



Course File Check List

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2020-21 ODD SEM

Tentative Academic Calendar of VTU, Belagavi for ODD Semester of 2020-2021

	I Sem B. E. / B. Tech. / B. Arch. / B. Plan	I sem M. Tech. / MBA / MCA / M. Arch.	III, V & VII Sem B. E. / B. Tech. / B. Plan / B. Arch. & IX Sem B. Arch.	III & V Sem MCA	III Sem MBA	III Sem M. Tech.	III Sem M. Arch.
Commencement of ODD Semester			01.09.2020	01.09.2020	01.09.2020	01.09.2020	01.09.2020
Last Working day of ODD Semester			17.12.2020	17.12.2020	17.12.2020	17.12.2020	17.12.2020
Practical Examinations			21.12.2020 To 31.12.2020	21.12.2020 To 31.12.2020	21.12.2020 To 31.12.2020	21.12.2020 To 31.12.2020	21.12.2020 To 31.12.2020
Theory Examinations			04.01.2021 To 23.01.2021	04.01.2021 To 23.01.2021	04.01.2021 To 23.01.2021	04.01.2021 To 23.01.2021	04.01.2021 To 23.01.2021
Internship Viva-Voce							
Professional training / Organization study							
Commencement of EVEN Semester			08.02.2021	08.02.2021	08.02.2021	22.02.2021	08.02.2021

Will be announced later

Will be announced later

NOTE

- VII Semester B. E / B. Tech students shall have to undergo INTERNSHIP as per circular of University VTU/Aca/2019-20/85, dated 12.05.2020.
- I Semester B. E / B. Tech / B. Arch Students shall compulsorily undergo Induction Program for a period of 3 Weeks as per the schedule given by VTU Belagavi
- The classroom sessions for all the higher semesters would be commencing from 01.09.2020(Tentative) in ONLINE mode until further orders.
- The Institute needs to function for six days a week with additional hours.
- The faculty/staff shall be available to undertake any work assigned by the university.
- If any of the above date is declared to be a holiday then the corresponding event will come into effect on the next working day.
- Notification regarding Calendar of Events relating to the conduct of University Examinations will be issued by the Registrar (Evaluation) from time to time.
- Academic Calendar may be modified based on guidelines/directions issued in future by MHRD/UGC/AICTE/State Government.


 REGISTRAR

Bapuji Institute of Engineering and Technology, Davangere-577004
CALENDER OF EVENTS - ODD SEMESTER: SEPTEMBER-JANUARY-2020-21 (Tentative)

PARTICULARS	I sem BE/B Tech	III, V BE/B Tech	VII sem BE/B Tech	III & V sem MCA	III sem MBA	III sem M.Tech
Commencement of ODD Sem	14-12-2020	01-09-2020	01-09-2020	01-09-2020	01-09-2020	01-09-2020
Last Working Day	25-03-2021	16-01-2021	16-01-2021	16-01-2021	16-01-2021	16-01-2021
1 st CIE Series	-----	19-10-2020 To 24-10-2020	19-10-2020 To 24-10-2020	19-10-2020 To 24-10-2020	15-10-2020 To 17-10-2020	19-10-2020 To 24-10-2020
2 nd CIE Series	-----	07-12-2020 To 09-12-2020	07-12-2020 To 09-12-2020	07-12-2020 To 09-12-2020	26-11-2020 To 28-11-2020	07-12-2020 To 09-12-2020
3 rd CIE Series	-----	11-01-2021 To 13-01-2021	11-01-2021 To 13-01-2021	11-01-2021 To 13-01-2021	7-01-2021 To 9-01-2021	11-01-2021 To 13-01-2021
Practical Examination	29-03-2021 Onwards #	21-01-2021 Onwards #	21-01-2021 Onwards #	08-02-2021 Onwards #	-----	21-01-2021 Onwards #
Theory Examination	12-04-2021 To 30-04-2021	08-02-2021 To 27-03-2021	08-02-2021 To 27-03-2021	21-01-2021 To 06-02-2021	21-01-2021 To 19-02-2021	28-01-2021 To 13-02-2021
Internship	-----	-----	29-03-2021 To 10-04-2021	-----	-----	-----
Internship Viva-Voce	-----	-----	-----	-----	-----	15-02-2021 To 22-02-2021
Professional Training/Organization Study	-----	-----	-----	-----	22-02-2021 To 03-04-2021	-----
Commencement of Even Semester	03-05-2021	29-03-2021	12-04-2021	15-02-2021	05-04-2021	23-02-2021

Notification regarding the calendar of events relating to the conduct of University Examination will be issued by the Registrar (Evaluation) from time to time.

Dear Academic

Principal



Vision of BIET

To be a center of excellence recognized nationally and internationally, in distinctive areas of engineering education and research, based on a culture of innovation and invention.

Mission of BIET

BIET contributes to the growth and development of its students by imparting a broad based engineering education and empowering them to be successful in their chosen field by inculcating in them positive approach, leadership qualities and ethical values



VISION OF THE DEPARTMENT

To train the students to become Civil Engineers with leadership qualities, having ability to take up professional assignments and research with a focus on innovative approaches to cater to the needs of the society.

MISSION OF THE DEPARTMENT

1. To provide quality education through updated curriculum and conducive teaching learning environment for the students to excel in higher studies, competitive examinations and professional career.
2. To impart soft skills, leadership qualities and professional ethics among the graduates to handle the projects independently with confidence.
3. To deal with the contemporary issues and to cater to the socio-economic needs.
4. To build industry-institute interaction and to establish good rapport with alumni.

PROGRAM EDUCATIONAL OBJECTIVES (PEOs)

PEO 1: Core Competence: Graduates will be able to plan, analyse, design and construct sustainable Civil Engineering Infrastructure.

PEO 2: Professional Skills: Graduates will be professional engineers with a sense of ethics, creativity, leadership, self-confidence and independent thinking to cater to the needs of the society.

PEO 3: Societal Needs: Graduates will be able to contribute effectively for the development of industry and professional bodies.

PEO 4: Cognitive Intelligence: Graduates will be able to take up competitive examinations, higher studies and involve in research and entrepreneurship activities.

PROGRAM SPECIFIC OUTCOMES (PSOs)

Students after the completion of the Program will be able to

1. Apply the fundamental concepts, software and codal provisions in the analysis, design and construction of sustainable civil engineering infrastructure.
2. Inculcate professional and leadership qualities, sense of ethics and confidence related to civil engineering.

Faculty will be able to

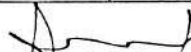
3. Contribute to the overall development of civil engineering community through the professional bodies and offer services to the society.
-

3D - CV301
 3B - CV305

Name of the Faculty : MER							
Time / Day	8 - 9	9 - 10	10.30 - 11.30	11.30 - 12.30	2 - 3	3 - 4	4 - 5
Mon	18CV32 - A				18CV35 - B		18CVL57 - T (B)
Tue		18CV32 - A					
Wed	18CV35 - B			18CV32 - A	18CVL57 - A3 (MER + SH)		
Thu		18CV35 - B	18CV32 - A		18CVL57 - B1 (MER + GNS)		
Fri		18CVL38 - A1 (MER + CPA)					
Sat		18CV32 - A					


 Time Table Coordinator


 HOD


 Principal

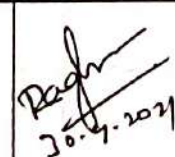

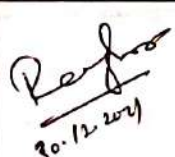
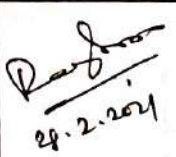
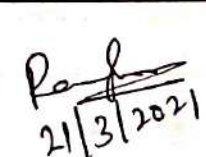




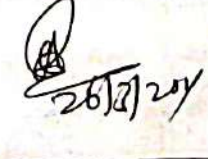

2020
 Period : From ... September To February - 2021 Semester : Odd/Even

Name of the Teacher : Raghu. M.E

Designation : Assistant Professor

Department : Civil Department

Sl. No.	Sem. / Sec. / Branch	Subject Name	Subject Code
1	III rd A Civil	Strength of materials	18CV32
2	III B Civil	Basic surveying	18CV35
3	III A A ₁ Civil	Basic material testing lab	18CV38
4			
5			
6			
7			

	Reviews at the end of the				End of Semester
	1st Month	2nd Month	3rd Month	4th Month	
Signature of Staff	 30.9.2021	 30.10.2021	 30.12.2021	 29.2.2021	 21/3/2021
Signature of the Head of Department	 30/9/21	 3/10/21	 30/12/21	 21/2/21	 26/3/21
Signature of the Principal	 PRINCIPAL Japuji Institute of Engineering & Technology DAYANGERE.				

Class : III A section Subject Code : 18CV32 Subject : Strength of Materials Total No. of Classes : 31

Sl. No.	USN	NAME	DATE	Days																															No. of Days Present	%	Test Marks			Average	Remarks														
				1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31			I	II	III																
62	Kandarp B N	Sandeep B N		9/10	12/14	15/16	15/21	19/22	23/25	25/27	27/29	29/31	31/33	33/35	35/37	37/39	39/41	41/43	43/45	45/47	47/49	49/51	51/53	53/55	55/57	57/59	59/61	61/63	63/65	65/67	67/69	69/71	71/73	73/75	75/77	77/79	79/81	81/83	83/85	85/87	87/89	89/91	91/93	93/95	95/97	97/99	99/100	10	10	29	30	05	22	32	
63	Shanwar B N	Shanwar B N		9/10	12/14	15/16	15/21	19/22	23/25	25/27	27/29	29/31	31/33	33/35	35/37	37/39	39/41	41/43	43/45	45/47	47/49	49/51	51/53	53/55	55/57	57/59	59/61	61/63	63/65	65/67	67/69	69/71	71/73	73/75	75/77	77/79	79/81	81/83	83/85	85/87	87/89	89/91	91/93	93/95	95/97	97/99	99/100	10	10	30	28	20	20	30	
64	Amrite	Amrita H. D.		9/10	12/14	15/16	15/21	19/22	23/25	25/27	27/29	29/31	31/33	33/35	35/37	37/39	39/41	41/43	43/45	45/47	47/49	49/51	51/53	53/55	55/57	57/59	59/61	61/63	63/65	65/67	67/69	69/71	71/73	73/75	75/77	77/79	79/81	81/83	83/85	85/87	87/89	89/91	91/93	93/95	95/97	97/99	99/100	10	10	30	25	21	26	36	
65	Triyeni. G.	Triyeni. G.		9/10	12/14	15/16	15/21	19/22	23/25	25/27	27/29	29/31	31/33	33/35	35/37	37/39	39/41	41/43	43/45	45/47	47/49	49/51	51/53	53/55	55/57	57/59	59/61	61/63	63/65	65/67	67/69	69/71	71/73	73/75	75/77	77/79	79/81	81/83	83/85	85/87	87/89	89/91	91/93	93/95	95/97	97/99	99/100	10	10	30	27	13	24	34	
66	Pallavi. O. X	Pallavi. O. X		9/10	12/14	15/16	15/21	19/22	23/25	25/27	27/29	29/31	31/33	33/35	35/37	37/39	39/41	41/43	43/45	45/47	47/49	49/51	51/53	53/55	55/57	57/59	59/61	61/63	63/65	65/67	67/69	69/71	71/73	73/75	75/77	77/79	79/81	81/83	83/85	85/87	87/89	89/91	91/93	93/95	95/97	97/99	99/100	10	10	30	30	20	27	37	
67	Harish. S. D. X	Harish. S. D. X		9/10	12/14	15/16	15/21	19/22	23/25	25/27	27/29	29/31	31/33	33/35	35/37	37/39	39/41	41/43	43/45	45/47	47/49	49/51	51/53	53/55	55/57	57/59	59/61	61/63	63/65	65/67	67/69	69/71	71/73	73/75	75/77	77/79	79/81	81/83	83/85	85/87	87/89	89/91	91/93	93/95	95/97	97/99	99/100	10	10	17	16	6	13	23	
68	Venug. B. Ghanshyamudra X	Venug. B. Ghanshyamudra X		9/10	12/14	15/16	15/21	19/22	23/25	25/27	27/29	29/31	31/33	33/35	35/37	37/39	39/41	41/43	43/45	45/47	47/49	49/51	51/53	53/55	55/57	57/59	59/61	61/63	63/65	65/67	67/69	69/71	71/73	73/75	75/77	77/79	79/81	81/83	83/85	85/87	87/89	89/91	91/93	93/95	95/97	97/99	99/100	10	10	30	30	62	21	31	
69	Harish. C. N. X	Harish. C. N. X		9/10	12/14	15/16	15/21	19/22	23/25	25/27	27/29	29/31	31/33	33/35	35/37	37/39	39/41	41/43	43/45	45/47	47/49	49/51	51/53	53/55	55/57	57/59	59/61	61/63	63/65	65/67	67/69	69/71	71/73	73/75	75/77	77/79	79/81	81/83	83/85	85/87	87/89	89/91	91/93	93/95	95/97	97/99	99/100	10	10	30	30	19	26	36	
70	Harish. K. S. X	Harish. K. S. X		9/10	12/14	15/16	15/21	19/22	23/25	25/27	27/29	29/31	31/33	33/35	35/37	37/39	39/41	41/43	43/45	45/47	47/49	49/51	51/53	53/55	55/57	57/59	59/61	61/63	63/65	65/67	67/69	69/71	71/73	73/75	75/77	77/79	79/81	81/83	83/85	85/87	87/89	89/91	91/93	93/95	95/97	97/99	99/100	10	10	30	27	01	20	30	
71	Triyeni. S. G. X	Triyeni. S. G. X		9/10	12/14	15/16	15/21	19/22	23/25	25/27	27/29	29/31	31/33	33/35	35/37	37/39	39/41	41/43	43/45	45/47	47/49	49/51	51/53	53/55	55/57	57/59	59/61	61/63	63/65	65/67	67/69	69/71	71/73	73/75	75/77	77/79	79/81	81/83	83/85	85/87	87/89	89/91	91/93	93/95	95/97	97/99	99/100	10	10	30	30	04	22	32	
72	Georg. L. X	Georg. L. X		9/10	12/14	15/16	15/21	19/22	23/25	25/27	27/29	29/31	31/33	33/35	35/37	37/39	39/41	41/43	43/45	45/47	47/49	49/51	51/53	53/55	55/57	57/59	59/61	61/63	63/65	65/67	67/69	69/71	71/73	73/75	75/77	77/79	79/81	81/83	83/85	85/87	87/89	89/91	91/93	93/95	95/97	97/99	99/100	10	10	30	25	11	23	33	
73	Erudasa Anjum. H. X	Erudasa Anjum. H. X		9/10	12/14	15/16	15/21	19/22	23/25	25/27	27/29	29/31	31/33	33/35	35/37	37/39	39/41	41/43	43/45	45/47	47/49	49/51	51/53	53/55	55/57	57/59	59/61	61/63	63/65	65/67	67/69	69/71	71/73	73/75	75/77	77/79	79/81	81/83	83/85	85/87	87/89	89/91	91/93	93/95	95/97	97/99	99/100	10	10	30	28	05	21	31	

Initials of Teacher
Initials of H.O.D.
Initial of Principal

Principal
Prin. Institute of Engineering & Technology
DHYANES

Class: **III B Section**

Subject Code: **18CV35**

Subject: **Basic Surveying**

Total No. of Classes: **28**

Sl No.	USN	NAME	DATE	10/20	11/20	12/20	13/20	14/20	15/20	16/20	17/20	18/20	19/20	20/20	21/20	22/20	23/20	24/20	25/20	26/20	27/20	28/20	No. of Days Present	%	Test Marks			Average	Remarks		
																									I	II	III				
1	UGBD19CV002	Abhinav K Chitharav	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A							
2	UGBD19CV004	Ajay A Sajjani	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A						
3	UGBD19CV008	AKHAY G M.	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A						
4	UGBD19CV010	AJ AKHAY	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A						
5	UGBD19CV012	ANURAG S.	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A						
6	UGBD19CV014	B Sayed Gamael Zahid	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A						
7	UGBD19CV016	Bharathgouda K.N.	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A						
8	UGBD19CV018	Chandan. B.V.	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A						
9	UGBD19CV022	Deeda P.	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A						
10	UGBD19CV024	Harish K.T.	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A						
11	UGBD19CV028	Karthik D Venkatar	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A						
12	UGBD19CV030	Lakshmi. B.S.	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A						
13	UGBD19CV032	MAHAKANJUN. T.M.	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A						
14	UGBD19CV034	MANOJ SUAMY K.M.	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A						
15	UGBD19CV036	Mohammed Ameerabhan	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A						
16	UGBD19CV038	Mohit B V	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A						
17	UGBD19CV040	MOHITH P RAM	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A						
18	UGBD19CV042	NANDINI. K.	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A						
19	UGBD19CV044	P N Mohammed Saibudhan	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A						
20	UGBD19CV046	POOJA. M. P.	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A						
21	UGBD19CV048	Pradeep K.B.	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A						
22	UGBD19CV050	Pragnaal Q.T. Matad	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A						
23	UGBD19CV052	Prerana. V.	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A						
24	UGBD19CV054	Raghu. Doddaman	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A						
25	UGBD19CV056	Rutera Banu. A.K.	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A						
26	UGBD19CV058	Sanjay. G.A.	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A						
27	UGBD19CV060	Shashi Kiran. E.P.	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A						
28	UGBD19CV062	Siraj Ahmed Y.B.	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A						
29	UGBD19CV064	Sunil N Raktod.	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A						
30	UGBD19CV066	Tegawani. M.	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A						

Initials of Teacher: **[Signature]**
 Initials of H.O.D.: **[Signature]**
 Initial of Principal: **[Signature]**
PRINCIPAL

Class : **C1 B Section**

Subject Code : **18CV35**

Subject : **Basic Surveying**

Total No. of Classes : **36**

Sl No.	USN	NAME	DATE	Test Marks			Average	Remarks
				I	II	III		
1	UGD19CV002	Abhinav K C I	15/10/20	10	24	21	10.17	27
2	UGD19CV004	Ajay A Jain	16/10/20	10	21	9	18	28
3	UGD19CV008	Akshay B M.	17/10/20	10	23	9	15	25
4	UGD19CV010	Alp Akbar	18/10/20	10	25	3	15	25
5	UGD19CV012	Anusha S.	19/10/20	10	21	6	15	25
6	UGD19CV014	B. Sayed Daw	20/10/20	10	23	6	16	26
7	UGD19CV016	Bharath Gowda	21/10/20	10	22	3	15	25
8	UGD19CV018	Chandan B.V	22/10/20	10	20	3	15	25
9	UGD19CV022	Deepa P.	23/10/20	10	22	3	15	25
10	UGD19CV024	Harish K.T.	24/10/20	10	22	3	15	25
11	UGD19CV028	Karthik D Ver	25/10/20	10	22	3	15	25
12	UGD19CV030	Lakshmi B.S	26/10/20	10	22	3	15	25
13	UGD19CV032	Mallikarjun	27/10/20	10	22	3	15	25
14	UGD19CV034	Manoj Swamy	28/10/20	10	22	3	15	25
15	UGD19CV036	Mohamed Ame	29/10/20	10	22	3	15	25
16	UGD19CV038	Mohit B V	30/10/20	10	22	3	15	25
17	UGD19CV040	Namrathya P E	31/10/20	10	22	3	15	25
18	UGD19CV042	Nandini K.	01/11/20	10	22	3	15	25
19	UGD19CV044	P N. Mohammed	02/11/20	10	22	3	15	25
20	UGD19CV046	Pooja M P	03/11/20	10	22	3	15	25
21	UGD19CV048	Pradeep K.B	04/11/20	10	22	3	15	25
22	UGD19CV050	Praveen Q.T	05/11/20	10	22	3	15	25
23	UGD19CV052	Preerna V.	06/11/20	10	22	3	15	25
24	UGD19CV054	Raghav Dadda	07/11/20	10	22	3	15	25
25	UGD19CV056	Rutkaranti Banti	08/11/20	10	22	3	15	25
26	UGD19CV058	Sangeetha E.T	09/11/20	10	22	3	15	25
27	UGD19CV060	Shashik K.	10/11/20	10	22	3	15	25
28	UGD19CV062	Siraj Ahmed	11/11/20	10	22	3	15	25
29	UGD19CV064	Sunil N Reddy	12/11/20	10	22	3	15	25
30	UGD19CV066	Tegawini M.	13/11/20	10	22	3	15	25

Initials of Teacher: **[Signature]**
 Initials of H.O.D.: **[Signature]**
 Initial of Principal: **[Signature]**

Class : 11 B

Subject Code : 18CV35

Sl No	USN	NAME	DATE	1	2	3	4	5	6	7	8	9	10	11	12
61		Maharaja P. D. S. S. S.	15/03/2024	A	A	A	A	A	A	A	A	A	A	A	A
62		Saradha K. S. S. S.	15/03/2024	A	A	A	A	A	A	A	A	A	A	A	A
63		Abhishek. H. S. S.	15/03/2024	A	A	A	A	A	A	A	A	A	A	A	A
64		Vinay K. V. S. S.	15/03/2024	A	A	A	A	A	A	A	A	A	A	A	A
65		Chidambara. H. S. S.	15/03/2024	A	A	A	A	A	A	A	A	A	A	A	A
66		Kalpana. M. S. S.	15/03/2024	A	A	A	A	A	A	A	A	A	A	A	A
67		Bhavana. M. S. S.	15/03/2024	A	A	A	A	A	A	A	A	A	A	A	A
68		Padma. L. S. S.	15/03/2024	A	A	A	A	A	A	A	A	A	A	A	A
69		Muralidharan. S. S.	15/03/2024	A	A	A	A	A	A	A	A	A	A	A	A
70		Mithun Chevan. S. S.	15/03/2024	A	A	A	A	A	A	A	A	A	A	A	A
71		Mohammed. S. S.	15/03/2024	A	A	A	A	A	A	A	A	A	A	A	A
72		Maharaja P. D. S. S.	15/03/2024	A	A	A	A	A	A	A	A	A	A	A	A

Initials of Teacher
Initials of H.O.D.
Initial of Principal

(Handwritten initials)

Class : 11 B

Sl No.	USN	NAME	No. of Days Present	%	Test Marks			Average	Remarks
					I	II	III		
61		Maharaja P. D. S. S.	10	30	23	12	22	32	
62		Saradha K. S. S. S.	10	30	23	12	22	32	
63		Abhishek. H. S. S.	10	30	23	12	22	32	
64		Vinay K. V. S. S.	10	30	23	12	22	32	
65		Chidambara. H. S. S.	10	30	23	12	22	32	
66		Kalpana. M. S. S.	10	30	23	12	22	32	
67		Bhavana. M. S. S.	10	30	23	12	22	32	
68		Padma. L. S. S.	10	30	23	12	22	32	
69		Muralidharan. S. S.	10	30	23	12	22	32	
70		Mithun Chevan. S. S.	10	30	23	12	22	32	
71		Mohammed. S. S.	10	30	23	12	22	32	
72		Maharaja P. D. S. S.	10	30	23	12	22	32	

Total No. of Classes : 56

Initials
Initials
Initial

(Handwritten initials)
PRINCIPAL
DAVANGERE.

Class :

3 A

Subject Code :

18CV32

Subject : Strength of Materials
Total No. of Classes : 94

Sl. No.	USN	NAME	DATE	Attendance																															No. of Days Present	%	Average	Remarks																																																								
				1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31					32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87
1	4B01SCV001		23/06/20	A	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94

Initials of Teacher
Initials of H.O.D.
Initial of Principal

Principal
Davanagere

Class : 9A
 Subject Code : 10CV32

Subject : Strength of Materials

Total No. of Classes : 94

Sl No.	USN	NAME	DATE	Attendance																															No. of Days Present	%	Test Marks			Average	Remarks	
				1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31			32	I	II			III
62		Sandeep. D.M.		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	38						
		Shankar Rao Kulkarni.C.		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	38						
63		Alankarika.H.J.		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	38						
64		Triveni.G.		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	38						
65		Pillvi.O.		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	38						
66		Harek.C.D		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	38						
67		Veen B Bhavramgoudy		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	38						
68		Harek.C.N.		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	38						
69		Harek.C.S.		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	38						
70		Harek.C.S.		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	38						
71		Trupti S. S. Lakshmi		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	38						
72		Deepak.L		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	38						
73		Priyanka Anjum.H.		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	38						

Initials of Teacher
 Initials of H.O.D.
 Initial of Principal

Principal
 Avinash M. S.
 Avinash M. S.
 Avinash M. S.

LESSON PLAN

Subject : _____ Subject Code : _____ Class : _____

Period	Date	Topics Planned	Date	Topics Covered	Remarks
35	03-11-2020	Rankine & Gordon's formulae questions Problems.	03-11-2020	Rankine & Gordon's formulae questions Problems	Covered
36	09-11-2020	problems of Euler's & Gordon's cr.	09-11-2020	Problems of Euler's & Gordon's cr.	Covered
37	11-11-2020	Module: 2 Compound Stress. Introduction	11-11-2020	Module: 2 Compound Stress Introduction	Covered.
38	12-11-2020	General two dimensional stress system Principal stress & planes	12-11-2020	General two dimensional stress system Principal stress & planes	Covered
39	13-11-2020	Problems on principal stress & planes	13-11-2020	Problems on principal stress & planes	Covered.
40	16-11-2020	Problems on principal stress & planes	16-11-2020	Problems on principal stress & planes	Covered
41	15-11-2020	Module: 3 Shear Force & Bending Moment Introduction	15-11-2020	Module: 3 Shear Force & Bending Moment : Introduction	Covered.
42	21-11-2020	Types of Beams, loading supports.	21-11-2020	Types of Beams, loading supports	Covered.
43	23-11-2020	Simply supported beam with point load & eccentric loading with problems	23-11-2020	Simply supported beam with point load & eccentric loading with problems	Covered.
44	24-11-2020	Simply supported beams with UDL with problems	24-11-2020	Simply supported beams with UDL with problems	Covered
45	25-11-2020	Simply supported beams with problems	25-11-2020	Simply supported beams with problems	Covered
46	26-11-2020	Problems on simply supported beam with different loading condition	26-11-2020	Problems on simply supported beam	Covered
47	28-11-2020	Problems ..	28-11-2020	Problems ..	Covered
48	30-11-2020	Overhanging beam Division to, with different loading	30-11-2020	Overhanging beams with different loading condition	Covered
49	11/12/20	Problems	11/12/20	Problems	Covered
50	21/12/20	Problems	21/12/20	Problems	Covered
51	19/12/20	Continuous beam with different loading conditions	19/12/20	Continuous beams with different loading conditions	Covered.

[Signature]

LESSON PLAN

Period	Date	Topics Planned	Date	Topics Covered	Remarks
52	14/12/20	problems	14/12/20	problems	covered
53	15/12/20	Module: 4 Bending stress Introduction	15/12/20	Bending stress ^{modulus} Introduction	covered
54	16/12/20	Derivation an equation for bending stress $m = \frac{E}{r} = \frac{b}{y}$	16/12/20	Derivation an equation for bending stress $m = \frac{E}{r} = \frac{b}{y}$	covered
55	21/12/20	Problem on Bending stress	21/12/20	Problem on Bending stress	covered
56	22/12/20	Problem on Bending stress	22/12/20	Problem on Bending stress	covered
57	23/12/20	Problem on Bending stress	23/12/20	Problem on Bending stress	covered
58	26/12/20	Deflection of Beams Module 5	26/12/20	Deflection of Beams	covered
59	29/12/20	Derivation of Beams. Simply supported Beams with point.	29/12/20	Derivation of Beams simply supported Beams with point	covered
60	29/12/20	UOL UOL Problem	29/12/20	UOL, UOL problem	covered


Text Books :

1. B. S. Badaavarejaiah, P. Mahalingappa. Strength of materials in SI units University of PUNE (India).
2. R. Subramanian. Strength of materials latest edition.

Reference Books :


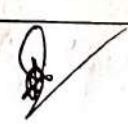
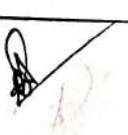
1. R. K. Bauli, A Text book of Strength of materials. 4th edition Laxmi Publication, 2010.
2. O. H. Young. S.P. Timoshenko. "elements of Strength of materials".
3. S. Ramamurtham. Strength of materials.
4. S.S. Bhavikatti. by Strength of materials.
5.


Signature of Faculty


HOD

TUTORIAL CLASSES

Date	Topics Discussed	Remarks
61	Problems on reflection	covered
62	Problem on deflection	covered.
63	Problems on reflection	covered
64	Problems on reflection	covered.
65	Problem on deflection	covered.
66	Problem on deflection	covered.
67	Module 2 (For Diploma) Thick & Thin cylinders Derivation of hoop stress & longitudinal stress	covered
68	Problem on same	covered
69	Problem on Thick cylinders	covered
70	Thick cylinders Derivations	covered
71	Problems on Thick cylinders	covered
72	Problems on thick cylinders	

Test	Date	Class Strength	No. of Students Appeared	No. of Students Scored < 15	Signature of the HOD
T1	24/10/20	70	70	0	
T2	07.12.2020	70	70	0	
T3	22.02.2024	70	67	50	

LESSON PLAN

Subject: Basic surveying Subject Code: L8CV35 Class: Bred B

Period	Date	Topics Planned	Date	Topics Covered	Remarks
1	10/9/20	Module 1. Definition of surveying objectives & Importance	10/9/20	Module 1. Definition of surveying objectives & importance	covered
2	14/9/20	Classification of surveying.	14/9/20	Classification of surveying	covered
3	15/9/20	Principles of surveying	15/9/20	Principles of surveying	covered
4	16/9/20	Applications of surveying unit & measurement	16/9/20	Applications of surveying unit & measurement	covered
5	21/9/20	Surveying measurement errors, types of errors precision	21/9/20	Surveying measurement errors, types of errors, precision	covered
6	23/9/20	Accuracy, classification of maps, map & numbering	23/9/20	Accuracy, classification of maps, map & numbering	covered
7	28/9/20	Measurement of horizontal distances, measuring tape & types, measurement using tape, Taping on	28/9/20	Measurement of horizontal distances, measuring tape measurement using tape,	covered
8	30/9/20	Level ground. + Direct ranging - Indirect ranging	30/9/20	Level ground. + Direct ranging + Indirect ranging.	covered
9	1/10/20	Sloping ground Direct & Indirect ranging	1/10/20	Sloping ground by direct & indirect ranging	covered
10	05/10/20	EDM, basic principles of use of tape survey work	05/10/20	EDM, basic principles of use of tape survey work	covered
11	07/10/20	conventional symbols, obstacles in tape surveying	07/10/20	conventional symbols, obstacles in tape surveying	covered
12	08/10/20	Problems	08/10/20	Problems	covered
13	12/10/20	Module 4 Plane table surveying accessories	12/10/20	Module 4 Plane table surveying accessories	covered
14	14/10/20	Setting up the plane table surveying	14/10/20	Setting up the plane table surveying	covered
15	15/10/20	Methods of plane table surveying Resection	15/10/20	Methods of plane table surveying Resection	covered
16	21/10/20	Intersection method, Traversing Advantages & Disadvantages of Plane table surveying	21/10/20	Intersection method, Traversing Advantages & Disadvantages of Plane table surveying	covered
17	22/10/20	Resection two point Problem, three point Problem	22/10/20	Resection - two point Problem & Three point Problem	covered

LESSON PLAN

Subject: Basic surveying Subject Code: 18CV35 Class: 3rd B

Period	Date	Topics Planned	Date	Topics Covered	Remarks
18	28.10.20	Three Point Problem	28.10.20	Three Point Problem	Covered
19	29.10.20	Module 1.5: Area & Volume Different Methods	29.10.20	Module 1.5: Area & Volume Introduction	Covered
20	30.10.20	Objects of Regular Intervals	30.10.20	Objects of Regular Intervals	Covered
21	01.11.20	Areas from coordinates with problems	01.11.20	Areas from coordinates with problems	Covered
22	02.11.20	Problem on Areas by Different methods	02.11.20	Problem on Areas by Different methods	Covered
23	03.11.20	Volume: Different Methods Problem on volume	03.11.20	Volume: Different Methods & Problem on volume	Covered
24	04.11.20	Problem on volume	04.11.20	Problem on volume	Covered
25	05.11.20	Module 2 Compass Surveying Introduction	05.11.20	Module 2 Compass Surveying Introduction	Covered
26	06.11.20	Definition of Bowditch's Rule, WCB, AB Problem	06.11.20	Definition of Bowditch's Rule, WCB, AB Problem	Covered
27	07.11.20	Magnetic Dip & Declination with problems Prismatic compass, Surveyor's compass with construction	07.11.20	Magnetic Dip & Declination with problems, Prismatic compass, Surveyor's compass with construction	Covered
28	08.11.20	Problem on Interior angles	08.11.20	Problem on Interior angles (Included angle)	Covered
29	09.11.20	Local attraction, Introduction Problem	09.11.20	Local attraction Problem	Covered
30	10.11.20	Problem on local attraction	10.11.20	Problem on local attraction	Covered
31	11.11.20	Private: computer Introduction	11.11.20	Private: computer	Covered
32	12.11.20	1. Dependent coordinates 2. Independent coordinates 3. Closing error with problems	12.11.20	1. Dependent coordinates 2. Independent coordinates 3. Closing error with problems	Covered
33	13.11.20	1. Bowditch's rule 2. Transit rule with problems	13.11.20	Problems	Covered
34	14.11.20	Omitted measurements Problem	14.11.20	Omitted measurements with problems	Covered

LESSON PLAN

Subject: Basic Surveying Subject Code: 18CV35 Class: 2nd B

Period	Date	Topics Planned	Date	Topics Covered	Remarks
35	30-12-20	Module: 3 Leveling: Basic definitions methods of leveling.	30-12-20	Module: 3 Leveling: Basic definitions methods of leveling	Covered
36	31-12-20	Dumpy level, Auto level, digital leveling & laser leveling	31-12-20	Dumpy level, Auto level, digital leveling & laser leveling	Covered
37	4-1-21	Curvature and refraction corrections,	4-1-21	Curvature & refraction corrections	Covered
38	5-1-21	Bearing & reduction of wells.	5-1-21	Bearing & reduction of wells & Problems	Covered
39	7-1-21	Problems on leveling	7-1-21	Problems on leveling	Covered
40	13-1-21	Problems on leveling Differential leveling	13-1-21	Problems on differential leveling	Covered
41	13-1-21	Problems on differential leveling	17-1-21	Problems on differential leveling	Covered
42	21-1-21	Profile leveling with problems	21-1-21	Profile leveling with problems	Covered
43	01-2-21	fly leveling with problems	01-2-21	fly leveling with problems.	Covered
44	3-2-2021	Reciprocal leveling	3-2-2021	Reciprocal leveling	Covered
45	4-2-2021	Problems on reciprocal leveling	4-2-2021	Problems on reciprocal leveling	Covered
46	10-2-2021	Module: 4 Plane table surveying accessories, setting up plane surveying	10-2-2021	Module: 4 Plane table surveying accessories setting up plane table surveying	Covered
47	11-2-2021	Method of plane table surveying	11-2-2021	Methods of plane table surveying	Covered
48	15-2-2021	Advantages & Disadvantages of plane table surveying Resection of two-point problems, three-point problems	15-2-2021	Advantages & Disadvantages of plane table surveying Resection of two-point three point problems	Covered
49	17-2-2021	Three point Problem	17-2-2021	Three point Problem	Covered
50	17-2-2021	Module: 5 Area & Volume Area - Methods of Area Calculation, offset at regular intervals.	17-2-2021	Module: 5 Area & Volume Area - methods of area calculation, offset at regular intervals	Covered
51	18-2-2021	Area from co-ordinates with problems, by different method, Volume: Different method	18-2-2021	Area from co-ordinates with problems, by different method, Volume: Different method.	Covered

LESSON PLAN

Period	Date	Topics Planned	Date	Topics Covered	Remarks


Text Books :

1. B. C. Punmia, "Surveying Vol-1". Lakmi Publications Pvt. Limited New Delhi - 2009.
2. Kanetkar T. P. & S. N. Kulkarni. Surveying & Levelling.

Reference Books :




1. S. K. Duggal. Surveying Volume 1.
2. K. R. Arora. Surveying Volume 1.
3. R. Subramanian. Surveying & Levelling.
4.
5.


Signature of Faculty


HOD

TUTORIAL CLASSES

Date	Topics Discussed	Remarks

Test	Date	Class Strength	No. of Students Appeared	No. of Students Scored < 15	Signature of the HOD
T1	23.10.2020	67	67	00	
T2	09.12.2020	67	67	00	
T3	23.02.2021	67	67	00	

TIME TABLE

RACHU MTE

Day	Time	1	2	SHORT BREAK		3	4	LUNCH BREAK		5	6	7
		8:00-9:00	9:00-10:00	10:00-10:30	10:30-11:30	11:30-12:30	12:30-2:00	2:00-3:00	3:00-4:00	4:00-5:00		
MONDAY		18CV32-A								18CV35-B		18CV35-T (B)
TUESDAY			18CV32-A									
WEDNESDAY		18CV35-B					18CV32-B			18CV35-T (B)	18CV35-T (B)	
THURSDAY			18CV35-B			18CV32-A				18CV35-T (B)	18CV35-T (B)	
FRIDAY			18CV35-B (M E N L E O A)			18CV38-A						
SATURDAY			18CV32-A									

Rachum
Sign. of the Staff

Rachum
Sign. of the HOD

Rachum
PRINCIPAL
Apul Institute of Engineering & Technology
DAVANGERE.

3rd P - Section
= Strength of Materials
3rd A - Section
= Basic Surveying
18CV35-T (B)
= Surveying Practice
18CV35-B1 & A3
- Surveying Practice.
18CV38-A1
Building materials Lab



Course File Check List

1. Contents
 2. Academic calendar of VTU, Institute and Department
 3. Vision, Mission statements of Institute
 4. Vision, Mission, PEOs, POs, PSOs statements of Department
 5. Individual time table
 6. Syllabus
 7. Course Articulation Matrix [CO-PO, CO-PSO mapping]
 8. Lesson plan
 9. Text books / Reference books referred
 10. Attendance register
 11. Course material
 - a) Notes
 - b) PPT
 - c) NPTEL / Youtube Videos
 12. Additional topics taken to meet the POs.
 - a) Site visits
 - b) Technical talks
 - c) Quiz
 - d) Group discussion
 - e) Blended learning
 - f) Model making competition
 - g) Modern tool (Computing tool) usage
 13. Exam question papers
 14. Test and Assignment question papers (with scheme of evaluation)
 15. Result analysis
 - a) Percentage CO covered / Percentage of CO addressed.
 - b) CO-PO and CO-PSO Attainment
 - c) Percentage of students passed
 16. Counselling report (Actions taken to improve Weak students / Slow learners)
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2020-21 Odd Sem

Tentative Academic Calendar of VTU, Belagavi for ODD Semester of 2020-2021

	I Sem B. E. / B. Tech. / B. Arch. / B. Plan	I sem M. Tech. / MBA / MCA / M. Arch.	III, V & VII Sem B. E. / B. Tech. / B. Plan / B. Arch. & IX Sem B. Arch.	III & V Sem MCA	III Sem MBA	III Sem M, Tech.	III Sem M. Arch.
Commencement of ODD Semester			01.09.2020	01.09.2020	01.09.2020	01.09.2020	01.09.2020
Last Working day of ODD Semester			17.12.2020	17.12.2020	17.12.2020	17.12.2020	17.12.2020
Practical Examinations			21.12.2020 To 31.12.2020	21.12.2020 To 31.12.2020	21.12.2020 To 31.12.2020	21.12.2020 To 31.12.2020	21.12.2020 To 31.12.2020
Theory Examinations			04.01.2021 To 23.01.2021	04.01.2021 To 23.01.2021	04.01.2021 To 23.01.2021	04.01.2021 To 23.01.2021	04.01.2021 To 23.01.2021
Internship Viva-Voce							
Professional training / Organization study							
Commencement of EVEN Semester			08.02.2021	08.02.2021	08.02.2021	22.02.2021	08.02.2021

Will be announced later

Will be announced later

NOTE

- VII Semester B. E / B. Tech students shall have to undergo INTERNSHIP as per circular of University VTU/Aca/2019-20/85, dated 12.05.2020.
- I Semester B. E / B. Tech / B. Arch Students shall compulsorily undergo Induction Program for a period of 3 Weeks as per the schedule given by VTU Belagavi
- The classroom sessions for all the higher semesters would be commencing from 01.09.2020(Tentative) in ONLINE mode until further orders.
- The Institute needs to function for six days a week with additional hours.
- The faculty/staff shall be available to undertake any work assigned by the university.
- If any of the above date is declared to be a holiday then the corresponding event will come into effect on the next working day.
- Notification regarding Calendar of Events relating to the conduct of University Examinations will be issued by the Registrar (Evaluation) from time to time.
- Academic Calendar may be modified based on guidelines/directions issued in future by MHRD/UGC/AICTE/State Government.


 REGISTRAR

Bapuji Institute of Engineering and Technology, Davangere-577004
CALENDER OF EVENTS - ODD SEMESTER: SEPTEMBER-JANUARY-2020-21 (Tentative)

PARTICULARS	I sem BE/B Tech	III, V BE/B Tech	VII sem BE/B Tech	III & V sem MCA	III sem MBA	III sem M.Tech
Commencement of ODD Sem	14-12-2020	01-09-2020	01-09-2020	01-09-2020	01-09-2020	01-09-2020
Last Working Day	25-03-2021	16-01-2021	16-01-2021	16-01-2021	16-01-2021	16-01-2021
1 st CIE Series	-----	19-10-2020 To 24-10-2020	19-10-2020 To 24-10-2020	19-10-2020 To 24-10-2020	15-10-2020 To 17-10-2020	19-10-2020 To 24-10-2020
2 nd CIE Series	-----	07-12-2020 To 09-12-2020	07-12-2020 To 09-12-2020	07-12-2020 To 09-12-2020	26-11-2020 To 28-11-2020	07-12-2020 To 09-12-2020
3 rd CIE Series	-----	11-01-2021 To 13-01-2021	11-01-2021 To 13-01-2021	11-01-2021 To 13-01-2021	7-01-2021 To 9-01-2021	11-01-2021 To 13-01-2021
Practical Examination	29-03-2021 Onwards #	21-01-2021 Onwards #	21-01-2021 Onwards #	08-02-2021 Onwards #	-----	21-01-2021 Onwards #
Theory Examination	12-04-2021 To 30-04-2021	08-02-2021 To 27-03-2021	08-02-2021 To 27-03-2021	21-01-2021 To 06-02-2021	21-01-2021 To 19-02-2021	28-01-2021 To 13-02-2021
Internship	-----	-----	29-03-2021 To 10-04-2021	-----	-----	-----
Internship Viva-Voce	-----	-----	-----	-----	-----	15-02-2021 To 22-02-2021
Professional Training/Organization Study	-----	-----	-----	-----	22-02-2021 To 03-04-2021	-----
Commencement of Even Semester	03-05-2021	29-03-2021	12-04-2021	15-02-2021	05-04-2021	23-02-2021

Notification regarding the calendar of events relating to the conduct of University Examination will be issued by the Registrar (Evaluation) from time to time.

Dear Academic

Principal



Vision of BIET

To be a center of excellence recognized nationally and internationally, in distinctive areas of engineering education and research, based on a culture of innovation and invention.

Mission of BIET

BIET contributes to the growth and development of its students by imparting a broad based engineering education and empowering them to be successful in their chosen field by inculcating in them positive approach, leadership qualities and ethical values



VISION OF THE DEPARTMENT

To train the students to become Civil Engineers with leadership qualities, having ability to take up professional assignments and research with a focus on innovative approaches to cater to the needs of the society.

MISSION OF THE DEPARTMENT

1. To provide quality education through updated curriculum and conducive teaching learning environment for the students to excel in higher studies, competitive examinations and professional career.
2. To impart soft skills, leadership qualities and professional ethics among the graduates to handle the projects independently with confidence.
3. To deal with the contemporary issues and to cater to the socio-economic needs.
4. To build industry-institute interaction and to establish good rapport with alumni.

PROGRAM EDUCATIONAL OBJECTIVES (PEOs)

PEO 1: Core Competence: Graduates will be able to plan, analyse, design and construct sustainable Civil Engineering Infrastructure.

PEO 2: Professional Skills: Graduates will be professional engineers with a sense of ethics, creativity, leadership, self-confidence and independent thinking to cater to the needs of the society.

PEO 3: Societal Needs: Graduates will be able to contribute effectively for the development of industry and professional bodies.

PEO 4: Cognitive Intelligence: Graduates will be able to take up competitive examinations, higher studies and involve in research and entrepreneurship activities.

PROGRAM SPECIFIC OUTCOMES (PSOs)

Students after the completion of the Program will be able to

1. Apply the fundamental concepts, software and codal provisions in the analysis, design and construction of sustainable civil engineering infrastructure.
2. Inculcate professional and leadership qualities, sense of ethics and confidence related to civil engineering.

Faculty will be able to

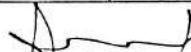
3. Contribute to the overall development of civil engineering community through the professional bodies and offer services to the society.
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3D - CV301
 3B - CV305

Name of the Faculty : MER							
Time / Day	8 - 9	9 - 10	10.30 - 11.30	11.30 - 12.30	2 - 3	3 - 4	4 - 5
Mon	18CV32 - A				18CV35 - B		18CVL57 - T (B)
Tue		18CV32 - A					
Wed	18CV35 - B			18CV32 - A	18CVL57 - A3 (MER + SH)		
Thu		18CV35 - B	18CV32 - A		18CVL57 - B1 (MER + GNS)		
Fri		18CVL38 - A1 (MER + CPA)					
Sat		18CV32 - A					


 Time Table Coordinator


 HOD


 Principal

B. E. CIVIL ENGINEERING
Choice Based Credit System (CBCS) and Outcome Based Education (OBE)
SEMESTER - III

STRENGTH OF MATERIALS

Course Code	18CV32	CIE Marks	40
Teaching Hours/Week (L:T:P)	(3:2:0)	SEE Marks	60
Credits	04	Exam Hours	03

Course Learning Objectives: This course will enable students

1. To understand the basic concepts of the stresses and strains for different materials and strength of structural elements.
2. To know the development of internal forces and resistance mechanism for one dimensional and two-dimensional structural elements.
3. To analyse and understand different internal forces and stresses induced due to representative loads on structural elements.
4. To determine slope and deflections of beams.
5. To evaluate the behaviour of torsion members, columns and struts.

Module-1

Simple Stresses and Strain: Introduction, Definition and concept and of stress and strain. Hooke's law, Stress-Strain diagrams for ferrous and non-ferrous materials, factor of safety, Elongation of tapering bars of circular and rectangular cross sections, Elongation due to self-weight. Saint Venant's principle, Compound bars, Temperature stresses, Compound section subjected to temperature stresses, state of simple shear, Elastic constants and their relationship.

Module-2

Compound Stresses: Introduction, state of stress at a point, General two dimensional stress system, Principal stresses and principal planes. Mohr's circle of stresses. Theory of failures: Max. Shear stress theory and Max. principal stress theory.

Thin and Thick Cylinders: Introduction, Thin cylinders subjected to internal pressure; Hoop stresses, Longitudinal stress and change in volume. Thick cylinders subjected to both internal and external pressure; Lame's equation, radial and hoop stress distribution.

Module-3

Shear Force and Bending Moment in Beams: Introduction to types of beams, supports and loadings. Definition of bending moment and shear force, Sign conventions, relationship between load intensity, bending moment and shear force. Shear force and bending moment diagrams for statically determinate beams subjected to point load, uniformly distributed loads, uniformly varying loads, couple and their combinations.

Module-4

Bending and Shear Stresses in Beams: Introduction, pure bending theory, Assumptions, derivation of bending equation, modulus of rupture, section modulus, flexural rigidity. Expression for transverse shear stress in beams, Bending and shear stress distribution diagrams for circular, rectangular, 'I', and 'T' sections. Shear centre (only concept).

Torsion in Circular Shaft: Introduction, pure torsion, Assumptions, derivation of torsion equation for circular shafts, torsional rigidity and polar modulus Power transmitted by a shaft.

Module-5

Deflection of Beams: Definition of slope, Deflection and curvature, Sign conventions, Derivation of moment-curvature equation. Double integration method and Macaulay's method: Slope and deflection for standard loading cases and for determinate prismatic beams subjected to point loads, UDL, UVL and couple.

Columns and Struts: Introduction, short and long columns. Euler's theory; Assumptions, Derivation for Euler's Buckling load for different end conditions, Limitations of Euler's theory. Rankine-Gordon's formula for columns.

Course outcomes: After studying this course, students will be able;

1. To evaluate the basic concepts of the stresses and strains for different materials and strength of structural elements.
2. To evaluate the development of internal forces and resistance mechanism for one dimensional and two dimensional structural elements.
3. To analyse different internal forces and stresses induced due to representative loads on structural elements.
4. To evaluate slope and deflections of beams.
5. To evaluate the behaviour of torsion members, columns and struts.

Question paper pattern:

- The question paper will have ten full questions carrying equal marks.
- Each full question will be for 20 marks.
- There will be two full questions (with a maximum of four sub- questions) from each module.
- Each full question will have sub- question covering all the topics under a module.
- The students will have to answer five full questions, selecting one full question from each module.

Textbooks:

1. B.S. Basavarajiah, P. Mahadevappa "Strength of Materials" in SI Units, University Press (India) Pvt. Ltd., 3rd Edition, 2010
2. Ferdinand P. Beer, E. Russell Johnston and Jr. John T. De Wolf "Mechanics of Materials", Tata McGraw-Hill, Third Edition, SI Units

Reference Books:

1. D.H. Young, S.P. Timoshenko "Elements of Strength of Materials" East West Press Pvt. Ltd., 5th Edition (Reprint 2014).
2. R K Bansal, "A Textbook of Strength of Materials", 4th Edition, Laxmi Publications, 2010.
3. S.S. Rattan "Strength of Materials" McGraw Hill Education (India) Pvt. Ltd., 2nd Edition (Sixth reprint 2013).
4. Vazirani, V N, Ratwani M M. and S K Duggal "Analysis of Structures Vol. I", 17th Edition, Khanna Publishers, New Delhi.

5. S. Ramamurtam
6. R. Subramanian
7. S.S. Bhavikatti

Title & Code	Strength of Materials (18CV32)
CO	Statement
18CV32.1	Evaluate the stresses and strains for ferrous and non-ferrous materials
18CV32.2	Evaluate the internal stresses developed in one dimensional, two dimensional structural elements and cylinders
18CV32.3	Evaluate the bending moment, shear force in prismatic beams and corresponding stresses
18CV32.4	Analyse and design the circular shafts subjected to torsion
18CV32.5	Evaluate the slope and deflection of prismatic beams
18CV32.6	Evaluate the failure loads for the columns and struts

Course Title		Strength of Materials										
CO	PO1	PO2	PO3	PO4	PO5	PO6	PO7	PO8	PO9	PO10	PO11	PO12
18CV32.1	2	2	-	1	-	-	-	-	-	-	-	2
18CV32.2	2	2	-	1	-	-	-	-	-	-	-	2
18CV32.3	2	2	-	1	-	-	-	-	-	-	-	2
18CV32.4	2	2	-	1	-	-	-	-	-	-	-	2
18CV32.5	2	2	-	1	-	-	-	-	-	-	-	2
18CV32.6	2	2	-	1	-	-	-	-	-	-	-	2
Average	2	2		1								2

CO	PSO1	PSO2
18CV32.1	2	2
18CV32.2	2	2
18CV32.3	2	2
18CV32.4	2	2
18CV32.5	2	2
18CV32.6	2	2
Average	2	2

CBCS SCHEME

18CV32

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Third Semester B.E. Degree Examination, Jan./Feb. 2021 Strength of Materials

Time: 3 hrs.

Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

- 1 a. Explain longitudinal strain and lateral strain. (04 Marks)
b. State and illustrate Saint Venant's principle. (06 Marks)
c. A tension test was conducted on mild steel bar and the following data was obtained from the test:
Diameter of the bar = 18mm
Gauge length of the bar = 82mm
Load at proportional limit = 75KN
Extension at a load of 62KN = 0.113mm
Load at failure = 82KN
Final gauge length of the bar = 106mm
Diameter of the bar at failure = 14mm
Determine the Young's modulus, proportional limit, true breaking stress, %elongation and percentage reduction in cross sectional area. (10 Marks)

OR

- 2 a. What are the elastic constants and explain them briefly. (06 Marks)
b. Obtain expression for temperature stress in a bar of uniform cross section when expansion or contraction is prevented partially. (04 Marks)
c. A weight of 390KN is supported by a short column of 250mm square in section. The column is reinforced with 8 steel bars of cross sectional area 2500mm². Find the stresses in steel and concrete if $E_s = 15E_c$.
If stress in concrete must not exceed 4.5MN/m², what area of steel is required in order that column may support a load of 480KN. (10 Marks)

Module-2

- 3 a. Derive Lamé's equation for the radial and hoop stress for thick cylinder subjected to internal and external fluid pressure. (08 Marks)
b. A 2-dimensional element has the tensile stresses of 600MN/m² and compressive stress of 400MN/m² acting on two mutually perpendicular planes and two equal shear stresses of 200MN/m² on their planes. Determine
i) Resultant stress on a plane inclined at 30° wrt x-axis.
ii) The magnitude and direction of principal stresses.
iii) Magnitude and direction of maximum shear stress. (12 Marks)

OR

- 4 a. Obtain expression for volumetric strain in thin cylinder subjected to internal pressure in the form of $e_v = \frac{pd}{2tE} \left[\frac{5}{2} - \frac{2}{m} \right]$. (08 Marks)
b. A cast iron pipe has 200mm internal diameter and 50mm metal thickness and carries water under a pressure of 5N/mm². Calculate the maximum and minimum intensities of circumferential stresses and sketch the distribution of circumferential stress intensity and the intensity of radial pressure across the section. (12 Marks)

Module-3

- 5 a. Define shear force, bending moment and point of contraflexure. Explain how to calculate them? (06 Marks)
 b. Develop shear force diagram and bending moment diagrams for the beam loaded shown in Fig. Q5(b) marking the values at salient points. Determine the position and magnitude of maximum bending moment.

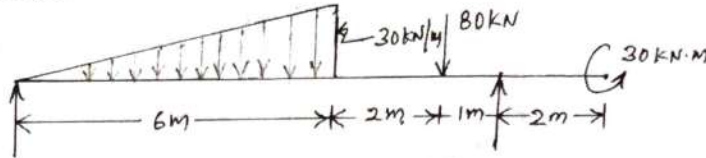


Fig. Q5(b)

(14 Marks)

OR

- 6 a. Obtain the relationship between udl, shear force and bending moment. (06 Marks)
 b. Construct SFD and BMD for the beam loaded shown in Fig. Q6(b). Also locate the point of contraflexure.

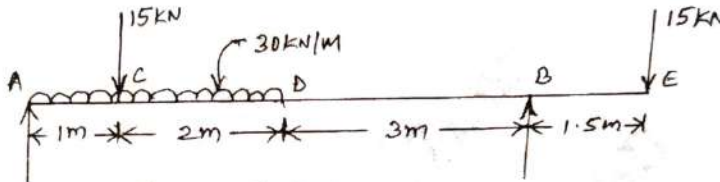


Fig. Q6(b)

(14 Marks)

Module-4

- 7 a. Derive torsional equation with usual notations. (06 Marks)
 b. A T-section of flange 120mm x 12mm and overall depth 200mm with 12mm web thickness is loaded such that at a section it has a bending moment of 20 kN.m and shear force of 120 kN. Sketch the bending and shear stress distribution diagram marking the salient values. (14 Marks)

OR

- 8 a. Derive Bernoulli-Euler bending equation with usual notations. (08 Marks)
 b. A solid circular shaft has to transmit power of 1000 kW at 120 rpm. Find the diameter of the shaft if the shear stress of the material is not to exceed 80 N/mm². The maximum torque is 1.25 times the mean torque. What percentage saving in material could be obtained if the shaft is replaced by a hollow one whose internal diameter is 0.6 times the external diameter? The length of the shaft, material and maximum shear stress being same. (12 Marks)

Module-5

- 9 a. Define slope, deflection and elastic curve. Explain Macaulay's method of determining slope and deflection. (10 Marks)
 b. Compare the crippling loads given by Euler's and Rankine's formula for a tubular steel column 2.5m long having outer and inner diameter as 40mm and 30mm respectively. The column is loaded through pin joints at the ends. Take permissible compressive stress as 320 N/mm², Rankine constant as $\frac{1}{7500}$ and $E=210$ GPa. For what length of the column of their cross section, does the Euler's formula cease to apply? (10 Marks)

OR

- 10 a. Differentiate between short and long column and what are the limitations of Euler's theory. (06 Marks)
- b. Calculate slope at A and deflection at D for the overhanging beam shown in Fig. Q10(b). Take $E = 200\text{GPa}$ and $I = 50 \times 10^6 \text{mm}^4$. (14 Marks)

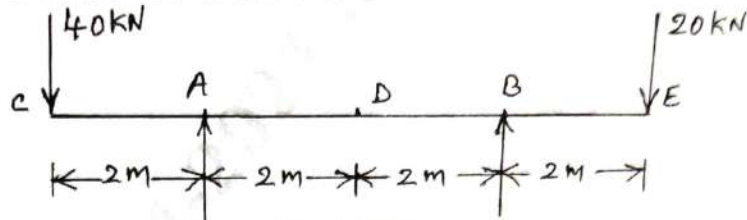


Fig. Q10(b).

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Third Semester B.E. Degree Examination, Jan./Feb. 2021 Strength of Materials

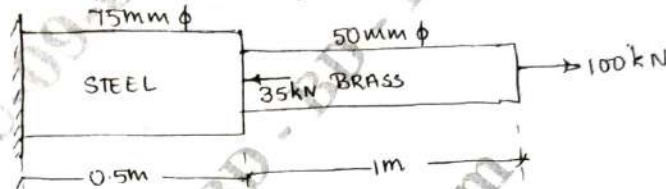
Time: 3 hrs.

Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

- 1 a. Define: (i) Young's modulus (ii) Bulk modulus (iii) Poisson's ratio. Derive a relationship between them. (10 Marks)
- b. Two solid cylindrical rods are connected and loaded as shown in Fig.Q1(b). Determine:
 - (i) Total deformation
 - (ii) Deformation at point B. $E_s = 200 \text{ GPa}$, $E_b = 100 \text{ GPa}$.



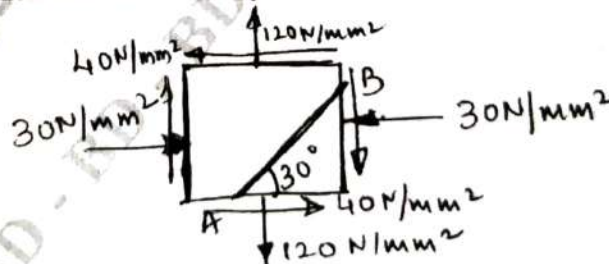
(10 Marks)

OR

- 2 a. A compound bar made of steel plate 60 mm wide and 10 mm thick to which a copper plate 60 mm wide and 5 mm thick are rigidly connected to each other. The length of the bar is 0.7 m. If the temperature is raised by 80°C . Determine the stress in each metal and the change in length. (12 Marks)
 $E_s = 200 \text{ GPa}$, $\alpha_s = 12 \times 10^{-6}/^\circ\text{C}$; $E_{cu} = 100 \text{ GPa}$, $\alpha_{cu} = 17 \times 10^{-6}/^\circ\text{C}$
- b. Derive an expression for extension of the bar due to its self weight only having area 'A' and length L suspended from its top. (04 Marks)
- c. Write a note on thermal stresses. (04 Marks)

Module-2

- 3 a. At a certain point in a strained material the stress condition shown in Fig.Q3(a) exists. Find:
 - (i) The normal and shear stress on the inclined plane AB
 - (ii) Principal stresses and principal planes
 - (iii) Maximum shear stresses and their planes



(12 Marks)

- b. Derive an expressions for volumetric strain in case of a thin cylindrical shell of diameter 'd' subjected to internal pressure 'p'. (05 Marks)
- c. Define: (i) Principal stresses (03 Marks)

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg. 42+8 = 50, will be treated as malpractice.

OR

- 4 a. A cylindrical shell is 3m long 1m internal diameter and is subjected to an internal pressure of 1 N/mm^2 . If thickness of the shell is 12mm, find the circumferential stress and the longitudinal stress. Also find maximum shear stress and the changes in the dimensions of the shell. Take $E = 200 \text{ kN/mm}^2$ and $\mu = 0.3$. (10 Marks)
- b. A thick metallic cylindrical shell of 150 mm, internal diameter is required to withstand an internal pressure of 8 MPa. Find the necessary thickness of cylinder, if permissible stress of the section is 20 MPa. (10 Marks)

Module-3

- 5 a. Derive relation between shear force, bending moment and load. (06 Marks)
- b. Calculate SF and BM at salient points and draw SFD and BMD for the beam shown in Fig.Q5(b).

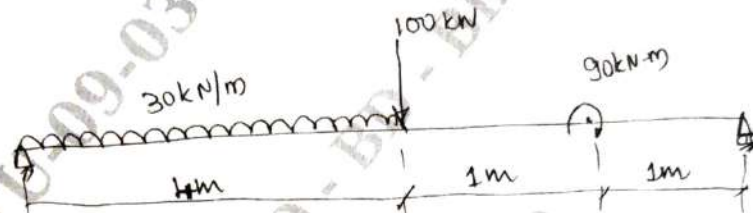


Fig.Q5(b)

(14 Marks)

OR

- 6 a. Define: (i) Bending moment (ii) Shear force (04 Marks)
- b. Draw SFD and BMD for beam shown in Fig.Q6(b).

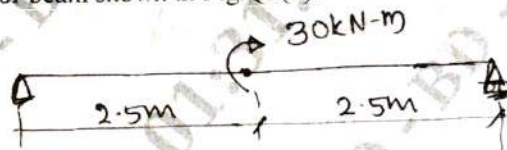


Fig.Q6(b)

(06 Marks)

- c. Draw SFD and BMD for simply supported beam of length L with point load ' P ' placed at a distance ' a ' from right support and ' b ' from left support. (10 Marks)

Module-4

- 7 a. Define: (i) Torsional strength (ii) Torsional stiffness (iii) Torsional rigidity (06 Marks)
- b. A shaft transmits 300 KW power at 120 rpm. Determine:
 (i) The necessary diameter of solid circular shaft.
 (ii) The necessary outer diameter of hollow circular section such that the inner diameter being $2/3$ of the outer diameter. Take allowable shear stress as 70 N/mm^2 . (14 Marks)

OR

- 8 Write short notes on any four:
 a. Maximum principal stress theory
 b. Maximum shear stress theory
 c. Maximum principal strain theory
 d. Maximum strain energy theory
 e. Maximum shear strain energy theory

(20 Marks)

Module-5

- 9 a. Show that for a rectangular cross section maximum shear stress is 1.5 times average shear stress. (06 Marks)
- b. A simply supported beam of span 6 m has a cross section as shown in Fig.Q9(b). It carries 2 point loads each of 30 kN at a distance of 2m from each support. Calculate the bending stress and shear stress for maximum values of bending moment and shear force respectively. Draw neat diagram of bending stress and shear stress distribution across the cross section.

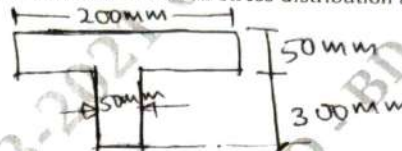


Fig.Q9(b)

(14 Marks)

OR

- 10 a. Derive an expression for Euler's buckling load for long column with one end fixed and other end free. (08 Marks)
- b. The cross section of a column is a hollow rectangular section with its external dimensions 200 mm \times 150 mm. The internal dimension are 150 \times 100 mm. The column is 5m long and fixed at both ends. If $E = 120$ GPa, calculate the critical load using Euler's formula. Compare the above load with the value obtained from Rankine's formula. The permissible compressive stress is 500 N/mm². The Rankine's constant is 1/6000. (12 Marks)

CBCS SCHEME

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18CV32

Third Semester B.E. Degree Examination, Aug./Sept.2020 Strength of Materials

Time: 3 hrs.

Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

- 1 a. Sketch a typical stress-strain curve for a ductile material and explain briefly the salient features of the curve. (05 Marks)
- b. Derive an expression for the deformation of a rectangular tapering bar of uniform thickness. (05 Marks)
- c. Determine the value of P that will not exceed a maximum deformation of 2mm or a stress of 120 MPa in steel, 80 MPa in Aluminium and 115 MPa in bronze (Fig.Q1(c)). Given the following data:
 $A_b = 600 \text{ mm}^2$, $E_b = 0.84 \times 10^5 \text{ N/mm}^2$
 $A_a = 800 \text{ mm}^2$, $E_a = 0.7 \times 10^5 \text{ N/mm}^2$
 $A_s = 400 \text{ mm}^2$, $E_s = 2.1 \times 10^5 \text{ N/mm}^2$

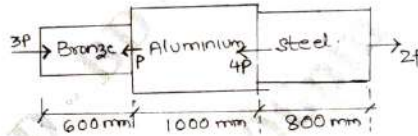


Fig.Q1(c)

(10 Marks)

OR

- 2 a. Derive the relationship between Young's modulus and bulk modulus. (05 Marks)
- b. A load of 270 kN is acting on a RCC column of size 200mm × 200mm. The column is reinforced with 10 bars of 12mm diameter each. Determine the stress in steel and concrete. $E_s = 16.5 E_c$. (05 Marks)
- c. A bar of brass 25mm diameter is enclosed in a steel tube of 50mm external diameter and 25mm internal diameter. The bar and tube are both initially 1m long and rigidly fastened at both the ends. Find the stresses in the two materials when the temperature rises from 10°C to 90°C.
 If the composite bar is then subjected to an axial tensile load of 60 kN, find the resulting stresses given that : $E_s = 200 \times 10^3 \text{ MPa}$, $E_b = 100 \times 10^3 \text{ MPa}$, $\alpha_s = 11.6 \times 10^{-6}/^\circ\text{C}$, $\alpha_b = 18.7 \times 10^{-6}/^\circ\text{C}$. (10 Marks)

Module-2

- 3 a. Explain the maximum shear stress theory. (05 Marks)
- b. Explain the procedure for determining stresses in a general two dimensional stress system using Mohr's circle. (05 Marks)
- c. At a point in a strained material, the state of stresses is as shown in Fig.Q3(c). Determine the principal stresses, maximum shear stress and sketch the orientation of the principal planes.

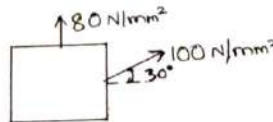


Fig.Q3(c)

(10 Marks)

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg. 42+8 = 50, will be treated as malpractice.

OR

- 4 a. In a thin cylinder, show that the hoop stress is twice the longitudinal stress. (08 Marks)
 b. The maximum stress permitted in a thick cylinder of internal diameter 100mm and external diameter 150mm is 16 N/mm^2 . If the internal pressure is 12 N/mm^2 , what external pressure can be applied? Plot curves showing the variation of Hoop stress and radial stress through the material. (12 Marks)

Module-3

- 5 a. Define the terms: (i) Bending Moment (ii) Point of Inflexion. (04 Marks)
 b. Draw SFD and BMD for the cantilever beam shown in Fig.Q5(b).

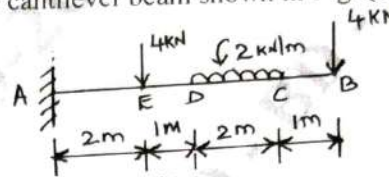


Fig.Q5(b)

- c. Draw SFD and BMD for a simply supported beam carrying two point loads of 12 kN at $1/3^{\text{rd}}$ span from either supports in addition to a UDL of 10 kN/m throughout span of beam is 6m. (10 Marks)

OR

- 6 a. Establish the relationship between shear force, bending moment and load intensity. (06 Marks)
 b. Draw SFD and BMD for the beam shown in Fig.Q6(b). Locate maximum shear force maximum bending moment and point of contraflexure.

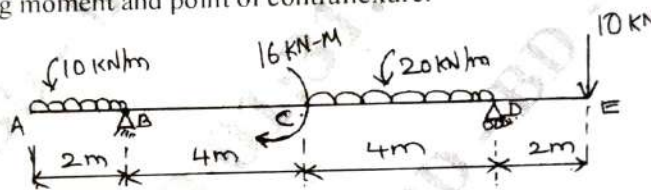


Fig.Q6(b)

(14 Marks)

Module-4

- 7 a. Derive the simple bending equation in the form $\frac{M}{I} = \frac{f}{y} = \frac{E}{R}$ with usual notations. (08 Marks)
 b. A beam of I section consists of $180\text{mm} \times 15\text{mm}$ flanges and a web of $280\text{mm} \times 15\text{mm}$. It is subjected to a bending moment of 120 kN-m and a shear force of 60 kN . Sketch the bending stress distribution and shear stress distribution along the depth of the section. (12 Marks)

OR

- 8 a. Derive the torsion equation for a circular shaft subjected to pure torsion. (10 Marks)
 b. A solid shaft of 60mm diameter is to be replaced by a hollow shaft of same length. The outer diameter of hollow shaft is same as that of solid shaft. If the angle of twist per unit torsional moment is the same in both cases, determine the inner diameter of hollow shaft. Take the modulus of rigidity of hollow shaft to be three times that of solid shaft. (10 Marks)

Module-5

- 9 a. Derive an expression for the slope and deflection of a simply supported beam carrying a central concentrated load. (08 Marks)
- b. A simply supported beam of constant cross section is 10m long. It is loaded with two point loads of 100 kN and 80 kN at points 2m and 6m from the left end respectively. Calculate the deflection under each load the maximum deflection. Take $E = 200 \text{ GPa}$ and $I = 18 \times 10^8 \text{ mm}^4$. (12 Marks)

OR

- 10 a. Distinguish between long and short columns. (04 Marks)
- b. What are the limitations of Euler's column theory? (04 Marks)
- c. A hollow cast iron column whose outside diameter is 200mm has a thickness of 20mm. It is 4.5m long and fixed at both ends. Calculate (i) Slenderness ratio (ii) Ratio of Euler's and Rankine's critical loads. Take $E = 100 \text{ GPa}$, $\alpha = \frac{1}{1600}$ and $\sigma_c = 550 \text{ N/mm}^2$. (12 Marks)

CBCS SCHEME

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17CV/CT32

Third Semester B.E. Degree Examination, Aug./Sept. 2020 Strength of Materials

Time: 3 hrs.

Max. Marks: 100

- Note:** 1. Answer any FIVE full questions, choosing ONE full question from each module.
2. Missing data, if any can be assumed.

Module-1

- 1 a. Define the four elastic constants. (08 Marks)
b. A steel rod of 30 mm in diameter is enclosed in an aluminium tube of 32 mm internal diameter and 60 mm external diameter. Both the bars are of length 750 mm and are rigidly connected to each other. The composite bar is subjected to an increase in temperature of 40°C. Compute the stresses in each material due to the temperature increase. If the bar is also subjected to a compression of 200 kN, compute the resultant stresses. Also, find the final deformation of the compound bar.
Material properties are : $E_S = 200 \text{ GPa}$, $\alpha_S = 12 \times 10^{-6} / ^\circ \text{C}$
 $E_A = 80 \text{ GPa}$, $\alpha_A = 22 \times 10^{-6} / ^\circ \text{C}$ (12 Marks)

OR

- 2 a. Sketch a typical stress strain curve for mild steel and briefly discuss the salient points on the curve. (06 Marks)
b. Derive an expression for elongation of a tapering rectangular plate of uniform thickness subjected to an axial load. (08 Marks)
c. A steel flat of thickness 25 mm tapers uniformly from 300 mm to 150 mm over a length of 750 mm. If the flat is subjected to an axial tension of 300 kN, compute the elongation of the flat. What is the % error if average area is used in calculating the extension?
 $E_S = 200 \text{ KN/mm}^2$. Also, compute the maximum stress. (06 Marks)

Module-2

- 3 a. Show that the sum of the normal stresses on any two perpendicular planes in a general two dimensional system is $(\sigma_x + \sigma_y)$. (06 Marks)
b. A closed cylindrical steel vessel 8 m long and 3.2 m internal diameter is subjected to an internal pressure of 5 MPa with thickness of vessel being 50 mm. Assuming $E = 200 \text{ GPa}$ and $\mu = 0.3$, compute hoop and longitudinal stresses, maximum shear stress and changes in length, diameter and volume. (08 Marks)
c. Compute the maximum and minimum hoop stress and plot their variation across the pipe thickness having an internal diameter of 500 mm and thickness 80 mm if the pipe is subjected to an internal fluid pressure of 10 MPa. (06 Marks)

OR

- 4 a. Derive expressions for circumferential and longitudinal stresses in a thin cylinder subjected to internal pressure, p. (06 Marks)
b. Direct stresses of magnitude 120 MPa tensile and 80 MPa compressive are acting at a point along with a shear stress of 50 N/mm². Compute the normal and tangential stresses on a plane inclined at 40° anticlockwise with the plane on which 120 MPa tensile stress is acting. Also, compute the magnitudes of principal stresses and planes. Sketch the stresses and their planes. (14 Marks)

Module-3

- 5 a. A Cantilever beam is subjected to a UDL of 20 kN/m throughout its length. Sketch SFD and BMD indicating salient values. Cantilever length = 3 m. (05 Marks)
- b. Sketch SFD and BMD for the beam shown in Fig. Q5 (b) indicating salient values (including point of contraflexure, maximum -ve and maximum +ve BMS and maximum SF). (15 Marks)

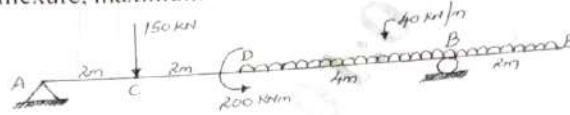


Fig. Q5 (b)

OR

- 6 a. A simply supported beam of span 8 m is carrying a concentrated load of 100 kN at a distance of 3 m from the left support. Sketch SFD and BMD indicating salient values. (05 Marks)
- b. Sketch SFD and BMD for the beam shown in Fig. Q6 (b) indicating salient values (including point of contraflexure, maximum -ve and maximum +ve BMS and maximum SF). (15 Marks)

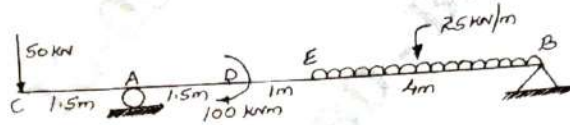


Fig. Q6 (b)

Module-4

- 7 a. Show that the strength of hollow shafts is greater than solid shaft having same material, length and weight. (08 Marks)
- b. Explain maximum shear stress theory of failure. (06 Marks)
- c. A steel shaft of diameter 150 mm transmits 250 kW at 200 rpm with $T_{max} = 1.35T_{mean}$. Compute the maximum shear stress and sketch the stress variation. (06 Marks)

OR

- 8 a. Explain maximum strain energy theory of failure. (06 Marks)
- b. A hollow circular shaft rotates at 200 rpm transmitting a power of 600 KW. Compute the diameters of the shaft if the external diameter is 1.5 times the internal diameter permissible shear stress in the material is 80 MPa and the angle of twist is 1.1° over a length of 3 m. $T_{max} = 1.35 T_{mean}$ and $G = 80$ GPa. Also, calculate the torque carried by a solid shaft of same length, cross sectional area and material as that of hollow shaft with the permissible shear stress and angle of twist being same. What is the percentage difference in torque carrying capacities? (14 Marks)

Module-5

- 9 a. Derive an expression for Euler's crippling load in a column with one end fixed and other end free. (10 Marks)
- b. An unsymmetrical I section with top flange 300×20 , bottom flange 150×15 and web thickness of 12 mm is used as a simply supported beam of span 6 m with a uniformly distributed load of 40 kN/m over its entire length. Overall depth of beam is 400 mm. Compute the maximum tensile and compressive stresses and sketch the bending stress distribution. Also, compute the shear stresses at salient points and sketch the shear stress distribution at support. (10 Marks)

OR

- 10 a. Derive an expression for shear stress in a beam with usual notations. (10 Marks)
- b. A hollow rectangular column having external dimensions of 250×375 with thickness = 10 mm is used as a column of length 3.5 m with both ends of the column being fixed. Compute the buckling load using both the formulae. $E = 200$ GPa, Rankine's constant are $\alpha = \frac{1}{7500}$ and $\sigma_c = 320$ N/mm². Comment on the formula giving larger load. (10 Marks)

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17CV/CT32

Third Semester B.E. Degree Examination, Aug./Sept. 2020
Strength of Materials

Time: 3 hrs.

Max. Marks: 100

Note: 1. Answer any FIVE full questions, choosing ONE full question from each module.
 2. Missing data, if any can be assumed.

Module-1

- 1 a. Define the four elastic constants. (08 Marks)
 b. A steel rod of 30 mm in diameter is enclosed in an aluminium tube of 32 mm internal diameter and 60 mm external diameter. Both the bars are of length 750 mm and are rigidly connected to each other. The composite bar is subjected to an increase in temperature of 40°C. Compute the stresses in each material due to the temperature increase. If the bar is also subjected to a compression of 200 kN, compute the resultant stresses. Also, find the final deformation of the compound bar.
 Material properties are : $E_S = 200 \text{ GPa}$, $\alpha_S = 12 \times 10^{-6} / ^\circ \text{C}$
 $E_A = 80 \text{ GPa}$, $\alpha_A = 22 \times 10^{-6} / ^\circ \text{C}$ (12 Marks)

OR

- 2 a. Sketch a typical stress strain curve for mild steel and briefly discuss the salient points on the curve. (06 Marks)
 b. Derive an expression for elongation of a tapering rectangular plate of uniform thickness subjected to an axial load. (08 Marks)
 c. A steel flat of thickness 25 mm tapers uniformly from 300 mm to 150 mm over a length of 750 mm. If the flat is subjected to an axial tension of 300 kN, compute the elongation of the flat. What is the % error if average area is used in calculating the extension? $E_S = 200 \text{ KN/mm}^2$. Also, compute the maximum stress. (06 Marks)

Module-2

- 3 a. Show that the sum of the normal stresses on any two perpendicular planes in a general two dimensional system is $(\sigma_x + \sigma_y)$. (06 Marks)
 b. A closed cylindrical steel vessel 8 m long and 3.2 m internal diameter is subjected to an internal pressure of 5 MPa with thickness of vessel being 50 mm. Assuming $E = 200 \text{ GPa}$ and $\mu = 0.3$, compute hoop and longitudinal stresses, maximum shear stress and changes in length, diameter and volume. (08 Marks)
 c. Compute the maximum and minimum hoop stress and plot their variation across the pipe thickness having an internal diameter of 500 mm and thickness 80 mm if the pipe is subjected to an internal fluid pressure of 10 MPa. (06 Marks)

OR

- 4 a. Derive expressions for circumferential and longitudinal stresses in a thin cylinder subjected to internal pressure, p . (06 Marks)
 b. Direct stresses of magnitude 120 MPa tensile and 80 MPa compressive are acting at a point along with a shear stress of 50 N/mm^2 . Compute the normal and tangential stresses on a plane inclined at 40° anticlockwise with the plane on which 120 MPa tensile stress is acting. Also, compute the magnitudes of principal stresses and planes. Sketch the stresses and their planes. (14 Marks)

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
 2. Any revealing of identification, appeal to evaluator and/or equations written eg. 42+8 = 50, will be treated as malpractice.

Module-3

- 5 a. A Cantilever beam is subjected to a UDL of 20 kN/m throughout its length. Sketch SFD and BMD indicating salient values. Cantilever length = 3 m. (05 Marks)
- b. Sketch SFD and BMD for the beam shown in Fig. Q5 (b) indicating salient values (including point of contraflexure, maximum -ve and maximum +ve BMS and maximum SF). (15 Marks)

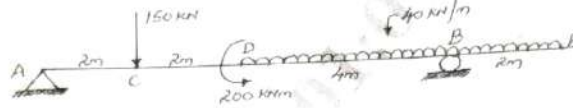


Fig. Q5 (b)

OR

- 6 a. A simply supported beam of span 8 m is carrying a concentrated load of 100 kN at a distance of 3 m from the left support. Sketch SFD and BMD indicating salient values. (05 Marks)
- b. Sketch SFD and BMD for the beam shown in Fig. Q6 (b) indicating salient values (including point of contraflexure, maximum -ve and maximum +ve BMS and maximum SF). (15 Marks)

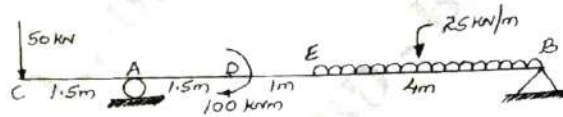


Fig. Q6 (b)

Module-4

- 7 a. Show that the strength of hollow shafts is greater than solid shaft having same material, length and weight. (08 Marks)
- b. Explain maximum shear stress theory of failure. (06 Marks)
- c. A steel shaft of diameter 150 mm transmits 250 kW at 200 rpm with $T_{max} = 1.35T_{mean}$. Compute the maximum shear stress and sketch the stress variation. (06 Marks)

OR

- 8 a. Explain maximum strain energy theory of failure. (06 Marks)
- b. A hollow circular shaft rotates at 200 rpm transmitting a power of 600 KW. Compute the diameters of the shaft if the external diameter is 1.5 times the internal diameter permissible shear stress in the material is 80 MPa and the angle of twist is 1.1° over a length of 3 m. $T_{max} = 1.35 T_{mean}$ and $G = 80$ GPa. Also, calculate the torque carried by a solid shaft of same length, cross sectional area and material as that of hollow shaft with the permissible shear stress and angle of twist being same. What is the percentage difference in torque carrying capacities? (14 Marks)

Module-5

- 9 a. Derive an expression for Euler's crippling load in a column with one end fixed and other end free. (10 Marks)
- b. An unsymmetrical I section with top flange 300×20 , bottom flange 150×15 and web thickness of 12 mm is used as a simply supported beam of span 6 m with a uniformly distributed load of 40 kN/m over its entire length. Overall depth of beam is 400 mm. Compute the maximum tensile and compressive stresses and sketch the bending stress distribution. Also, compute the shear stresses at salient points and sketch the shear stress distribution at support. (10 Marks)

OR

- 10 a. Derive an expression for shear stress in a beam with usual notations. (10 Marks)
- b. A hollow rectangular column having external dimensions of 250×375 with thickness = 10 mm is used as a column of length 3.5 m with both ends of the column being fixed. Compute the buckling load using both the formulae. $E = 200$ GPa, Rankine's constant are $\alpha = \frac{1}{7500}$ and $\sigma_c = 320$ N/mm². Comment on the formula giving larger load. (10 Marks)

CBCS SCHEME

15CV/CT32

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Third Semester B.E. Degree Examination, June/July 2019

Strength of Materials

Max. Marks: 80

Time: 3 hrs.

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

- Define : (i) Modulus of Rigidity (ii) Poisson's ratio (04 Marks)
 - Prove that the total extension of a uniformly tapering rod of diameter D_1 and D_2 , when the rod is subjected to an axial load 'P' is given by $dl = \frac{4PL}{\pi E D_1 D_2}$ (06 Marks)
 - An axial pull of 40,000 N is acting on a bar consisting of three sections of length 300mm, 250mm and 200mm and of diameters 20mm, 40mm and 50mm respectively. If the Young's modulus = $2 \times 10^5 \text{ N/mm}^2$, determine (i) Stress in each section (ii) total extension of the bar. (06 Marks)

OR

- Explain elasticity and elastic limit. (04 Marks)
 - A steel bar 300mm long, 50mm wide and 40mm thick is subjected to a pull of 300 kN in the direction of its length. Determine the change in volume. Take $E = 2 \times 10^5 \text{ N/mm}^2$ and Poisson's ratio = 0.25. (06 Marks)
 - A reinforced short concrete column 250mm \times 250mm in section is reinforced with 8 steel bars. The total area of steel bars is 2500 mm^2 . The column carries a load of 390 kN. If the modulus of elasticity for steel is 15 times that of concrete. Find the stresses in concrete and steel. (06 Marks)

Module-2

- Differentiate between thin cylinder and a thick cylinder. Find an expression for the radial pressure and hoop stress at any point in case of a thick cylinder. (10 Marks)
 - A rectangular bar of cross section area of 11,000 mm^2 is subjected to a tensile load 'P' as shown in Fig.Q3(b). The permissible normal and shear stresses on the oblique plane BC are given as 7 N/mm^2 and 3.5 N/mm^2 respectively. Determine the safe value of 'P'. (06 Marks)



Fig.Q3(b)

OR

- Determine the maximum and minimum hoop stress across the section of a pipe 400mm internal diameter and 100mm thick, when the pipe contains a fluid at a pressure of 8 N/mm^2 . Also sketch the radial pressure distribution and hoop stress distribution across the section. (08 Marks)
 - At a point in a strained material the principal tensile stresses across two perpendicular planes are 80 N/mm^2 and 40 N/mm^2 . Determine normal stress, shear stress and the resultant stress on a plane inclined at 20° with the major principal plane. Determine also the obliquity. (08 Marks)

Module-3

- 5 a. Define (i) Shear force (ii) Bending moment. (02 Marks)
 b. Draw the SF and BM diagrams for a cantilever of length 'L' carrying a point load 'W' at the free end. (04 Marks)
 c. Draw the SF and BM diagrams of a simply supported beam of length 7 m carrying uniformly distributed loads as shown in Fig.Q5(c). (10 Marks)

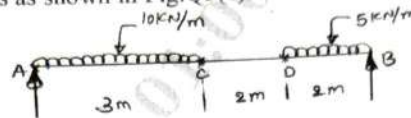


Fig.Q5(c)

OR

- 6 A horizontal beam 10m long is carrying a uniformly distributed load of 1 kN/m. The beam is supported on two supports 6 m apart. Find the position of the supports, so that bending moment on the beam is as small as possible. Also draw the SF and BM diagram. (16 Marks)

Module-4

- 7 a. Define the terms : (i) Neutral axis (ii) Section modulus. (04 Marks)
 b. A hollow mild steel tube 6m long 40mm internal diameter and 5mm thick is used as a strut with both ends hinged. Find the crippling load and safe load taking factor of safety as 3. Take $E = 2 \times 10^5 \text{ N/mm}^2$. (06 Marks)
 c. The external and internal diameter of a hollow cast iron column are 50mm and 40mm respectively. If the length of this column is 3m and both of its ends are fixed, determine the crippling load using Rankine's formula. Take the values of $\sigma_c = 550 \text{ N/mm}^2$ and $\alpha = \frac{1}{1600}$ in Rankine's formula. (06 Marks)

OR

- 8 a. Define (i) Buckling load (ii) Slenderness ratio. (04 Marks)
 b. A timber beam of rectangular section of length 8m is simply supported. The beam carries a U.D.L. of 12 kN/m run over the entire length and a point load of 10 kN at 3m from the left support. If the depth is two times the width and the stress in the timber is not to exceed 8 N/mm^2 , find the suitable dimensions of the section. (12 Marks)

Module-5

- 9 a. List the theories of failures. (04 Marks)
 b. A hollow shaft of external diameter 120mm transmits 300 kW power at 200 r.p.m. Determine the maximum internal diameter if the maximum stress in the shaft is not to exceed 60 N/mm^2 . (06 Marks)
 c. Determine the diameter of a solid steel shaft which will transmit 90 kW at 160 r.p.m. Also determine the length of the shaft if the twist must not exceed 1° over the entire length. The maximum shear stress is limited to 60 N/mm^2 . Take the value of modulus of rigidity $= 8 \times 10^4 \text{ N/mm}^2$. (06 Marks)

OR

- 10 a. Derive the relation for a circular shaft when subjected to a torsion as given below:

$$\frac{T}{J} = \frac{\tau}{R} = \frac{C\theta}{L}$$

(08 Marks)

- b. State and explain theory of maximum principal strain theory.

