2\right) \qquad \text{MW} -----\rightarrow (7)$$

Laplace transformation of the equation (7) gives,

$$\Delta P_{12}(s) = \frac{T}{s}\big(\Delta\omega_1(s) - \Delta\omega_2(s)\big) \ \text{ or } \ \Delta P_{12}(s) = \frac{2\,\pi\,T}{s}\big(\Delta f_1(s) - \Delta f_2(s)\big) \ ---\rightarrow (8)$$

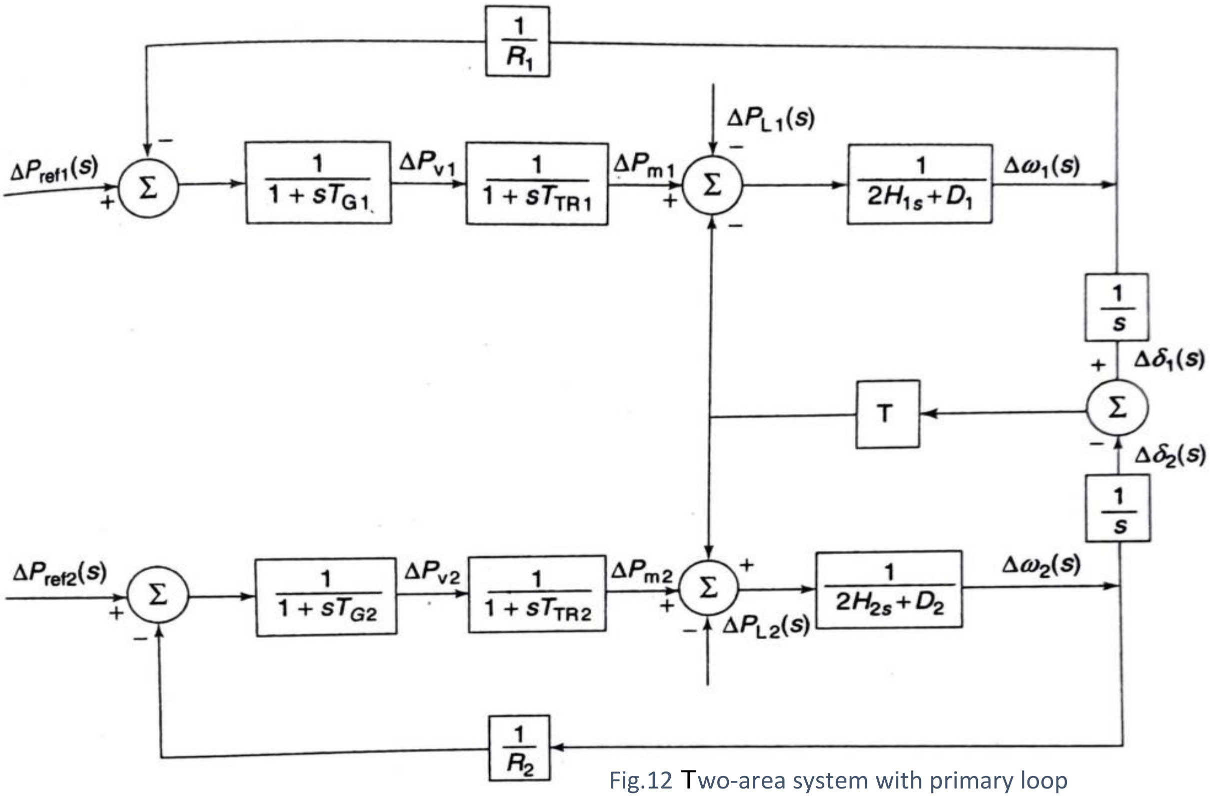

Fig.12 Two-area system with primary loop

The block diagram representation of two-area system is shown in fig. (12). P_{12} represents tie-line power in the direction 1 → 2 & P_{21} represents tie-line power in the direction 2 → 1. Thus if losses are neglected, $\Delta P_{21} = -\Delta P_{12}$. In multi-area system, it is always convenient to consider the tie-line power (ΔP_{12}) positive in the direction out from area-1 to area-2. It is equivalent to increasing the load of area-1 and decreasing the load of area-2, therefore feedback of ΔP_{12} has a negative sign for area-1 and a positive sign for area-2.

