

Bapuji Educational Association (Regd.)
Bapuji Institute of Engineering and Technology, Davangere - 04
DEPARTMENT OF MATHEMATICS
Question Bank

Complex analysis, probability distribution and statistical methods (18MAT41)

- 1) Define analytic function and obtain C-R equations in the Cartesian form.
- 2) Define limit and continuity of $f(z)$. Derive C-R equations in polar form.
- 3) Determine the analytic function .
- 4) Determine the analytic function
- 5) Determine the analytic function
- 6) Show that the function is harmonic. And find the harmonic conjugate.
- 7) Show that the function is harmonic. And find the harmonic conjugate.
- 8) If $f(z)$ is a regular function of z , then prove that .
- 9) If represents the complex potential of an electrostatic field and , determine.
- 10) Discuss the transformation.
- 11) Discuss the transformation .
- 12) Discuss the transformation
- 13) Find the bilinear transformation that map onto the points respectively
- 14) Find the Bilinear transformation which maps the points $z=1, i, -1$ onto the points $w=i, 0, -i$.
- 15) Evaluate where c is the circle .
- 16) Derive Cauchy's integral formula.
- 17) Define Random variable, Probability mass function, Probability density function. The p.d.f. of a variate X is given by the following table

| | | | | | | | |
|------|---|---|----|----|---|---|-----|
| x | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
| P(x) | 0 | k | 2k | 3k | | 2 | 7+k |

Find k , and evaluate $P(x < 6)$, $P(0 < x < 5)$

- 18) The sales per day in a shop is exponentially distributed with the average sale amounting to Rs. 100 and net profit is 8%. Find the probability that the net profit exceeds Rs. 30 on two consecutive days.
- 19) In a Normal distribution, 7% of the items are under 35 and 89% are under 63. Find the mean and std. deviation. Given that $A(1.23) = 0.39$ and $A(1.48) = 0.43$.

- 20) Certain tubes manufactured by a company have mean life time of 800 hours and S.D of 60 hours. Find the probability that a random sample of 16 tubes taken from groups will have mean life time (i) Between 770 hours and 830 hours (ii) Less than 785 hours (iii) More than 820 hours. Given

- 21) The probability density function of a variate x is

| | | | | | | | |
|--------|-----|------|------|------|------|-------|-------|
| x | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
| $P(x)$ | k | $3k$ | $5k$ | $7k$ | $9k$ | $11k$ | $13k$ |

Find k , $p(x < 4)$, and $p(3 < x \leq 6)$.

- 22) If the mean and S.D of the number of correctly answered questions in attest given to 4096 students are 2.5 and find an estimate if the number of candidates answering correctly (i) 8 or more questions (ii) 2 or less (iii) 5 questions
- 23) In a Normal distribution 31% of the items are under 45 and 8% over 64. Find the mean and standard deviation, given that $A(0.5) = 0.19$, $A(1.4) = 0.42$ where $A(z)$ is the area under the standard normal curve.
- 24) In a certain town the duration of a shower is exponentially distributed with mean minutes. What is the probability that a shower will last for i) 10 minutes or more, ii) between 10 and 12 minutes.
- 25) Given that 2% of the fuses manufactured by a firm are defective, find by using Poisson distribution, the probability that a box containing 200 fuses has a) no defective fuses, b) 3 or more defective fuses c) at least one defective fuse.
- 26) In a Normal distribution, 7% of the items are under 35 and 89% are under 63. Find the mean and std. deviation. Given that $A(1.23) = 0.39$ and $A(1.48) = 0.43$.
- 27) The number of accidents in a year to taxi drivers in a city follows a poison distribution with mean 3. Out of 1000 taxi drivers find approximately the number of the drivers with i) no accident in a year ii) more than 3 accidents in a year.
- 28) If x is an exponential variate with mean 5, find
- 29) In a test of electric bulbs, it was found that the life time of a particular brand was distributed normally with an average life of 2000 hours and S.D of 60 hours. If a firm purchase 2500 bulbs find the number of bulbs that are likely to last for (i) More than 2100 hours (ii) Less than 1950 hours (iii) Between 1900 to 2100 hours
- 30) Compute the co-efficient of Co-relation and Lines of Regression for the data

| | | | | | | | |
|-----|---|---|---|---|---|---|---|
| x | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|-----|---|---|---|---|---|---|---|

| | | | | | | | |
|---|---|---|----|----|----|----|----|
| y | 9 | 8 | 10 | 12 | 11 | 13 | 14 |
|---|---|---|----|----|----|----|----|

- 31) Compute the co-efficient of Co-relation of the two groups

| | | | | | | | | | | |
|---|----|----|----|----|----|----|----|----|----|----|
| x | 92 | 89 | 87 | 86 | 83 | 77 | 71 | 63 | 53 | 50 |
| y | 86 | 83 | 91 | 77 | 68 | 85 | 52 | 82 | 37 | 57 |

- 32) Compute the co-efficient of Co-relation between x and y for the following data. Also obtain Regression lines

| | | | | | | | | | | |
|---|----|----|----|----|----|----|----|----|----|----|
| x | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| y | 10 | 12 | 16 | 28 | 25 | 36 | 41 | 49 | 40 | 50 |

- 33) Show that if θ is the angle between the lines of regression, then

- 34) Two ladies were asked to rank 7 different types of lipsticks the rank given by them are as follows.

| Lipsticks | A | B | C | D | E | F | G |
|-----------|---|---|---|---|---|---|---|
| Neelu | 2 | 1 | 4 | 3 | 5 | 7 | 6 |
| Neema | 1 | 3 | 2 | 4 | 5 | 6 | 7 |

Calculate the Rank correlation coefficient

- 35) Calculate the Rank correlation coefficient between the marks assigned to 10 students by judge A and B in a certain test as shown below.