(08 Marks)

2 of 2

Harshit N.
 Chanakya H-Pak 1

CBCS SCHEME

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15CV/CT32

Third Semester B.E. Degree Examination, Jan./Feb. 2021 Strength of Materials

Time: 3 hrs.

Max. Marks: 80

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

- 1 a. State and explain Elastic constants. (04 Marks)
 b. A bar of 20mm is tested in tension. It is observed that when a load of 40kN is applied, the extension measured over a gauge length of 200mm is 0.12mm and contraction in diameter is 0.0036mm. Find Poisson's ratio and elastic constants E, C, K. (12 Marks)

OR

- 2 a. Define temperature stresses and state its importance. (06 Marks)
 b. A composite bar is rigidly fitted at the supports A and B as shown in the Fig.Q.2(b). Determine the reactions at the supports when temperature rises by 20°C. Take $E_a = 70 \text{ GN/m}^2$, $E_s = 200 \text{ GN/m}^2$, $\alpha_a = 11 \times 10^{-6}/^\circ\text{C}$ and $\alpha_s = 12 \times 10^{-6}/^\circ\text{C}$. (10 Marks)

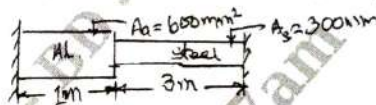


Fig.Q.2(b)

Module-2

- 3 a. Define principal planes and principal stresses. (04 Marks)
 b. Stresses acting at a point in a two dimensional stress system shown in the Fig.Q.3(b), find:
 i) Normal and shear stresses on the inclined plane
 ii) Principal stresses and their planes
 iii) Maximum shear stresses and their planes.

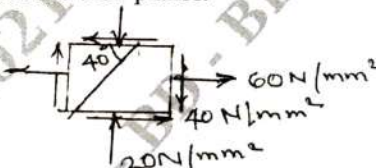


Fig.Q.3(b)

(12 Marks)

OR

- 4 a. Derive expressions for hoop stress and longitudinal stress in a thin cylinder. (06 Marks)
 b. A cylindrical thin shell 800mm diameter and 3m long is having 10mm metal thickness. The shell is subjected to an internal pressure of 2.5N/mm². Determine:
 i) Change in diameter
 ii) Change in length
 iii) Change in volume
 Take $E = 2 \times 10^5 \text{ N/mm}^2$ $\mu = 0.3$ (10 Marks)

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
 2. Any revealing of identification, appeal to evaluator and /or equations written eg. 42+8 = 50, will be treated as malpractice.

Module-3

- 5 a. Derive the relationship between intensity of load, shear force and bending moment. (06 Marks)
- b. Draw shear force and bending moment diagrams for the beam shown in the Fig.Q.5(b).

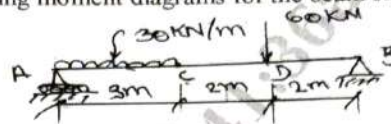


Fig.Q.5(b)

(10 Marks)

OR

- 6 a. Explain:
- Sagging bending moment
 - Hogging bending moment
 - Point of contra flexure.
- b. Draw shear force and bending moment diagrams for the beam shown in the Fig.Q.6(b). Locate the points of contra flexure. (10 Marks)

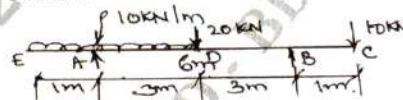


Fig.Q.6(b)

Module-4

- 7 a. What are assumptions made in bending theory? (04 Marks)
- b. Derive the bending equation $\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}$ with usual notations. (06 Marks)
- c. Prove that maximum shear stress is 1.5 times the average shear stress in rectangular section. (06 Marks)

OR

- 8 a. What is effective length of column? How it is related with end conditions of column and explain with neat sketches. (08 Marks)
- b. A hollow cast iron column whose outside diameter is 200mm and has a thickness of 20mm, 4.5m long and is fixed at both ends. Evaluate Rankine's crippling load using $f_c = 550\text{N/mm}^2$. Take Rankines constant $\frac{1}{1600}$. (08 Marks)

Module-5

- 9 a. Derive the Torsion equation $\frac{I}{J} = \frac{\tau}{R} = \frac{C\theta}{L}$ with usual notation. (06 Marks)
- b. A solid circular shaft is to be designed to transmit 440kW power at 280rpm. If the maximum shear stress is not to exceed 40N/mm^2 and the angle of twist is not to exceed 1° per meter length, determine the diameter of the shaft. Take modulus of rigidity 84kN/mm^2 . (10 Marks)

OR

- 10 a. Explain: i) Maximum principal stress theory ii) Maximum shear stress theory. (06 Marks)
- b. A bolt is required to resist an axial tension of 25kN and a transverse shear of 20kN. Find the size of the bolt by using i) Maximum principal stress theory ii) Maximum shear stress theory $\sigma_e = 300\text{N/mm}^2$, F.S = 3 (10 Marks)



Assignment

Date	25	10	20
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Assignment No.	01	Maximum Marks	10
Course/Subject Title	Strength of Materials	Course/Subject Code	18CV32
Semester	III	Scheme	CBCS - 18

Course Outcome Statements : After the successful completion of the course, the students will be able to Outcome Statements

18CV32.1	Evaluate the stresses and strains for ferrous and non-ferrous materials
18CV32.2	Evaluate the internal stresses developed in one dimensional, two dimensional structural elements and cylinders
18CV32.3	Evaluate the bending moment, shear force in prismatic beams and corresponding stresses
18CV32.4	Analyse and design the circular shafts subjected to torsion
18CV32.5	Evaluate the slope and deflection of prismatic beams
18CV32.6	Evaluate the failure loads for the columns and struts

Note :

Q. No.	Question	Marks	RBT Level	CO
Module -1				
1	Draw stress versus strain curve for mild steel specimen subjected to axial tension indicating the salient points. Define the elastic constants.		L1	CO1
2	Derive the expression for elongation of tapering circular bar due to axial load P. Use standard notations.		L2	CO1
3	A brass bar having cross-sectional area of 1000mm^2 , is subjected to axial forces as shown in fig. Find the total elongation of the bar. Take $E = 1.05 \times 10^5 \text{N/mm}^2$.		L3	CO1
4	A compound tube consists of a steel tube 140mm internal diameter and 160mm external diameter and an outer brass tube 160mm internal diameter and 180mm external diameter. The two tubes are of the same length. The compound tube carries an axial load of 900KN. find the stress and the load carried by each tube and the amount it shortens. Length of each tube is 140mm. take young's modulus for the steel as $2 \times 10^5 \text{N/mm}^2$ and for brass as $1 \times 10^5 \text{N/mm}^2$.		L3	CO1
5	A Steel rail is 12.6m long and is laid at a temperature of 24°C the maximum temperature expected is 44°C a) Estimate the minimum gap between two rails to be left so that temperature stress does not develop. b) Calculate the thermal stress developed in the rails, if 1) No expansion joint is prevented. 2) If a 2mm gap is provided for expansion. c) If the stress developed is 20MN/m^2 , what is the gap b/w the rails? Take $E = 2 \times 10^5 \text{N/mm}^2$ and $\alpha = 12 \times 10^{-6} / ^\circ\text{C}$		L3	CO1
6	Derive the relationship b/w Modulus of elasticity (E) and Bulk modulus (K).		L3	CO1
7	A Steel flat of thickness 25mm tapers uniformly from 300mm to 150mm over a length of 750mm. if the flat is subjected to an axial tension of 300kN, compute the elongation of the flat. What is the % error if average area is used in calculating the extension?		L3	CO1
Module -2				
8	Define Thick and thin cylinders. Derive an expression for hoop stress.		L4	CO2
9	A thin cylindrical shell 1m in diameter & 3m long has a metal thickness of 10mm. if it is subjected to an internal pressure of 3Mpa. Determine hoop stress, longitudinal stress, change in length, change in diameter and change in volume. If $E = 210\text{GPa}$ & $\mu = 0.3$.		L4	CO2
10	Derive the lame's equations for radial and hoop stresses for thick cylinder to internal		L2	CO2



Assignment

Date	25	10	20
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	& external fluid pressure.			
11	A Thick cylinder of internal diameter 200mm is subjected to an internal fluid pressure of 40mpa. If the allowable stress in tension for the material is 120Mpa. Find the thickness required.		L2	CO2
Module -4				
12	Derive the pure torsion equation with usual notations.		L2	CO4
13	Prove that a hallow circular shaft is stronger than a solid circular shaft in torsion. Which have same materials, length and weight?		L2	CO4
14.	Determine the diameter of solid shaft which will transmit 90Kn at 160rpm, if the shear stress in the shaft is limited to 60Mpa. Find also length of the shaft. If the twist is limited to 1°. Given $C = 8 \times 10^4$ Mpa.		L4	CO4
15	A 150mm diameter solid steel shaft is transmitting 450kw power at 90rpm. Compute the maximum shear stress. Find the change that would occur in the shearing stress. If the speed increased to 360rpm.		L4	CO4

Last date for submission	30	11	2020
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RBT (Revised Bloom's Taxonomy) Levels : Cognitive Domain		
L1 : Remembering	L2 : Understanding	L3 : Applying
L4 : Analysing	L5 : Evaluating	L6 : Creating


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Assignment

Date	10	12	2020
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Assignment No.	02	Maximum Marks	10
Course/Subject Title	Strength of Materials	Course/Subject Code	18CV32
Semester	III	Scheme	CBCS - 18

Course Outcome Statements : After the successful completion of the course, the students will be able to Outcome Statements

18CV32.1	Evaluate the stresses and strains for ferrous and non-ferrous materials
18CV32.2	Evaluate the internal stresses developed in one dimensional, two dimensional structural elements and cylinders
18CV32.3	Evaluate the bending moment, shear force in prismatic beams and corresponding stresses
18CV32.4	Analyse and design the circular shafts subjected to torsion
18CV32.5	Evaluate the slope and deflection of prismatic beams
18CV32.6	Evaluate the failure loads for the columns and struts

Note :

Q. No.	Question	Marks	RBT Level	CO
Module -2				
1	Derive an Expression for two dimensional stress system.			
2	A rectangular block of material is subjected to a tensile stress of 110 N/mm^2 on one plane and a tensile stress of 47 N/mm^2 on the right angles to the former. Each of the above stress is accomplished by a shear stress of 63 N/mm^2 and that associated with the former tensile stress tends to rotate the block in anticlockwise. Find 1) the direction and magnitude of principal stress. 2) Magnitude of the greatest Shear Stress.		L2 L3	CO2 CO2
3	Explain the procedure for determining stresses in a general two dimensional stress system using Mohr's circle		L2	CO2
4	Explain the a) Maximum Shear stress theory. b) Maximum shear strain theory		L2	CO2
Module -3				
5	Define Shear force, bending moment and point of Inflexion.		L1	CO3
6	Establish the relationship between shear force, bending moment and load intensity.		L2	CO3
7	Draw the shear force and bending moment for cantilever beam subjected to uniformly varying load.		L2	CO3
8	Draw the shear force and bending moment diagram for the simply supported beam of length 7m carrying a uniformly distributed load as shown in fig 		L4	CO3
9	Draw the shear force and bending moment diagram for the cantilever beam as shown in fig 		L4	CO2
10	Draw the shear force and bending moment diagram for the cantilever beam as shown in fig		L3	CO3



Assignment

Date	10	12	2020
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Module - 5			
11	Distinguish between long and short columns.		L2 CO6
12	Derive an expression for Euler's crippling load when both ends of the column are hinged.		L2 CO6
13	What are the limitations of Euler's column theory?		L2 CO6
14	A steel bar of rectangular section 30mm×40mm pinned at both ends is subjected to axial compression. The bar is 1.75m long. Determine the buckling load and corresponding axial stress using Euler's formula. Determine the minimum length for which Euler's equation, if proportionality limit of material is 200Mpa.		L3 CO6
15.	A hallow CI column whose outside diameter is 200mm as a thickness of 20mm. it is 4.5m long and fixed at both ends. 1) Calculate safe load by Rankine's formula using FOS=4 2) Calculate the ratio of Euler's and Rankine's critical load and slenderness ratio. Take $f_c = 550\text{N/mm}^2$ $a = 1/1600$ in Rankine's formulae and $E = 9.4 \times 10^4 \text{N/mm}^2$.		L3 CO6

Last date for submission	30	12	2020
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RBT (Revised Bloom's Taxonomy) Levels : Cognitive Domain		
L1 : Remembering	L2 : Understanding	L3 : Applying
L4 : Analysing	L5 : Evaluating	L6 : Creating

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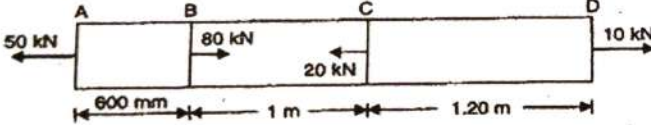
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Subject Title	Strength of Materials	Subject Code	18CV32
Semester	III A	Scheme	CBCS
Date	24/10/2020	IA No.	01
Time	9:30 -10:30	Max. Marks	30

Course Outcome Statements	
18CV32.1	Evaluate the stresses and strains for ferrous and non-ferrous materials
18CV32.2	Evaluate the internal stresses developed in one dimensional, two dimensional structural elements and cylinders
18CV32.3	Evaluate the bending moment, shear force in prismatic beams and corresponding stresses
18CV32.4	Analyse and design the circular shafts subjected to torsion
18CV32.5	Evaluate the slope and deflection of prismatic beams
18CV32.6	Evaluate the failure loads for the columns and struts


NOTE: Answer any five questions. Each questions carries equal marks

Q. No.	Questions	Marks	RBT Level	CO
1	Draw stress versus strain curve for mild steel specimen subjected to axial tension indicating the salient points. Define the elastic constants.	6	L1	CO1
2	Derive the expression for elongation of tapering circular bar due to axial load P. Use standard notations.	6	L2	CO1
3	A bras bar having cross- sectional area of 1000mm ² , is subjected to axial forces as shown in fig. Find the total elongation of the bar. Take E= 1.05×10 ⁵ N/mm ² . 	6	L3	CO1
4	A compound tube consists of a steel tube 140mm internal diameter and 160mm external diameter and an outer brass tube 160mm internal diameter and 180mm external diameter. The two tubes are of the same length. The compound tube carries an axial load of 900KN. find the stress and the load carried by each tube and the amount it shortens. Length of each tube is 140mm. take young's modulus for the steel as 2 × 10 ⁵ N/mm ² and for brass as 1×10 ⁵ N/mm ² .	6	L3	CO1
5	A Steel rail is 12.6m long and is laid at a temperature of 24°c the maximum temperature expected is 44°c a) Estimate the minimum gap between two rails to be left so that temperature stress does not develop. b) Calculate the thermal stress developed in the rails, if 1) No expansion joint is prevented. 2) If a 2mm gap is provided for expansion. c) If the stress developed is 20MN/m ² , what is the gap b/w the rails? Take E = 2×10 ⁵ N/mm ² and α = 12×10 ⁻⁶ / °c	6	L3	CO1
6	Derive the relationship b/w Modulus of elasticity (E) and Bulk modulus (K).	6	L3	CO1

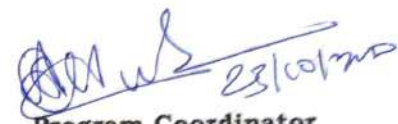


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RBT (Revised Bloom's Taxonomy) Levels		
L1 : Remembering	L2 : Understanding	L3 : Applying
L4 : Analysing	L5 : Evaluating	L6 : Creating


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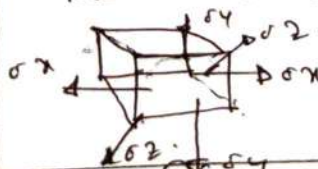
Scheme of Valuation

Course/Subject Title	Strength of Materials	Course/Subject Code	18CV32
Semester	III A	CIE No.	01
Date	24/10/2020	Max. Marks	30

Q.	Solution	Marks
1.	<p>Answer any five questions.</p> <p> $A = \text{proportionality limit}$ $B = \text{upper yield point}$ $C = \text{lower yield point}$ $D = \text{ultimate strength}$ $E = \text{breaking point}$ </p> <p>Elastic constant $E \times 1/2 = 2m$</p>	<p>5m</p> <p>6m</p>
02	<p>Tapering circular bar due to axial load P.</p> <div style="border: 1px solid black; padding: 5px; display: inline-block;"> $A = \frac{4PL}{\pi E d_1 d_2}$ </div> <p>Explanation</p>	6m
03	<p> $\Delta_{AB} = \frac{P_1 \times L_1}{A E} = \frac{50 \times 1000}{1000 \times 1.5 \times 10^8} \times 6000 = 0.2 \text{ mm}$ </p> <p> $\Delta_{BC} = \frac{P_2 \times L_2}{A E} = \frac{20 \times 10^3 \times 10000}{1000 \times 1.5 \times 10^8} = 0.133 \text{ mm}$ </p> <p> $\Delta_{CD} = \frac{P_3 \times L_3}{A E} = \frac{10000 \times 10000}{1000 \times 1.5 \times 10^8} = 0.222 \text{ mm}$ </p>	



Scheme of Valuation

Q.	Solution	Marks
04	<p>total elongation of bar = $0.245 - 0.1904 + 0.2095$</p> <p>Area of steel tube = $\pi/4 (160^2 - 140^2) = 4712.39 \text{ mm}^2$</p> <p>Area of brass tube = $\pi/4 (180^2 - 160^2) = 5340.70 \text{ mm}^2$</p> <p>$\frac{\sigma_s}{E_s} = \frac{\sigma_b}{E_b}$ } 1M $\sigma_s = 2\sigma_b$ } 1M</p> <p>$\sigma_s \times A_s + \sigma_b \times A_b = 900 \times 10^3$</p> <p>$\sigma_b = 60.95 \text{ N/mm}^2$ - 1M $\sigma_s = 121.9 \text{ N/mm}^2$ - 1M</p> <p>$P_b = \sigma_b \times A_b = 60.95 \times 5340.7$ $P_b = 325.515 \text{ kN}$ - 1M</p> <p>$P_s = \sigma_s \times A_s = 121.9 \times 4712.4$ $P_s = 574.44 \text{ kN}$ - 1M</p> <p>Decrease in length of compound tube = $\frac{\sigma_b}{E_b} \times L$ $= \frac{60.95}{2 \times 10^5} \times 140$ } 1M $= 0.0853 \text{ mm}$ } 1M</p>	6M
05	<p>$E = 2 \times 10^5 \text{ N/mm}^2$</p> <p>1) Free Expansion of rail - $\Delta = \alpha t L$ $\Delta = 3.024 \text{ mm}$ - 1M</p> <p>2) If no expansion joint is provided, free expansion prevented is equal to 3.024 $\frac{PL}{AE} = 3.024$ (a) $\sigma = \alpha t E$ $\sigma = 48 \text{ N/mm}^2$ - 2M</p> <p>3) If a gap of 2mm is provided, free expansion $\Delta = \alpha t L - \delta$ $\sigma = 16.253 \text{ N/mm}^2$ - 1M</p> <p>4) $\sigma_t = 20 \text{ N/mm}^2$ $\Delta = \frac{PL}{AE} = \Delta = 1.26 \text{ mm}$ $\Delta = \alpha t L - \delta$ $\sigma = 1.76 \text{ mm}$ - 2M</p>	6M
06	<p>Modulus of elasticity (E) & Bulk modulus (K)</p>  <p>$\epsilon_v = \epsilon_x + \epsilon_y + \epsilon_z$ - 1M</p> <p>$\frac{\delta V}{V} = \frac{3\sigma}{E} (1 - 2\mu)$ - 2M</p> <p>Bulk modulus = $\frac{\sigma}{\frac{\delta V}{V}}$ $3K(1 - 2\mu) = E$ - 2M</p>	6M

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 23/10/2020



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Subject Title	Strength of Materials	Subject Code	18CV32
Semester	III A	Scheme	CBCS
Date	07/12/2020	IA No.	02
Time	3:00 -04:00 PM	Max. Marks	30

Course Outcome Statements	
18CV32.1	Evaluate the stresses and strains for ferrous and non-ferrous materials
18CV32.2	Evaluate the internal stresses developed in one dimensional, two dimensional structural elements and cylinders
18CV32.3	Evaluate the bending moment, shear force in prismatic beams and corresponding stresses
18CV32.4	Analyse and design the circular shafts subjected to torsion
18CV32.5	Evaluate the slope and deflection of prismatic beams
18CV32.6	Evaluate the failure loads for the columns and struts

NOTE: Answer any one full question from each part.				
Q. No.	Questions	Marks	RBT Level	CO
Part - A				
1a)	Derive an Expression for two dimensional stress system.	8	L2	CO2
1b)	A rectangular block of material is subjected to a tensile stress of 110 N/mm^2 on one plane and a tensile stress of 47 N/mm^2 on the right angles to the former. Each of the above stress is accomplished by a shear stress of 63 N/mm^2 and that associated with the former tensile stress tends to rotate the block in anticlockwise. Find 1) the direction and magnitude of principal stress. 2) Magnitude of the greatest Shear Stress.	7	L3	CO2
OR				
2a)	Define Thick and thin cylinders. Derive an expression for hoop stress.	8	L2	CO2
2b)	Find the thickness of metal necessary for a cylindrical shell of internal diameter 160 mm to withstand a pressure of 8 N/mm^2 . The maximum hoop stress in the section is not to exceed 35 mpa .	7	L3	CO2
Part - B				
3a)	Derive the pure torsion equation with usual notations.	8	L2	CO4
3b)	A steel bar of rectangular section $30 \text{ mm} \times 40 \text{ mm}$ pinned at both ends is subjected to axial compression. The bar is 1.75 m long. Determine the buckling load and corresponding axial stress using Euler's formula. Determine the minimum length for which Euler's equation, if proportionality limit of material is 200 Mpa .	7	L3	CO6
OR				
4a)	Derive an expression for Euler's crippling load when both ends of the column are hinged.	8	L2	CO6
4b)	Determine the diameter of solid shaft which will transmit 90 kN at 160 rpm , if the shear stress in the shaft is limited to 60 Mpa . Find also length of the shaft. If the twist is limited to 1° . Given $C = 8 \times 10^4 \text{ Mpa}$.	7	L3	CO4




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RBT (Revised Bloom's Taxonomy) Levels		
L1 : Remembering	L2 : Understanding	L3 : Applying
L4 : Analysing	L5 : Evaluating	L6 : Creating


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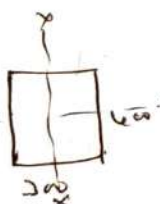
Scheme of Valuation

Course/Subject Title	Strengths of Materials	Course/Subject Code	18CV32
Semester	Vth A.	CIE No.	02
Date	7/12/2020.	Max. Marks	30

Q.	Solution	Marks
1.	<p>Answer any one full question from each part Part - A.</p> <p>Two dimensional system.</p> <p>Direction with n is σ_n</p> $\sigma_n = \left(\frac{\sigma_x + \sigma_y}{2} \right) + \cos 2\theta \left(\frac{\sigma_y - \sigma_x}{2} \right) + \tau \sin 2\theta$ $\sigma_t = \left(\frac{\sigma_y - \sigma_x}{2} \right) \sin 2\theta - \tau \cos 2\theta$	8m
10	<p>1. Major principal stress</p> $\sigma_{n1} = \left(\frac{\sigma_x + \sigma_y}{2} \right) + \sqrt{\left(\frac{\sigma_y - \sigma_x}{2} \right)^2 + \tau^2}$ $\sigma_{n1} = 148.93 \text{ N/mm}^2$ <p>2. Minor principal stress</p> $\sigma_{n2} = \left(\frac{\sigma_x + \sigma_y}{2} \right) - \sqrt{\left(\frac{\sigma_y - \sigma_x}{2} \right)^2 + \tau^2}$ $\sigma_{n2} = 8.07 \text{ N/mm}^2$ <p>3. Direction.</p> $\tan 2\theta = \frac{2\tau}{\sigma_y - \sigma_x}$ $\theta = 31.63^\circ$ <p>4. Magnitude</p> $\sigma_t(\text{max}) = \frac{1}{2} \sqrt{(\sigma_x - \sigma_y)^2 + 4\tau^2}$ $\sigma_t(\text{min}) = 70.63 \text{ N/mm}^2$	2 1/2 2 1/2
15	<p>Thin cylinders: If the thickness of the wall of the cylindrical vessel is less than $\frac{1}{10}$ to $\frac{1}{20}$ of its internal diameter.</p> <p>Thick cylinders: If the thickness of the wall of the cylindrical vessel is more than $\frac{1}{10}$ of its internal diameter.</p>	7m 2m



Scheme of Valuation

Q.	Solution	Marks
	<p>Expression of hoop stress $\sigma_1 = \frac{pd}{2t}$ Hoop with h's. - 3m</p> <p>Longitudinal stress $\sigma_2 = \frac{pd}{4t}$ with h's. - 3m</p>	<p>3m</p> <p>3m</p> <hr/> <p>8m</p>
2 (b)	<p>$Px = \frac{b}{2r} \rightarrow \sigma_x = \frac{b}{2r} + 1 \rightarrow \text{---} - 2m$</p> <p>$6400 a + b = 51200 \rightarrow \text{---} (11)$</p> <p>$6400 a + b = 22400 \rightarrow \text{---} (12)$</p> <p>$q = 13.1$ $b = 13760$ - 4m</p> <p>$t = 20.96 \text{ mm}$ - 1m</p>	<p>7m</p>
3 (a)	<p>Derive the <u>Part B</u> Torsion equation with usual notation.</p> <p>h's - 4m</p> <p>Explanation - 3m</p> <div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 10px auto;"> $\frac{T}{J} = \frac{C\theta}{L} = \frac{6S}{R}$ </div> <p> $J_{xx} = \frac{64b^3}{12} = 160 \times 10^3 \text{ mm}^4$ $J_{yy} = \frac{64bd^3}{12} = 90 \times 10^3 \text{ mm}^4$ $k_{min} = \sqrt{\frac{J_{min}}{A}} = 8.66 \text{ mm}$ $l = 1.75 \text{ m}$ $l_c = l = 1.75 \text{ m}$ $E = 2 \times 10^5 \text{ MPa}$ </p>	<p>8m</p>
3 (b)	<p>  </p> <p> $P_{cr} = \frac{\pi^2 EI_{min}}{l_c^2}$ $P_{cr} = 51.09 \times 10^3 \text{ N}$ - 2m </p> <p> Axial stress $b = \frac{W \cos \theta}{A_{ax}} = \frac{P_{cr}}{A} = \frac{56 \times 10^3}{30840}$ $b = 48.33 \text{ MPa}$ - 2m </p> <p> <u>Case: 2</u> $P_{cr} = \frac{\pi^2 EI}{l_c^2} = \frac{\pi^2 E (Ak^2)}{l_c^2}$ </p> <p> $b_{cr} = \frac{\pi^2 Ek^2}{l_c^2}$ $l_c = 99.34 \times k_{min}$ $l_c = 99.860.33 \text{ mm}$ - 2m </p>	<p>4m</p> <p>2m</p>



Scheme of Valuation

Q.	Solution	Marks
2 (b)	<p>Expansion & hoop stress $\sigma_1 = \frac{pd}{2t}$ Ag with h's. - 3m</p> <p>longitudinal stress $\sigma_2 = \frac{pd}{4t}$ with h's. - 3m</p> <p>$P \times \frac{b}{22} - 9 \rightarrow \sigma_2 = \frac{b}{22} + 9 \rightarrow -2m$</p> <p>$6400a + b = 51200$ (11)</p> <p>$64009 + b = 224000$ (12)</p> <p>$q = 13.1$ $b = 13760$ - 4m</p> <p>$t = 20.56mm$ - 1m</p>	<p>3m</p> <p>3m</p> <hr/> <p>8m</p> <hr/> <p>7m</p>
3 (a)	<p>Derive the <u>part B</u> torsion equation with correct notation.</p> <p>h's - pm</p> <p>Explanation - 3m</p> <p>$\frac{T}{J} = \frac{C\theta}{L} = \frac{6S}{R}$</p>	<p>8m</p>
3 (b)	<p>Diagram of a rectangular cross-section with dimensions 200×400 mm.</p> <p>$I_{xx} = \frac{60 \times 3}{12} = 160 \times 10^3 mm^4$</p> <p>$I_{yy} = \frac{b^3 d}{12} = 90 \times 10^3 mm^4$</p> <p>$k_{min} = \sqrt{\frac{I_{min}}{A}} = 8.66mm$</p> <p>$l = 1.75m$ $l_c = l = 1.75m$ $E = 2 \times 10^5 MPa$</p> <p>Calc (1) axial stress</p> <p>$P_{CR} = \frac{n^2 E I_{min}}{l_c^2}$ $P_{CR} = 58.99 \times 10^3 N$ - 2m</p> <p>Axial stress $b = \frac{W_{SD}}{A_{res}} = \frac{P_{CR}}{A} = \frac{58 \times 10^3}{30000}$ $b = 48.33 MPa$ - 2m</p> <p>Calc: 2</p> <p>$P_{CR} = \frac{n^2 E I}{l_c^2} = \frac{n^2 E (A k^2)}{l_c^2}$</p> <p>$b_{CR} = \frac{n^2 E k^2}{l_c^2}$</p> <p>$l_c = 99.34 \times k_{min}$</p> <p>$l_c = 98.860.33mm$</p>	<p>4m</p> <hr/> <p>2m</p> <hr/> <p>2m</p>



Scheme of Valuation

Course/Subject Title		Course/Subject Code	
Semester		CIE No.	
Date		Max. Marks	

Q.	Solution	Marks
45	<p>Both ends of the column are hinged</p> <div style="border: 1px solid black; padding: 5px; display: inline-block;"> $P = \frac{n^2 E I}{L^2}$ </div> <div style="border: 1px solid black; padding: 5px; display: inline-block; margin-left: 20px;"> $L = L_e$ </div> <p>Derivation with fig. + 8 marks</p>	5m
46	<p>$P = \frac{2n^2 T}{60}$</p> <p>$T = 5.371 \times 10^6 \text{ N-mm}$</p> <p>$\frac{T}{S} = \frac{b \Delta}{R}$</p> <div style="border: 1px solid black; padding: 5px; display: inline-block;"> $d = 76.96 \text{ mm}$ </div> <p style="margin-left: 20px;">- 3.5m</p> <p>$\frac{T}{S} = \frac{C \theta}{L}$</p> <div style="border: 1px solid black; padding: 5px; display: inline-block;"> $L = 892.57 \text{ mm}$ </div> <p style="margin-left: 20px;">- 3.5m</p>	7m

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Program Coordinator
 (HOD, Civil)

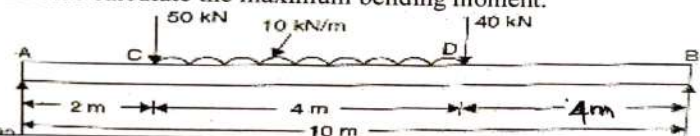
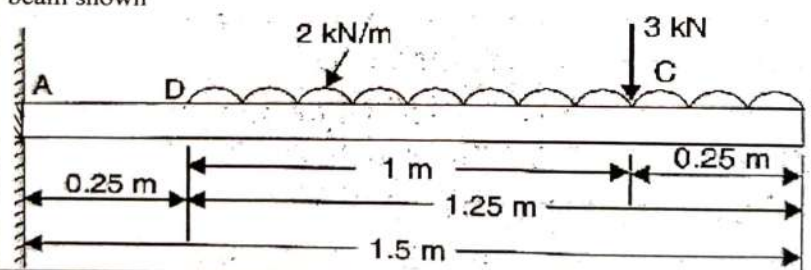


USN										
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Course/Subject Title	Strength of Materials	Course/Subject Code	18CV32
Semester	III A	Scheme	CBCS - 18
Date	23/02/2021	CIE No.	3
Time	3:00to 4:00 PM	Max. Marks	30

Course Outcome Statements : After the successful completion of the course, the students will be able to	
CO1	Evaluate the stresses and strains for ferrous and non-ferrous materials
CO2	Evaluate the internal stresses developed in one dimensional, two dimensional structural elements and cylinders
CO3	Evaluate the bending moment, shear force in prismatic beams and corresponding stresses
CO4	Analyse and design the circular shafts subjected to torsion
CO5	Evaluate the slope and deflection of prismatic beams
CO6	Evaluate the failure loads for the columns and struts

Note : Answer all Questions

Q. No.	Question	Marks	RBT Level	CO
Part A				
1 a)	Establish the relationship between shear force, bending moment and load intensity.	5	L2	CO3
1 b)	A Simply Supported beam of length 10m, carries the uniformly distributed load and two point load as shown in fig. Draw SFD and BMD for the beam and also calculate the maximum bending moment. 	10	L3	CO3
OR				
2 a)	A Cantilever 1.5m long is loaded with a uniformly distributed load of 2kN/m run over a length of 1.25m from the free end. It also carries a point load of 3kN at a distance of 0.25m from the free end. Draw SFD and BMD for the beam shown 	5	L3	CO3
2 b)	Draw SFD and BMD for the beam shown	10	L3	CO3




Note : Answer all Questions				
Q. No.	Question	Marks	RBT Level	CO
Part B				
3 a)	List the assumptions made in theory of bending and Derive the equation of pure bending with usual notations.	5	L2	CO3
3 b)	Determine the deflection at the mid-span and slope at the supports for a simply supported beam of span L,m subjected to concentrated load W at mid-span.	10	L3	CO5
OR				
4 a)	Derive the Euler's-Bernoulli differential equation for flexure.	5	L2	CO5
4 b)	A timber beam 200mm×300mm is simply supported over a span of 10m. What UDL can it carry if the maximum permissible stress is 7.5Mpa. Find the maximum bending stress and radius of curvature at a section 1m from left support. Take $E=12.6 \times 10^3 \text{ N/mm}^2$.	10	L3	CO3

RBT (Revised Bloom's Taxonomy) Levels : Cognitive Domain		
L1 : Remembering	L2 : Understanding	L3 : Applying
L4 : Analysing	L5 : Evaluating	L6 : Creating


 Course Coordinator
 (Raghu, M E)
ASSISTANT PROFESSOR
 Civil Engineering Department
 B.I.E.T., Davangere.


 Coordinator
 DQAC


 Program Coordinator
 (HOD, Civil)

Scheme of Valuation

Course/Subject Title	Strength of Materials	Course/Subject Code	18CV32
Semester	IIIrd sem.	CIE No.	03
Date	23-02-2021	Max. Marks	30

1a)

Derivation $\cos 45^\circ = \frac{5}{10}$
 $\sin 45^\circ = \frac{5}{10}$

1b)

Explanation - 4m

$R_{AV} + R_{BV} = 130 \text{ kN}$

$R_{BV} = 50 \text{ kN}$

$R_{AV} = 80 \text{ kN}$

SF) A = +80 kN

SF) C = 30 kN

left

SF) C = 30 kN

right

SF) D = -10 kN

left

SF) D = -50 kN

right

SF) B = -50 kN

$x = 5 \text{ m}$

$x = 5 \text{ m}$

BM

BM) A = 0

BM) B = 0

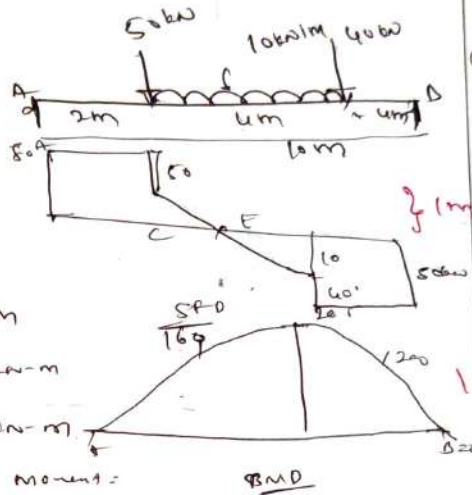
BM) C = 160 kN-m

BM) D = 205 kN-m

BM) E = 205 kN-m

max Bending moment =

$= 205 \text{ kN-m}$



05

10m

1m

2a)

SF) B = 0

SF) C = $2 \times 0.25 = 0.5 \text{ kN}$

left

SF) C = $0.5 + 3 = 3.5 \text{ kN}$

right

SF) D = $3.5 + 2 \times 1 = 5.5 \text{ kN}$

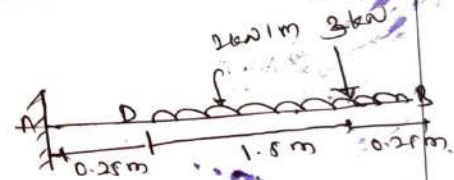
SF) A = 5.5 kN

BM) B = 0

BM) D = $-2 \times 0.25 \times \frac{0.25}{2} = -0.0625 \text{ kN-m}$

BM) E = $-2 \times 1.25 \times \frac{1.25}{2} + (3 \times 1) = -4.5625 \text{ kN-m}$

BM) A = $-2 \times (1.25 \times \frac{1.25}{2} + 0.25) - 3 \times (1 \times 1) = -5.9375 \text{ kN-m}$

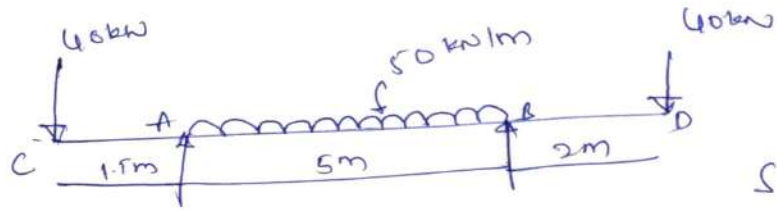


2kN

2kN

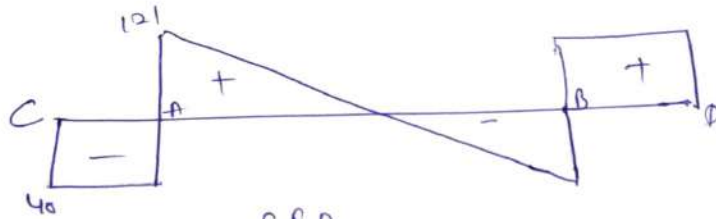
5m

2 (5)

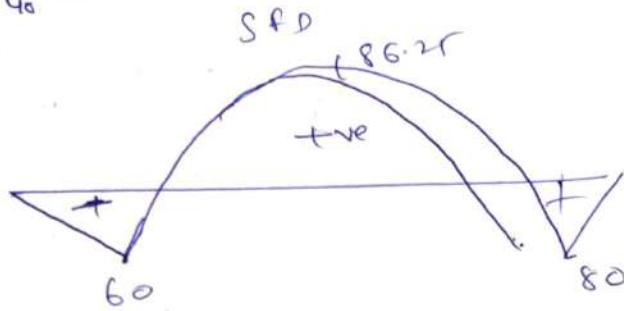


$$\begin{aligned} SF|_C &= -40 \text{ kN} \\ SF|_A &= 12 \text{ kN} \\ SF|_B &= -129 + 163 \\ &= 40 \text{ kN} \\ SF|_D &= +40 \text{ kN} \end{aligned}$$

-5m



-5m



$$\begin{aligned} BM|_C &= 0 \\ BM|_A &= -60 \text{ kNm} \\ BM|_E &= 86.25 \text{ kNm} \\ BM|_B &= -80 \text{ kNm} \\ BM|_D &= 0 \end{aligned}$$

10m

Scheme of Valuation

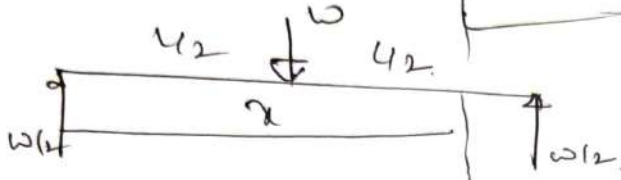
35

Assumption - 1m

Derivation - 4m

$$\frac{M}{L} = \frac{F}{4} = \frac{E}{L}$$

35



35

Assumption - 1m

Derivation - 4m

$$\frac{M}{L} = \frac{F}{4} = \frac{E}{L}$$

$$BM(x-x) = \frac{wx}{2} - w(x-\frac{L}{2})$$

$$EI \frac{d^2y}{dx^2} = -\frac{wx}{2} + w(x-\frac{L}{2})$$

$$EI \frac{dy}{dx} = -\frac{wx^2}{4} + \frac{w}{2} (x-\frac{L}{2})^2 + C_1$$

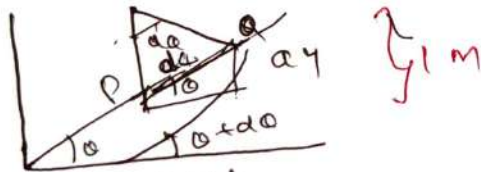
$$EI y = \frac{-wx^3}{12} + \frac{w}{6} (x-\frac{L}{2})^3 + C_1 x + C_2$$

Boundary condition
 $C_2 = 0, C_1 = \frac{wL^2}{16}$

Slope $\frac{dy}{dx} \Big|_A = \frac{wL^2}{16EI}, \frac{dy}{dx} \Big|_B = \frac{-wL^2}{16EI}$

Deflection $y_{max} = \frac{wL^2}{48EI}$

15



$$ds = R(d\alpha)$$

$$\frac{1}{R} = \frac{d\alpha}{dy}$$

$$\tan \alpha = \frac{dy}{dx} \quad \sec \alpha = \frac{d\alpha}{dx}$$

$$\frac{dy}{dx} = \tan \alpha$$

Scheme of Valuation

$$\frac{d^2y}{dx^2} = \sec^2 \theta \cdot \frac{d\theta}{dx}$$

$$\frac{d\theta}{dx} = \frac{d\theta}{dy} \cdot \frac{dy}{dx} = \frac{1}{R} \sec \theta$$

$$\frac{d^2y}{dx^2} = \sec^2 \theta \left(\frac{1}{R} \right) \sec \theta = \sec^3 \theta \left(\frac{1}{R} \right)$$

$$\frac{1}{R} = \frac{d^2y (dx^2)}{(\sec^3 \theta)^{3/2}}$$

$$\frac{1}{R} = \frac{d^2y (dx^2)}{\left(1 + \frac{dy}{dx} \right)^{3/2}}$$

$$\frac{1}{R} = \frac{d^2y}{dx^2}$$

$$M = EI \cdot \frac{d^2y}{dx^2}$$

⑤ $I = 450 \times 10^6 \text{ mm}^4 \rightarrow \textcircled{1}$

$$\frac{M}{I} = \frac{d^2y}{dx^2} = \frac{E}{R}$$

$$M = \frac{EI}{R} = \frac{7.5 \times 450 \times 10^6}{(20/2)} = 22.5 \times 10^6 \text{ N-mm} \quad \left. \begin{array}{l} 2 \text{ m} \\ 1 \text{ m} \end{array} \right\}$$

$$M = \frac{wL^2}{8} = 22.5 \times 10^6 \quad \left. \begin{array}{l} 1 \text{ m} \\ 2 \text{ m} \end{array} \right\}$$

$$w = 1.8 \text{ kN/m}$$

$$\text{Bm at } 1 \text{ m} = 9 \times 1 - \frac{1.8 \times 1^2}{2} = 8.1 \text{ kN-m} \quad \left. \begin{array}{l} 2 \text{ m} \\ 1 \text{ m} \end{array} \right\}$$

Bending Stress at 1 m = $\frac{M}{I} = \frac{1}{4}$

$$t_{1 \text{ m}} = \frac{M \times y}{I} = 2.7 \text{ MPa} \quad \left. \begin{array}{l} 2 \text{ m} \\ 1 \text{ m} \end{array} \right\}$$

$$R \text{ at } m =$$

$$\frac{M}{I} = \frac{E}{R}$$

$$R = 700 \times 10^3 \text{ mm}^3 \quad \left. \begin{array}{l} 2 \text{ m} \\ 1 \text{ m} \end{array} \right\}$$

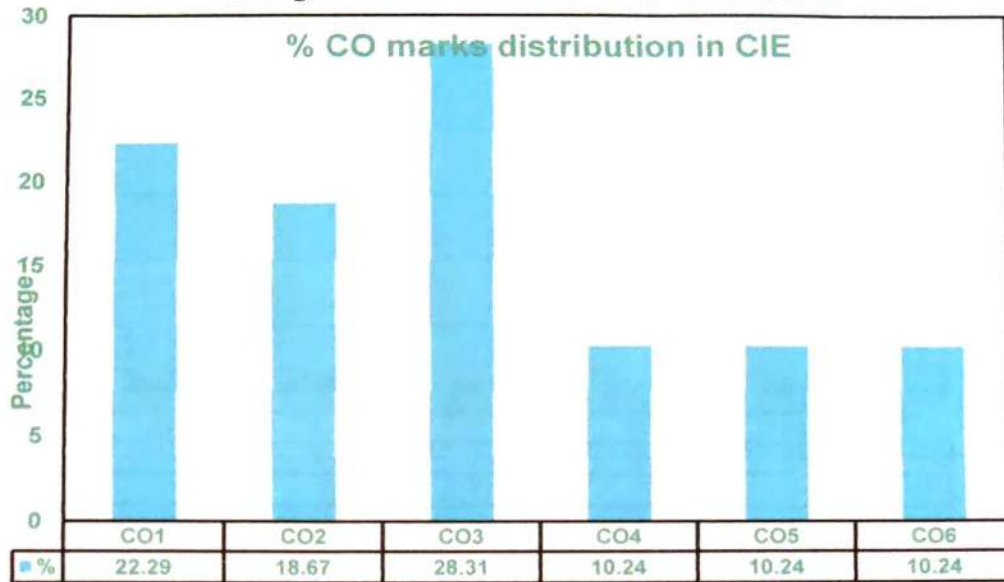
[Signature]
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 Course Coordinator
 Civil Engineering Department
 B.I.E.T., Davanagere.

[Signature]
 Coordinator
 DQAC

[Signature]
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 (HOD, Civil)

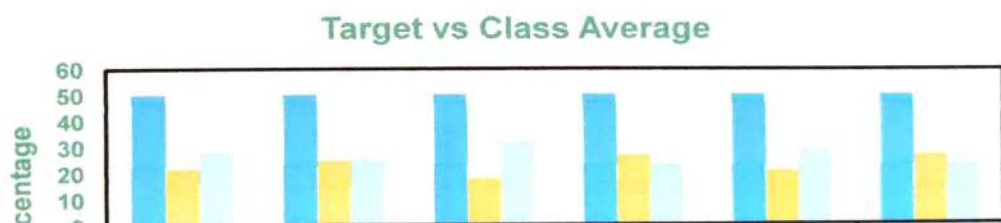
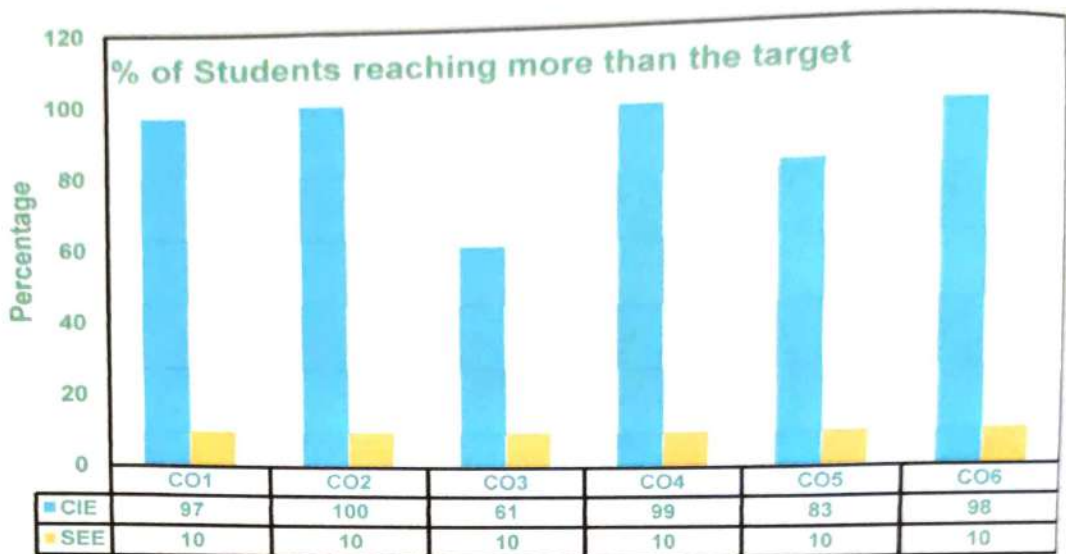
Attainment

Strength of materials 18CV32 ODD SEM 2020-21

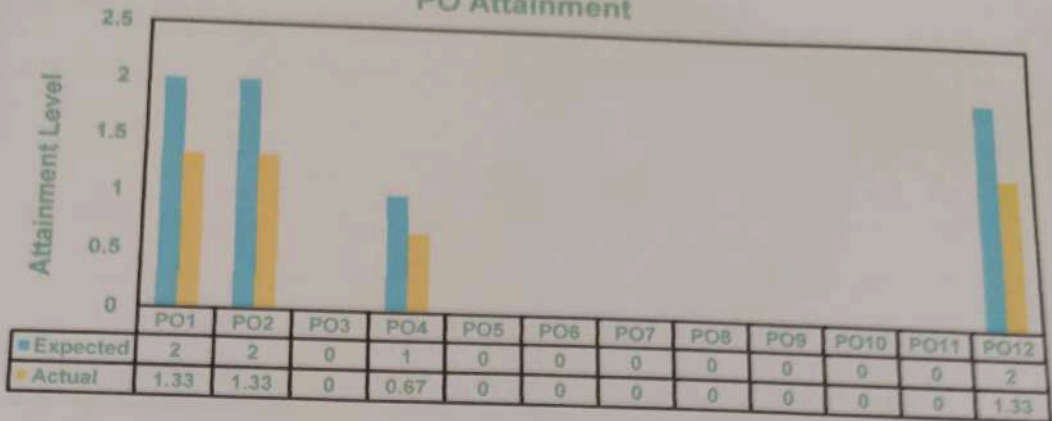


20-2021

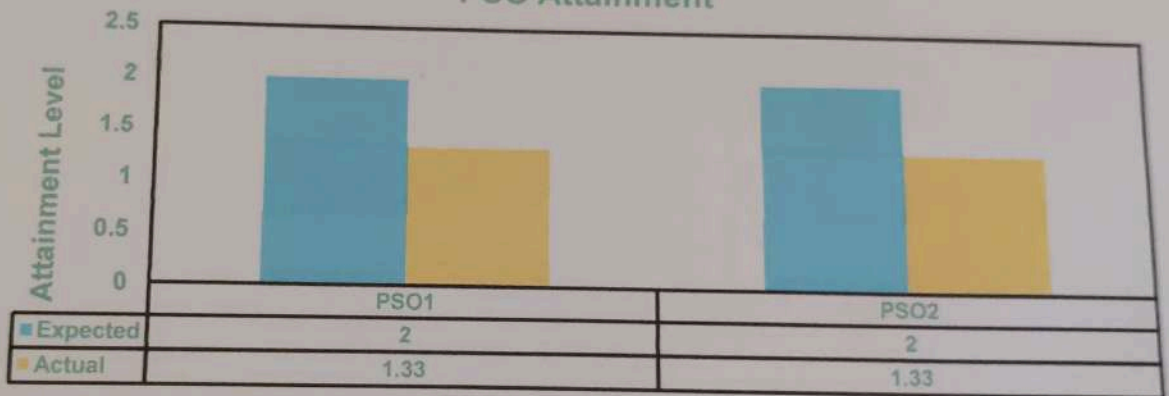
**Bapuji Educational Association @
Bapuji Institute of Engineering and Technology, Davangere-577 004
Department of Civil Engineering**



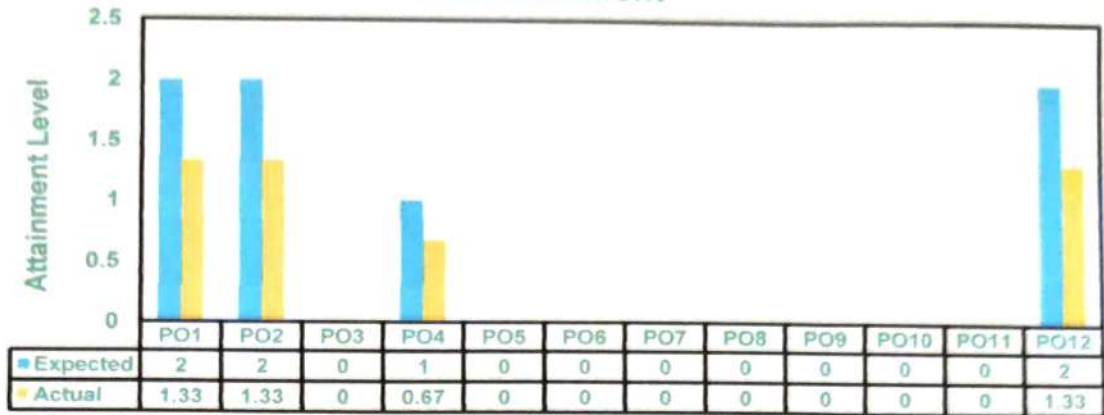
PO Attainment



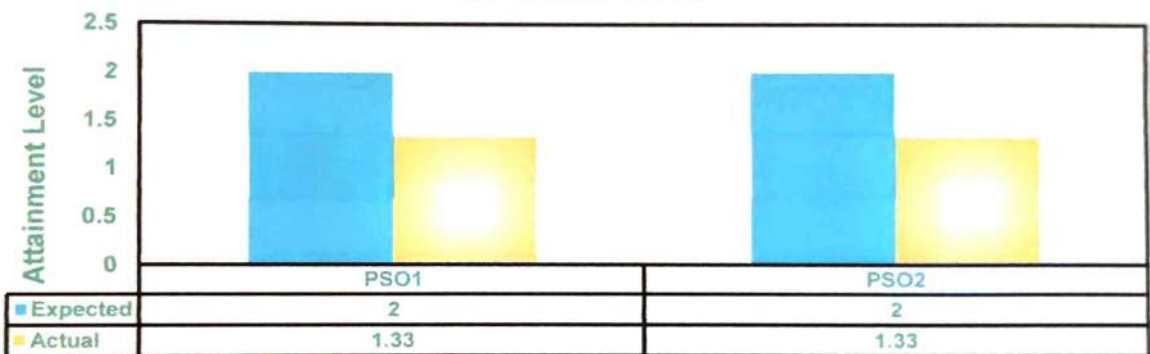
PSO Attainment



PO Attainment



PSO Attainment



Bapuji Educational Association ®
Bapuji Institute of Engineering and Technology, Davangere-577 004
Department of Civil Engineering

Result Analysis Academic Year : 2020 -2021 ODD SEM

Name of the Faculty		Raghu M E		Academic Year : 2020 - 2021 ODD SEM	
Sl. No.	Subject Title	Subject Code	Total No. of students appeared for the exam	No. of Students passed	Pass Percentage
1	Strength of materials	18CV32	70	41	59


Staff in Charge


HOD Civil





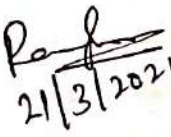




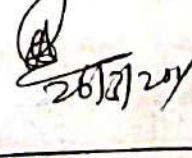
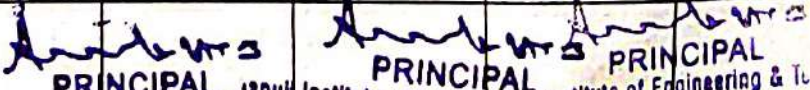
2020
 Period : From ... September To February - 2021 Semester : Odd/Even

Name of the Teacher : Raghu. M.E

Designation : Assistant Professor

Department : Civil Department

Sl. No.	Sem. / Sec. / Branch	Subject Name	Subject Code
1	III rd A Civil	Strength of materials	18CV32
2	III B Civil	Basic surveying	18CV35
3	III A A ₁ Civil	Basic material testing lab	18CV38
4			
5			
6			
7			

	Reviews at the end of the				End of Semester
	1st Month	2nd Month	3rd Month	4th Month	
Signature of Staff	 30.9.2021	 30.10.2021	 30.12.2021	 28.2.2021	 21/3/2021
Signature of the Head of Department	 30/9/21	 3/10/21	 30/12/21	 28/2/21	 26/3/21
Signature of the Principal	 PRINCIPAL Japuji Institute of Engineering & Technology DAYANGERE.				

Class : II A

Subject Code : 18CV32

Subject : Surveying - 2, Mathematics - I

Total No. of Classes : 56

Sl. No.	USN	NAME	DATE												No. of Days Present	%	Test Marks			Average	Remarks		
			28/10/20	29/10/20	30/10/20	31/10/20	01/11/20	02/11/20	03/11/20	04/11/20	05/11/20	06/11/20	07/11/20	08/11/20			I	II	III				
1		Abaran Shajib	A	A	A	A	A	A	A	A	A	A	A	A	A	1	10	24	23	08	19	29	
2		Altal Noorsabhaney	A	A	A	A	A	A	A	A	A	A	A	A	A	2	10	30	30	12	24	34	
3		Akshad R Andanuv	A	A	A	A	A	A	A	A	A	A	A	A	A	3	10	30	30	21	27	37	
4		Akhil C Aluia	A	A	A	A	A	A	A	A	A	A	A	A	A	4	10	30	30	19	27	37	
5		Akshath Ashoka P	A	A	A	A	A	A	A	A	A	A	A	A	A	5	10	30	30	15	25	35	
6		Akshaya Kumara-L-M	A	A	A	A	A	A	A	A	A	A	A	A	A	6	10	27	30	18	26	36	
7		Amritha K.M	A	A	A	A	A	A	A	A	A	A	A	A	A	7	10	24	30	14	25	33	
8		ANPithy Baskaraj's Patil	A	A	A	A	A	A	A	A	A	A	A	A	A	8	10	30	30	10	23	33	
9		Banvaraj S Patil	A	A	A	A	A	A	A	A	A	A	A	A	A	9	10	30	30	67	23	33	
10		Bhumiya P.	A	A	A	A	A	A	A	A	A	A	A	A	A	10	10	30	30	67	23	33	
11		Chinnayee K.	A	A	A	A	A	A	A	A	A	A	A	A	A	11	10	30	30	27	27	37	
12		Darshan V.M	A	A	A	A	A	A	A	A	A	A	A	A	A	12	10	34	24	9	14	29	
13		Devesh. U.S.	A	A	A	A	A	A	A	A	A	A	A	A	A	13	10	30	30	30	28	35	
14		Hemanth Kumar D.V.	A	A	A	A	A	A	A	A	A	A	A	A	A	14	10	24	23	08	18	28	
15		Jhanavi D.	A	A	A	A	A	A	A	A	A	A	A	A	A	15	10	30	30	30	26	36	
16		Krutika S.P.	A	A	A	A	A	A	A	A	A	A	A	A	A	16	10	30	30	12	26	36	
17		M-Syed Anwar	A	A	A	A	A	A	A	A	A	A	A	A	A	17	10	30	30	24	28	38	
18		Maujunnats M	A	A	A	A	A	A	A	A	A	A	A	A	A	18	10	30	30	24	28	38	
19		Mithun V.	A	A	A	A	A	A	A	A	A	A	A	A	A	19	10	21	19	02	14	24	
20		Mohammed Faizan Manabadi	A	A	A	A	A	A	A	A	A	A	A	A	A	20	10	25	23	13	21	31	
21		Nadun Ks. A.2.	A	A	A	A	A	A	A	A	A	A	A	A	A	21	10	25	23	13	21	31	
22		Nanda K.B	A	A	A	A	A	A	A	A	A	A	A	A	A	22	10	30	30	30	21	37	
23		Nirafson G.M.	A	A	A	A	A	A	A	A	A	A	A	A	A	23	10	25	30	05	22	32	
24		Paaja P.	A	A	A	A	A	A	A	A	A	A	A	A	A	24	10	25	30	00	19	29	
25		Pradeep P.N.	A	A	A	A	A	A	A	A	A	A	A	A	A	25	10	23	21	04	16	26	
26		Prakharth N.E.	A	A	A	A	A	A	A	A	A	A	A	A	A	26	10	24	23	08	12	22	
27		Ragheevendra V.	A	A	A	A	A	A	A	A	A	A	A	A	A	27	10	30	30	04	22	32	
28		Raveen H.N.	A	A	A	A	A	A	A	A	A	A	A	A	A	28	10	25	21	12	20	30	
29		Sambram Snaith	A	A	A	A	A	A	A	A	A	A	A	A	A	29	10	26	30	48	15	24	
30		Sevath	A	A	A	A	A	A	A	A	A	A	A	A	A	30	10	30	23	08	21	31	

Initials of Teacher
 Initials of H.O.D.
 Initial of Principal

PRINCIPAL
 Institute of Engineering & Technology
 DAVVANGERE

Sl No.	USN	NAME	DATE
31	UBD19CV061	Shritha D.K. X	A
32	UBD19CV063	Soumdarya . N.M X	1 2 3 4 5 6 7 8 9 10 A
33	UBD19CV065	Suaree V Kate X	1 2 3 4 5 6 7 8 9 10 A
34	UBD19CV067	U madhukhye	1 2 3 4 5 6 7 8 9 10 11
35	UBD19CV069	Vafarmani . U.	1 2 3 4 5 6 7 8 9 10 11
36	UBD19CV070	Veeran . R.K. X	1 2 3 4 5 6 7 8 9 10 11
37	UBD19CV073	Vijayalakshmi R.Telkar	1 2 3 4 5 6 7 8 9 10 11
38	UBD19CV075	Vikas s.m.	1 2 3 4 5 6 7 8 9 10 11
39	UBD19CV077	Vibekandanada. H.M X	1 2 3 4 5 6 7 8 9 10 11
40	UBD19CV079	Vijith B.John X	A A A A A A A A A A A
41	UBD19CV089	Vinay Kumar M	A 1 A A A A A A A A A
42	UBD19CV093	Mohammed Azhar Razith	A A A A A A A A A A A
43		Trupti . S.R. X	A A A A A A A A A A A
44	UBD19CV097	Harekith. N.	A A A A A A A A A A A
45		Chananya. H. Patil X	A A A A A A A A A A A
46	UBD19CV076	A Shivaraj K. T X	A A A A A A A A A A A
47		Bharathi Reddy N X	A A A A A A A A A A A
48		A Mohamed bhagir X	A A A A A A A A A A A
49		Abhilesh S.V X	A A A A A A A A A A A
50		Dhanush R. X	A A A A A A A A A A A
51		Sunabhi S. Bharadwaj	A A A A A A A A A A A
52		Chetan Satyraj X	A A A A A A A A A A A
53		Vinod Kumar Buddini	A A A A A A A A A A A
54		Nandesh M Mattoo	A A A A A A A A A A A
55		Santosh S Rajgor X	A A A A A A A A A A A
56	UBD19CV100	A Sufitha R	A A A A A A A A A A A
57		Rajitha A X	A A A A A A A A A A A
58		Deepti G. Girvat. Kiran M.M X	A A A A A A A A A A A
59		Suhasi . N. Patkar X	A A A A A A A A A A A
60		Munugharajendhu. V.V	A A A A A A A A A A A
61		Shree Harshitha R X	A A A A A A A A A A A

Sl No.	USN	NAME	DATE	No. of Days Present	%	Test Marks			Average	Remarks
						I	II	III		
31	UBD19CV061	Shritha D.K. X	A	1	100					
32	UBD19CV063	Soumdarya . N.M X	1 2 3 4 5 6 7 8 9 10 A	10	100					
33	UBD19CV065	Suaree V Kate X	1 2 3 4 5 6 7 8 9 10 A	10	100					
34	UBD19CV067	U madhukhye	1 2 3 4 5 6 7 8 9 10 11	11	100					
35	UBD19CV069	Vafarmani . U.	1 2 3 4 5 6 7 8 9 10 11	11	100					
36	UBD19CV070	Veeran . R.K. X	1 2 3 4 5 6 7 8 9 10 11	11	100					
37	UBD19CV073	Vijayalakshmi R.Telkar	1 2 3 4 5 6 7 8 9 10 11	11	100					
38	UBD19CV075	Vikas s.m.	1 2 3 4 5 6 7 8 9 10 11	11	100					
39	UBD19CV077	Vibekandanada. H.M X	1 2 3 4 5 6 7 8 9 10 11	11	100					
40	UBD19CV079	Vijith B.John X	A A A A A A A A A A A	11	100					
41	UBD19CV089	Vinay Kumar M	A 1 A A A A A A A A A	11	100					
42	UBD19CV093	Mohammed Azhar Razith	A A A A A A A A A A A	11	100					
43		Trupti . S.R. X	A A A A A A A A A A A	11	100					
44	UBD19CV097	Harekith. N.	A A A A A A A A A A A	11	100					
45		Chananya. H. Patil X	A A A A A A A A A A A	11	100					
46	UBD19CV076	A Shivaraj K. T X	A A A A A A A A A A A	11	100					
47		Bharathi Reddy N X	A A A A A A A A A A A	11	100					
48		A Mohamed bhagir X	A A A A A A A A A A A	11	100					
49		Abhilesh S.V X	A A A A A A A A A A A	11	100					
50		Dhanush R. X	A A A A A A A A A A A	11	100					
51		Sunabhi S. Bharadwaj	A A A A A A A A A A A	11	100					
52		Chetan Satyraj X	A A A A A A A A A A A	11	100					
53		Vinod Kumar Buddini	A A A A A A A A A A A	11	100					
54		Nandesh M Mattoo	A A A A A A A A A A A	11	100					
55		Santosh S Rajgor X	A A A A A A A A A A A	11	100					
56	UBD19CV100	A Sufitha R	A A A A A A A A A A A	11	100					
57		Rajitha A X	A A A A A A A A A A A	11	100					
58		Deepti G. Girvat. Kiran M.M X	A A A A A A A A A A A	11	100					
59		Suhasi . N. Patkar X	A A A A A A A A A A A	11	100					
60		Munugharajendhu. V.V	A A A A A A A A A A A	11	100					
61		Shree Harshitha R X	A A A A A A A A A A A	11	100					

Class: Unit 9 A-Section Subject Code: 18CV32 Subject: Strength of Materials Total No. of Classes: 56

Sl. No.	USN	NAME	DATE	18/10/22	19/10/22	20/10/22	21/10/22	22/10/22	23/10/22	24/10/22	25/10/22	26/10/22	27/10/22	28/10/22	29/10/22	30/10/22	31/10/22	01/11/22	02/11/22	03/11/22	04/11/22	05/11/22	06/11/22	07/11/22	08/11/22	09/11/22	10/11/22	11/11/22	12/11/22	13/11/22	14/11/22	15/11/22	No. of Days Present	%	Test Marks	Average	Remarks			
31		Shreyas D.K	26/10/22	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	10	24	23	27	27		
32		Soujanya N.M	26/10/22	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	10	20	30	19	27		
33		Suvarshi V.W	27/10/22	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	10	30	30	11	24	34	
34		U Madhukar	26/10/22	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	10	30	38	28	26	32	
35		Vaishnavi V.	26/10/22	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	10	30	30	20	27	37	
36		Veeresh K.R.	27/10/22	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	10	30	30	05	20	30	
37		Vijayalakshmi R. Telkar	27/10/22	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	10	30	30	20	27	37	
38		Vikas S.M.	27/10/22	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	10	19	30	60	16	26	
39		Vivekananda H.M.	27/10/22	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	10	30	28	01	18	28	
40		Yash B. Jain	27/10/22	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	10	26	30	12	25	33	
41		Yash Kumar M.	27/10/22	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	10	24	23	05	19	27	
42		Mohammed Ashar Raikhi	27/10/22	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	10	16	21	25	06	18	28
43		TRUPTI S.D.	27/10/22	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	10	30	30	02	21	31	
44		HAYASHI N.	27/10/22	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	10	30	30	07	23	33	
45		Chananeh H. DATE	27/10/22	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	10	28	30	01	20	38	
46		Shivraj W.H.	27/10/22	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	10	18	27	05	15	25	
47		Bhavath Dedy	27/10/22	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	10	27	29	63	20	30	
48		Mohammed Ibrahim	27/10/22	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	10	30	30	06	22	32	
49		Abhishek S V	27/10/22	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	10	30	30	04	22	32		
50		Dhanush R.	27/10/22	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	10	28	24	01	18	28		
51		Surathi S. Bharendra	27/10/22	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	10	30	30	09	20	30		
52		Chethan Katrevala	27/10/22	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	10	24	27	06	19	29		
53		Mihel Kumar M. Kalyan	27/10/22	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	10	30	20	02	17	27		
54		NALAKSHI M. MATHAS	27/10/22	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	10	30	30	07	23	33	
55		Georgina B. Bhandari	27/10/22	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	10	30	30	07	22	32	
56		Georgina B. Bhandari	27/10/22	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	10	30	30	07	22	32		
57		Georgina B. Bhandari	27/10/22	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	10	30	30	07	22	32		
58		Georgina B. Bhandari	27/10/22	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	10	30	30	07	22	32		
59		Georgina B. Bhandari	27/10/22	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	10	30	30	07	22	32		
60		Georgina B. Bhandari	27/10/22	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	10	30	30	07	22	32		
61		Shree Namini S. Shrinidhi	27/10/22	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	10	30	30	03	21	31		

PRINCIPAL
Jyotif Institute of Engineering & Technology
DAVANGERE

Sl No.	USN	NAME	DATE	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	No. of Days Present	%	Test Marks			Average	Remarks															
																																				I	II	III																	
1	UGD19CV002	Abhinav K Chittaggar	10/10	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A																			
2	UGD19CV004	Asay A Sajuni	10/10	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A																
3	UGD19CV008	AKHAY G.M.	10/10	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A														
4	UGD19CV010	AJ AKHAY	10/10	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A													
5	UGD19CV012	Anuoka S.	10/10	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A													
6	UGD19CV014	B Sayed Sameer Zakiya	10/10	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A												
7	UGD19CV018	Bharathgouda K.N.	10/10	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A												
8	UGD19CV018	Chandan B.V.	10/10	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A												
9	UGD19CV022	Deeda P.	10/10	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A											
10	UGD19CV034	Harith K.T.	10/10	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A										
11	UGD19CV028	Karthik D Vennekari	10/10	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A										
12	UGD19CV030	Lakshmi B.S.	10/10	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A										
13	UGD19CV032	Malikarjun T.M.	10/10	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A										
14	UGD19CV034	Manoj suamy K.M.	10/10	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A									
15	UGD19CV036	Mohammed Amerkhan D	10/10	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A									
16	UGD19CV038	Mohit G.V.	10/10	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A									
17	UGD19CV044	Namrath P Ram	10/10	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A									
18	UGD19CV042	Nandini K.	10/10	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A									
19	UGD19CV044	P.N. Mohammed Saibuddin	10/10	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A									
20	UGD19CV046	Pooja M.P.	10/10	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A									
21	UGD19CV048	Pradeep K.B.	10/10	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A									
22	UGD19CV050	Pragnaal Q.T. Matad	10/10	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A								
23	UGD19CV052	Prerana V.	10/10	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A								
24	UGD19CV054	Raghu. Doddamanani	10/10	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A								
25	UGD19CV054	Ravihar Banu A.K.S.	10/10	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A								
26	UGD19CV058	Sanjay G.A.	10/10	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A								
27	UGD19CV060	Shashi Kiran E.P.	10/10	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A								
28	UGD19CV062	Siraj Ahmed Y.B.	10/10	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A								
29	UGD19CV064	Sunil N Redkar	10/10	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A							
30	UGD19CV066	Tegawani M.	10/10	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A							

Initials of Teacher
Initials of H.O.D.
Initial of Principal

Japuji Institute of Engineering & Technology
DAVANGERE
PRINCIPAL

Class : **C1 B Section**

Subject Code : **18CV35**

Subject : **Basic Surveying**

Total No. of Classes : **36**

Sl No.	USN	NAME	DATE	Test Marks			Average	Remarks
				I	II	III		
1	UGD19CV002	Abhinav K C I	01/03/20	10	24	21	10.19	27
2	UGD19CV004	Ajay A Jain	02/03/20	10	21	9	18.28	28
3	UGD19CV008	Akshay B M.	03/03/20	10	23	9	15.25	25
4	UGD19CV010	Alp Akbar	04/03/20	10	25	3	15.48	25
5	UGD19CV012	Anusha S.	05/03/20	10	24	6	15.25	25
6	UGD19CV014	B. Sayed Daw	06/03/20	10	25	6	16.23	26
7	UGD19CV016	GhanathSouda	07/03/20	10	22	3	15.25	25
8	UGD19CV018	Chandan B.V	08/03/20	10	20	3	15.25	25
9	UGD19CV022	Deepa P.	09/03/20	10	22	3	15.25	25
10	UGD19CV024	Harish K.T.	10/03/20	10	22	3	15.25	25
11	UGD19CV028	Karthik D ver	11/03/20	10	22	3	15.25	25
12	UGD19CV030	Lakshmi B.S	12/03/20	10	22	3	15.25	25
13	UGD19CV032	MallikaJun	13/03/20	10	22	3	15.25	25
14	UGD19CV034	Manoj Swamy	14/03/20	10	22	3	15.25	25
15	UGD19CV036	Mohamed Ame	15/03/20	10	22	3	15.25	25
16	UGD19CV038	Mohit B V	16/03/20	10	22	3	15.25	25
17	UGD19CV040	Namrathya P E	17/03/20	10	22	3	15.25	25
18	UGD19CV042	Nandini K.	18/03/20	10	22	3	15.25	25
19	UGD19CV044	P N. Mohammed	19/03/20	10	22	3	15.25	25
20	UGD19CV046	Pooja M P	20/03/20	10	22	3	15.25	25
21	UGD19CV048	Pradeep K.B	21/03/20	10	22	3	15.25	25
22	UGD19CV050	Pravejal Q.T	22/03/20	10	22	3	15.25	25
23	UGD19CV052	Preerna V.	23/03/20	10	22	3	15.25	25
24	UGD19CV054	Raghav Dadda	24/03/20	10	22	3	15.25	25
25	UGD19CV056	Rutkar Bani	25/03/20	10	22	3	15.25	25
26	UGD19CV058	Sangeetha E.T	26/03/20	10	22	3	15.25	25
27	UGD19CV060	Shashik K.	27/03/20	10	22	3	15.25	25
28	UGD19CV062	Siraj Ahmed	28/03/20	10	22	3	15.25	25
29	UGD19CV064	Sunil N Reddy	29/03/20	10	22	3	15.25	25
30	UGD19CV066	Tegawini M.	30/03/20	10	22	3	15.25	25

Initials of Teacher
Initials of H.O.D.
Initial of Principal

PRINCIPAL
DAVANAGERE

Total No. of Classes : 28

Subject: Basic Surveying

Class : UI B Section

Subject Code : 18CV35

Sl No.	USN	NAME	DATE	Days																												No. of Days Present	%	Test Marks			Average	Remarks
				1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28			I	II	III		
31	UGD19CV066	Varikhanvi. V.	15/10/20	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	10	100			
32	UGD19CV028	Veeran. G. R. S.V.	15/10/20	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	10	100			
23	UGD19CV072	Veman M. Elvadi	15/10/20	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	10	100			
34	UGD19CV074	Utkar D. Nadav	15/10/20	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	10	100			
35	UGD19CV074	Vivek. A.P.	15/10/20	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	10	100			
32	UGD19CV078	Yasharajini. M.V.	15/10/20	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	10	100			
37	UGD19CV080	Zaiba Anjum	15/10/20	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	10	100			
38	UGD19CV090	T.Joseph. B.R.	15/10/20	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	10	100			
39	UGD19CV065	Pavan K. C.D.	15/10/20	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	10	100			
40	UGD18CV024	Evanesh. B.E.	15/10/20	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	10	100			
41	UGD19CV047	Mohan. D. D.	15/10/20	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	10	100			
42	UGD19CV078	Siddarth. Ganes. M.S.	15/10/20	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	10	100			
43	UGD19CV002	Uday. K.H.	15/10/20	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	10	100			
44	UGD19CV025	Dishantha	15/10/20	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	10	100			
45	UGD19CV002	Adarsh. N.C.	15/10/20	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	10	100			
46	UGD19CV078	Shankar. P.	15/10/20	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	10	100			
47	UGD19CV078	Shankar. N.A.	15/10/20	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	10	100			
48	UGD19CV078	Mohan Kumar. S.E.	15/10/20	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	10	100			
49	UGD19CV078	Sharath. K.R.	15/10/20	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	10	100			
50	UGD19CV078	Chetan. S.	15/10/20	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	10	100			
51	UGD19CV078	Chirudeep. J.	15/10/20	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	10	100			
52	UGD19CV078	Vinay. K.	15/10/20	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	10	100			
53	UGD19CV078	Hyla Kesavay. K.	15/10/20	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	10	100			
54	UGD19CV078	Pranav. R. P. R.	15/10/20	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	10	100			
55	UGD19CV078	Hemanth. H. V.	15/10/20	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	10	100			
56	UGD19CV078	Prasanna. K. S.	15/10/20	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	10	100			
57	UGD19CV078	Sobha. S.	15/10/20	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	10	100			
58	UGD19CV078	Rajitha. Kumar. N. L.	15/10/20	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	10	100			
59	UGD19CV078	Vinay. D. V.	15/10/20	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	10	100			
60	UGD19CV078	Deeptika. P. Pragy. H. S.	15/10/20	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	10	100			
61	UGD19CV078	Prakash. D.	15/10/20	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	A	10	100			

Class :		Subject Code :	
11 B		18CV35	
Sl No	USN	NAME	DATE
61		Maharaja P O	12/01/2021
62		Saravathi K	12/01/2021
63		Rajalakshmi H	12/01/2021
64		Vinay K	12/01/2021
65		Chidambaram H	12/01/2021
66		Valent. Man	12/01/2021
67		Ghavana. M	12/01/2021
68		Pandey L	12/01/2021
69		Muralidharan B	12/01/2021
70		Mysuru Chavan	12/01/2021
71		Mohammed Saad	12/01/2021
72		Mohammed Saad	12/01/2021
73		Mohammed Saad	12/01/2021
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Class :		Subject Code :		Total No. of Classes :			
11 B		18CV35		56			
Sl No	USN	NAME	No. of Days Present	% Present	Test Marks	Average	Remarks
					I II III		
61		Maharaja P O	10	25	23 12	22	32
62		Saravathi K	10	25	16	23	33
63		Rajalakshmi H	10	25	30	28	35
64		Vinay K	10	25	5	21	31
65		Chidambaram H	10	25	18	24	34
66		Valent. Man	10	25	12	15	29
67		Ghavana. M	10	25	12	23	35
68		Pandey L	10	25	16	22	32
69		Muralidharan B	10	25	26	25	35
70		Mysuru Chavan	10	25	16	22	32
71		Mohammed Saad	10	25	20	24	34
72		Mohammed Saad	10	25	14	22	32
73		Mohammed Saad	10	25	14	22	32
74							
75							
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Initials of Teacher: [Signature]
 Initials of H.O.D.: [Signature]
 Initials of Principal: [Signature]
 DAVANGERE

Class : II A

Subject Code : 18CVL38

Subject : Basic Mathematics Texting part

Total No. of Classes : 10

Sl No.	USN	NAME	DATE											No. of Days Present	%	Test Marks			Average	Remarks																																	
				1	2	3	4	5	6	7	8	9	10			I	II	III																																			
1	LB019CV001	Aboon Shariff	1/2/24	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50
2	UB019CV003	Abdul Noor Sabhanavar	1/2/24	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50
3	UB019CV005	Akashdeep E. Anandhar	1/2/24	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50
4	UB019CV006	Akhil C. AKULA	1/2/24	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50
5	UB019CV007	Akshaya Arunbabu Sathyanarayana	1/2/24	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50
6	UB019CV009	Akshaya Kumara C.M.	1/2/24	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50
7	UB019CV011	Amita K.M.	1/2/24	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50
8	UB019CV013	Arpita Basappa S.Pattil	1/2/24	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50
9	UB019CV015	Balvraj S.Pattil	1/2/24	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50
10	UB019CV017	Bhumiya P.	1/2/24	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50
11	UB019CV019	Chinnahyre.K.	1/2/24	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50
12	UB019CV021	Darshan V.N.	1/2/24	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50
13	UB019CV023	Eanesh U Shirgereg	1/2/24	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50
14	UB019CV025	Hemanth Kumar.D.V.	1/2/24	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50
15	UB019CV029	Vinay Kumar M?	1/2/24	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50
16	UB019CV027	Harkith N ?	1/2/24	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50
17	UB019CV027	Bhavana Reddy N	1/2/24	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50
18	UB019CV027	Dhanush.D	1/2/24	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50
19	UB019CV027	Vinod Kumar Gudhishl	1/2/24	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50
20	UB019CV027	Sajitha R. kutt	1/2/24	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50
21	UB019CV027	Sureef N. Dakkar	1/2/24	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50
22	UB019CV027	Sandeep B.N	1/2/24	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50
23	UB019CV027	TRIVHENA	1/2/24	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50
24	UB019CV027	Veena B. Bharamangadatti	1/2/24	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50
25	UB019CV027	Trupti Balikai ?	1/2/24	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50

Initials of Teacher
Initials of H.O.D.
Initial of Principal

Principal
DAVANGERE

Class : 3-A Subject Code : 18CV32

Subject : Statistics Total No. of Classes : 94

Sl No.	USN	NAME	DATE	Attendance												No. of Days Present	%	Test Marks			Average	Remarks																																																																								
				1	2	3	4	5	6	7	8	9	10	11	12			I	II	III																																																																										
1	4801SCV001		23/10/20	A	S	E	F	S	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94

Initials of Teacher
Initials of H.O.D.
Initial of Principal

Principal
DAVANAGERE

Class :

Subject Code :

18CV52

Subject : Strength of Materials

Total No. of Classes :

94

Sl No.	USN	NAME	DATE	No. of Days Present		Test Marks			Average	Remarks
				I	II	I	II	III		
31	UGD019RV021		4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31	0	0					
32	UGD019RV021		4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31	0	0					
33	UGD019RV021		4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31	0	0					
34	UGD019RV021		4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31	0	0					
35	UGD019RV021		4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31	0	0					
36	UGD019RV021		4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31	0	0					
37	UGD019RV021		4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31	0	0					
38	UGD019RV021		4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31	0	0					
39	UGD019RV021		4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31	0	0					
40	UGD019RV021		4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31	0	0					
41	UGD019RV021		4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31	0	0					
42	UGD019RV021		4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31	0	0					
43	UGD019RV021		4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31	0	0					
44	UGD019RV021		4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31	0	0					
45	UGD019RV021		4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31	0	0					
46	UGD019RV021		4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31	0	0					
47	UGD019RV021		4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31	0	0					
48	UGD019RV021		4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31	0	0					
49	UGD019RV021		4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31	0	0					
50	UGD019RV021		4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31	0	0					
51	UGD019RV021		4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31	0	0					
52	UGD019RV021		4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31	0	0					
53	UGD019RV021		4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31	0	0					
54	UGD019RV021		4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31	0	0					
55	UGD019RV021		4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31	0	0					
56	UGD019RV021		4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31	0	0					
57	UGD019RV021		4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31	0	0					
58	UGD019RV021		4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31	0	0					
59	UGD019RV021		4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31	0	0					
60	UGD019RV021		4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31	0	0					
61	UGD019RV021		4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31	0	0					

Initials of Teacher
Initials of H.O.D.
Initial of Principal

PRINCIPAL
Jyothi Institute of Engineering & Technology
DAVANGERE

Subject Code : 19CV32

Subject : Strength of Materials

Total No. of Classes : 94

94

Sl No.	USN	NAME	DATE											No. of Days Present	%	Test Marks			Average	Remarks																																																																													
				1	2	3	4	5	6	7	8	9	10			I	II	III																																																																															
62		Sandhya. D. N.	12/25	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94
63		Shankar Rao Kulkarni. C.	12/25	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94
64		Alankar. H. J.	12/25	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94
65		TRIVANI. G.	12/25	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94
66		PILLAI. O.	12/25	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94
67		HARSH. C. B.	12/25	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94
68		Veena B Bharamgoudar	12/25	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94
69		HARSH. C. N.	12/25	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94
70		HARSH. C. S.	12/25	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94
71		TRIVANI. S. Srikar	12/25	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94
72		DePa. L	12/25	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94
73		Priyanka Anjum. H.	12/25	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94

Initials of Teacher
Initials of H.O.D.
Initial of Principal

PRINCIPAL
DAVANGERE
April Institute of Engineering & Technology

LESSON PLAN

Subject : _____ Subject Code : _____ Class : _____

Period	Date	Topics Planned	Date	Topics Covered	Remarks
35	03-11-2020	Rankine & Gordon's formulae questions Problems.	03-11-2020	Rankine & Gordon's formulae questions Problems	Covered
36	09-11-2020	problems of Euler's & Gordon's cr.	09-11-2020	Problems of Euler's & Gordon's cr.	Covered
37	11-11-2020	Module: 2 Compound Stress. Introduction	11-11-2020	Module: 2 Compound Stress Introduction	Covered.
38	12-11-2020	General two dimensional stress system Principal stress & planes	12-11-2020	General two dimensional stress system Principal stress & planes	Covered
39	13-11-2020	Problems on principal stress & planes	13-11-2020	Problems on principal stress & planes	Covered.
40	16-11-2020	Problems on principal stress & planes	16-11-2020	Problems on principal stress & planes	Covered
41	15-11-2020	Module: 3 Shear Force & Bending Moment Introduction	15-11-2020	Module: 3 Shear Force & Bending Moment : Introduction	Covered.
42	21-11-2020	Types of Beams, loading supports.	21-11-2020	Types of Beams, loading supports	Covered.
43	23-11-2020	Simply supported beam with point load & eccentric loading with problems	23-11-2020	Simply supported beam with point load & eccentric loading with problems	Covered.
44	24-11-2020	Simply supported beam with UDL with problems	24-11-2020	Simply supported beam with UDL with problems	Covered
45	25-11-2020	Simply supported beam with problems	25-11-2020	Simply supported beam with problems	Covered
46	26-11-2020	Problems on simply supported beam with different loading condition	26-11-2020	Problems on simply supported beam	Covered
47	28-11-2020	Problems ..	28-11-2020	Problems ..	Covered
48	30-11-2020	Overhanging beam Division to, with different loading	30-11-2020	Overhanging beam with different loading condition	Covered
49	11/12/20	Problems	11/12/20	Problems	Covered
50	21/12/20	Problems	21/12/20	Problems	Covered
51	19/12/20	Continuous beam with different loading conditions	19/12/20	Continuous beam with different loading conditions	Covered.

[Signature]

LESSON PLAN

Period	Date	Topics Planned	Date	Topics Covered	Remarks
52	14/12/20	problems	14/12/20	problems	covered
53	15/12/20	Module: 4 Bending stress Introduction	15/12/20	Bending stress ^{moderate} Introduction	covered
54	16/12/20	Derivation an equation for bending stress $m = \frac{E}{r} = \frac{b}{y}$	16/12/20	Derivation an equation for bending stress $m = \frac{E}{r} = \frac{b}{y}$	covered
55	21/12/20	Problem on Bending stress	21/12/20	Problem on Bending stress	covered
56	22/12/20	Problem on Bending stress	22/12/20	Problem on Bending stress	covered
57	23/12/20	Problem on Bending stress	23/12/20	Problem on Bending stress	covered
58	26/12/20	Deflection of Beams Module 5	26/12/20	Deflection of Beams	covered
59	29/12/20	Derivation of Beams. Simply supported Beams with point.	29/12/20	Derivation of Beams simply supported Beams with point	covered
60	29/12/20	UOL UOL Problem	29/12/20	UOL, UOL problem	covered


Text Books :

1. B. S. Badaavarejaiah, P. Mahalingappa. Strength of materials in SI units University of PUNE (India).
2. R. Subramanian. Strength of materials latest edition.