*Change of load in area-1:

Consider a load change of ΔP_{L1} in area-1. When the system has reached a steady state, both areas will have same steady state frequency deviations. Therefore, $\Delta\omega = \Delta\omega_1 = \Delta\omega_2$.

First investigate the response of the two-area system with fixed speed changer positions, that is, $\Delta P_{ref,1} = \Delta P_{ref,2} = 0$, the tie-line and rotating masses exhibit damped oscillations called synchronising oscillations.

By adding the powers at the summing junctions, we obtain,

for area-1: $\Delta P_{m1} - \Delta P_{12} - \Delta P_{L1} = \Delta\omega\, D_1 ----\rightarrow (9)$

for area-2: $\Delta P_{m2} + \Delta P_{12} = \Delta\omega\, D_2 ----\rightarrow (10)$

Since the incremental increase in turbine dynamics is determined by the static loop gains:

$$\Delta P_{m1} = \frac{-\Delta\omega}{R_1} \quad ---\rightarrow (11) \qquad \text{and} \qquad \Delta P_{m2} = \frac{-\Delta\omega}{R_2} \quad ---\rightarrow (12)$$

Substituting equations (11) & (12) in equations (9) & (10), we get,

$$-\Delta P_{12} - \Delta P_{L1} = \Delta\omega\left(\frac{1}{R_1} + D_1\right) \text{ and } \Delta P_{12} = \Delta\omega\left(\frac{1}{R_2} + D_2\right)$$

$$\therefore\ \Delta\omega = \frac{-\Delta P_{L1}}{\left(\frac{1}{R_1} + \frac{1}{R_2}\right) + (D_1 + D_2)} = \frac{-\Delta P_{L1}}{\beta_1 + \beta_2} \quad ----\rightarrow (13)$$

and $$\Delta P_{12} = \Delta\omega\left(\frac{1}{R_2} + D_2\right) = \frac{-\Delta P_{L1}\left(\frac{1}{R_2} + D_2\right)}{\left(\frac{1}{R_1} + \frac{1}{R_2}\right) + (D_1 + D_2)} = \frac{-\Delta P_{L1}\,\beta_2}{\beta_1 + \beta_2} \quad ----\rightarrow (14)$$

where $\beta_1 = D_1 + \frac{1}{R_1}$ and $\beta_2 = D_2 + \frac{1}{R_2}$ are the composite or area frequency response characteristics (AFRC) of area-1 and area-2 respectively.

*Change of load in area-2:

Consider a load change of ΔP_{L2} in area-2. When the system has reached a steady state, both areas will have same steady state frequency deviations. Therefore, $\Delta\omega = \Delta\omega_1 = \Delta\omega_2$.

First investigate the response of the two-area system with fixed speed changer positions, that is, $\Delta P_{ref,1} = \Delta P_{ref,2} = 0$, the tie-line and rotating masses exhibit damped oscillations called synchronising oscillations.

By adding the powers at the summing junctions, we obtain,

for area-1: $\Delta P_{m1} - \Delta P_{12} = \Delta\omega\, D_1 ----\rightarrow (15)$

for area-2: $\Delta P_{m2} + \Delta P_{12} - \Delta P_{L2} = \Delta\omega\, D_2 ----\rightarrow (16)$