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|------------|----|----|----|----|----|----|----|----|----|----|
| Marks A | 52 | 53 | 42 | 60 | 45 | 41 | 37 | 38 | 25 | 27 |
| Marks B | 65 | 68 | 43 | 38 | 77 | 48 | 35 | 30 | 25 | 50 |

- 36) Find the Rank correlation coefficient from the following.

| | | | | | | | | | | |
|---|----|----|----|---|----|----|----|----|----|----|
| x | 48 | 33 | 40 | 9 | 16 | 16 | 65 | 24 | 16 | 57 |
| y | 13 | 13 | 24 | 6 | 15 | 4 | 20 | 9 | 6 | 19 |

- 37) Given

| | x-series | y-series |
|------|----------|----------|
| Mean | 18 | 100 |
| S.D | 14 | 20 |

And $r = 0.8$, write down the equation of the lines of regression and hence find the most probable value of y when $x = 70$

38) Prove that

39) Fit a Straight line for the following data

| | | | | | | | | |
|---|---|---|---|---|---|---|----|----|
| x | 1 | 3 | 4 | 6 | 8 | 9 | 11 | 14 |
| y | 1 | 2 | 4 | 4 | 5 | 7 | 8 | 9 |

40) Fit a Second degree Parabola in the least square sense for the following data and hence estimate

| | | | | | |
|---|----|----|----|----|----|
| x | 1 | 2 | 3 | 4 | 5 |
| y | 10 | 12 | 13 | 16 | 19 |

41) An experiment on lifetime t of cutting tool at different cutting speeds v (units) are given below

| | | | | |
|---------------|-----|-----|-----|-----|
| Speed (v) | 350 | 400 | 500 | 600 |
| Life(t) | 61 | 26 | 7 | 2.6 |

Fit a relation of the form

42) The joint distribution of two random variables x and y is as follows.

| | | | |
|---|-----|-----|-----|
| y | -4 | 2 | 7 |
| x | | | |
| 1 | 1/8 | 1/4 | 1/8 |
| 5 | 1/4 | 1/8 | 1/8 |

Find Marginal distribution of x and y , $E(x)$, $E(y)$, $Cov(x, y)$ and (x, y) .

43) The joint distribution of two random variables x and y is as follows.

| | | | |
|---|-----|-----|-----|
| y | -3 | 2 | 4 |
| x | | | |
| 1 | 0.1 | 0.2 | 0.2 |
| 3 | 0.3 | 0.1 | 0.1 |

Find the co variance of (x, y) and correlation co-efficient of (x, y) .

44) The joint distribution of two random variables X and Y is given by $f(x,y)=k(2x+y)$ where x and y are integers such that (i) Find the value of constant k (ii) Find the Marginal probability distributions of X and Y .

- 45) Explain the following terms: Type I and Type II errors, Null hypothesis, Point estimate and Interval estimate.
- 46) A die is thrown 9000 times and a throw of 3 or 4 was observed 3240 times. Show that the die can not be regarded as an unbiased one.
- 47) A coin is tossed 1000 times and head turns up 540 times. Decide on the hypothesis that the coin is unbiased.
- 48) In 324 throws of a six faced die, an odd number turned up 181 times. is it reasonable to think that the die is unbiased one?
- 49) Two horses A and B were tested according to the time (in seconds) to run a particular race with a following results

| | | | | | | | |
|---------|----|----|----|----|----|----|----|
| Horse A | 28 | 30 | 32 | 33 | 33 | 29 | 34 |
| Horse B | 29 | 30 | 30 | 24 | 27 | 29 | |

Test whether you can discriminate between the two horses.

- 50) The number of accidents per day (x) as recorded in a textile industry over a period of 400 days is given below. Test the goodness of fit in respect of Poisson Distribution of fit to the given data ()

| | | | | | | |
|-----|-----|-----|----|----|---|---|
| x | 0 | 1 | 2 | 3 | 4 | 5 |
| f | 173 | 168 | 37 | 18 | 3 | 1 |

- 51) A set of five similar coins is tossed 320 times and the result is

| | | | | | | |
|--------------|---|----|----|-----|----|----|
| No. of heads | 0 | 1 | 2 | 3 | 4 | 5 |
| Frequency | 6 | 27 | 72 | 112 | 71 | 32 |

Fit a Binomial distribution and test for its goodness of fit at 5% level of Significance (