Reference Books :


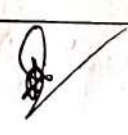
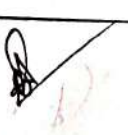
1. R. K. Bansal, A Text book of Strength of materials. 4th edition Laxmi Publication, 2010.
2. O. H. Young. S.P. Timoshenko. "elements of Strength of materials".
3. S. Ramamurtham. Strength of materials.
4. S.S. Bhavikatti. by Strength of materials.
5.


 Signature of Faculty


 HOD

TUTORIAL CLASSES

Date	Topics Discussed	Remarks
61	Problems on reflection	covered
62	Problem on deflection	covered.
63	Problem on reflection	covered
64	Problem on reflection	covered.
65	Problem on deflection	covered.
66	Problem on deflection	covered.
67	Module 2 (For Diploma) Thick & Thin cylinders Derivation of hoop stress & longitudinal stress	covered
68	Problem on same	covered
69	Problem on Thick cylinders	covered
70	Thick cylinders Derivations	covered
71	Problems on Thick cylinders	covered
72	Problems on thick cylinders	

Test	Date	Class Strength	No. of Students Appeared	No. of Students Scored < 15	Signature of the HOD
T1	24/10/20	70	70	0	
T2	07.12.2020	70	70	0	
T3	22.02.2024	70	67	50	

LESSON PLAN

Subject: Basic surveying Subject Code: L8CV35 Class: Bred B

Period	Date	Topics Planned	Date	Topics Covered	Remarks
1	10/9/20	Module 1. Definition of surveying objectives & Importance	10/9/20	Module 1. Definition of surveying objectives & importance	covered
2	14/9/20	Classification of surveying.	14/9/20	Classification of surveying	covered
3	15/9/20	Principles of surveying	15/9/20	Principles of surveying	covered
4	16/9/20	Applications of surveying unit & measurement	16/9/20	Applications of surveying unit & measurement	covered
5	21/9/20	Surveying measurement errors, types of errors precision	21/9/20	Surveying measurement errors, types of errors, precision	covered
6	23/9/20	Accuracy, classification of maps, map & numbering	23/9/20	Accuracy, classification of maps, map & numbering	covered
7	28/9/20	Measurement of horizontal distances, measuring tape & types, measurement using tapes, Taping on	28/9/20	Measurement of horizontal distances, measuring tape measurement using tapes,	covered
8	30/9/20	Level ground. + Direct ranging - Indirect ranging	30/9/20	Level ground. + Direct ranging + Indirect ranging.	covered
9	1/10/20	Sloping ground Direct & Indirect ranging	1/10/20	Sloping ground by direct & indirect ranging	covered
10	05/10/20	EDM, basic principles of levelling & tape survey work	05/10/20	EDM, basic principles of levelling & tape survey work	covered
11	07/10/20	conventional symbols, obstacles in tape surveying	07/10/20	conventional symbols, obstacles in tape surveying	covered
12	08/10/20	Problems	08/10/20	Problems	covered
13	12/10/20	Module 4 Plane table surveying accessories	12/10/20	Module 4 Plane table surveying accessories	covered
14	14/10/20	Setting up the plane table surveying	14/10/20	Setting up the plane table surveying	covered
15	15/10/20	Methods of plane table surveying Resection	15/10/20	Methods of plane table surveying Resection	covered
16	21/10/20	Intersection method, Traversing Advantages & Disadvantages of plane table surveying	21/10/20	Intersection method, Traversing Advantages & Disadvantages of plane table surveying	covered
17	22/10/20	Resection two point Problem, three point Problem	22/10/20	Resection - two point Problem & Three point Problem	covered

LESSON PLAN

Subject: Basic Surveying Subject Code: 18CV35 Class: 3rd B

Period	Date	Topics Planned	Date	Topics Covered	Remarks
18	28.10.20	Three Point Problem	28.10.20	Three Point Problem	Covered
19	29.10.20	Module 1.5: Area & Volume Differentiation - method of area calculation	29.10.20	Module 1.5: Area & volume. Introduction	Covered
20	30.10.20	Objects of Regular intervals	30.10.20	Objects of Regular Intervals	Covered
21	05.11.20	Areas from co-ordinates with problems	05.11.20	Areas from co-ordinates with problems	Covered
22	9.11.20	Problem on Area by Different methods	9.11.20	Problem on Area by Different methods	Covered
23	11.11.20	Volume: Different methods Problem on volume	11.11.20	Volume: Different methods & Problem on volume	Covered
24	18.11.20	Problem on volume	18.11.20	Problem on volume	Covered
25	15.11.20	Module 2: Compass Surveying Introduction	15.11.20	Module 2: Compass Surveying Introduction	Covered
26	23.11.20	Definition of Bowditch's Rule, WCB, AB problems	23.11.20	Definition of Bowditch's Rule, WCB, AB problems	Covered
27	25.11.20	Magnetic Dip & Declination with problems. Prismatic compass, surveyor compass with construction	25.11.20	Magnetic Dip & Declination with problems. Prismatic compass, surveyor compass with construction	Covered
28	26.11.20	Problem on Interior angles	26.11.20	Problem on Interior angles (Included angle)	Covered
29	27.11.20	Local attraction, Introduction Problem	27.11.20	Local attraction Problems	Covered
30	28.11.20	Problem on local attraction	28.11.20	Problem on local attraction	Covered
31	28.11.20	Private: computer Introduction	28.11.20	Private: computer	Covered
32	21.11.20	1. Dependent co-ordinates 2. Independent co-ordinates 3. closing error with problems	21.11.20	1. Dependent co-ordinates 2. Independent co-ordinates 3. closing error with problems	Covered
33	23.11.20	1. Bowditch rule 2. Transit rule with problems	23.11.20	Problems	Covered
34	28.11.20	Omitted measurements Problem	28.11.20	Omitted measurements with problems	Covered

LESSON PLAN

Subject: Basic Surveying Subject Code: 18CV35 Class: 2nd B

Period	Date	Topics Planned	Date	Topics Covered	Remarks
35	30-12-20	Module: 3 Leveling: Basic definitions methods of leveling.	30-12-20	Module: 3 Leveling: Basic definitions methods of leveling	Covered
36	31-12-20	Dumpy level, Auto level, digital leveling & laser leveling	31-12-20	Dumpy level, Auto level, digital leveling & laser leveling	Covered
37	4-1-21	Curvature and refraction corrections,	4-1-21	Curvature & refraction corrections	Covered
38	5-1-21	Bearing & reduction of wells.	5-1-21	Bearing & reduction of wells & Problems	Covered
39	7-1-21	Problems on leveling	7-1-21	Problems on leveling	Covered
40	13-1-21	Problems on leveling Differential leveling	13-1-21	Problems on differential leveling	Covered
41	13-1-21	Problems on differential leveling	17-1-21	Problems on differential leveling	Covered
42	21-1-21	Profile leveling with problems	21-1-21	Profile leveling with problems	Covered
43	01-2-21	fly leveling with problems	01-2-21	fly leveling with problems.	Covered
44	3-2-2021	Reciprocal leveling	3-2-2021	Reciprocal leveling	Covered
45	4-2-2021	Problems on reciprocal leveling	4-2-2021	Problems on reciprocal leveling	Covered
46	10-2-2021	Module: 4 Plane table surveying accessories, setting up plane surveying	10-2-2021	Module: 4 Plane table surveying accessories setting up plane table surveying	Covered
47	11-2-2021	Method of plane table surveying	11-2-2021	Methods of plane table surveying	Covered
48	15-2-2021	Advantages & Disadvantages of plane table surveying Resection of two-point problems, three-point problems	15-2-2021	Advantages & Disadvantages of plane table surveying Resection of two-point three point problems	Covered
49	17-2-2021	Three point Problem	17-2-2021	Three point Problem	Covered
50	17-2-2021	Module: 5 Area & Volume Area - Methods of Area Calculation, offset at regular intervals.	17-2-2021	Module: 5 Area & Volume Area - methods of area calculation, offset at regular intervals	Covered
51	18-2-2021	Area from co-ordinates with problems, by different method, Volume: Different method	18-2-2021	Area from co-ordinates with problems, by different method, Volume: Different method.	Covered

LESSON PLAN

Period	Date	Topics Planned	Date	Topics Covered	Remarks

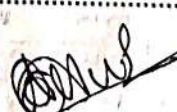
Text Books :

1. B. C. Punmia, "Surveying Vol. 1". Lakmi Publications Pvt. Limited New Delhi - 2009.
2. Kanetkar T. P. & S. N. Kulkarni. Surveying & Levelling.

Reference Books :

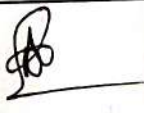


1. S. K. Duggal. Surveying Volume 1.
2. K. R. Arora. Surveying Volume 1.
3. R. Subramanian. Surveying & Levelling.
4.
5.


Signature of Faculty


HOD

TUTORIAL CLASSES

Date	Topics Discussed	Remarks

Test	Date	Class Strength	No. of Students Appeared	No. of Students Scored < 15	Signature of the HOD
T1	23.10.2020	67	67	00	
T2	09.12.2020	67	67	00	
T3	23.02.2024	67	67	00	

TIME TABLE

RACHU MTE

Day	Time	1	2	SHORT BREAK		3	4	LUNCH BREAK		5	6	7
		8:00-9:00	9:00-10:00	10:00-10:30	10:30-11:30	11:30-12:30	12:30-2:00	2:00-3:00	3:00-4:00	4:00-5:00		
MONDAY		18CV32-A								18CV35-B		18CV35-T (B)
TUESDAY			18CV32-A									
WEDNESDAY		18CV35-B					18CV32-B			18CV35-B	18CV35-T (B) (MEET SH)	
THURSDAY			18CV35-B			18CV32-A				18CV35-B (MEET END)		
FRIDAY			18CV35-B (MEET END)			18CV38-A (MEET END)						
SATURDAY			18CV32-A									

Rachum
Sign. of the Staff

Rachum
Sign. of the HOD

Rachum
PRINCIPAL
Apur Institute of Engineering & Technology
DAVANAGERE.

3rd P-Section
= Strength of Materials
3rd A-Section
= Basic Surveying
18CV35-T (B)
= Surveying Practice
18CV35-B1 & A3
- Surveying Practice.
18CV38-A1
Building materials Lab

B. E. CIVIL ENGINEERING
Choice Based Credit System (CBCS) and Outcome Based Education (OBE)
SEMESTER - III

STRENGTH OF MATERIALS

Course Code	18CV32	CIE Marks	40
Teaching Hours/Week (L:T:P)	(3:2:0)	SEE Marks	60
Credits	04	Exam Hours	03

Course Learning Objectives: This course will enable students

1. To understand the basic concepts of the stresses and strains for different materials and strength of structural elements.
2. To know the development of internal forces and resistance mechanism for one dimensional and two-dimensional structural elements.
3. To analyse and understand different internal forces and stresses induced due to representative loads on structural elements.
4. To determine slope and deflections of beams.
5. To evaluate the behaviour of torsion members, columns and struts.

Module-1

Simple Stresses and Strain: Introduction, Definition and concept and of stress and strain. Hooke's law, Stress-Strain diagrams for ferrous and non-ferrous materials, factor of safety, Elongation of tapering bars of circular and rectangular cross sections, Elongation due to self-weight. Saint Venant's principle, Compound bars, Temperature stresses, Compound section subjected to temperature stresses, state of simple shear, Elastic constants and their relationship.

Module-2

Compound Stresses: Introduction, state of stress at a point, General two dimensional stress system, Principal stresses and principal planes. Mohr's circle of stresses. Theory of failures: Max. Shear stress theory and Max. principal stress theory.

Thin and Thick Cylinders: Introduction, Thin cylinders subjected to internal pressure; Hoop stresses, Longitudinal stress and change in volume. Thick cylinders subjected to both internal and external pressure; Lamé's equation, radial and hoop stress distribution.

Module-3

Shear Force and Bending Moment in Beams: Introduction to types of beams, supports and loadings. Definition of bending moment and shear force, Sign conventions, relationship between load intensity, bending moment and shear force. Shear force and bending moment diagrams for statically determinate beams subjected to point load, uniformly distributed loads, uniformly varying loads, couple and their combinations.

Module-4

Bending and Shear Stresses in Beams: Introduction, pure bending theory, Assumptions, derivation of bending equation, modulus of rupture, section modulus, flexural rigidity. Expression for transverse shear stress in beams, Bending and shear stress distribution diagrams for circular, rectangular, 'I', and 'T' sections. Shear centre (only concept).

Torsion in Circular Shaft: Introduction, pure torsion, Assumptions, derivation of torsion equation for circular shafts, torsional rigidity and polar modulus Power transmitted by a shaft.

Module-5

Deflection of Beams: Definition of slope, Deflection and curvature, Sign conventions, Derivation of moment-curvature equation. Double integration method and Macaulay's method: Slope and deflection for standard loading cases and for determinate prismatic beams subjected to point loads, UDL, UVL and couple.

Columns and Struts: Introduction, short and long columns. Euler's theory; Assumptions, Derivation for Euler's Buckling load for different end conditions, Limitations of Euler's theory. Rankine-Gordon's formula for columns.

Course outcomes: After studying this course, students will be able;

1. To evaluate the basic concepts of the stresses and strains for different materials and strength of structural elements.
2. To evaluate the development of internal forces and resistance mechanism for one dimensional and two dimensional structural elements.
3. To analyse different internal forces and stresses induced due to representative loads on structural elements.
4. To evaluate slope and deflections of beams.
5. To evaluate the behaviour of torsion members, columns and struts.

Question paper pattern:

- The question paper will have ten full questions carrying equal marks.
- Each full question will be for 20 marks.
- There will be two full questions (with a maximum of four sub- questions) from each module.
- Each full question will have sub- question covering all the topics under a module.
- The students will have to answer five full questions, selecting one full question from each module.

Textbooks:

1. B.S. Basavarajaiah, P. Mahadevappa "Strength of Materials" in SI Units, University Press (India) Pvt. Ltd., 3rd Edition, 2010
2. Ferdinand P. Beer, E. Russell Johnston and Jr. John T. De Wolf "Mechanics of Materials", Tata McGraw-Hill, Third Edition, SI Units

Reference Books:

1. D.H. Young, S.P. Timoshenko "Elements of Strength of Materials" East West Press Pvt. Ltd., 5th Edition (Reprint 2014).
2. R K Bansal, "A Textbook of Strength of Materials", 4th Edition, Laxmi Publications, 2010.
3. S.S. Rattan "Strength of Materials" McGraw Hill Education (India) Pvt. Ltd., 2nd Edition (Sixth reprint 2013).
4. Vazirani, V N, Ratwani M M. and S K Duggal "Analysis of Structures Vol. I", 17th Edition, Khanna Publishers, New Delhi.

Module 1

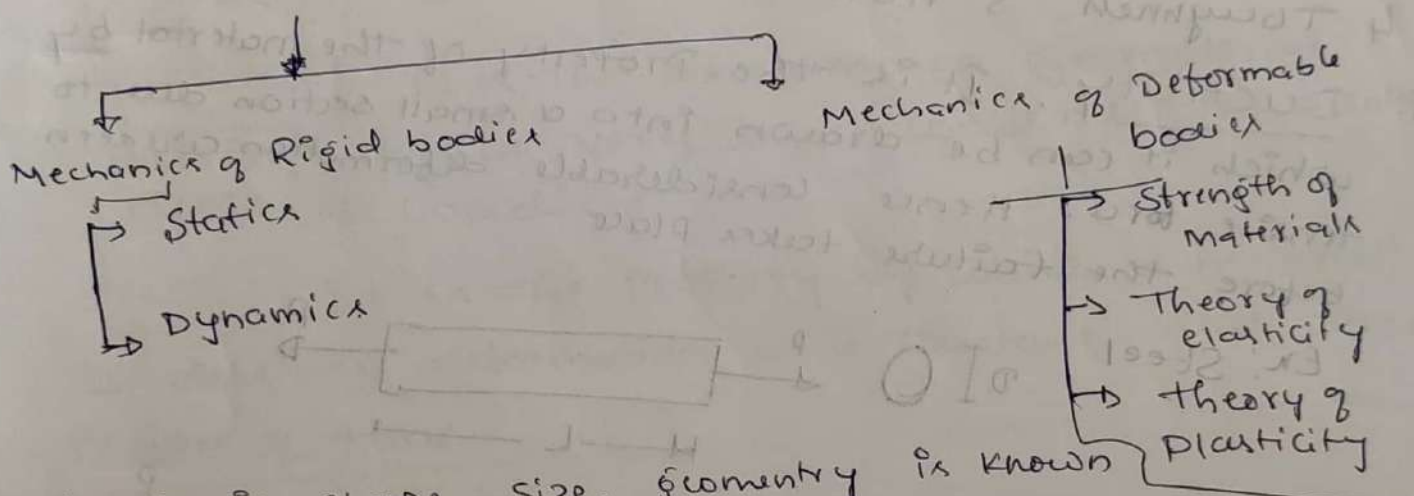
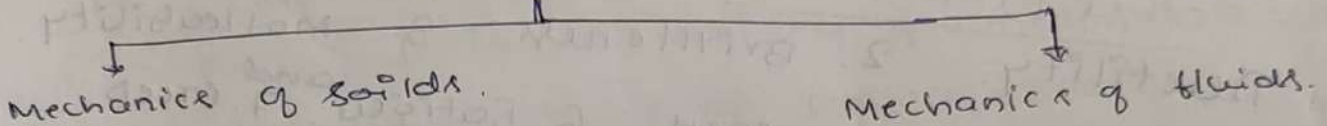
Simple stresses & strain

Introduction

M. E. Raghur.
Assistant Professor.
B.P. ET. Davanagere.

When an external force acts on a body, the body tends to undergo some deformation. Due to cohesion b/w the molecules, the body resists deformation. The resistance by which materials of the body opposes the deformation is known as "strength of materials."

Engineering Mechanics



Change in shape, size, geometry is known as Deformation.

Materials are classified into

- 1. elastic - [tennis / rubber ball]
 - 2. plastic - [chewing gum]
 - 3. Rigid - [stone]
- } Deformation.

1. Elasticity :

When an external force acts on a body, the body tends to undergo some deformation. If the external force is removed, then the body comes back to its original shape & size. [which means the deformation disappears completely]. The body is known as "elastic body". Then that material is referred as elastic & this property is called elasticity. [$\delta = \checkmark$ | $\delta = 0$]

① Plastic : Due to external loading, if there is permanent deformation in the material, (i.e. material will not regain original size or shape or dimension) even after the removal of the load, then the material is said to be plastic & this property is called plasticity.

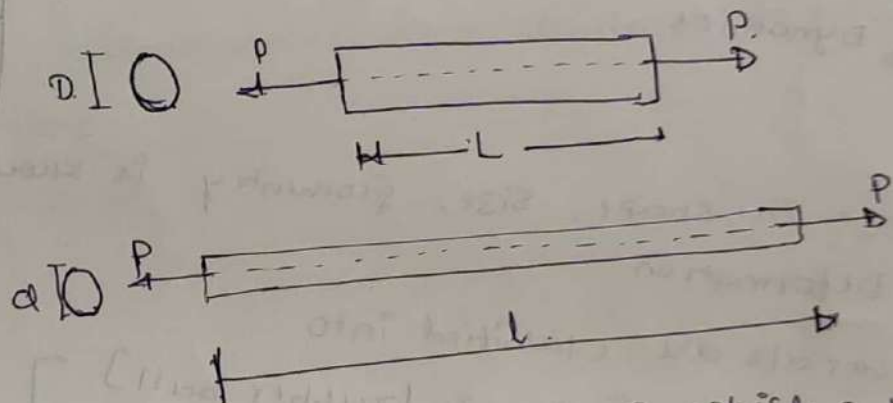
A Rigid material is one in which deformation is not seen when subjected to external loading [d=0]

Other Properties of Materials are

1. Ductility.
2. Brittleness
3. Malleability,
4. Toughness,
5. Hardness,
6. Fatigue, and
7. Creep.

1. Ductility : It is the property of the material by which it can be drawn into a small section due to tensile force. Hence considerable deformation are seen before the failure takes place.

Ex: Steel



② Brittleness - Brittleness is a property in which a material breaks without any significant deformation under the action of external load. A brittle material shows little (negligible) plastic deformation before the fracture takes place.

Ex: Glass, Ceramic materials.

When a material cannot be drawn into a smaller section due to a tensile force, then it is referred as brittle. Brittleness is therefore called as a lack of ductility. In such type of material failure takes place even at small deformation, which is highly undesirable for engineering application. Ex: Cast Iron.

- ③ Malleability: It is the property by which a material can be uniformly extended in a direction without rupture due to a tensile force. Hence a malleable will have high degree of plasticity.
Ex: Gold [due to this it may be drawn into wire] (a) sheets]
- ④ Toughness: It is the property of the material which enables it to absorb energy without failure.
Ex: Impact loading.
- ⑤ Hardness: It is the ability of a material to resist indentation (a) scratching (a) surface abrasion.
- ⑥ Fatigue: It is the phenomenon of a material failing under very small load due to repeated cycle of loading.
- ⑦ Creep: It is the property by which a material undergoes deformation at a constant load over a period of time.

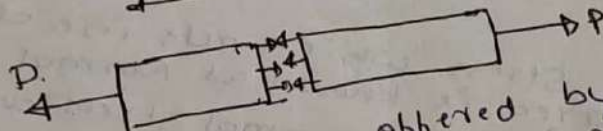
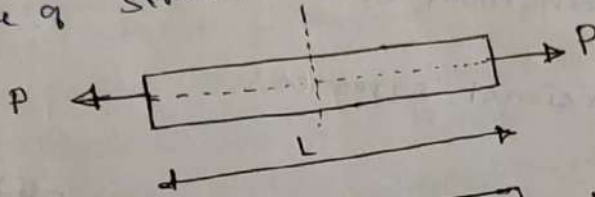
longitudinal strain.

Strength of Materials

When an external force acts on a body, the body tends to undergo some deformation. Due to cohesion between the molecules, the body resists deformation. This resistance by which materials of the body oppose the deformation is known as Strength of Materials.

Stress:

When a material is subjected to load, internal resisting forces are developed and then it is said to be in a state of stress.



Stress is the resistance offered by the body to the deformation and is measured as the intensity of load per unit area and is denoted by f , σ with unit as N/mm^2 or KN/m^2 $\text{\textcircled{a}}$ Pascal

$$1 \text{ Pascal} = 1 \text{ N/m}^2$$

$$\text{Stress} = \frac{\text{load}}{\text{Area}}$$

$$f = \frac{P}{A}$$

$f, \sigma =$ Stress [also called Intensity of stress]

$P =$ External force $\text{\textcircled{a}}$ load

$A =$ Area of cross section.

Unit of Stress

$$1 \text{ Pa} = 1 \text{ N/m}^2$$

$$1 \text{ kPa} = 1 \times 1000 \text{ N/m}^2 = 1 \times 10^3 \text{ N/m}^2$$

$$1 \text{ MPa} = 1 \times 1000 \times 1000 \text{ N/m}^2 = 1 \times 10^6 \text{ N/m}^2$$

$$1 \text{ GPa} = 1 \times 1000 \times 1000 \times 1000 \text{ N/m}^2 = 1 \times 10^9 \text{ N/m}^2$$

$$1 \text{ TPa} = 1 \times 1000 \times 1000 \times 1000 \times 1000 \text{ N/m}^2 = 1 \times 10^{12} \text{ N/m}^2$$

$$1 \text{ MPa} = 1 \times 10^6 \frac{\text{N}}{\text{m}^2} = \frac{1 \times 10^6}{(1000)^2} = \frac{\text{N}}{\text{mm}^2}$$

$$1 \text{ MPa} = 1 \text{ N/mm}^2$$

$$1 \text{ m} = 1000 \text{ mm}$$

Based on nature of forces, stresses are classified as (or)

Types of Stresses

The following are the different types of stress

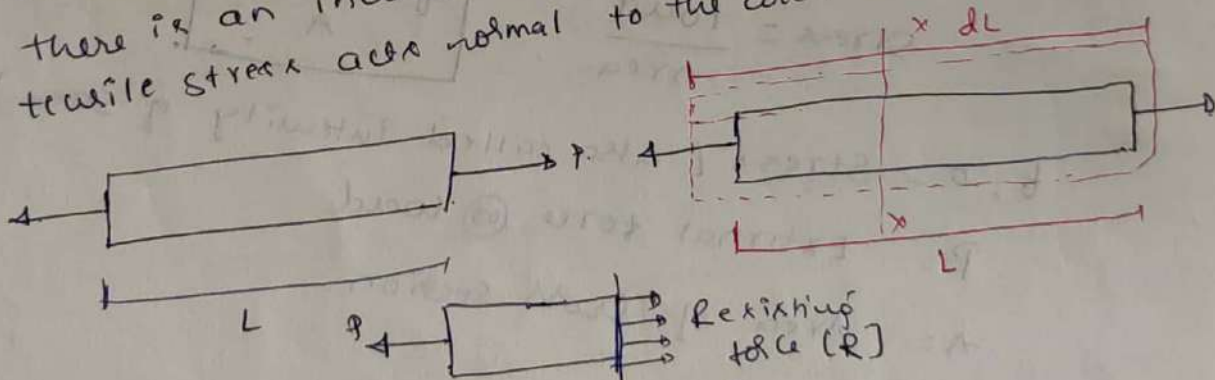
1. Normal stress.
 - (a) Tensile stress
 - (b) compressive stresses.
2. shear stress (or) Tangential stresses.
3. Bending stress
4. Twisting (or) Torsional stresses.
5. Bearing stresses.

(1) Normal Stress:

The stress which acts in a direction perpendicular to the area is known as Normal stress. It is denoted by sigma (i.e. σ). The normal stresses are further divided into Tensile stress and compressive stress.

(a) Tensile Stress:

The stress induced in a body, when subjected to two equal and opposite pulls as shown in fig. as a result of which there is an increase in length is known as Tensile stress. The tensile stress acts normal to the area & it pulls on the area.



Let P = Pull (or) force acting on the body.

A = Area or cross-sectional area of the body.

L = original length of the body.

dl = Increase in length of the body due to pull P acting on the body.

e = strain (i.e. tensile strain)

σ = stress induced in the body.

Tensile stress: $\sigma = \frac{\text{Resisting force (R)}}{\text{Cross sectional area (A)}} = \frac{P}{A} = \frac{\text{Tensile load } P}{\text{Area (A)}}$

$$\sigma = \frac{P}{A} \quad \therefore (R=P)$$

Tensile strain is given by

$$e = \frac{\text{Increase in length}}{\text{original length}}$$

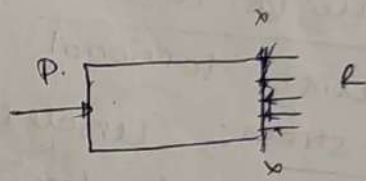
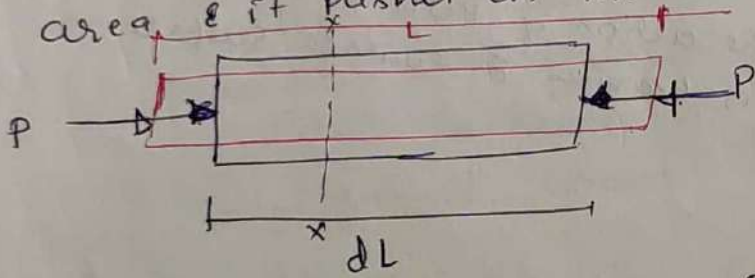
i.e.

$$e = \frac{dL}{L}$$

$$e = \frac{dL}{L}$$

② Compressive stress:

The stress induced in a body when subjected to two equal & opposite pushes as shown in fig as a result of which there is a decrease in length of the body is known as compressive stress. The compressive stress acts normal to the area & it pushes on the area.



Let an axial push 'P' acting on a body in cross sectional Area 'A' due to external push P. Let the original length L of the body decreases by dL.

Then Compressive stress = $\sigma = \frac{\text{Resisting force (R)}}{\text{Area (A)}} = \frac{\text{Push (P)}}{\text{Area}}$

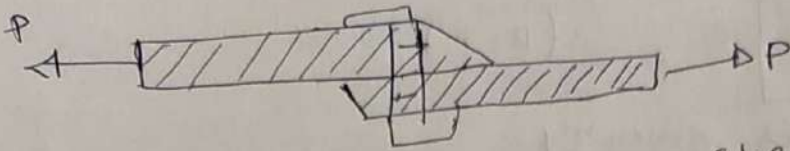
$$\sigma = \frac{P}{A}$$

Compressive strain is given by $e = \frac{\text{Decrease in length}}{\text{original length}}$

$$e = \frac{dL}{L}$$

③ Shear stress (or) tangential stress:

The stress induced in a body when subjected to two equal & opposite forces which are acting tangentially across the resisting section as shown in fig, as a result of which the body tends to shear off across the section is known as shear stress & the corresponding strain is known as shear strain. It is denoted by τ (Tow) Strain



The shear stress is defined as Shear force per unit area
 (or) It is the ratio of shear resistance to shear area is called shear stress. It is denoted by τ (tau)

$$\text{Shear stress} = \frac{\text{Shear force}}{\text{Shear Area}} = F/A$$

Bending stress: The stress produced at the section to resist the bending moment in beam is known as Bending stress.

Twisting (or) Torsion: The stress produced at the section to resist torsional moment (or) twisting moment in shaft.

Bearing stress: Under the action of pull P the 2 plates press against the limit in bearing & contact surface.

Types of strain:

Longitudinal strain:

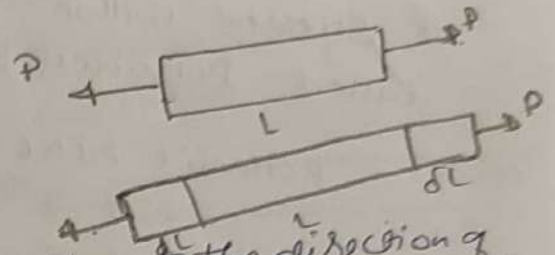
When a body is subjected to an axial tensile load. There is an increase in the length of the body. But at the same time there is a decrease in other dimension of the body at the right angles to the line of action of the applied load. Thus the body is having axial deformation & also deformation at right angles to the line of action of the applied load.

The ratio of axial deformation to the original length of the body is known as longitudinal (or) linear strain (or)

The longitudinal strain is also defined as the deformation of the body per unit length in the direction of the applied load.

Let. L = length of the body,
 P = Tensile force acting on the body.
 δL = Increase in length of the body in the direction of P .

$$\text{longitudinal strain} = \frac{\delta L}{L}$$



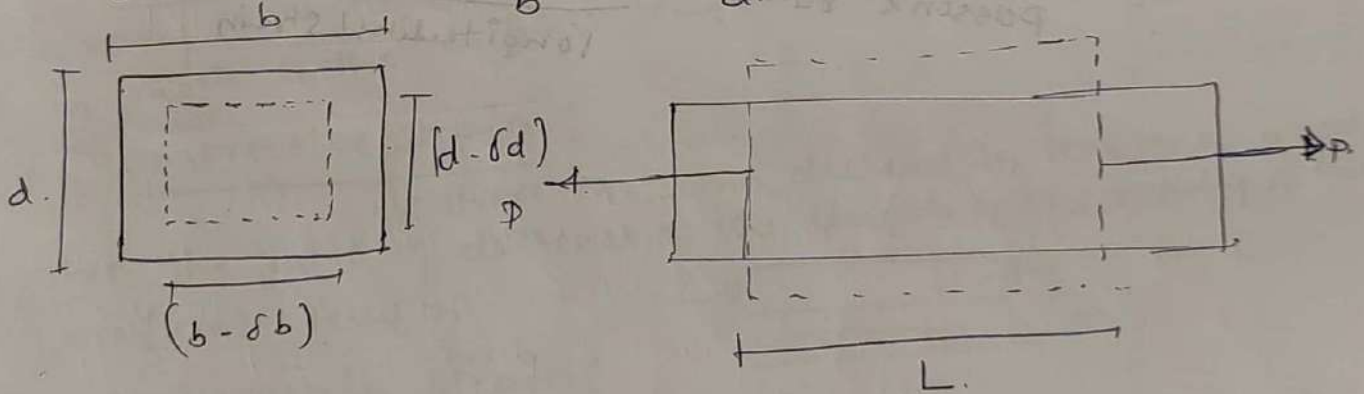
Lateral Strain:

The strain at right angles to the direction of applied load is known as lateral strain. Let a rectangular bar of length L , breadth b , & depth d is subjected to an axial load P as shown in fig. The length of the bar will increase while the breadth & depth will decrease.

Let δL = Increase in length
 δb = Decrease in breadth,
 δd = Decrease in depth.

$$\text{longitudinal strain} = \frac{\delta L}{L}$$

$$\epsilon \text{ (lateral strain)} = \frac{\delta b}{b} \text{ or } \frac{\delta d}{d}$$



Note: If longitudinal strain is tensile, the lateral strain will be compression.
 If longitudinal strain is compression & the lateral strain will be tensile.

Poisson's Ratio:

The ratio of lateral strain to the longitudinal strain is a constant for a given material, when the material is stressed within the elastic limit. This ratio is called Poisson's ratio & generally denoted by μ .

$$\text{Poisson's ratio } \mu = \frac{\text{lateral strain}}{\text{longitudinal strain}}$$

$$\textcircled{\infty} \text{ lateral strain} = \mu \times \text{longitudinal strain}$$

As lateral strain is opposite in sign to longitudinal strain hence algebraically lateral strain is written as

$$\text{lateral strain} = -\mu \times \text{longitudinal strain}$$

$\textcircled{\infty}$

Poisson's ratio is defined as the ratio of lateral strain to the longitudinal strain & is denoted by μ (μ).

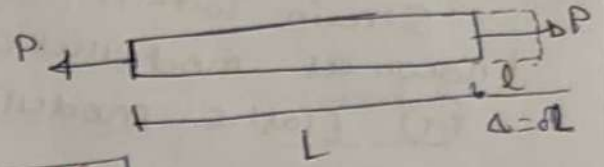
$$\therefore \text{Poisson's ratio} = \frac{\text{lateral strain}}{\text{longitudinal strain}}$$

Strain

When a body is subjected to some external force, there is some change of dimension of the body. The ratio of change of dimension of the body to the original dimension is known as strain.

Strain can be represented or denoted by ϵ or e . Strain is unit-less or dimensionless.

$$\epsilon = \frac{\text{Change in dimension}}{\text{Original dimension}}$$

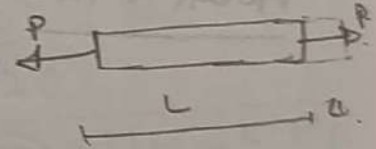


$$\epsilon = \frac{(L + \Delta) - L}{L} = \frac{\Delta}{L} = \frac{\delta l}{l}$$

Strain may be:

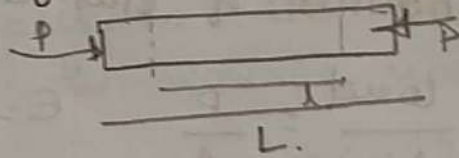
1. Tensile strain.
2. Compressive strain.
3. Volumetric strain and
4. Shear strain.

1. Tensile strain: If there is some increase in length of a body due to external force, then the ratio of increase in length to the original length of the body.



2. Compressive strain:

If there is some decrease in length of a body, then the ratio of decrease of the length of the body to the original length.



3. Volumetric strain:

The ratio of change of volume of the body to the original volume.

4. Shear strain:

The strain produced by the shear stress is known as shear strain.

$$\epsilon = \frac{\delta V}{V}$$

Hook's law and Elastic moduli

When a material is loaded within elastic limit, Stress is proportional to the strain produced by the stress.

That means stress is directly proportional to the strain within the elastic limit. This constant is known as modulus of elasticity, (or) modulus of rigidity (or) Elastic moduli.

modulus of elasticity (or young's modulus)

The ratio of tensile stress (or) compressive stress to the corresponding strain is a constant.

This ratio is known as young's modulus (or) modulus of elasticity and is denoted by E.

$$E = \frac{\text{Tensile stress}}{\text{Tensile strain}} \quad \text{(or)}$$

$$\frac{\text{compressive stress}}{\text{compressive strain}}$$
$$E = \frac{\sigma}{\epsilon}$$

Hook's law

Stress & strain,

$$f \propto \epsilon$$

$$f = E \epsilon$$

where E = constant E is known as young's modulus (or) modulus of elasticity.

$$f = \frac{\text{load}}{\text{Area}} = \frac{P}{A} \quad \epsilon = \frac{\Delta L}{L} = \frac{a}{L}$$

$$\frac{P}{A} = E \frac{a}{L}$$

$$\frac{PL}{Aa} = E$$

$$E = \frac{PL}{Aa}$$

N/mm².

Where E = young's modulus

P = load (or) KN.

L = length in m (or) mm

A = Area of cross section mm².

a = change in length mm

For a given material young's modulus E remains constant E it varies from material to material.

Problem

1. A circular rod of steel 10mm diameter is subjected to a tensile load of 11kN. for which the total extension on a 300mm length was 0.20mm. find the value of young's modulus.

→ diameter $d = 10\text{mm}$

$$A = \frac{\pi (10)^2}{4} = 78.53 \text{ mm}^2$$

$$P = 11 \times 10^3 \text{ N}$$

$$L = 300 \text{ mm}$$

$$\Delta = 0.20 \text{ mm}$$

$$E = ?$$

$$\text{Stress} = \frac{\text{Load}}{\text{Area}}$$

$$= \frac{11 \times 10^3 \text{ N}}{78.53 \text{ mm}^2}$$

$$\text{Stress } f = 140.05 \text{ N/mm}^2$$

$$\text{Strain} = \frac{\Delta}{L} = \frac{0.20 \text{ mm}}{300 \text{ mm}}$$

$$E = 6.66 \times 10^{-4}$$

$$E = \frac{PL}{A\Delta} = \frac{11 \times 10^3 \times 300}{78.53 \times 0.20}$$

$$E = 2.10 \times 10^5 \text{ MPa } (\text{or}) \text{ N/mm}^2$$

- 02) length of an aluminium rod is 400mm with diameter as 10mm. when it is subjected to a tensile force of 2kN. the length increases to 400.15mm. Find the Stress in the bar and the young's modulus.

→ length of rod $L = 400\text{mm}$

diameter of rod $d = 10\text{mm}$

$$P = 2 \text{ kN} = 2 \times 10^3 \text{ N}$$

$$A = \frac{\pi (d)^2}{4} = \frac{\pi (10)^2}{4}$$

$$A = 78.53 \text{ mm}^2$$

$$E = ? \quad f = ?$$

$$L' \text{ [final length]} = 400.15 \text{ mm}$$

$$L \text{ [original length]} = 400 \text{ mm}$$

$$\Delta = L' - L = 400.15 - 400$$

$$\Delta = 0.15 \text{ mm}$$

Stress - Strain Relation for Structural Steel

$$\text{Stress } f = \frac{P}{A} = \frac{2 \times 10^3}{78.55} = 25.46 \text{ N/mm}^2 \quad [\text{Tensile Stress}]$$

$$\text{Strain} = \frac{\Delta}{L} = \frac{0.15}{400} = 3.75 \times 10^{-4}$$

$$E = \frac{f}{\epsilon} = \frac{PL}{A\Delta} = \frac{2 \times 10^3 \times 400}{78.55 \times 0.15} =$$

$$E = 67897.30 \text{ N/mm}^2$$

$$E = \boxed{67897.30 \text{ N/mm}^2}$$

③ A uniform steel rod 6mm diameter & 0.5m length is subjected to a tensile force of 3kN. find stress in the bar & its elongation if $E = 200 \text{ GPa}$.

$$\hookrightarrow D = 6 \text{ mm}$$

$$L = 0.5 \text{ m} = 500 \text{ mm}$$

$$P = 3 \times 10^3 \text{ N}$$

$$E = 200 \text{ GPa} = 200 \times 10^9 \text{ Pa}$$

$$f = ? \quad \Delta = ?$$

$$E = 200 \times 10^9 \text{ N/m}^2 = 200 \times 10^9 \text{ (1000)}^2$$

$$= 200 \times 10^3 \text{ MPa} \text{ (GPa)}$$

$$\hookrightarrow \text{Stress } f = \frac{\text{Load}}{A_{\text{req}}}$$

$$E = \boxed{200 \times 10^3 \text{ N/mm}^2}$$

$$A = \frac{\pi (d)^2}{4} = \frac{\pi \times (6)^2}{4} = 28.274 \text{ mm}^2$$

$$\begin{aligned} & 200 \text{ GPa} \\ & \frac{10^3 \times 10^6}{10^3 \times \text{MPa}} \\ & = 200 \times 10^3 \text{ N/mm}^2 \end{aligned}$$

$$E = \frac{PL}{A\Delta} \Rightarrow \Delta = \frac{PL}{EA} = \frac{3 \times 10^3 \times 500}{28.274 \times 200 \times 10^3}$$

$$\Delta = \frac{3 \times 10^3 \times 500}{28.274 \times 200 \times 10^3}$$

$$\Delta = \boxed{0.269 \text{ mm}}$$

$$\Delta = \frac{kN \times mm}{mm^2 \times N/mm^2}$$

$$\Delta = mm$$

$$\text{Stress } f = \frac{P}{A} = \frac{3 \times 10^3 \text{ N}}{28.274 \text{ mm}^2} = \underline{\underline{106.104 \text{ N/mm}^2}}$$

Stress - Strain Relation for Structural Steel

(Mild steel or Ductile material)
(Ferritic material)

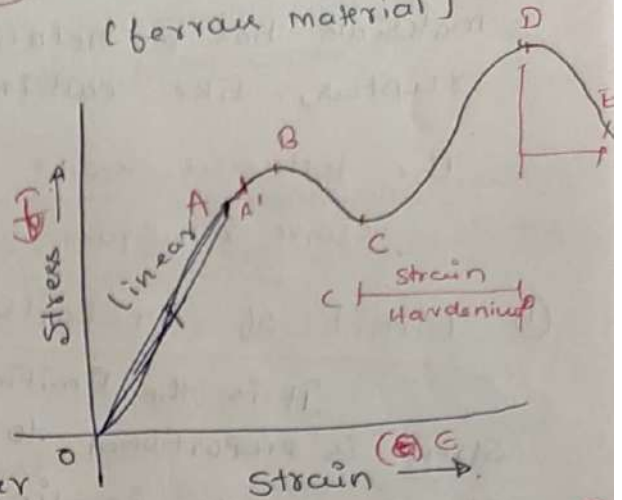
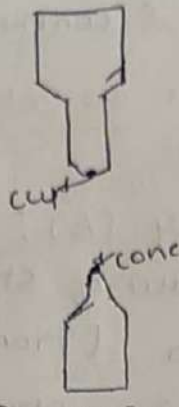
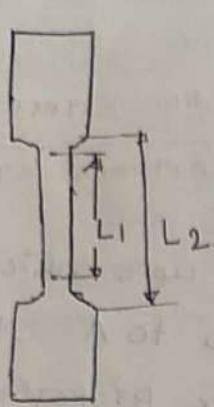


Fig (a) Test Specimen Fig (b) Specimen after breaking

- A = Proportionality Limit, A' = elastic limit.
- B = Upper yield Point.
- C = Yield Point / lower yield Point.
- D = Ultimate strength / Ultimate stress.
- E = Breaking Point / Rupture strength.

Fig (a) shows a typical tensile test specimen of mild steel. Its ends are gripped into universal testing machine. Extensometer is fitted to the test specimen which measures extension over the length L_1 as shown in fig (a). The length over which extension is measured is called gauge length. The load is applied gradually & at regular interval of load extension is measured the process is continued until failure of the specimen takes place.

will give the stress (σ) load divided by original cross-sectional Area. Similarly strain is obtained by dividing the extensometer reading by gauge length of extensometer (L_1) & by dividing scale reading by grip to grip (L_2) knowing the original cross-sectional Area & length of the specimen, the normal stress (σ) & the strain (ϵ) can be obtained. The graph of these quantities i.e. stress (σ) along y-axis & strain along x-axis is called the stress-strain diagram.

The stress strain diagram differs in form for various materials. The engineering materials are classified as either ductile or brittle materials. A ductile material is one having relatively large tensile strains upto the point of rupture.

like structural steel & aluminium, whereas brittle materials has a relatively small strain upto the point of rupture like cast iron & concrete.

The following salient points are observed on stress-strain curve diagram. [proportional limit]

① Limit of Proportionality (A):
 It is the limiting value of stress upto which the stress is proportional to strain [from 'o' to 'A' stress-strain curve is linear i.e. stress is proportional to strain upto point A. & is called Proportionality limit.]

stress \propto strain
 $f \propto e$
 $f = Ee$ $E = \text{Young's modulus (N/mm}^2\text{)}$

② Elastic limit (A')
 The elastic limit is the limit beyond which the material will no longer go back to its original shape when the load is removed.

(or)
 This is the limiting value of stress upto which if the material is stressed & then released (unloaded) strain (deformation) disappears completely & the original length is regained. This point is slightly beyond the limit of proportionality.

③ Upper yield point (B)
 This is the stress at which the load starts reducing & the extension increases. This phenomenon is known as yielding of materials.

④ Lower yield point (C)
 At this stage the stress remains same but strain increases for some time.
Yield point: It is a point at which the material will have appreciable elongation (or) yielding without any increase in load. & diagram becomes a curve at C.
 The stress corresponding to the point C is called yield stress [lower yield point]

5) Ultimate strength (D)

This is the maximum stress the material can resist at this stage c/s area at a particular section starts reducing rapidly. This is called neck formation.

6) Break Point (E)

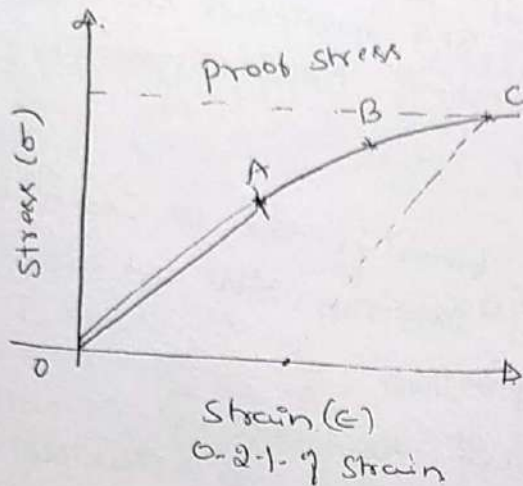
The stress at which finally the specimen fails is called breaking point (or) σ_t is the point at which the specimen fails (or) breaks.

The load corresponding to breaking is called breaking load E the stress corresponding to breaking load is taken as nominal breaking stress (or) ~~actual~~ breaking stress.

$$\text{Nominal breaking stress} = \frac{\text{Breaking load}}{\text{original c/s Area}}$$

$$\text{breaking stress} = \frac{\text{Breaking load}}{\text{final c/s area}}$$

Stress-strain curve for non-ferrous materials



A = proportionality limit.
 B = elastic limit.
 C = proof stress.

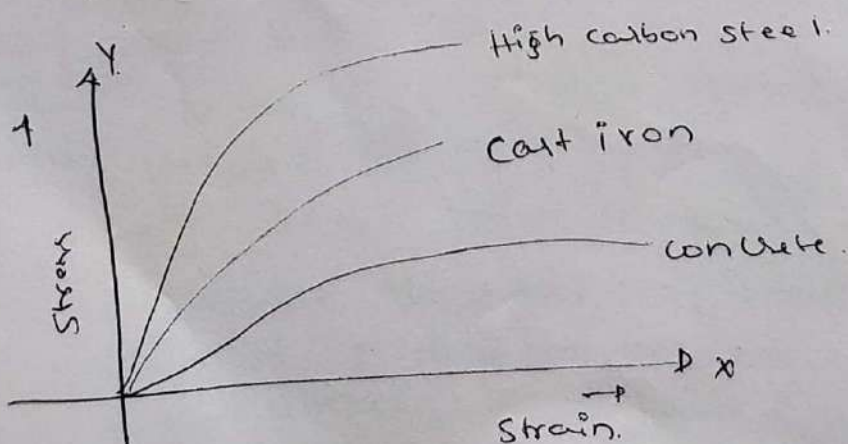
From the curve it is observed that the stress is proportional to strain upto point 'A' & is called proportionality limit. i.e. Hooke's law is valid upto point 'A'. But the material retains its elastic property upto point 'B' & is called as "Stress at elastic limit".

To decide a failure stress for non-ferrous material as per Indian standard (IS) we consider 0.2% of strain along x-axis. From that point we draw the line parallel to linear portion of curve. The line meets the curve at point C is called as proof stress [failure stress].

Non-ferrous material (or) non-elastic materials / brittle materials

These materials don't show specific yield point & their failure is sudden.

Ex: Copper, Zinc, concrete, aluminium, cast iron etc.



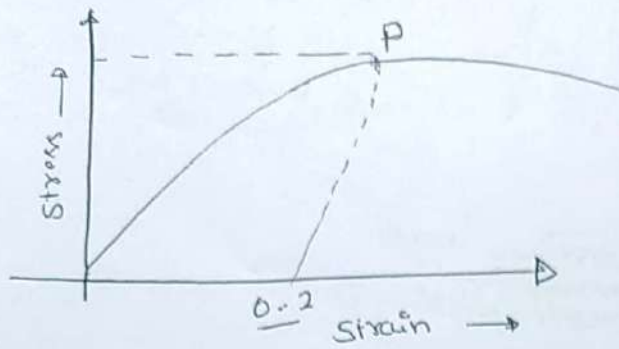
Stress-strain relation in aluminium & high strength steel.

fig.

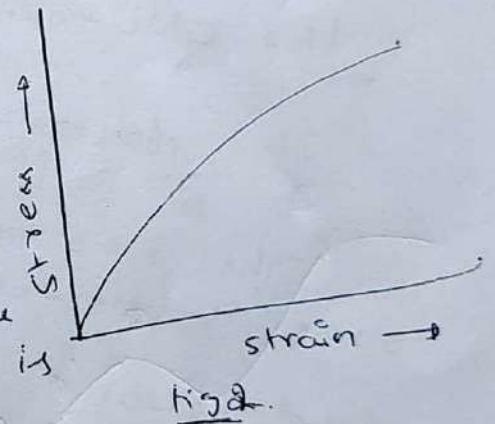
In These elastic materials there is no clear cut yield point. The necking takes place at ultimate stress & eventually the breaking point is lower than the ultimate point.

The stress 'P' at which if unloading is made there will be 0.2% permanent set is known as 0.2% proof stress & this point is treated as yield point for all practical purposes.

Stress-strain relation in brittle materials.

The typical stress-strain relation in a brittle material like cast iron is as shown in fig(2)

In these materials there is no appreciable change in rate of strain. There is no yield point & no necking takes place. Ultimate & breaking point are one & the same. The strain at failure is very small.

Important Definitions

Yield stress: It is the ratio of yield load to the cross sectional area is known as yield stress.

$$\text{Yield Stress} = \frac{\text{Yield Load}}{\text{Cross-sectional area}}$$

Ultimate stress: It is the ratio of ultimate load (or) maximum load to the cross sectional area.

$$\text{Ultimate Stress} = \frac{\text{Ultimate Load}}{\text{Cross-sectional Area}}$$

Working stress (or) allowable stress (or) safe stress
It is the ratio of working load (or) service load to the gross sectional area.

The working stress is also known as allowable stress & safe stress.

$$\text{Working stress} = \frac{\text{Working load}}{\text{Gross-sectional Area}}$$

Factor of safety [FOS]

It is the ratio of yield stress (or) ultimate stress to the working stress is known as F.O.S (or)

The greatest stress to which a material may be subjected in practice must be less than the ultimate stress. This corresponding stress is called working stress.

$$\text{factor of safety} = \frac{\text{ultimate stress}}{\text{working stress}} > 1$$

The FOS must always be such that the working stress is below elastic limit.

Load factor :

It is the ratio of load at failure to working load is called load factor.

$$\text{load factor} = \frac{\text{Load failure}}{\text{working load}}$$

It is the ratio of change in area to the total area is known as % of reduction in Area.

$$\% \text{ of reduction in Area} = \frac{\text{Change in Area} \times 100}{\text{total Area}}$$

$$= \frac{\Delta A}{A} \times 100$$

Subjected to compressive force:

$$\% \text{ of reduction in Area} = \frac{\text{original Area} - \text{final Area}}{\text{original Area}}$$

Subjected to ~~to~~ tensile force:

$$\% \text{ of reduction in Area} = \frac{\text{final Area} - \text{original Area}}{\text{original Area}}$$

Modulus of Rigidity (G) Shear modulus:

It is the ratio of shear stress to the corresponding shear strain within elastic limit is known as elastic modulus of Rigidity (G) Shear modulus.

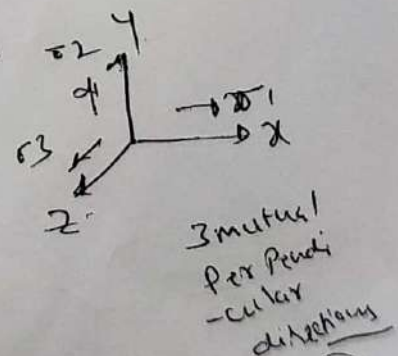
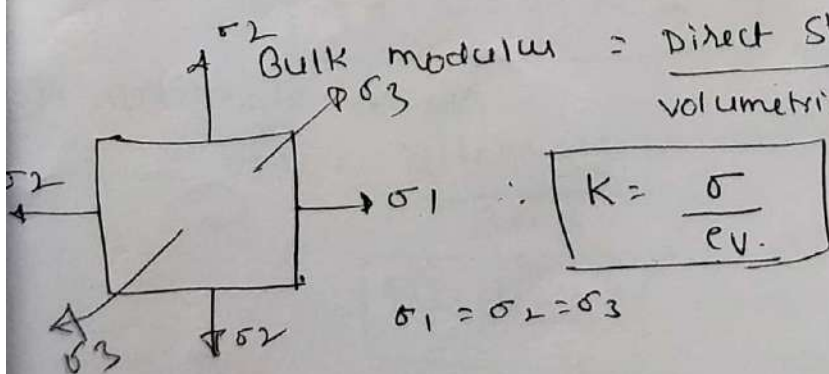
It is denoted by G or C or N

$$C \text{ or } G \text{ or } N = \frac{\text{Shear Stress}}{\text{Shear Strain}} = \frac{\tau}{\phi}$$

$$C = \frac{\tau}{\phi}$$

Bulk modulus:

It is defined as the ratio of direct stress to the corresponding volumetric strain. when the body is subjected to 3 mutual str perpendicular stress of same intensity is known as bulk modulus. It is denoted by K .



The following data referred to a mild steel specimen tested in a laboratory.

- (1) Diameter of the specimen = 25mm.
- (2) Length of the specimen = 300mm
- (3) Extension under a load of 15kN = 0.045mm
- (4) Load at yield point = 127.65kN.
- (5) maximum load = 208.6kN
- (6) Length of the specimen after failure = 375mm.
- (7) Neck diameter = 17.75mm.

Determine Young's modulus, yield point, ultimate stress, Percentage of elongation Percentage of reduction in area, safe stress adopting factor of safety.

Given. $d = 25\text{mm}$, $L = 300\text{mm}$, $\delta L = 0.045\text{mm}$, $P = 15\text{kN}$
 $P = 15 \times 10^3\text{N}$

$$E = \frac{\text{Stress}}{\text{Strain}} = \frac{\sigma}{\epsilon}$$

$$\text{Stress} = \frac{\text{Load}}{\text{Area}} = \frac{15 \times 10^3}{\frac{\pi}{4} (25)^2} = 30.55 \text{ N/mm}^2$$

$$\text{Strain} = \frac{\text{Change in length}}{\text{Original length}} = \frac{\delta L}{L} = \frac{0.045}{300} = 1.5 \times 10^{-4}$$

$$E = \frac{\text{Stress}}{\text{Strain}} = \frac{30.55}{1.5 \times 10^{-4}} = 2.03 \times 10^5 \text{ N/mm}^2$$

Stress at Yield Point (or) yield stress

$$f_y = \frac{\text{Yield load}}{\text{Area}} = \frac{127.65 \times 10^3}{\frac{\pi \times (25)^2}{4}}$$

$$f_y = 260.04 \text{ N/mm}^2$$

ultimate stress

$$f_u = \frac{\text{ultimate load (or) maximum load}}{\text{Area}} = \frac{208.6 \times 10^3}{\frac{\pi \times (25)^2}{4}}$$

$$f_u = 424.95 \text{ N/mm}^2$$

(12)

Percentage of Elongation = $\frac{375 - 300}{300} \times 100$ Final length - Initial length / Original length

$= 25\%$

Percentage Reduction in Area.

$$= \frac{\frac{\pi}{4} (d^2 - d_1^2)}{\frac{\pi}{4} d^2} \times 100$$

$d = 25$
 $d_1 = 17.75$
 (neck diameter)

$$= \frac{d^2 - d_1^2}{d^2} \times 100$$

$$= \frac{(25)^2 - (17.75)^2}{25^2} \times 100$$

$$= 0.4959 \times 100$$

$$= 49.59\%$$

Stress = $\frac{\text{Yield Stress}}{F.S} = \frac{260 \times 17}{2} = 130.085 \text{ N}$

Q2) In a tension testing Machine the following result

- obtained
- ① Dis of the specimen = 12mm.
 - ② Tensile load = 100kN.
 - ③ Change in length = 0.012mm
 - ④ length of specimen = 200mm
 - ⑤ ultimate load = 260kN.
 - ⑥ Breaking load = 230kN.
 - ⑦ The load behind the stress is not proportional to strain = 100kN.
 - ⑧ Extension of bar was 62.5mm & dia was 6.5mm at fracture.

- estimate
- ⑨ Young's modulus
 - ⑩ ultimate stress
 - ⑪ % of reduction area
 - ⑫ % of elongation
 - ⑬ Break stress
 - ⑭ working stress if FOS is 2

Given $d = 12 \text{ mm}$, $P = 100 \times 10^3 \text{ N}$, $\Delta L = 0.012 \text{ mm}$,
 $L = 240 \text{ mm}$, Ultimate load = 260 kN, Breaking load = 230 kN
 Extension of bar = 62.5 mm.

Area of specimen $A = \frac{\pi (6.5)^2}{4} = 33.18 \text{ mm}^2$

Area of the specimen $A = \frac{\pi (12)^2}{4} = 113.097 \text{ mm}^2$

Stress $\sigma = \frac{P}{A} = \frac{100 \times 10^3}{113.09} = 884.4 \text{ N/mm}^2$

Strain $e = \frac{\Delta L}{L} = \frac{0.012}{240} = 5 \times 10^{-5}$

Young's modulus (E) $E = \frac{\sigma}{e} = \frac{884.4}{5 \times 10^{-5}}$

$E = 17.688 \times 10^6 \text{ N/mm}^2$

Ultimate stress

Ultimate stress = $\frac{\text{ultimate load}}{\text{Cross sectional Area}}$
 $= \frac{260 \times 10^3}{113.09} = 2.299 \times 10^3 \text{ N/mm}^2$

% elongation = $\frac{\text{Total increase in length}}{\text{Original length}} \times 100$
 $= \frac{62.5}{240} \times 100$

% elongation = 26.04%

% Area reduction = $\frac{\text{Initial Area} - \text{Final Area}}{\text{Initial Area}} \times 100$
 $= \frac{113.11 - 33.18}{113.11} \times 100$
 $= 70.66 \%$

Breaking stress

i.e. Breaking stress = $\frac{\text{Breaking load}}{\text{Area}}$
 $= \frac{230 \times 10^3}{113.18} = 2.032 \times 10^3 \text{ N/mm}^2$

⑥ Working stress if FOS = 2.

$$F.O.S = \frac{\text{Yield stress}}{\text{Working stress}}$$

$$\text{Yield stress} = \frac{\text{Yield load}}{\text{Area}} = \frac{884100 \times 10^3}{113.11}$$

$$\text{Yield stress} = 8840.09 \text{ N/mm}^2$$

$$\text{Working stress} = \frac{\text{Yield stress}}{F.O.S}$$

$$= \frac{8840.09}{2}$$

$$\text{Working stress} = 4420.047 \text{ N/mm}^2$$

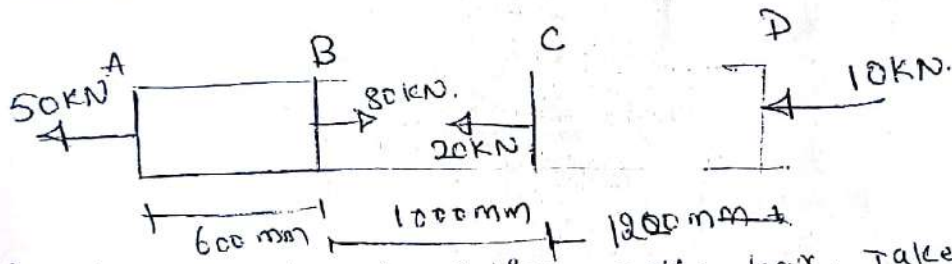
Principle of Superposition:

When a no of loads are acting on a body, the resulting strain, according to the principle of superposition will be the algebraic sum of strains caused by the individual loads.

While using this principle for an elastic body which is subjected to a number of direct forces [tensile (or) compressive] at different sections along the length of the body, first the free body diagram of individual section is drawn. Then the deformation of each section is obtained. The total deformation of the body will be then equal to the algebraic sum of deformation of the individual sections.

Problem:

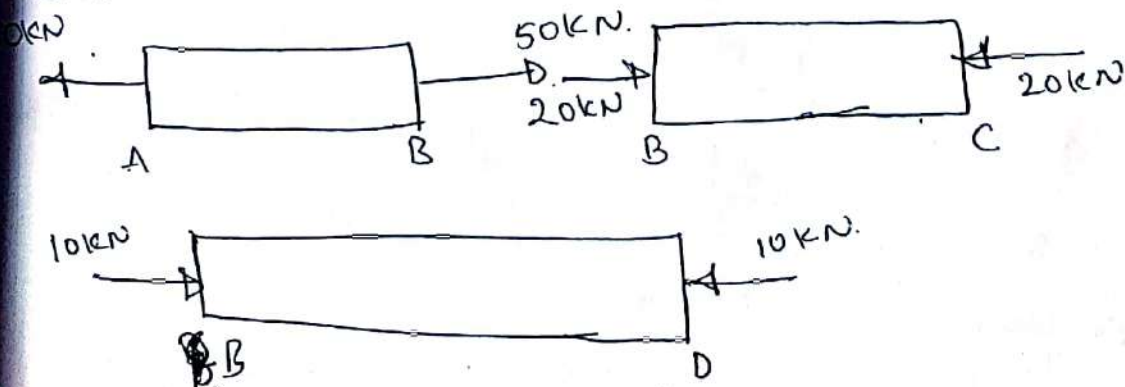
A brass bar, having cross-sectional area of 1000mm^2 , is subjected to axial forces as shown in fig.



Find the total elongation of the bar, take $E = 1.05 \times 10^5 \text{N/mm}^2$

$A = 1000\text{mm}^2$, $E = 1.05 \times 10^5 \text{N/mm}^2$, $\Delta L = \text{Total elongation of the bar.}$

The total of 80kN acting at B is split up into three parts of 50kN , 20kN , and 10kN . Then the part AB of the bar will be subjected to a tensile load of 50kN part BC is subjected to a compressive load of 20kN and part BD is subjected to a compressive load of 10kN in its.



The part AB is subjected to a tensile load of 50kN
Hence there will be ~~subjected to a tensile load of 50kN~~
~~Part BC is subjected to a~~ increase in length of this part

Increase in length of AB =

$$\Delta = \frac{P_1}{AE} \times L_1$$

$$P_1 = 50,000 \text{ N}$$

$$L_1 = 600 \text{ mm}$$

$$= \frac{50 \times 1000}{10000 \times 1.05 \times 10^5} \times 600$$

$$\Delta_1 = 0.2857 \text{ mm}$$

Increase in length
+ve Tension

The part BC is subjected to a compressive load of 20kN
(OR) 20,000N. Hence there will be decrease in length of the part.
Decrease in length

$$\Delta_2 = \frac{P_2}{A_2 E} \times L_2$$

$$P_2 = 20,000 \text{ N}$$

$$L_2 = 1 \text{ m} = 1000 \text{ mm}$$

$$\Delta_2 = \frac{20,000}{10000 \times 1.05 \times 10^5} \times 1000$$

$$\Delta_2 = 0.1904 \text{ mm}$$

Decrease in length
-ve

The part CD: This part is subjected to a compressive load of 10kN. Hence there will be decrease in length.

$$\Delta_3 = \frac{P_3}{AE} \times L_3$$

$$P_3 = 10,000 \text{ N}$$

$$L_3 = 1.2 + 1 = 2.2 \text{ m}$$

$$\Delta_3 = \frac{10,000}{10000 \times 1.05 \times 10^5} \times 2200$$

$$L_3 = 2200 \text{ mm}$$

$$\Delta_3 = 0.2095 \text{ mm}$$

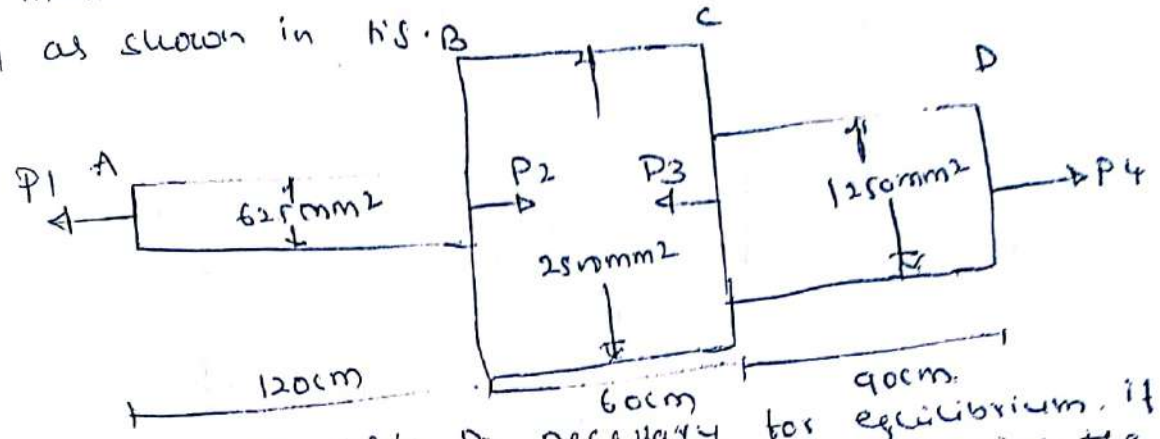
-ve [Decrease in length]

Total elongation of the bar = 0.2857 - 0.1904 - 0.2095

$$\Delta = -0.1142 \text{ mm}$$

Negative sign shows that there will be decrease in length of the bar.

A member ABCD is subjected to point loads P_1, P_2, P_3 & P_4 as shown in N.S.B



calculate the force P_2 necessary for equilibrium, if $P_1 = 45\text{KN}$, $P_3 = 450\text{KN}$ and $P_4 = 130\text{KN}$. Determine the total elongation of the member, assuming the modulus of elasticity to be $2.1 \times 10^5 \text{ N/mm}^2$

- Part AB $A_1 = 625\text{mm}^2$ $L_1 = 120\text{cm} = 1200\text{mm}$
 - Part BC $A_2 = 250\text{mm}^2$ $L_2 = 60\text{cm} = 600\text{mm}$
 - Part CD $A_3 = 1250\text{mm}^2$ $L_3 = 90\text{cm} = 900\text{mm}$
- $E = 2.1 \times 10^5 \text{ N/mm}^2$

Value of P_2 is necessary for equilibrium.

Resolve the forces on the rod about its axis (i.e. equating the forces acting towards left to those acting towards right) we get.

$$P_1 + P_3 = P_2 + P_4$$

But $P_1 = 45\text{KN}$, $P_3 = 450\text{KN}$, $P_4 = 130\text{KN}$.

$$45 + 450 = P_2 + 130$$

$$495 = P_2 + 130$$

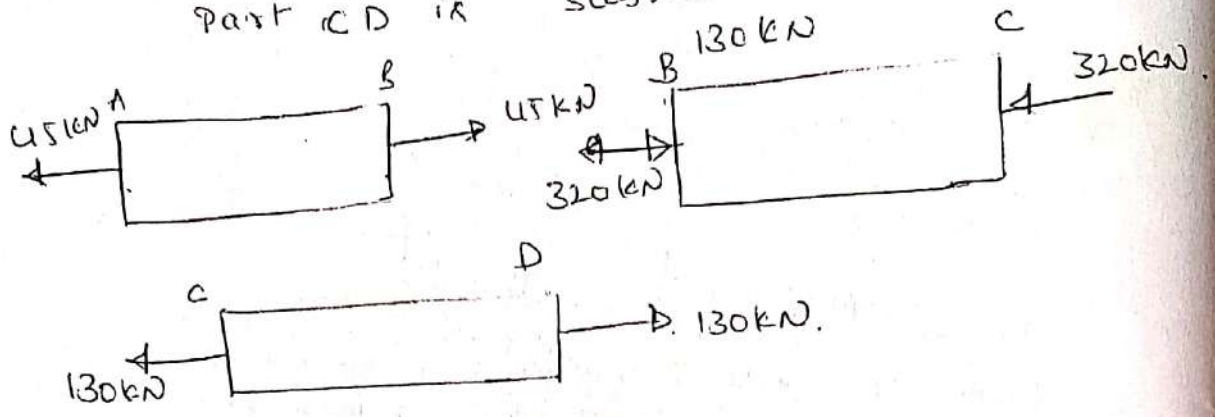
$$495 - 130 = P_2$$

$$P_2 = 365\text{KN}$$

The force of 365KN acting at B is split into two forces of 45KN and 320KN (i.e. $365 - 45 = 320\text{KN}$)

The force of 450KN acting at C is split into two forces of 320KN and 130KN (i.e. $450 - 320 = 130\text{KN}$)

It is clear that Part AB is subjected to tensile force
 Part BC is subjected to compressive force
 Part CD is subjected to tensile force



Part AB

$$\Delta_1 = \frac{P_1 \times L_1}{A_1 E} = \frac{45000 \times 1200}{625 \times 2.1 \times 10^5}$$

$$\Delta_1 = 0.4114 \text{ mm}$$

(+ve)
 (Increase in length)

Part BC

$$\Delta_2 = \frac{P_2 \times L_2}{A_2 E} = \frac{320,000 \times 600}{2500 \times 2.1 \times 10^5}$$

$$\Delta_2 = 0.3657 \text{ mm}$$

(-ve)

Part CD

$$\Delta_3 = \frac{P_3 \times L_3}{A_3 E} = \frac{130,000 \times 900}{1250 \times 2.1 \times 10^5}$$

$$\Delta_3 = 0.4457 \text{ mm}$$

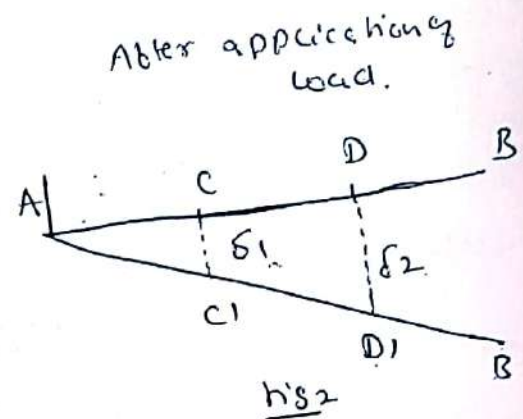
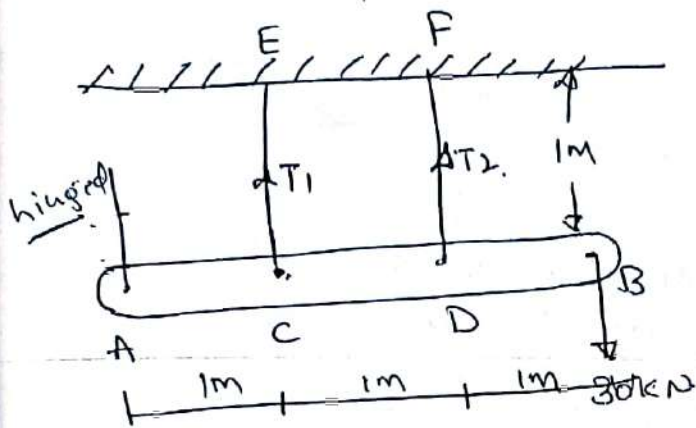
+ve

Total change in length $\Delta = \Delta_1 - \Delta_2 + \Delta_3$
 $= 0.4114 - 0.3657 + 0.4457$

$$\Delta = 0.4914 \text{ mm}$$

EXTENSION

A rigid bar ACDB is hinged at A and supported in a horizontal position by two identical steel wires as shown in fig. A vertical load of 30kN is applied at B. Find the tensile forces T_1 and T_2 induced in these wires by the vertical load.



Before application of load h/s_1

Two identical steel wires means the area of the cross-sections, lengths and value of E for both wires is same
 $A_1 = A_2$ $L_1 = L_2$ & $E_1 = E_2$
 Load at B = 30kN = 30,000 N.

b) T_1 = Tension in the first wire δ_1 = Extension of first wire

T_2 = Tension in the second wire δ_2 = —||— Second —||

Since the rigid bar remains straight.

hence extensions δ_1 & δ_2 are given by

$$\frac{\delta_1}{\delta_2} = \frac{AC}{AD} = \frac{1}{2}$$

But δ_1 is the extension of the wire EC $2\delta_1 = \delta_2$ (✓)

$$\sigma_1 = \frac{\text{Stress in EC} \times L_1}{E_1} = \left[\frac{T_1}{A_1} \right] \times L_1$$

$$\delta_1 = \frac{T_1 \times L_1}{A_1 \times E_1}$$

$$\delta_2 = \frac{T_2 \times L_2}{A_2 \times E_2}$$

Substituting values of δ_1 & δ_2 in equation (1)

$$2\delta_1 = \delta_2 \quad \rightarrow (1)$$

$$\frac{2 \times T_1 \times L_1}{A_1 \times E_1} = \frac{T_2 \times L_2}{A_2 \times E_2}$$

But $A_1 = A_2$ $L_1 = L_2$ $E_1 = E_2$, Hence above equation becomes

$$2T_1 = T_2 \quad \rightarrow (2)$$

Now taking the moments of all the forces on the rigid bar about A, we get

$$T_1 \times 1 + T_2 \times 2 = 30 \times 3$$

Substituting the value of T_2 from equation (2) into equation (3) we get

$$T_1 + 2T_2 = 90 \quad \rightarrow (3)$$

$$T_1 + 2(2T_1) = 90$$

$$5T_1 = 90$$

$$T_1 = 18 \text{ kN}$$

from equation

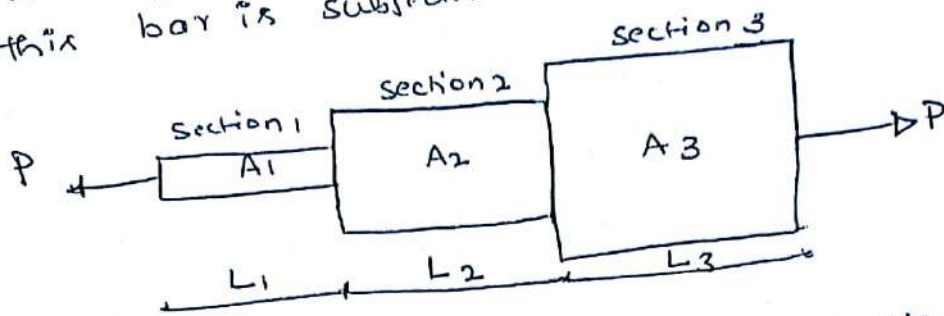
$$2T_1 = T_2$$

$$2 \times 18 = T_2$$

$$T_2 = 36 \text{ kN}$$

Analysis of Bars of Varying Sections

A bar of different lengths & of different diameters and hence of different cross sectional areas is shown in fig. (1). Let this bar is subjected to an axial load P.



The stress, strain and change in length will be different. The total change in length will be obtained by adding the changes in length of individual section.

Let \$P\$ = Axial load acting on the bar,

\$L_1 \& A_1\$ = Length and Area of cross section 1

\$L_2 \& A_2\$ = length and Area of cross section 2

\$L_3 \& A_3\$ = Length and Area of cross section 3

\$E\$ = Young's modulus for the bar.

Then stress for the section 1

$$\sigma_1 = \frac{\text{load}}{\text{Area of cross section 1}} = \frac{P}{A_1}$$

$$\sigma_1 = \frac{P}{A_1}$$

Similarly (2) & (3)

$$\sigma_2 = \frac{\text{load}}{\text{Area of cross section 2}} = \frac{P}{A_2}$$

$$\sigma_2 = \frac{P}{A_2}$$

$$\sigma_3 = \frac{P}{A_3}$$

$$E = \frac{\sigma}{\epsilon}$$

Young's modulus = $\frac{\text{Tensile Stress}}{\text{Tensile Strain}} = \frac{\sigma}{\epsilon}$ The strain in

different sections are obtained.

Strain of section 1 $e_1 = \frac{\sigma_1}{E} = \frac{P}{A_1 E}$

$\sigma_1 = \frac{P}{A_1}$

$$e_1 = \frac{P}{A_1 E}$$

Strain of section 2 & 3

$$e_2 = \frac{P}{A_2 E}$$

$$e_3 = \frac{P}{A_3 E}$$

But strain in section 1 = $\frac{\text{change in length of section 1}}{\text{length of this section 1}}$

where $e_1 = \frac{dL_1}{L_1}$
 $dL_1 = \text{change in length of section 1}$

$$e_1 = \frac{dL_1}{L_1}$$

$$dL_1 = e_1 L_1$$

where $e_1 = \frac{P}{A_1 E}$

$$dL_1 = \frac{P L_1}{A_1 E}$$

Similarly strain in section 2 & 3

$$dL_2 = \frac{P L_2}{A_2 E}$$

$$dL_3 = \frac{P L_3}{A_3 E}$$

total change in length of the bar

$$dL = dL_1 + dL_2 + dL_3$$

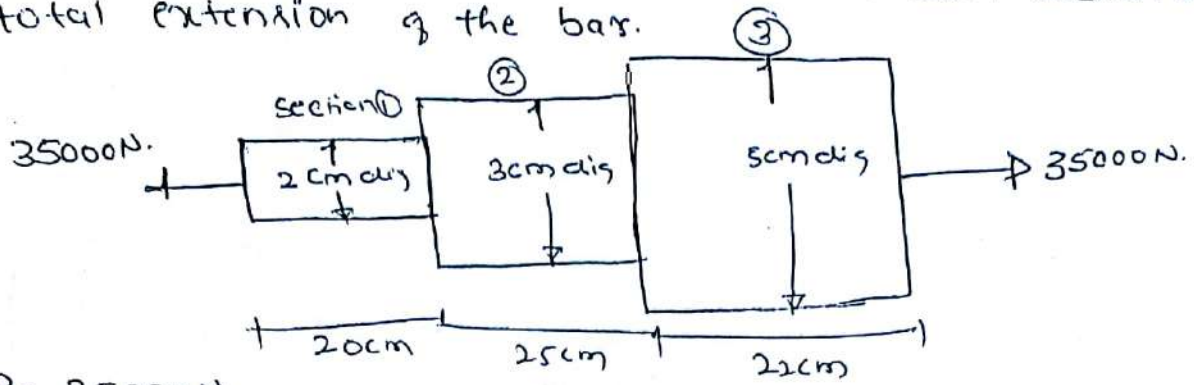
$$= \frac{P L_1}{A_1 E} + \frac{P L_2}{A_2 E} + \frac{P L_3}{A_3 E}$$

$$dL = \frac{P}{E} \left[\frac{L_1}{A_1} + \frac{L_2}{A_2} + \frac{L_3}{A_3} \right]$$

This equation is used when the young's modulus of a section is same. If the young's modulus of different section different, then total change in length of the bar is given by

$$dL = P \left[\frac{L_1}{A_1 E_1} + \frac{L_2}{A_2 E_2} + \frac{L_3}{A_3 E_3} \right]$$

Consisting of three lengths as shown in fig. If the young's modulus = $2.1 \times 10^5 \text{ N/mm}^2$. determine: (1) Stresser in each section and (2) total extension of the bar.



Given

$$P = 35000 \text{ N}$$

$$L_1 = 20 \text{ cm} = 200 \text{ mm}$$

$$L_2 = 25 \text{ cm} = 250 \text{ mm}$$

$$D_1 = 2 \text{ cm} = 20 \text{ mm}$$

$$D_2 = 3 \text{ cm} = 30 \text{ mm}$$

$$A_1 = \frac{\pi D_1^2}{4} = \frac{\pi (20)^2}{4} = 314.15 \text{ mm}^2 \quad A_2 = \frac{\pi D_2^2}{4} = \frac{\pi (30)^2}{4} = 706.85 \text{ mm}^2$$

$$L_3 = 22 \text{ cm} = 220 \text{ mm}$$

$$D_3 = 5 \text{ cm} = 50 \text{ mm}$$

$$E = 2.1 \times 10^5 \text{ N/mm}^2$$

$$A_3 = \frac{\pi D_3^2}{4} = \frac{\pi (50)^2}{4} = 1963.49 \text{ mm}^2$$

(1) Stresser in each section

$$\text{section ①} \quad \sigma_1 = \frac{\text{Axial load}}{\text{Area of section ①}} = \frac{35000 \text{ N}}{314.15 \text{ mm}^2} = 111.41 \text{ N/mm}^2$$

$$\text{section ②} \quad \sigma_2 = \frac{\text{Axial load}}{\text{Area of section ②}} = \frac{35000 \text{ N}}{706.85 \text{ mm}^2} = 49.51 \text{ N/mm}^2$$

$$\text{section ③} \quad \sigma_3 = \frac{35000}{1963.49} = 17.825 \text{ N/mm}^2$$

(2) Total Extension of the bar

$$\text{Total Extension } \Delta = \frac{P}{E} \left[\frac{L_1}{A_1} + \frac{L_2}{A_2} + \frac{L_3}{A_3} \right]$$

$$\Delta = \frac{35000}{2.1 \times 10^5} \left[\frac{200}{314.15} + \frac{250}{706.85} + \frac{220}{1963.49} \right]$$

$$\Delta = 0.1667 \times 10^{-1} [0.636 + 0.353 + 0.1120]$$

$$\Delta = 1.835 \text{ mm} \quad \text{or } 0.1835 \text{ mm}$$

3. A member formed by connecting a steel bar in shown in fig. Assuming that the bars are prevented from buckling sideways, calculate the magnitude of force P that will cause the total length of the member to decrease 0.25mm. The value of elastic modulus for steel and aluminium are $2.1 \times 10^5 \text{ N/mm}^2$ & $7 \times 10^4 \text{ N/mm}^2$ respectively.

Given

Length of steel bar $L_1 = 30 \text{ cm} = 300 \text{ mm}$

Area of steel bar $A_1 = 5 \text{ cm} \times 5 \text{ cm}$

$$= 50 \text{ mm} \times 50 \text{ mm} = 2500 \text{ mm}^2$$

elastic modulus

$$E_1 = 2.1 \times 10^5 \text{ N/mm}^2$$

Length of aluminium $L_2 = 38 \text{ cm} = 380 \text{ mm}$

Area of Aluminium $A_2 = 10 \text{ cm} \times 10 \text{ cm}$

$$= 100 \text{ mm} \times 100 \text{ mm}$$

elastic modulus

$$E_2 = 7 \times 10^4 \text{ N/mm}^2$$

Total decrease in length, $\Delta L = 0.25 \text{ mm}$

P = Required force.

$$\Delta L = P \left[\frac{L_1}{A_1 E_1} + \frac{L_2}{A_2 E_2} \right]$$

$$0.25 = P \left[\frac{300}{2500 \times 2.1 \times 10^5} + \frac{380}{10000 \times 7 \times 10^4} \right]$$

$$0.25 = P \left[5.714 \times 10^{-7} + 5.428 \times 10^{-7} \right]$$

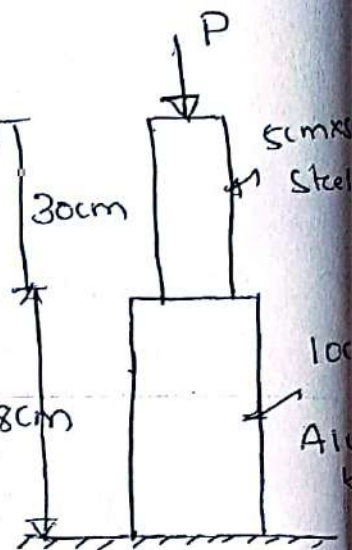
$$0.25 = P \left[1.114 \times 10^{-6} \right]$$

$$P = \frac{0.25}{1.114 \times 10^{-6}}$$

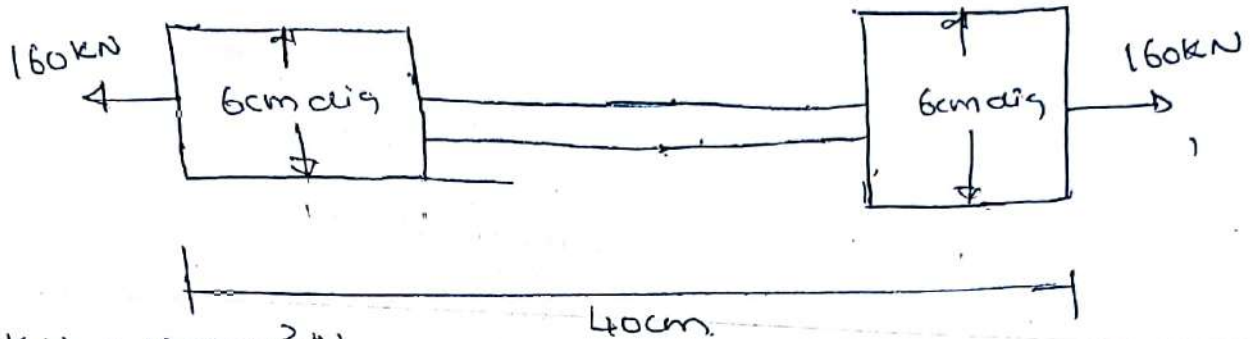
$$P = 224376.23 \text{ N}$$

$$P = 2.243 \times 10^5$$

$$P = 224.37 \text{ kN}$$



The bar shown in fig. is subjected to a tensile load of 160 kN. If the stress in the middle portion is limited to 150 N/mm^2 . determine the diameter of the middle portion. Find also the length of the middle portion. If the total elongation of the bar is to be 0.2 mm. Young's modulus given as equal to $2.1 \times 10^5 \text{ N/mm}^2$



$$= 160 \text{ kN} = 160 \times 10^3 \text{ N},$$

$$\text{stress in middle portion} = 150 \text{ N/mm}^2$$

$$\text{Total elongation } \Delta L = 0.2 \text{ mm.}$$

$$\text{Total length of the bar } L = 40 \text{ cm} = 400 \text{ mm}$$

$$E = 2.1 \times 10^5 \text{ N/mm}^2$$

$$D_1 = 6 \text{ cm} = 60 \text{ mm.}$$

$$A_1 = \frac{\pi D_1^2}{4} = \frac{\pi (60)^2}{4} = 2827.43 \text{ mm}^2.$$

$$D_2 = \text{Diameter of the middle portion.}$$

$$L_2 = \text{length of the middle portion in mm.}$$

$$\text{Length of both end portions of the bar}$$

$$L_1 = (400 - L_2) \text{ mm.}$$

$$\text{Stress} = \frac{\text{load}}{A_{\text{req}}}$$

middle portion

$$\sigma_2 = \frac{P}{A_2}$$

$$150 = \frac{160 \times 10^3}{A_2}$$

$$A_2 = \frac{160 \times 10^3}{150}$$

$$D_2^2 = \frac{4 \times 160 \times 10^3}{150 \times \pi}$$

$$D_2^2 = 1358.12 \text{ mm}^2$$

$$D_2 = 36.85 \text{ mm}$$

$$D_2 = 3.68 \text{ cm}$$

Area of cross section of middle portion

$$A_2 = \frac{\pi}{4} \times D_2^2$$

$$A_2 = \frac{\pi}{4} \times 36.85^2$$

$$A_2 = 1066.6 \text{ mm}^2$$

$$\text{Total def extension } \Delta L = \frac{P}{E} \left[\frac{L_1}{A_1} + \frac{L_2}{A_2} \right]$$

$$0.2 = \frac{160 \times 10^3}{2.1 \times 10^5} \left[\frac{(400 - L_2)}{2827.43} + \frac{L_2}{1066} \right]$$

$$\frac{0.2 \times 2.1 \times 10^5}{160 \times 10^3} = \frac{(400 - L_2)}{2827.43} + \frac{L_2}{1066}$$

$$0.2625 = \frac{1066(400 - L_2) + 2827.43L_2}{2827.43 \times 1066}$$

$$791185.55 = 426400 - 1066L_2 + 2827.43L_2$$

$$791185.55 = 426400 - 1066L_2 + 2827.43L_2$$

$$791185.55 - 426400 = L_2 [2827 - 1066]$$

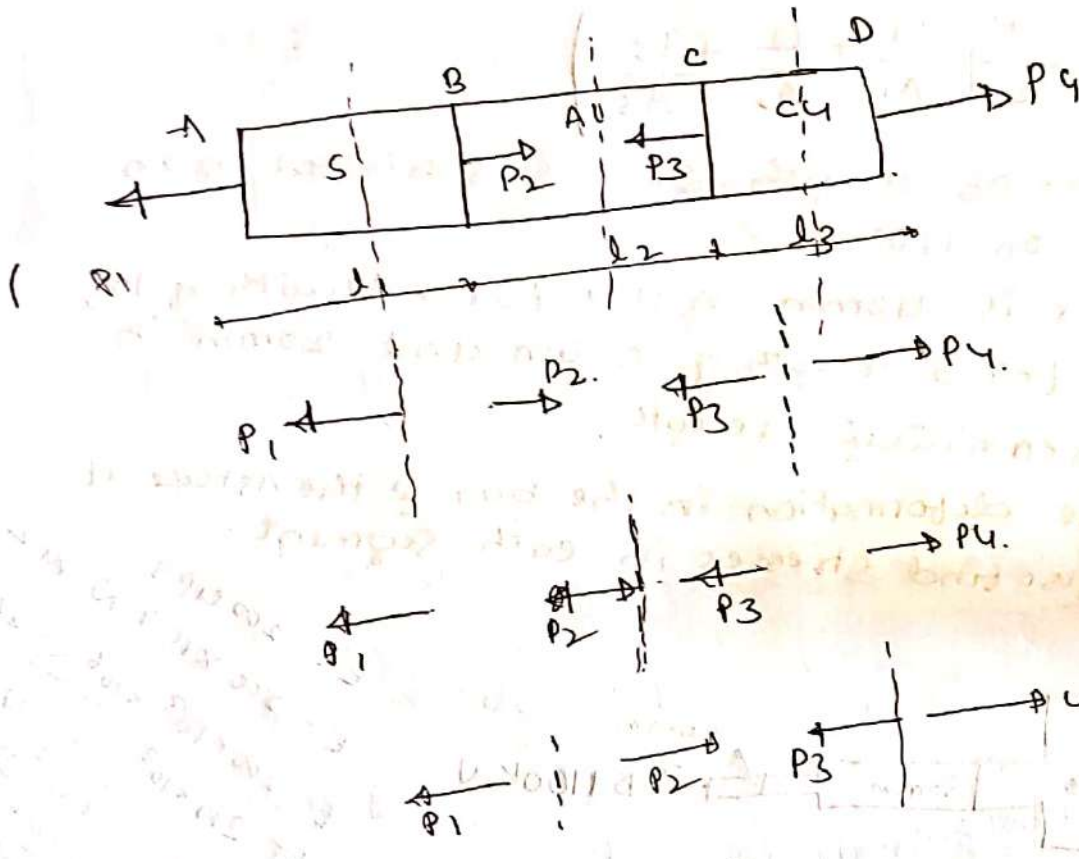
$$364785.55 = 1761L_2$$

$$L_2 = 207.14 \text{ mm}$$

$$L_2 = 20.714 \text{ cm}$$

Principle of Superposition:

When a number of loads are acting on a body in equilibrium, the resulting strain will be the algebraic sum of the strain caused by individual forces, and is known as principle of superposition.