Substituting equations (11) & (12) in equations (15) & (16), we get,

$$-\Delta P_{12} = \Delta\omega\left(\frac{1}{R_1} + D_1\right) \text{ and } \Delta P_{12} - \Delta P_{L2} = \Delta\omega\left(\frac{1}{R_2} + D_2\right)$$

$$\therefore\ \Delta\omega = \frac{-\Delta P_{L2}}{\left(\frac{1}{R_1} + \frac{1}{R_2}\right) + (D_1 + D_2)} = \frac{-\Delta P_{L2}}{\beta_1 + \beta_2} \quad ----\rightarrow (17)$$

and $$\Delta P_{12} = -\Delta\omega\left(\frac{1}{R_1} + D_1\right) = -\frac{-\Delta P_{L2}\left(\frac{1}{R_1} + D_1\right)}{\left(\frac{1}{R_1} + \frac{1}{R_2}\right) + (D_1 + D_2)} = \frac{\Delta P_{L2}\,\beta_1}{\beta_1 + \beta_2} = -\Delta P_{21} \quad ----\rightarrow (18)$$

*Change of load in both areas: If we have a simultaneous change of load in both areas, we get,

for area-1: $\Delta P_{m1} - \Delta P_{12} - \Delta P_{L1} = \Delta\omega\, D_1 ----\rightarrow (19)$

for area-2: $\Delta P_{m2} + \Delta P_{12} - \Delta P_{L2} = \Delta\omega\, D_2 ----\rightarrow (20)$

Adding equations (19) & (20) we get,

$$-\Delta P_{L1} - \Delta P_{L2} = (\Delta\omega\, D_1 - \Delta P_{m1}) + (\Delta\omega\, D_2 - \Delta P_{m2})$$
$$= \Delta\omega\left(\frac{1}{R_1} + D_1\right) + \Delta\omega\left(\frac{1}{R_2} + D_2\right) = \Delta\omega(\beta_1 + \beta_2)$$

$$\therefore\ \Delta\omega = \frac{-(\Delta P_{L1} + \Delta P_{L2})}{\beta_1 + \beta_2} ---\rightarrow (21)$$

$$\Delta P_{12} = \frac{-\Delta P_{L1}\,\beta_2}{\beta_1 + \beta_2} + \frac{\Delta P_{L2}\,\beta_1}{\beta_1 + \beta_2} ----\rightarrow (22)$$

We can observe from the above equations that with only primary governor control, load change in either of the areas will lead to a steady state deviation in frequency of both areas.

Frequency Bias Tie-Line Control:

