

DEPARTMENT OF MATHEMATICS

Transform Calculus, Fourier Series and Numerical Techniques (18MAT31)

Assignment -I

- 1) Find the Laplace transform of i) $\left(\frac{\sin 2t}{\sqrt{t}}\right)^2$ ii) $\left(\frac{4t+5}{e^{2t}}\right)^2$
- 2) Find $L[f(t)]$ of triangular wave of period $2a$ given by $f(t) = \begin{cases} t & 0 < t < a \\ 2a-t & a < t < 2a \end{cases}$
- 3) Find the Laplace transform of periodic function of period $\frac{2\pi}{\omega}$, $f(t) = \begin{cases} \sin \omega t, & 0 < t < \frac{\pi}{\omega} \\ 0, & \frac{\pi}{\omega} < t < \frac{2\pi}{\omega} \end{cases}$
- 4) If $f(t) = \begin{cases} t & 0 < t < a \\ 2a-t & a < t < 2a \end{cases}$ show that $L[f(t)] = \frac{1}{s^2} \tanh\left[\frac{as}{2}\right]$
- 5) Express $f(t)$ in terms of unit step function and hence find $L[f(t)]$ where $f(t) = \begin{cases} 1 & 0 \leq t \leq 1 \\ t & 1 < t < 2 \\ t^2 & t > 2 \end{cases}$
- 6) Find i) $L^{-1}\left\{\frac{2s-1}{s^2+4s+29}\right\}$ ii) $L^{-1}\left\{\log\left(\frac{s+a}{s+b}\right)\right\}$
iii) $L^{-1}\left\{\frac{2s^2-6s+5}{s^3-6s^2+11s-6}\right\}$ iv) $L^{-1}\left\{\frac{1}{s+2} + \frac{3}{2s+5} - \frac{4}{3s-2}\right\}$
- 7) Find $f(t)$ from the equation $f(t) = 1 + 2 \int_0^t f(t-u)e^{-2u} du$.
- 8) State Convolution theorem and evaluate $L^{-1}\left\{\frac{1}{(s^2+a^2)^2}\right\}$
- 9) Find the inverse Laplace transform of $\frac{1}{s(s^2+1)}$ using convolution theorem
- 10) Solve the differential equation by Laplace Transform method $y'' + 4y' + 3y = e^{-t}$ given $y(0) = y'(0) = 1$
- 11) Obtain the Fourier series expansion of $f(x) = x - x^2$ in $(-\pi, \pi)$ and Hence deduce that $\frac{\pi^2}{12} = \frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} \dots$

12) Expand $f(x) = |x|$ as Fourier series in $-\pi < x < \pi$ and deduce that $\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} - \dots$

13) Find the half range sine series of e^x in the interval $0 \leq x \leq 1$.

14) Find the half range cosine series of $f(x) = (x - 1)^2$ in the interval $0 \leq x \leq 1$

15) Compute the first two harmonics of the Fourier Series of $f(x)$ given the following table:

x°	0	60°	120°	180°	240°	300°
y	7.9	7.2	3.6	0.5	0.9	6.8

16) Obtain the Fourier series upto first harmonic given:

x	0	1	2	3	4	5	6
y	9	18	24	28	26	20	9


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Assignment -II

1) Find the complex Fourier transform of the function

$$f(x) = \begin{cases} 1 & \text{for } |x| \leq a \\ 0 & \text{for } |x| > a \end{cases}, \text{ Hence evaluate } \int_0^{\infty} \frac{\sin x}{x} dx$$

2) Find the Fourier Sine transform of $f(x) = e^{-|x|}$ and Hence evaluate $\int_0^{\infty} \frac{x \sin mx}{1+x^2} dx \quad m > 0$

3) Find the infinite Fourier transform of $e^{-|x|}$

4) Find the inverse Z-transform of $\frac{4z^2 - 2z}{z^3 - 5z^2 + 8z - 4}$

5) Solve: $u_{n+2} - 3u_{n+1} + 2u_n = 3^n$, given $u_0 = u_n = 0$.

6) Solve: $\frac{dy}{dx} = e^x - y$, $y(0) = 2$ using Taylor's Series method upto 4th degree terms and find the value of $y(1.1)$.

7) Use Runge -Kutta method of fourth order to solve

$$\frac{dy}{dx} + y = 2x \text{ at } x = 1.1 \text{ given } y(1) = 3 (\text{Take } h = 0.1)$$

8) Apply Milne's predictor-corrector formulae to compute $y(0.4)$ given $\frac{dy}{dx} = 2e^x y$, with

x	0	0.1	0.2	0.3
y	2.4	2.473	3.129	4.059

9) Using Adams-Bashforth method, find $y(4.4)$ given $5x \frac{dy}{dx} + y^2 = 2$ with

x	4	4.1	4.2	4.3
y	1	1.0049	1.0097	1.0143

10) Solve by Runge Kutta method $\frac{d^2y}{dx^2} = x \left(\frac{dy}{dx}\right)^2 - y^2$ for $x=0.2$ correct 4 decimal places using initial conditions $y(0)=1, y'(0) = 0, h = 0.2$.


11) Apply Milne's predictor-corrector method to compute $\frac{d^2y}{dx^2} = 1 + \frac{dy}{dx}$ and the following table of initial values:

X	0	0.1	0.2	0.3
Y	1	1.1103	1.2427	1.3990
y'	1	1.2103	1.4427	1.6990

12) Derive Euler's equation in the standard form $\frac{\partial f}{\partial y} - \frac{d}{dx} \left[\frac{\partial f}{\partial y'} \right] = 0$

13) Find the extremal of the of the functional $\int_0^{\frac{\pi}{2}} [y^2 - (y')^2 + 2y \sin x] dx; y\left(\frac{\pi}{2}\right) = 1, y(0)=0$.

14) Prove that geodesics of a plane surface are straight lines.


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