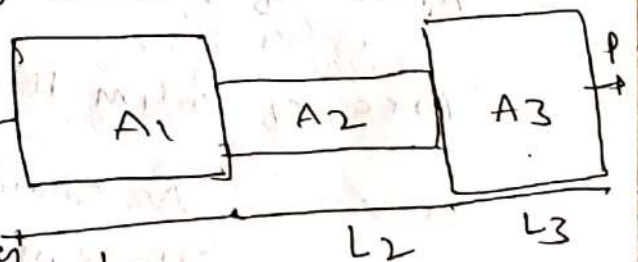


Young's modulus $E = \frac{\pm \sigma_{AB} \pm \sigma_{BC} \pm \sigma_{CD}}$

Strain Elongation $\Delta = \pm \Delta_{AB} \pm \Delta_{BC} \pm \Delta_{CD}$

Bars of Varying Sections

If an axial force P (tensile \odot compressive) is acting on a bar of varying cross section, the total elongation \odot contraction of the bar will be equal to the sum of L_1 , L_2 , L_3 elongation \odot contraction of each section, under the action of axial force P .



Thus referring fig.

The total elongation Δ will be given by

$$\Delta = \Delta_1 + \Delta_2 + \Delta_3$$

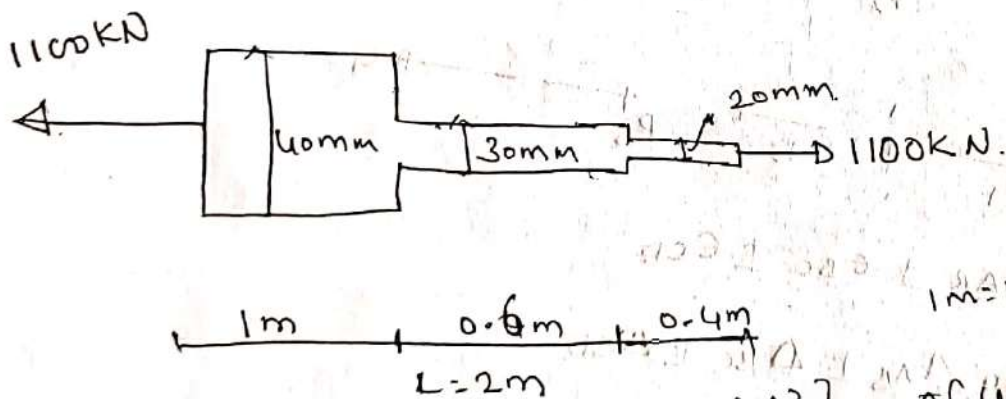
where Δ_1 , Δ_2 & Δ_3 are the the elongations of the three portions.

$$\text{Thus } \Delta = \frac{P}{E} \left[\frac{L_1}{A_1} + \frac{L_2}{A_2} + \frac{L_3}{A_3} \right]$$

① A steel bar of length 2m is subjected to an axial pull of 1100kN.

② The bar is 40mm in dia for a length of 1m, 30mm dia for a length of 0.6m and 20mm in dia for remaining length.

③ find the deformation in the bar & the stress if $E = 200 \text{ GPa}$. also find stresses in each segment.



$E = 200 \text{ GPa}$
 $E = 200 \times 10^9 \text{ N/m}^2$
 $E = 200 \times 10^3 \times 10^6 \text{ N/m}^2$
 $E = 200 \times 10^3 \times \frac{10^6}{1000} \text{ N/mm}^2$
 $E = 200 \times 10^3 \times 1000 \text{ N/mm}^2$
 $E = 200 \times 10^6 \text{ N/mm}^2$

1. Area of 1m length = $\frac{\pi [d^2]}{4} = \frac{\pi [40]^2}{4} = 1256.63 \text{ mm}^2$
2. Area of 0.6m length = $\frac{\pi [30]^2}{4} = 706.85 \text{ mm}^2$
3. Area of 0.4m length = $\frac{\pi [20]^2}{4} = 314.15 \text{ mm}^2$

Stresses - As single tensile force is acting, each segment will have same load.

i.e. $P_1 = P_2 = P_3 = 1100 \times 10^3 \text{ N}$

Stress in each segment $\sigma_1 = \frac{P_1}{A_1} = \frac{1100 \times 10^3}{1256.6} = 875.37 \text{ N/mm}^2$

Stress in 2nd segment $\sigma_2 = \frac{P_2}{A_2} = \frac{1100 \times 10^3}{706.8} = 1556.31 \text{ N/mm}^2$

Stress in 3rd segment $\sigma_3 = \frac{P_3}{A_3} = \frac{1100 \times 10^3}{314.15} = 3497.6 \text{ N/mm}^2$

⑤ Deformation in segment

$$\Delta_{AB} = \left[\frac{PL}{AE} \right]_{AB} = \left[\frac{1100 \times 1000 \times 10^3}{1256.6 \times 200 \times 10^3} \right] = 4.37 \text{ mm}$$

$$\Delta_{BC} = \left[\frac{PL}{AE} \right]_{BC} = \left[\frac{1100 \times 10^3 \times 600}{706.8 \times 200 \times 10^3} \right] = 4.67 \text{ mm}$$

$$\Delta_{CD} = \left[\frac{PL}{AE} \right]_{CD} = \left[\frac{1100 \times 10^3 \times 400}{314.15 \times 200 \times 10^3} \right] = 7.00 \text{ mm}$$

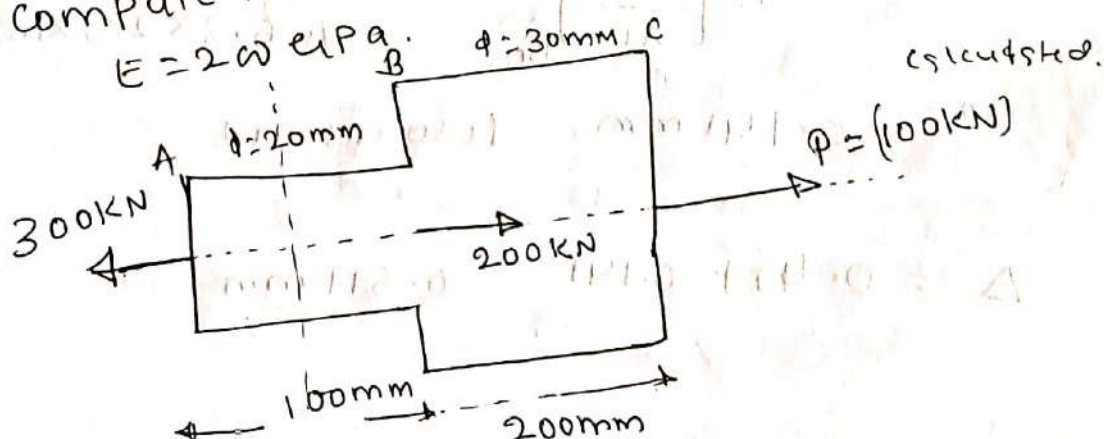
$$\Delta = \Delta_{AB} + \Delta_{BC} + \Delta_{CD}$$

$$\Delta = 4.37 + 4.67 + 7.00$$

$\Delta = 16.04 \text{ mm}$ - total elongation.

② Determine the stress in different segments of a circular bar shown in fig

③ Compute the total deformation of the part. if $E = 200 \text{ GPa}$.



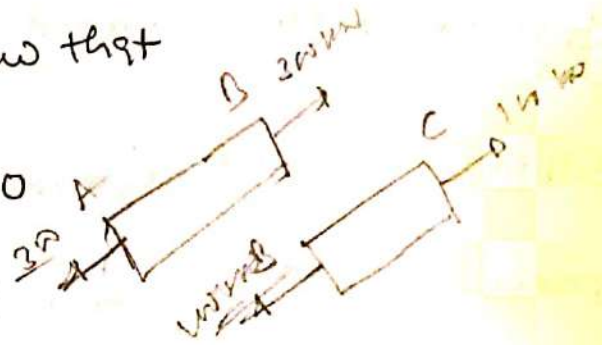
(21)

For equilibrium we know that

$$\sum H = 0$$

$$-300 + 200 + P = 0$$

$$P = 100 \text{ kN}$$



$$\text{Area } A_{AB} = \frac{\pi d^2}{4} = \frac{\pi (20)^2}{4} = 314.15 \text{ mm}^2$$

$$A_{BC} = \frac{\pi (30)^2}{4} = 706.8 \text{ mm}^2$$

Stress in segment AB

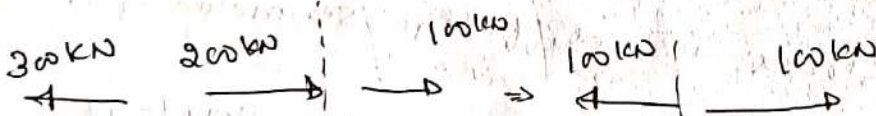
$$f_{AB} = \frac{P}{A_{AB}} = \frac{300 \times 10^3}{314.15} = 955.10 \text{ N/mm}^2 \text{ (T)}$$

$$\Delta_{AB} = \left[\frac{PL}{AE} \right]_{AB} = \frac{300 \times 10^3 \times 200}{314.15 \times 200 \times 10^3} = 0.47 \text{ mm}$$

$\Delta_{AB} = 0.47 \text{ mm}$ Elongation

Stress in segment BC

$$f_{BC} = \frac{P_{BC}}{A} = \frac{100 \times 10^3}{706.8} = 141.48 \text{ N/mm}^2 \text{ (T)}$$

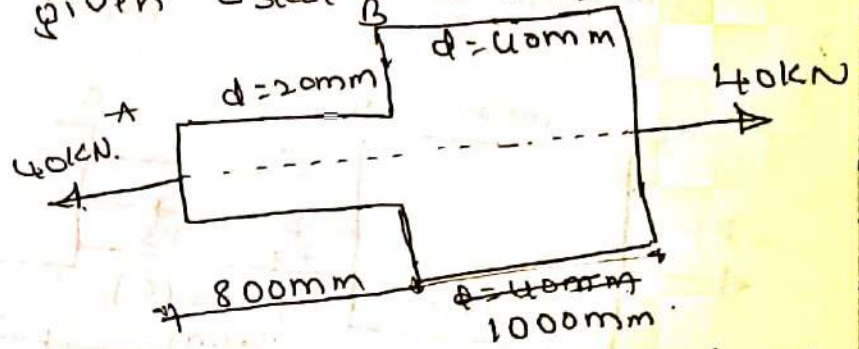


$$\Delta_{BC} = \left[\frac{PL}{AE} \right]_{BC} = \frac{100 \times 10^3 \times 200}{706.8 \times 200 \times 10^3}$$

$$\Delta_{BC} = 0.141 \text{ mm (Elongation)}$$

$$\Delta = 0.47 + 0.141 = 0.611 \text{ mm}$$

3) A steel rod 20mm dia and 800mm long is rigidly attached to an aluminum rod, 40mm dia and 1m long. The combination is subjected to a tensile load of 40kN. Find the stress in the material and total elongation of the bar. Given $E_{\text{steel}} = 200 \text{ GPa}$, $E_{\text{aluminum}} = 70 \text{ GPa}$.



$E_s = 200 \times 10^3 \text{ N/mm}^2$
 $E_{al} = 70 \times 10^3 \text{ N/mm}^2$

$A_{AB} = \frac{\pi (20)^2}{4} = 314.15 \text{ mm}^2$

$A_{BC} = \frac{\pi (40)^2}{4} = 1256.63 \text{ mm}^2$

Stress in segment AB is

$\sigma_{AB} = \frac{P_{AB}}{A_{AB}} = \frac{40 \times 10^3}{314.15}$

$\sigma_{AB} = 127.34 \text{ MPa}$

Deformation in AB

$\Delta_{AB} = \left[\frac{PL}{AE} \right]_{AB} = \frac{40 \times 10^3 \times 800}{314.15 \times 200 \times 10^3}$

$\Delta_{AB} = \frac{40 \times 10^3 \times 800}{314.15 \times 200 \times 10^3}$

$\Delta_{AB} = 0.509 \text{ mm}$ Elongation

Stress in segment BC is

$\sigma_{BC} = \frac{P_{BC}}{A_{BC}} = \frac{40 \times 10^3}{1256.63} = 31.83 \text{ N/mm}^2$

Deformation in BC

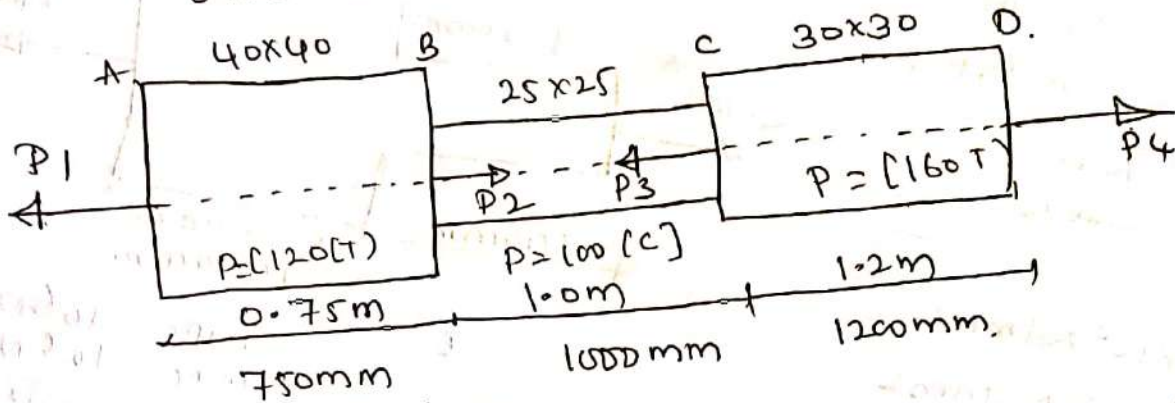
$\Delta_{BC} = \left[\frac{PL}{AE} \right]_{BC} = \frac{40 \times 10^3 \times 1000}{1256.63 \times 70 \times 10^3} = 0.45 \text{ mm}$

Total Deflection Deformation

$\Delta = \Delta_{AB} + \Delta_{BC} = 0.509 + 0.45$

$\Delta = 0.963 \text{ mm}$ Elongation

(4) A member ABCD is subjected to point load P_1, P_2, P_3 & P_4 as shown in fig. calculate the force P_3 for equilibrium if $P_1 = 120\text{ kN}, P_2 = 220\text{ kN}, P_4 = 160\text{ kN}$. Determine also the net change in length of the bar take $E = 2 \times 10^5 \text{ N/mm}^2$.



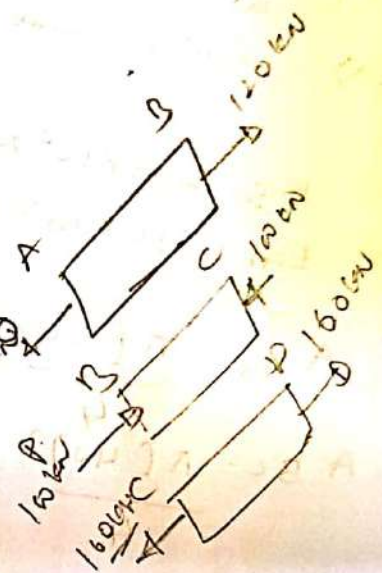
For Equilibrium

$$\sum H = 0$$

$$-P_1 + P_2 - P_3 + P_4 = 0$$

$$-120 + 220 - P_3 + 160 = 0$$

$$P_3 = 260\text{ kN}$$



Deformation in AB

$$\Delta_{AB} = \left[\frac{P_1 L}{AE} \right]_{AB} = \frac{120 \times 750}{40 \times 40 \times 2 \times 10^5}$$

$$\Delta_{AB} = 0.281\text{ mm} \quad \text{Elongation}$$

$$A_{AB} = 40 \times 40 = 1600\text{ mm}^2$$

$$\Delta_{BC} = \left[\frac{P_2 L}{AE} \right]_{BC} = \frac{100 \times 10^3 \times 1000}{25 \times 25 \times 2 \times 10^5}$$

$$\Delta_{BC} = 0.8\text{ mm} \quad \text{[compression]}$$

$$\Delta_{CD} = \left[\frac{P_4 L}{AE} \right]_{CD} = \frac{160 \times 10^3 \times 1200}{900 \times 2 \times 10^5}$$

$$\Delta_{CD} = 1.066\text{ mm}$$

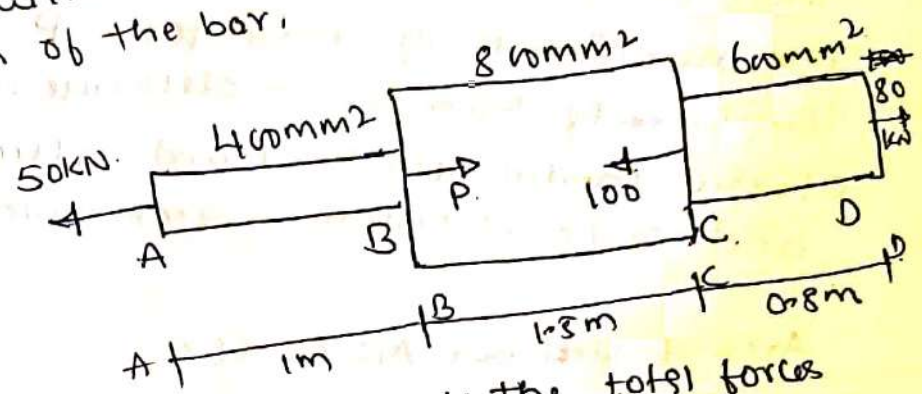
$$A_{CD} = 30 \times 30 = 900\text{ mm}^2$$

$$\Delta = 0.281 - 0.8 + 1.066$$

$$\text{Total Deformation } \Delta = 0.547\text{ mm}$$

5. A steel bar ABCD of varying sections is subjected to the axial forces as shown in fig. Find out the value of P necessary for bar equilibrium. If $E = 210 \text{ kN/mm}^2$, determine the total Elongation of the bar.

$E = 210 \text{ kN/mm}^2$
 $= 210 \times 10^3 \text{ N/mm}^2$

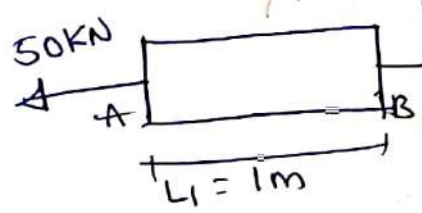


For the equilibrium of the bar equate the total forces acting towards right to the total forces acting to the left

$$-50 + P - 100 + 80 = 0$$

$$P = 70 \text{ kN}$$

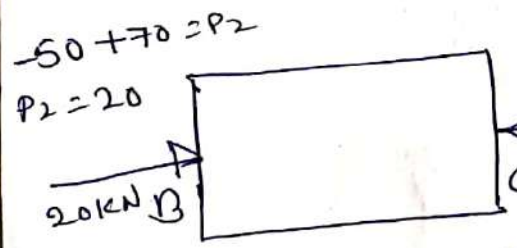
Now consider equilibrium of each part of the parts AB, BC, and CD.



$$P_1 = [70 - 100 + 80]$$

$$\Delta_{AB} = \frac{P_1 L_1}{A_1 E}$$

$$\Delta_{AB} = \frac{50 \times 10^3 \times 1000}{400 \times 210 \times 10^3} = 0.595 \text{ mm [T]}$$



$$-50 + 70 = P_2$$

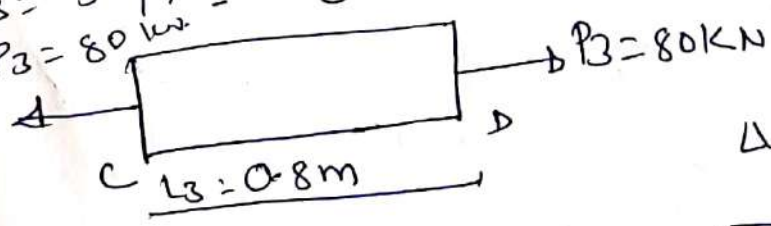
$$P_2 = 20$$

$$P_3 = 80 - 100$$

$$P_3 = 20 \text{ kN}$$

$$\Delta_{BC} = \frac{P_2 L_2}{A_2 E}$$

$$\Delta_{BC} = \frac{20 \times 10^3 \times 1500}{800 \times 210 \times 10^3} = 0.178 \text{ mm [Compression]}$$



$$P_3 = -50 + 70 + 100$$

$$P_3 = 80 \text{ kN}$$

$$\Delta_{CD} = \frac{P_3 L_3}{A_3 E}$$

$$\Delta_{CD} = \frac{80 \times 10^3 \times 800}{600 \times 210 \times 10^3}$$

$$\Delta_{CD} = 0.507 \text{ mm [T]}$$

Total Deformation $\Delta = \Delta_{AB} - \Delta_{BC} + \Delta_{CD} = 0.595 - 0.178 + 0.507 = 0.924 \text{ mm}$

Expression for deformation in case of uniformly tapering sections.

Case: 1 Uniformly tapering rod (or) Circular bar

Consider a uniformly tapering rod (or) circular bar from d_1 to d_2 over a length L subjected to pull P as shown in fig with E as young's modulus.

M. E. Rajhu,
Assistant Professor.
B. P. E T.
Davanagere

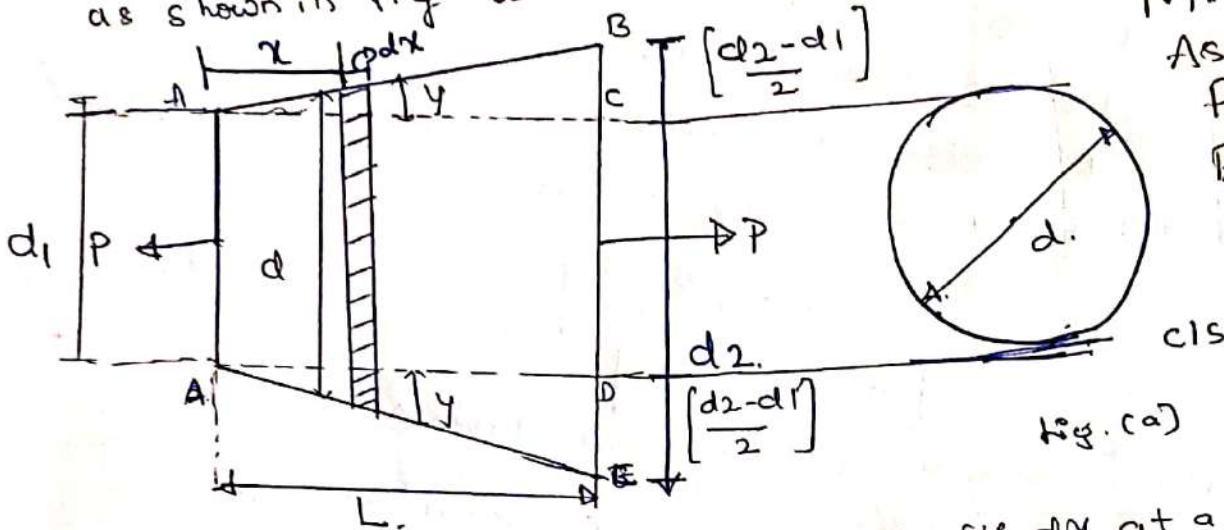


fig. (a)

Consider an elemental strip of length dx at a distance x as shown and let the diameter of the section be d , as the section is uniformly tapering.

$$d = d_1 + y + y$$

$$d = d_1 + 2y$$

From similar triangles,

$$\frac{d_2 - d_1}{2} = \frac{L}{x} \cdot y$$

$$y = \frac{d_2 - d_1}{2L} \cdot x \cdot x$$

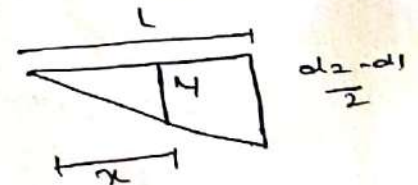
$$d = d_1 + 2y$$

$$d = d_1 + 2 \left[\frac{d_2 - d_1}{2L} \right] \cdot x \cdot x$$

$$d = d_1 + \left[\frac{d_2 - d_1}{L} \right] \cdot x \cdot x$$

$$d = d_1 + k(x^2) \quad \text{--- (1)}$$

$$\text{Let } k = \frac{d_2 - d_1}{L}$$



Let 'P' be the axial load on a circular bar of length 'L' as shown in fig. The bar of length 'L' has a diameter d_1 at one end & d_2 at the other end.

Consider an elementary strip of thickness dx at a distance 'x' from end A.

Let d_1 = Diameter at the smaller end.

d_2 = Diameter at the larger end.

L = length of the bar.

E = Young's modulus.

from fig & by considering similar triangles we get

$$d = d_1 + \left[\frac{d_2 - d_1}{L} \right] \cdot x$$

$$k = \frac{d_2 - d_1}{L}$$

$$d = d_1 + k \cdot x$$

Area of section at distance 'x' from A

i.e. $A = \frac{\pi d^2}{4}$

(Circular section)

Length of elementary section $L = dx$

w.k.t. $\Delta = \frac{PL}{AE}$

Let δ or Δ = deformation in the elementary strip.

∴ Extension in the elementary strip = $\frac{P \cdot dx}{\frac{\pi d^2}{4} \cdot E}$

$$\text{Extension in the elementary strip} = \frac{4 P \cdot dx}{\pi d^2 \cdot E}$$

Total Extension of the bar is obtained by integrating the above equation b/w the limits 0 to L

i.e. $\Delta = \int_0^L \frac{4 P \cdot dx}{\pi d^2 \cdot E}$

$$\Delta = \int_0^L \frac{4 P \cdot dx}{\pi \cdot (d_1 + k \cdot x)^2 \cdot E}$$

$$\Delta = \frac{4P}{\pi E} \int_0^L \frac{dx}{(d_1 + kx)^2}$$

$$\int (ax + b)^n \cdot dx$$

$$= \frac{1}{a} \frac{(ax + b)^{n+1}}{n+1}$$

$n \neq -1$

(or) $\int_0^L \frac{1}{(a + bx)^2} dx$

$$\Delta = \frac{4P}{\pi E} \left[-\frac{1}{k} \left[\frac{1}{a_1 + kx} \right] \right]_0^L$$

$$= \left(-\frac{1}{b} \right) \left[\frac{1}{a + bx} \right]_0^L$$

$$\Delta = \frac{-4PL}{\pi E (d_2 - d_1)} \left[\frac{1}{d_1 + \frac{(d_2 - d_1)x}{L}} \right]_0^L$$

$$= \frac{-4PL}{\pi E (d_2 - d_1)} \left[\frac{1}{d_1 + \frac{(d_2 - d_1)L}{L}} - \frac{1}{d_1 + 0} \right]$$

$$\Delta = \frac{-4PL}{\pi E (d_2 - d_1)} \left[\frac{1}{d_2} - \frac{1}{d_1} \right]$$

$$= \frac{-4PL}{\pi E (d_2 - d_1)} \left[\frac{d_1 - d_2}{d_2 d_1} \right]$$

taking -ve inside.

$$\frac{4PL}{\pi E (d_2 - d_1)} \left[\frac{d_2 - d_1}{d_1 d_2} \right]$$

$$\Delta = \frac{4PL}{\pi E d_1 d_2}$$

$$\Delta = \frac{4PL}{\pi E d^2}$$

1. A stepped bar is subjected to an external loading as shown in fig. calculate the change in length of the bar. $E_s = 200 \text{ GPa}$, $E_{cu} = 100 \text{ GPa}$, $\phi = 50 \text{ mm}$.

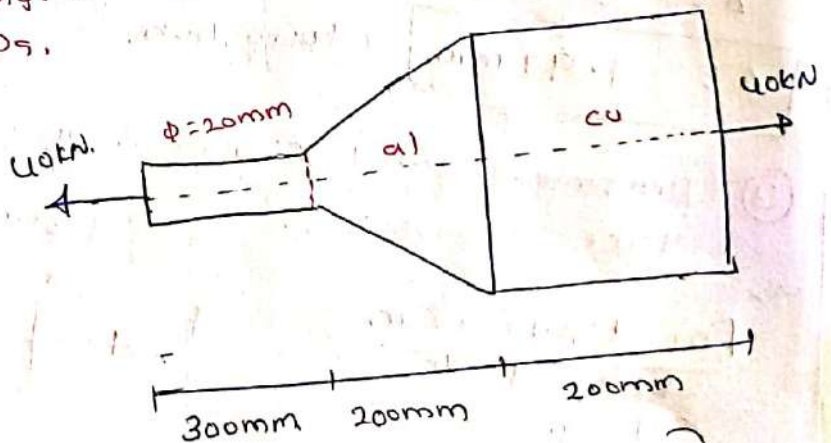
$$\Delta = \Delta_s + \Delta_{q1} + \Delta_{cu}$$

$$\Delta = \frac{PL}{AE} + \frac{4PL}{\pi E d_1 d_2} + \frac{PL}{AE}$$

$$= P \left[\frac{L}{AE} + \frac{4L}{\pi E d_1 d_2} + \frac{L}{AE} \right]$$

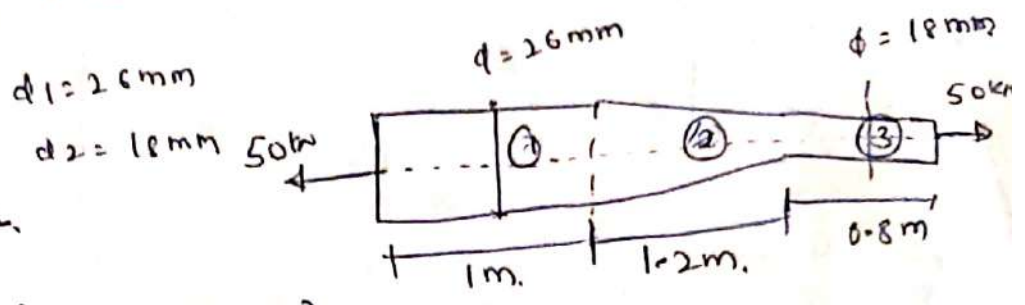
$$\Delta = 40 \times 10^3 \left[\frac{300}{\pi \left[\frac{20}{4} \right]^2 \times (200 \times 10^3)} + \frac{4 \times 200}{\pi \times 70 \times 10^3 \times 20 \times 50} + \frac{200}{\pi (50)^2 \times (100 \times 10^3)} \right]$$

$$\Delta = 0.37 \text{ mm}$$



② Find the total elongation of the bar shown in fig. subjected to an axial load of 50kN. Take $E = 2.1 \times 10^5 \text{ MPa}$.

$\Rightarrow P = 50 \times 10^3 \text{ N}$
 $E = 2.1 \times 10^5 \text{ MPa}$
 $E = 2.1 \times 10^5 \text{ N/mm}^2$



$$A_1 = \frac{\pi d_1^2}{4} = \frac{\pi [26]^2}{4} = 530.92 \text{ mm}^2$$

$$A_2 = \frac{\pi d_2^2}{4} = \frac{\pi [18]^2}{4} = 254.46 \text{ mm}^2$$

$1\text{m} = 1000\text{mm}$
 $1.2 = 1200\text{mm}$

The total elongation $\Delta = \Delta_1 + \Delta_2 + \Delta_3$

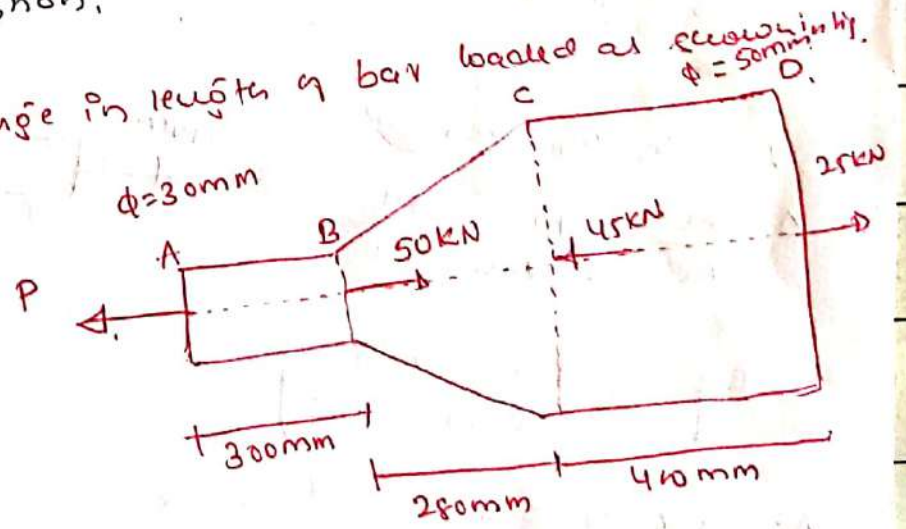
$$\Delta = \left[\frac{PL}{AE} \right]_1 + \left[\frac{4PL}{\pi E d_1 d_2} \right]_2 + \left[\frac{PL}{AE} \right]_3$$

$$\Delta = \left[\frac{50 \times 10^3 \times 1 \times 10^3}{530.9 \times 2.1 \times 10^5} \right] + \left[\frac{4 \times 50 \times 10^3 \times 1.2 \times 10^3}{\pi \times 2.1 \times 10^5 \times 26 \times 18} \right] + \left[\frac{50 \times 10^3 \times 0.8 \times 10^3}{254.46 \times 2.1 \times 10^5} \right]$$

$$\Delta = 0.448 + 0.777 + 0.748$$

$\Delta = 1.97 \text{ mm}$. Elongation.

③ Determine total change in length of bar loaded at section in fig. Take $E = 200 \text{ GPa}$.



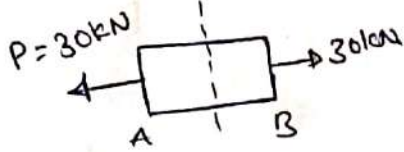
For Equilibrium
 $\rightarrow +ve \quad \leftarrow -ve$
 $\Sigma H = 0$

$$-P + 50 - 45 + 25 = 0$$

$$-P + 30 = 0$$

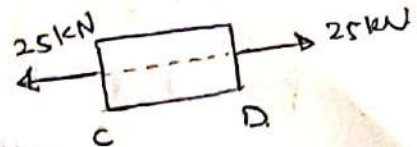
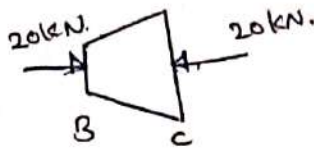
$P = 30 \text{ kN}$.

At segment ①



$$30 - 50 = -20$$

$$45 - 25 = 20$$



total Deformation $\Delta = \pm \Delta_{AB} \pm \Delta_{BC} \pm \Delta_{CD}$

$$\Delta = \left[\frac{PL}{AE} \right]_{AB} + \left[\frac{4PL}{\pi E d_1 d_2} \right]_{BC} + \left[\frac{PL}{AE} \right]_{CD}$$

Tension = +ve
Compression = -ve

$$A_1 = \frac{\pi [d_1]^2}{4} = \frac{\pi [30]^2}{4} = 706.85 \text{ mm}^2$$

$$A_2 = \frac{\pi [d_2]^2}{4} = \frac{\pi [50]^2}{4} = 1963.49 \text{ mm}^2$$

$$200 \text{ GPa} \\ 200 \times 10^9 \text{ N/mm}^2$$

$$E_{DS} = 10^3 \text{ (MPa)}$$

$$E = 200 \times 10^3 \text{ N/mm}^2$$

$$\Delta = \left[\frac{30 \times 10^3 \times 3}{706.85 \times 200 \times 10^3} \right]_{AB} - \left[\frac{4 \times 20 \times 10^3 \times 3}{\pi \times 200 \times 10^3 \times 30 \times 50} \right]_{BC} + \left[\frac{25 \times 10^3 \times 4}{1963.49 \times 200 \times 10^3} \right]_{CD}$$

$$\Delta = (0.063)_{AB} - (0.023)_{BC} + (0.025)_{CD}$$

$\Delta =$ Elongation - shortening + Elongation

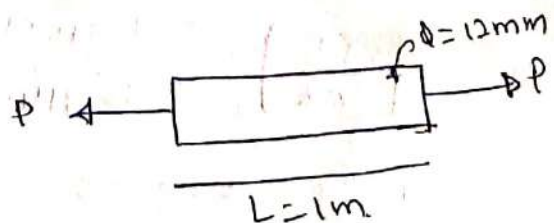
$$\Delta = 0.065 \text{ mm}$$

Ⓢ A rod of 12mm dia and 1m long is subjected to a tensile load of P in such a way that elongation should not be more than 0.4mm. Find the value of P if $E = 2 \times 10^5 \text{ N/mm}^2$

\Rightarrow $d = 12 \text{ mm}$, $L = 1 \text{ m} = 1000 \text{ mm}$ $P = ?$ $\Delta = 0.4 \text{ mm}$
 $E = 2 \times 10^5 \text{ N/mm}^2$

$$\Delta = \frac{PL}{AE}$$

$$0.4 = \frac{P \times 1000 \text{ mm}}{113.09 \text{ mm}^2 \times 2 \times 10^5 \text{ N/mm}^2}$$



$$A = \frac{\pi (12)^2}{4} = 113.09 \text{ mm}^2$$

$$P = \frac{\Delta AE}{L}$$

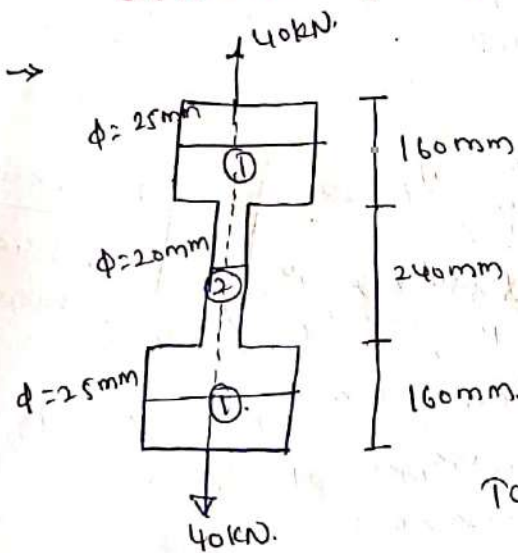
$$P = \frac{0.4 \times 113.09 \times 2 \times 10^5}{1000}$$

$$P = 9047.2 \text{ N}$$

$$P = 9.047 \times 10^3 \text{ N}$$

$$P = 9.047 \text{ kN}$$

Q. A bar shown in fig is tested in UTM, it is observed that a load of 40 kN. total extension is 0.285 mm. Determine young's modulus.



$$P = 40 \text{ kN.}$$

$$\Delta = 0.285 \text{ (mm)}$$

$$E = ?$$

$$A_1 = \frac{\pi [d_1]^2}{4} = \frac{\pi (25)^2}{4} = 490.87 \text{ mm}^2$$

$$A_2 = \frac{\pi [d_2]^2}{4} = \frac{\pi (20)^2}{4} = 314.15 \text{ mm}^2$$

$$L_1 = L_3 = 160 \text{ mm, } d_1 = 25 \text{ mm}$$

$$L_2 = 240 \text{ mm, } d_2 = 20 \text{ mm.}$$

$$\text{Total Deformation } \Delta = \Delta_1 + \Delta_2 + \Delta_3$$

$$\Delta = 0.285 \text{ mm}$$

but $E = ?$

$$\Delta = \frac{PL}{AE}$$

$$E_1 = \left[\frac{PL}{A\Delta} \right]_1 = \left[\frac{40 \times 10^3 \times 160}{490.8 \times 0.285} \right]_1 = 45754.15 \text{ N/mm}^2$$

$$E_2 = \left[\frac{PL}{A\Delta} \right]_2 = \left[\frac{40 \times 10^3 \times 240}{314.15 \times 0.285} \right]_2 = 107223.33 \text{ N/mm}^2$$

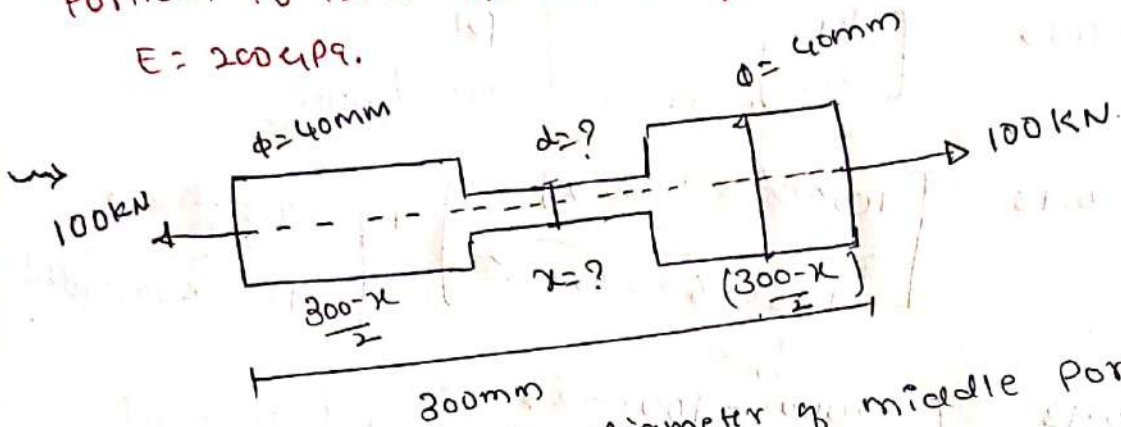
$$E = E_1 + E_2 + E_3$$

$$E = 45754.15 + 107223.33 + 45754.15$$

$$E = 198731.63 \text{ MPa.}$$

$$E = 1.98 \times 10^5 \text{ N/mm}^2$$

5) A bar shown in fig is subjected to loading. find the diameter of the middle portion. if the stress there is to be limited to 100 N/mm^2 . Also find the length of the middle portion. if total extension of the bar is to be 0.13 mm . give $E = 200 \text{ GPa}$.



Case 1: Let 'd' be the diameter of middle portion

if $\sigma = 100 \text{ N/mm}^2$ $P = 100 \text{ kN}$.

$$\sigma = \frac{P}{A} = \frac{100 \times 10^3}{\frac{\pi d^2}{4}}$$

$$100 = \frac{100 \times 10^3 \times 4}{\pi d^2}$$

$$d^2 = \frac{100 \times 10^3 \times 4}{100 \times \pi}$$

$$d^2 = 1273.2$$

$$d = \sqrt{1273.2}$$

$$d = 35.68 \text{ mm}$$

$$A_1 = \frac{\pi (40)^2}{4} = 1256.63 \text{ mm}^2$$

$$A_2 = \frac{\pi (35.68)^2}{4} = 999.86 \text{ mm}^2 = 1000 \text{ mm}^2$$

Case 2

Let x be the length of the middle portion

$$\Delta = \Delta_1 + \Delta_2 + \Delta_3 = 0.13 \text{ m}$$

$$E = 200 \text{ GPa} = 200 \times 10^3 \text{ N/mm}^2$$

total elongation $\Delta = \left[\text{Elongation of outer segments} \right] + \left[\text{Elongation of middle segment} \right]$

$$0.13 = \left[\frac{PL}{AE} \right]_{\text{outer}} + \left[\frac{PL}{AE} \right]_{\text{inner middle}}$$

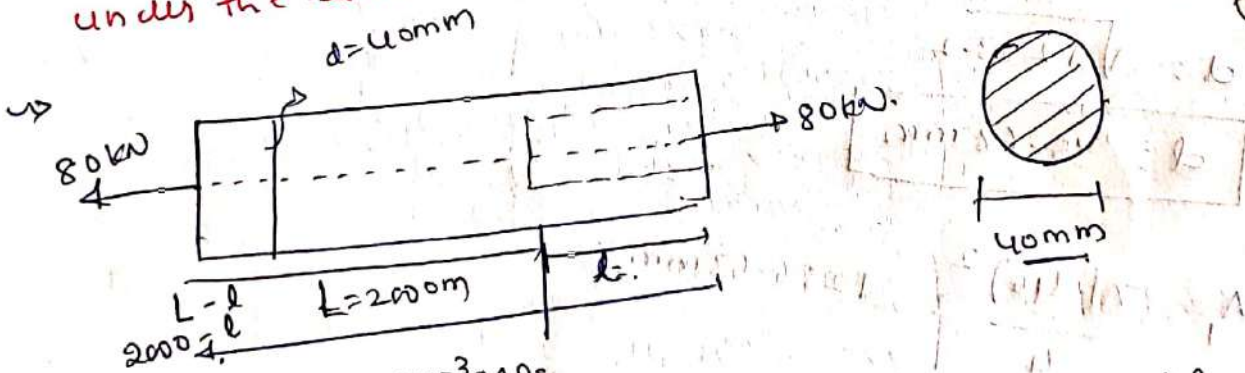
$$0.13 = \left[\frac{100 \times 10^3 \times [300 - x]}{1256.6 \times 200 \times 10^3} \right] + \left[\frac{100 \times 10^3 \times x}{1000 \times 200 \times 10^3} \right]$$

$$0.13 = \left[\frac{(30000 - 100 \times 10^3 \times x) \times 10^3}{1256.6 \times 200 \times 10^3} \right] + \left[\frac{100 \times 10^3 \times x}{1000 \times 200 \times 10^3} \right]$$

check

$$0.13 = \left[\frac{0.0115x + 30000 - 100x}{251320} \right] + \left[\frac{100x}{2 \times 10^5} \right]$$

⑥ A steel tie bar of 40mm diameter and 2m long is subjected to a pull of 80kN. calculate length of pore of diameter 20mm to be drilled in the rod, so that final extension of the entire bar under the same pull increased by 20%. $E = 200 \text{ GPa}$.



$$E = 200 \text{ GPa} = 200 \times 10^3 \text{ MPa}$$

Case: 1

Deformation in the solid bar [before pore drilling]

$$\Delta = \frac{PL}{AE} = \frac{80 \times 10^3 [2000]}{\frac{\pi (40)^2}{4} \times [200 \times 10^3]} = 0.63 \text{ mm}$$

$$\Delta = 0.63 \text{ mm}$$

Case: 2

Deformation in the bar due to a ϕ dia 20mm.

$$\Delta = \frac{PK}{AE}$$

$$\Delta = \Delta + (20\% \cdot \Delta)$$

$$= 0.63 + 20\% \cdot (0.63)$$

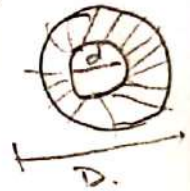
$$\Delta = 0.63 + 0.126$$

$$\Delta = 0.756 \text{ mm}$$

Let x be the length of the bore drilled

$$\left[\begin{array}{l} \text{Deformation in} \\ \text{Undrilled position} \\ \text{(Solid portion)} \end{array} \right] + \left[\begin{array}{l} \text{Deformation in} \\ \text{drilled position} \end{array} \right] = 0.756$$

$$\frac{P(2000-x)}{\frac{\pi(40)^2}{4} \times 200 \times 10^3} + \frac{P(x)}{\frac{\pi(40^2 - 20^2)}{4} \times 200 \times 10^3} = 0.756$$



$$\frac{80 \times 10^3 (2000-x)}{\frac{\pi(40)^2}{4} \times 200 \times 10^3} + \frac{80 \times 10^3 x}{\frac{\pi(40^2 - 20^2)}{4} \times 200 \times 10^3} = 0.756$$

$$A = \frac{\pi D^2}{4}$$

$$A = \frac{\pi d^2}{4}$$

$$A = \frac{\pi}{4} (D^2 - d^2)$$

$$\frac{160 \times 10^6 - 80 \times 10^3 x}{1256 \times 200 \times 10^3} + \frac{80 \times 10^3 x}{942.477 \times 200 \times 10^3} = 0.756$$

$$\frac{160 \times 10^6 - 80 \times 10^3 x}{1256 \times 200 \times 10^3} + \frac{4 \cdot 244 \times 10^4 x}{1256 \times 200 \times 10^3} = 0.756$$

$$0.6366x - 3 \cdot 1847 \times 10^4 x + 4 \cdot 244 \times 10^4 x = 0.756 - 0.63 + 1$$

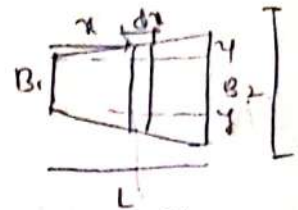
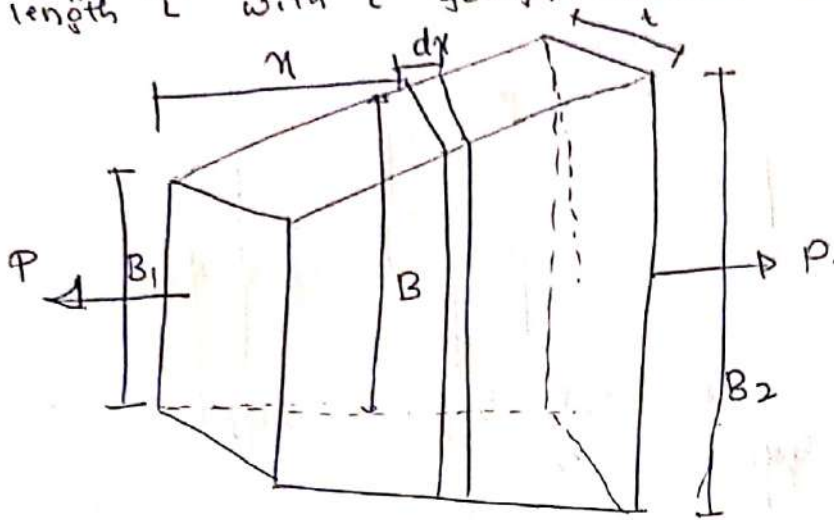
$$7.4287 \times 10^4 x = 0.756 - 0.63 + 1$$

$$x = \frac{0.1194}{7.4287 \times 10^4}$$

$$x = \dots$$

* Deformation in case of a uniformly tapering plate (Rectangular section)

Consider a plate of constant thickness t tapering uniformly from breadth B_1 to B_2 subjected to an axial pull P over a length L with E young's modulus as shown in fig.

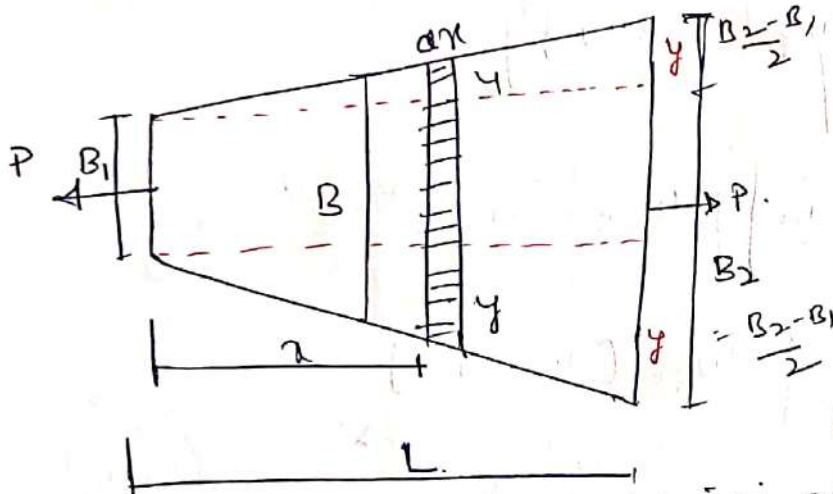


$$B = B_1 + B_2$$

$$B = B_1 + y + y$$

$$B = B_1 + 2y$$

$$y = \frac{B_2 - B_1}{2}$$



$$L \Rightarrow \left[\frac{B_2 - B_1}{2} \right]$$

$$x \rightarrow y$$

$$y = \left(\frac{B_2 - B_1}{2} \right) \left(\frac{x}{L} \right)$$

$$y = \left(\frac{B_2 - B_1}{2} \right) \frac{x}{L}$$

Consider an elemental length dx at a distance x in which from the B_1 . Let deformation δ @ x and breadth as B .

from the fig

$$B = B_1 + 2y$$

$$B = B_1 + y + y$$

$$= B_1 + 2 \left[\frac{B_2 - B_1}{2} \right] \frac{x}{L}$$

where $k = \frac{B_2 - B_1}{L}$

$$= B_1 + \left[\frac{B_2 - B_1}{L} \right] x$$

$$B = B_1 + kx$$

Hence

$$\delta = \frac{PL}{AE}$$

$$\delta = \frac{P \times (dx)}{(B \times t) E}$$

$$= \frac{P dx}{(t \times E) (B)}$$

Area of Rectangular section $A = B \times t$ = Breadth \times thickness

Substituting the B in this equation

$$\delta = \frac{P dx}{Et(B_1 + kx)} \quad \delta \text{ for Elemental area}$$

Hence Deformation for complete plate. L

$$\Delta = \int_0^L \frac{P}{Et(B_1 + kx)} dx = \frac{P}{Et} \int_0^L \frac{1}{(B_1 + kx)} dx$$

w.k.T. $\int_0^L \frac{1}{(a+bx)} dx = \frac{1}{b} \log_e [a+bx]$

$$\Delta = \frac{P}{Et} \left[\frac{1}{k} \log_e [B_1 + kx] \right]_0^L$$

$$\Delta = \frac{P}{Et} \left[\left(\frac{1}{\frac{B_2 - B_1}{L}} \right) \log_e \left[B_1 + \left(\frac{B_2 - B_1}{L} \right) x \right] \right]_0^L$$

$$\Delta = \frac{PL}{Et(B_2 - B_1)} \left[\log_e \left(\frac{B_2 - B_1}{B_1} \right) \right] \quad \Delta = \frac{P}{Et} \left[\frac{L}{(B_2 - B_1)} \right]$$

$$\Delta = \frac{PL}{Et(B_2 - B_1)} \log_e \left(\frac{B_2}{B_1} \right) \quad \log_e \left(\frac{B_2 - B_1}{B_1} \right) = \log_e \left(\frac{B_2}{B_1} \right) - \log_e B_1$$

$$\Delta = \frac{2.303 PL}{Et(B_2 - B_1)} \log_{10} \left(\frac{B_2}{B_1} \right)$$

$$= \frac{PL}{Et(B_2 - B_1)} \times \log_e (B_2 - B_1) - \log_e B_1$$

$$= \frac{PL}{Et(B_2 - B_1)} \times \log_e (B_2 - B_1)$$

$$\Delta = \frac{PL}{Et(B_2 - B_1)} \log_e \left(\frac{B_2}{B_1} \right)$$

$$A = b_1 + kx \dots \dots \text{Where } b = b_1 + kx$$

$$A = (b_1 + k \cdot x) t$$

length of elementary section $l = dx$

w.k.r $\Delta = \frac{PL}{AE}$

$$k = \frac{b_2 - b_1}{L}$$

$$\Delta = \frac{P \cdot dx}{(b_1 + k \cdot x) \cdot E}$$

$$\Delta = \frac{P \cdot dx}{(b_1 + k \cdot x) \cdot t \cdot E}$$

Total Extension of the bar is obtained by integrating the above equation b/w the limits 0 to L

$$\Delta = \int_0^L \frac{P \cdot dx}{(b_1 + k \cdot x) \cdot t \cdot E}$$

$$\Delta = \frac{P}{t \cdot E} \int_0^L \frac{dx}{(b_1 + k \cdot x)}$$

$$\int \frac{1}{(a + bx)} dx = \frac{1}{b} \log(a + bx)$$

$$= \frac{1}{b} \log(a + bx)$$

$$\Delta = \frac{P}{t \cdot E} \left[\frac{1}{k} \times \log(b_1 + kx) \right]_0^L$$

$$\Delta = \frac{P}{t \cdot E \cdot k} \left[\log e^{b_1 + k \cdot L} - \log e^{b_1} \right]$$

$$\log(a - b) = \log\left(\frac{a}{b}\right)$$

$$\Delta = \frac{P}{t \cdot E \cdot k} \left[\log e \left[\frac{b_1 + k \cdot L}{b_1} \right] \right]$$

$$\Delta = \frac{P}{t \cdot E \cdot k} \left[\log e \left[\frac{b_1 + \frac{b_2 - b_1}{L} \times k \cdot L}{b_1} \right] \right]$$

$$\Delta = \frac{P}{t \cdot E \cdot k} \left[\log e \left[\frac{b_2}{b_1} \right] \right] \quad \text{--- (6)}$$

$$\Delta = \frac{P}{t \cdot E \left[\frac{b_2 - b_1}{L} \right]} \left[\log e \left[\frac{b_2}{b_1} \right] \right] \quad \text{--- (7)}$$

$$\Delta = \frac{PL}{t \cdot E \cdot (b_2 - b_1)} \times \omega \phi_e \left[\frac{b_2}{b_1} \right]$$

$$\Delta = \frac{2.303 \cdot PL}{t \cdot E \cdot (b_2 - b_1)} \omega \phi_{10} \left[\frac{b_2}{b_1} \right]$$

1. A brass plate of uniform thickness 6mm varies in width from 100mm to 180mm over a length of 600mm with a tensile load of 40kN. Find the elongation of the bar if $E = 82 \text{ GPa}$.

$\Rightarrow t = 6 \text{ mm}$

$B_1 = 100 \text{ mm}$

$B_2 = 180 \text{ mm}$

$L = 600 \text{ mm}$

$E = 82 \text{ GPa} = 82 \times 10^3 \text{ MPa} = 82 \times 10^3 \text{ N/mm}^2$

$P = 40 \text{ kN}$

$\Delta = ?$

$$\Delta = \frac{2.303 PL}{E \left[\log \left(\frac{B_2}{B_1} \right) \right]}$$

$$\Delta = \frac{2.303 \times 40 \times 10^3 \times 600}{82 \times 10^3 \times 6 \times \left[\log \left(\frac{180}{100} \right) \right]}$$

$\Delta = 0.358 \text{ mm}$

2. A rectangular bar made of steel is 2.8m long & 15mm thick. The rod is subjected to an axial tensile load of 40kN. The width of the rod varies from 75mm at one end to 30mm at the other. Find the extension of the rod if $E = 2 \times 10^5 \text{ N/mm}^2$.

$\Rightarrow L = 2.8 \text{ m} = 2800 \text{ mm}$

$t = 15 \text{ mm}$

$P = 40 \text{ kN} = 40,000 \text{ N}$

$B_2 = B_1 = 75 \text{ mm}$

$B_1 = B_2 = 30 \text{ mm}$

$\Delta = ?$

$E = 2 \times 10^5 \text{ N/mm}^2$

$$\Delta = \frac{2.303 \times P L}{E \left[\log \left(\frac{B_2}{B_1} \right) \right]}$$

$$\Delta = \frac{2.303 \times 40,000 \times 2800}{2 \times 10^5 \times 15 \left[\log \left(\frac{75}{30} \right) \right]}$$

$\Delta = 0.76 \text{ mm}$

3. The extension in a rectangular steel bar of length 400mm & thickness 10mm, is found to be 0.21mm. The bar tapers uniformly in width from 100mm to 50mm. If E for the bar is $2 \times 10^5 \text{ N/mm}^2$. determine the axial load on the bar.

$L = 400\text{mm}$

$t = 10\text{mm}$

$\delta = 0.21\text{mm}$

$B_2 = 100\text{mm}$

$B_1 = 50\text{mm}$

$E = 2 \times 10^5 \text{ N/mm}^2$

$P = ?$

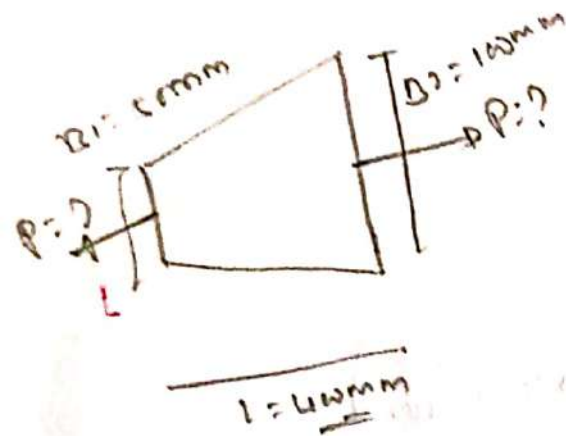
$$\delta = \frac{2.303 PL}{E t (B_2 - B_1)} \log_{10} \left[\frac{B_2}{B_1} \right]$$

$$0.21 = \frac{2.303 \times P \times 400}{2 \times 10^5 \times 10 (100 - 50)} \log_{10} \left[\frac{100}{50} \right]$$

$$0.21 = \frac{921.2 P}{100 \times 10^6} \times 0.30$$

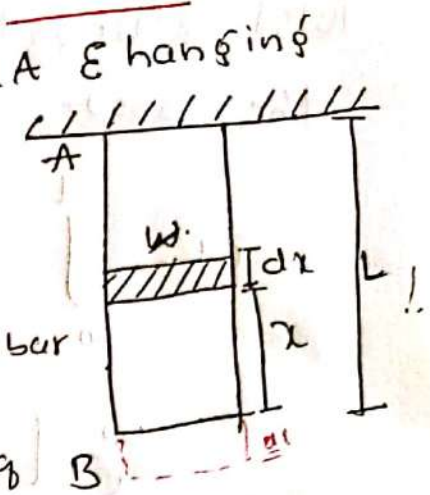
$$0.21 = [9.212 \times 10^6] P \times 0.30$$

$$P = 75.98 \text{ kN}$$



Elongation of a Bar due to its own weight

Fig shows a bar AB fixed at A and hanging freely under its own weight.



- Let L = length of the bar
- A = Area of cross section.
- E = Young's modulus for the bar material.
- w = weight per unit volume of the bar material.

Consider a small strip of thickness dx at a distance x from the lower end (B).

Weight of the bar for a length of x is given by
 $P = \text{Specific weight} \times \text{Volume of bar up to length } x$

$$P = w \times A \times x$$

This means that on the strip, a weight of $w \times A \times x$ is acting in the downward direction. Due to this weight, there will be some increase in the length of element. But length of the element is dx .

Now stress on the element = $\frac{\text{weight acting on element}}{\text{Area of cross section}}$

$$= \frac{w \times A \times x}{A} = w \cdot x$$

The above equation shows the stress due to self weight is not uniform. It depends on x . The stress increases with increase of x .

$$\text{Strain in the element} = \frac{\text{Stress}}{E} = \frac{w \cdot x}{E}$$

$$\begin{aligned} \therefore \text{Elongation of the element} &= \text{strain} \times \text{length of element} \\ &= \frac{w \cdot x}{E} \cdot dx \\ &= \frac{w \cdot x}{E} \cdot dx \end{aligned}$$

Total elongation of the bar is obtained by integrating the above equation b/w limits 0 to L

$$\delta L = \int_0^L \frac{W \times x}{E} dx = \frac{W}{E} \int_0^L \cancel{W} \times x \cdot dx$$

$$= \frac{W}{E} \left[\frac{x^2}{2} \right]_0^L$$

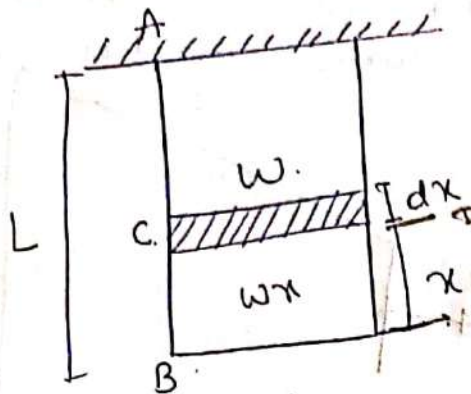
$$= \frac{W}{E} \left[\frac{L^2}{2} \right]$$

$$W = w \times L$$

$$\delta L = \frac{WL}{2E}$$

Elongation due to self weight?

Consider a bar AB of length 'L' hanging freely under its own weight 'W' as shown in fig. Let 'A' be the uniform cross section.



Consider a small strip dx at a distance x from 'B'. with the weight of the portion as W_x , the deformation in this Element/strip

$$\delta = \frac{(A \times x)(P) dx}{A E}$$

$$\delta = \frac{P x \cdot dx}{E}$$

Deformation in complete bar.

$$\Delta = \int_0^L \frac{P x \cdot dx}{E} = \frac{P}{E} \left[\frac{x^2}{2} \right]_0^L = \frac{P L^2}{2 E}$$

$$\Delta = \frac{W L^2}{2 E V} = \frac{W L^2}{2 E (A \cdot L)}$$

$$\Delta = \frac{W L}{2 A E}$$

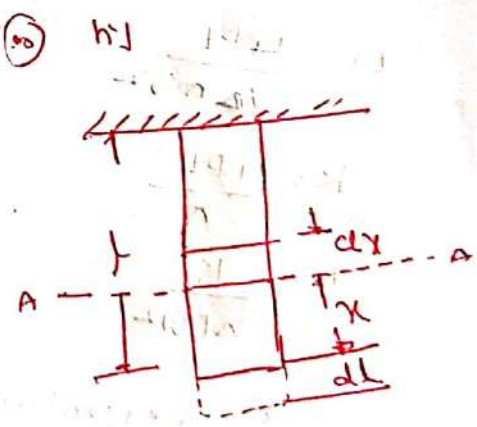
Volume of the bar = $A \times L$

w.k.t.
 $wt = \frac{\text{Volume} \times \rho}{\text{Volume} \times \text{density}}$
 Specific weight = $\frac{wt}{\text{Volume}}$

$$W = (A \times L) \rho$$

$$W_x = (A \times x) \rho$$

$$\rho = \frac{W_x}{V_x}$$



Thus the deformation of the bar under its own weight is equal to half the deformation. If the body is subjected to the direct load equal to the weight of the body.

$$\Delta = \frac{1}{2} \frac{P L}{A E}$$

$$\Delta = \frac{1}{2} \frac{P L}{A E}$$

Analysis of Bars of Composite Section

A bar made up of two or more bars of equal lengths but of different materials rigidly fixed with each other and behaving as one unit for extension or compression when subjected to an axial tensile or compressive load is called a composite bar.

Let P = total load on the composite bar.

L = length of composite bar.

A_1 = Area of cross section of bar 1

A_2 = ———— || ————
bar 2

E_1 = Young's modulus of bar 1

E_2 = Young's modulus of bar 2

P_1 = load shared by bar 1.

P_2 = load shared by bar 2.

σ_1 = Stress induced in bar 1 σ_2 = Stress induced in bar 2

Now the total load on the composite bar is equal to the sum of the load carried by the two bars.

$$P = P_1 + P_2 \quad \rightarrow \textcircled{I}$$

The stress in bar 1

$$\sigma_1 = \frac{\text{Load carried by bar 1}}{\text{Area of cross section of bar 1}} = \frac{P_1}{A_1}$$

$$P_1 = \sigma_1 A_1 \quad \rightarrow \textcircled{II}$$

The stress in bar 2

$$\sigma_2 = \frac{P_2}{A_2}$$

$$P_2 = \sigma_2 A_2 \quad \rightarrow \textcircled{III}$$

Substituting the values of P_1 and P_2 in eq (I)

We get

$$P = P_1 + P_2$$

$$P = \sigma_1 A_1 + \sigma_2 A_2 \quad \rightarrow \textcircled{IV}$$

Since the ends of the two bars are rigidly connected each bar will change in length by same amount. Also the length of each bar is same and hence the ratio of change in length to the original length.

But strain in bar 1 = strain bar ①

$$= \frac{\text{Young's modulus of bar ①}}{\sigma_1}$$

$$\text{Strain in bar 2} = \frac{\sigma_2}{E_2}$$

But strain in bar 1 = strain in bar 2

$$\frac{\sigma_1}{E_1} = \frac{\sigma_2}{E_2} \rightarrow \text{⑤}$$

From eq ④ & ⑤, the stresses σ_1 & σ_2 can be determined. By substituting the values of σ_1 & σ_2 in equation ① & ③, the load carried by different materials may be computed.

Modular ratio.

It is the ratio of Young's modulus of one material to another (i.e. E_1/E_2) & is denoted by M .

is constant.
From compatibility condition.

We have $\frac{\sigma_1}{E_1} = \frac{\sigma_2}{E_2}$

$$\sigma_1 = \sigma_2 \left[\frac{E_1}{E_2} \right]$$

$$M = \frac{E_1}{E_2}$$

Problems on composite sections:

① A steel rod of 3cm diameter is enclosed centrally in a hollow copper tube of external diameter 5cm and internal diameter of 4cm. The composite bar is then subjected to an axial pull of 45000N. If the length of each bar is equal to 15cm, determine.

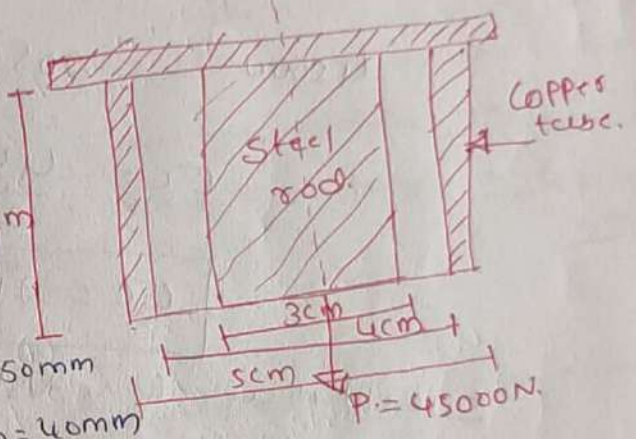
1. Stress in the rod and tube.
 2. Load carried by each bar.
- take $E_{\text{steel}} = 2.1 \times 10^5 \text{ N/mm}^2$ & $E_{\text{copper}} = 1.1 \times 10^5 \text{ N/mm}^2$

Given
 \rightarrow

Diameter of steel rod = 3cm = 30mm

Area of steel rod $A_s = \frac{\pi (30)^2}{4}$

$$A_s = 706.85 \text{ mm}^2$$



External dia of copper tube = 5cm = 50mm

Internal dia of copper tube = 4cm = 40mm

Area of copper tube $A_c = \frac{\pi (50^2 - 40^2)}{4} = 706.85 \text{ mm}^2$

Axial pull of composite bar $P = 45000 \text{ N}$

Length of each bar $L = 15 \text{ cm} = 150 \text{ mm}$

Young's modulus of steel $E_s = 2.1 \times 10^5 \text{ N/mm}^2$

Young's modulus of copper $E_c = 1.1 \times 10^5 \text{ N/mm}^2$

① Stress in the rod and tube.

Let $\sigma_s =$ stress in steel, $\sigma_c =$ stress in copper.
 $P_s =$ load carried by steel rod, $P_c =$ load carried by copper tube.

Now strain in steel = strain in copper

$$\frac{\sigma}{E} = \text{strain}$$

$$\frac{\sigma_s}{E_s} = \frac{\sigma_c}{E_c}$$

$$\frac{\sigma}{E} = \epsilon$$

$$\sigma_s = \frac{E_s}{E_c} \sigma_c$$

$$\sigma_s = \frac{2.1 \times 10^5}{1.1 \times 10^5} \times \sigma_c$$

$$\sigma_s = 1.909 \sigma_c \quad \rightarrow \text{①}$$

Now stress = $\frac{\text{Load}}{\text{Area}}$

Load = stress \times Area

Total Load = Load on steel + Load on copper.

$P = \sigma_s \times A_s + \sigma_c \times A_c$
 $P = 1.909 \sigma_c \times 706.86 + \sigma_c \times 706.86$

$45000 = (1.909 \sigma_c \times 706.86) + (\sigma_c \times 706.86)$
 $= 1349.395 \sigma_c + 706.86 \sigma_c$

$45000 = 2056.25 \sigma_c$

$\sigma_c = \frac{45000}{2056.25} = 21.88$

$\sigma_c = 21.88 \text{ N/mm}^2$

Substituting this value in eq ①

we get

$\sigma_s = 1.909 \sigma_c$

$\sigma_s = 1.909 \times 21.88$

$\sigma_s = 41.77 \text{ N/mm}^2$

② Load carried by each bar:

As Load = Stress \times Area

Load carried by Steel rod.

$P_s = \text{Stress} \times A_s$

$P_s = 41.77 \times 706.86$

$P_s = 29530.2952554 \text{ N}$

$P_s = 29.525 \text{ kN}$

Load carried by Copper tube.

$P_c = \text{Stress} (\sigma_c) \times A_c$

$= 21.88 \times 706.86$

$P_c = 15466.09 \text{ N}$

$P_c = 15.466 \text{ kN}$

Q2

A compound tube consists of a steel tube 140mm internal diameter and 160mm external diameter and an outer brass tube 160mm internal diameter and 180mm external diameter. The two tubes are of the same length. The compound tube carries an axial load of 900kN. Find the stresses and the load carried by each tube and the amount it shortens. Length of each tube is 140mm. Take E for steel as $2 \times 10^5 \text{ N/mm}^2$ and for brass as $1 \times 10^5 \text{ N/mm}^2$.

Internal dia of steel = 140mm.
Outer dia of steel = 160mm

$$\text{Area of steel tube} = \frac{\pi}{4} (160^2 - 140^2) = 4712.38 \text{ mm}^2$$

Internal dia of brass = 160mm
Outer dia of brass = 180mm
Area of brass tube = $\frac{\pi}{4} (180^2 - 160^2) = 5340.70 \text{ mm}^2$

Axial load $P = 900 \text{ kN} = 900 \times 10^3 \text{ N}$.

Length of each tube = 140mm.

$E_s = 2 \times 10^5 \text{ N/mm}^2$ $E_b = 1 \times 10^5 \text{ N/mm}^2$

Let σ_s = stress in steel, σ_b = stress in brass

Now strain in steel = strain in brass

$$\frac{\sigma_s}{E_s} = \frac{\sigma_b}{E_b}$$

$$\text{Strain} = \frac{\text{Stress}}{E}$$

$$\sigma_s = \frac{E_s}{E_b} \cdot \sigma_b$$

$$\sigma_s = \frac{2 \times 10^5}{1 \times 10^5} \sigma_b$$

$$\sigma_s = 2 \sigma_b \rightarrow (1)$$

$$\text{Stress} = \frac{\text{Load}}{A}$$

$$\text{Stress} \times A$$

Now,

Load in steel + load in brass = total load.

$$\sigma_s \times A_s + \sigma_b \times A_b = 900 \times 10^3$$

$$2 \times \sigma_b \times 4712.38 + \sigma_b \times 5340.70 = 900 \times 10^3$$

$$9424.76 \sigma_b + 5340.70 \sigma_b = 900 \times 10^3$$

$$\sigma_b = 60.95 \text{ N/mm}^2$$

Stress in brass $\sigma_b = 60.95 \text{ N/mm}^2$

Substituting this σ_b in eq (1) we get.

$$\sigma_s = 2 \times 60.95$$

$$\sigma_s = 121.9 \text{ N/mm}^2$$

load carried by brass tube = $\frac{\text{stress}}{\text{strain}} \times \text{Area}$
 $= \sigma_b \times A_b$
 $= 60.95 \times 5340.7$
 $= 325515.66 \text{ N}$

$$P_b = 325.515 \text{ kN}$$

load carried by steel tube = $\text{stress} \times \text{Area}$
 $= \sigma_s \times A_s$
 $= 121.9 \times 4712.4$
 $= 574441.56 \text{ N}$

$$P_s = 574.441 \text{ kN}$$

Decrease in length of compound tube.

= Decrease in length of either of the tubes.

= Decrease in length of brass tube.

= Strain in brass tube \times Original length

Strain = $\frac{\delta L}{L}$
 $\delta L = \text{strain} \times \text{length (original)}$

$$\frac{\delta L}{L} = \frac{\text{strain} \times L}{L}$$

$$\delta L = \frac{\text{stress} \times L}{E_b}$$

$$= \frac{\sigma_b \times L}{E_b}$$

$$= \frac{60.95 \times 1160}{1 \times 10^5}$$

$$= 0.0853 \text{ mm}$$

$$\frac{\sigma_s \times L}{E_s}$$

$$\frac{121.9 \times 1160}{2 \times 10^5}$$

$$= 0.0653 \text{ mm}$$

③ A reinforced concrete column is 300mm x 300mm in section. The column is provided with 8 bars of 20mm dia. The column carries a load of 360kN. Find the stress in concrete and steel if $E_s = 2.1 \times 10^5 \text{ N/mm}^2$ and $E_c = 0.14 \times 10^5 \text{ N/mm}^2$

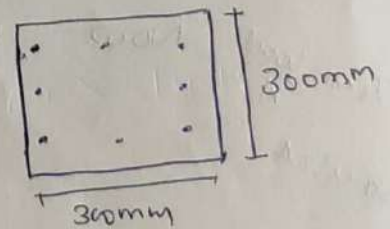
Area of column = 300×300
 $= 90000 \text{ mm}^2$

Area of steel = $8 \times \frac{\pi}{4} (20)^2$

$$A_s = 2513.27 \text{ mm}^2$$

Area of concrete = $A - A_s$
 $= 90000 - 2513.27$

$$A_c = 87486.72 \text{ mm}^2$$



W.K.T.

for equilibrium condition

$$P = P_s + P_c$$

$$360 \times 10^3 = \sigma_s \times A_s + \sigma_c \times A_c \rightarrow \textcircled{1}$$

$$\sigma = \frac{P}{A}$$

$$P = \sigma \times A$$

$$\text{Strain} = \frac{\text{Stress}}{E}$$

Strain in steel = strain in concrete

$$\frac{\sigma_s}{E_s} = \frac{\sigma_c}{E_c}$$

$$\sigma_s = \sigma_c \left[\frac{E_s}{E_c} \right]$$

$$\sigma_s = \sigma_c \left[\frac{2.1 \times 10^5}{0.14 \times 10^5} \right]$$

$$\sigma_s = 15 \sigma_c \rightarrow \textcircled{2}$$

substituting the σ_s in eq $\textcircled{1}$ we get

$$360 \times 10^3 = 15 \sigma_c \times 2513.27 + \sigma_c \times 87486.72$$

$$360 \times 10^3 = \sigma_c [125185.77]$$

$$\sigma_c = \frac{360 \times 10^3}{125185.77}$$

$$\sigma_c = 2.87 \text{ N/mm}^2$$

stress in concrete

$$\sigma_s = 15 \times \sigma_c = 15 \times 2.87$$

$$\sigma_s = 43.03 \text{ N/mm}^2$$

stress in steel

④ Two vertical rods one of steel and the other of copper are each rigidly fixed at the top and 50cm apart. Diameters and lengths of each rod are 2cm and 4m respectively. A cross bar fixed to the rods at the lower ends carries a load of 5000N such that the cross bar remains horizontal even after loading. Find the stress in each rod the position of the load on the bar. Take E for steel $2 \times 10^5 \text{ N/mm}^2$ and E for copper $1 \times 10^5 \text{ N/mm}^2$.

∴ Distance b/w the rods = 50cm
= 500mm

Dia of steel rod = Dia of copper rod
= 2cm = 20mm

Area of steel rod = Area of copper rod
 $A_s = A_c = \frac{\pi}{4} (20)^2 = 314.15 \text{ mm}^2$

Length of each rod = 4m = 4000mm, $P = 5000\text{N}$

Total $P = P_s + P_c$

$$5000\text{N} = \sigma_s \times A_s + \sigma_c \times A_c \quad \rightarrow \textcircled{1}$$

σ_s = stress in steel, σ_c = stress in copper,

Cross bar remains horizontal, the extensions of the steel and copper rod are equal. Also the rods have the same original length. Hence the strain of these rods are equal.

Strain in steel = strain in copper

$$\frac{\sigma_s}{E_s} = \frac{\sigma_c}{E_c}$$

$$\sigma_s = \left(\frac{E_s}{E_c} \right) \sigma_c$$

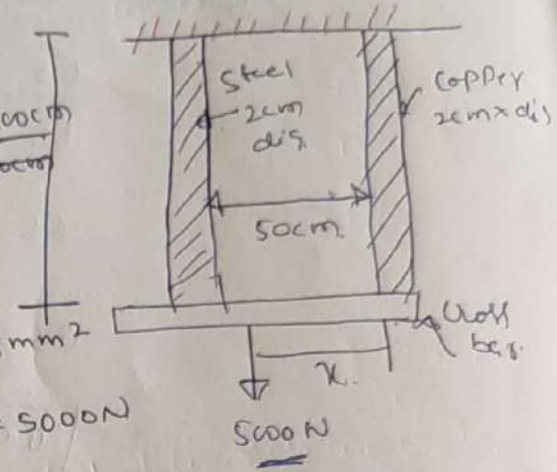
$$\sigma_s = \left(\frac{2 \times 10^5}{1 \times 10^5} \right) \sigma_c$$

$$\sigma_s = 2\sigma_c \quad \rightarrow \textcircled{2}$$

Substituting the σ_s in eq ① we get

$$5000 = 2\sigma_c \times 314.15 + \sigma_c \times 314.15$$

$$5000 = \sigma_c [942.46]$$



Steel = $\frac{\text{load}}{A_{\text{rod}}}$
Steel $\times A_{\text{rod}} = \text{load}$

$$E = \frac{\text{Stress}}{\text{Strain}}$$

$$\text{Strain} = \frac{\sigma}{E}$$

$$\sigma_c = 5.30 \text{ N/mm}^2$$

Substituting σ_c in eq (2)

$$\sigma_s = \sigma_c \times 2$$

$$\sigma_s = 2 \times 5.30$$

$$\sigma_s = 10.61 \text{ N/mm}^2$$

Position of load of 5000N on cross bar
 Let x = The distance of 5000N load from the copper rod
 (i.e. from the right hand.)

Load carried by each rod
 steel rod. $P_s = \sigma_s \times A_s$

$$= 10.61 \times 314.15$$

$$P_s = 3329.99 \text{ N}$$

Copper rod $P_c = \sigma_c \times A_c$

$$= 5.3 \times 314.15$$

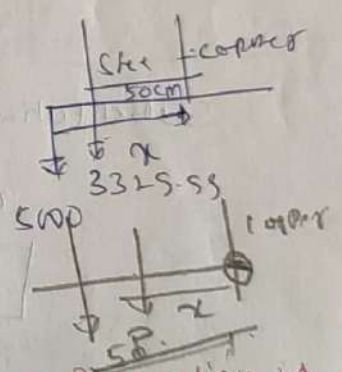
$$P_c = 1664.99 \text{ N}$$

Taking moment about the copper rod and equating the same we get

$$5000 \times x = P_s \times 50$$

$$5000 \times x = 3329.99 \times 50$$

$$x = 33.29 \text{ cm}$$

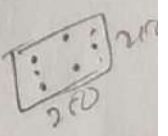


A reinforced short column 250mm x 250mm in section is reinforced with 8 steel bars. The total area of steel bars is 2500mm². The column carries a load of 390kN. If the modulus of elasticity for steel is 15 times that of concrete. Find the stresses in concrete and steel.

Area of concrete column = $250 \times 250 = 62,500 \text{ mm}^2$

Area of steel bars $A_s = 2500 \text{ mm}^2$

Area of concrete $A_c = 62,500 - 2500 = 60,000 \text{ mm}^2$



Total load on column $P = 390 \text{ kN} = 390,000 \text{ N}$.