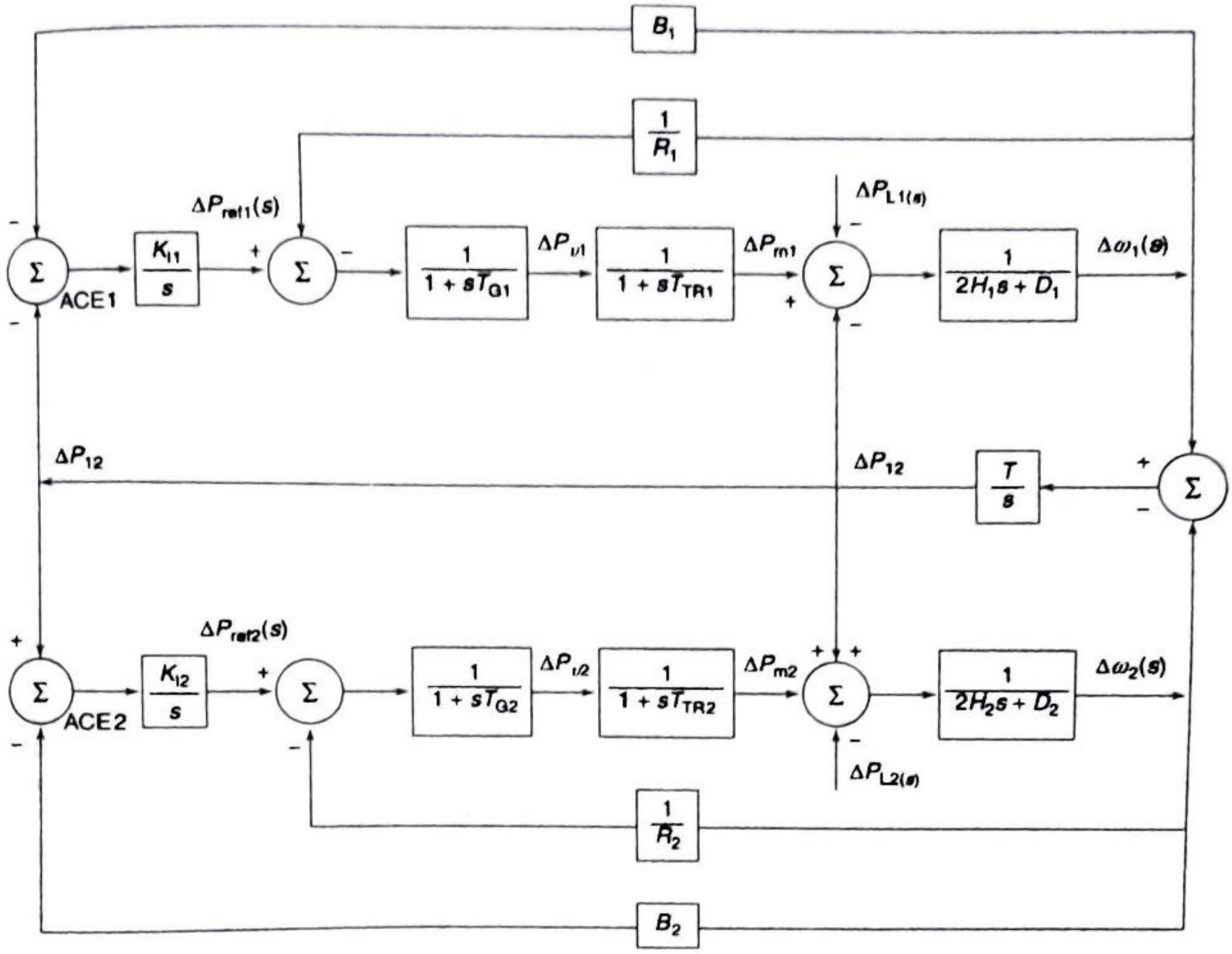

Fig.13 Block diagram of AGC for a two-area system with supplementary control

Some form of reset integral control must be added to the two area system to reduce both persistent static frequency error and static error in tie-line flow to zero. This is the supplementary control action as shown in fig.(13). Supplementary controls are provided to restore balance between generation and load of each area and restore frequency to a nominal value. In general, there are three modes in which interconnected operation can be carried out.

1. Flat frequency mode: when a system responds to frequency changes only, it cannot have control over the power flow in the tie-lines.
2. Flat tie-line mode: when a system responds to tie-line changes and changes its generation to maintain the scheduled tie-line interchanges, it cannot respond to frequency changes.
3. Both of the above methods have disadvantages. So a combined control is used, called the frequency bias control.

The basic objective of this control is to restore balance between each area load and generation when the control action maintains: frequency at scheduled value, net interchange power with neighbouring areas at scheduled value and each area in normal steady state absorbs its own load.

Let us consider identical areas, with area-1 supplying, say 150 MW, over the tie-line to area-2. Assume that an increase in load by 50 MW in area-2 occurs. With primary control acting, and identical characteristics, each area will increase its generation by 25 MW, and the tie-line power increases to 175 MW. If now area-1 had an agreement (contract) to sell only 150 MW to area-2, then the production cost of the extra 25 MW would be unbilled. This increase in generation occurred, because of interconnection between the two areas. Therefore, the supplementary control in a given area should change the generation only for changes in load in the same control area.

A control signal made up of tie-line flow deviation added to frequency deviation weighted by a bias factor can achieve the desired objective of restoring frequency to a nominal value and holding the tie-line power flow at the scheduled value. This control signal is called the area control error (ACE) and is defined as: the net power interchange, together with a gain, B (MW/0.1 Hz), called the frequency bias, as a multiplier on the frequency deviation.

$$\mathrm{ACE} = \sum_{k=1}^{k} \mathrm{P_k} - \mathrm{P_s} + 10\ \mathrm{B}\ (\mathrm{f_{act}} - \mathrm{f_0}) \quad \text{in MW}$$

Where P_k = MW tie flow defined as positive out of the area, P_s = Scheduled MW interchange between the areas, f_{act} = System actual frequency in Hz, and f_0 = Scheduled base frequency in Hz.

