# Algebraic Approach to Targeting Direct recycle

Material recycle pinch diagrams is a graphical tool to identify targets for direct recycle problems. Inspite of usefulness of graphical methods, it is useful to develop an algebraic procedure ~~~~ in the following cases

1) Numerous sources and sinks.
   As the number of sources and sinks increase, it becomes more convenient to use algebraic calculations

2) Scaling problems
   If there is significant difference in values of flowrates/loads for some sources/sinks, the graphical representation becomes inaccurate

3) If the targeting is tied with a broader design task handled through algebraic calculations, it is desirable to use consistent algebraic tools.

## Problem statement

Given a process with a number of process sources ($N_{sources}$) considered for recycle. Each source, $i$, has a flow rate, $W_i$, and composition of targeted species, $y_i$.
The sinks ($N_{sinks}$) are process units that use fresh resource. Each sink, $j$, requires a flow rate, $G_j^{in}$, and an inlet composition, $z_j^{in}$ must satisfy

$$0 \le z_j^{in} \le z_j^{max} \qquad \text{where } j = 1, 2, \ldots N_{sinks}$$

The objective is to develop an algebraic procedure to minimize the purchase of fresh resource, maximise usage of process sources, and minimize waste discharge

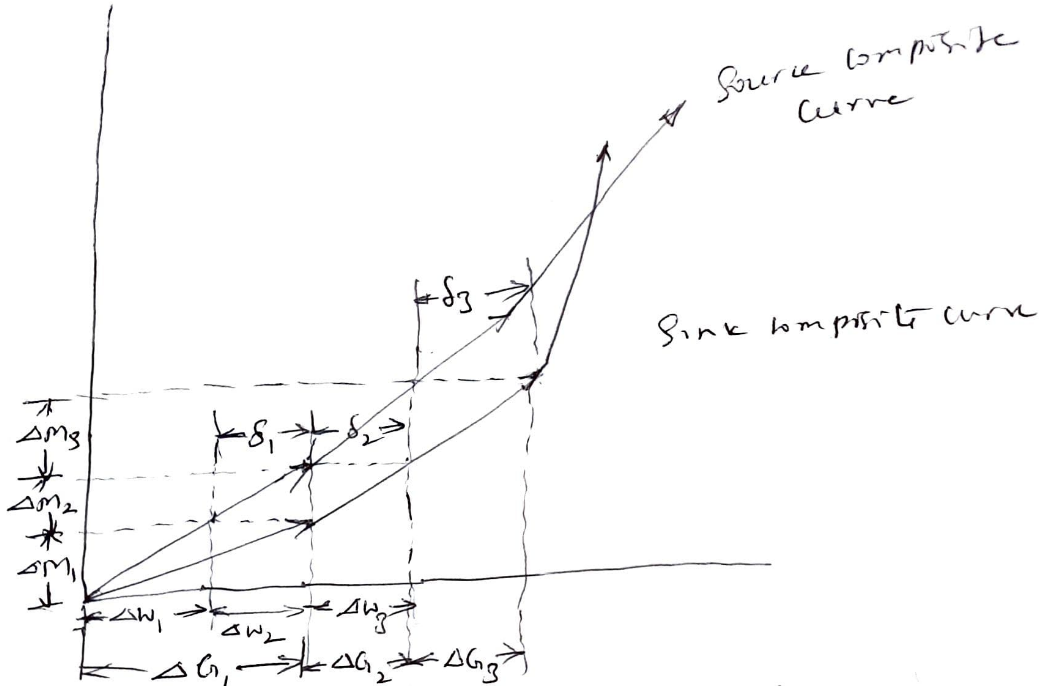


Fig 1. Load intervals, flows and residuals

Calculate the flow rate of the source and the sink within each load interval. These flow rate correspond to the horizontal distances

$$\Delta W_K = \frac{\Delta m_K}{y_{\text{source in interval } K}} \qquad \text{—— (1)}$$

and

$$\Delta G_K = \frac{\Delta m_K}{Z^{max}_{\text{sink in interval } K}} \qquad \text{—— (2)}$$

$$\delta_K = \sum_{K=1}^{E} W_K - \sum_{K=1}^{E} G_K \qquad \text{—— (3)}$$

Horizontal distance between source and sink composite curves $(\delta_K)$ is the difference in cumulative flow rates,

For first interval

$$\delta_1 = \Delta W_1 - \Delta G_1 \qquad \text{—— (4)}$$

For second interval

$$\delta_2 = \Delta W_1 + \Delta W_2 - \Delta G_1 - \Delta G_2 \qquad \text{—— (5)}$$

Substituting in Eqn ④ in Eqn ⑤

$$\delta_2 = \delta_1 + \Delta W_2 - \Delta G_2 \qquad (6)$$

and, for $k^{th}$ interval

$$\delta_K = \delta_{K-1} + \Delta W_K - \Delta G_K \qquad (7)$$

Cascade diagram.