E for steel = $15 \times E$ for concrete

$$E_s = 15 E_c$$

Let σ_s = stress in steel N/mm^2

σ_c = stress in concrete N/mm^2

$$\frac{E_s}{E_c} = 15$$

Now strain in steel = strain in concrete

$$\frac{\sigma_s}{E_s} = \frac{\sigma_c}{E_c}$$

$$\sigma_s = \left(\frac{E_s}{E_c} \right) \sigma_c$$

$$\sigma_s = 15 \sigma_c \quad \text{--- (1)}$$

$$\frac{\text{stress}}{\text{strain}} = E$$

$$\frac{\text{stress}}{E} = \text{strain}$$

$$\frac{\sigma}{E} = \epsilon$$

Also we know that

Load = stress \times Area

Total load = load on steel + load on concrete.
 $P = \sigma_s \times A_s + \sigma_c \times A_c$

$$390,000 = 15 \sigma_c \times 2500 + \sigma_c \times 60,000$$

$$390,000 = 97500 \sigma_c$$

$$\sigma_c = 4.0 \text{ N/mm}^2 \quad \text{--- (2)}$$

Substituting the σ_c in eq (1) we get

$$\sigma_s = 15 \times 4.0$$

$$\sigma_s = 60 \text{ N/mm}^2$$

(b) A steel rod and two copper rods together support a load of 370 kN as shown in fig. The cross-sectional area of steel rod is 2500 mm² and of each copper rod is 1600 mm². Find the stresses in the rods. Take E for steel = 2×10^5 N/mm² and for copper = 1×10^5 N/mm².

Given $P = 370 \text{ kN} = 370,000 \text{ N}$

Steel $A_s = 2500 \text{ mm}^2$

Copper $A_c = 2 \times 1600$

$$A_c = 3200 \text{ mm}^2$$

E_s for steel $E_s = 2 \times 10^5 \text{ N/mm}^2$

E for copper $E_c = 1 \times 10^5 \text{ N/mm}^2$

Length of the steel rod = $15 \text{ cm} + 10 \text{ cm} = 25 \text{ cm} = 250 \text{ mm}$

Length of the copper rod = $15 \text{ cm} = 150 \text{ mm}$

Let $\sigma_s =$ stress in steel N/mm² $\sigma_c =$ stress in copper N/mm²

We know that decrease in length of steel rod is equal to the decrease in length of copper rods.

But decrease in length of the steel rod

$$= \text{Strain in steel} \times \text{length of steel rod}$$

$$\therefore \text{Strain} = \frac{\text{Stress}}{E}$$

$$E = \frac{\sigma L}{\delta L}$$

$$\delta L = \frac{\sigma L}{E}$$

$$\text{Determination} = \frac{\text{Stress in steel} \times L_s}{E_s}$$

$$= \frac{\sigma_s}{2 \times 10^5} \times 250 \rightarrow (1)$$

$$\text{Strain} = \frac{\text{Stress}}{E}$$

$$\frac{\delta L}{L} = \left(\frac{\text{Stress}}{E} \right)$$

Only decrease in length of the copper rod

$$= \text{Strain in copper rod} \times \text{length of copper rod}$$

$$= \frac{\text{Stress in copper} \times L_c}{E_c}$$

$$= \frac{\sigma_c}{1 \times 10^5} \times 150 \rightarrow (2)$$

Equating the decrease in length of steel rod to the decrease in the length of copper rods, we get

$$\frac{\sigma_s}{2 \times 10^5} \times 250 = \frac{\sigma_c}{1 \times 10^5} \times 150$$

$$\sigma_s = \sigma_c \times \left(\frac{2 \times 10^5}{1 \times 10^5} \right) \times \left[\frac{150}{250} \right]$$

$$\sigma_s = \sigma_c \times 1.2$$

$$\sigma_s = 1.2 \sigma_c \quad \rightarrow \textcircled{3}$$

Also we know that

(load = stress \times area) load on steel + load on copper = (total) load applied

$$\sigma_s \times A_s + \sigma_c \times A_c = 370,000$$

$$1.2 \times \sigma_c \times 250 + \sigma_c \times 320 = 370,000$$

$$\sigma_c = \frac{370,000}{620}$$

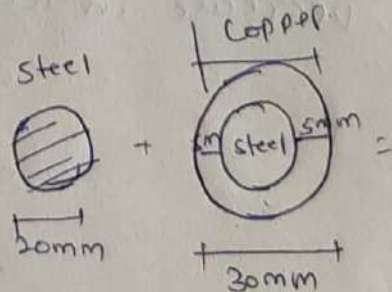
$$\sigma_c = 59.67 \text{ N/mm}^2$$

Substituting σ_c in eq. (3) we get:

$$\sigma_s = 1.2 \times 59.67$$

$$\sigma_s = 71.612 \text{ N/mm}^2$$

⑦ A compound bar consist of a circular rod of steel of dia 20mm rigidly fitted into a copper tube of internal diameter 20mm & thickness 5mm. If the bar is subjected to a load of 100kN. find the stress developed. $E_s = 200 \text{ GPa}$ and $E_{cu} = 120 \text{ GPa}$.



$$\rightarrow E_s = 200 \times 10^3 \text{ N/mm}^2$$

$$E_{cu} = 120 \times 10^3 \text{ N/mm}^2$$

$$P = 100 \text{ kN} = 100,000 \text{ N}$$

$$A_s = \frac{\pi}{4} [20]^2 = 314.15 \text{ mm}^2$$

$$A_{cu} = \frac{\pi}{4} [30^2 - 20^2]$$

$$A_{cu} = 392.69 \text{ mm}^2$$

We know that

Load on steel + load on copper = Total load

$$\sigma_s \times A_s + \sigma_{cu} \times A_{cu} = P \rightarrow \textcircled{1}$$

$$\text{Stress} = \frac{\text{Load}}{A_{res}}$$

$$\text{Load} = \text{Stress} \times A_{res}$$

Now

Strain in steel = strain in copper

$$\text{Strain} = \frac{\text{Stress}}{E}$$

$$\frac{\sigma_s}{E_s} = \frac{\sigma_c}{E_c}$$

$$\sigma_s = \left(\frac{E_s}{E_c} \right) \sigma_c$$

$$\sigma_s = \left[\frac{200 \times 10^3}{120 \times 10^3} \right] \sigma_c$$

$$\sigma_s = 1.667 \sigma_c \rightarrow \textcircled{2}$$

Substituting σ_s in eq $\textcircled{1}$ we get

$$1.667 \times \sigma_c \times 314.15 + \sigma_c \times 392.69 = 100,000$$

$$\sigma_c = 109.12 \text{ N/mm}^2$$

Substituting σ_c in eq $\textcircled{2}$ we get

$$\sigma_s = 1.667 \times 109.12$$

$$\sigma_s = 181.91 \text{ N/mm}^2$$

$$P_c = \sigma_c \times A_c = 109.12 \times 392.69$$

$$P_c = 42.85 \text{ kN}$$

$$P_s = \sigma_s \times A_s = 181.91 \times 314.15 = 57.147 \text{ kN}$$

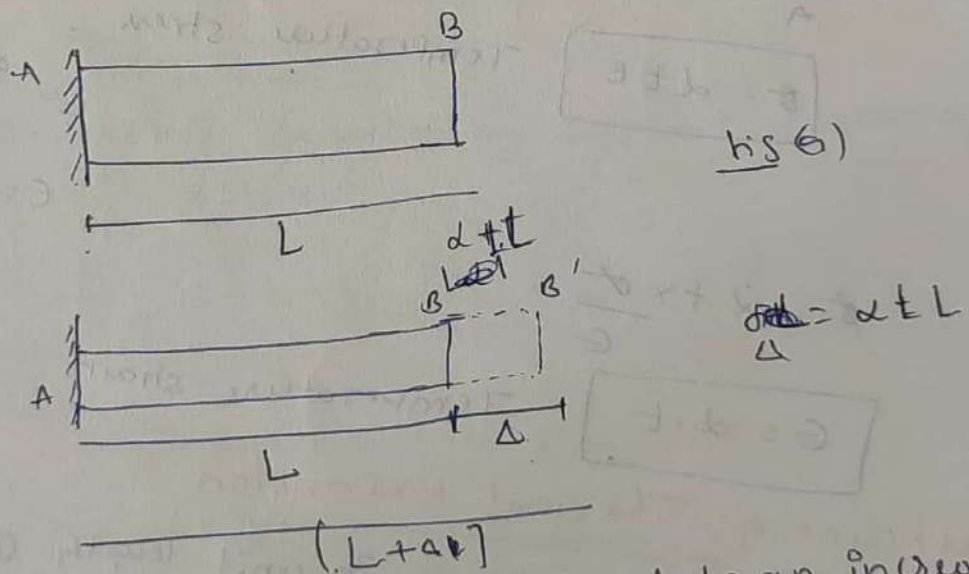
Temperature stress:

Rashu. M.E.
Assistant Professor.

Most of the Engineering materials when subjected to variation of temperature either expands or contracts. If the expansion or contraction is restricted thermal stresses are developed. If bar is allowed to expand or contract freely thermal stress donot develop.

Thermal stress are the stress induced in a body due to the change in temperature. Thermal stress are set up in a body when a temperature of a body is raised or lower. [Expansion or contraction] & the body is not allowed to expand or contract freely. But if the body is allowed to expand or contract freely than no temperature stress is induced or set up in the body.

The change in length due to change in temperature is found to be directly proportional to length of the member & also to change in temperature.



Consider a bar of length 'L' subjected to an increase in temperature 't'. If the bar is free to expand then the increase in length Δ is given by
change in length $\Delta = \alpha t L$ $1/8 \times 8 \times \text{mm}$

- α = co-efficient of Thermal Expansion.
- t = Rise in temperature / change in temperature.
- L = original length.
- Δ = change in length.

① If full Expansion is prevented.

Case 1

If the changes due to temperature are permitted freely no stress develop in the member as shown in fig. Due to increase in temperature by $(t^{\circ}C)$ the bar will extend by " ΔL_t " ϵ due to this change no stress is induced.

W.K.T

$$\Delta = \frac{PL}{AE}$$

$$\Delta L_t = \frac{P}{A} \times \frac{L}{E}$$

$$\frac{\Delta L_t + \epsilon}{k} = \frac{P}{A}$$

$$\frac{P}{A} = \Delta L_t + \epsilon$$

$$\sigma = \Delta L_t + \epsilon$$

Temperature stress

$$E = \frac{\text{Stress}}{\text{Strain}}$$

$$E = \frac{\sigma}{\epsilon}$$

$$E = \frac{\sigma}{\epsilon}$$

$$\Delta = \Delta L_t$$

$$E = \frac{\text{Stress}}{\text{Strain}} = \frac{\sigma}{\epsilon}$$

$$E = \frac{\sigma}{\epsilon}$$

②

$$\epsilon = \alpha \cdot t \times \frac{\sigma}{E}$$

$$\epsilon = \alpha \cdot t$$

Temperature strain

co-efficient of Thermal Expansion.

It is defined as change in unit length of the member material due to unit change in temperature. This value is different for different materials.

co-efficient of Thermal Expansion for some of the

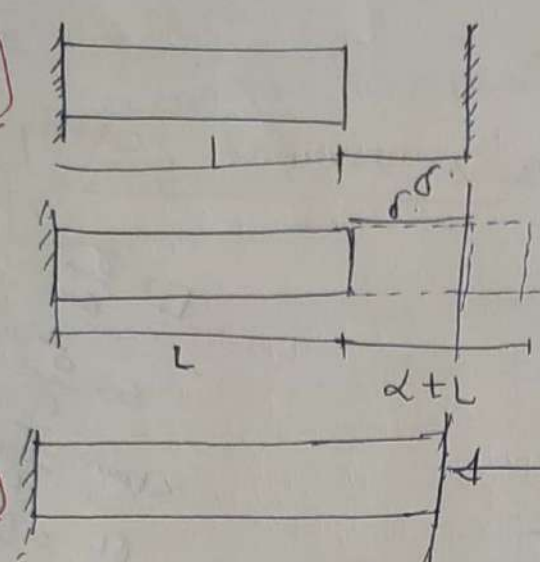
Commonly used engineering materials	co-efficient of thermal expansion
1. Steel	$12 \times 10^{-6} / ^{\circ}C$
2. Copper	$17.5 \times 10^{-6} / ^{\circ}C$
3. Stainless Steel	$18 \times 10^{-6} / ^{\circ}C$
4. Brass, Bronze.	$19 \times 10^{-6} / ^{\circ}C$
5. Aluminium	$23 \times 10^{-6} / ^{\circ}C$

Case II

If free expansion is prevented partly

$$\delta = \frac{\sigma}{E}$$

W.K.T -
 Strain $\epsilon = \frac{\delta}{L}$
 $\delta L = L \cdot \frac{\sigma}{E}$
 $\Delta = \delta L$
 $2tL - \delta = \frac{L \cdot \sigma}{E}$



$$\Delta = 2tL - \delta$$

$$\frac{PL}{AE} = 2tL - \delta$$

$$\frac{PL}{AE} = 2tL - \frac{\sigma}{E}$$

$$\frac{P \cdot L}{A \cdot E} = \frac{2tL \cdot E - \sigma}{E}$$

If the bar is free to extend when temperature is increased by t degree its extension (free expansion) would have been " $2tL$ ". But this extension is completely prevented in this case by forces developed at supports. This support force P is such that it causes shortening (Δ) of the bar by " $2tL$ ".

Thus shortening caused by support reaction P is given by $\Delta = 2tL - \delta$

W.K.T $\Delta = \frac{PL}{AE} \therefore \frac{PL}{AE} = 2tL - \delta$

(or)

Case I If full expansion is prevented

W.K.T $\Delta = 2tL$

Also $E = \frac{\sigma}{\epsilon} = \frac{\text{Stress}}{\text{Strain}}$

$\epsilon = \frac{\sigma}{E}$ but strain $\epsilon = \frac{\delta L}{L}$

$$\frac{\delta L}{L} = \frac{\sigma}{E}$$

$$\delta L = L \cdot \frac{\sigma}{E} \quad [\delta L = \Delta]$$

$$\Delta = L \cdot \frac{\sigma}{E}$$

$$2tL = L \cdot \frac{\sigma}{E}$$

$$\frac{\Delta t L}{L} = \frac{\sigma}{E}$$

$$\sigma = \alpha \cdot t \cdot E$$

Case: 2

Free Expansion is prevented partly

$$\Delta = \alpha t L - \delta$$

w.k.T

$$\sigma L = L \cdot \frac{\sigma}{E}$$

$$\Delta = L \cdot \frac{\sigma}{E}$$

$$\alpha t L - \delta = L \cdot \frac{\sigma}{E}$$

$$\frac{(\alpha t L - \delta) E = \sigma L}{L} \Rightarrow \sigma = \left(\frac{\alpha t L - \delta}{L} \right) \cdot E$$

$$\sigma = \left(\alpha t - \frac{\delta}{L} \right) E$$

$$E = \frac{\sigma}{\epsilon}$$

$$\epsilon = \frac{\sigma}{E}$$

$$\text{Strain} = \frac{\sigma}{E}$$

$$\frac{\Delta L}{L} = \frac{\sigma}{E}$$

$$\Delta L = L \cdot \frac{\sigma}{E}$$

$$\sigma = \left(\alpha t L - \delta \right) \left[\frac{E}{L} \right]$$

Problems

① A rod is 2m long at 10°C. find the expansion of the rod. when temperature is raised to 80°C. If this expansion is prevented, find the stress in the material. given $E = 1 \times 10^5 \text{ MPa}$. $\alpha = 0.000012 / ^\circ\text{C}$

↪ $L = 2\text{m} = 2000\text{mm}$

$E = 1 \times 10^5 \text{ N/mm}^2$

$\alpha = 0.000012 / ^\circ\text{C}$

$T = T_f - T_i = 80^\circ - 10^\circ = 70^\circ\text{C}$

$t_c = 70^\circ\text{C}$

Change in Temperature

$t_c = \text{final Temperature} - \text{Initial Temp}$

$t_c = 80^\circ - 10^\circ$

$t_c = 70^\circ\text{C}$

① case

expansion of the rod

$\Delta = \alpha t L$

$= 0.000012 \times 70 \times 2000$

$\Delta = 1.68\text{mm}$

Case: 2 Stresses due to prevention of Expansion

$$\epsilon = \alpha \cdot t \cdot E$$

$$= 0.000012 \times 70 \times 1 \times 10^5$$

$$\sigma = 84 \text{ mPa.} \quad \text{Compressive.}$$

Jan: 2018 (29) A rod of steel is 20m at 10°C. find free expansion of rod when temperature is fixed at 65°C. find temperature stress produced when expansion is prevented & when the rod is permitted to expand by 5.8mm $E = 2 \times 10^5 \text{ mPa}$, $\alpha = 12 \times 10^{-6} / ^\circ\text{C}$

→ $L = 20\text{m} = 20 \times 10^3 \text{ mm.}$ (1) $\Delta = L \alpha t$

$t_{in} = 10^\circ\text{C}$

$t_{final} = 65^\circ\text{C}$

$$\Delta = 20 \times 10^3 \times 12 \times 10^{-6} \times 55$$

$$\Delta = 13.2 \text{ mm}$$

$\Delta = 5.8 \text{ mm}$

$E = 2 \times 10^5 \text{ mPa.}$

$\alpha = 12 \times 10^{-6} / ^\circ\text{C}$

$t_f = t_{final} - t_{in}$

$= 65 - 10$

$$t = 55$$

Case 2 : Stresses due to prevention of Expansion

$$\sigma = \alpha \cdot T \cdot E$$

$$\sigma = 12 \times 10^{-6} \times 55 \times 2 \times 10^5$$

$$\sigma = 132 \text{ mPa}$$

$$\sigma = (L \alpha T - \Delta) \left[\frac{E}{L} \right]$$

$$= 20 \times 10^3 \times 12 \times 10^{-6} \times 55 - 5.8 \left[\frac{2 \times 10^5}{20 \times 10^3} \right]$$

$$\sigma = 74 \text{ mPa.}$$

(3) Explain the reason for development of stress in bars, when their temperature rises (or) falls.

Accordingly calculate the nature & magnitude of stress induced in the rod of 2m length & 20mm diameter when its temperature rises by 70°C with both ends constrained. Take $E = 1 \times 10^5 \text{ mPa}$ $\alpha = 1.2 \times 10^{-5} / ^\circ\text{C}$

→ $L = 2\text{m} = 2 \times 10^3 \text{ mm}$

$T = 70^\circ\text{C}$

$E = 1 \times 10^5 \text{ N/mm}^2$

$\alpha = 1.2 \times 10^{-5} / ^\circ\text{C.}$

$d = 20 \text{ mm.}$

$$\sigma = \alpha T E$$

$$= 1.2 \times 10^{-5} \times 70 \times 1 \times 10^5$$

$$\sigma = 84 \text{ MPa} \quad [\text{compressive}]$$

$$t_w = \Delta$$

$$\Delta = 20 \times 10^{-6} \times 22 \times 10^3 \times 22$$

$$\Delta = 9.68 \text{ mm}$$

Case 2: Shear stress to deformation of specimen

$$E = 9 \text{ TPa}$$

$$L = 10 \times 10^{-3} \times 22 \times 10^3 \times 22$$

$$\sigma = 13 \text{ MPa}$$

$$\sigma = \left(\frac{L}{L_0} - 1 \right) \left(\frac{F}{A} \right)$$

$$13 \times 10^6 = \left(\frac{10 \times 10^{-3}}{L_0} - 1 \right) \left(\frac{F}{22 \times 10^3 \times 22} \right)$$

$$F = 4 \text{ N}$$

$$L = 20 \text{ mm} = 20 \times 10^{-3} \text{ m}$$

$$T = 10^\circ \text{C}$$

$$T_2 = 20^\circ \text{C}$$

$$\Delta = 2 \text{ mm}$$

$$E = 9 \times 10^{10} \text{ MPa}$$

$$\sigma = 13 \times 10^6 \text{ Pa}$$

$$F = 4 \text{ N}$$

$$L = 10$$

$$F = 4$$

Temperature Stress in Compound Bars: (Composite)

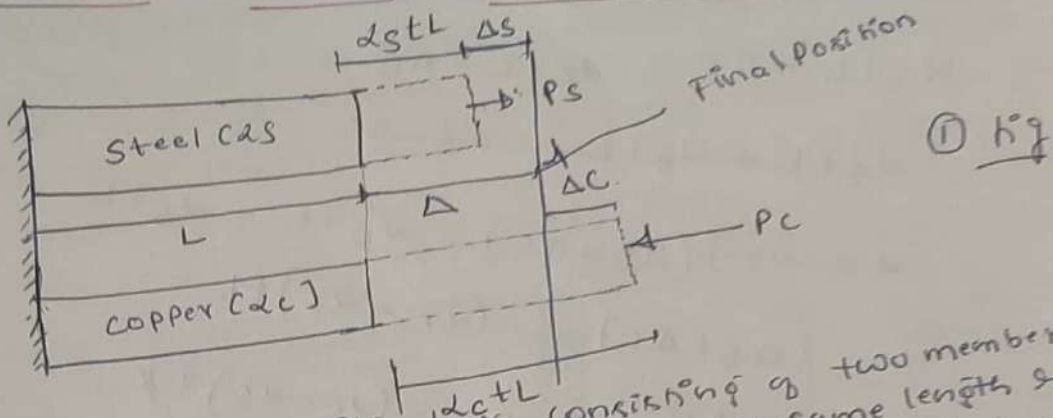


Fig shows a composite bar consisting of two members. A bar of steel & another bar of copper of same length rigidly connected at their ends, having different coefficient of expansion. Let the composite bar is subjected to a temperature rise of $t^\circ\text{C}$. Then the two opposite kinds of stress, i.e. tensile & compressive will be set up in the two materials. If the bars are free to expand, increase in length of copper $(\alpha_c L)$ will be more than the steel $(\alpha_s L)$. This is because of ~~more~~ copper is having more coefficient of thermal expansion (α_c) than that of steel (α_s) . & it will elongate more than the steel.

[But both the members are rigidly connected to form a composite bar, the composite bar should expand as a whole by the same amount Δ . In order to compensate this there will be slightly increase in length of the steel (α_s) due to compensative force is induced in the steel & slightly decrease in the length of the copper (α_c) due to the force P_c induced in the copper].

In order to have common expansion Δ , tensile force will be developed in steel & compressive force in copper.

\therefore Tensile force in steel = compressive force in copper
 $P_s = P_c = P$ (say)

From compatibility equation
 final extension of copper = final extension of steel
 $\Delta_c = \Delta_s = \Delta$ (say)

from fig 1 $\Delta = \alpha_s L + \Delta_s \rightarrow$ ① for steel

$\Delta = \alpha_c L - \Delta_c \rightarrow$ ② for copper.

From eq (1) & (2) we get

$$\alpha_s t L + \Delta S = \alpha_c t L - \Delta C$$

$$\alpha_s t L - \alpha_c t L = \Delta S - \Delta C$$

$$\alpha_s t L - \alpha_c t L = \Delta S - \Delta C$$

$$(\Delta S + \Delta C) = (\alpha_c - \alpha_s) t L$$

$$\frac{P_s \cdot L}{A_s \cdot E_s} + \frac{P_c \cdot L}{A_c \cdot E_c} = (\alpha_c - \alpha_s) t L$$

since $P_s = P_c = P$

$$\frac{P}{A_s \cdot E_s} + \frac{P}{A_c \cdot E_c} = (\alpha_c - \alpha_s) t \rightarrow (9)$$

$$(\alpha_c - \alpha_s) t = P \left[\frac{1}{A_s \cdot E_s} + \frac{1}{A_c \cdot E_c} \right] \rightarrow (1)$$

$$P = \frac{t (\alpha_c - \alpha_s)}{\frac{1}{A_s \cdot E_s} + \frac{1}{A_c \cdot E_c}} \rightarrow (2)$$

If we know the value of P then equation (1) reduces to

$$i.e. \sigma_s = \frac{P_s}{A_s} \quad \epsilon \sigma_c = \frac{P_c}{A_c} \quad P_s = \sigma_s \cdot A_s$$

$$P_c = \sigma_c \cdot A_c$$

$$(\alpha_c - \alpha_s) t = \left[\frac{\sigma_s \cdot A_s}{A_s \cdot E_s} + \frac{\sigma_c \cdot A_c}{A_c \cdot E_c} \right]$$

$$(\alpha_c - \alpha_s) t = \left[\frac{\sigma_s}{E_s} + \frac{\sigma_c}{E_c} \right]$$

But $\frac{\sigma}{E} = \epsilon \quad \therefore \frac{\sigma_s}{E_s} = \epsilon_s$ [strain in steel]

if $\frac{\sigma_c}{E_c} = \epsilon_c$ [strain in copper]

Therefore equation (2) becomes

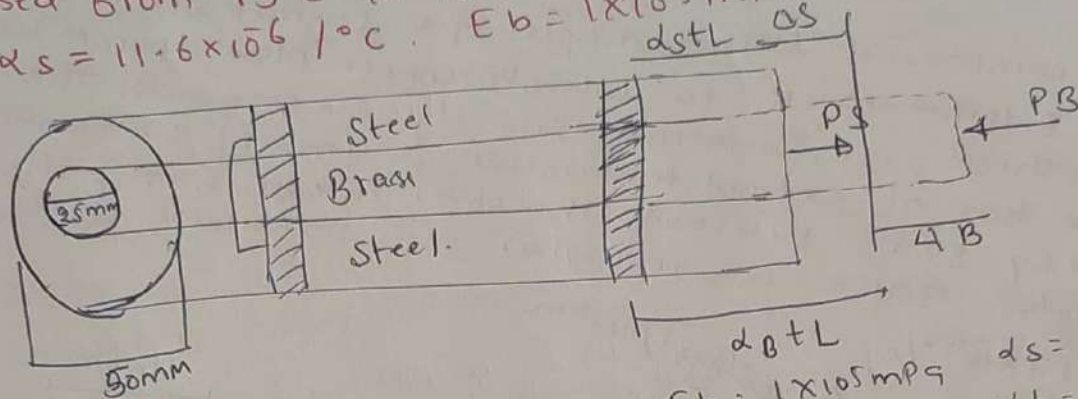
$$\epsilon_s + \epsilon_c = (\alpha_c - \alpha_s) t \rightarrow (3)$$

(3) gives strain developed in the two materials

Dec. 2013 - Jan 2014

① A bar of brass 25mm dia is enclosed in a steel tube of 25mm internal dia & 50mm external diameter. The bar & the tube are rigidly connected at both ends. Find the stress in two materials, if the temperature of the system is raised from 15°C to 95°C. Assume $E_s = 2 \times 10^5 \text{ MPa}$, $\alpha_s = 11.6 \times 10^{-6} / ^\circ\text{C}$, $E_b = 1 \times 10^5 \text{ MPa}$, $\alpha_b = 18.7 \times 10^{-6} / ^\circ\text{C}$.

Sol



Area of bar:

$$A_b = \frac{\pi}{4} (25)^2 = 490.87 \text{ mm}^2$$

$$A_s = \frac{\pi}{4} (50^2 - 25^2) = 1472.62 \text{ mm}^2$$

$L_b = L_s = L$ $P_b = P_s = P$

$E_b = 1 \times 10^5 \text{ MPa}$ $E_s = 2 \times 10^5 \text{ MPa}$
 $\alpha_s = 11.6 \times 10^{-6} / ^\circ\text{C}$ $\alpha_b = 18.7 \times 10^{-6} / ^\circ\text{C}$
 $t_1 = 15^\circ\text{C}$ $t_2 = 95^\circ\text{C}$
 $\Delta t = 95 - 15 = 80^\circ\text{C}$

From fig.

Elongation of steel = $\Delta_{st}L + \Delta_s$ $\rightarrow \text{①}$
 Elongation of brass = $\Delta_{bt}L - \Delta_c$ $\rightarrow \text{②}$

eq ① & ②

$$\Delta_{st}L + \Delta_s = \Delta_{bt}L - \Delta_c$$

$$A_b \Delta_s + A_s \Delta_{st} = A_b \Delta_{bt}L - A_b \Delta_c$$

$$A_b \Delta_s = (A_b \alpha_b - A_s \alpha_s) t L$$

$(A_b \alpha_b - A_s \alpha_s) t L = \frac{P_b L_b}{A_b E_b} + \frac{P_s L_s}{A_s E_s}$ since $P_b = P_s = P$
 $(A_b \alpha_b - A_s \alpha_s) t L = P \left[\frac{1}{A_b E_b} + \frac{1}{A_s E_s} \right]$ ($L_b = L_s = L$)

$$[18.7 \times 10^{-6} - 11.6] \times 10^{-6} \times 80 = P \left[\frac{1}{490.87 \times 2 \times 10^5} + \frac{1}{1472.62 \times 10^5} \right]$$

$P = 23.89 \text{ kN}$

$$(A_b \alpha_b - A_s \alpha_s) t = P \left[\frac{1}{A_b E_b} + \frac{1}{A_s E_s} \right]$$

$$(18.7 \times 10^{-6} - 11.6 \times 10^{-6}) 80 = P \left[\frac{1}{490.87 \times 2 \times 10^5} + \frac{1}{1472.62 \times 10^5} \right]$$

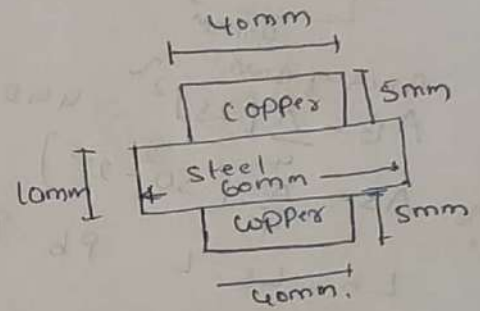
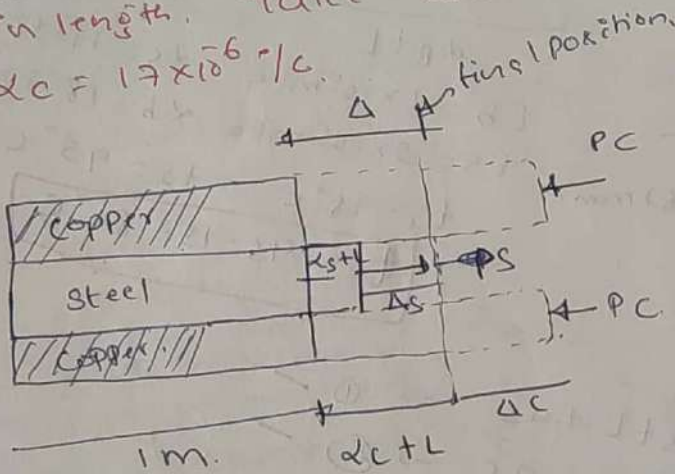
$$5.68 \times 10^{-4} = P [2.376 \times 10^{-8}]$$

$P = 23.89 \text{ kN}$

Stress in steel $\sigma_s = \frac{P}{A_s} = \frac{23.89 \times 10^3}{1472.62} = 16.22 \text{ N/mm}^2$

Stress in Brass $\sigma_B = \frac{P}{A_B} = \frac{23.89 \times 10^3}{490.87} = 48.68 \text{ N/mm}^2$

Q2) A compound bar is made of a central steel plate 60mm wide & 10mm thick to which copper plate 40mm wide and 5mm thick are connected rigidly on each side. The length of the bar at normal temperature is 1m. If the temperature is raised by 80°C . Determine the stress in each bar & the change in length. Take $E_s = 200 \text{ GPa}$, $E_c = 100 \text{ GPa}$, $\alpha_s = 12 \times 10^{-6} / ^\circ\text{C}$, $\alpha_c = 17 \times 10^{-6} / ^\circ\text{C}$.



③ A steel rail is 12.6 m long & is laid at a temperature of 24°C. The maximum temperature expected is 44°C.

① Estimate the min gap b/w two rails to be left so that temperature stress do not develop.

② Calculate the thermal stress developed in the rails if

- (a) No expansion joint is provided.
- (b) If a 2 mm gap is provided for expansion.

③ If the stress developed is 20 N/mm². What is the gap b/w the rails?

Take $E = 2 \times 10^5 \text{ MN/m}^2$ $\alpha = 12 \times 10^{-6} / ^\circ\text{C}$

Given

$$L = 12.6 \text{ m} = 12.6 \times 10^3 \text{ mm}$$

$$t_1 = 24^\circ\text{C} \quad t_2 = 44^\circ\text{C}$$

$$t = t_2 - t_1 = 44^\circ\text{C} - 24^\circ\text{C} = 20^\circ\text{C}$$

$$E = 2 \times 10^5 \text{ MN/m}^2$$

$$= 2 \times 10^5 \times 10^6 \text{ N/mm}^2$$

$$= 2 \times 10^5 \times 10^6 \text{ N/mm}^2$$

$$E = 2 \times 10^5 \text{ N/mm}^2$$

$$\alpha = 12 \times 10^{-6} / ^\circ\text{C}$$

$$\begin{aligned} E &= 2 \times 10^5 \text{ MN/m}^2 \\ &= 2 \times 10^5 \times 10^6 \text{ N/m}^2 \\ &= 2 \times 10^5 \times 10^6 / 10^6 \text{ N/mm}^2 \\ &= 2 \times 10^5 \text{ N/mm}^2 \end{aligned}$$

① Free expansion of rail $\Delta = \alpha t L$

$$\Delta = 12 \times 10^{-6} \times 20 \times 12.6 \times 10^3$$

$$\Delta = 3.024 \text{ mm}$$

Provided, free expansion prevented

(ii) If no expansion joint is provided, $\Delta = 3.024 \text{ mm}$

$$\Delta = 3.024 \text{ mm}$$

$$\frac{PL}{AE} = 3.024$$

$$\frac{P}{A} = \frac{3.024 \times E}{L}$$

$$\frac{P}{A} = \frac{3.024 \times 2 \times 10^5}{12.6 \times 10^3}$$

$$\frac{P}{A} = 48 \text{ N/mm}^2$$

$$\sigma_T = 48 \text{ N/mm}^2$$

(or)

$$\sigma = \alpha t E$$

$$\sigma = 12 \times 10^{-6} \times 20 \times 2 \times 10^5$$

$$\sigma = 48 \text{ N/mm}^2$$

② If a gap of 2mm is provided, free expansion is prevented is

$$\Delta = \alpha t L - \delta$$

$$\Delta = 3.024 - 2 = 1.024 \text{ mm}$$

$$\frac{PL}{AE} = 1.024$$

$$\frac{P}{A} = \frac{1.024 \times E}{L}$$

$$\sigma = \frac{1.024 \times 2 \times 10^5}{12.6 \times 10^3}$$

$$\sigma = 16.253 \text{ N/mm}^2$$

③ Given $\sigma_t = 20 \text{ N/mm}^2$, $\Delta = ?$
 W.K. that $\Delta = \frac{PL}{AE}$

$$\frac{P}{A} = \frac{\Delta E}{L}$$

$$(\sigma_t) = \frac{\Delta \times E}{L}$$

$$\Delta = \frac{\sigma_t \times L}{E} = \frac{20 \times 12.6 \times 10^3}{2 \times 10^5}$$

$$\Delta = 1.26 \text{ mm}$$

Also

$$\Delta = \alpha t L - \delta$$

$$\Delta = 3.024 - \delta$$

$$\delta = 3.024 - 1.26$$

$$\delta = 1.764 \text{ mm}$$

$$\Delta = \frac{PL}{AE}$$

$$\frac{\Delta \times E}{L} = \frac{P}{A}$$

$$\frac{\Delta \times E}{L} = \sigma$$

$$\Delta = \frac{\sigma \times L}{E}$$

$$\Delta = \frac{20 \times 12.6 \times 10^3}{2 \times 10^5}$$

$$\Delta = 1.26 \text{ mm}$$

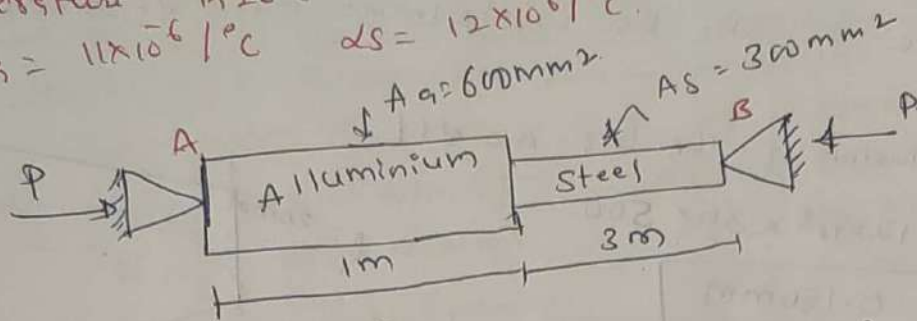
$$\Delta = \alpha t L - \delta$$

$$\Delta = 3.024 - \delta$$

$$= 3.024 - 1.26$$

$$\Delta = 1.764 \text{ mm}$$

(4) A composite bar is rigidly fitted at the supports A & B as shown in fig. Determine the reaction at the supports when temperature is 20°C .
 Take $E = 70 \text{ GN/m}^2$, $E_s = 200 \text{ GN/m}^2$,
 $\alpha_s = 11 \times 10^{-6} / ^\circ\text{C}$, $\alpha_s = 12 \times 10^{-6} / ^\circ\text{C}$.



$E_a = 70 \text{ GN/m}^2$
 $E_s = 200 \times 10^3 \text{ N/mm}^2$

Given
 $t = 20^\circ\text{C}$, $P = ?$
 $E_a = 70 \text{ GN/m}^2$
 $= \frac{70 \times 10^9}{10^6} \text{ N/mm}^2$
 $E_s = 70 \times 10^3 \text{ N/mm}^2$

$E_s = 200 \text{ GN/m}^2$
 $E_s = 200 \times 10^3 \text{ N/mm}^2$
 $\alpha_a = 11 \times 10^{-6} / ^\circ\text{C}$, $L_a = 1\text{m} = 1000\text{mm}$
 $\alpha_s = 12 \times 10^{-6} / ^\circ\text{C}$, $L_s = 3\text{m} = 3000\text{mm}$

W.K.T. Total Deformation $\Delta = \Delta_a + \Delta_s$

$$\Delta = \alpha_a t L_a + \alpha_s t L_s$$

$$\Delta = (11 \times 10^{-6}) (20 \times 1000) + (12 \times 10^{-6}) (20 \times 3000)$$

$$\Delta = 0.94 \text{ mm}$$

If P is the support reaction then.

$$\Delta = \Delta_a + \Delta_s$$

$$0.94 = \frac{P \cdot L_a}{A_a \cdot E_a} + \frac{P \cdot L_s}{A_s \cdot E_s}$$

$$P \left[\frac{L_a}{A_a \cdot E_a} + \frac{L_s}{A_s \cdot E_s} \right] = 0.94$$

$$P \left[\frac{1000}{600 \times 70 \times 10^3} + \frac{3000}{300 \times 200 \times 10^3} \right] = 0.94$$

$$P [2.380 \times 10^{-5} + 5 \times 10^{-5}] = 0.94$$

$$P = 12,737.12 \text{ N}$$

$$P = 12.737 \text{ kN}$$

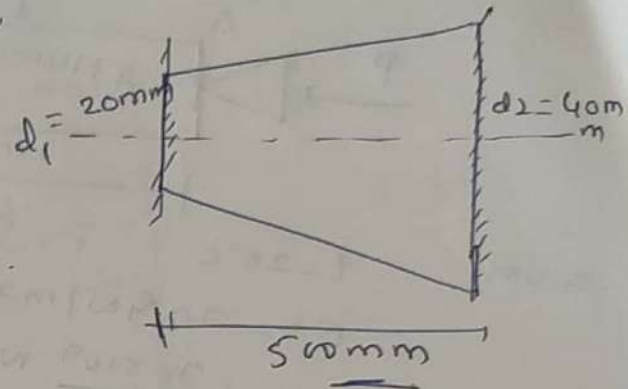
$E_a = 70 \text{ GN/m}^2$
 $= \frac{70 \times 10^9}{10^6} \text{ N/mm}^2$
 $= 70 \times 10^3 \text{ N/mm}^2$
 $E_s = 200 \times 10^3 \text{ N/mm}^2$

* A steel bar of uniform varying diameter as shown in fig is held b/w two rigid unyielding supports at room temperature. What is the maximum stress induced in the bar if the temperature rises by 30°C ? Take $E_s = 2 \times 10^5 \text{ N/mm}^2$ and $\alpha_s = 12 \times 10^{-6} / ^\circ\text{C}$

⇒ Free Expansion of the bar $\Delta = \alpha t L$

$$\Delta = 12 \times 10^{-6} \times 30 \times 500$$

$$\Delta = 0.180 \text{ mm}$$



w.k.r. for uniform varying diameter.

$$\Delta = \frac{4PL}{\pi E d_1 d_2}$$

$$0.180 = \frac{4 \times P \times 500}{\pi \times 2 \times 10^5 \times 20 \times 40}$$

$$\frac{0.180 \times \pi \times 2 \times 10^5 \times 20 \times 40}{4 \times 500} = P$$

$$P = 45.23 \text{ kN}$$

Stress calculation.

for 20mm dia, w.k.r. $\sigma = P/A = \frac{45.23 \times 10^3}{\pi \times 20^2 / 4}$

$$\sigma = 143.97 \text{ N/mm}^2$$

for 40mm dia. bar

$$\sigma = P/A = \frac{45.23 \times 10^3}{\pi \times 40^2 / 4}$$

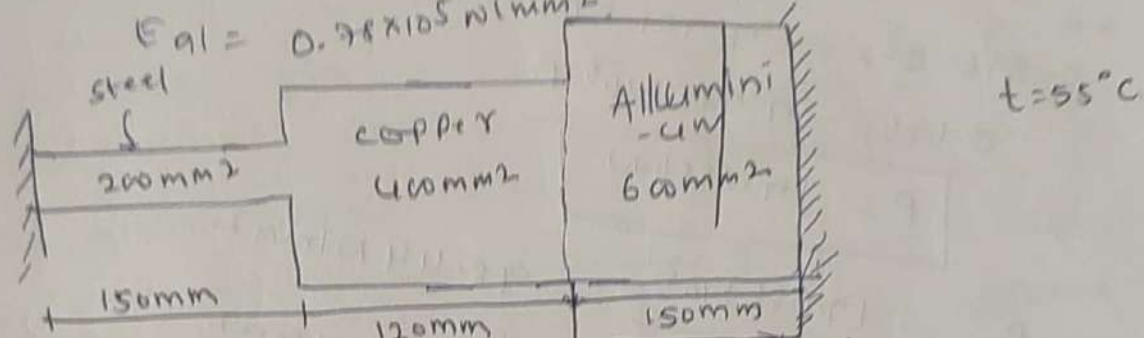
$$\sigma = 35.96 \text{ N/mm}^2$$

∴ Max stress $\sigma = 144 \text{ N/mm}^2$

6) A rod is composed of 3 segments as shown in fig. The rod is held b/w rigid supports, find the stress developed in each material when the temperature of the system is raised to 55°C . under different conditions.

- (a) when the supports are unyielding.
 (b) when supports are yields by 0.2mm.

Given $E_s = 2 \times 10^5 \text{ N/mm}^2$, $\alpha_s = 1.2 \times 10^{-5} / ^\circ\text{C}$
 $E_{cu} = 1 \times 10^5 \text{ N/mm}^2$, $\alpha_c = 1.75 \times 10^{-5} / ^\circ\text{C}$
 $E_{al} = 0.78 \times 10^5 \text{ N/mm}^2$, $\alpha_{al} = 2.2 \times 10^{-5} / ^\circ\text{C}$



Sol case (a) when supports are unyielding.

Total Deformation $\Delta = \Delta_s + \Delta_{cu} + \Delta_{al}$

$$\Delta = \alpha_s L_s + \alpha_{cu} L_{cu} + \alpha_{al} L_{al}$$

$$\Delta = [1.2 \times 10^{-5} \times 55 \times 150] + [1.75 \times 10^{-5} \times 55 \times 120] + [2.2 \times 10^{-5} \times 55 \times 150]$$

$$\Delta = 0.396 \text{ mm}$$

Also $\Delta = \Delta_s + \Delta_{cu} + \Delta_{al}$

$$0.396 = \left[\frac{P \times L_s}{A_s \times E_s} + \frac{P \times L_{cu}}{A_{cu} \times E_{cu}} + \frac{P \times L_{al}}{A_{al} \times E_{al}} \right]$$

$$0.396 = P \left[\frac{L_s}{A_s \times E_s} + \frac{L_{cu}}{A_{cu} \times E_{cu}} + \frac{L_{al}}{A_{al} \times E_{al}} \right]$$

$$0.396 = P \left[\frac{150}{200 \times 2 \times 10^5} + \frac{120}{400 \times 1 \times 10^5} + \frac{150}{600 \times 0.78 \times 10^5} \right]$$

$$0.396 = P [3.75 \times 10^{-6} + 3 \times 10^{-6} + 3.205 \times 10^{-6}]$$

$$0.396 = P [9.955 \times 10^{-6}]$$

$$P = 39.778 \text{ kN}$$

Stress $\sigma_s = \frac{P}{A_s} = \frac{39.778 \times 10^3}{200} = 198.89 \text{ N/mm}^2$

$$\sigma_c = \frac{P}{A_c} = \frac{39.778 \times 10^3}{400} = 99.445 \text{ N/mm}^2$$

$$\sigma_A = \frac{P}{A_A} = \frac{39.778 \times 10^3}{600} = 66.2966 \text{ N/mm}^2$$

Ques 2) When supports yields by 0.2mm

from equation 1

$$\Delta - \delta = 9.955 \times 10^6 P$$

$$0.396 - 0.2 = 9.955 \times 10^6 P$$

$$0.196 = 9.955 \times 10^6 P$$

$$P = 19.688 \text{ kN}$$

$$\sigma_s = \frac{P}{A_s} = \frac{19.688 \times 10^3}{200} = 98.44 \text{ N/mm}^2$$

$$\sigma_r = \frac{P}{A_c} = \frac{19.688 \times 10^3}{400} = 49.22 \text{ N/mm}^2$$

$$\sigma_{Al} = \frac{P}{A_l} = \frac{19.688 \times 10^3}{600} = 32.81 \text{ N/mm}^2$$

BA

Poisson's ratio

It is the ratio of lateral strain to the longitudinal strain. This ratio is known as Poisson's ratio. It is denoted by μ .

$$\text{Poisson's ratio, } \mu = \frac{\text{lateral strain}}{\text{longitudinal strain}}$$

$$\therefore \text{Lateral strain} = \mu \times \text{longitudinal strain}$$

As lateral strain is opposite in sign to the longitudinal strain, we can write as

$$\text{lateral strain} = -\mu \times \text{longitudinal strain}$$

Volumetric strain

It is the ratio of change in volume to the original volume of the body. It is denoted by e_v .

$$e_v = \frac{\delta V}{V}$$

$$\delta V = \frac{\text{Change in volume}}{\text{original volume}}$$

Elastic constants

Elastic constants are those factors which determine the deformations produced by a given stress system within the limits for which Hooke's law are constant. Various elastic constants are.

- ① modulus of elasticity (E)
- ② Poisson's ratio (μ)
- ③ modulus of rigidity (μ or K)
- ④ Bulk modulus (K).

① modulus of elasticity (E):

It is the ratio of Normal stress to the longitudinal strain is known as Young's modulus. It is denoted by 'E'. Young's modulus is also known as modulus of elasticity.

$$E = \frac{\sigma}{\epsilon}$$

where $\sigma = \text{Stress}$
 $\epsilon = \text{Strain}$

② Poisson's ratio (μ) $\left(\frac{1}{m} \right)$

$$\mu = \frac{\text{Lateral strain}}{\text{Longitudinal strain}}$$

Lateral strain = $\mu \times$ longitudinal strain.

③ modulus of Rigidity (G or N)

It is defined as the ratio of shear stress to the corresponding shear strain within the elastic limit & is usually denoted by G or N . Thus

$$G = \frac{\tau}{\phi}$$

where $G = \text{modulus of Rigidity}$
 $\tau = \text{shearing stress}$
 $\phi = \text{shearing strain}$

④ Bulk modulus (K) ?

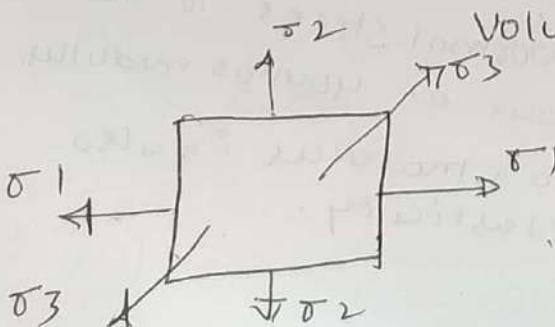
When a body is subjected to identical stresses (σ) in three mutually perpendicular directions, the body undergoes uniform changes in three directions without undergoing distortion of shape. The ratio of change in volume to the original volume is defined as Volumetric Strain (ϵ_v). Then the bulk modulus K is defined as

As the ratio of identical stress acting in three mutually perpendicular directions to the corresponding volumetric strain. ϵ_v is denoted by K .

$$K = \frac{\sigma}{\epsilon_v}$$

where $\sigma = \text{Identical Stress}$
in three mutually perpendicular directions
 $\epsilon_v = \text{volumetric strain}$

$$\text{Bulk modulus } K = \frac{\text{Direct Stress}}{\text{Volumetric strain}} = \frac{\sigma}{\frac{\Delta V}{V}} \left(\frac{\sigma}{\frac{\Delta V}{V}} \right)$$



$$\sigma_1 = \sigma_2 = \sigma_3 = \sigma$$

Poisson's Ratio:

$$\mu = \frac{\text{Lateral strain}}{\text{Longitudinal strain}}$$

$$\text{Lateral strain} = \mu \times \text{Longitudinal strain}$$

① Determine the change in length, Breadth and thickness of a steel bar which is 4m long, 30mm wide and 20mm thick & is subjected to an axial pull of 30kN in the direction of its length. Take $E = 2 \times 10^5 \text{ N/mm}^2$ and Poisson's ratio = 0.3

$$L = 4\text{m} = 4000\text{mm}, \quad b = 30\text{mm}$$

$$t = 20\text{mm}$$

$$A = b \times t = 30 \times 20 = 600\text{mm}^2$$

$$P = 30\text{kN} \quad E = 2 \times 10^5 \text{ N/mm}^2 \quad \mu = 0.3$$

Now strain in the direction of load [Longitudinal strain]

$$\text{Strain} = \frac{\text{Stress}}{E}$$

$$\text{Stress} = \frac{\text{Load}}{A_{\text{res}}}$$

$$\text{Strain} = \frac{\delta L}{L}$$

$$\frac{\delta L}{L} = \frac{\text{Load}}{A \times E}$$

$$\text{Longitudinal strain} = \frac{30000}{600 \times 2 \times 10^5} = 2.5 \times 10^{-4}$$

$$\text{But longitudinal strain} = \frac{\delta L}{L}$$

$$\frac{\delta L}{L} = 2.5 \times 10^{-4}$$

$$(\delta L) \text{ @ change in length} = 2.5 \times 10^{-4} \times 4000$$

$$= \underline{\underline{1\text{mm}}}$$

$$\text{Poisson's ratio} = \frac{\text{lateral strain}}{\text{longitudinal strain}}$$

$$0.3 = \frac{\text{lateral strain}}{2.5 \times 10^{-4}}$$

$$\text{Lateral strain} = 7.5 \times 10^{-5}$$

We know that

$$\text{Lateral strain} = \frac{\delta b}{b} \text{ (or) } \frac{\delta d}{d} \text{ (or) } \frac{\delta t}{t}$$

$$\delta b = b \times \text{lateral strain}$$

$$\delta b = 30 \times 0.000075 = 0.00225 \text{ mm}$$

$$\begin{aligned} \text{Similarly: } \delta t &= \delta t \times t \times \text{lateral strain} \\ &= 20 \times 0.000075 \\ &= 0.0015 \text{ mm} \end{aligned}$$

② Determine the value of Young's modulus and Poisson's ratio of a metallic bar of length 30cm, breadth 4cm and depth 4cm when the bar is subjected to an axial compressive load of 400kN. The decrease in length is given as 0.075cm and increase in breadth is 0.003cm.

Given $E = ?$ $E = ?$ $L = 30 \text{ cm} = 300 \text{ mm}$, $B = 4 \text{ cm} = 40 \text{ mm}$ $d = 4 \text{ cm} = 40 \text{ mm}$
 $P = 400 \text{ kN}$, $\delta L = 0.075 \text{ cm}$ (decrease), $\delta b = 0.003 \text{ cm}$ (increase)

$$\text{Area of cross } b \times d = 40 \times 40 = 1600 \text{ mm}^2$$

$$\text{Longitudinal strain} = \frac{\delta L}{L} = \frac{0.075}{30} = 0.0025$$

$$\text{Lateral strain} = \frac{\delta b}{b} = \frac{0.003}{4} = 0.00075$$

$$\text{Poisson's ratio} = \frac{\text{Lateral strain}}{\text{Longitudinal strain}} = \frac{0.00075}{0.0025}$$

$$\boxed{\mu = 0.3}$$

$$\text{Longitudinal strain} = \frac{\text{stress}}{E}$$

$$\left(\frac{\delta L}{L}\right) = \frac{P}{A \times E}$$

$$0.0025 = \frac{400 \times 10^3}{1600 \times E}$$

$$E = 100 \times 10^3 \text{ N/mm}^2$$

$$\boxed{E = 1 \times 10^5 \text{ N/mm}^2}$$

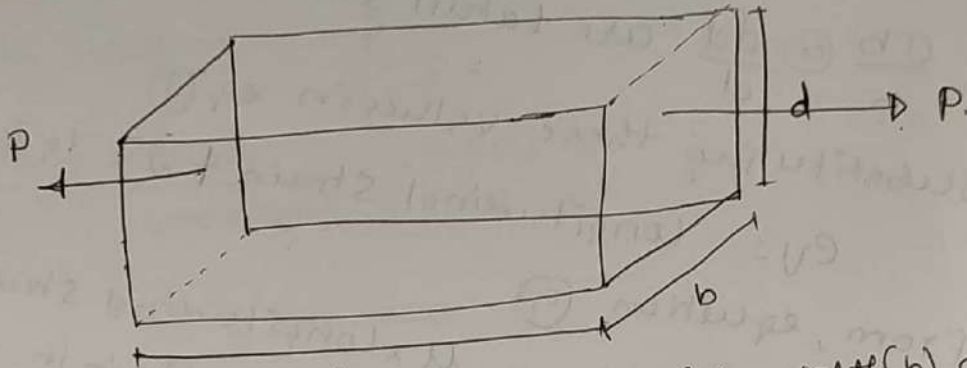
Volumetric Strain. :

The ratio of change in volume to the original volume of a body [When a body is subjected to a single force system of forces] is called volumetric strain. It is denoted by e_v

$$e_v = \frac{\delta V}{V} \quad \text{where, } \delta V = \text{change in volume}$$

$$V = \text{original volume.}$$

Volumetric strain of a Rectangular Bar which is subjected to an axial load 'P' in the direction of its length. :



Consider a Rectangular bar of length (L) , width (b) and depth (d) which is subjected to an axial load $'P'$ in the direction of its length as shown in fig.

Let $\delta L =$ change in length, $\delta b =$ change in breadth,
 $\delta d =$ change in depth.

Final length of the bar = $L + \delta L$

Final breadth of the bar = $b + \delta b$

Final depth of the bar = $d + \delta d$

Now original volume of the bar $V = L \cdot b \cdot d$.

$$\text{Final volume} = [L + \delta L] [b + \delta b] [d + \delta d]$$

$$= L \cdot b \cdot d + b \cdot d \cdot \delta L + L \cdot d \cdot \delta b + L \cdot b \cdot \delta d$$

(ignoring the small quantities)

Change in volume $\delta V =$ Final volume - original volume.

$$\delta V = [L b d + b \cdot d \cdot \delta L + L b \delta d + L d \cdot \delta b] - L b d$$

$$\delta V = b d \delta L + L b \delta d + L d \cdot \delta b$$

Volumetric strain $e_v = \frac{\delta V}{V}$

$$e_v = \frac{b \cdot d \delta L + L b \delta d + L d \delta b}{L b d}$$

$$e_v = \frac{\delta L}{L} + \frac{\delta d}{d} + \frac{\delta b}{b} \longrightarrow \textcircled{1}$$

But $\frac{\delta L}{L}$ = Longitudinal strain

$\frac{\delta b}{b}$ & $\frac{\delta d}{d}$ are lateral strain.

Substituting these values in eq (1)

$$e_v = \text{Longitudinal strain} + 2 \times \text{lateral strain}$$

From, equation (2)

$$\text{lateral strain} = -\mu \times \text{Longitudinal strain}$$

Substituting the value of lateral strain in equation (1) we get

$$e_v = \text{Longitudinal strain} - 2 \times \mu \times \text{Longitudinal strain}$$

$$= \text{Longitudinal strain} [1 - 2\mu]$$

$$e_v = \frac{\delta L}{L} [1 - 2\mu]$$

lateral strain = $-\mu \times$ longitudinal strain.

① Determine the volumetric strain and final volume of the given steel bar which is 4m long, 30 mm wide and 20 mm thick and is subjected to an axial pull of 30 kN in the direction of its length. Take $E = 2 \times 10^5 \text{ N/mm}^2$ and Poisson's ratio = 0.3

$\rightarrow L = 4\text{m} = 4000\text{mm}$ $b = 30\text{mm}$ $d = 20\text{mm}$ $P = 30\text{kN}$ $E = 2 \times 10^5 \text{ N/mm}^2$
 $\mu = 0.3$

Original Volume $V = L \cdot b \cdot d = 4000 \times 30 \times 20 = 2400000 \text{ mm}^3$

The value of longitudinal strain is $\left[\frac{\delta L}{L} \right]$

Strain = $\frac{\text{stress}}{E} = \frac{P}{A \times E}$

$\frac{\delta L}{L} = \frac{30000}{600 \times 2 \times 10^5} = 0.00025$

$\frac{\delta L}{L} = 0.00025$

Volumetric strain $e_v = \frac{\delta V}{V} = \frac{\delta L}{L} [1 - 2\mu]$

$\frac{\delta V}{V} = 0.00025 [1 - 2 \times 0.3]$

$\frac{\delta V}{V} = 0.0001$

$\delta V = 0.0001 \times V$

$= 0.0001 \times 2400000$

$\delta V = 240 \text{ mm}^3$

Final Volume = original volume + δV

$= 2400000 + 240$

$= 2400240 \text{ mm}^3$

② A steel bar 300 mm long, 50 mm wide and 40 mm thick is subjected to a pull of 30 kN in the direction of its length. Determine the change in volume. Take $E = 2 \times 10^5 \text{ N/mm}^2$ and $\mu = 0.25$

\rightarrow Given $L = 300\text{mm}$ $b = 50\text{mm}$ $t = 40\text{mm}$ $P = 30\text{kN}$
 $E = 2 \times 10^5 \text{ N/mm}^2$ $\mu = 0.25$

original volume = $L \times B \times t$

$$= 300 \times 50 \times 40 = 600000 \text{ mm}^3$$

Longitudinal strain = $\frac{\delta L}{L}$

$$\frac{\delta L}{L} = \frac{P}{AE}$$

$$\frac{\delta L}{L} = \frac{300 \times 10^3}{50 \times 40 \times 2 \times 10^5}$$

$$\frac{\delta L}{L} = 0.00075$$

Now volumetric strain is given by equation

$$e_v = \frac{\delta V}{V} [1 - 2\mu]$$

$$\frac{\delta V}{V} = 0.00075 [1 - 2 \times 0.25]$$

$$\frac{\delta V}{V} = 0.000375$$

Let δV = change in volume,

then $\frac{\delta V}{V}$ represents volumetric strain

$$\frac{\delta V}{V} = 0.000375$$

$$\delta V = 0.000375 \times V$$

$$\delta V = 0.000375 \times 600000$$

$$\delta V = \underline{225 \text{ mm}^3}$$

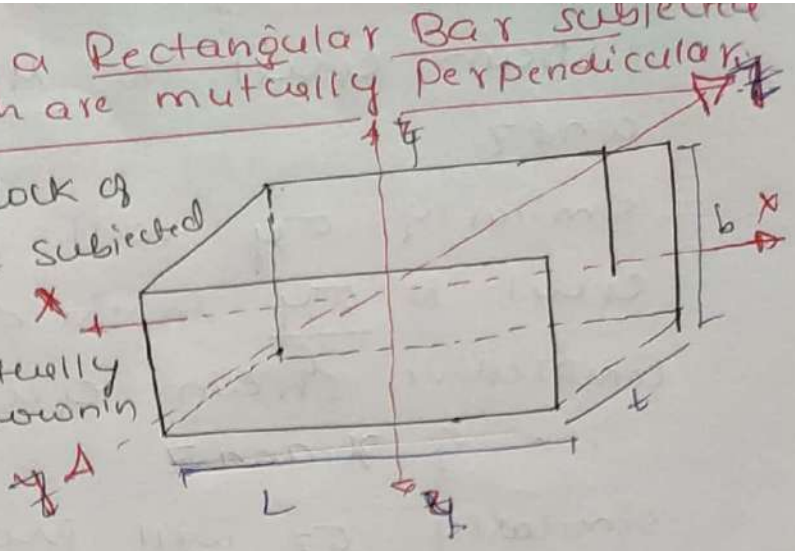
Final volume = original volume + δV

$$= 600000 + 225$$

$$\underline{\text{Final volume}} = \underline{600225 \text{ mm}^3}$$

Volumetric Strain of a Rectangular Bar subjected to three forces which are mutually perpendicular.

Consider a rectangular block of dimensions x, y and z subjected to three direct tensile stresses along three mutually perpendicular axis as shown in fig.



Then volume $V = xyz$

Taking \log on both sides

$$\log V = \log x + \log y + \log z$$

Differentiating the above equation w.r.t

$$\frac{1}{V} \cdot dV = \frac{1}{x} \cdot dx + \frac{1}{y} \cdot dy + \frac{1}{z} \cdot dz$$

$$\textcircled{a} \quad \frac{dV}{V} = \frac{dx}{x} + \frac{dy}{y} + \frac{dz}{z} \quad \rightarrow \textcircled{1}$$

But $\frac{dV}{V} = \frac{\text{Change of Volume}}{\text{Original Volume}} = \text{Volumetric strain}$

$\frac{dx}{x} = \frac{\text{Change of dimension } x}{\text{Original dimension of } x} = \text{Strain in } x \text{ direction} = e_x$

Similarly $\frac{dy}{y} = \text{Strain in } y \text{ direction} = e_y$

and $\frac{dz}{z} = \text{Strain in } z \text{ direction} = e_z$

Substituting these values in eq (1)

w.e get
$$\frac{dV}{V} = e_x + e_y + e_z$$

Now, let σ_x, σ_y & σ_z , Tensile stress in $x-x, y-y$ & $z-z$ direction respectively.
 $E = \text{Young's modulus}$. $\mu = \text{Poisson's ratio}$.

Now σ_x will produce a tensile strain equal to $\frac{\sigma_x}{E}$ in the direction of x , and a compressive

Longitudinal stress $\rightarrow \frac{\sigma_x}{E}$

strain equal to $\frac{\mu \sigma_x}{E}$ in the direction of y and z,

similarly, σ_y will produce a tensile strain equal to $\frac{\sigma_y}{E}$ in the direction of y and a compressive strain equal to $\frac{\mu \sigma_y}{E}$ in the direction of x and z,

similarly, σ_z will produce a tensile strain equal to $\frac{\sigma_z}{E}$ in the direction of z and a compressive strain equal to $\frac{\mu \sigma_z}{E}$ in the direction of x and y.

Hence σ_y and σ_z will produce a compressive strain equal to $\frac{\mu \sigma_y}{E}$ and $\frac{\mu \sigma_z}{E}$ in the direction of x.

Now tensile strain along x-direction is given by.

$$e_x = \frac{\sigma_x}{E} - \frac{\mu \sigma_y}{E} - \frac{\mu \sigma_z}{E}$$

$$e_x = \frac{\sigma_x}{E} - \mu \left[\frac{\sigma_y + \sigma_z}{E} \right]$$

$$e_y = \frac{\sigma_y}{E} - \frac{\mu \sigma_z}{E} - \frac{\mu \sigma_x}{E}$$

$$e_z = \frac{\sigma_z}{E} - \frac{\mu \sigma_x}{E} - \frac{\mu \sigma_y}{E}$$

Similarly

$$e_y = \frac{\sigma_y}{E} - \mu \left[\frac{\sigma_z + \sigma_x}{E} \right]$$

$$e_z = \frac{\sigma_z}{E} - \mu \left[\frac{\sigma_x + \sigma_y}{E} \right]$$

Adding all the strains we get

$$e_x + e_y + e_z = \frac{1}{E} (\sigma_x + \sigma_y + \sigma_z) - \frac{2\mu}{E} (\sigma_x + \sigma_y + \sigma_z)$$

$$= \frac{1}{E} (\sigma_x + \sigma_y + \sigma_z) (1 - 2\mu)$$

But

$$e_x + e_y + e_z = \text{volumetric strain} = \frac{dV}{V}$$

$$\frac{dV}{V} = \frac{1}{E} (\sigma_x + \sigma_y + \sigma_z) (1 - 2\mu) \quad \text{--- (70)}$$

Problems

- ① A metallic bar $300\text{mm} \times 100\text{mm} \times 40\text{mm}$ is subjected to a force of 5kN (tensile) 6kN (tensile) and 4kN (tensile) along x , y and z directions respectively. Determine the change in volume of the block. Take $E = 2 \times 10^5 \text{ N/mm}^2$ and Poisson's ratio $= 0.25$.

$\Rightarrow x = 300\text{mm}, y = 100\text{mm}$ and $z = 40\text{mm}$

$$\begin{aligned} \text{Volume} &= xyz \\ &= 300 \times 100 \times 40 \\ &= 1200000 \text{mm}^3 \end{aligned}$$

Load in the direction of $x = 5\text{kN}$

$y = 6\text{kN}$
 $z = 4\text{kN}$

$$E = 2 \times 10^5 \text{ N/mm}^2$$

$$\mu = 0.25$$

$$\sigma_x = \frac{\text{load in } x\text{-direction}}{y \times z}$$

$$\sigma_x = \frac{5000}{100 \times 40}$$

$$\sigma_x = 1.25 \text{ N/mm}^2$$

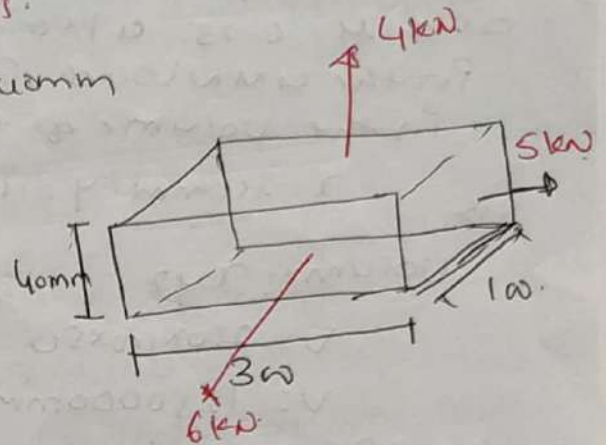
$$\sigma_z = \frac{\text{load in } z\text{-direction}}{x \times y} = \frac{4000}{300 \times 100}$$

$$\sigma_z = 0.133 \text{ N/mm}^2$$

$$\sigma_y = \frac{\text{load in } y\text{-direction}}{x \times z}$$

$$\sigma_y = \frac{6000}{300 \times 40} = 0.5 \text{ N/mm}^2$$

$$\sigma_y = 0.5 \text{ N/mm}^2$$



$$\frac{\Delta V}{V} = \frac{1}{E} (\sigma_x + \sigma_y + \sigma_z) (1 - 2\mu)$$

$$\frac{\Delta V}{V} = \frac{1}{2 \times 10^5} [1.25 + 0.5 + 0.133] (1 - 2 \times 0.25)$$

$$\Delta V = 47803.33 \times V$$

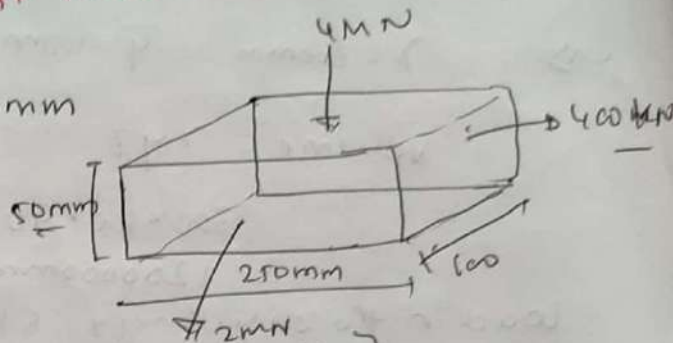
$$\Delta V = 47803.33 \times 1200000 = 5.65 \text{ mm}^3$$

(55)

$$\Delta V = 5.64 \text{ mm}^3$$

Q. A metallic bar $250\text{mm} \times 100\text{mm} \times 50\text{mm}$ is loaded as shown in fig. find the change in volume. Take $E = 2 \times 10^5 \text{ N/mm}^2$ and $\mu = 0.25$. also find the change that should be made in the 4MN load. In order that there should be no change in the volume of the bar.

$$x = 250\text{mm} \quad y = 100\text{mm} \quad z = 50\text{mm}$$



$$\text{Volume} = xyz$$

$$V = 250 \times 100 \times 50$$

$$V = 1250000 \text{ mm}^3$$

$$\text{Load in } x\text{-direction} = 400\text{kN} = 400 \times 10^3 \text{ N (tension)}$$

$$\text{Load in } y\text{-direction} = 2\text{MN} = 2 \times 10^6 \text{ N (tension)}$$

$$\text{Load in } z\text{-direction} = 2\text{MN} = 2 \times 10^6 \text{ N (compression)}$$

$$E = 2 \times 10^5 \text{ N/mm}^2, \quad \mu = 0.25$$

$$\sigma_x = \text{Stress in } x\text{-direction}$$

$$\sigma_x = \frac{\text{Load in } x\text{-direction}}{\text{Area of cross-section } (y \times z)}$$

$$\sigma_x = \frac{400 \times 10^3}{100 \times 50}$$

$$\sigma_x = 80 \text{ N/mm}^2 \quad (T)$$

$$\sigma_y = \frac{\text{Load in } y\text{-direction}}{(z \times x)}$$

$$\sigma_y = \frac{2 \times 10^6}{50 \times 250}$$

$$\sigma_y = 160 \text{ N/mm}^2 \quad (T)$$

$$\sigma_z = \frac{\text{Load in } z\text{-direction}}{(x \times y)}$$

$$= \frac{4 \times 10^6}{(250 \times 100)}$$

$$\sigma_z = 160 \text{ N/mm}^2 \quad \text{Compression.}$$

Volumetric Strain

$$\frac{\Delta V}{V} = \frac{1}{E} (\sigma_x + \sigma_y + \sigma_z) (1 - 2\mu)$$

$$\frac{\Delta V}{V} = \frac{1}{2 \times 10^5} [80 + 160 - 160] [1 - 2 \times 0.25]$$

$$\Delta V = 0.0002 V.$$

Change in volume

$$dV = 0.0002 \times V$$
$$= 0.0002 \times 1250000$$

$$dV = 250 \text{ mm}^3$$

Change in the 4MN load when there is no change in volume of bar

$$\frac{dV}{V} = \frac{1}{E} (\sigma_x + \sigma_y + \sigma_z) (1 - 2\mu)$$

If there is no change in volume, then $\frac{dV}{V} = 0$.

$$\frac{1}{E} (\sigma_x + \sigma_y + \sigma_z) (1 - 2\mu) = 0$$

But for most of materials, the value of μ lies b/w 0.25 and 0.33 and hence the term $(1 - 2\mu)$ is never zero.

$$\sigma_x + \sigma_y + \sigma_z = 0$$

The stress σ_x and σ_y are not to be changed. Only the stress corresponding to the load 4MN i.e. stress in z-direction is to be changed.

$$\sigma_z = -\sigma_x - \sigma_y = -80 - 180 = -240 \text{ N/mm}^2 \text{ (C)}$$

$$\sigma_z = \frac{\text{load}}{\text{Area (xy)}} = \frac{\text{load}}{250 \times 100}$$

$$240 = \frac{\text{load}}{250 \times 100}$$

$$\text{load} = 240 \times (250 \times 100)$$
$$= 6 \times 10^6 \text{ N}$$

$$\text{load} = \underline{6 \text{ MN}}$$

But already a compressive load of 4MN is given

\therefore Additional load that must be added

$$= 6 \text{ MN} - 4 \text{ MN}$$

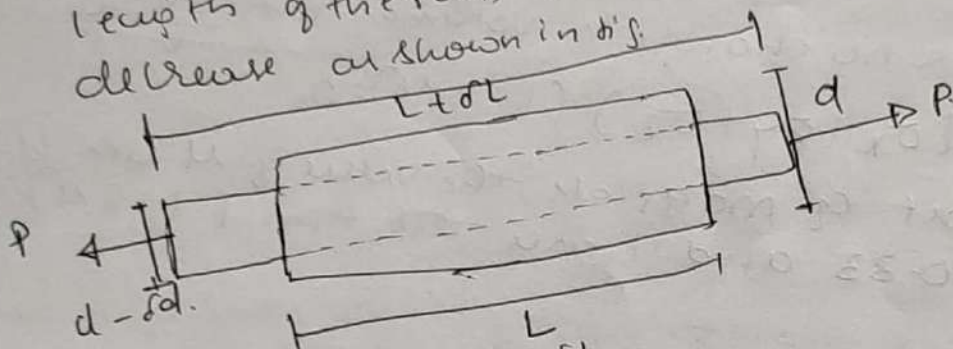
$$= 2 \text{ MN (compressive)}$$

Volumetric strain of a cylindrical rod.

Consider a cylindrical rod which is subjected to an axial tensile load P .

Let d = diameter of the rod,
 L = length of the rod.

Due to tensile load P , there will be an increase in length of the rod, but the diameter of the rod will decrease as shown in fig.



Final length = $L + \delta L$,
 Final Diameter = $d - \delta d$.

Now original volume of the rod,
 $V = \frac{\pi}{4} d^2 \times L$

$$\begin{aligned} \text{Final volume} &= \frac{\pi}{4} (d - \delta d)^2 [L + \delta L] \\ &= \frac{\pi}{4} [d^2 + \delta d^2 - 2d \times \delta d] [L + \delta L] \\ &= \frac{\pi}{4} [d^2 \times L + \delta d^2 \times L - 2d \times L \times \delta d + d^2 \times \delta L \\ &\quad + \delta d^2 \times \delta L - 2d \times \delta d \times \delta L] \\ &= \frac{\pi}{4} [d^2 \times L + \delta d^2 \times L - 2dL\delta d + d^2 \delta L + \delta d^2 \delta L \\ &\quad - 2\delta dL\delta d] \end{aligned}$$

$$= \frac{\pi}{4} [d^2 L - 2dL\delta d + d^2 \delta L]$$

Neglecting the product and higher powers of two small quantities,

$$\therefore \text{Change in volume, } \delta V = \text{Final Volume} - \text{original Volume}$$

=

$$= \frac{\pi}{4} (d^2 \times L - 2d \times L \times \delta d + d^2 \delta L) - \frac{\pi}{4} d^2 L$$

$$= \frac{\pi}{4} [d^2 \delta L - 2dL\delta d]$$

Volumetric strain $e_v = \frac{\text{Change in volume}}{\text{original volume}} = \frac{\delta V}{V}$

$$= \frac{\pi (d^2 \delta L - 2dL\delta d)}{4}$$

$$\frac{\pi}{4} d^2 L$$

$$= \frac{\delta L}{L} - \frac{2\delta d}{d} \rightarrow \textcircled{1}$$

Where $\frac{\delta L}{L}$ is the strain of length and $\frac{\delta d}{d}$ is strain of diameter.

\therefore Volumetric strain = strain in length - Twice the strain of diameter.

Problem

① A steel rod of 5m length and 30mm in diameter is subjected to an axial load of 50kN. Determine the change in length, diameter and volume of the rod. Take $E = 2 \times 10^5 \text{ N/mm}^2$, and Poisson's ratio = 0.25.

length $L = 5\text{m} = 5 \times 10^3 \text{ mm}$, $d = 30\text{mm}$, volume $V = \frac{\pi}{4} d^2 L$

$$V = \frac{\pi}{4} \times 30^2 \times 5 \times 10^3$$

$$V = 35.34 \times 10^5$$

$P = 50 \times 10^3 \text{ N}$, $\Delta L = ?$, $E = 2 \times 10^5 \text{ N/mm}^2$, $\mu = 0.25$

Let $\delta d =$ change in diameter,

$\delta L =$ change in length, $\delta V =$ change in volume.

$$\text{Strain in length} = \frac{\text{stress}}{E}$$

$$= \frac{\text{load}}{\text{Area}} \times \frac{1}{E}$$

$$= \frac{P}{\frac{\pi}{4} d^2} \times \frac{1}{E}$$

$$= \frac{50 \times 10^3}{\frac{\pi}{4} \times 30^2} \times \frac{1}{2 \times 10^5}$$

$$\Delta L = \underline{\underline{0.0003536 \text{ mm}}}$$

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But strain of length = $\frac{\delta L}{L}$

$$\frac{\delta L}{L} = 0.0003536$$

Longitudinal strain $\delta L = 0.0003536 \times 5 \times 10^3$

Now strain $\delta L = 1.768 \text{ mm}$

Now Poisson's ratio = $\frac{\text{lateral strain}}{\text{longitudinal strain}}$

$$\text{Lateral strain} = \text{Poisson's ratio} \times \text{longitudinal strain}$$

$$= \mu \times \delta L$$

$$= 0.25 \times 0.0003536$$

$$= 0.0000884$$

But lateral strain = $\frac{\delta d}{d}$

$$\frac{\delta d}{d} = 0.0000884$$

$$\delta d = 0.0000884 \times d$$

$$\delta d = 0.0000884 \times 30$$

$$\delta d = 0.002652 \text{ mm}$$

$$\text{Volumetric strain } \frac{\delta V}{V} = \frac{\delta L}{L} - \frac{2\delta d}{d}$$

$$= 0.0003536 - 2[0.0000884]$$

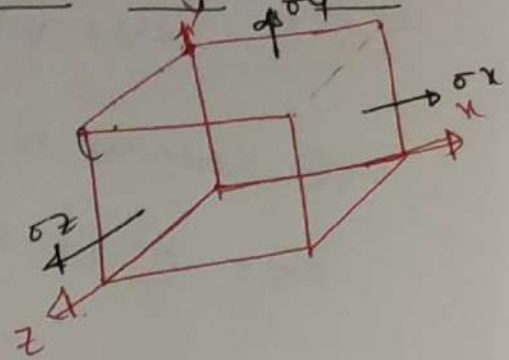
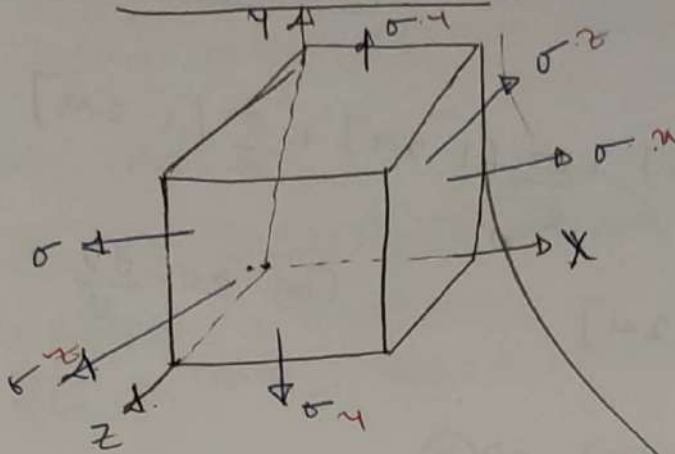
$$= 0.0001768$$

$$\delta V = 0.0001768 \times V$$

$$= 0.0001768 \times 35.343 \times 10^5$$

$$\delta V = 624.86 \text{ mm}^3$$

Relationship b/w modulus of elasticity [E] and Bulk modulus [K]



Consider a Rectangular [cube] which is subjected to three mutually \perp tensile stress of "equal intensity"

- Let L = length of the cube
- dL = change in length of the cube.
- E = young's modulus of the material of the cube.
- σ = Tensile stress along on the faces
- μ = Poisson's ratio.

When σ_x produces a tensile strain equal to $\frac{\sigma_x}{E}$ in the direction of x on the other hand σ_y & σ_z will produce a compressive strain equal to $\mu \frac{\sigma_y}{E}$ & $\mu \frac{\sigma_z}{E}$ in the direction x .

Since in this case when the stress is subjected to mutual \perp tensile stress of equal intensity.

Tens i.e, $\sigma_x = \sigma_y = \sigma_z = \sigma$
Tensile strain along x -direction is given by

$$e_x = \frac{\sigma}{E} - \mu \frac{\sigma}{E} - \mu \frac{\sigma}{E}$$

$$e_x = \frac{\sigma}{E} [1 - 2\mu] \rightarrow \text{a)}$$

ii) y Tensile strain along y -direction

$$e_y = \frac{\sigma}{E} [1 - 2\mu] \rightarrow \text{b)}$$

iii) z Tensile strain along z -direction

$$e_z = \frac{\sigma}{E} [1 - 2\mu] \rightarrow \text{c)}$$

W.K.T. $e_v \Rightarrow$ Volumetric strain $e_v = e_x + e_y + e_z \rightarrow (1)$

of (1) reduces to.

$$e_v = \frac{\sigma}{E} (1-2\mu) + \frac{\sigma}{E} (1-2\mu) + \frac{\sigma}{E} (1-2\mu)$$

$$e_v = \frac{3\sigma}{E} (1-2\mu)$$

$$(2) \quad e_v = \frac{\delta V}{V}$$

$$\frac{\delta V}{V} = \frac{3\sigma}{E} (1-2\mu) \rightarrow (2)$$

But Bulk modulus $k = \frac{\text{Direct stress}}{\text{Volumetric strain}} = \frac{\sigma}{e_v}$

$$k = \frac{\sigma}{\frac{\delta V}{V}} \rightarrow (3)$$

Substituting eq (2) in eq (3) we get

$$k = \frac{\sigma}{\frac{\delta V}{V}}$$

$$k = \frac{\sigma}{\frac{3\sigma(1-2\mu)}{E}}$$

$$k = \frac{E}{3(1-2\mu)}$$

$$3k(1-2\mu) = E$$

Relationship b/w E & k

- ① A bar of 30mm diameter is subjected to a pull of 60kN. The measured extension on gauge length of 200mm, is 0.1mm and change in diameter is 0.004mm, calculate.

① Young's modulus, ② Poisson's ratio, ③ Bulk modulus.

→ Diam = 30mm

Area of bar = $\frac{\pi}{4} (30)^2 = 706.85 \text{ mm}^2$

$P = 60 \text{ kN}$ $L = 200 \text{ mm}$,

$\delta L = 0.1 \text{ mm}$ $\delta d = 0.004 \text{ mm}$.

① Young's modulus $E = \frac{\text{Tensile stress}}{\text{Longitudinal strain}}$

Tensile stress $\sigma = \frac{P}{A} = \frac{60 \times 10^3}{706.85} = 84.87 \text{ N/mm}^2$

Longitudinal strain $\epsilon = \frac{\delta L}{L} = \frac{0.1}{200} = 0.0005$

$E = \frac{\text{Tensile stress}}{\text{Longitudinal strain}}$

$E = \frac{\sigma}{\epsilon} = \frac{84.87}{0.0005}$

$E = 1.69 \times 10^5 \text{ N/mm}^2$

② Poisson's ratio

$\mu = \frac{\text{Lateral strain}}{\text{Longitudinal strain}}$

$\mu = \frac{\left(\frac{\delta d}{d}\right)}{0.0005} = \frac{\left(\frac{0.004}{30}\right)}{0.0005}$

$\mu = 0.266$

③ Bulk modulus (K)

$K = \frac{E}{3(1-2\mu)} = \frac{1.69 \times 10^5}{3(1-2 \times 0.266)}$

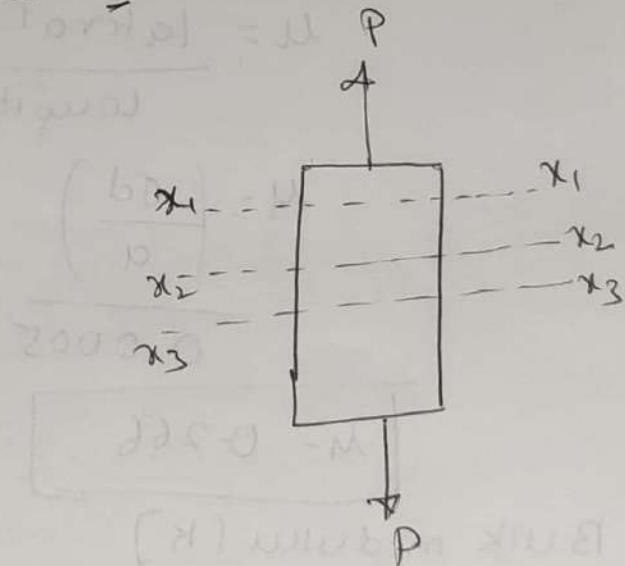
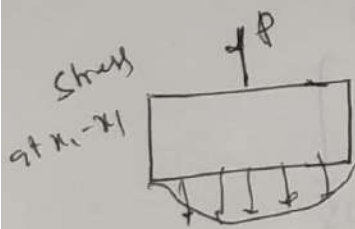
$K = 1.209 \times 10^5 \text{ N/mm}^2$

Assumptions made in the properties of Materials

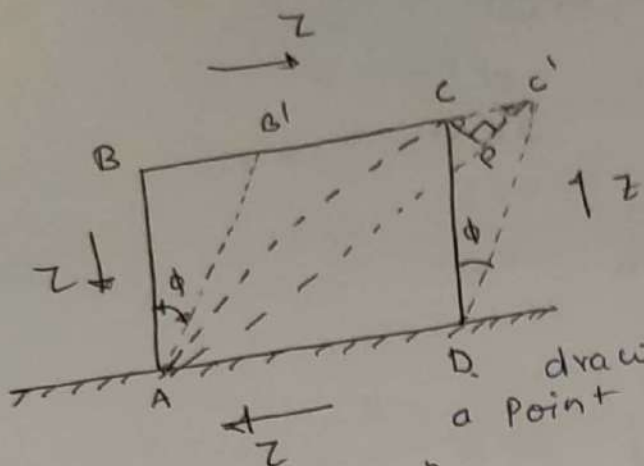
- ① The material is homogeneous having same composition through out the body.
- ② The elastic properties are same at each every point in the body.
- ③ The material is isotropic, which means that the material is equally elastic in all the directions.
- ④ The material of the body has a solid structure.
- ⑤ Initially there are number of internal holes in the body.
- ⑥ Principle of superposition holds good for internal hole in the body.

Saint Venant's Principle:

It is assumed that the distribution of stress over the cross section is uniform [$f = P/A$]. This assumption is based on Saint Venant's principle. The principle states that "except in the region of extreme ends of a bar under direct loading, the stress distribution over a cross-section is uniform."



Relationship b/w modulus of elasticity (E) and Modulus of Rigidity (C or G)



Consider a body ABCD of length thickness with the face AD is rigid and shear force is applied on the side BC as shown in fig. Due to the shear force the body takes a new plane AB'C'D. Draw diagonal AC' and draw a line from C to meet at a point P.