Applying the supplementary control action, the control error for each area consists of a linear combination of frequency and tie-line errors:

$$\mathrm{ACE_1} = \Delta \mathrm{P_{12}} + \mathrm{B_1}\ \Delta\omega \quad \text{and} \quad \mathrm{ACE_2} = \Delta \mathrm{P_{21}} + \mathrm{B_2}\ \Delta\omega$$

B_1 and B_2 are the bias factors and determine the amount of interaction during a disturbance in the other area. It has been found that the most suitable bias factor for an area is its frequency response characteristic β. Thus,

$$\mathrm{ACE_1} = \Delta \mathrm{P_{12}} + \beta_1\ \Delta\omega \quad \text{and} \quad \mathrm{ACE_2} = \Delta \mathrm{P_{21}} + \beta_2\ \Delta\omega$$

where $\beta_1 = \mathrm{D_1} + \frac{1}{\mathrm{R_1}}$ and $\beta_2 = \mathrm{D_2} + \frac{1}{\mathrm{R_2}}$.

$\Delta \mathrm{P_{12}}$ *and* $\Delta \mathrm{P_{21}}$ are the deviations from the scheduled tie-line interchanges.

The speed-changer commands (reference power set points) will be of the form:

$$\Delta \mathrm{P_{ref,1}} = -\,\mathrm{K_{I\,1}} \int (\Delta \mathrm{P_{12}} + \mathrm{B_1}\Delta\omega)\ \mathrm{dt} \quad \text{and} \quad \Delta \mathrm{P_{ref,2}} = -\,\mathrm{K_{I\,2}} \int (\Delta \mathrm{P_{21}} + \mathrm{B_2}\Delta\omega)\ \mathrm{dt}$$

The constants K_{I1} and K_{I2} are integrator gains. The minus signs must be included since each area should increase its generation if either its frequency error or its tie-line power increment is negative. ACE actuates changes in the reference power set points. When steady state is reached $\Delta \mathrm{P_{12}}$ and $\Delta\omega$ will be zero, since the integral controller will bring back the frequency to nominal value. The gain of integral controller should not be high enough to cause instability. The supplementary ACE control is applied only to selected units in a power system area.

***Choice of Bias Factors:** The ACE signals in a general form are given by, $\mathrm{ACE_1} = \mathrm{K_1}\Delta \mathrm{P_{12}} + \mathrm{B_1}\ \Delta\omega = 0$ & $\mathrm{ACE_2} = \mathrm{K_2}\Delta \mathrm{P_{21}} + \mathrm{B_2}\ \Delta\omega = 0$. These equations results in $\Delta \mathrm{P_{12}} = 0$ & $\Delta\omega = 0$ for all non-zero values of K_1, K_2, B_1 and B_2. The composition of ACE signals is more from dynamic performance conditions. Assume that a sudden change of $\Delta \mathrm{P_{L1}}$ in area-1. The frequency deviation $\Delta\omega$ is determined by the regulation characteristics of both areas. Consider the following cases to examine the performance of supplementary control:

$$\Delta\omega = \frac{-\Delta \mathrm{P_{L1}}}{\beta_1 + \beta_2} \quad \text{and} \quad \Delta \mathrm{P_{12}} = \frac{-\Delta \mathrm{P_{L1}}\,\beta_2}{\beta_1 + \beta_2} = -\Delta \mathrm{P_{21}}$$

Case-1: $K_1 = K_2 = 1$, $B_1 = \beta_1$ and $B_2 = \beta_2$ → $\mathrm{ACE_1} = -\Delta \mathrm{P_{L1}}$ and $\mathrm{ACE_2} = 0$.
Thus, only the supplementary control in area-1 will change the load reference set point to meet the load change in area-1 & bring ACE-1 to zero. The supplementary control of area-2 will not be affected.