The flow balances can be carried out for all the intervals resulting in cascade diagram shown in Fig.2

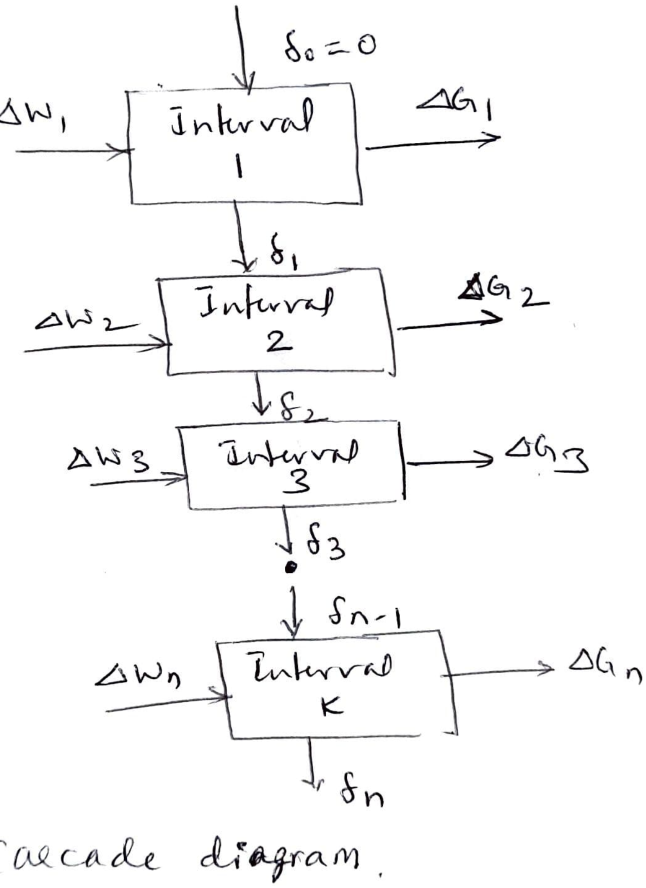


Fig.2. Cascade diagram.

The most negative value of $\delta$ on the cascade diagram ($\delta_{max}$) represents the target for minimum fresh consumptions. In order to remove infeasibilities, a flow rate of fresh resource equal to $\delta_{max}$ is added to the top of the cascade (i.e $\delta_0 = \delta_{max}$). The residuals are accordingly increased by $\delta_{max}$, i.e

$$\delta'_K = \delta_K + \delta_{max} \qquad (8)$$

where $\delta'_K$ is revised residual flow leaving the $K^{th}$ interval. Consequently the most negative residual becomes zero thereby designating the pinch location.

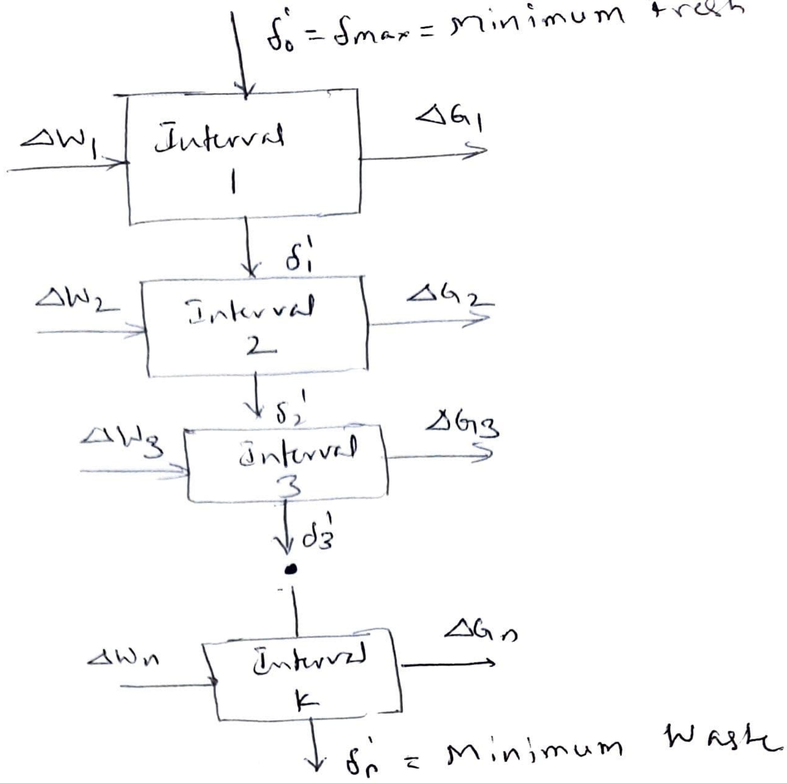


Fig.3. Revised cascade diagram.

The revised residual leaving the last interval is the target for minimum waste discharge since it represents unrecycled/unreused flowrates of the sources. The ~~above~~ fig.3 is an illustration of the revised cascade diagram.

The following residuals determine the targets

$\delta_{max}$ = Target for minimum fresh usage

$\delta'_n$ = Target for minimum waste discharge.