From ΔAPC
 $AC = AP$ ✓

$\angle PC'C = 45^\circ$ [Since angle of deformation is very small we can assume that it is 45°]

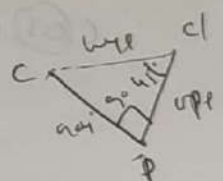
Since the deformation of the diagonal member AC is given by strain $EAC = \frac{AC' - AC}{AC} = \frac{PC'}{AC} \rightarrow (1)$

From $\Delta PC'C$

$$\sin 45 = \frac{PC'}{CC'}$$

$$\frac{1}{\sqrt{2}} = \frac{PC'}{CC'}$$

$$PC' = CC' \times \frac{1}{\sqrt{2}} \rightarrow (2)$$



From ΔACD

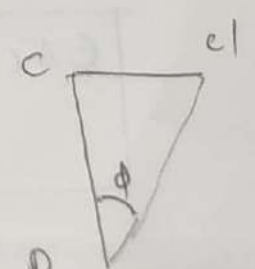
$$AC = \sqrt{AD^2 + CD^2}$$

Assume that the section is square $AD = CD$.

$$AP = AC = \sqrt{CD^2 + CD^2} = \sqrt{2} CD^2 = \sqrt{2} \cdot CD \rightarrow (3)$$

Shear strain for the $\Delta PC'C$

i.e $\tan \phi = \frac{CC'}{CD}$



For small angle, $\tan \phi = \phi$

$$\therefore \phi = \frac{cd}{CD} \rightarrow (4)$$

Substituting (4) & (3) in eq (1)

$$EAC = \frac{cd \times \frac{1}{\sqrt{2}}}{\sqrt{2} \cdot CD} = \frac{cd}{2 \cdot CD}$$

from eq (4) $d = \frac{cd}{CD}$

$$EAC = \frac{cd}{CD}$$

$$E_{AC} = \frac{cd}{2 \cdot CD}$$

$$= \frac{1}{2} \times \frac{cd}{CD}$$

$$EAC = \frac{\phi}{2} \rightarrow (5)$$

W.K.T. Modulus of Rigidity

$$G = \frac{\tau}{\phi} = \frac{Z}{2 \cdot EAC}$$

$$G = \frac{Z}{2 \cdot EAC} \rightarrow (6)$$

$$EAC = \frac{Z}{2G} \rightarrow (7)$$

An element oriented along the diagonal AC & BD subjected to tensile stress & compressive stress in the fig. Hence the strain along the member AC is given by

$$EAC = \frac{\sigma}{E} - \left(-\mu \frac{\sigma}{E} \right)$$

$$EAC = \frac{\sigma}{E} + \mu \frac{\sigma}{E}$$

$$EAC = \frac{\sigma}{E} (1 + \mu) \rightarrow (8)$$

In equation (2) strain ϵ
 $EAC = \frac{Z}{E} (1+\mu) \rightarrow (8)$

Comparing (6) & (8)

$$\frac{Z}{2a} = \frac{Z}{E} (1+\mu)$$

$$E = 2a(1+\mu)$$

Relationship b/w modulus of elasticity (E) modulus of Rigidity (a) and Bulk modulus (K)

w.k.t = $E = 2a(1+\mu) \rightarrow (1)$

$E = 3K(1-2\mu) \rightarrow (2)$

from eq (1) $E = 2a(1+\mu)$

$$\frac{E}{2a} = (1+\mu)$$

$$\mu = \frac{E}{2a} - 1 \rightarrow (3)$$

Substituting the value of μ in eq (2) we get

$$E = 3K \left[1 - 2 \left(\frac{E}{2a} - 1 \right) \right]$$

$$E = 3K \left[1 - \frac{2E}{2a} + 2 \right]$$

$$E = 3K \left[1 - \frac{E}{a} + 2 \right]$$

$$E = 3K \left(3 - \frac{E}{a} \right)$$

$$E = 9K - \frac{3KE}{a}$$

$$E + \frac{3KE}{a} = 9K$$

$$E \left[1 + \frac{3K}{a} \right] = 9K$$

for which Young's modulus is $1.2 \times 10^5 \text{ N/mm}^2$ and bulk modulus of a material, rigidity is $4.9 \times 10^4 \text{ N/mm}^2$

$\mu = ? \quad K = ? \quad E = 1.2 \times 10^5 \text{ N/mm}^2 \quad G = 4.9 \times 10^4 \text{ N/mm}^2$

$$E = 2G(1 + \mu)$$

$$1.2 \times 10^5 = 2 \times 4.9 \times 10^4 (1 + \mu)$$

$$(1 + \mu) = \frac{1.2 \times 10^5}{2 \times 4.9 \times 10^4}$$

$$1 + \mu = 1.25$$

$$\mu = 1.25 - 1$$

$$\boxed{\mu = 0.25}$$

Bulk modulus is given by by equation

$$K = \frac{E}{3(1 - 2\mu)}$$

$$K = \frac{1.2 \times 10^5}{3(1 - 2 \times 0.25)}$$

$$\boxed{K = 8 \times 10^4 \text{ N/mm}^2}$$

2) A bar of cross section $8 \text{ mm} \times 8 \text{ mm}$ is subjected to an axial pull of 7000 N . The lateral dimension of the bar is found to be changed to $7.9985 \text{ mm} \times 7.9985 \text{ mm}$. If the modulus of rigidity of the material is $0.8 \times 10^5 \text{ N/mm}^2$, determine the Poisson's ratio and modulus of elasticity.

Given Cross-section $8 \text{ mm} \times 8 \text{ mm}$ $P = 7000 \text{ N}$,
 lateral dimension of bar = $7.9985 \text{ mm} \times 7.9985 \text{ mm}$
 Volume of $G = 0.8 \times 10^5 \text{ N/mm}^2$

Let $\mu = ? \quad E = ?$

Now lateral strain = $\frac{\text{change in lateral dimension}}{\text{original lateral dimension}}$

$$= \frac{8 - 7.9985}{8} = \frac{0.0015}{8}$$

$$= 0.0001875$$

To find the value of Poisson's ratio, we have to know the value of longitudinal strain. But in this problem the length of the bar and the axial tension is not given. Hence the longitudinal strain cannot be calculated but axial stress can be calculated. This longitudinal strain will be equal to the axial stress divided by E .

$$\text{axial stress} = \frac{P}{\text{Area}} = \frac{7000}{64} = 109.375 \text{ N/mm}^2$$

$$\text{longitudinal strain} = \frac{\sigma}{E}$$

$$\text{But lateral strain} = \mu \times \text{longitudinal strain} = \mu \times \frac{\sigma}{E}$$

$$0.0001875 = \frac{\mu \times 109.375}{E}$$

$$\frac{E}{\mu} = \frac{109.375}{0.0001875}$$

$$E = 583333.34$$

(c) using equation,

$$C = \frac{E}{2(1+\mu)}$$

$$E = 2C(1+\mu)$$

$$= 2 \times 0.8 \times 10^5 (1+\mu)$$

$$583333.34 = 2 \times 0.8 \times 10^5 (1+\mu)$$

$$(1+\mu) = \frac{583333.34}{2 \times 0.8 \times 10^5}$$

$$1+\mu = 3.64584$$

$$1 = 3.64584 - \mu$$

$$1 = 3.64584 - \mu$$

$$1 = 2.64584$$

$$\mu = \frac{1}{2.64584} = 0.378$$

Modulus of elasticity (E)

$$E = 583333.3 \text{ N/mm}^2$$

$$E = 583333.3 \times 0.378$$

$$E = 2.2049 \times 10^5 \text{ N/mm}^2$$

③ calculate the modulus of rigidity and bulk modulus of a cylindrical bar of diameter 30mm and of length 1.5m if the longitudinal strain in a bar during a tensile stress is four times the lateral strain. Find the change in volume when the bar is subjected to a hydrostatic pressure of 100 N/mm^2 . Take $E = 1 \times 10^5 \text{ N/mm}^2$.

→ Dia = 30mm.

$$L = 1.5 \text{ m} = 1.5 \times 1000 = 1500 \text{ mm}$$

$$\begin{aligned} \text{Volume of bar } V &= \frac{\pi d^2 L}{4} \\ &= \frac{\pi (30)^2 \times 1500}{4} \\ &= 1060287.52 \text{ mm}^3 \end{aligned}$$

$$\text{Longitudinal strain} = 4 \times \text{lateral strain}$$

$$P = 100 \text{ N/mm}^2$$

$$\mu = \frac{\text{lateral strain}}{\text{longitudinal strain}} = \frac{1}{4} = 0.25$$

$$\mu = 0.25$$

C = modulus of rigidity.

K = Bulk modulus, $E = \text{Young's modulus} = 1 \times 10^5 \text{ N/mm}^2$

$$E = 2C(1 + \mu)$$

$$1 \times 10^5 = 2C(1 + 0.25)$$

$$C = \frac{1 \times 10^5}{2 \times 1.25}$$

$$C = 4 \times 10^4 \text{ N/mm}^2$$

$$E = 3K(1 - 2\mu)$$

$$1 \times 10^5 = 3K(1 - 2 \times 0.25)$$

$$K = \frac{1 \times 10^5}{3 \times 0.5}$$

$$K = 0.667 \times 10^5 \text{ N/mm}^2$$

$$K = \frac{P}{\text{volumetric strain}} = \frac{P}{\left(\frac{\Delta V}{V}\right)}$$

$$0.667 \times 10^5 = \frac{100}{\frac{\Delta V}{V}}$$

$$\frac{\Delta V}{V} = \frac{100}{0.667 \times 10^5} = 1.5 \times 10^{-3}$$

$$\Delta V = 1.5 \times 10^{-3} \times V = 1590.43 \text{ mm}^3$$

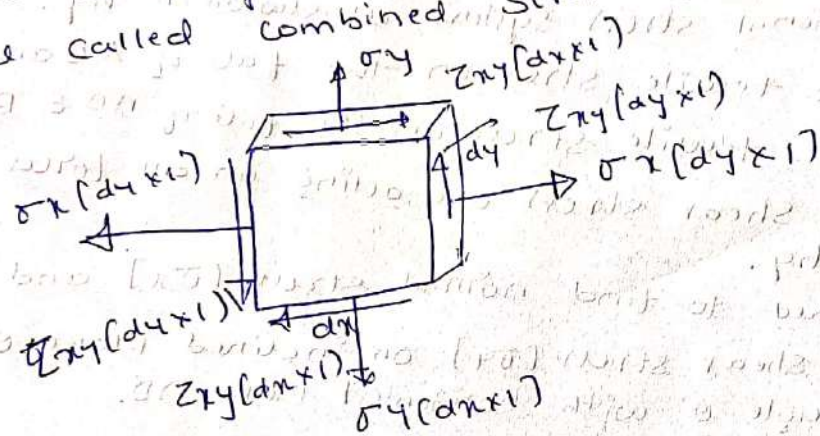
Module: 2

Compound stresses

Introduction, state of stress at a point, General two dimensional stress system, Principal stresses and principal Planes, Mohr's circle of stresses, Theory of failure, Max. Shear stress theory and max principle stress theory.

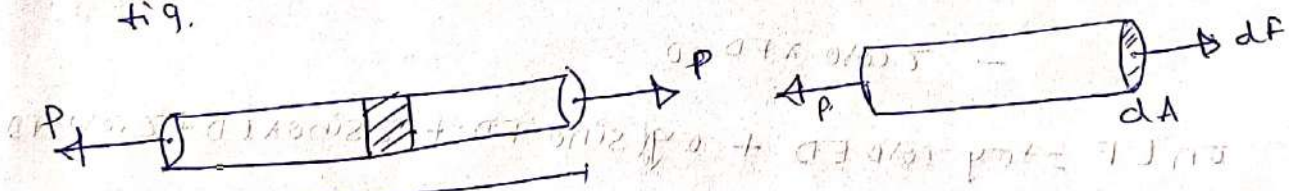
Compound stresses:

The stress system in which both Direct stresses (σ) and shearing stresses (τ) are acting simultaneously are called combined stresses (σ) compound stresses.



Stress at a point:

Consider an elastic body subjected to the forces as shown in fig. Consider an element as shown in fig.



Force on element = dF

Area of the element = dA

Stress at a point OR element = $\frac{dF}{dA}$

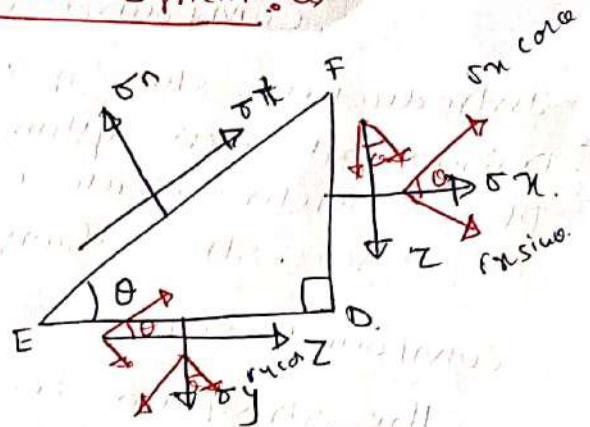
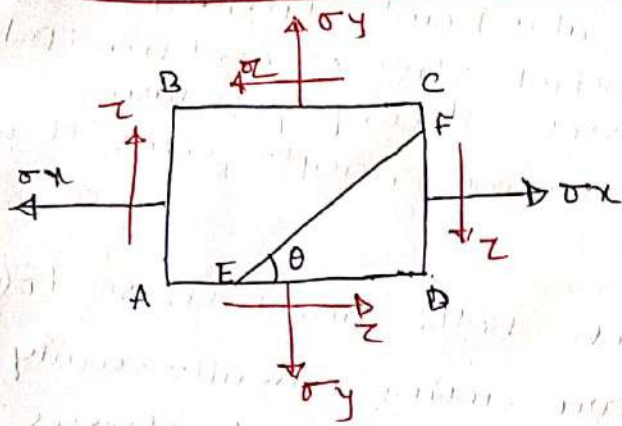
As $dA \rightarrow 0$

Stress = $\lim_{dA \rightarrow 0} \frac{dF}{dA}$

$$dF = \frac{\partial P}{\partial A}$$

- Stress at a point.

General two dimensional stress system



Consider a rectangular block ABCD of unit thickness under two dimensional stress system as shown in fig.

Let σ_x be the tensile stress on the face of AB and CD, σ_y be the tensile stress on the face of AD and BC, τ be the shear stress all acting on all faces as shown in fig.

It is required to find normal stress $[\sigma_n]$ and tangential (or) shear stress $[\sigma_t]$ on inclined plane θ making an angle θ with horizontal plane AD.

Considering portion EDF.

Resolving all forces along T_n & T_t direction

Along T_n direction.

$$\sigma_n \times [EF \cos \theta] - \sigma_y [\cos \theta (ED \cos \theta)] - \sigma_x \sin \theta (FD \sin \theta) - \tau \sin \theta \times ED$$

$$- \tau \cos \theta \times FD = 0$$

$$\sigma_n EF = \sigma_y \cos \theta ED + \sigma_x \sin \theta FD + \tau \sin \theta ED + \tau \cos \theta FD$$

$$\sigma_n = \sigma_y \cos \theta \left(\frac{ED}{EF} \right) + \sigma_x \sin \theta \left(\frac{FD}{EF} \right) + \tau \sin \theta \left(\frac{ED}{EF} \right) + \tau \cos \theta \left(\frac{FD}{EF} \right)$$

$$\frac{ED}{EF} = \cos \theta \quad \frac{FD}{EF} = \sin \theta$$

Substituting these values in the above eq.

$$\sigma_n = \sigma_y \cos \theta \times \cos \theta + \sigma_x \sin \theta \times \sin \theta + \tau \sin \theta \times \cos \theta + \tau \cos \theta \times \sin \theta$$

$$\sigma_n = \sigma_x \sin^2 \theta + \sigma_y \cos^2 \theta + 2\tau \sin \theta \cos \theta$$

$$\sigma_x = \sigma_x \left[\frac{1 - \cos 2\theta}{2} \right] + \sigma_y \left[\frac{1 + \cos 2\theta}{2} \right] + z \sin 2\theta$$

$$\sigma_x = \frac{\sigma_x}{2} - \frac{\sigma_x \cos 2\theta}{2} + \frac{\sigma_y}{2} + \frac{\sigma_y \cos 2\theta}{2} + z \sin 2\theta$$

$$\sigma_x = \left(\frac{\sigma_x + \sigma_y}{2} \right) + \cos 2\theta \left[\frac{\sigma_y - \sigma_x}{2} \right] + z \sin 2\theta \rightarrow \textcircled{1}$$

Along σ_T direction

$$\sigma_t(EF) + \sigma_x \cos \theta \times \frac{DF}{EF} - \sigma_y \sin \theta \times ED + z \cos \theta \times ED - z \sin \theta \times FD = 0$$

$$\sigma_t(EF) = -\sigma_x \cos \theta \times \frac{DF}{EF} + \sigma_y \sin \theta \times ED - z \cos \theta \times ED + z \sin \theta \times FD$$

$$\sigma_t = -\sigma_x \cos \theta \times \left(\frac{DF}{EF} \right) + \sigma_y \sin \theta \times \left(\frac{ED}{EF} \right) - z \cos \theta \left(\frac{ED}{EF} \right) + z \sin \theta \left(\frac{FD}{EF} \right)$$

$$\frac{DF}{EF} = \sin \theta \quad \frac{ED}{EF} = \cos \theta$$

$$\sigma_t = -\sigma_x \cos \theta \times \sin \theta + \sigma_y \sin \theta \cos \theta - z \cos \theta \cos \theta + z \sin \theta \sin \theta$$

$$\sigma_t = (\sigma_y - \sigma_x) \sin \theta \cos \theta - z \cos 2\theta + z \sin 2\theta$$

$$\sigma_t = \frac{(\sigma_y - \sigma_x)}{2} \sin 2\theta - z (\cos 2\theta - \sin 2\theta)$$

$$\sigma_t = \left[\frac{\sigma_y - \sigma_x}{2} \right] \sin 2\theta - z (\cos 2\theta) \rightarrow \textcircled{2}$$

$$\begin{aligned} &\cos 2\theta - \sin 2\theta \\ &\frac{\cos 2\theta + \sin 2\theta}{2} - \frac{\cos 2\theta - \sin 2\theta}{2} \\ &= \frac{\cos 2\theta}{2} \end{aligned}$$

Equation is normal stress on inclined plane where
Eq ② is shear ③ Tangential stress on inclined plane

The planes on which the shear stress (Tangential stress) is zero are known as principal planes. And the stresses acting on the principal planes are known as principal stress.

∴ For principal plane.

$$\sigma_t = 0$$

$$\left[\frac{\sigma_y - \sigma_x}{2} \right] \sin 2\theta - z (\cos 2\theta) = 0$$

$$\frac{(\sigma_y - \sigma_x)}{2} \sin 2\theta = \tau \cos 2\theta$$

$$(\sigma_y - \sigma_x) \sin 2\theta = 2\tau \cos 2\theta$$

$$\tan 2\theta = \frac{2\tau}{\sigma_y - \sigma_x}$$

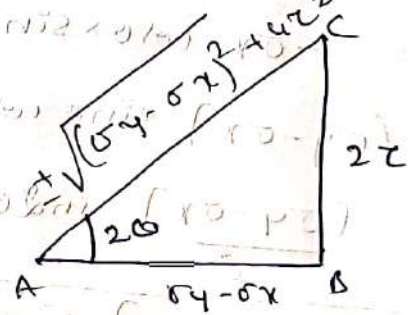
$$\tan 2\theta = \frac{2\tau}{\sigma_y - \sigma_x}$$

Principal stresses and principal planes.

In case of two dimensional stress system there always exists two planes separated by 90°, on which the shearing stresses are zero. These shear free planes are called principal planes and the corresponding values of normal stress (σ_n) acting on these planes are called principal stresses, of which one is major principal stress and minor principal stress.

We know that

$$\tan 2\theta = \frac{2\tau}{\sigma_y - \sigma_x}$$



$$AB = \sigma_y - \sigma_x$$

$$BC = 2\tau$$

$$\text{Then } AC = \pm \sqrt{AB^2 + BC^2}$$

$$AC = \pm \sqrt{(\sigma_y - \sigma_x)^2 + (2\tau)^2}$$

$$AC = + \sqrt{(\sigma_y - \sigma_x)^2 + 4\tau^2}$$

$$AC = - \sqrt{(\sigma_y - \sigma_x)^2 + 4\tau^2}$$

from fig

$$\sin 2\theta = \frac{\text{opp}}{\text{hyp}} = \frac{BC}{AC} = \pm \frac{2\tau}{\sqrt{(\sigma_y - \sigma_x)^2 + 4\tau^2}}$$

$$\cos 2\theta = \frac{\text{adj}}{\text{hyp}} = \frac{AB}{AC} = \pm \frac{(\sigma_y - \sigma_x)}{\sqrt{(\sigma_y - \sigma_x)^2 + 4\tau^2}}$$

$$\sigma_{n_1} \text{ or } \sigma_{n_2} = \left[\frac{\sigma_x + \sigma_y}{2} \right] \pm \frac{\cos 2\theta}{2} \left[\frac{\sigma_y - \sigma_x}{2} \right] + \tau \sin 2\theta$$

$$= \left[\frac{\sigma_x + \sigma_y}{2} \right] \pm \left(\frac{\sigma_y - \sigma_x}{2} \right) \frac{(\sigma_y - \sigma_x)}{\sqrt{(\sigma_y - \sigma_x)^2 + 4\tau^2}} \pm \frac{\tau \times 2\tau}{\sqrt{(\sigma_y - \sigma_x)^2 + 4\tau^2}}$$

$$\sigma_{n_1} \text{ or } \sigma_{n_2} = \left[\frac{\sigma_x + \sigma_y}{2} \right] \pm \frac{(\sigma_y - \sigma_x)^2}{2\sqrt{(\sigma_y - \sigma_x)^2 + 4\tau^2}} \pm \frac{2\tau^2}{\sqrt{(\sigma_y - \sigma_x)^2 + 4\tau^2}}$$

$$= \left[\frac{\sigma_x + \sigma_y}{2} \right] \pm \frac{(\sigma_y - \sigma_x)^2 + 4\tau^2}{2\sqrt{(\sigma_y - \sigma_x)^2 + 4\tau^2}}$$

$$= \left[\frac{\sigma_x + \sigma_y}{2} \right] \pm \frac{\sqrt{(\sigma_y - \sigma_x)^2 + 4\tau^2}}{2}$$

$$\sigma_{n_1} \text{ or } \sigma_{n_2} = \left[\frac{\sigma_x + \sigma_y}{2} \right] \pm \sqrt{\left(\frac{\sigma_y - \sigma_x}{2} \right)^2 + \tau^2}$$

Major Principal stress $\sigma_{n_1} = \left[\frac{\sigma_x + \sigma_y}{2} \right] + \sqrt{\left(\frac{\sigma_y - \sigma_x}{2} \right)^2 + \tau^2}$

Minor Principal stress $\sigma_{n_2} = \left[\frac{\sigma_x + \sigma_y}{2} \right] - \sqrt{\left(\frac{\sigma_y - \sigma_x}{2} \right)^2 + \tau^2}$

Maximum shear stress:

$$\sigma_t = \left[\frac{\sigma_y - \sigma_x}{2} \right] \sin 2\theta - z \cos 2\theta = 0$$

The plane when shear is maximum

when $\frac{d}{d\theta} (\sigma_t) = 0$.

$$0 = \frac{d}{d\theta} \left[\left[\frac{\sigma_y - \sigma_x}{2} \right] \sin 2\theta - z \cos 2\theta \right]$$

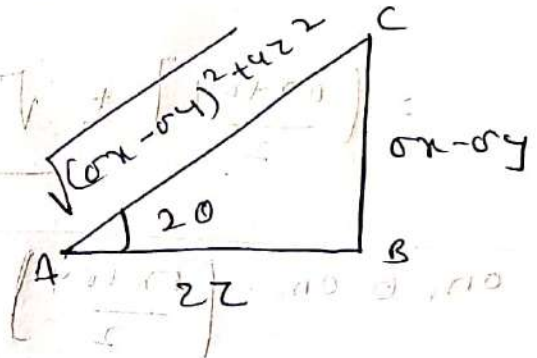
$$0 = \left[\frac{\sigma_y - \sigma_x}{2} \right] \cos 2\theta \times 2 - z \sin 2\theta (-2)$$

$$0 = \left[\frac{\sigma_y - \sigma_x}{2} \right] \cos 2\theta + 2z \sin 2\theta$$

$$0 = \left[\frac{\sigma_x - \sigma_y}{2} \right] \cos 2\theta + 2z \sin 2\theta$$

$$\tan 2\theta = \frac{\sigma_x - \sigma_y}{2z}$$

$$\sin 2\theta = \frac{\pm (\sigma_x - \sigma_y)}{\sqrt{(\sigma_x - \sigma_y)^2 + 4z^2}}$$



$$\cos 2\theta = \frac{\pm 2z}{\sqrt{(\sigma_x - \sigma_y)^2 + 4z^2}}$$

$$\sigma_t = \left[\frac{\sigma_y - \sigma_x}{2} \right] \sin 2\theta - z \cos 2\theta$$

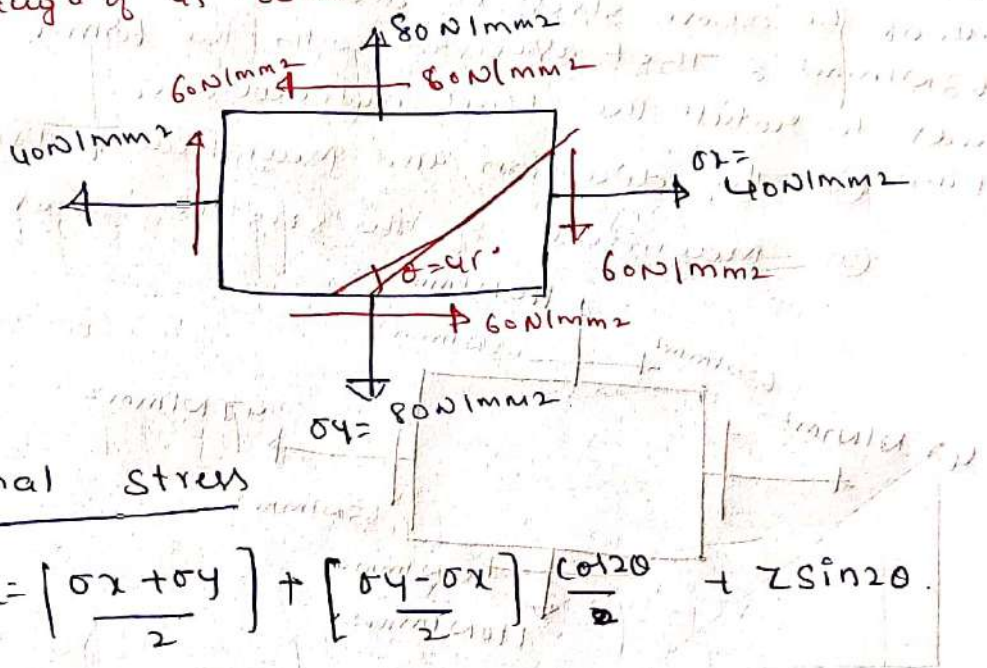
$$\sigma_t = \pm \left[\frac{\sigma_y - \sigma_x}{2} \right] \frac{(\sigma_x - \sigma_y)}{\sqrt{(\sigma_x - \sigma_y)^2 + 4z^2}} \mp \frac{z \times 2z}{\sqrt{(\sigma_x - \sigma_y)^2 + 4z^2}}$$

$$\sigma_t = \mp \frac{(\sigma_x - \sigma_y)^2 + 4z^2}{2\sqrt{(\sigma_x - \sigma_y)^2 + 4z^2}}$$

$$\sigma_t]_{\max} = \pm \frac{1}{2} \sqrt{(\sigma_x - \sigma_y)^2 + 4\tau^2}$$

Problems

1. At a point within a body subjected to two mutually perpendicular directions, the stresses are 80 N/mm^2 tensile and 40 N/mm^2 tensile. Each of the above stress is accompanied by a shear stress of 60 N/mm^2 . Determine normal stress shear stress & Resultant stress on an oblique plane inclined at an angle of 45° with the axis of minor tensile stress.



1 Normal stress

$$\sigma_n = \left[\frac{\sigma_x + \sigma_y}{2} \right] + \left[\frac{\sigma_y - \sigma_x}{2} \right] \cos 2\theta + \tau \sin 2\theta$$

$$\sigma_n = \left[\frac{40 + 80}{2} \right] + \left[\frac{80 - 40}{2} \right] \cos 2 \times 45^\circ + 60 \sin 2 \times 45^\circ$$

$$\sigma_n = 60 + 0 + 60$$

$$\sigma_n = 120 \text{ N/mm}^2$$

② Shear stress

$$\sigma_t = \left[\frac{\sigma_y - \sigma_x}{2} \right] \sin 2\theta - \tau \cos 2\theta$$

$$\sigma_t = \left[\frac{80 - 40}{2} \right] \sin 2 \times 45^\circ - 60 \cos 2 \times 45^\circ$$

$$\sigma_t = 30 - 20 = 10$$

$$\sigma_t = 20 \text{ N/mm}^2$$

③ Resultant stress

$$\sigma_R = \sqrt{\sigma_n^2 + \sigma_t^2}$$

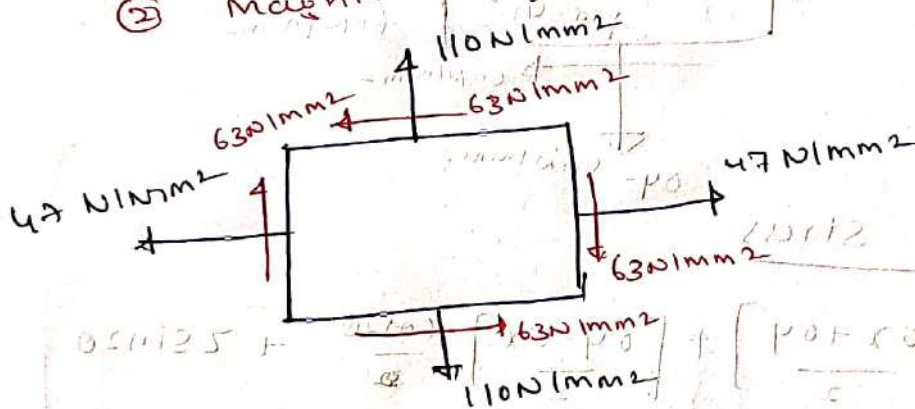
$$= \sqrt{120^2 + 20^2}$$

$$\sigma_R = 121.65 \text{ N/mm}^2$$

② A rectangular block of material is subjected to a tensile stress of 110 N/mm^2 on one plane & a tensile stress of 47 N/mm^2 on the plane at right angles to the former. Each of the above stress is accompanied by a shear stress of 63 N/mm^2 & this associated with the former tensile stress tends to rotate the block anticlockwise.

Find ① The direction and magnitude of principal stress.

② Magnitude of the greatest shear stress.



④ Major Principal stress:

$$\sigma_{n1} = \left[\frac{\sigma_x + \sigma_y}{2} \right] + \sqrt{\left[\frac{\sigma_y - \sigma_x}{2} \right]^2 + \tau^2}$$

$$\sigma_{n1} = \left[\frac{47 + 110}{2} \right] + \sqrt{\left[\frac{110 - 47}{2} \right]^2 + 63^2}$$

$$= 78.5 + \sqrt{992.25 + 3969}$$

$$= 78.5 + 70.43$$

$$\sigma_{n1} = 148.93 \text{ N/mm}^2$$

Minor Principal Stress:

$$\sigma_{n2} = \left[\frac{\sigma_x + \sigma_y}{2} \right] - \sqrt{\left[\frac{\sigma_x - \sigma_y}{2} \right]^2 + \tau^2}$$

$$\sigma_{n2} = \left[\frac{47 + 110}{2} \right] - \sqrt{\left[\frac{110 - 47}{2} \right]^2 + 63^2}$$

$$= 78.5 - 70.43$$

$$\sigma_{n2} = 8.07 \text{ N/mm}^2$$

Direction

$$\tan 2\theta = \frac{2\tau}{\sigma_y - \sigma_x}$$

$$\tan 2\theta = \frac{2 \times 63}{110 - 47}$$

$$2\theta = \tan^{-1}(2)$$

$$\theta = \frac{63.43}{2}$$

$$\theta = 31.71$$

$$\theta = 31.431$$

Magnitude of greatest shear.

$$\sigma_{t \max} = \frac{1}{2} \sqrt{(\sigma_x - \sigma_y)^2 + 4\tau^2}$$

$$= \frac{1}{2} \sqrt{(47 - 110)^2 + 4 \times 63^2}$$

$$\frac{1}{2} \times 140.8$$

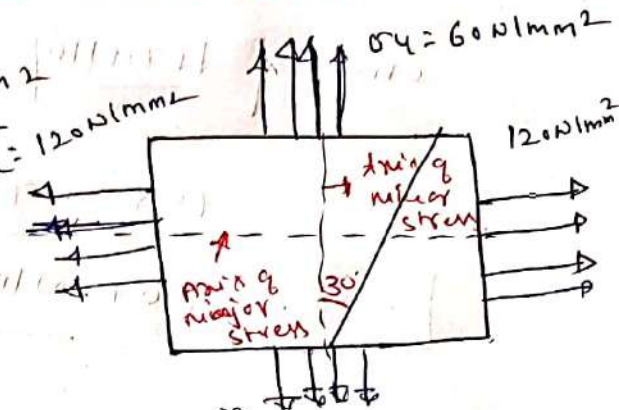
$$\sigma_{t \max} = 70.43 \text{ N/mm}^2$$

Q3) The tensile stresses at a point across two mutually perpendicular planes are 120 N/mm^2 and 60 N/mm^2 . Determine the normal, tangential and resultant stresses on a plane inclined at 30° to the axis of the minor stress.

Major principal stress $\sigma_x = 120 \text{ N/mm}^2$

Minor principal stress $\sigma_y = 60 \text{ N/mm}^2$

Angle of oblique plane with the axis of minor plane. $\theta = 30^\circ$



1. Normal stress

$$\sigma_n = \frac{\sigma_x + \sigma_y}{2} + \left(\frac{\sigma_x - \sigma_y}{2} \right) \cos 2\theta + \tau \sin 2\theta$$

$$\sigma_n = \frac{120 + 60}{2} + \left[\frac{120 - 60}{2} \right] \cos 2 \times 30^\circ$$

$$\sigma_n = 90 + 30 \cos 60^\circ = 105 \text{ N/mm}^2$$

$$\sigma_n = 105 \text{ N/mm}^2$$

2. Tangential stress:

$$\sigma_t = \left(\frac{\sigma_x - \sigma_y}{2} \right) \sin 2\theta$$

$$= \left(\frac{120 - 60}{2} \right) \sin 2 \times 30^\circ$$

$$\sigma_t = 25.98 \text{ N/mm}^2$$

Resultant stress

$$\sigma_R = \sqrt{\sigma_n^2 + \sigma_t^2}$$

$$= \sqrt{105^2 + 25.98^2}$$

$$\sigma_R = 108.16 \text{ N/mm}^2$$

Fig. 3.15(b).

Example 3.9 At a point in a strained material there is tensile stress of 80 N/mm^2 on a horizontal plane and a compressive stress of 40 N/mm^2 on a vertical plane. There is also a shear stress of 48 N/mm^2 on each of these planes.

Determine the planes of maximum shear stress at the point. Determine also the resultant stress on the planes of maximum shear stress.

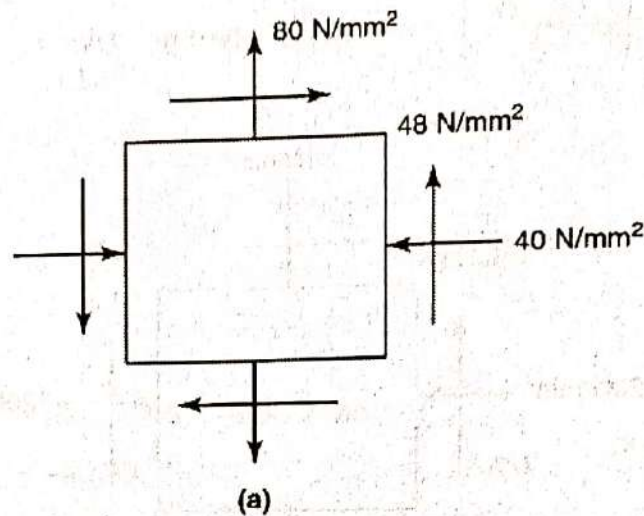


Fig 3.16

Solution

The state of stress is as shown in Fig. 3.16(a).

Taking horizontal axis as x and vertical axis as y , we have $P_x = -40 \text{ N/mm}^2$, $P_y = 80 \text{ N/mm}^2$ and $q = 48 \text{ N/mm}^2$.

$$q_{\max} = \sqrt{\left(\frac{P_x - P_y}{2}\right)^2 + q^2} = \sqrt{\left(\frac{-40 - 80}{2}\right)^2 + 48^2} = 76.84 \text{ N/mm}^2$$

Inclination of principal stress to the plane of p_x (vertical) is given by

$$\tan 2\theta = \frac{2q}{P_x - P_y}$$

$$\tan 2\theta = \frac{2 \times 48}{-40 - 80} = \frac{96}{-120}$$

$$2\theta = -38.66^\circ \text{ and } 141.34^\circ$$

$$\text{or } \theta = -19.33^\circ \text{ and } 70.67^\circ$$

Inclination of maximum shear stress is $-19.33^\circ + 45^\circ$ and $70.67^\circ + 45^\circ$, i.e., 25.67° and 115.67° .

The corresponding normal stress is given by

$$\begin{aligned} P_n &= \frac{P_x + P_y}{2} + \frac{P_x - P_y}{2} \cos 2\theta' + q \sin 2\theta' \\ &= \frac{-40 + 80}{2} + \frac{-40 - 80}{2} \cos 2(25.67) + 48 \sin 2(25.67) \end{aligned}$$

$$\begin{aligned}
 P_n &= \frac{P_x + P_y}{2} + \frac{P_x - P_y}{2} \cos 2\theta' + q \sin 2\theta' \\
 &= \frac{-40 + 80}{2} + \frac{-40 - 80}{2} \cos 2(25.67) + 48 \sin 2(25.67) \\
 &= +20 \text{ N/mm}^2
 \end{aligned}$$

$$\begin{aligned}
 \text{Resultant stress} &= \sqrt{p_n^2 + p_t^2} \\
 &= \sqrt{(20)^2 + 76.84^2}, \quad (\text{Since } p_t = q_{\max}) \\
 &= 79.4 \text{ N/mm}^2 \text{ (Ans)}
 \end{aligned}$$

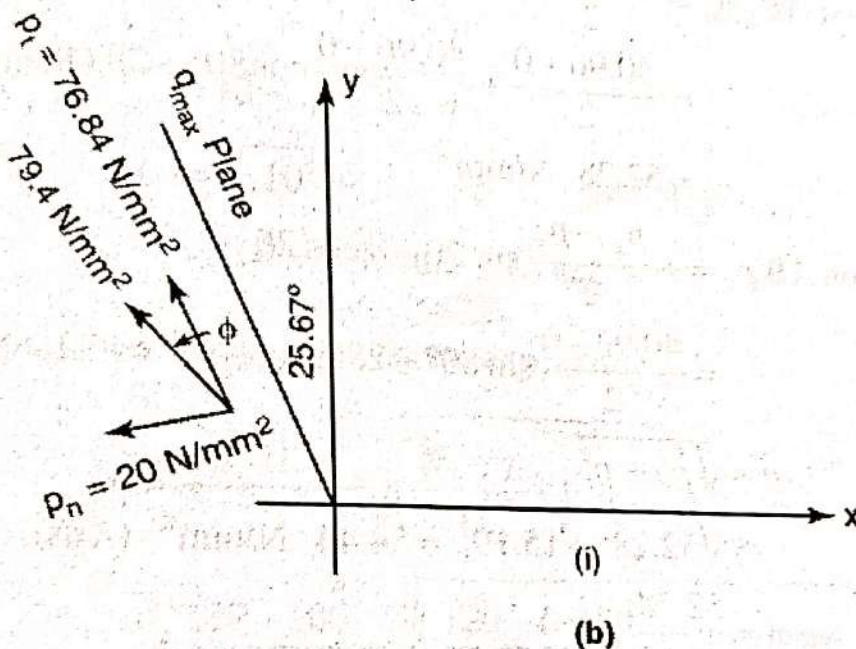


Fig. 3.16

Referring to Fig. 3.16(b), we get

$$\tan \phi = \frac{20}{76.84}$$

$$\phi = 14.59^\circ \text{ as shown in Fig. (Ans)}$$

∴ Its inclination to the plane of p_x (i.e., vertical plane) is given by

$$\alpha = 25.67 + 14.59 = 41.26^\circ$$

Example 3.6 The state of stress at a point in a strained material is as shown in Fig. 3.13(a). Determine

- The direction of the principal planes
- The magnitude of principal stresses, and
- The magnitude of maximum shear stress and its direction. Indicate all the above planes by a sketch

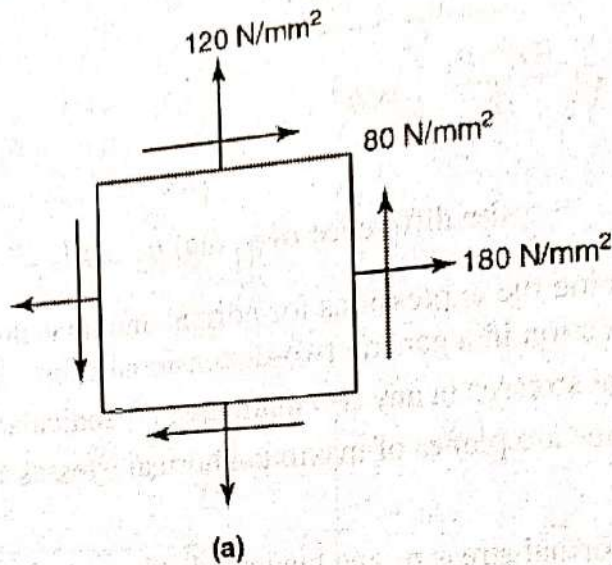


Fig. 3.13

Solution

Let θ be the inclination of principal plane to the plane of 180 N/mm^2 . Then

$$p_x = 180 \text{ N/mm}^2, \quad p_y = 120 \text{ N/mm}^2, \quad q = 80 \text{ N/mm}^2.$$

$$\therefore \tan 2\theta = \frac{2q}{p_x - p_y} = \frac{2 \times 80}{180 - 120} = 2.6667$$

$$\therefore 2\theta = 69.444^\circ \quad \text{and} \quad 69.444 + 180^\circ$$

$$\text{i.e.,} \quad \theta = 34.722^\circ \quad \text{and} \quad 124.722^\circ \quad (\text{Ans})$$

$$p_1 = \frac{p_x + p_y}{2} + \sqrt{\left(\frac{p_x - p_y}{2}\right)^2 + q^2}$$

$$= \frac{180 + 120}{2} + \sqrt{\left(\frac{180 - 120}{2}\right)^2 + 80^2}$$

$$= 150 + 85.44 = 235.44 \text{ N/mm}^2 \text{ (tensile)} \quad (\text{Ans})$$

$$p_2 = \frac{p_x + p_y}{2} - \sqrt{\left(\frac{p_x - p_y}{2}\right)^2 + q^2}$$

$$= 150 - 85.44 = 65.56 \text{ N/mm}^2 \text{ (Tensile)} \quad (\text{Ans})$$

$$q_{\max} = \sqrt{\left(\frac{p_x - p_y}{2}\right)^2 + q^2} = 85.44 \text{ N/mm}^2 \quad (\text{Ans})$$

Planes of maximum shearing stresses are at 45° to principle planes. All these are shown in Fig. 3.13(b).

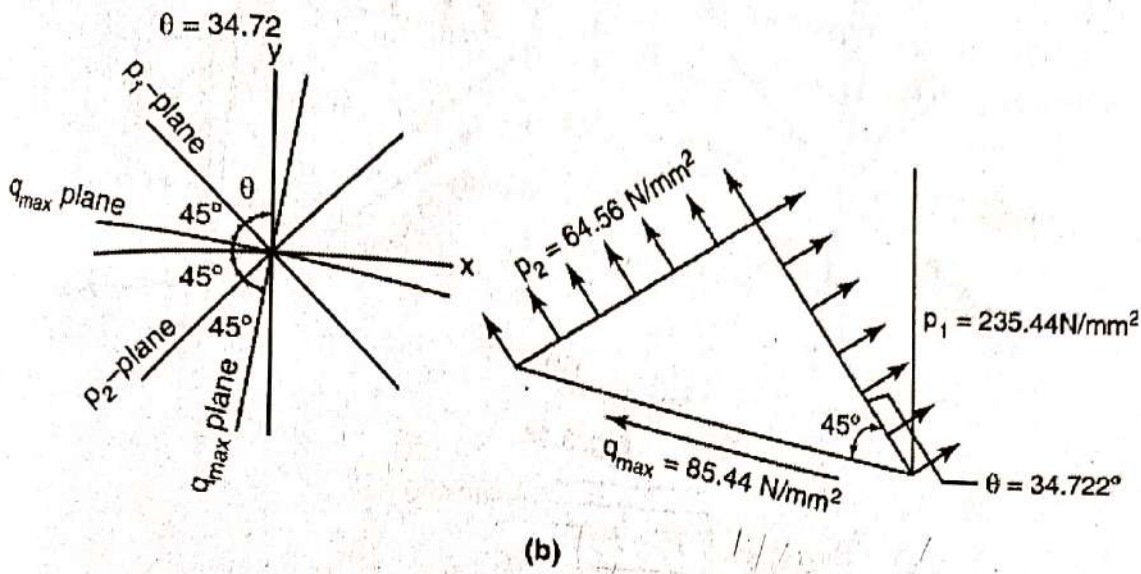


Fig. 3.13

Example 3.7 A plane element is subjected to stresses as shown in Fig. 3.14(a). Determine principal stresses, maximum shear stress and their planes. Sketch the planes determined.

Solution
 Taking x and y coordinates, as shown in Fig. 3.14(a), we have $p_x = 60 \text{ N/mm}^2$, $p_y = -40 \text{ N/mm}^2$ and $q = 10 \text{ N/mm}^2$.

$$\begin{aligned} \text{Then, } p_1 &= \frac{p_x + p_y}{2} + \sqrt{\left(\frac{p_x - p_y}{2}\right)^2 + q^2} = \frac{60 - 40}{2} + \sqrt{\left(\frac{60 - (-40)}{2}\right)^2 + 10^2} \\ &= 10 + 50.99 = 60.99 \text{ N/mm}^2 \text{ (Ans)} \end{aligned}$$

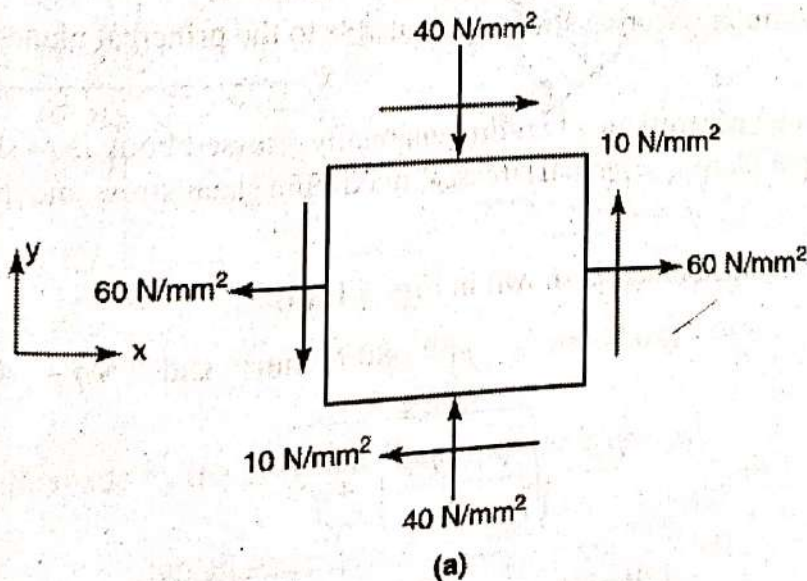


Fig. 3.14

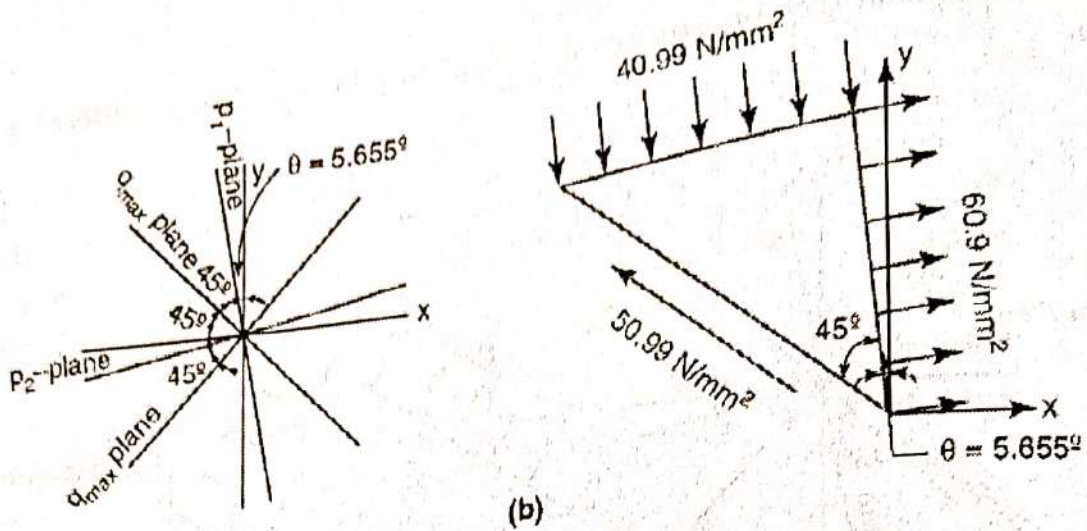


Fig. 3.14

$$P_2 = \frac{P_x + P_y}{2} - \sqrt{\left(\frac{P_x - P_y}{2}\right)^2 + q^2}$$

$$= 10 - 50.99 = -40.99 \text{ N/mm}^2$$

$$= -40.99 \text{ N/mm}^2 \quad (\text{Comp}) \quad (\text{Ans})$$

$$q_{\max} = \sqrt{\left(\frac{P_x - P_y}{2}\right)^2 + q^2} = 50.99 \text{ N/mm}^2 \quad (\text{Ans})$$

Let θ be the inclination of principal stress to the plane of 60 N/mm^2 . Then

$$\tan 2\theta = \frac{2q}{p_x - P_y}$$

$$= \frac{2 \times 10}{60 - (-40)} = \frac{1}{5}$$

$$2\theta = 11.31^\circ \text{ and } 191.31^\circ$$

$$\therefore \theta = 5.655^\circ \text{ and } 95.655^\circ \quad (\text{Ans})$$

The planes of maximum shearing stresses are at 45° to the principal planes. These are shown in Fig. 3.14(b).

Engineers have been trying to predict failure loads for various structural elements so that they can improve their designs. Most of the structural elements are in complex state of stress and it is not easy to conduct laboratory tests on models with complex loading. The procedure followed by engineers is to conduct uniaxial tests on specimens and get the failure condition for the material. From a few test results they arrive at failure criteria for structural element in a complex condition. They use these criteria for predicting failure load in any structural element. The following five theories have been put forth by researches:

1. Maximum principal stress theory
2. Maximum shear stress theory
3. Maximum principal strain theory
4. Maximum strain energy theory
5. Maximum distortions energy theory

These theories of failure are briefly explained in this chapter and suitability of these theories for different types of materials are mentioned.

4.1 MAXIMUM PRINCIPAL STRESS THEORY

This theory is also known as **Rankine's theory**. According to it, a material in complex state of stress fails, when the maximum principal stress in it reaches the value of stress at elastic limit in simple tension.

Thus, in a two-dimensional general stress condition, the failure criteria is

$$p_1 = \sqrt{\frac{p_x + p_y}{2}} + \sqrt{\left(\frac{p_x - p_y}{2}\right)^2 + q^2} = p_e \quad (4.1)$$

where p_e is the stress at elastic limit in uniaxial tension test.

$\approx f_y$, Yield stress.

This theory is found to be reasonably good for brittle materials.

4.2 MAXIMUM SHEAR STRESS THEORY

This theory is known as **Coulomb's theory** as it was originally proposed by CA Coulomb. But it is also associated with the names of J Guest and H Tresca. According to this theory, a material in complex state of stress fails when the maximum shearing stress in it reaches the value of shearing stress at elastic limit in uniaxial tension test. In a general two-dimensional stress system maximum shearing stress is given by

$$q_{\max} = \sqrt{\left(\frac{p_x - p_y}{2}\right)^2 + q^2}$$

In uniaxial tension test, maximum shearing stress at elastic limit is

$$q_{\max} = \sqrt{\left(\frac{p_e - 0}{2}\right)^2 + 0} = \frac{p_e}{2}$$

∴ According to this theory, failure criterion is

$$\sqrt{\left(\frac{p_x - p_y}{2}\right)^2 + q^2} = \frac{p_e}{2}$$

In case of three-dimensional problems, maximum shear stress at a point may be taken as $(p_1 - p_3)/2$, where p_1 is the maximum principal stress and p_3 is minimum principal stress. Hence, the failure criterion is $(p_1 - p_3)/2 = p_e/2$, or

$$(p_1 - p_3) = p_e \quad (4.2)$$

This theory gives better results for ductile materials with elastic limit same in tension and in compression.

4.3 MAXIMUM STRAIN THEORY

This theory is also known as **St. Venant's theory**. According to it, failure in a complex system occurs when the maximum strain in it reaches the value of the strain in uniaxial stress at elastic limit.

If p_1 is the maximum stress in a complex state and p_2 and p_3 are the stresses in other two mutually perpendicular directions, then

$$\text{Maximum strain } e_{\max} = \frac{p_1 - \mu(p_2 + p_3)}{E}$$

$$\text{Strain in uniaxial tension at elastic limit} = \frac{p_e}{E}$$

$$\therefore \text{The failure criterion is } \frac{p_1 - \mu(p_2 + p_3)}{E} = \frac{p_e}{E}$$

or

$$p_1 - \mu(p_2 + p_3) = p_e \quad (4.3)$$

This theory is considered to be good for materials failing with brittle fractures.

4.4 MAXIMUM STRAIN ENERGY THEORY

This theory is known as **Beltrami and Haigh's theory** also. According to this theory, a material in complex stress system fails when the maximum strain energy per unit volume at a point reaches the value of strain energy per unit volume at elastic limit in simple tension test.

Consider an element of unit side in a three-dimensional stress system subjected to principal stresses p_1, p_2 and p_3 as shown in Fig. 4.1.

$$\text{Strain energy in it is } = \frac{1}{2} p_1 e_1 + \frac{1}{2} p_2 e_2 + \frac{1}{2} p_3 e_3$$

Now,

$$e_1 = \frac{1}{E} [p_1 - \mu(p_2 + p_3)],$$

$$e_2 = \frac{1}{E} [p_2 - \mu(p_1 + p_3)] \quad \text{and} \quad e_3 = \frac{1}{E} [p_3 - \mu(p_1 + p_2)]$$

nence, this theory is used by designers for all ductile materials.

Example 4.1 A bolt is subjected to an axial pull of 12 kN together with a transverse shear force of 6 kN. Determine the required diameter of the bolt by using

- (i) Maximum principal stress theory
- (ii) Maximum strain theory
- (iii) Maximum shear stress theory

Use the following data:

Elastic limit in tension = 300 N/mm², Factor of safety = 3, Poisson's ratio = 0.3

Solution

Let the diameter of the bolt be 'd'. Then the direct stress

$$p_x = \frac{12 \times 10^3}{\pi/4 d^2} = \frac{48 \times 10^3}{\pi d^2}$$

Shear stress at the centre of the bolt is

$$\begin{aligned} q &= \frac{4}{3} \times q_{av} \\ &= \frac{4}{3} \times \frac{6 \times 10^3}{\pi/4 d^2} \\ &= \frac{32 \times 10^3}{\pi d^2} \end{aligned}$$

$$\begin{aligned} \therefore \text{The principal stresses are } p_1 &= \frac{p_x}{2} + \sqrt{\left(\frac{p_x}{2}\right)^2 + q^2} \\ &= \frac{24 \times 10^3}{\pi d^2} + \sqrt{\left(\frac{24 \times 10^3}{\pi d^2}\right)^2 + \left(\frac{32 \times 10^3}{\pi d^2}\right)^2} \\ &= \frac{24 \times 10^3}{\pi d^2} \left(1 + \sqrt{1 + \left(\frac{32}{24}\right)^2}\right) \\ &= \frac{24 \times 10^3}{\pi d^2} (1 + 1.66667) \end{aligned}$$

$$\begin{aligned}
 &= \frac{20371.833}{d^2} \\
 p_2 &= \frac{p_x}{2} - \sqrt{\left(\frac{p_x}{2}\right)^2 + q^2} \\
 &= \frac{24 \times 10^3}{\pi \pi d^2} (1 - 1.66667) \\
 &= -\frac{5092.984}{d^2} \\
 q_{\max} &= \sqrt{\left(\frac{p_x}{2}\right)^2 + q^2} \\
 &= \frac{24 \times 10^3}{\pi d^2} \times 1.66667 \\
 &= \frac{12732.421}{d^2}
 \end{aligned}$$

(i) From maximum principal stress theory:

$$\text{Permissible stress in tension} = \frac{300}{3} = 100 \text{ N/mm}^2$$

$$\therefore p_1 = 100$$

$$\text{i.e., } \frac{20371.833}{d^2} = 100$$

$$\text{or, } d = 14.273 \text{ mm (Ans)}$$

(ii) Maximum strain theory:

$$\begin{aligned}
 e_{\max} &= \frac{p_1}{E} - \mu \frac{p_2}{E} \\
 &= \frac{20371.833}{d^2 E} - 0.3 \left(-\frac{5092.984}{d^2 E} \right) \\
 &= \frac{21899.728}{d^2 \times E}
 \end{aligned}$$

According to this theory, the design condition is

$$e_{\max} = \frac{P_e}{E \times \text{Factor of safety}}$$

$$\text{or, } \frac{21899.728}{d^2 \times E} = \frac{300}{E \times 3}$$

$$\text{or, } d = 14.799 \text{ mm}$$

(iii) Maximum shear stress theory: The design condition is

$$e_{\max} = \frac{\text{Shear stress at elastic limit}}{\text{Factor of safety}}$$

$$\frac{12732.421}{d^2} = \frac{300/2}{3}$$

$$d = 15.958 \text{ mm}$$

Hence, maximum shear stress theory governs the design. Use at least 15.958 mm diameter bolt. The practical size is 16 mm diameter. (Ans)

IMPORTANT CONCEPTS AND FORMULAE

- Theories of failure are developed to predict failure criteria in a material under complex state of stress in terms of test results obtained in uniaxial tests.
- Maximum principal theory is also known as Rankine's theory. According to this theory, failure condition is

$$p_1 = \frac{p_x + p_y}{2} + \sqrt{\left(\frac{p_x - p_y}{2}\right)^2 + q^2} = p_e$$

- Maximum shear stress theory is known as Coulomb's theory. In this the failure criterion is

$$p_1 - p_3 = p_e$$

- Maximum strain theory is known as St. Venant's theory and in this failure criterion works out to be

$$p_1 - \mu(p_2 + p_3) = p_e$$

- Maximum strain energy theory is also known as Beltrami and Haigh's theory. In this, failure criterion works out to be

$$p_1^2 + p_2^2 + p_3^2 - 2\mu(p_1p_2 + p_2p_3 + p_3p_1) = p_e^2$$

- Maximum distortion energy theory is known as Von-Mises criteria for failures. In this case the failure criteria works out to be

$$(p_1 - p_2)^2 + (p_2 - p_3)^2 + (p_3 - p_1)^2 = 2p_e^2$$

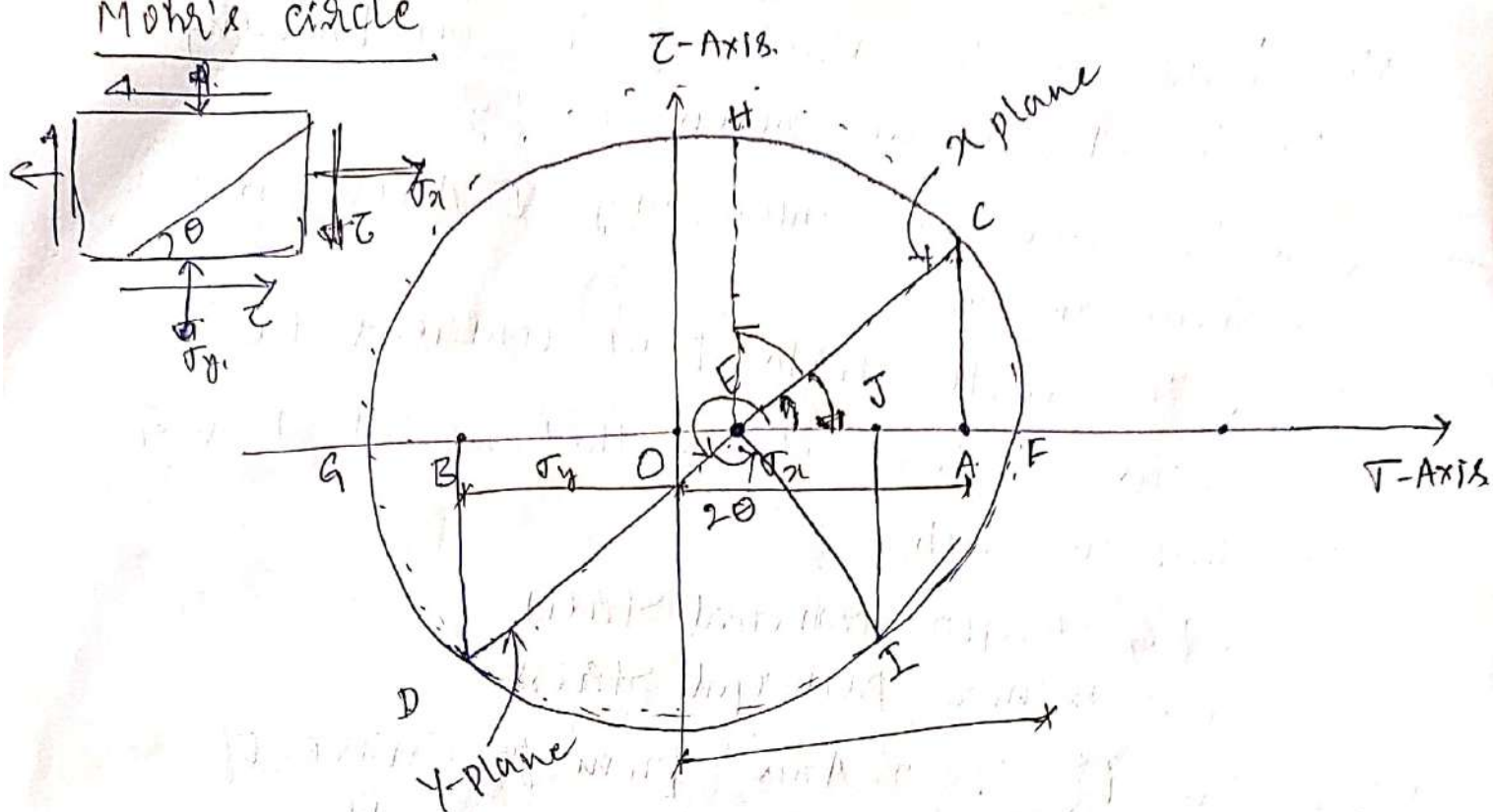
PROBLEMS FOR EXERCISE

- A bolt is required to resist an axial tension of 25 kN and a transverse shear of 20 kN. Find the safe size of the bolt by
 - The maximum principal stress theory
 - The maximum shear stress theory
 - The maximum distortion energy theory

The elastic limit of the material is 300 N/mm². Poisson's ratio = 0.3 and factor of safety = 3.0.

[Ans (i) 23.11 mm (ii) 27.00 mm (iii) 23.11 mm]

Mohr's circle



- 1) Draw Horizontal Axis indicating σ -Axis.
 - 2) Draw Vertical line τ -Axis intersecting σ -Axis at 90° .
 - 3) Take intersecting point of σ -Axis & τ -Axis as origin 'O'.
 - 4) If the Normal stress (σ_x or σ_y) is tensile. Mark a point on σ -axis as 'A' by taking proper scale. If the σ is to the right side of the origin, If the normal stress is negative mark the point on σ -axis to the left of origin 'B'.
 - 5) Now see whether whether the Shear stress on the plane of normal stress, τ producing rotating the plane clock wise or anticlock wise. If the τ rotating the plane clock wise then Shear value is taken as +ve & vice versa.
- Draw τ on point A

Now draw plot the value of τ by drawing \perp^r to A & B as shown in fig

5) Join the line CD intersecting σ -Axis on E as shown in fig.

6) Draw the circle with E as centre & EC as radius, so that it cut the τ -axis at F & G as shown in graph.

OF = Major principal stress
OG = minor principal stress.

7) Draw the \perp^r to σ -Axis from the centre of circle E to meet the circle at H,
EH = Max. Shear Stress.

~~8) Now observe that the inclined plane~~
8) ~~Observe whether~~ If the plane inclined plane making an angle θ with σ -plane OR τ -plane ~~Draw the~~ ^{with} either clock wise OR anticlockwise in element.

~~If the inclined~~
Now draw the line making angle ' 2θ ' with ~~either a~~ plane to meet the circle at I, as shown

9) Draw \perp^r line ~~to~~ from σ -Axis to meet I.
OJ is normal stress on inclined plane
OIJ is Tangential stress OR Shear stress on Inclined plane.

Module: 2

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Thin and thick cylinder

Curved structural forms like pipes, boilers, fluid storage tanks are referred as cylinders and they exhibit greater strength by virtue of shape rather than material used.

The vessels such as boilers, compressed air receivers etc. are of cylindrical and spherical forms. These vessels are generally used for storing fluids [liquid or gas] under pressure. The walls of such vessels are thin as compared to their diameters.

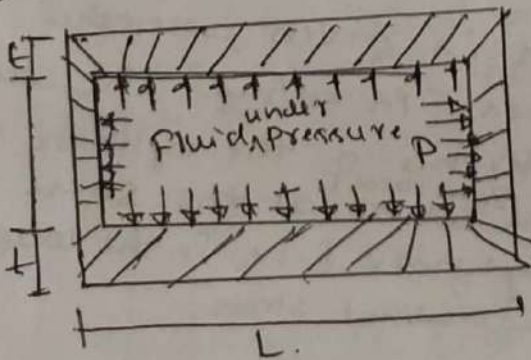
Thin cylinder: If the thickness of the wall of the cylindrical vessel is less than $\frac{1}{15}$ to $\frac{1}{20}$ of its internal diameter, the cylindrical vessel is known as a thin cylinder.

In case of thin cylinders, the stress distribution is assumed uniform over the thickness of the wall.

1. The thin cylinders are subjected to circumferential stress, sometimes they are referred as Hoop stress (σ_r)
2. Longitudinal stress (σ_l) acting longitudinally [Radial stress are neglected in case of thin cylinder]
Ex: cycle tube.

Thin cylindrical vessel subjected to internal pressure:

Fig shows a thin cylindrical vessel in which a fluid under pressure is stored.



Let d = Internal diameter of the thin cylinder
 t = thickness of the wall of the cylinder
 P = Internal pressure of the fluid.
 L = length of the cylinder.

On the account of Internal pressure P , the cylindrical vessel may fail by splitting up in any one of the two ways as shown in fig (a) & (b).

The forces due to pressure of the fluid acting vertically upwards and downwards on the thin cylinder, tend to burst the cylinder as shown in fig (a).

The forces due to pressure of the fluid, acting at the ends of the thin cylinder, tends to burst the thin cylinder as shown in fig (b).

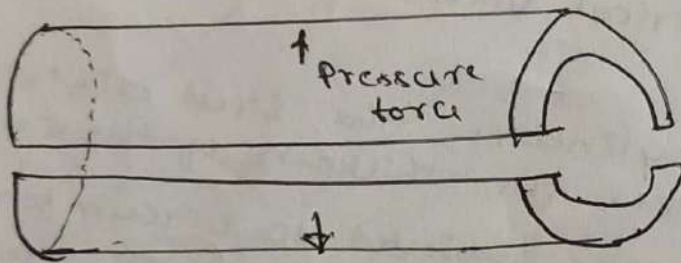


fig (a)

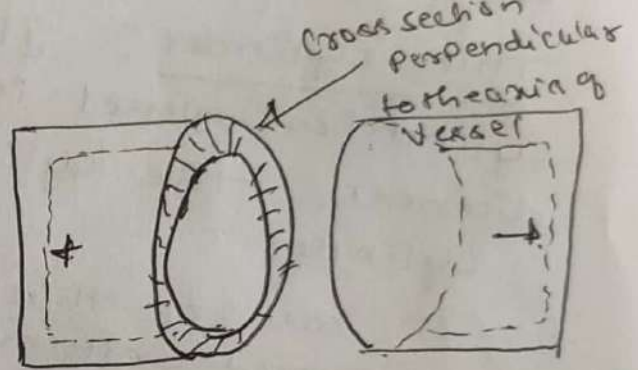


fig (b)

Stresses in a thin cylindrical vessel subjected to Internal Pressure?

When a thin cylindrical vessel is subjected to internal fluid pressure, the stresses in the wall of the cylinder on the cross section along the axis and on the cross section perpendicular to the axis are set up.

- These stresses are tensile and are known as:
1. Circumferential stress [or hoop stress] and [a]
 2. Longitudinal stress. (fig b)

The name of the stress is given according to the direction in which the stress is acting. The stress acting along the circumference of the cylinder is called circumferential stress, ^[hoop stress] where at the stress acting along the length of the cylinder [i.e. in the longitudinal direction] is known as longitudinal stress.

Expression for Circumferential Stress (σ_1) [Hoop stress] (2)

Consider a thin cylindrical vessel subjected to an internal fluid pressure. The circumferential stress will be set up in the material of the cylinder. If the bursting of the cylinder takes place as shown in fig (a).

The expression for hoop stress (σ_1) circumferential stress is obtained as ^{given} below

Let P = Internal Pressure of fluid.

d = Internal diameter of the cylinder

t = Thickness of the wall of the cylinder.

σ_1 = Circumferential (σ_1) hoop stress in the material

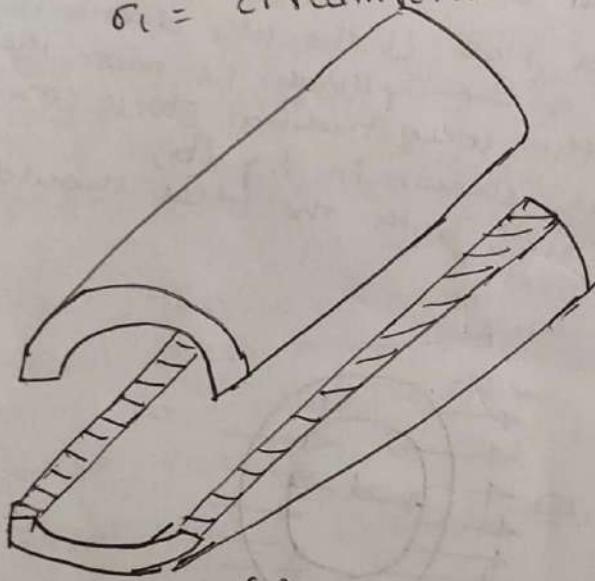


fig (a)

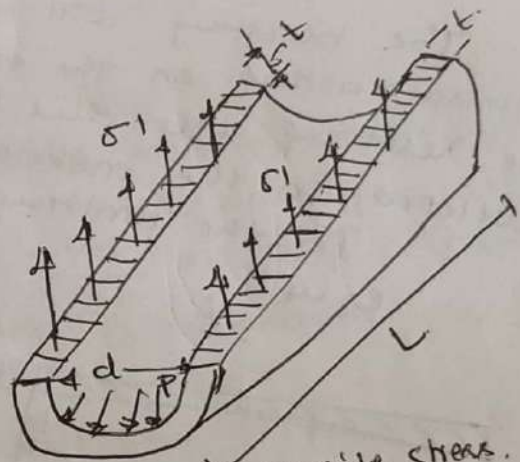


fig (b)

Tensile stress.

The bursting will take place if the force due to fluid pressure is more than the resisting force due to circumferential stress set up in the material.

In the limiting case, the two forces should be equal

$$\text{Force due to fluid pressure} = P \times \text{Area on which } P \text{ is acting} \rightarrow (1)$$

$$\therefore [P \text{ is acting on projected area } d \times L]$$

$$\text{Force due to circumferential stress} = \sigma_1 \times \text{Area on which } \sigma_1 \text{ is acting}$$

$$= \sigma_1 \times [L \times t + L \times t]$$

$$= \sigma_1 \times 2Lt = 2\sigma_1 Lt \rightarrow (2)$$

Equating (1) & (2) we get

$$P \times [d \times L] = 2\sigma_1 \times Lt$$

$$\frac{PdL}{Lt} = 2\sigma_1$$

$$\sigma_1 = \frac{Pd}{2t}$$

(3)

Expression for longitudinal stress:

Consider a thin cylindrical vessel subjected to internal fluid pressure. The longitudinal stress will be set up in the material of the cylinder. If the bursting of the cylinder takes place along the section A-B of fig (a).

The longitudinal stress σ_2 developed in the material is obtained as

Let P = Internal pressure of fluid stored in thin cylinder.

d = Internal diameter of cylinder.

t = thickness of cylinder.

σ_2 = longitudinal stress in the material.

The bursting will take place if the force due to fluid pressure acting on the end of the cylinder is more than the resisting force due to the longitudinal stress (σ_2) developed in the material as shown in fig. (b).

In the limiting case, both the forces should be equal.

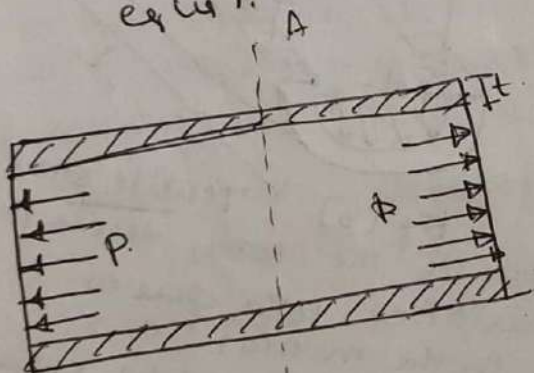


fig (a)

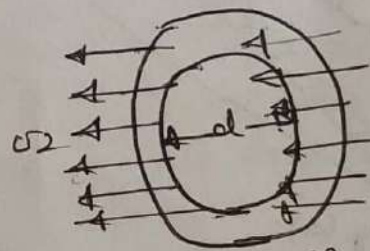


fig (b)

longitudinal stress (σ_2) develop.

Force due to fluid pressure = $P \times \text{Area on which it is acting}$

(Applied force) $P \times \frac{\pi d^2}{4}$

$$= P \times \frac{\pi d^2}{4}$$

Resisting force = $\sigma_2 \times \text{Area on which } \sigma_2 \text{ is acting}$

(Stress \times Area)

$$= \sigma_2 \times \pi \cdot d \cdot t$$

\therefore Hence in limiting case.

Force due to fluid pressure = Resisting force.

$$P \times \frac{\pi d^2}{4} = \sigma_2 \times \pi \cdot d \cdot t$$

$$\sigma_2 = \frac{P \times \frac{\pi d^2}{4}}{\pi \cdot d \cdot t} = \frac{Pd}{4t}$$

The stress σ_2 is tensile.

The equation σ_2 can be written as

$$\sigma_2 = \frac{P \cdot d}{2 \cdot 2t}$$

where $\sigma_1 = \frac{Pd}{2t}$

$$\sigma_2 = \frac{1}{2} \sigma_1$$

longitudinal stress = $\frac{1}{2}$ of circumferential stress [Hoop Stress]

This means $\sigma_1 = 2\sigma_2$

Hence in the material of the cylinder the permissible stress should be less than the circumferential stress. In other words, the circumferential stress should not be greater than the permissible stress.

Maximum shear stress:

At any point in the material of the cylindrical shell, there are two principal stresses, namely a circumferential stress of magnitude $\sigma_1 = \frac{Pd}{2t}$ acting circumferentially and a longitudinal stress of magnitude $\sigma_2 = \frac{Pd}{4t}$ acting parallel to the axis of the shell.

These two stresses are tensile and perpendicular to each other

\therefore Maximum shear stress $\tau_{max} = \frac{\sigma_1 - \sigma_2}{2}$

$$\tau_{max} = \frac{\frac{Pd}{2t} - \frac{Pd}{4t}}{2}$$

$$= \frac{2Pd - Pd}{8t} = \frac{Pd}{8t}$$

$$\tau_{max} = \frac{Pd}{8t}$$

Problems on thin cylinders

- ① A cylindrical pipe of diameter 1.5 m and thickness 1.5 cm is subjected to an internal fluid pressure of 1.2 N/mm². Determine
- ① longitudinal stress developed in the pipe
 - ② circumferential stress developed in the pipe

∴ given $d = 1.5 \text{ m}$ $t = 1.5 \text{ cm} = 1.5 \times 10^{-2} \text{ m}$

$P = 1.2 \text{ N/mm}^2$

As the ratio $\frac{t}{d} = \frac{1.5 \times 10^{-2}}{1.5} = 0.01 < \frac{1}{20} = \frac{1}{20}$ which is less than $\frac{1}{20}$, hence this is a case of thin cylinder

① longitudinal stress $\sigma_2 = \frac{Pd}{4t}$

$\sigma_2 = \frac{Pd}{4t} = \frac{1.2 \times 1.5}{4 \times 1.5 \times 10^{-2}} = \frac{1.8}{0.06} = 30 \text{ N/mm}^2$

$\sigma_2 = \frac{1.2 \times 1.5}{4 \times 1.5 \times 10^{-2}}$

$\sigma_2 = 30 \text{ N/mm}^2$

② circumferential stress $\sigma_1 = \frac{Pd}{2t}$

$\sigma_1 = \frac{1.2 \times 1.5}{2 \times 1.5 \times 10^{-2}}$

$\sigma_1 = 60 \text{ N/mm}^2$

- ② A cylinder of internal diameter 2.5 m & of thickness 5 cm contains a gas. If the tensile stress in the material is not exceed 80 N/mm², determine the internal pressure of the gas.

∴ $d = 2.5 \text{ m}$ $t = 5 \text{ cm} = 5 \times 10^{-2} \text{ m}$

Maximum permissible stress $\sigma_1 = 80 \text{ N/mm}^2$.

P. Internal pressure = ?

$\sigma_1 = \frac{P \cdot d}{2t}$

$\frac{\sigma_1 \cdot 2t}{d} = P$

$\frac{80 \times 2 \times 5 \times 10^{-2}}{2.5} = P$

$P = 3.2 \text{ N/mm}^2$

③ A thin cylinder of internal diameter 1.25m contains a fluid at an internal pressure of 2N/mm². Determine the maximum thickness of the cylinder if.

① The longitudinal stress is not to exceed 30N/mm².

② The circumferential stress is not to exceed 45N/mm².

Given Data $D = 1.25\text{m}$ $P = 2\text{N/mm}^2$ $t = ?$

Longitudinal stress $\sigma_2 = 30\text{N/mm}^2$

Circumferential stress $\sigma_1 = 45\text{N/mm}^2$

$$\textcircled{1} \quad \sigma_2 = \frac{P \cdot d}{4t} = 30 = \frac{2 \times 1.25}{4t}$$

$$t = \frac{2 \times 1.25}{4 \times 30}$$

$$t = 0.020\text{m}$$

② Circumferential stress σ_1

$$\sigma_1 = \frac{P \cdot d}{2t} \quad 45 = \frac{2 \times 1.25}{2t}$$

$$t = \frac{2 \times 1.25}{45 \times 2}$$

$$t = 0.027\text{m}$$

The longitudinal (a) circumferential stresses induced in the material are inversely proportional to the thickness (t) of the cylinder.

Hence the stress induced will be less if the value of t is more. Hence the maximum value of t is

$$t = 0.027\text{m} = 2.77\text{cm}$$

Effect of Internal Pressure on the Dimensions of a thin cylindrical shell.

When a fluid having internal pressure (P) is stored in a thin cylindrical shell, due to internal pressure of the fluid the stresses set up at any point of the material of the shell are

- (1) Hoop stress (σ_1) acting on longitudinal section.
- (2) Longitudinal stress (σ_2) acting on the circumferential section.

These stresses are principal stresses, as they are acting on the principal planes. The stress in the third principal plane is zero at the thickness (t) of the cylinder is very small. Actually the third stress in the third principal plane is radial stress which is very small for thin cylinders & can be neglected.

Let

- P = Internal pressure of fluid,
- L = length of cylindrical shell.
- d = diameter of the cylindrical shell.
- t = thickness of the cylindrical shell.
- E = modulus of elasticity for the material of the shell.
- σ_1 = Hoop stress or circumferential stress in the material.
- σ_2 = Longitudinal stress in the material.
- μ = Poisson ratio.
- δd = change in diameter due to the stress in the material.
- δL = change in length.
- δV = change in volume.

The values of σ_1 & σ_2 are given

$$\sigma_1 = \frac{Pd}{2t} \quad \text{and} \quad \sigma_2 = \frac{Pd}{4t}$$

Let e_1 & e_2 are ~~the~~ circumferential strain & longitudinal strain

Then Circumferential strain

$$e_1 = \frac{\sigma_1}{E_1} - \mu \frac{\sigma_2}{E_2}$$

Substituting the values of σ_1 & σ_2

$$e_1 = \frac{Pd}{2t} - \frac{\mu Pd}{4t}$$

where $E = E_1 = E_2$

$$e_1 = \frac{Pd}{2tE} - \frac{\mu Pd}{4tE}$$

$$e_1 = \frac{Pd}{2tE} \left[1 - \frac{\mu}{2} \right]$$

Circumferential strain

→ (a)

Longitudinal strain

$$e_2 = \frac{\sigma_2}{E} - \frac{\mu \sigma_1}{E} = \frac{Pd}{4tE} - \mu \frac{Pd}{2tE} = \frac{Pd}{4tE} - \frac{\mu Pd}{2tE}$$

$$e_2 = \frac{Pd}{2tE} \left[\frac{1}{2} - \mu \right]$$

$$e_2 = \frac{Pd}{2tE} \left[\frac{1}{2} - \mu \right]$$

→ (c)

→ Longitudinal strain

But circumferential strain is also given as

$$e_1 = \frac{\text{change in circumferential stress due to pressure}}{\text{original circumference}}$$

$$= \frac{\text{Final circumference} - \text{original circumference}}{\text{original circumference}}$$

$$= \frac{\pi(d + \delta d) - \pi d}{\pi d} = \frac{\pi d + \pi \delta d - \pi d}{\pi d} = \frac{\pi \delta d}{\pi d}$$

$$e_1 = \frac{\delta d}{d} \left[\text{or} = \frac{\text{change in diameter}}{\text{original diameter}} \right]$$

Equating the two values of e_1 is given by equation (a) & (b)

$$\frac{\delta d}{d} = \frac{Pd}{2tE} \left[1 - \frac{\mu}{2} \right] \Rightarrow \frac{\delta d}{d} = \frac{Pd^2}{2tE} \left[1 - \frac{\mu}{2} \right]$$

Similarly longitudinal strain is also given as

$$e_2 = \frac{\text{change in length due pressure}}{\text{original length}} = \frac{\sigma L}{L} \rightarrow (d)$$

Equating the two values of e_2 is given by equation (c) & (d)

$$\frac{\sigma L}{L} = \frac{Pd}{2tE} \left[\frac{1}{2} - \mu \right] \Rightarrow \sigma L = \frac{PdL}{2tE} \left[\frac{1}{2} - \mu \right]$$

change in length.

(9)

Change in volume of a thin cylinder:

Volumetric strain: It is defined as change in volume divided by original volume

$$\text{Volumetric strain} = \frac{\delta V}{V}$$

But change in volume $[\delta V] = \text{Final volume} - \text{Original volume}$

$$\begin{aligned} \text{Original volume } [V] &= \text{Area of the cylindrical shell} \times \text{Length} \\ &= \frac{\pi}{4} d^2 \times L \end{aligned}$$

$$\text{Final volume} = \left[\text{Final Area of the cross section} \right] \times \text{Final length}$$

$$= \frac{\pi}{4} (d + \delta d)^2 \times [L + \delta L]$$

$$= \frac{\pi}{4} [d^2 + (\delta d)^2 + 2d \cdot \delta d] \times [L + \delta L]$$

$$= \frac{\pi}{4} [d^2 L + (\delta d)^2 L + 2d \cdot \delta d \cdot L + \delta L d^2 + \delta L (\delta d)^2 + 2d \cdot \delta d \cdot \delta L]$$

Neglecting the smaller quantities such as $(\delta d)^2 L$, $\delta L (\delta d)^2$ and $2d \delta d \delta L$. we get

$$= \frac{\pi}{4} [d^2 L + 2d \cdot \delta d \cdot L + \delta L d^2] \quad \checkmark$$

$$\text{Final volume} = \frac{\pi}{4} [d^2 L + 2d \cdot \delta d \cdot L + \delta L d^2]$$

Change in volume (δV)

$$\delta V = \frac{\pi}{4} [d^2 L + 2d \cdot \delta d \cdot L + \delta L d^2] - \frac{\pi}{4} d^2 \times L$$

$$= \frac{\pi}{4} [2dL \cdot \delta d + \delta L \cdot d^2]$$

$$\text{Volumetric Strain} = \frac{\delta V}{V} = \frac{\frac{\pi}{4} [2dL \cdot \delta d + \delta L \cdot d^2]}{\frac{\pi}{4} d^2 \times L}$$

$$= \frac{2 \cdot \delta d}{d} + \frac{\delta L}{L}$$

$$= 2e_1 + e_2$$

Where circumferential strain

$$e_1 = \frac{\sigma_1}{E} - \mu \frac{\sigma_2}{E}$$

$$= \frac{Pd}{2t} \frac{1}{E} - \mu \frac{Pd}{4t} \frac{1}{E}$$

$$e_1 = \frac{Pd}{2tE} \left[1 - \frac{\mu}{2} \right]$$

$$\left[\sigma_1 = \frac{Pd}{2t} \right.$$

$$\left. \sigma_2 = \frac{Pd}{4t} \right]$$

& longitudinal strain

$$e_2 = \frac{\sigma_2}{E} - \mu \frac{\sigma_1}{E}$$

$$= \frac{Pd}{4t} \frac{1}{E} - \mu \frac{Pd}{2t} \frac{1}{E}$$

$$= \frac{Pd}{4tE} - \frac{\mu Pd}{2tE}$$

$$e_2 = \frac{Pd}{4tE} \left[\frac{1}{2} - \mu \right]$$

$$\therefore \Delta V = 2e_1 + e_2$$

$$V = 2 \frac{Pd}{2tE} \left[1 - \frac{\mu}{2} \right] + \frac{Pd}{4tE} \left[\frac{1}{2} - \mu \right]$$

$$= \frac{Pd}{2tE} \left[2 - \frac{2\mu}{2} + \frac{1}{2} - \mu \right]$$

$$= \frac{Pd}{2tE} \left[2 + \frac{1}{2} - \mu - \mu \right]$$

$$= \frac{Pd}{2tE} \left[2 + \frac{1}{2} - \mu - \mu \right]$$

$$= \frac{Pd}{2tE} \left[\frac{5}{2} - 2\mu \right]$$

$$V = \frac{Pd}{4tE} \left[5 - 2\mu \right]$$

Also change in
Volume (ΔV) = $V(2e_1 + e_2)$

① A thin cylindrical shell 1m in diameter & 3m long has a metal thickness of 10mm. If it is subjected to an internal pressure of 3MPa. Determine hoop stress, longitudinal stress, change in length, change in diameter and change in volume. (δL , δd , δV) if $E = 210 \text{ GPa}$, $\mu = 0.3$.

u) $d = 1000 \text{ mm}$ $L = 3 \text{ m} = 3000 \text{ mm}$ $t = 10 \text{ mm}$, $P = 3 \text{ N/mm}^2$

① hoop stress $\sigma_1 = \frac{Pd}{2t}$ $\sigma_1 = \frac{3 \times 1000}{2 \times 10}$

$\sigma_1 = 150 \text{ MPa}$ ① 150 N/mm^2

② longitudinal stress $\sigma_2 = \frac{Pd}{4t} \Rightarrow \frac{\sigma_1}{2} = \frac{150}{2}$

$\sigma_2 = 75 \text{ N/mm}^2$ ② 75 MPa

③ change in length is given by

$$\delta L = \frac{PdL}{2tE} \left[\frac{1}{2} - \mu \right] = \frac{3 \times 1000 \times 3000}{2 \times 10 \times 210 \times 10^3} \left[\frac{1}{2} - 0.3 \right]$$

$$\delta L = 2.1428 (0.5 - 0.3)$$

$\delta L = 0.428 \text{ mm}$

④ change in diameter (d) is given by

$$\delta d = \frac{Pd^2}{2tE} \left[1 - \frac{\mu}{2} \right] = \frac{3 \times 1000^2}{2 \times 10 \times 210 \times 10^3} \left[1 - \frac{0.3}{2} \right]$$

$\delta d = 0.607 \text{ mm}$

⑤ change in volume (δV) is given by equation

$$V = \frac{\pi d^2 L}{4} \quad \delta V = V [2e_1 + e_2] \quad \therefore [e_1 = \frac{\delta d}{d}, e_2 = \frac{\delta L}{L}]$$

$$\delta V = V \left[2 \times \frac{\delta d}{d} + \frac{\delta L}{L} \right]$$

Substituting the values of δd , δL , and V

$$\delta V = V \left[2 \times \frac{0.607}{1000} + \frac{0.428}{3000} \right] =$$

$\delta V = 1.35 \times 10^{-3} V$

$$V = \text{original volume} = \frac{\pi}{4} d^2 L = \frac{\pi}{4} \times 1000^2 \times 3000$$

$$\delta V = 1.35 \times 10^{-3} \times 2.356 \times 10^9$$

$V = 2.356 \times 10^9 \text{ mm}^3$

$\delta V = 3.1806 \times 10^6 \text{ mm}^3$

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- Q) Calculate (1) the change in diameter, (2) change in length & subjected to internal pressure of 30 N/mm^2 . Take the value of (3) change in volume of a thin cylindrical shell 100 cm diameter, 1 cm thick and 5 m long when subjected to internal pressure of 30 N/mm^2 . Take the value of $E = 2 \times 10^5 \text{ N/mm}^2$ and poisson's ratio $\mu = 0.3$.