Case-2: $K_1 = K_2 = 1$, $B_1 = 2\beta_1$ and $B_2 = 2\beta_2$ → $\mathrm{ACE_1} = -\Delta \mathrm{P_{L1}}\left(1 + \frac{\beta_1}{\beta_1+\beta_2}\right)$ and $\mathrm{ACE_2} = \frac{-\Delta \mathrm{P_{L1}}\,\beta_2}{\beta_1 + \beta_2}$.
Thus, both area-1 and area-2 supplementary controls would respond and correct the frequency deviation twice as fast.

Case-3: If we set the bias factors significantly lower than the respective areas β's, a situation opposite to the above would exist (degradation of system frequency control). At values significantly higher than the area β, the control action may become unstable.

State-Space Models:
The state space model of a system allows a design of controller using all the state variables of the system. This technique is extremely useful in multivariable systems.
State space model of an isolated system:

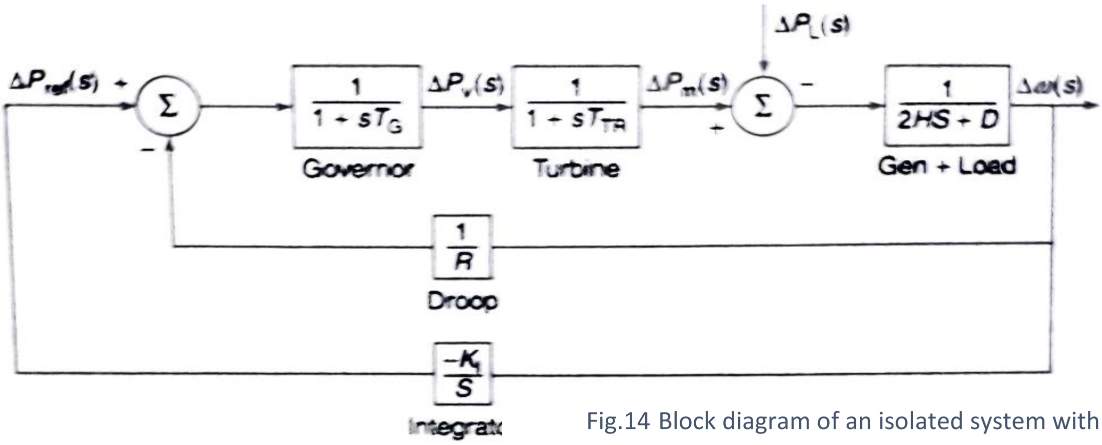

Fig.14 Block diagram of an isolated system with AGC

The s-domain equations are as follows,

For generator and load, $\left[\frac{1}{2\,\mathrm{H\,s} + \mathrm{D}}\right](\Delta \mathrm{P_m(s)} - \Delta \mathrm{P_L(s)}) = \Delta\omega(\mathrm{s})$

$$\Delta\omega(\mathrm{s})(2\,\mathrm{H\,s} + \mathrm{D}) = \Delta \mathrm{P_m(s)} - \Delta \mathrm{P_L(s)}$$

$$\mathrm{s}\,\Delta\omega(\mathrm{s}) = \frac{-\mathrm{D}}{2\mathrm{H}}\Delta\omega(\mathrm{s}) + \frac{1}{2\mathrm{H}}\Delta \mathrm{P_m(s)} - \frac{1}{2\mathrm{H}}\Delta \mathrm{P_L(s)} --\rightarrow (1)$$

For the turbine,

$$\frac{\Delta \mathrm{P_m(s)}}{\Delta \mathrm{P_v(s)}} = \frac{1}{1 + \mathrm{s\,T_{TR}}}$$

$$(1 + \mathrm{s\,T_{TR}})\,\Delta \mathrm{P_m(s)} = \Delta \mathrm{P_v(s)}$$

$$\mathrm{s}\,\Delta \mathrm{P_m(s)} = \frac{-1}{\mathrm{T_{TR}}}\Delta \mathrm{P_m(s)} + \frac{1}{\mathrm{T_{TR}}}\Delta \mathrm{P_v(s)} \; ---\rightarrow (2)$$

For the governor,

$$\frac{\Delta \mathrm{P_v(s)}}{\Delta \mathrm{P_g(s)}} = \frac{1}{1 + \mathrm{s\,T_G}} \quad \text{and} \quad \Delta \mathrm{P_g(s)} = \Delta \mathrm{P_{ref}(s)} - \frac{1}{\mathrm{R}}\,\Delta\omega\,(\mathrm{s})$$

$$\left[\Delta \mathrm{P_{ref}(s)} - \frac{1}{\mathrm{R}}\,\Delta\omega\,(\mathrm{s})\right]\left[\frac{1}{1 + \mathrm{s\,T_G}}\right] = \Delta \mathrm{P_v(s)}$$

$$\Delta \mathrm{P_v(s)}(1 + \mathrm{s\,T_G}) = \Delta \mathrm{P_{ref}(s)} - \frac{1}{\mathrm{R}}\,\Delta\omega\,(\mathrm{s})$$

$$\mathrm{s}\,\Delta \mathrm{P_v(s)} = \frac{-1}{\mathrm{T_G}}\Delta \mathrm{P_v(s)} + \frac{1}{\mathrm{T_G}}\Delta \mathrm{P_{ref}(s)} - \frac{1}{\mathrm{RT_G}}\,\Delta\omega\,(\mathrm{s}) --\rightarrow (3)$$

For the integrator,

$$\Delta \mathrm{P_{ref}(s)} = -\frac{\mathrm{K_I}}{\mathrm{s}}\,\Delta\omega\,(\mathrm{s})$$

$$\mathrm{s}\;\Delta \mathrm{P_{ref}(s)} = -\,\mathrm{K_I}\Delta\omega\,(\mathrm{s}) \; ---\rightarrow (4)$$

Equations (1) to (4) are the required state equations. These equations are transformed into time domain and written in a matrix form.

The standard form of state space model: $\dot{\mathrm{X}}(\mathrm{t}) = \mathrm{A\,X(t)} + \mathrm{B\,u(t)}$

$$\mathrm{y(t)} = \mathrm{C\,X(t)} + \mathrm{D\,u(t)} \; ---\rightarrow (5)$$

$$\begin{bmatrix} \Delta\dot{\omega} \\ \Delta\dot{P}_m \\ \Delta\dot{P}_v \\ \Delta\dot{P}_{ref} \end{bmatrix} = \begin{bmatrix} \frac{-D}{2H} & \frac{1}{2H} & 0 & 0 \\ 0 & \frac{-1}{T_{TR}} & \frac{1}{T_{TR}} & 0 \\ \frac{-1}{RT_G} & 0 & \frac{-1}{T_G} & \frac{1}{T_G} \\ -\ K_I & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta\omega \\ \Delta P_m \\ \Delta P_v \\ \Delta P_{ref} \end{bmatrix} + \begin{bmatrix} \frac{-1}{2H} \\ 0 \\ 0 \\ 0 \end{bmatrix} \Delta P_L \ --\rightarrow (6)$$

In time domain, state vector = $X(t) = [\Delta\omega \ \ \Delta P_m \ \ \Delta P_v \ \ \Delta P_{ref}]^T \ ---\rightarrow (7)$

Input vector = $u(t) = \Delta P_L$

Output vector = $y(t) = \Delta\omega$

$$A = \begin{bmatrix} \frac{-D}{2H} & \frac{1}{2H} & 0 & 0 \\ 0 & \frac{-1}{T_{TR}} & \frac{1}{T_{TR}} & 0 \\ \frac{-1}{RT_G} & 0 & \frac{-1}{T_G} & \frac{1}{T_G} \\ -\ K_I & 0 & 0 & 0 \end{bmatrix} \qquad B = \begin{bmatrix} \frac{-1}{2H} \\ 0 \\ 0 \\ 0 \end{bmatrix} \qquad C = [\,1 \quad 0 \quad 0 \quad 0\,] \quad \text{and} \quad D = 0$$