As given

Diameter of shell $d = 100 \text{ cm}$
 Thickness of shell $t = 1 \text{ cm}$
 Length of shell $L = 5 \text{ m} = 5 \times 100 = 500 \text{ cm}$
 Internal pressure $P = 30 \text{ N/mm}^2$
 $E = 2 \times 10^5 \text{ N/mm}^2$
 $\mu = 0.3$

(1) Change in diameter (δd) is given by equation

$$\delta d = \frac{Pd^2}{2tE} \left[1 - \frac{\mu}{2} \right]$$

$$\delta d = \frac{3 \times 100^2}{2 \times 1 \times 2 \times 10^5} \left[1 - \frac{0.3}{2} \right]$$

$$\delta d = \frac{2 \times 1 \times 2 \times 10^5}{7.5 \times 10^4} (0.85)$$

$$\delta d = 6.375 \times 10^{-4} \times 0.85$$

$$\delta d = 0.06375 \text{ cm}$$

(2) Change in length (δL) is given by equation

$$\delta L = \frac{PdL}{2tE} \left[\frac{1}{2} - \mu \right] =$$

$$\delta L = \frac{3 \times 100 \times 500}{2 \times 1 \times 2 \times 10^5} \left[\frac{1}{2} - 0.3 \right]$$

$$\delta L = 0.375 (0.2)$$

$$\delta L = 0.075 \text{ cm}$$

(3) Change in volume (δV) is given by equation.

$$\delta V = V (2e_1 + e_2)$$

$$= V \left(2 \cdot \frac{\delta d}{d} + \frac{\delta L}{L} \right)$$

Substituting the values $\frac{\delta d}{d}$, $\frac{\delta L}{L}$, E & d we get

$$\delta V = V \left[2 \times \frac{0.06375}{100} + \frac{0.075}{500} \right]$$

$$\delta V = 1.425 \times 10^3 \text{ V}$$

$$V = \text{original volume} = \frac{\pi}{4} d^2 x L$$

$$V = \frac{\pi}{4} \times 100^2 \times 500$$

$$V = 3.9269 \times 10^6 \text{ cm}^3$$

$$\delta V = 1.425 \times 10^{-3} \times V$$

$$= 1.425 \times 10^{-3} \times 3.9269 \times 10^6$$

$$\delta V = 5595.96 \text{ cm}^3$$

18 CHAPTER

THICK CYLINDERS AND SPHERES

18.1. INTRODUCTION

In the last chapter, we have mentioned that if the ratio of thickness to internal diameter of a cylindrical shell is less than about $1/20$, the cylindrical shell is known as thin cylinders. For them it may be assumed with reasonable accuracy that the hoop and longitudinal stresses are constant over the thickness and the radial stress is small and can be neglected. If the ratio of thickness to internal diameter is more than $1/20$, then cylindrical shell is known as thick cylinders.

The hoop stress in case of a thick cylinder will not be uniform across the thickness. Actually the hoop stress will vary from a maximum value at the inner circumference to a minimum value at the outer circumference.

18.2. STRESSES IN A THICK CYLINDRICAL SHELL

Fig. 18.1 (a) shows a thick cylinder subjected to an internal fluid pressure.

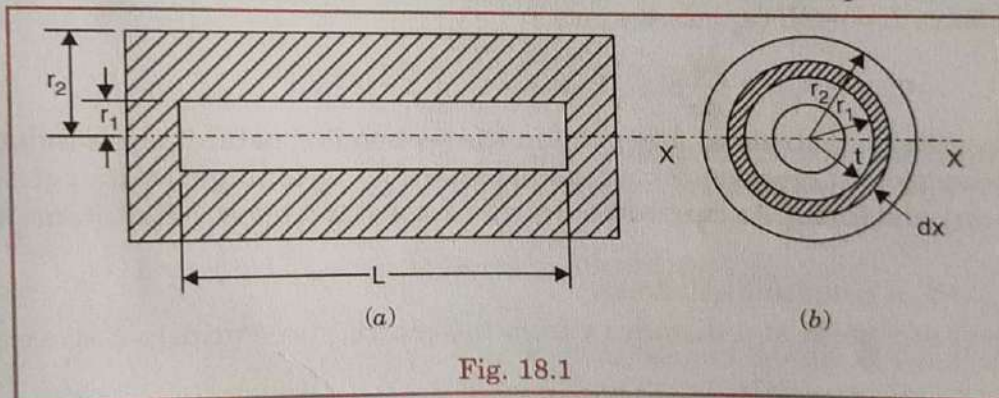


Fig. 18.1

Let r_2 = External radius of the cylinder,
 r_1 = Internal radius of the cylinder, and
 L = Length of cylinder.

Consider an elementary ring of the cylinder of radius x and thickness dx as shown in Figs. 18.1 (b) and 18.2.

Let p_x = Radial pressure on the inner surface of the ring
 $p_x + dp_x$ = Radial pressure on the outer surface of the ring
 σ_x = Hoop stress induced in the ring.

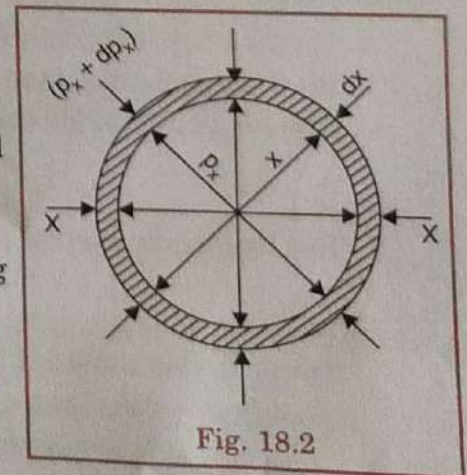


Fig. 18.2

STRENGTH OF MATERIALS

Take a longitudinal section $x-x$ and consider the equilibrium of half of the ring as shown in Fig. 18.2 (a) or in Fig. 18.2 (b).

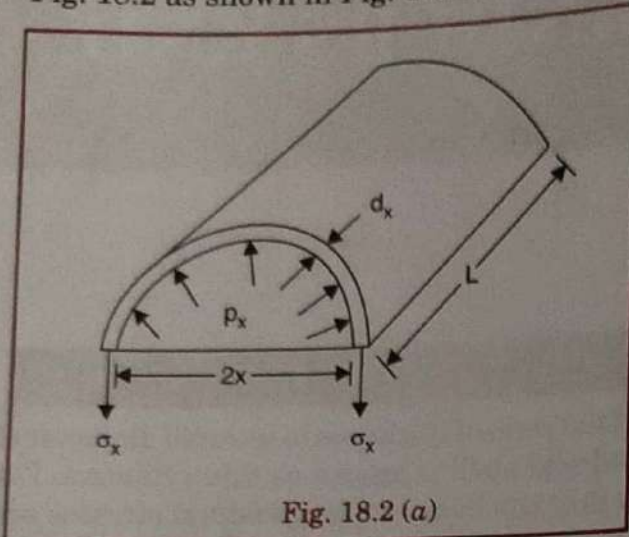


Fig. 18.2 (a)

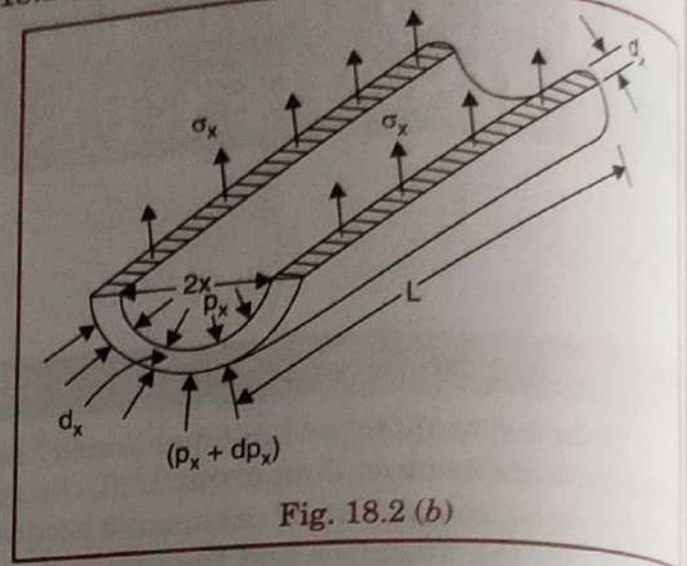


Fig. 18.2 (b)

Bursting force

$$\begin{aligned}
 &= p_x (2xL) - (p_x + dp_x) \times 2(x + dx) \cdot L \\
 &= 2L [p_x \cdot x - (p_x \cdot x + p_x \cdot dx + x \cdot dp_x + dp_x \cdot dx)] \\
 &= 2L [-p_x \cdot dx - x \cdot dp_x] \quad \text{(Neglecting } dp_x \cdot dx \text{ which is a small quantity)} \\
 &= -2L (p_x dx + x \cdot dp_x)
 \end{aligned}$$

Resisting force = Hoop stress \times Area on which it acts = $\sigma_x \times 2dx \cdot L$

Equating the resisting force to the bursting force, we get

$$\sigma_x \times 2dx \cdot L = -2L (p_x \cdot dx + x \cdot dp_x)$$

or
$$\sigma_x = -p_x - x \frac{dp_x}{dx}$$

The longitudinal strain at any point in the section is constant and is independent of the radius. This means that cross-sections remain plane after straining and this is true for sections remote from any end fixing. As longitudinal strain is constant, hence longitudinal stress will also be constant.

Let $\sigma_2 =$ Longitudinal stress.

Hence at any point at a distance x from the centre, three principal stresses are acting. They are :

- (i) the radial compressive stress, p_x
- (ii) the hoop (or circumferential) tensile stress, σ_x
- (iii) the longitudinal tensile stress σ_2 .

The longitudinal strain (e_2) at this point is given by,

$$e_2 = \frac{\sigma_2}{E} - \frac{\mu\sigma_x}{E} + \frac{\mu p_x}{E}$$

But longitudinal strain is constant.

$$\therefore \frac{\sigma_2}{E} - \frac{\mu\sigma_x}{E} + \frac{\mu p_x}{E} = \text{constant}$$

But σ_2 is also constant, and for the material of the cylinder E and μ are constant.

$$\begin{aligned}
 \therefore \sigma_x - p_x &= \text{constant} \\
 &= 2a \text{ where } a \text{ is constant}
 \end{aligned}$$

\therefore
Equating the two values of σ_x given by equations (iii) and (iv), we get

$$\sigma_x = p_x + 2a \quad \dots(iv)$$

$$p_x + 2a = -p_x - x \frac{dp_x}{dx}$$

or

$$x \cdot \frac{dp_x}{dx} = -p_x - p_x - 2a = -2p_x - 2a$$

or

$$\frac{dp_x}{dx} = -\frac{2p_x}{x} - \frac{2a}{x} = \frac{-2(p_x + a)}{x}$$

or

$$\frac{dp_x}{(p_x + a)} = -\frac{2dx}{x}$$

Integrating the above equation, we get

$$\log_e (p_x + a) = -2 \log_e x + \log_e b$$

where $\log_e b$ is a constant of integration.

The above equation can also be written as

$$\begin{aligned} \log_e (p_x + a) &= -\log_e x^2 + \log_e b \\ &= \log_e \frac{b}{x^2} \end{aligned}$$

 \therefore

$$p_x + a = \frac{b}{x^2}$$

or

$$p_x = \frac{b}{x^2} - a \quad \dots(18.1)$$

Substituting the values of p_x in equation (iv), we get

$$\sigma_x = \frac{b}{x^2} - a + 2a = \frac{b}{x^2} + a \quad \dots(18.2)$$

Equation (18.1) gives the radial pressure p_x and equation (18.2) gives the hoop stress at any radius x . These two equations are called *Lame's equations*. The constants 'a' and 'b' are obtained from boundary conditions, which are :

- (i) at $x = r_1$, $p_x = p_0$ or the pressure of fluid inside the cylinder, and
- (ii) at $x = r_2$, $p_x = 0$ or atmosphere pressure.

After knowing the values of 'a' and 'b', the hoop stress can be calculated at any radius.

After knowing the values of 'a' and 'b', the hoop stress can be calculated at any radius. In case of thick

Thick cylinders problems

① Determine the maximum and minimum hoop stress across the section of a pipe of 400mm internal diameter and 10mm thick, when the pipe contains a fluid at a pressure of 8 N/mm². Also sketch the radial pressure distribution and hoop stress distribution across the section.

∴ Internal dia = $d_i = 400\text{mm}$ $t = 10\text{mm}$,
 Internal radius $r_1 = \frac{400}{2} = 200\text{mm}$
 External dia = $d_e = 400 + 10 + 10 = 600\text{mm}$
 External radius = $r_2 = \frac{600}{2} = 300\text{mm}$
 Fluid pressure $p_0 = 8\text{N/mm}^2$.
 at $x = r_1$, $p_r = p_0 = 8\text{N/mm}^2$.

The radial pressure (p_r) is given by

$$p_r = \frac{b}{x^2} - a \quad \rightarrow \textcircled{1}$$

Now apply the boundary conditions to the above equation.
 The boundary conditions are,

① At $x = r_1 = 200\text{mm}$, $p_r = 8\text{N/mm}^2$

② At $x = r_2 = 300\text{mm}$, $p_r = 0$.

Substituting these boundary conditions in equation ①

We get

$$\textcircled{1} \quad 8 = \frac{b}{200^2} - a = \frac{b}{40000} - a \quad \rightarrow \textcircled{2}$$

$$\textcircled{2} \quad 0 = \frac{b}{300^2} - a = \frac{b}{90000} - a \quad \rightarrow \textcircled{iii}$$

and subtracting equation ③ from equation ② we get

$$8 = \frac{b}{40000} - \frac{b}{90000} = \frac{9b - 4b}{360000} = \frac{5b}{360000}$$

$$b = \frac{360000 \times 8}{5} = 576000$$

Substituting this value in equation (ii) we get

$$0 = \frac{576000}{90000} - a$$

$$a = 6.4$$

The value of a and b are ~~now~~ substituted in the hoop stress:

Now hoop stress at any radius r is given by equation,

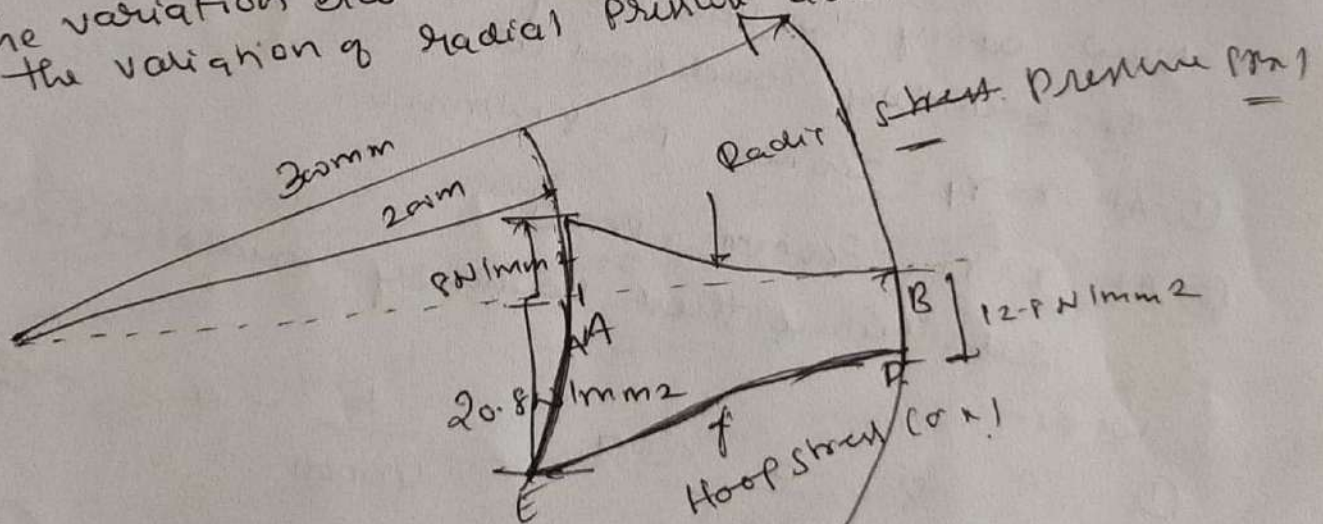
$$\sigma_r = \frac{b}{r^2} + a$$

$$\sigma_r = \frac{576000}{r^2} + 6.4$$

At $r = 20 \text{ mm}$, $\sigma_{20} = \frac{576000}{20^2} + 6.4 = 14.4 + 6.4 = 20.8 \text{ N/mm}^2$

At $r = 30 \text{ mm}$, $\sigma_{30} = \frac{576000}{30^2} + 6.4 = 6.4 + 6.4 = 12.8 \text{ N/mm}^2$

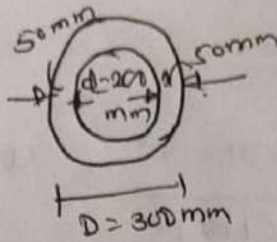
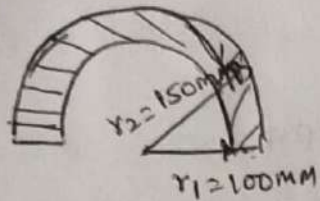
The radial pressure distribution and hoop stress distribution across the section. AB is taken a horizontal line. AC = $P \text{ N/mm}^2$. The variation b/w B and C is parabolic. The curve BC shows the variation of radial pressure across AB.



The curve DE which is also parabolic, $BD = 12.8 \text{ N/mm}^2$, $AE = 20.8 \text{ N/mm}^2$.

The radial pressure is compressive whereas the hoop stress is tensile.

A thick pipe of outer dia 300mm where thickness of metal has 50mm is subjected to an internal fluid pressure of 40MPa and an external pressure of 2.5MPa. find maximum & minimum circumferential & radial stress in pipe thickness and plot stress variation.



$$p_r = \frac{b}{r^2} - a$$

$$\sigma_r = \frac{b}{r^2} + a$$

$d_o = 300\text{mm}$
 $r_o = 150\text{mm}$
 $t = 50\text{mm}$
 $d_i = 200\text{mm}$
 $r_i = 100\text{mm}$

Case ① $p_r = 40\text{ MPa}$ at $r_i = 100\text{mm}$

② $p_r = 2.5\text{ MPa}$ at $r_o = 150\text{mm}$.

Case ① $p_r = -a + \frac{b}{r^2}$

$$40 = -a + \frac{b}{(100)^2}$$

$$40 = -a + \frac{b}{10000}$$

$$400000 = \frac{-10000a + b}{1} \quad \text{--- (1)}$$

$$-10000a + b - 400000 = 0$$

Case ② $2.5 = -a + \frac{b}{150^2}$

$$2.5 = -a + \frac{b}{22500}$$

$$56250 = -22500a + b \quad \text{--- (2)}$$

$$-22500a + b - 56250 = 0 \quad \text{--- (2)}$$

Solving (1) & (2)

Let $a = 27.5$ $b = 6.75 \times 10^5$

To find Hoop stresses

$$\sigma_r]_{r=100} = a + \frac{b}{r^2} = 27.5 + \frac{6.75 \times 10^5}{100^2} = 95 \text{ MPa}$$

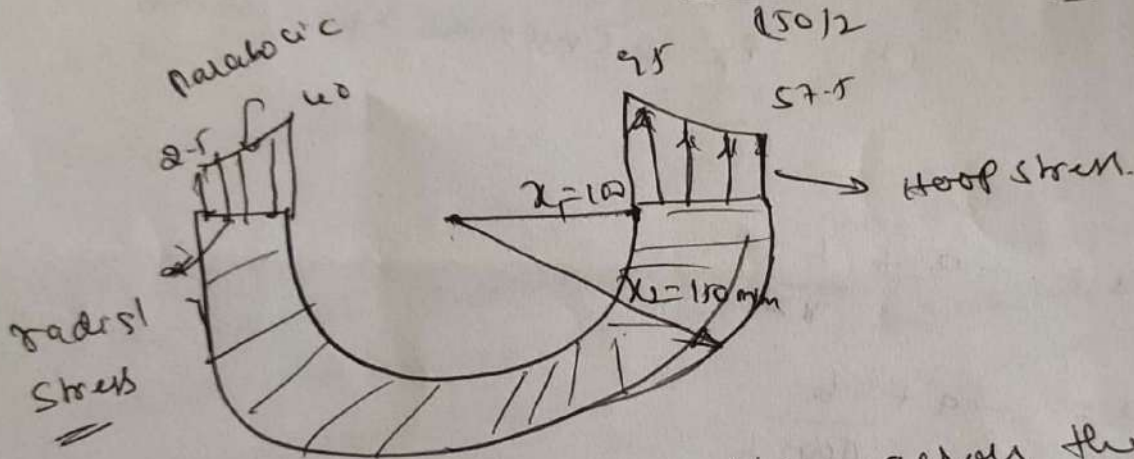
$$\sigma_r]_{r=150} = a + \frac{b}{r^2} = 27.5 + \frac{6.75 \times 10^5}{(150)^2} = 57.5 \text{ MPa}$$

Radial stress

$$p_r = -a + \frac{b}{r^2}$$

$$p_r]_{r=100} = -27.5 + \frac{6.75 \times 10^5}{(100)^2} = 40 \text{ MPa}$$

$$p_r]_{r=150} = -27.5 + \frac{6.75 \times 10^5}{(150)^2} = 2.5 \text{ MPa}$$



Stress variation across the thickness

Note:

For design of thick cylinders, always hoop stress (σ_r) at inner surface is considered.

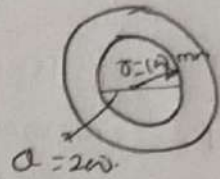
As the maximum stress is absorbed at the inner surface it is also equated to allowable stress in tension.

A thick cylinder of internal dia 200mm. is subjected to an internal fluid pressure of 40MPa. if the allowable stress in tension for the material is 120MPa. find the thickness required.

→ $d_i = 200\text{mm}$ $r_i = 100\text{mm}$

$p_r = 40\text{MPa}$ at $r = 100$

$\sigma_r = 120\text{MPa}$ at $r = 100$



$$\sigma_r = a + \frac{b}{r^2}$$

$$p_r = -a + \frac{b}{r^2}$$

$$120 = a + \frac{b}{100^2}$$

$$40 = -a + \frac{b}{100^2}$$

$$120 = a + \frac{b}{10000}$$

$$40 = -a + \frac{b}{100^2}$$

$$120 \times 10^4 = 10000a + b \quad \text{--- (1)} \quad 40 \times 10^4 = -10000a + b \quad \text{--- (2)}$$

$$10000a + b = 120 \times 10^4 \quad \text{--- (1)}$$

$$-10000a + b = 40 \times 10^4 \quad \text{--- (2)}$$

$$a = 40$$

$$b = 8 \times 10^5$$

let t be the thickness of cylinder.

given. $p_r = 0 = -a + \frac{b}{r^2}$ at $(r = (100 + t))$

$$0 = -40 + \frac{8 \times 10^5}{(100 + t)^2}$$

$$0 = -40(100 + t)^2 + 8 \times 10^5$$

$$-4 \times 10^5 - 40t^2 - 8 \times 10^3 t + 8 \times 10^5 = 0$$

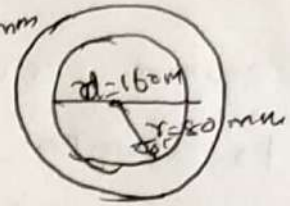
$$4 \times 10^5 - 40t^2 - 8 \times 10^3 t = 0$$

$$t = 41.62\text{mm}$$

Find the thickness of metal necessary for a cylindrical shell of internal dia 160mm to withstand a pressure of 8 N/mm². The maximum hoop stress in the section is not to exceed 35 MPa.

us $t = ?$ $d_i = 160 \text{ mm}$ $r_i = 80 \text{ mm}$ $r_i = \frac{160}{2} = 80 \text{ mm}$

$P_x = 8 \text{ N/mm}^2$ ($P_m = 8 \text{ mm}$)
 $\sigma_x = 35 \text{ MPa}$ (hoop stress)



Case 1 $P_x = 8 \text{ N/mm}^2$ at $r = 80 \text{ mm}$

Case $\sigma_x = 35 \text{ N/mm}^2$ at $r = 80 \text{ mm}$

Case 2

$$\sigma_r = a + \frac{b}{r^2}$$

$$35 = a + \frac{b}{80^2}$$

$$22400 = 6400a + b$$

$$6400a + b = 22400 \quad \text{--- (1)}$$

Case 1

$$P_x = -a + \frac{b}{r^2}$$

$$8 = -a + \frac{b}{80^2}$$

$$51200 = -6400a + b$$

$$-6400a + b = 51200 \quad \text{--- (2)}$$

The maximum hoop stress σ_x at the inner radius of the shell

Let $r_2 =$ External radius.

The radial pressure and hoop stress at any radius r are given by equation

$$P_x = \frac{b}{r^2} - a \quad \text{--- (1)}$$

$$\sigma_x = \frac{b}{r^2} + a \quad \text{--- (2)}$$

Let us now apply the boundary conditions. The boundary conditions are at $r = 80 \text{ mm}$, $P_x = 8 \text{ N/mm}^2$, & $\sigma_x = 35 \text{ N/mm}^2$

Substituting the values

$$P_x = \frac{b}{r^2} - a \quad r = 80 \text{ mm}, P_x = 8 \text{ N/mm}^2 \text{ in eq (1)}$$

$$8 = -a + \frac{b}{80^2}$$

$$-6400a + b = 51200 \quad \text{--- (1)}$$

$$\sigma_x = 35 \text{ N/mm}^2 \text{ in eq (2)}$$

$$35 = \frac{b}{80^2} + a \quad \text{--- (2)}$$

$$\sigma_x = \frac{b}{r^2} + a$$

$$6400a + b = 22400 \quad \text{--- (3)}$$

$$6400a + b = 22400 \quad \text{--- (4)}$$

we get $27 = 29$
 $a = 13.5$

Substituting the value of a in equation (ii) we get

$$8 = \frac{b}{6400} - 13.5$$

$$b = (8 + 13.5) \times 6400$$

$$b = 21.5 \times 6400$$

$$\boxed{b = 137600}$$

Substituting the values of a & b in eq (i)

$$P_r = \frac{21.5 \times 6400}{r^2} - 13.5$$

But at the outer surface, the pressure is zero.
Hence at $r = r_2$, $P_r = 0$, substituting these values in the above equation we get

$$0 = \frac{21.5 \times 6400}{r_2^2} - 13.5$$

$$r_2^2 = \frac{21.5 \times 6400}{13.5}$$

$$r_2 = \sqrt{\frac{21.5 \times 6400}{13.5}} = 100.96 \text{ mm}$$

Thickness of shell $t = r_2 - r_1$

$$100.96 - 80$$

$$= \underline{20.96 \text{ mm}}$$

Module: 3.

Shear Force and Bending moment in Beams.

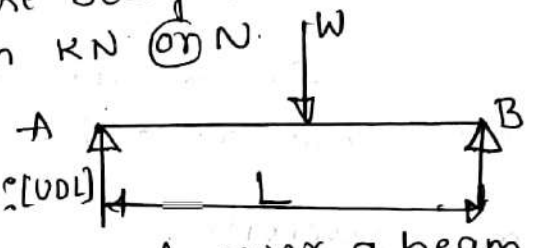
Introduction

types of loading, supports, and types of beams.

Loadings are classified based on the portion and type of which it is being applied on the body.

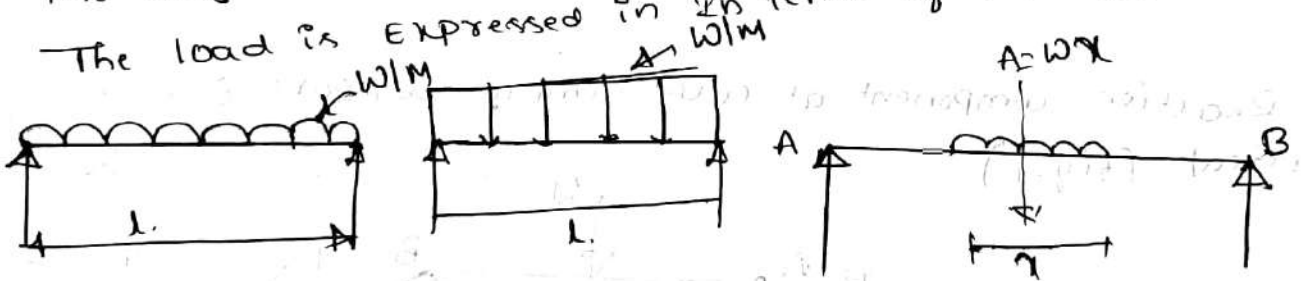
a) Point load (or) Concentrated load :

In this type, the total load is assumed to be concentrated at a point i.e. it is not covering any distance (or) area of the body on which it is being applied. The load is in KN (or) N.



b) Uniformly Distributed load [UDL]

It is the one which is spread over a beam in such a manner that rate of loading w is uniform along the length [i.e. each unit length is loaded to the same rate]. The load is expressed in terms of N/m (or) KN/m .



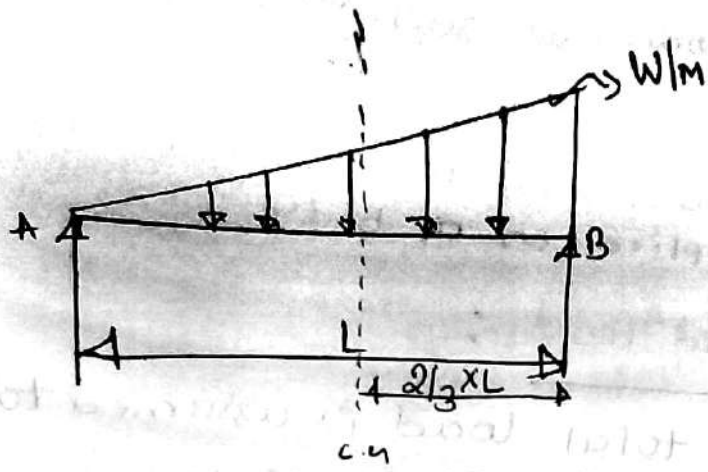
For calculation purpose it is treated as equivalent concentrated load [ECL] acting at mid point of the portion (or) at its centroid with its area has magnitude.

c) Uniformly Varying load [UVL]

It is the one which is spread over a beam in such a manner that rate of loading varies from point to point along the beam. as shown in fig. In which the load is zero at one end and increases uniformly to the other end. Such load is known as triangular load.

For solving the problems the total load is equal to the area of the triangle and this total load is assumed to be acting at the C.G. of the triangle i.e. at a distance of $\frac{2}{3}$ of the length from the zero load end.

$\frac{2}{3}$ of the total length of the beam from the left end.

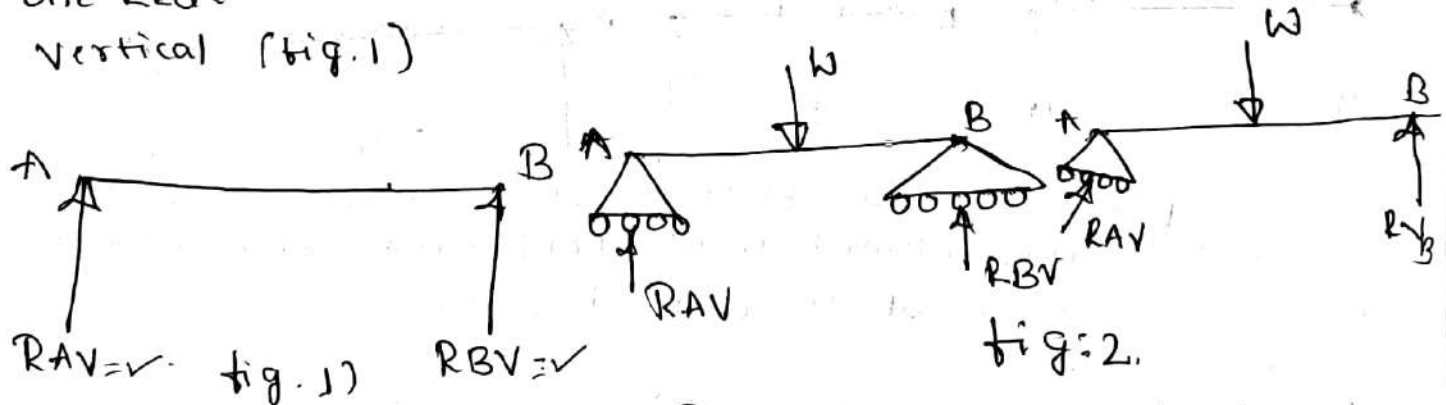


* Supports

Supports are classified based on number of reaction reactions that are going to develop at the contact ~~start~~ points.

① Simply supports:
The ends of the beam rest simply on a rigid body without any fixing is called simply support.

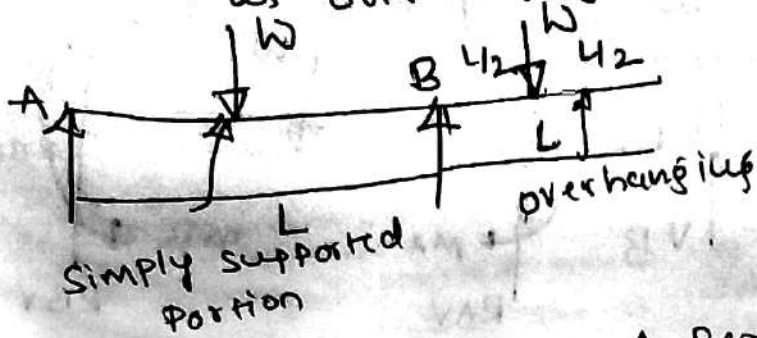
One Reaction component at each simple support & is acting vertical (fig. 1)



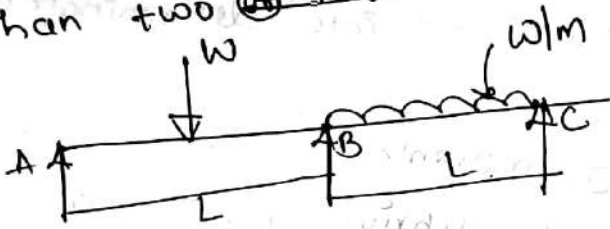
② Roller Support: Vertical Reaction component always acting perpendicular to the line of rollers (fig 2)

③ Hinged support (or) Pinned support: two unknowns in x & y direction.
 fig: 3

③ Overhanging Beam: If the end portion of the beam is extended beyond the support such beam is known as overhanging beam.



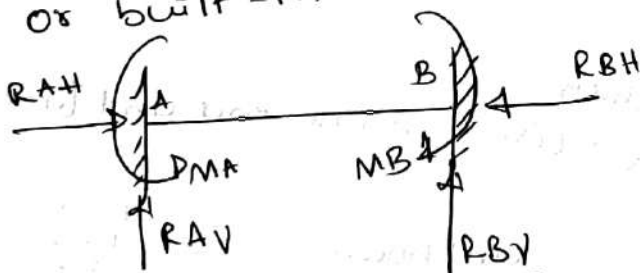
④ Continuous Beams: A beam which is provided more than two ~~or more~~ supports



⑤ In determinate beams all the unknowns such as reaction force or moments cannot be evaluated by using the equilibrium conditions

Examples

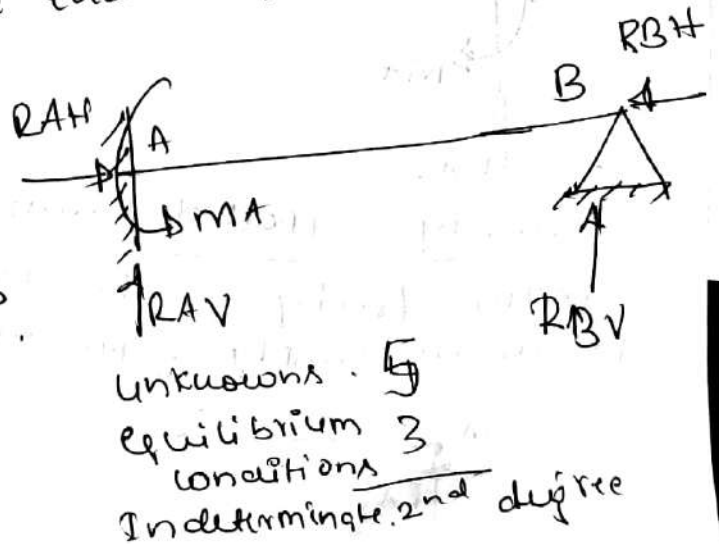
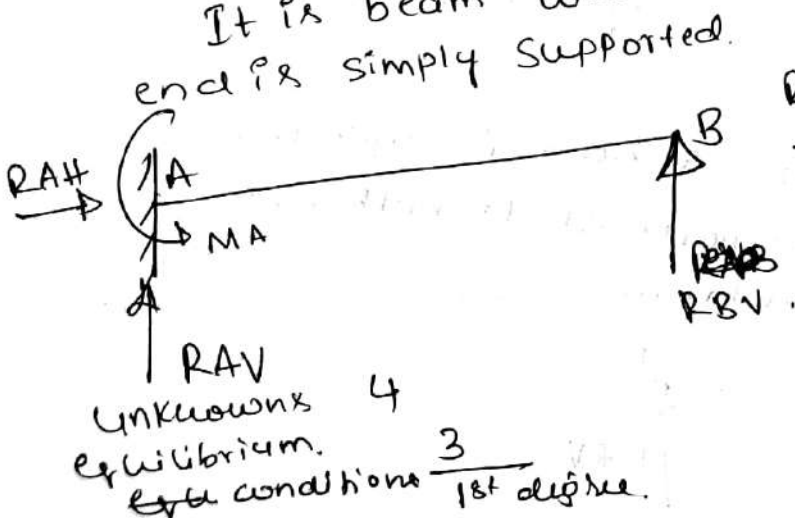
⑥ Fixed Beams: A beam whose both ends are fixed or built-in walls is known as fixed beam.



Unknowns = 6
Equilibrium conditions = 3
Indeterminate = 3rd degree.

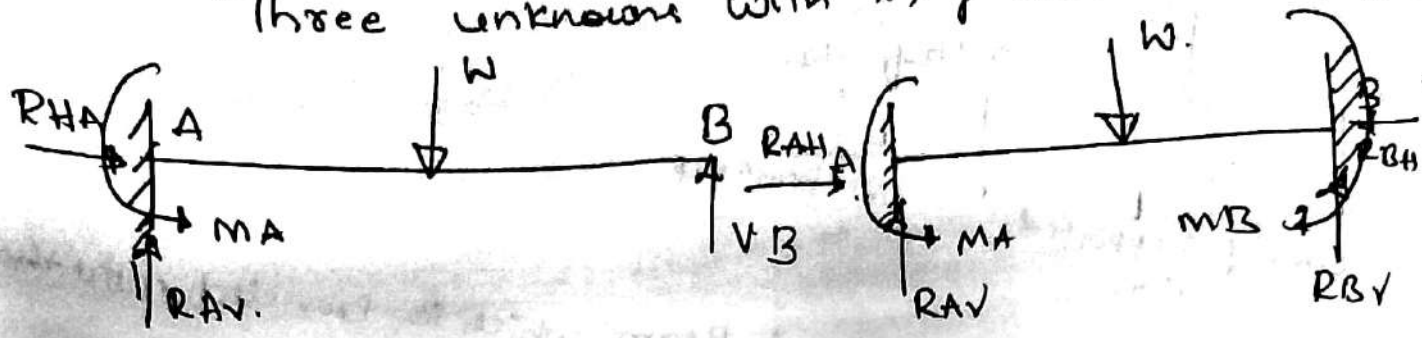
⑦ Propped Cantilever Beams:

It is beam whose one end is fixed and other end is simply supported.



fixed or built in support.

Three unknowns with x , y and z moment.



Beams: Beam is a horizontal member.

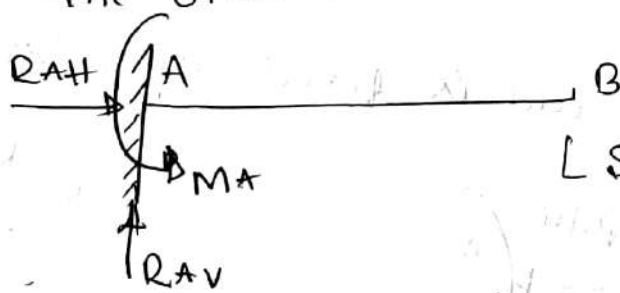
Beams are broadly classified into determinate and indeterminate beams.

- In case of Determinate beams all the unknowns such as reaction forces and moments can be evaluated by using the three static equilibrium conditions.
 - i.e. $\sum H = 0$, $\sum V = 0$, $\sum M = 0$

Types of Beams:

- Cantilever beam,
- Simply supported beam
- Overhanging beam
- Fixed beam and
- Continuous beam.

1. Cantilever beam: A beam is fixed at one end and free at the other end.

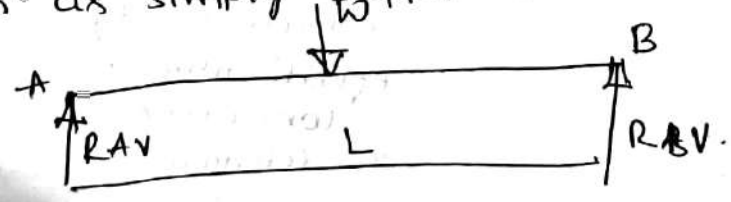


Unknowns = 3
RAH
RAV
MA

Equilibrium conditions 3

[Statically Determinate] Beam

2) Simply supported beam: A beam is supported resting freely on the supports at its both ends is known as simply supported beam.



Unknowns = 2
RAV
RBV

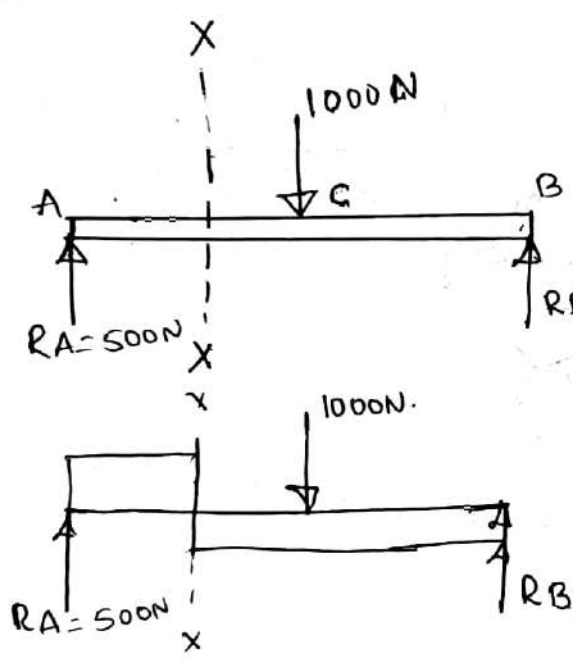
Equilibrium conditions 3

Shear Force (S.F)

The algebraic sum of the vertical force at any section of a beam to the right or left of the section is known as shear force.

Sign convention

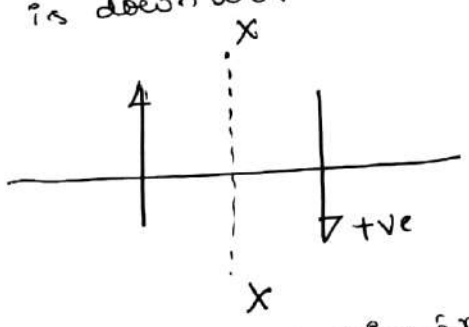
A simply supported beam AB carrying a load of 1000N at its middle point. The reactions at the supports will be equal to 500N, Hence $R_A = R_B = 500N$



The beam is divided into two portions by the section X-X. The resultant of the load and reaction to the left of X-X is 500N vertically upwards.
 (There is no load to the left of X-X). And the resultant of the load and reaction to the right of X-X is $[1000\downarrow - 500\uparrow = 500\downarrow N]$ 500N downwards.

The resultant force acting on any one of the parts normal to the axis of the beam is called the shear force at the section X-X. Here the shear force at the section is upwards and

500N. The shear force at a section will be considered as positive when the resultant of the forces including reaction is upward \oplus to the left of the section and is downward \ominus to the right of the section.



Shear force diagram:
 Plot which shows the length of beam.

A shear force diagram is the variation of shear force along the

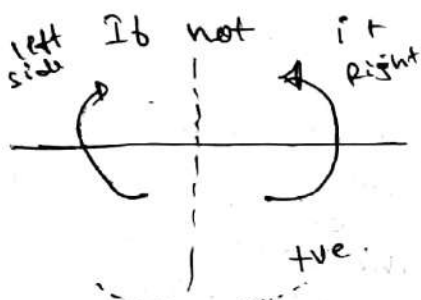
Bending moment

The algebraic sum of the moments of all the forces acting to the left (or) right of the section is known as Bending moment.

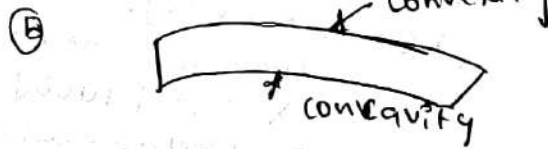
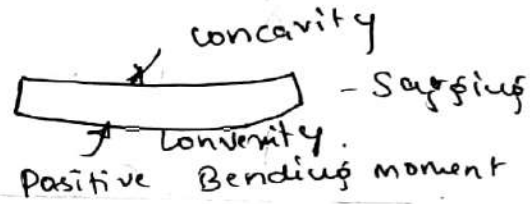
Bending moment at a section is constant and varies (or changes) as the distance varies.

Sign Convention

If the algebraic sum of moments to the left of the section is clockwise (or) right of the section is anticlockwise then it is positive bending moment i.e. sagging of the member.



If not it is negative bending moment i.e. hogging.



Bending moment Diagram (BMD)

A BMD is the plot which shows the variation of bending moment along the length of the beam.

Note:

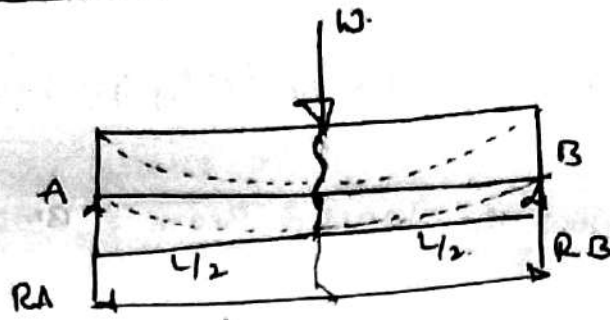
Bending moment at free end, simple support, roller support, hinged support end is zero.

The Bending moment develops only in case of fixed supports and over hanging portion.

Ex 1.5

Shear force and Bending moment

addition of forces to the left or right of the section



Breaking
① shear force

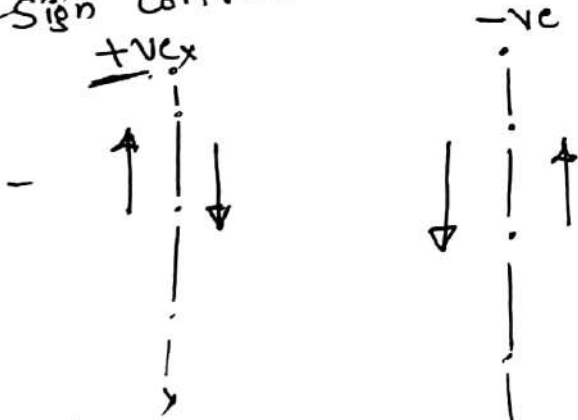
Shear Force: It is defined as the algebraic sum of forces acting either on left hand side or right hand side of the section.

Its unit will be N or kN .

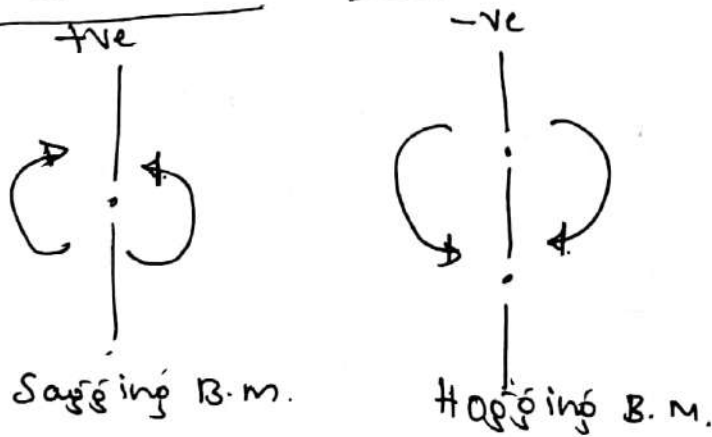
Bending moment: It is defined as the algebraic sum of moment of forces acting on the left side or right side of the section.

Its unit will be $N\cdot mm$ or $kN\cdot mm$.

Imp Sign convention for shear force



Sign convention for bending moment

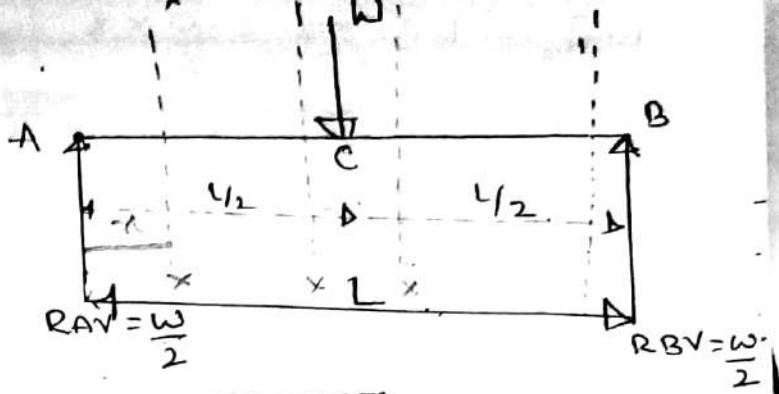


Important points to be noted while drawing Shear force & Bending Moment

1. Length of SFD & BMD must be equal to the span of the beam.
2. SFD is drawn below the loaded beam & BMD is drawn below SFD.
3. For simply supported beam, B.M is zero at the supports.
4. For cantilever beam, B.M is zero at the free end.
5. Calculate the SF & B.M at all critical points.
6. If no load is present b/w the two points then SF will be constant.

Simply supported Beam Point load at the centre.

Consider the beam shown in fig. which is simply supported at A and B and carries a vertical load W at the distance $\frac{L}{2}$ from the support.



Reactions $\Sigma F_x = 0$
 $\Sigma F_y = 0$

$+ R_{AV} - W + R_{BV} = 0$

$R_{AV} + R_{BV} = W$

$\Sigma M_A = 0$

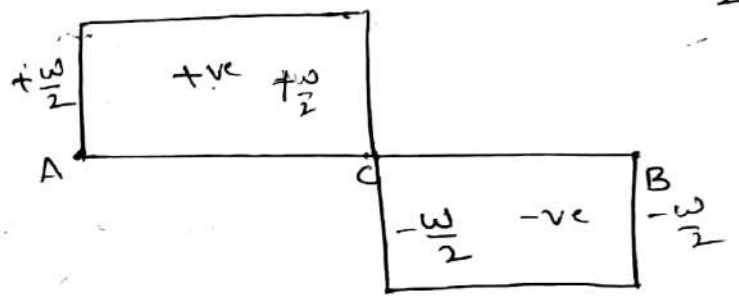
$R_{AV} \times 0 + W \times \frac{L}{2} - R_{BV} \times L = 0$

$R_{BV} = \frac{W}{2}$

$\Sigma M_B = 0$

$+ R_{BV} \times 0 - W \times \frac{L}{2} + R_{AV} \times L = 0$

$R_{AV} = \frac{W}{2}$



Shear Force Diagram

To plot the Shear Force Diagram.

$SF]_A = + R_{AV} = + \frac{W}{2}$

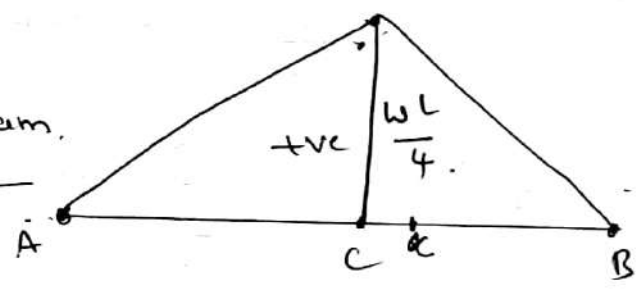
$SF]_{C \text{ left}} = + R_{AV} = + \frac{W}{2}$

$SF]_{C \text{ right}} = + R_{AV} - W$
 $= + \frac{W}{2} - W$

$SF]_C = - \frac{W}{2}$

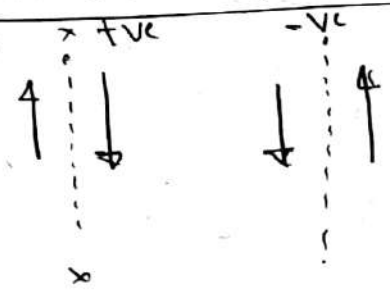
$SF]_B = + R_{AV} - W$
 $= + \frac{W}{2} - W$

$SF]_B = - \frac{W}{2}$



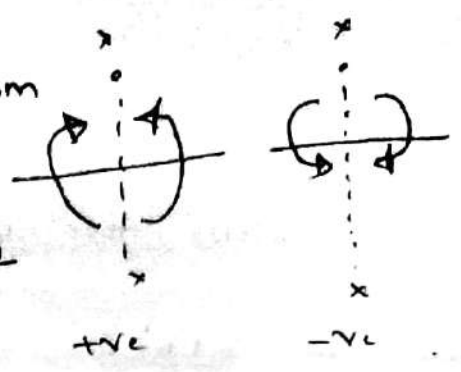
Bending Moment Diagram

Shear force sign conventions



6) TO Plot the Bending Moment Diagram

7) Consider a section a distance x from the support A



$$BM]_x = RAV \times x = \frac{w}{2} \times x \quad 0 < x < L/2$$

$$BM]_A = \frac{w}{2} \times x = \frac{w}{2} \times 0 = 0$$

$x = 0$

$$BM]_C = \frac{w}{2} \times x = \frac{w}{2} \times \frac{L}{2} = +\frac{wL}{4}$$

$$x = \frac{L}{2}$$

Hence Bending varies linearly from zero at support A to a maximum value below the point load.

8) The bending moment at any section b/w C & B at distance x from the end A is given by.

$$BM]_x = RA \cdot x - w \times (x - \frac{L}{2})$$

$$= \frac{w}{2} \cdot x - w \cdot x + \frac{wL}{2}$$

$$BM]_x = \frac{wL}{2} - \frac{wx}{2}$$

$$\text{At } \frac{wL}{2} \quad x = \frac{L}{2} = \frac{wL}{2} - \frac{wx}{2} = \frac{wL}{2} - \frac{w}{2} \times \frac{L}{2} = \frac{wL}{4}$$

$$BM]_C = +\frac{wL}{4}$$

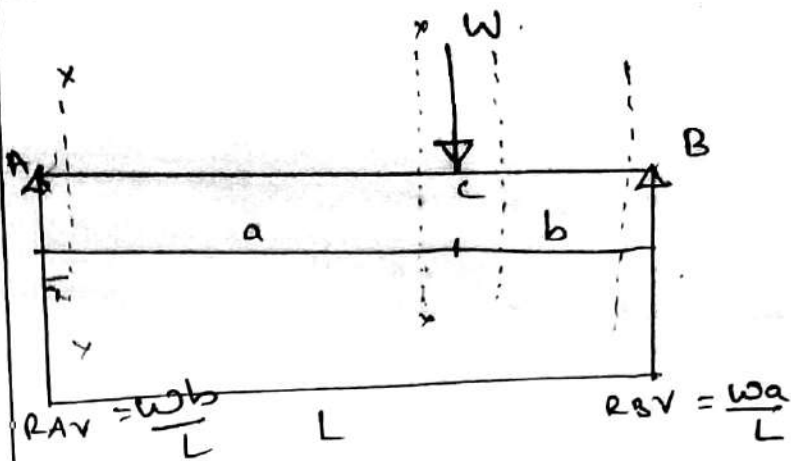
$x = L/2$

$$\text{At } BM]_B \quad x = L = \frac{wL}{2} - \frac{wx}{2} = \frac{wL}{2} - \frac{w}{2} \times L = 0$$

$$BM]_B = 0$$

Simply supported Beam subjected to e centric loading

Consider the simply supported beam at A and B carrying a vertical load w at a distance a from the left end



Reactions

$$\sum F_y = 0$$

$$+R_{AV} - w + R_{BV} = 0$$

$$R_{AV} + R_{BV} = w$$

$$\sum M_A = 0$$

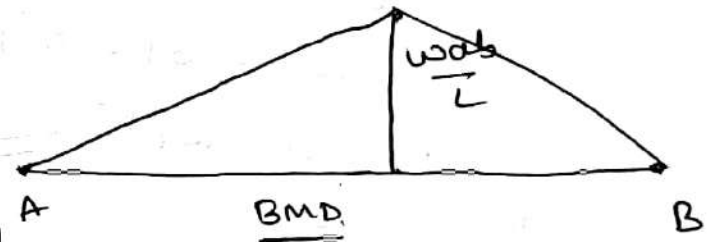
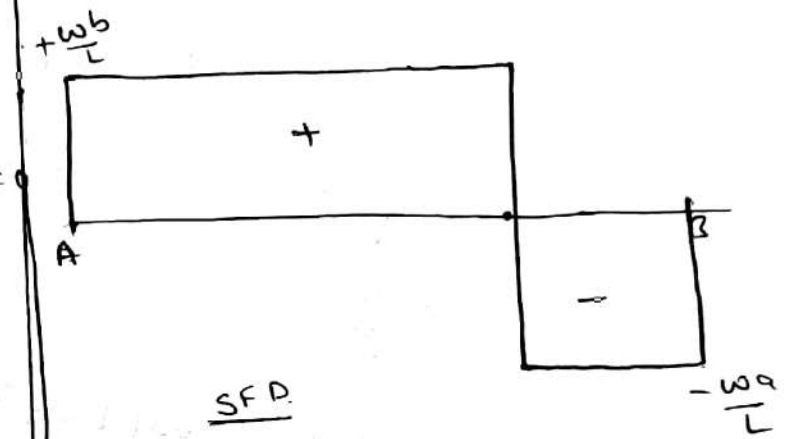
$$+R_{AV} \times 0 + w \times a - R_{BV} \times (a+b) = 0$$

$$R_{BV} = \frac{wa}{L}$$

$$\sum M_B = 0$$

$$+R_{AV} \times L - w \times b + R_{BV} \times 0 = 0$$

$$R_{AV} = \frac{wb}{L}$$



Calculation of the SF

$$SF]_A = + R_{AV} = + \frac{wb}{L}$$

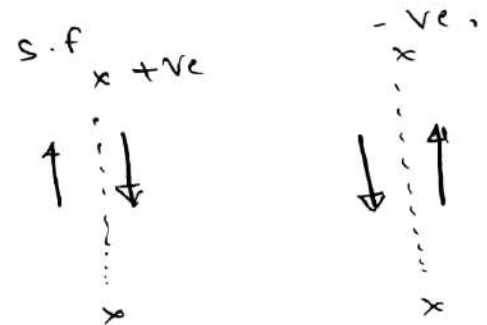
$$SF]_c \text{ Left} = + R_{AV} = + \frac{wb}{L}$$

$$SF]_c \text{ Right} = + \frac{wb}{L} - w = w \left[\frac{b}{L} - 1 \right] = -w \left[\frac{L-b}{L} \right]$$

where $L-b = a$

$$SF]_c \text{ Right} = - \frac{wa}{L}$$

$$SF]_B = - \frac{wa}{L}$$



BMD calculation:

$$Bm)_x =$$

$$M(x) = RAV \times x$$

$$M(x) = +\frac{wb}{L} \times x$$

When $x=0$

$$Bm)_A = \frac{wb}{L} \times 0 = 0$$

$x=0$

$$Bm)_c = \frac{wb}{L} \times a = +\frac{wab}{L}$$

wt

$x=a$

$$Bm)_c = RAV \times x - w(x-a)$$

Right

$$x=a \Rightarrow = RAV \frac{wb}{L} \times a - w(a-a)$$

$$Bm)_c = \frac{wab}{L}$$

Right

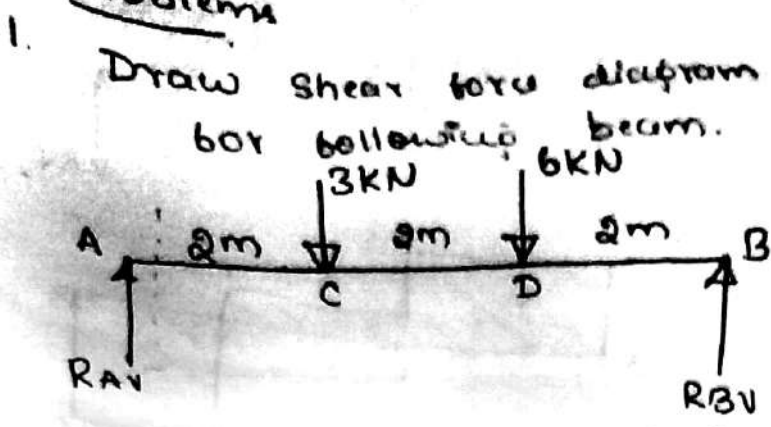
$$Bm)_B = \frac{wb}{L} \times L - w(L-a)$$

$$= +wb - w(L-a)$$

$$= +wb - w[b]$$

$$Bm)_B = \underline{\underline{0}}$$

Problems



It is a simply supported beam. Let R_{AV} and R_{BV} be the support reactions as shown in fig.

W.K.T Applying equilibrium conditions

$$\sum F_y = 0$$

$$+R_{AV} - 3 - 6 + R_{BV} = 0$$

$$\boxed{R_{AV} + R_{BV} = 9 \text{ KN}}$$

$$\sum M_A = 0$$

$$-R_{BV} \times 6 + 6 \times 4 + 3 \times 2 = 0$$

$$+R_{BV} \times 6 = +30$$

$$R_{BV} = \frac{30}{6}$$

$$\boxed{R_{BV} = 5 \text{ KN}}$$

$$\sum M_B = 0$$

$$+R_{AV} \times 6 - 3 \times 4 - 6 \times 2 = 0$$

$$R_{AV} \times 6 = 24$$

$$\boxed{R_{AV} = 4 \text{ KN}}$$

Shear force calculations

$$SF)_A = +4 \text{ KN} = +V_A$$

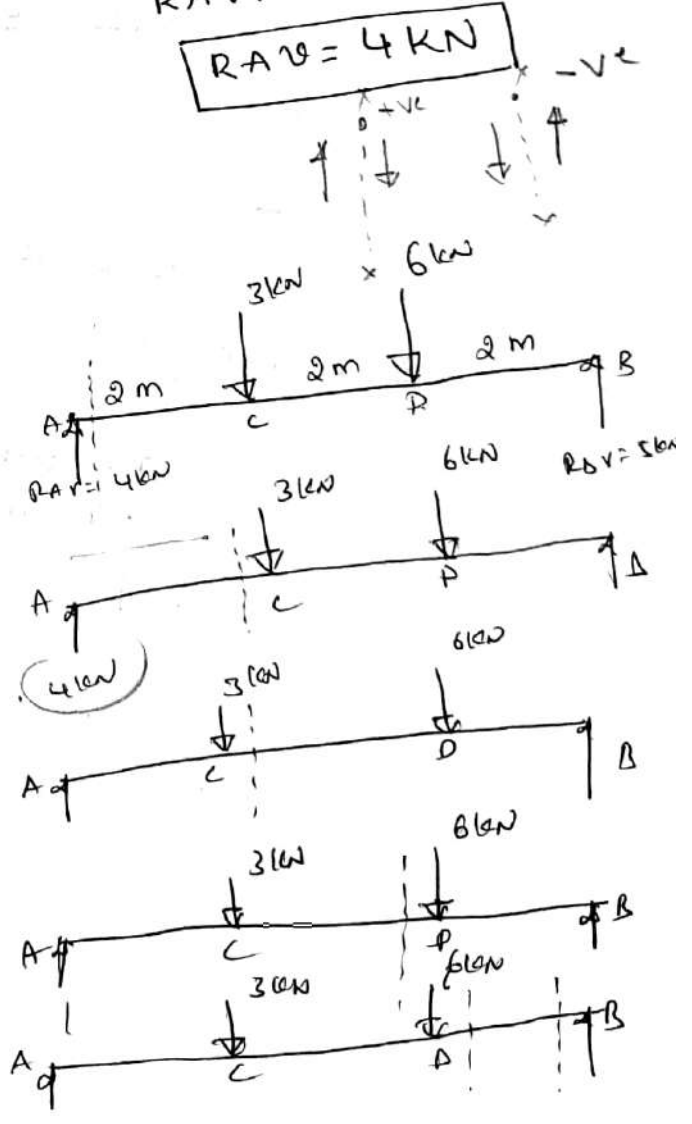
$$SF)_C = +4 \text{ KN}$$

$$SF)_C \text{ R.C} = +4 \text{ KN} - 3 \text{ KN} = 1 \text{ KN}$$

$$SF)_D \text{ L.C} = 4 \text{ KN} - 3 = 1 \text{ KN}$$

$$SF)_D \text{ R.C} = 4 - 3 - 6 = -5 \text{ KN}$$

$$SF)_B = 4 - 3 - 6 = -5 \text{ KN}$$



Calculation of Bending moment:

As the supports A & B are simply supported the bending moments at these points are zero.

$Bm)_A = 0$
 $Bm)_B = 0$

$Bm)_C = +R_{AV} \times 2 = 4 \times 2 = 8 \text{ kN-m}$
 left support

$Bm)_C = +R_{AV} \times 4 + 3 \times 2 = 16 - 6 = 10 \text{ kN-m}$
 Right support
 $R_{BV} \times 4 - 6 \times 2 = 0$
 $5 \times 4 - 12 = 8 \text{ kN-m}$

$Bm)_C = 8 \text{ kN-m}$
 Right support

$Bm)_D = (R_{AV} \times 4) - 3 \times 2 = 16 - 6 = 10 \text{ kN-m}$
 left

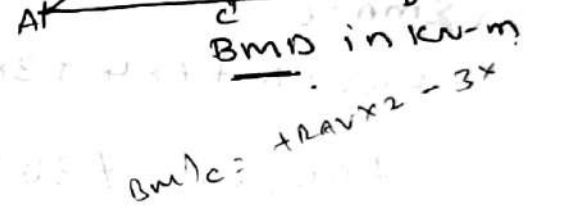
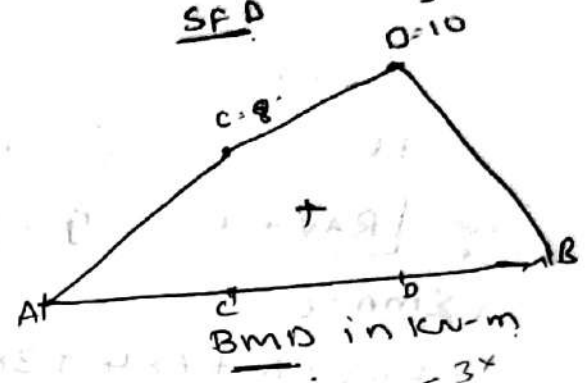
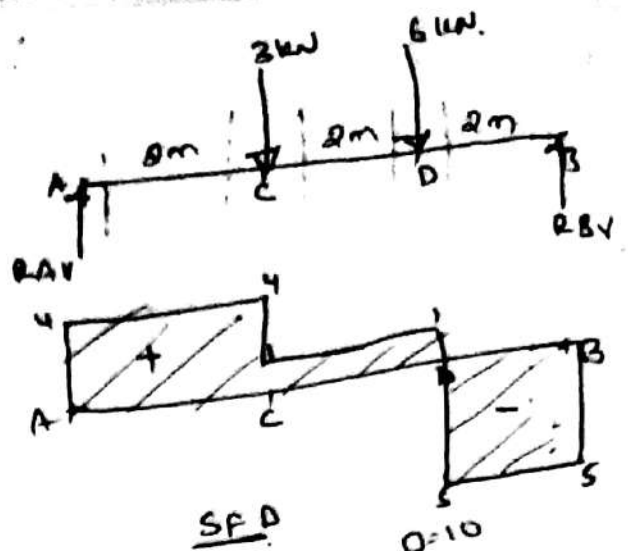
$Bm)_D = +5 \times 2 = 10 \text{ kN-m}$
 right

∴ In case of BMD for point source the lines will be combination of vertical and horizontal lines i.e. inclined lines.]

$Bm)_C = +R_{AV} \times 2 - 3 \times 0 = 8 \text{ kN-m}$
 Right

$Bm)_D = R_{AV} \times 4 - 3 \times 2 - 6 \times 0 = 16 - 6 = 10 \text{ kN-m}$
 Right

$Bm)_B = +R_{AV} \times 6 - 3 \times 4 - 6 \times 2 = 24 - 12 - 12 = 0$



Draw the shear force and Bending moment Diagram for uniformly Distributed load on the simply supported beam.

$$\sum V = 0$$

$$R_{AV} + R_{BV} - WL = 0$$

$$R_{AV} + R_{BV} = WL$$

$$\sum M_A = 0$$

$$-R_{BV} \times L + WL \times \frac{L}{2} = 0$$

$$+R_{BV} \times L = +\frac{WL \times L}{2}$$

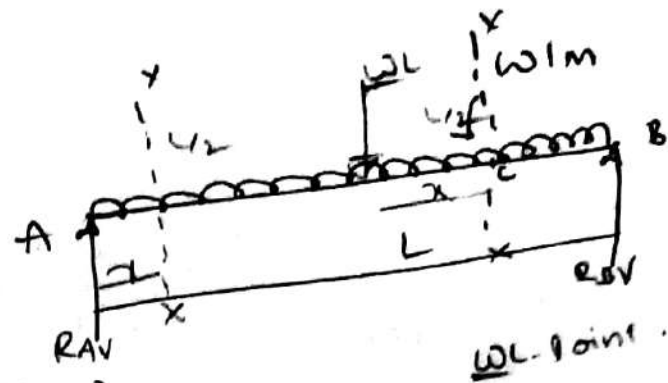
$$R_{BV} = \frac{WL}{2}$$

$$\sum M_B = 0$$

$$+R_{AV} \times L - \frac{WL \times L}{2} = 0$$

$$R_{AV} \times L = \frac{WL \times L}{2}$$

$$R_{AV} = \frac{WL}{2}$$



Shear force calculation

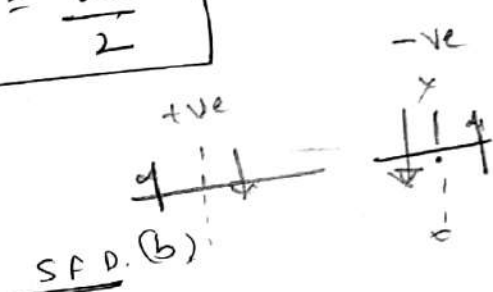
$$SF]_A = \frac{WL \times 0}{2} = +\frac{WL}{2} = +V_A$$

$$SF]_B = -\frac{WL}{2} = -\frac{WL}{2} = -V_B$$

$$SF]_C = +V_A - w \times \frac{L}{2}$$

$$= +\frac{WL}{2} - w \times \frac{L}{2} = +\frac{WL}{2} - \frac{WL}{2} = 0$$

$$SF]_C = 0$$



Bending Moment calculations at the section X at a distance x from left end A is given by

$$M_x = +R_{AV} \cdot x - w \cdot x \cdot \frac{x}{2}$$

$$= \frac{WL}{2} x - \frac{w \cdot x^2}{2}$$

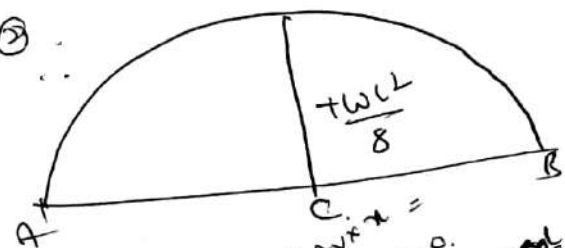
$$\therefore R_A = \left(\frac{WL}{2}\right) \text{ --- (2)}$$

From eq (2) it is clear that BM varies according to parabolic law.

The values of B.M at different points

$$BM \text{ at } A \quad x=0 \text{ hence } M_A = \frac{WL}{2} \times 0 - \frac{w \cdot 0^2}{2} = 0$$

$$BM \text{ at } B \quad x=L \text{ hence } M_B = \frac{WL}{2} \cdot L - \frac{w \cdot L^2}{2} = \frac{WL^2}{2} - \frac{WL^2}{2} = 0$$



$$BM]_A = R_{AV} \times x = R_{AV} \times 0 = 0$$

$$BM]_C = R_{AV} \times \frac{L}{2} - w \times \frac{L}{2} \times \frac{L}{2}$$

$$= +\frac{WL}{2} \times \frac{L}{2} - \frac{WL^2}{2} = \frac{WL^2}{8} - \frac{WL^2}{8} = 0$$

At c, $x = L/2$ hence $M_c = \frac{\omega \cdot L}{2} \cdot \frac{L}{2} - \frac{\omega}{2} \left(\frac{L}{2}\right)^2$

$$= \frac{\omega L^2}{4} - \frac{\omega L^2}{8} = + \frac{\omega L^2}{8}$$

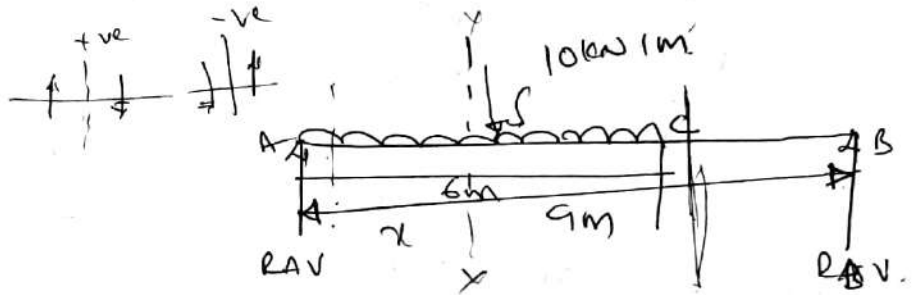
Thus, the B.M increases according to parabolic law from zero at A to $\frac{\omega L^2}{8}$ at middle point of the beam and from this value the B.M. decreases to zero at B according to the parabolic law.

- ① Draw the shear force and bending moment diagram for a simply supported beam of length 9m and carrying a UDL of 10kN/m for a distance of 6m from the left end. Also calculate the maximum B.M on this section.

$\Sigma V = 0$

$R_{AV} + R_{BV} - 60 = 0$

$R_{AV} + R_{BV} = 60 \text{ kN.}$



$\Sigma M_B = 0$

$\Sigma M_A = 0$

$+ R_{AV} \times 9 - 60 \times 6 = 0$

$R_{AV} = \frac{360}{9}$

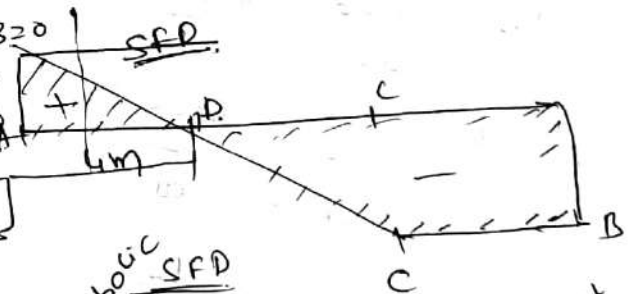
$R_{AV} = 40 \text{ kN}$

$- R_{BV} \times 9 + 60 \times 3 = 0$

$- R_{BV} \times 9 = -180$

$R_{BV} = \frac{180}{9}$

$R_{BV} = 20 \text{ kN}$



Shear force calculation:

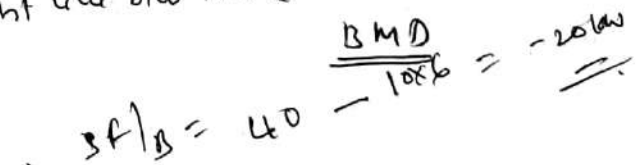
Consider any section at a distance x from A b/w A to C. The shear force at this section is given by:

$F_x = + R_{AV} - 10 \times x$
 $= 40 - 10 \times x$

① Show the SFD varies by a straight line b/w A to C.

at SFD $A = x=0 = 40 - 10 \times 0 = 40 \text{ kN.}$

at SFD $B = x=6 = 40 - 10 \times 6 = -20 \text{ kN.}$



The shear force at A is 40kN and at C is -20kN. Also shear force b/w A to C varies by a straight line. This means that somewhere b/w A to C, the shear force is zero let the S.F. is at x meters from A. Then substituting the value of $F_x = 0$

$$\Sigma F_x = 40 - 10x$$

$$0 = 40 - 10x$$

$$SF)_0 = \underline{-20 \text{ kN}}$$

$$10x = 40$$

$$x = \frac{40}{10}$$

$$x = 4 \text{ m}$$

Hence shear force is zero at a distance 4m from A.

Bending Moment Diagram

The B.M. at any section b/w A and C is given by

$$M_x = +RAV \times x - 10 \cdot x \cdot \frac{x}{2} = 40x - \frac{10x^2}{2} = 40x - 5x^2 \rightarrow \text{(ii)}$$

eq (2) shows that B.M. varies according to parabolic law b/w A and C

$$BM)_A \quad x=0 \text{ hence } M_A = 40 \times 0 - 5 \times 0^2 = 40 \times 0 - 5 \times 0 = 0$$

$$BM)_C \quad x=6 \quad M_C = 40 \times 6 - 5 \times 6^2 = 40 \times 6 - 5 \times 6^2 = +60 \text{ kNm}$$

$$BM)_D \quad x=4 \text{ m} \quad M_D = 40 \times 4 - 5 \times 4^2 = 40 \times 4 - 5 \times 4^2 = +80 \text{ kNm}$$

$$BM)_B \quad x=0 \text{ m} \quad M_B = 40 \times 0 - 5 \times 0^2 = 0 \text{ kNm}$$

The Bending Moment b/w C and B varies according to linear law. B.M. is zero whereas at C it is 60 kNm.

Maximum Bending moment :

The B.M. is maximum at a point where the shear force sign changes. This means that the point where shear force becomes zero from positive to negative value \odot or vice versa. The B.M. at that point will be maximum. From the shear force diagram, we know that at point D, the SF is zero after changing the sign. The B.M. is maximum at D.

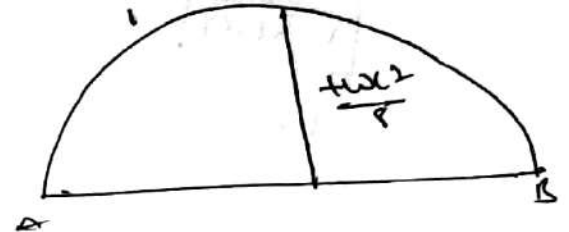
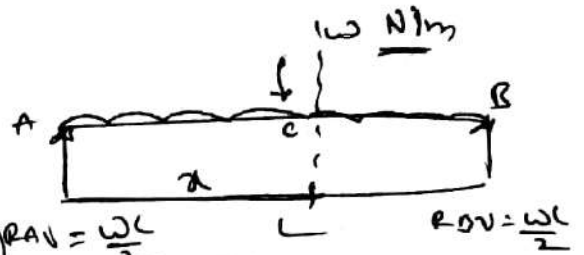
$$BM)_D = \underline{+80 \text{ kNm}}$$

Bending moment calculations for UDL over the span

$$BMD) M_x = +R_A \times x - w \times x \cdot \frac{x}{2}$$

$$= \frac{wL}{2} \times x - \frac{w \cdot x^2}{2}$$

$$BMD)_x = \frac{wL}{2} \times x - \frac{w \cdot x^2}{2} \quad \text{(Parabolic variation)} \quad R_A = \frac{wL}{2} \quad R_B = \frac{wL}{2}$$



$$BMD)_A = \frac{wL}{2} \times 0 - \frac{w \cdot 0^2}{2}$$

$$BMD)_A = 0$$

$$BMD)_C = \frac{wL}{2} \times x - \frac{w \cdot x^2}{2}$$

$$x = L/2 = \frac{wL}{2} \times (L/2) - \frac{w \cdot (L/2)^2}{2}$$

$$= \frac{wL^2}{4} - \frac{wL^2}{8}$$

$$BMD)_C = \frac{wL^2}{8}$$

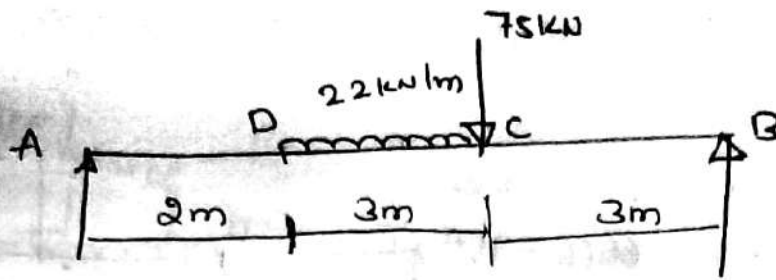
$$BMD)_B = \frac{wL}{2} \times x - \frac{w \cdot x^2}{2}$$

$$x = L = \frac{wL}{2} \times L - \frac{w \cdot L^2}{2}$$

$$= \frac{wL^2}{2} - \frac{wL^2}{2}$$

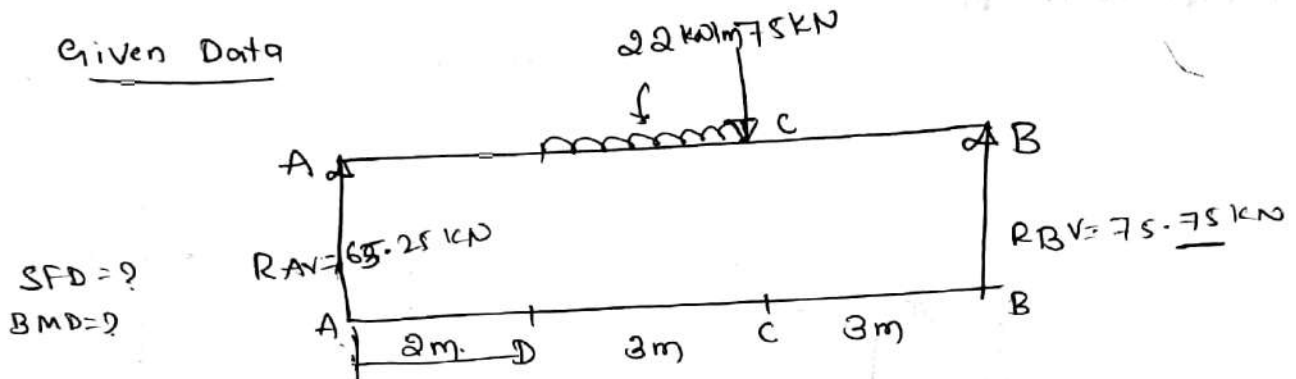
$$BMD)_B = 0$$

1 For the beam loaded as shown in fig.



Draw SFD and BMD.

→ Given Data



Soln ① Step No 1: Calculation of support reaction.

$$\sum F_y = 0 \quad \uparrow +ve \quad \downarrow -ve$$

$$R_A + R_B - 75 - [22 \times 3] = 0$$

$$\boxed{R_A + R_B = 141 \text{ kN}} \quad \rightarrow \text{①}$$

② $\sum M_A = 0$ ↺ +ve ↻ -ve

$$-R_B \times 8 + 75 \times 5 + 66 \times 3.5 = 0$$

$$\boxed{R_B = 75.75 \text{ kN}}$$

R_B value in eq ① we get .

$$R_A + 75.75 = 141$$

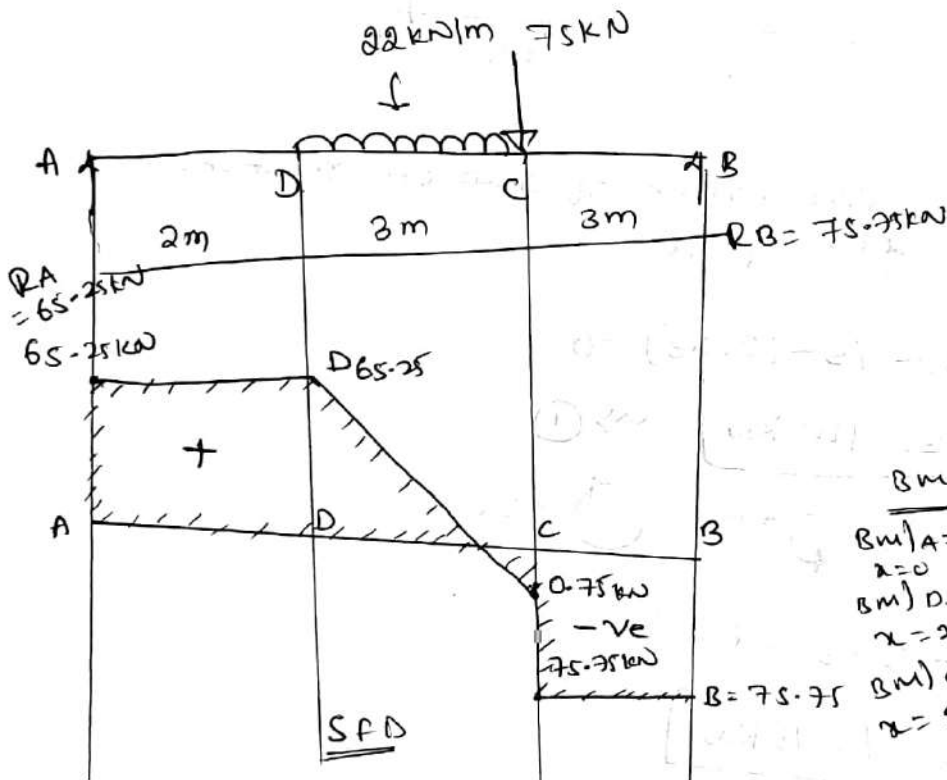
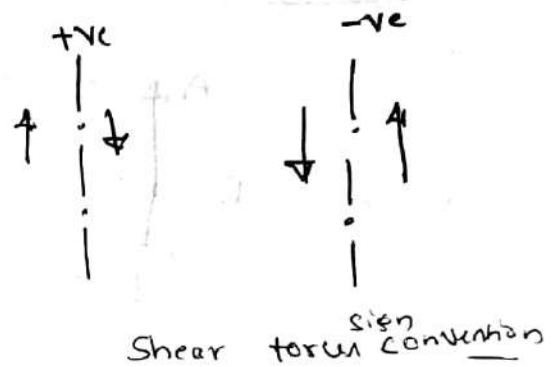
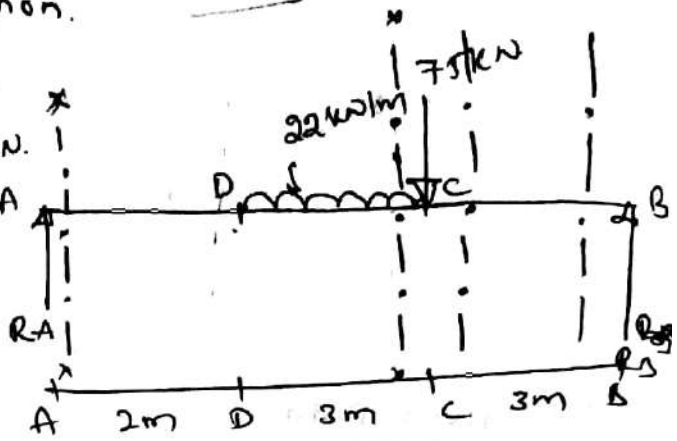
$$\boxed{R_A = 65.25 \text{ kN}}$$

Step No 2

Shear Force calculation.

UDL Point
 $22 \times 3 = 66$

- ① S.F at Point A = $+65.25 \text{ KN}$.
- S.F at Point D = 65.25 KN
- ② S.F at Point C = $+65.25 - 66 = -0.75 \text{ KN}$.
- ③ S.F at Point B = $65.25 - 66 - 75 = -75.75 \text{ KN}$.
- ④ S.F at Point B = -75.75 KN .



BMD calculations

UDL = Inclined
 constant = straight line

BM) A = $RA \times 0 = 0$
 $x=0$

BM) D = $RA \times 2 = 65.25 \times 2 = 130.5 \text{ kNm}$
 $x=2$

BM) C = $RA \times 5 - 22 \times 3 \times \frac{3}{2} - 75 \times 0$
 $x=5 = 65.25 \times 5 - 99 = 0$
 $= 326.25 - 99$
 $= 227.25$

BM) B = 0
 $x=0$ (at B)

Simply supported beam

Step No: 3 Bending moment calculations

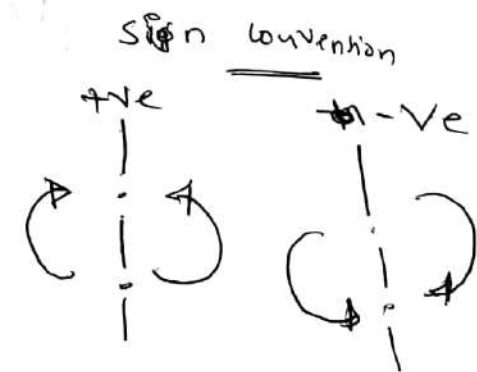
$MA = 0$ $MB = 0$ [Simply supported beam]

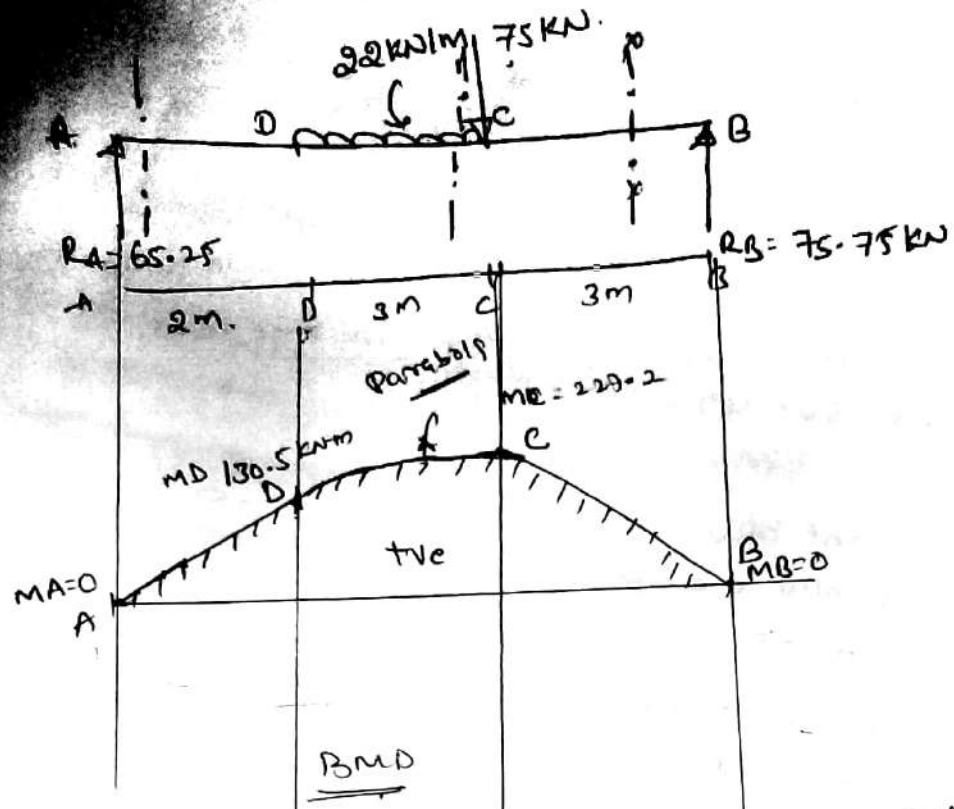
$MC = + 75.75 \times 3 = 227.25 \text{ kNm}$
 $- RA \times 5 - 22 \times 3 \times \frac{3}{2} = 65.25 \times 5 - 22 \times 3 \times \frac{3}{2}$

$MD = 65.25 \times 2 = 130.5 \text{ kNm}$
 $= + RA \times 2 = 65.25 \times 2 = 130.5 \text{ kNm}$

BM) A = $+ RA \times 0 = 0$
 $x=0$

BM) B = $+ RA \times 8 - 66 \times 8$





Where the SF is zero
BM is maximum

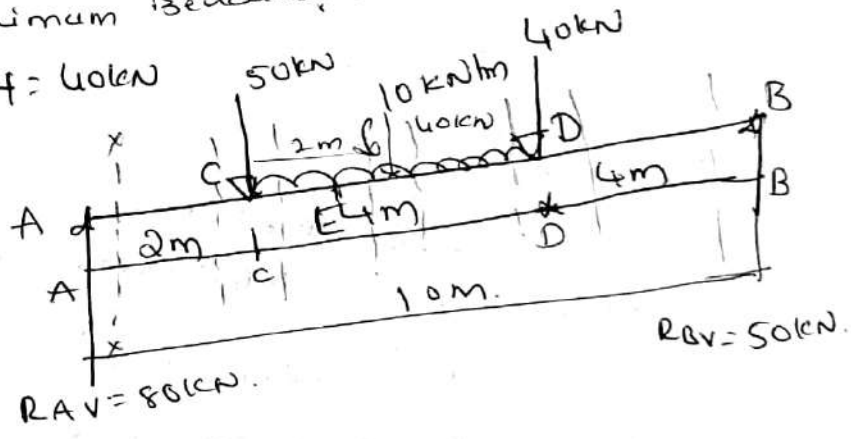
Where the S.F. is constant
BM is straight

Q) A simply supported beam AB length 10m, carries the uniformly distributed load and two point loads as shown in fig. Draw a S.F. and B.M. diagram for the beam. Also calculate the maximum bending moment.

$$\sum V = 0$$

$$R_{AV} - 50 - 40 - 40 + R_{BV} = 0$$

$$R_{AV} + R_{BV} = 130 \text{ kN}$$



$$\sum M_A = 0$$

$$+50 \times 2 + 40 \times 6 + 40 \times 6 - R_{BV} \times 10 = 0$$

$$R_{BV} \times 10 = 500$$

$$R_{BV} = \frac{500}{10}$$

$$R_{BV} = 50 \text{ kN}$$

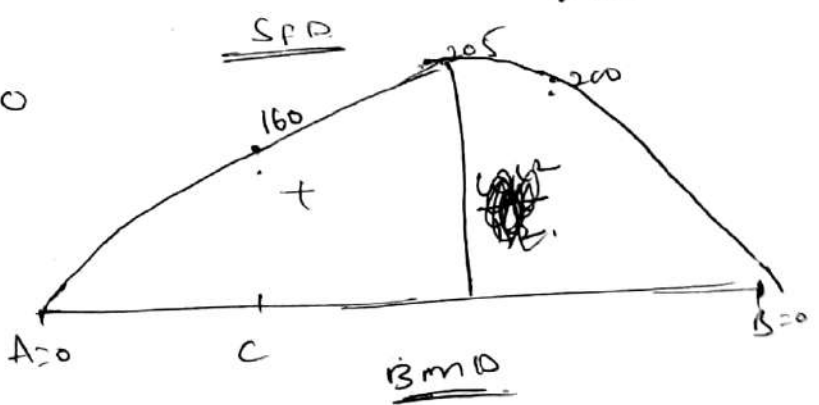
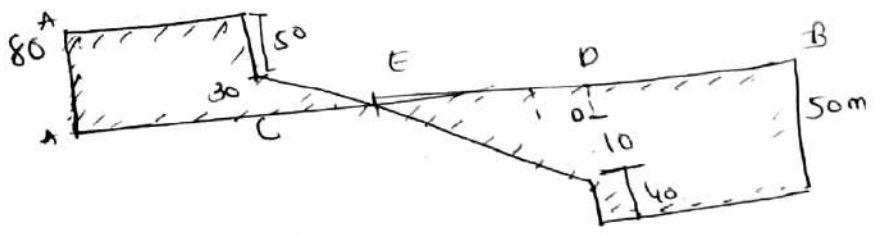
$$\sum M_B = 0$$

$$-40 \times 4 - 40 \times 6 - 50 \times 8 + R_{AV} \times 10 = 0$$

$$+800 = +R_{AV} \times 10$$

$$\frac{800}{10} = R_{AV}$$

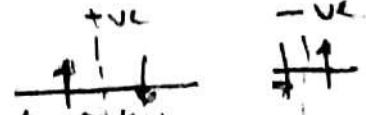
$$R_{AV} = 80 \text{ kN}$$



* S.F. Diagram.

$$SF_A = +RAV = +80 \text{ kN}$$

SF_C will remain constant from A to C & equal to 80 kN.



Left
 SF_C Just on R.H.S of C = $RAV - 50 = 80 - 50 = 30 \text{ kN}$

SF_D Just on L.H.S of D = $RA - 50 - 10 \times 4 = 80 - 50 - 40 = -10 \text{ kN}$.
 The S.F. b/w C and D varies according to straight line law.

SF_D Just on R.H.S of D = $RA - 50 - 40 - 40 = -50 \text{ kN}$.

SF at B = -50 kN

The S.F. remains constant b/w D and B & equal to -50 kN .
 The S.F. is zero at point E b/w C and D.

Now shear force at E = $RA - 50 - 10 \times (x - 2)$
 $= 80 - 50 - 10x + 20$
 $= 50 - 10x$

But shear force E = 0.

$$50 - 10x = 0$$

$$50 = 10x$$

$$\frac{50}{10} = x$$

$$x = 5 \text{ m}$$

B.M. Diagram

B.M. at A is zero

B.M. at B is zero

B.M. at C, M.C = $+RA \times 2 = 80 \times 2 = 160 \text{ kN-m}$

B.M. at D, $M_D = RA \times 6 - 50 \times 4 - 40 \times \frac{4}{2}$
 $= 80 \times 6 - 50 \times 4 - 40 \times 2$
 $= 200 \text{ kN-m}$

At E, $x = 5 \text{ m}$, hence B.M. at E.

$$M_E = +RA \times 5 - 50[5 - 2] - 10 \times [5 - 2] \times \left(\frac{5 - 2}{2}\right)$$

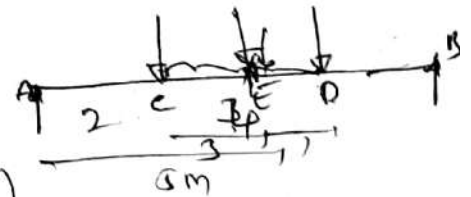
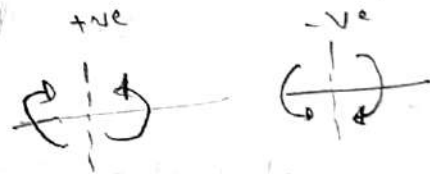
$$= 80 \times 5 - 50 \times 3 - 10 \times 3 \times \frac{3}{2}$$

$$= 400 - 150 - 45$$

$$= 205 \text{ kNm}$$

The maximum B.M. is at E, where S.F. becomes zero after checking the sign.

Max B.M. = 205 kNm



Simply supported Beam subjected to uniformly varying load.

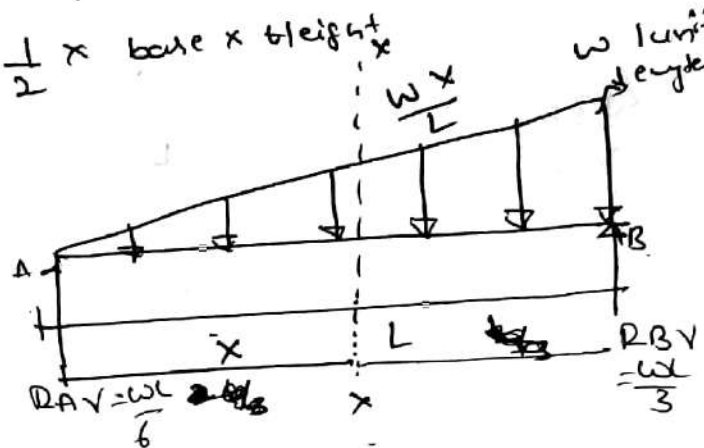
Let the load from zero at support A to w unit length at support B. Linearly as shown in fig.

Total load on the beam is $= \frac{1}{2} \times \text{base} \times \text{height}$

$$= \frac{1}{2} \times L \times w$$

$$= \frac{wL}{2}$$

Its centroid from B is at $L/3$



$$\sum M_B = 0$$

$$R_A \times L - \frac{wL}{2} \times \frac{L}{3} = 0$$

$$R_A \times L = \frac{wL^2}{6}$$

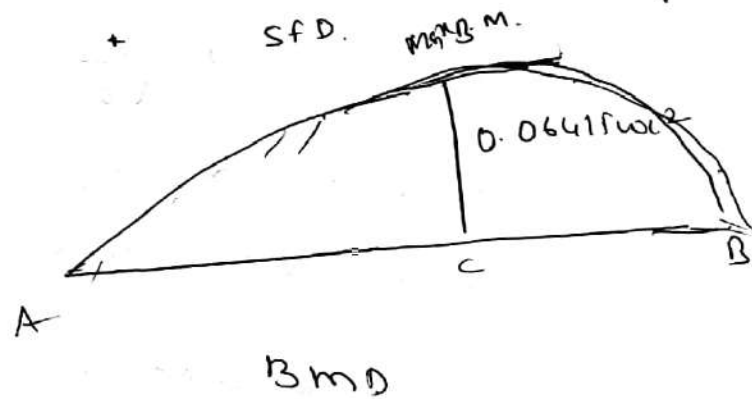
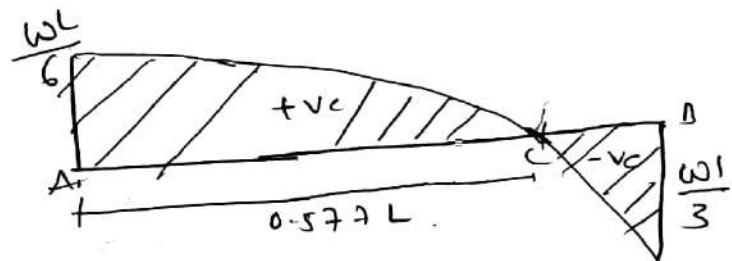
$$R_A = \frac{wL}{6}$$

$$\sum M_A = 0$$

$$-R_B \times L + \frac{wL}{2} \times \frac{2L}{3} = 0$$

$$R_B \times L = \frac{wL^2}{3}$$

$$R_B = \frac{wL}{3}$$



Load intensity at section $x-x$ distance

x from A is $\frac{wx}{L}$

Total load on left hand portion of the section

$$= \frac{1}{2} \times \frac{wx}{L} \times x = \frac{wx^2}{2L}$$

Its centroid from the section $x-x$ is at $\frac{x}{3}$

$F = R_A - \text{load on the left portion}$

$$= \frac{wL}{6} - \frac{wx^2}{2L} \quad (\text{parabolic variation})$$

$$SF|_A = \frac{wL}{6}$$

$$SF|_B = \frac{wL}{6} - \frac{wL^2}{2L}$$

$$= \frac{wL}{6} - \frac{wL^2}{2L} = \frac{2wL - 6wL}{12} = -\frac{4wL}{12} = -\frac{wL}{3}$$

$$SF|_B = -\frac{wL}{3}$$

Shear force is zero at x , where x is given by

$$0 = \frac{wL}{6} - \frac{wx^2}{2L}$$

$$x^2 = \frac{L^2}{3}$$

$$x = \frac{L}{\sqrt{3}} = 0.577L$$

$$\frac{wx^2}{2L} = \frac{wL}{6}$$

$$x^2 = \frac{wL^2 \cdot x^2}{w \cdot 6}$$

BMD at section $x-x$ is given by

$M = R_A x$ - moment of load on left hand portion.

$$= \frac{wL}{6} x - \frac{wx^2}{2L} \cdot \frac{x}{3} \quad (\text{Cubic variation})$$

$$x^2 = \frac{L^2}{3}$$

$$x = \frac{L}{\sqrt{3}}$$

$$BM|_A = 0$$

$$BM|_B = \frac{wL}{6} \times L - \frac{wL^2}{2L} \cdot \frac{L}{3}$$

$$= \frac{wL^2}{6} - \frac{wL^2}{6}$$

$$BM|_B = 0$$

Maximum moment occurs at the point where Shear force is zero. Since $m = \frac{df}{dx}$.

Maximum moment is at $x = \frac{L}{\sqrt{3}}$

$$M_{max} = \frac{wL}{6} \frac{L}{\sqrt{3}} - \frac{w}{6L} \left(\frac{L}{\sqrt{3}} \right)^2$$

$$= \frac{wL^2}{6\sqrt{3}} \left[1 - \frac{1}{3} \right]$$

$$= 0.0645 wL^2$$

Simply supported Beam subjected to external moment

Consider the beam AB of span L subjected to an external clockwise moment at a point, distance 'a' from support A as shown in fig.

Taking moment about B, $\Sigma M_B = 0$

$$-R_A \times L + M_0 = 0$$

$$R_A = \frac{M_0}{L}$$

$$\Sigma M_A = 0$$

$$-R_B \times L + M_0 = 0$$

$$R_B = \frac{M_0}{L}$$

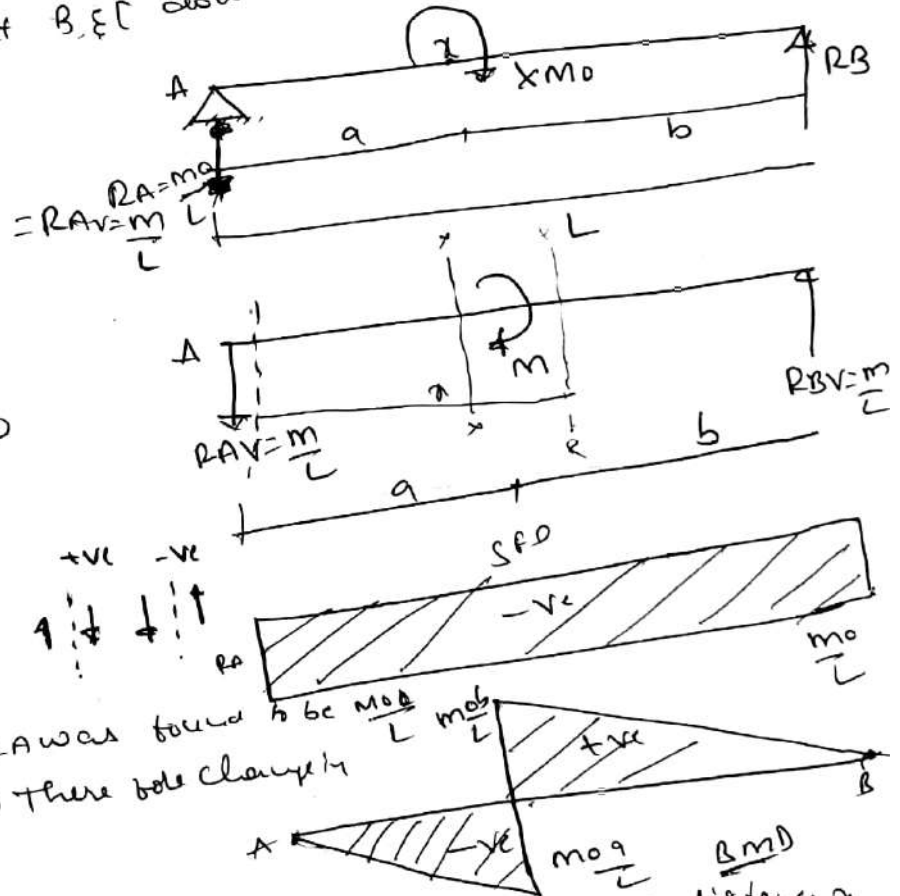
$$R_B = \frac{m}{L}$$

Assumed direction of R_A was found to be wrong (since R_A was found to be $\frac{M_0}{L}$ there both change the direction.)

Shear force calculation

$$SF|_A = -R_A = -$$

$$SF|_x = -R_A = -\frac{M_0}{L} \text{ (constant.)}$$



Now at any section at distance x from A.

BMD

At section $x-x$, in portion left of the section

$$M = -RAx = -\frac{M_0}{L} \cdot x \quad [\text{Linear variation}]$$

$$\text{At } x=0$$

$$M=0$$

$$\text{At } x=a$$

$$M = -\frac{M_0 a}{L}$$

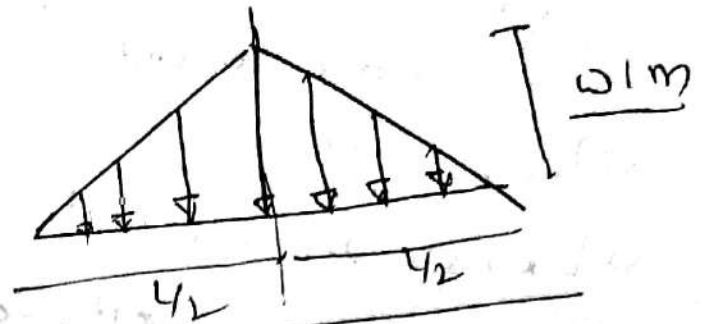
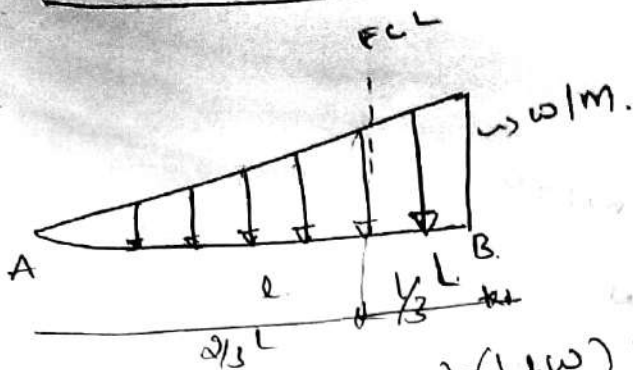
to the right of the section $x-x$.

$$M = -\frac{M_0}{L}x + M_0 = M_0 \left[1 - \frac{x}{L} \right] \quad [\text{Linear variation}]$$

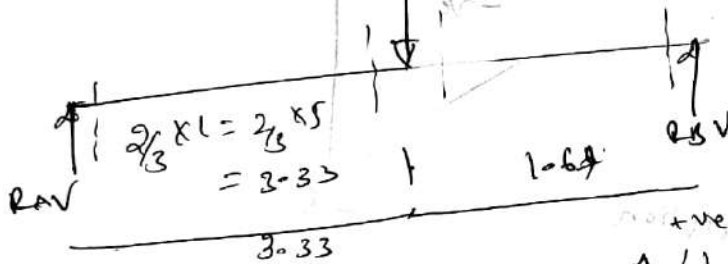
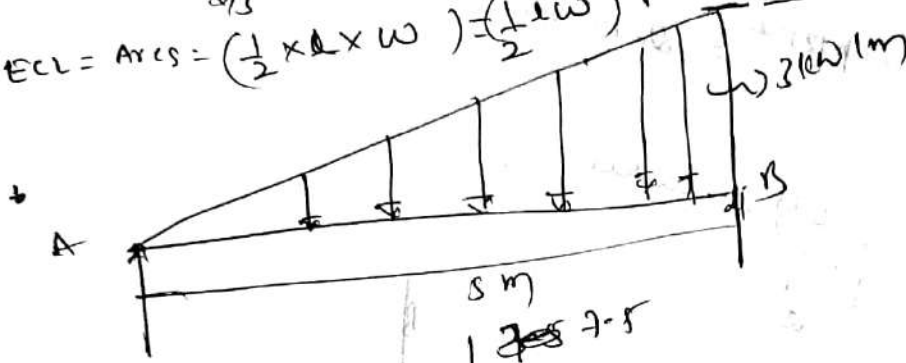
$$\text{At } x=b, \quad M_0 \left[1 - \frac{a}{L} \right] = \frac{M_0 b}{L}$$

$$\text{At } x=L \quad M_0 \left[1 - \frac{L}{L} \right] = 0$$

Beams subjected to uniformly varying load.



$$ECL = \text{Area} = \left(\frac{1}{2} \times L \times w\right) = \frac{1}{2} Lw \text{ N or kN}$$



$$\sum F_x = 0$$

$$R_{AV} + R_{BV} - 7.5 = 0$$

$$R_{AV} + R_{BV} = 7.5 \text{ kN}$$

$$\sum M_A = 0$$

$$+ 7.5 \times 3.33 - R_{BV} \times 5 = 0$$

$$7.5 \times 3.33 = R_{BV} \times 5$$

$$R_{BV} = 4.995 \text{ kN}$$

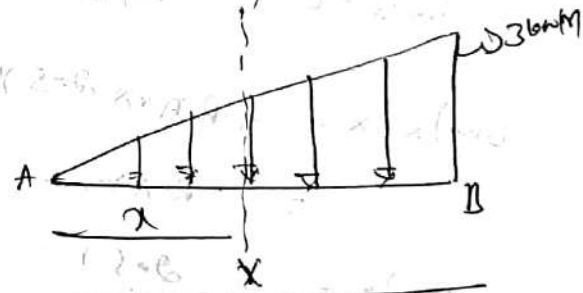
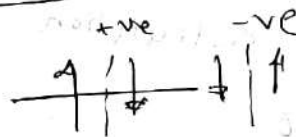
$$R_{AV} = 2.505 \text{ kN}$$

$$SF)_A = +R_{AV} = 2.505 \text{ kN}$$

$$SF)_C = +R_{AV} = 2.505 \text{ kN}$$

$$SF)_R = R_{AV} - 7.5 = 2.505 - 7.5 = -4.995$$

$$SF)_B = -R_{BV} = -4.995 \text{ kN}$$



By applying similar triangles

$$5m - 3$$

$$x = y$$

$$y = \frac{3x}{5}$$

$$ECL = \frac{1}{2} \times x \times \left(\frac{3x}{5}\right)$$

$$= \frac{1}{2} \times x^2 \times \left(\frac{3}{5}\right)$$

$$= 0.3x^2$$

$$SF)_{x=x} = +VA - \left[\frac{1}{2} x \left(\frac{3x}{5} \right) \right]$$

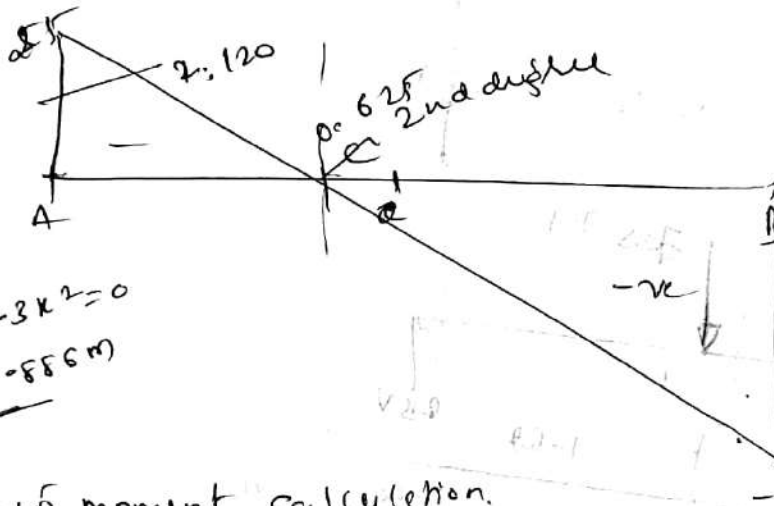
$$= 2.5 - 0.3x^2$$

$$SF)_{A, x=0} = 2.5$$

$$SF)_{C, x=2.5} = 2.5 - 0.3(2.5)^2 = 0.625 \text{ kN}$$

$$SF)_{B, x=5} = 2.5 - 0.3(5)^2 = -5 \text{ kN}$$

$$SF)_{1.125} = 2.5 - 0.3(1.125)^2 = 2.126 \text{ kN}$$



$$SF = 2.5 - 0.3x^2 = 0$$

$$x = 2.886 \text{ m}$$

Bending moment calculation.

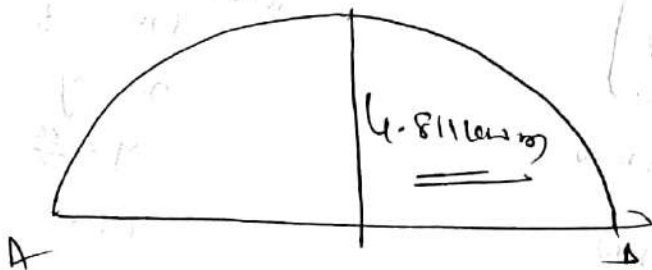
$$BM)_{A} = 0 \quad BM)_{B} = 0$$

$$BM)_{x-x} = +R_A \times x - \left[\frac{1}{2} x \left(\frac{3x}{5} \right) \right] \times \left[\frac{1}{3} x \right]$$

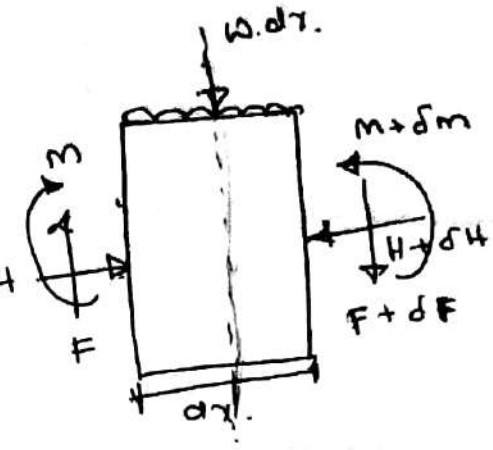
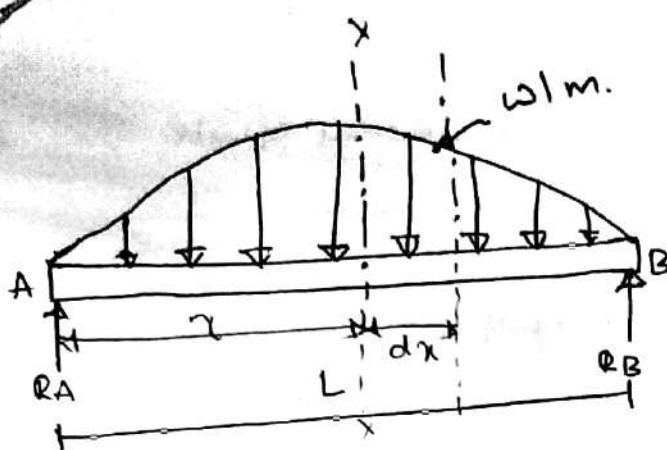
$$= R_A x - 0.1x^3$$

$$BM)_{max} = 2.5x - 0.1x^3$$

$$2.886 = 4.811 \text{ kNm}$$



Derive the Relationship between Intensity of load (W), Shear force and bending moment (M).



Consider a simply supported beam loaded as shown in fig. Let consider an elemental strip of length dx of the beam at a distance x from the left hand support A. As the length of strip dx is very small the load intensity can be considered to be uniform over this length.

When the section dx of the beam is separated the internal stresses will appear on both ends of the system. These internal stresses the resultants are M, H and F on the left hand side section and $m+dM, H+dH$ & $f+dF$ at the right hand side section. dM, dH & dF shows only the increment or decrement in the values from those at the left.

We have equations of equilibrium conditions

$$\sum V = 0, \quad \sum H = 0, \quad \text{and } \sum M = 0 \text{ for this free body}$$

$$\sum V = 0 \quad \text{ive } \text{cancelled}$$

$$- (W \times dx) - (F + dF) = 0$$

$$- W dx - dF = 0$$

$$\boxed{\frac{dF}{dx} = -W}$$

By taking moment on the elemental strip at the right side $\sum M = 0$ +ve same point.

$$+ F dx + M - (M + dM) - W dx \times \frac{dx}{2} = 0$$

Neglecting the small quantity of higher order

We get

$$F dx + M^0 - M^0 - dm = 0$$

$$F dx = dm$$

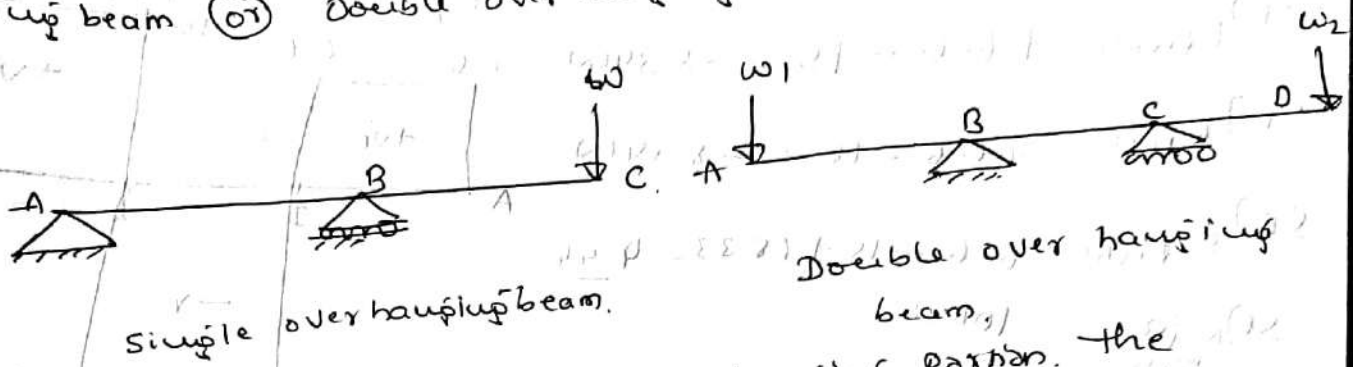
$$F = \frac{dm}{dx}$$

Hence rate of change of moment is always equal to shear force.

Overhanging Beams

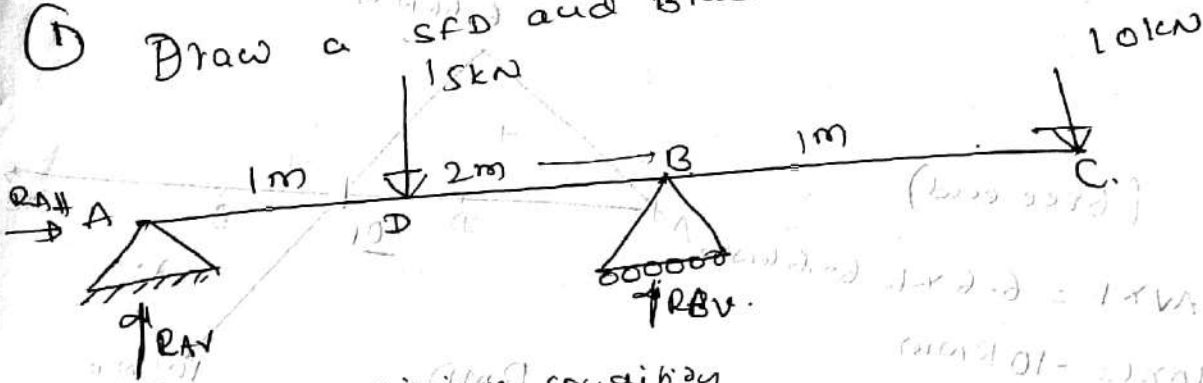
When a beam is extended beyond a support, it is referred as overhanging beam & it may be single overhanging beam or double overhanging beam.

Ex



If any load is acting on the overhanging portion, the moment at the support will not be zero.

① Draw a SFD and BMD for the overhanging beam.



Apply the equilibrium condition

$$\sum F_x = 0 \quad R_{AH} = 0$$

$$\sum F_y = 0 \quad +R_{AV} - 15 + R_{BV} - 10 = 0$$

$$\boxed{R_{AV} + R_{BV} = 25 \text{ kN}} \quad \rightarrow \text{①}$$

$$\sum M_A = 0$$

$$+15 \times 1 - R_{BV} \times 3 + 10 \times 4 = 0$$

$$15 + 40 = R_{BV} \times 3 \quad (10 \times 4 = 40)$$

$$55 = R_{BV} \times 3$$

$$\boxed{R_{BV} = 18.33 \text{ kN}}$$

$$R_{AV} + 18.33 = 25$$

$$\boxed{R_{AV} = 6.66 \text{ kN}}$$

Shear force calculation

$$SF_A = +RAV = 6.6 \text{ kN}$$

$$SF)_{D \text{ left}} = +6.6 \text{ kN}$$

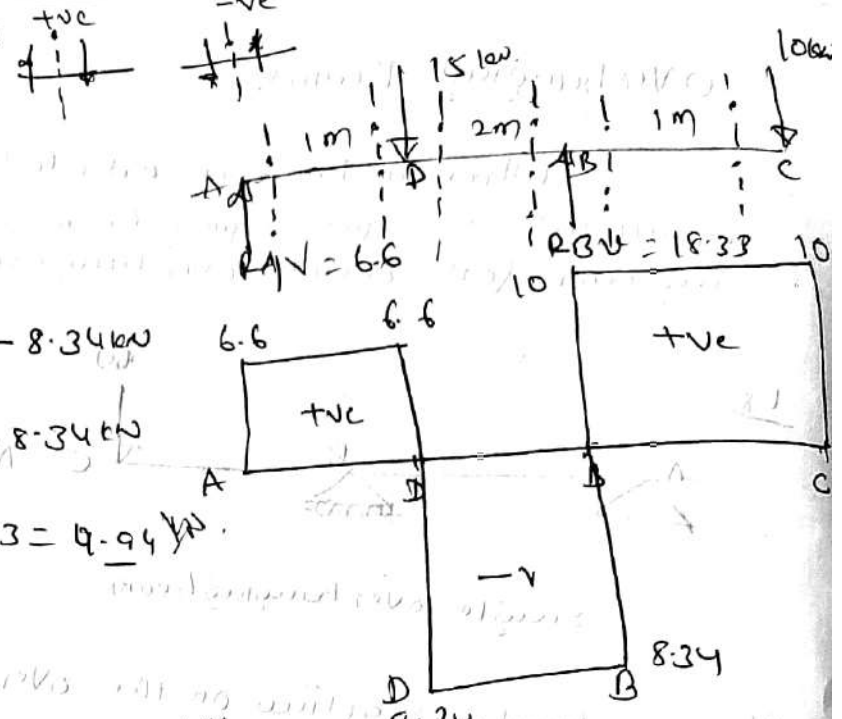
$$SF)_{D \text{ right}} = +6.6 - 15 = -8.34 \text{ kN}$$

$$SF)_{B \text{ left}} = +6.6 - 15 = -8.34 \text{ kN}$$

$$SF)_{B \text{ right}} = 6.6 - 15 + 18.33 = 9.94 \text{ kN}$$

$$SF)_{C \text{ right}} = 10 \text{ kN}$$

$$SF)_{C} = 10 \text{ kN}$$



Bending moment calculation

$$BM)_A = 0$$

$$BM)_C = 0 \text{ (free end)}$$

$$BM)_D = +RAV \times 1 = 6.6 \times 1 = 6.6 \text{ kNm}$$

$$BM)_B = -10 \times 1 = -10 \text{ kNm}$$

$$\textcircled{0} \quad \begin{aligned} RAV \times 3 - 15 \times 2 &= 0 \\ 6.6 \times 3 - 30 &= -10 \text{ kNm} \end{aligned}$$

Note: O_1 is the point of zero bending moment which can be located with the conditions

$$BM)_{O_1} \text{ (or) } BM)_{x} = 0$$

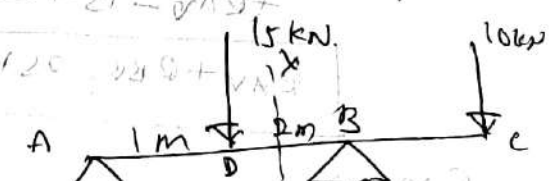
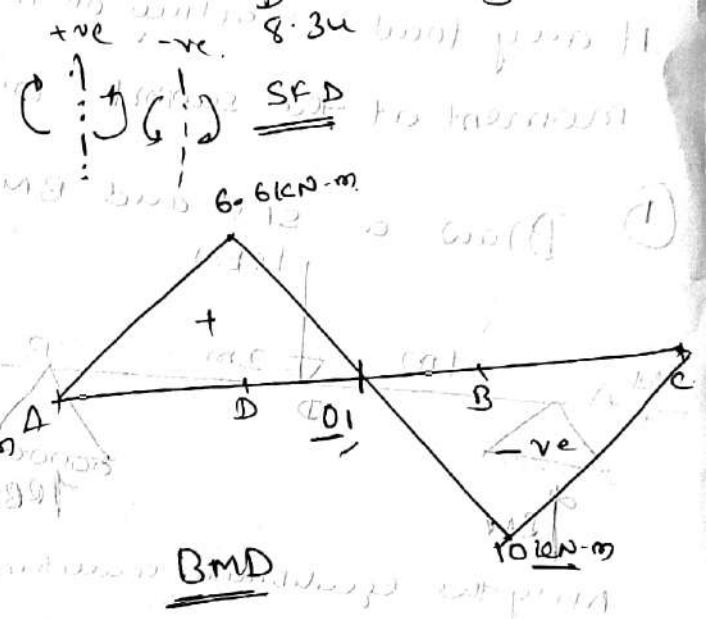
$$RAV \times x - 15 \times (x-1) = 0$$

$$6.6x - 15x + 15 = 0$$

$$+ \uparrow 15x - 15 = 6.6x$$

$$8.4x = 15$$

$$x = 1.8 \text{ m}$$

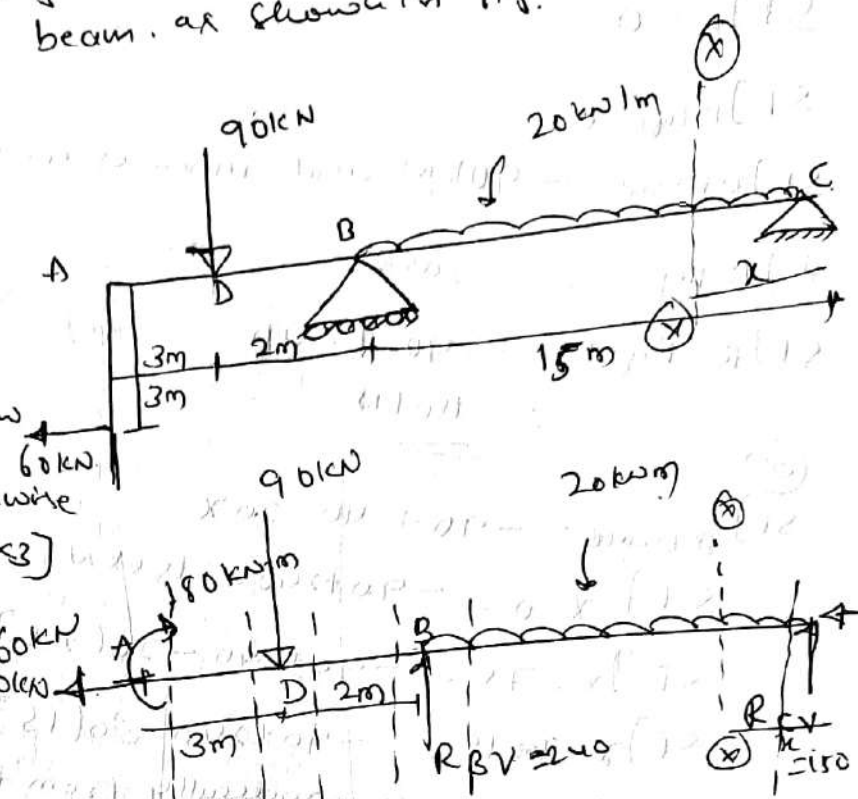


$$\begin{aligned} RAV \times x - 15(x-1) &= 0 \\ 6.6x - 15x + 15 &= 0 \\ + \uparrow 15x - 15 &= 6.6x \\ 8.4x &= 15 \\ x &= 1.8 \text{ m} \end{aligned}$$

2) Draw the bending moment & shearing force diagram for the overhanging beam, as shown in fig.

sol

A horizontal force of 60kN acting horizontally 3m below A is replaced by a clockwise moment of magnitude $(60 \times 3) = 180 \text{ kNm}$ together with a horizontal force of 60kN at A as shown in fig.



$$\sum F_x = 0$$

$$-60 + R_{AH} = 0$$

$$R_{AH} = -60 \text{ kN}$$

$$\sum F_y = 0$$

$$-90 + R_{BV} + R_{CV} - 300 = 0$$

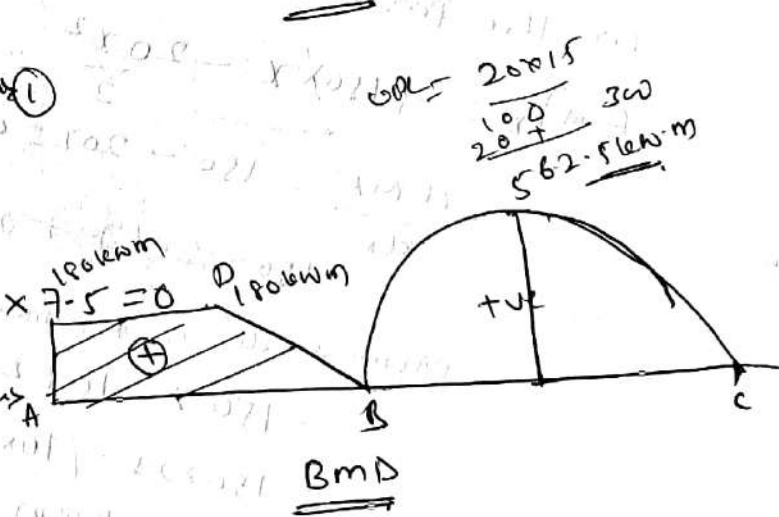
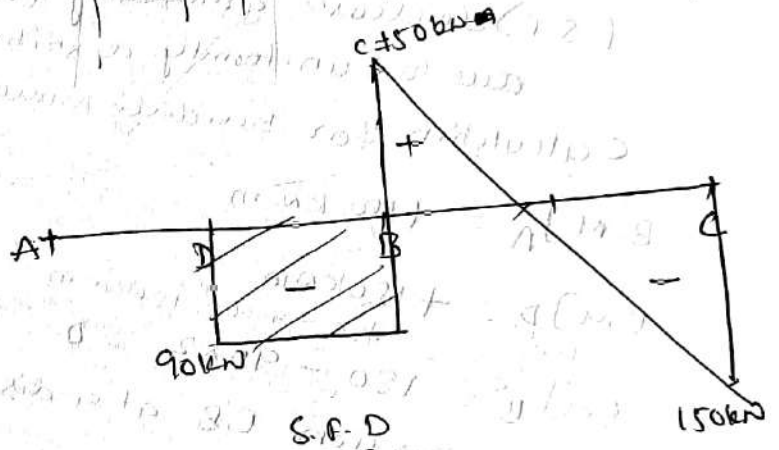
$$R_{BV} + R_{CV} = 390 \text{ kN} \quad (1)$$

$$\sum M_c = 0$$

$$+180 - 90 \times 17 + R_{BV} \times 15 - 20 \times 15 \times 7.5 = 0$$

$$R_{BV} = 240 \text{ kN}$$

$$R_{CV} = 150 \text{ kN}$$



Calculation for SFD

SF]A = 0

SF]D left = 0

SF]D Right = -90 kN and remains constant up to B

SF]B left = -90 kN

SF]B Right = -90 + 240
= 150 kN

(00)

SF] x from B = -90 + 240 - 20x

S.F] x=0 = -90 + 240 = 150 kN

S.F] x=7.5 = -90 + 240 - 20(7.5) = 0

S.F] x=15 = -90 + 240 - 20(15) = -150 kN

[S.F] decreases gradually from +150 kN at B to -150 kN at C due to uniformly distributed load.

Calculation for Bending moment.

B.M]A = +180 kN-m

B.M]D = +180 kN-m
(left) = 180 - 90x2 = 0

B.M]B = 180 - 90x2 = 0

In the portion CB at a distance x from the right end C

B.M]x = +150x - 20x^2/2 = 150x - 10x^2

dM/dx = 150 - 20x = 0

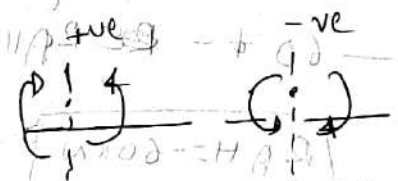
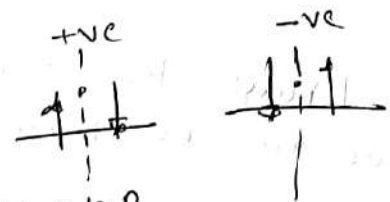
Magnitude of Maximum Bending moment

= 150x - 10x^2

= 150x7.5 - 10x7.5^2 = 562.5 kN-m

B.M]x =

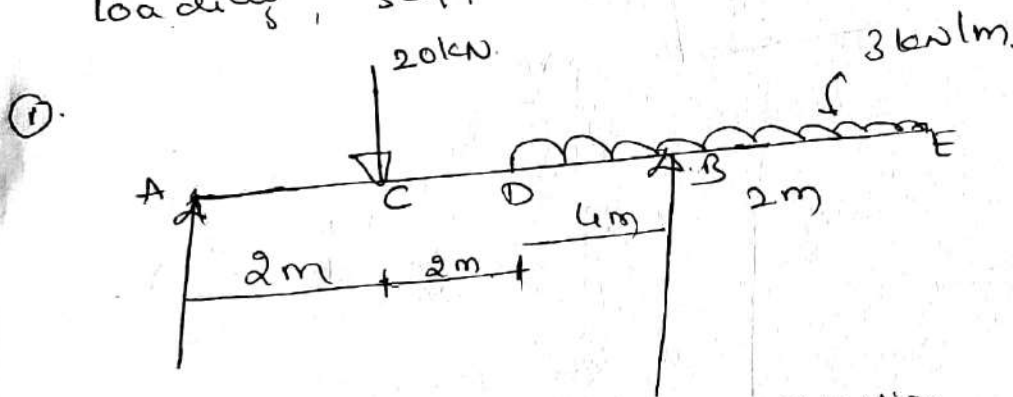
B.M]x = 150x - 20x^2/2
= 150x - 10x^2



Point of contraflexure [Inflexion] moment

It's a point for a bending moment diagram where the bending moment changes its sign (i.e. negative to positive) or [Positive to negative]. It is also referred as the point of inflexion. The bending moment at this point will be zero. i.e. The bending moment changes from sagging to humping (or) vice versa.

Point of contraflexure may be depending on type of loading, support and the type of beam.



Point
 $3 \times 6 = 18 \text{ kN}$

Let R_{AV} and R_{BV} are the support reaction

Apply equilibrium condition

$$\sum F_y = 0$$

$$+R_{AV} - 20 + R_{BV} - 18 = 0$$

$$\boxed{R_{AV} + R_{BV} = 38 \text{ kN}} \rightarrow \text{①}$$

Making moment about A.

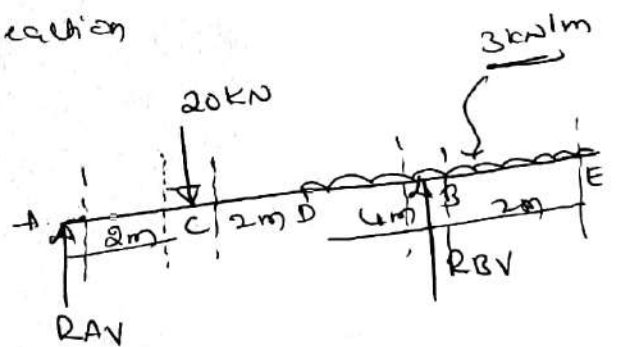
$$+ 20 \times 2 - R_{BV} \times 8 + 18 \times 7 = 0$$

$$\cancel{R_{BV} \times 8} = \boxed{R_{BV} = 20.75 \text{ kN}}$$

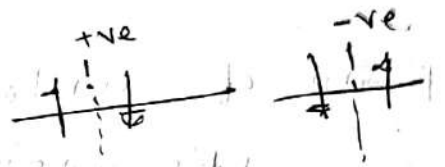
Sub in eq ① we get

$$R_{AV} + 20.75 = 38$$

$$\boxed{R_{AV} = 17.25 \text{ kN}}$$



S. F calculations



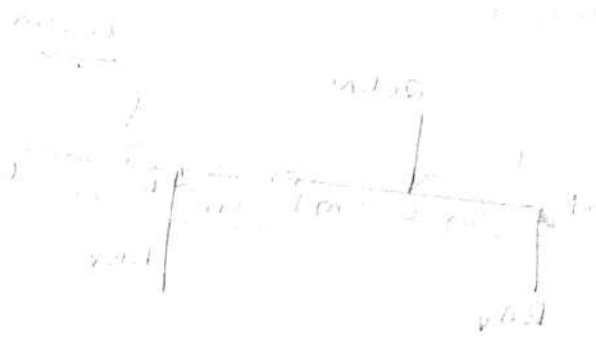
$SF]_A = 17.25 \text{ kN}$

$SF]_C \text{ left} = 17.25 \text{ kN}$ constant from A to C

$SF]_C \text{ right} = 17.25 - 20 = -2.75 \text{ kN}$

$SF]_D \text{ left} = 17.25 - 20$

Point of contraflexure is where the bending moment is zero.

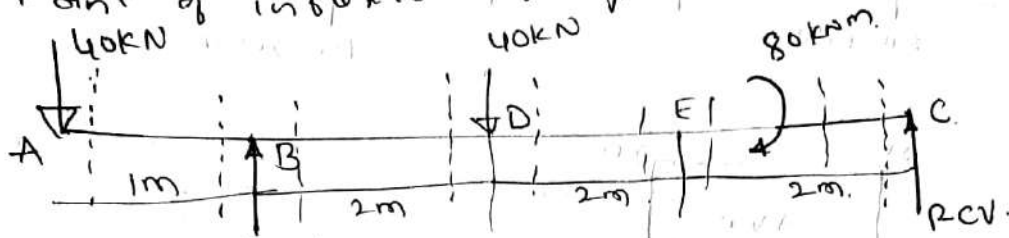


By balance of forces in vertical direction
 $\sum F_v = 0$
 $R_A - 20 = 0$
 $R_A = 20 \text{ kN}$

By balance of moments about A
 $\sum M_A = 0$
 $M_C - 20 \times 2 = 0$
 $M_C = 40 \text{ kNm}$

②

Draw the B.M & S.F. diagrams for the beam as shown in fig. indicating the salient values on them. Locate the Point of inflexion if any.



Applying the RBV equilibrium condition
 $\sum F_y = 0$

$$-40 + R_{BV} - 40 + R_{CV} = 0$$

$$R_{BV} + R_{CV} = 80 \text{ kN} \quad \text{--- (1)}$$

$\sum M_C = 0$

$$+80 - 40 \times 7 + R_{BV} \times 6 - 40 \times 4 = 0$$

$$R_{BV} \times 6 = 440 + 80$$

$$R_{BV} = 60 \text{ kN}$$

$$60 + R_{CV} = 80$$

$$R_{CV} = 20 \text{ kN}$$

Shear force calculations.

$$SF]_A = -40 \text{ kN}$$

$$SF]_{B \text{ left}} = -40 \text{ kN}$$

$$SF]_{B \text{ right}} = -40 + R_{BV} = -40 + 60 = 20 \text{ kN}$$

$$SF]_{D \text{ left}} = 20 \text{ kN}$$

$$SF]_{D \text{ right}} = -40 + 60 - 40 = -20 \text{ kN}$$

$$SF]_C = -20 \text{ kN}$$

Bending Moment calculations

$$BM]_A = 0$$

$$BM]_B = -40 \times 1 = -40 \text{ kNm}$$

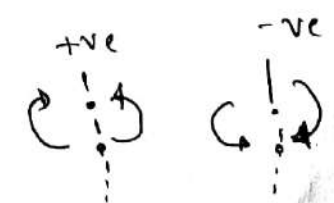
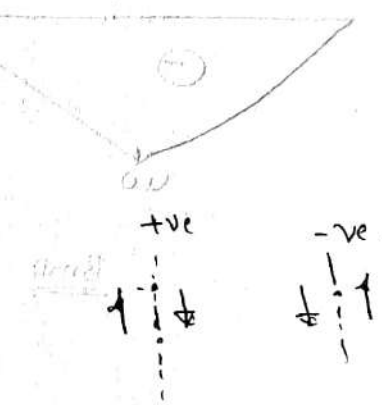
$$BM]_D = -40 \times 3 + R_{BV} \times 2 = -120 + 60 \times 2 = 0$$

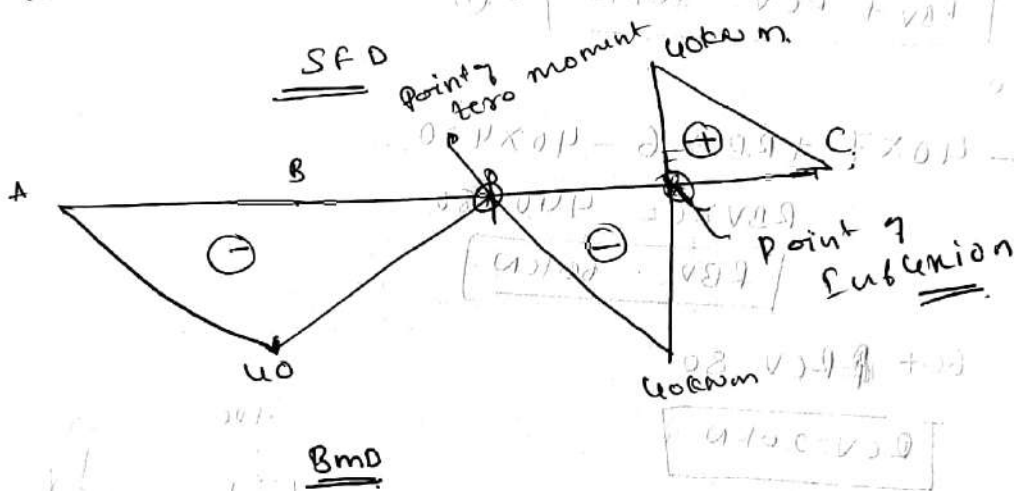
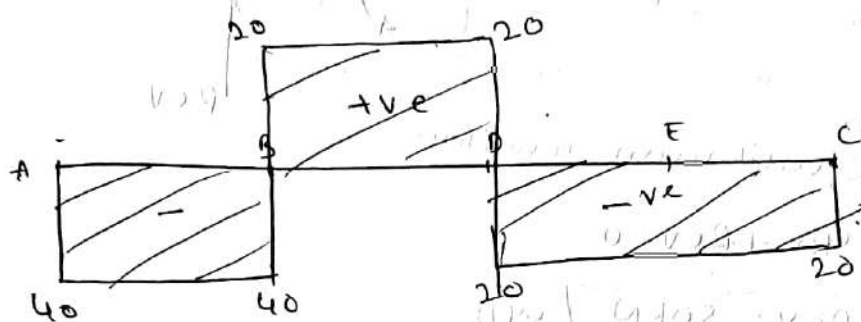
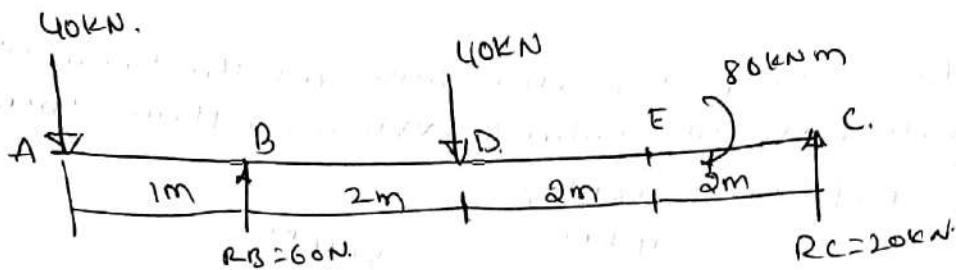
$$BM]_{E \text{ towards left}} = -40 \times 5 + 60 \times 4 - 40 \times 2 = -40 \text{ kNm}$$

$$BM]_{E \text{ towards right}} = +60 \times 4 - 40 \times 5 + 40 \times 2 + 80 = 40 \text{ kNm}$$

$$BM]_C = 0$$

$$= +40 \text{ kNm}$$





Draw the bending moment & shearing force diagrams for the beam shown in fig. Determine the position of maximum shearing force and maximum bending moment and the position of the point of contraflexure.

Apply the equilibrium condition

$$\sum F_y = 0$$

$$-40 + R_{AV} + R_{BV} - 160 - 20 = 0$$

$$R_{AV} + R_{BV} = 220 \text{ kN}$$

$$\sum M_B = 0$$

$$-40 \times 11 + R_{AV} \times 8 - 160 \times 4 + 20 \times 2 = 0$$

$$R_{AV} = 130 \text{ kN}$$

Substituting R_{AV} value in eq (1)

We get

$$130 + R_{BV} = 220$$

$$R_{BV} = 220 - 130$$

$$R_{BV} = 90 \text{ kN}$$

Shear force calculations

$$SF)_C = -40 \text{ kN}$$

$$SF)_A = -40 \text{ kN}$$

$$SF)_A \text{ Right} = -40 + 130 = 90 \text{ kN}$$

$$SF)_B \text{ left} = 90 + 20 = 110 \text{ kN}$$

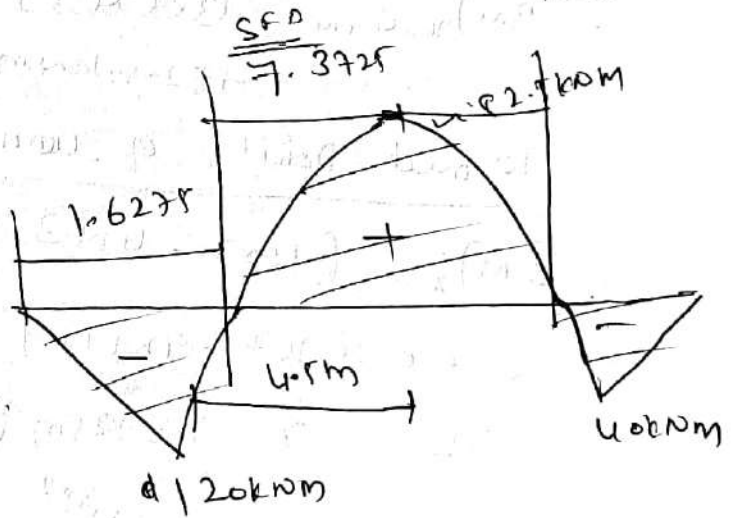
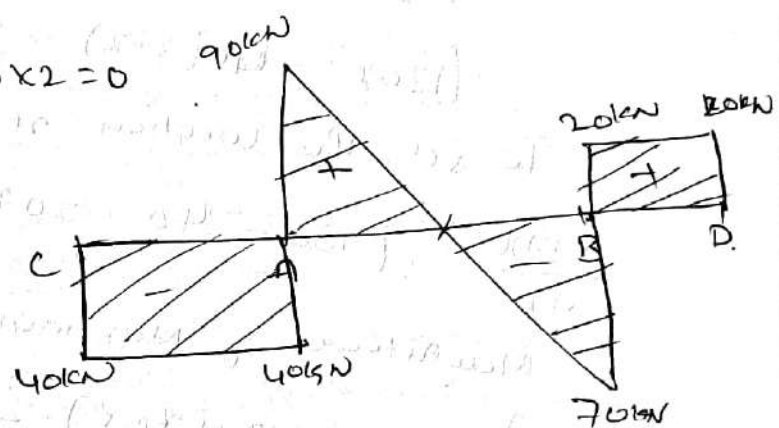
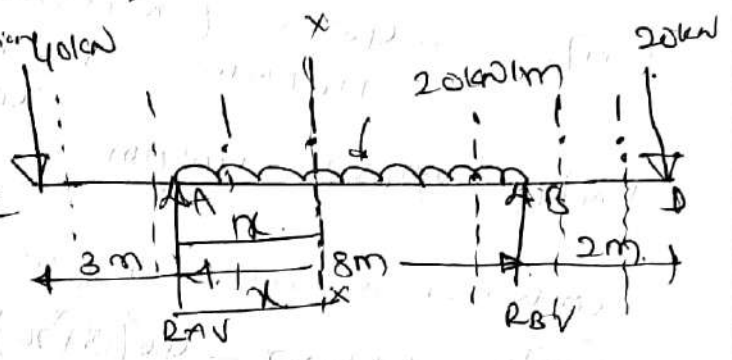
$$SF)_B \text{ right} = 20$$

$$SF)_B \text{ left} = +20 - 90 = -70 \text{ kN}$$

$$SF)_B \text{ right} = -40 + 130 - 160 + 90 = 20 \text{ kN}$$

$$SF)_D = 20 \text{ kN}$$

$$SF)_D \text{ left} = -40 + 130 - 160 + 90 = 20 \text{ kN}$$



$$SF)_B \text{ left} = -40 + 130 - 160 + 90 = 20 \text{ kN}$$

$$SF)_B \text{ right} = -40 + 130 - 160 + 90 = 20 \text{ kN}$$

Bending moment calculations

$$B.M]_C = B.M]_D = 0 \quad [\text{Since } C \text{ and } D \text{ are free ends}]$$

$$B.M]_A = -40 \times 3 = -120 \text{ kNm} \quad \left[\begin{array}{l} \text{Negative since to the left of the section,} \\ \text{the moment acting is anticlockwise} \\ \text{direction} \end{array} \right]$$

$$B.M]_B = +20 \times 2 = 40 \text{ kNm} +$$

Consider a section xx at a distance x from support A,

$$B.M]_x = +130x - 40(3+x) - 20(x) \left(\frac{x}{2} \right) \quad 3 < x < 8$$

$$= (130x - 40(3+x) - 10x^2)$$

To get the location at which bending moment is maximum

$$\frac{dM_x}{dx} = (130 - 40 - 20x) = 0 \quad \text{i.e. } x = 4.5 \text{ m}$$

Magnitude of maximum bending moment.

$$B.M]_{x=4.5 \text{ m}} = 130(4.5) - 40(3+4.5) - 10(4.5)^2$$

$$= +82.5 \text{ kNm}$$

To find point of contraflexure.

$$B.M]_x = (130x - 40(3+x) - 10x^2) = 0 = x^2 + 9x - 12$$

$$\text{i.e. } (x^2 - 9x + 12) = 0$$

$$\text{Sol} \quad x_1 = 7.372 \text{ m} \quad \text{and} \quad x_2 = 1.627 \text{ m}$$

$$130x - 40(3+x) - 10x^2 = 0$$

$$130x - 120 - 40x - 10x^2 = 0$$

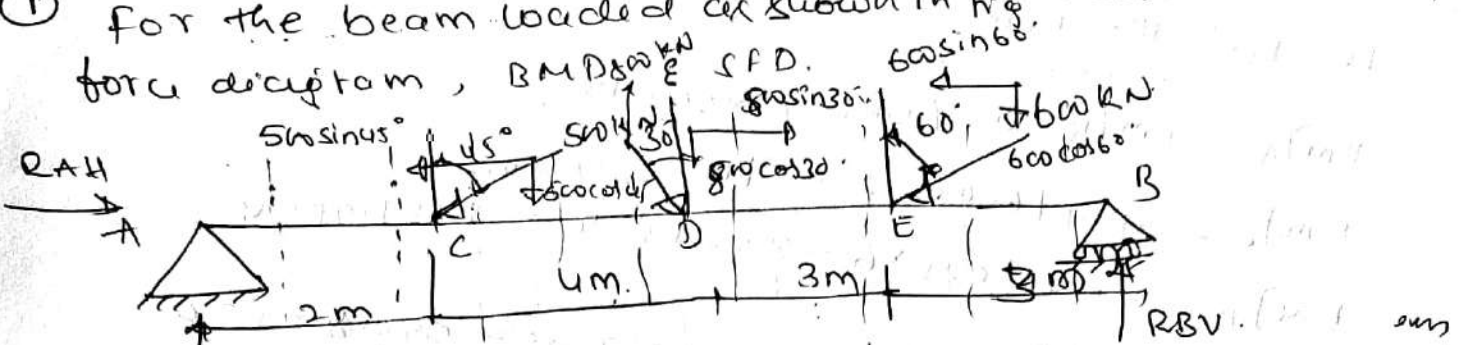
$$+10x^2 - 90x - 120 = 0$$

$$130x - 40(3+x) - 10x^2 = 0$$

Bending moment at free end is zero

Inclined loading on Beams

① For the beam loaded as shown in fig. draw the axis force diagram, BMD and SFD.



Applying the equilibrium.

$$\sum F_y = 0$$

$$R_{AV} + R_{BV} - 500 \cos 45 - 800 \cos 30 - 600 \cos 60 = 0$$

$$R_{AV} + R_{BV} = 1346.37 \text{ kN} \quad \text{--- (1)}$$

$$\sum M_A = 0$$

$$500 \cos 45 \times 2 + 800 \cos 30 \times 6 + 600 \times 0.866 \times 9 - R_{BV} \times 12 = 0$$

$$(353.55 \times 2) + (692.82 \times 6) + (300 \times 9) = R_{BV} \times 12$$

$$R_{BV} = 630.325 \text{ kN}$$

Substituting R_{BV} in eq (1)

$$R_{AV} + 630.325 = 1346.37$$

$$R_{AV} = 716.025 \text{ kN}$$

$$\sum F_x = 0$$

$$R_{AH} = 500 \sin 45 + 800 \sin 30 + 600 \sin 60 = 0$$

$$R_{AH} = 473.16 \text{ kN}$$

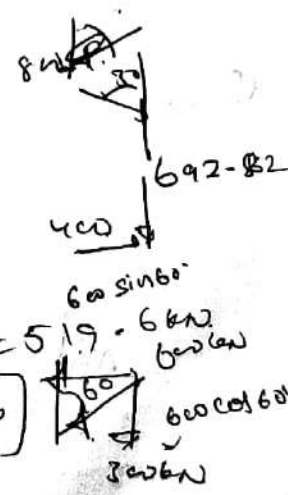
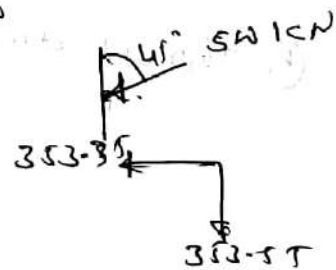
Calculation for S.F.D.

$$SF]_A = +R_{AV} = +716.025 \text{ kN}$$

$$SF]_{C \text{ right}} = +R_{AV} - 353.55 = +362.475 \text{ kN}$$

$$SF]_{D \text{ right}} = +362.475 - 692.82 = -330.325 \text{ kN}$$

$$SF]_{E \text{ right}} = -330.325 - 300 = -630.325 \text{ kN}$$



To plot the thrust diagram.

(Thrust at support B) = 0 (B is the roller support)

$$\text{Thrust}]_E = 519.6 \text{ kN} \text{ \& \#x2013; constant upto E}$$

$$\text{Thrust}]_D = +519.6 - 400 = +119.6 \text{ kN}$$

$$\text{Thrust}]_C = +119.6 \text{ kN} + 353.55 = 473.15 \text{ kN}$$

constant upto C

Thrust $T_A = 473.16 \text{ kN}$.

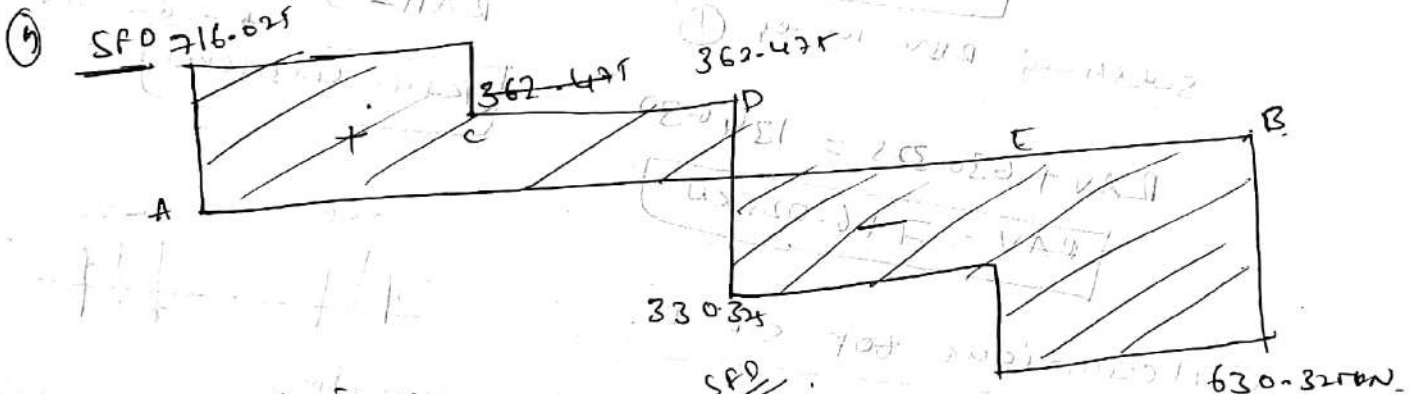
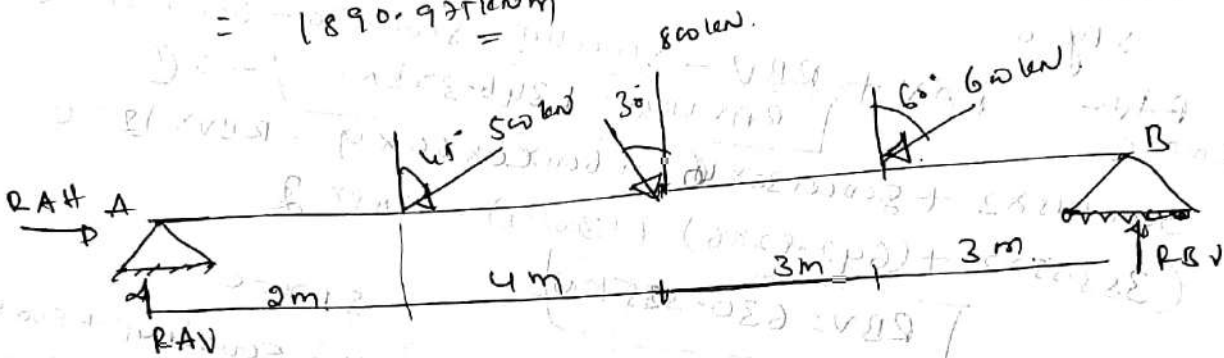
To Plot the BMD.

$BM_A = BM_B = 0$ [Simply supported]

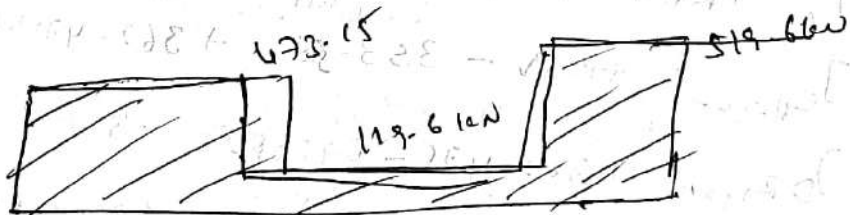
$BM_C = +716.025 \times 2 = 1432.05 \text{ kNm}$

$BM_D = +716.025 \times 6 - 353.55 \times 4 = 2881.95 \text{ kNm}$

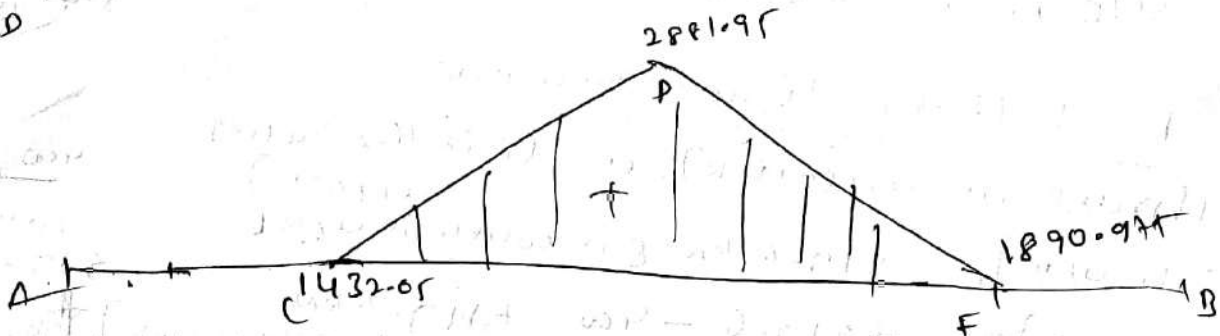
$BM_E = +630$
 $716.025 \times 9 - 353.55 \times 7 - 692.8 \times 3 = 0$
 $= 1890.975 \text{ kNm}$

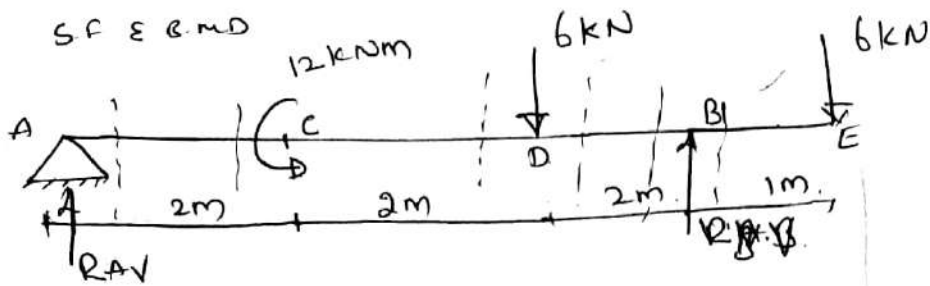


⑤ Thrust diagram



⑥ BMD





$$\sum F_y = 0$$

$$+R_{AV} + R_{BV} - 6 - 6 = 0$$

$$R_{AV} + R_{BV} = 12 \text{ kN}$$

$$\sum M_A = 0$$

$$-12 + (6 \times 4) - R_{BV} \times 6 + (6 \times 7) = 0$$

$$-12 + 24 + 42 - R_{BV} \times 6 = 0$$

$$R_{BV} = 9 \text{ kN}$$

$$R_{AV} + 9 = 12$$

$$R_{AV} = 3 \text{ kN}$$

SF calculations

$$SF)_A = +3 \text{ kN}$$

$$SF)_D = +3 \text{ kN}$$

$$SF)_D = +3 - 6 = -3 \text{ kN}$$

$$SF)_B = -3 \text{ kN}$$

$$SF)_D = +3 - 6 + 9 = +6 \text{ kN}$$

$$SF)_B = +6 - 9 = -3 \text{ kN}$$

$$SF)_E = 6 \text{ kN}$$

BM calculations

$$BM)_A = 0 \quad BM)_E = 0$$

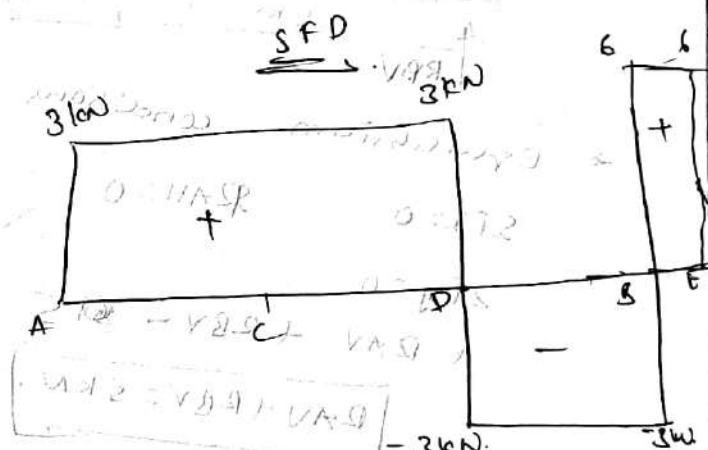
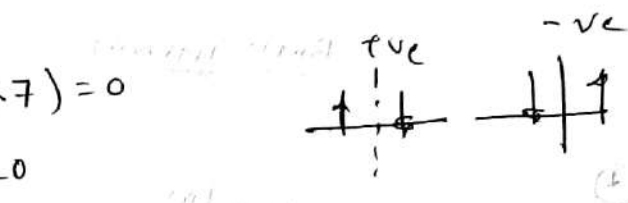
$$BM)_C = +R_{AV} \times 2 = 3 \times 2 = 6 \text{ kNm}$$

$$BM)_C = 6 - 12 = -6 \text{ kNm}$$

$$BM)_D = +R_{AV} \times 4 - 12 = 0$$

$$= 3 \times 4 - 12 = 12 - 12 = 0$$

$$BM)_D = R_{AV} \times 4 - 12 - 6 \times 0 = 0$$

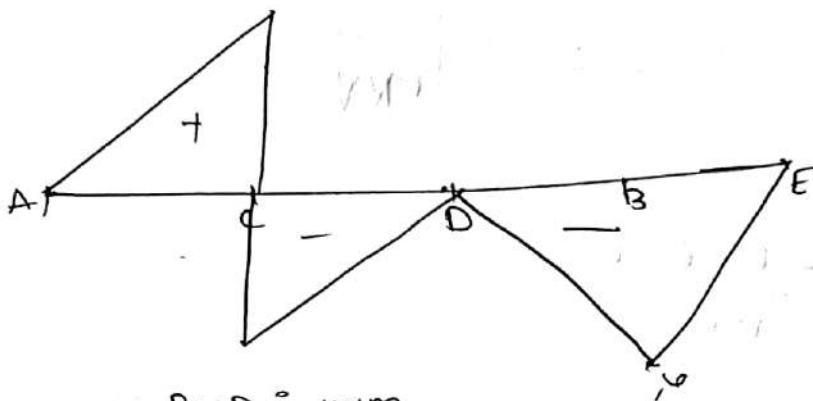


$$BM)_B = R_{AV} \times 6 - 12 - 6 \times 2$$

$$= 3 \times 6 - 12 - 12$$

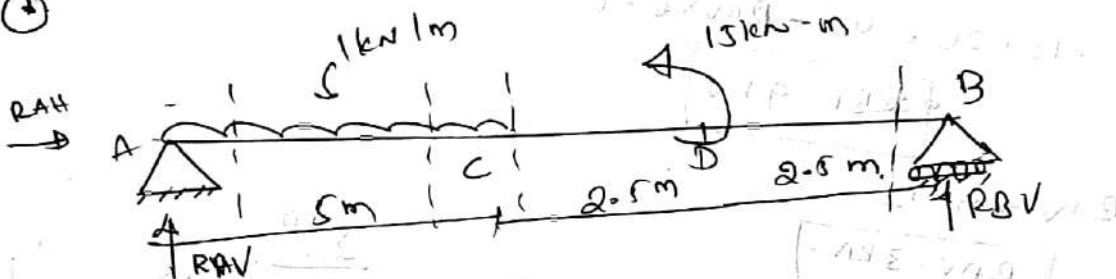
$$= 18 - 12 - 12 = -6 \text{ kNm}$$

$$BM)_B = -6 \text{ kNm}$$



BMD in kNm

①



→ Equilibrium conditions

$$\sum F_x = 0 \quad R_{AH} = 0$$

$$\sum F_y = 0$$

$$+ R_{AV} + R_{BV} - 5 = 0$$

$$\boxed{R_{AV} + R_{BV} = 5 \text{ kN}}$$

$$\sum M_A = 0$$

$$5 \times 2.5 - 15 - R_{BV} \times 10 = 0$$

$$\boxed{R_{BV} = -0.25 \text{ kN}}$$

$$\boxed{R_{AV} = 5.25 \text{ kN}}$$

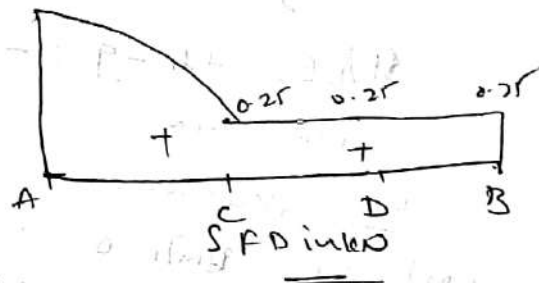
SF calculations

$$SF)_A = + R_{AV} = +5.25 \text{ kN}$$

$$SF)_C = + R_{AV} = 5.25 - 5 = 0.25 \text{ kN}$$

$$SF)_D = +0.25 \text{ kN}$$

$$SF)_B = +0.25 - R_{BV} = +0.25 - (-0.25) = 0.25 \text{ kN}$$



BMD calculations

Bm) A = 0

Bm) B = 0

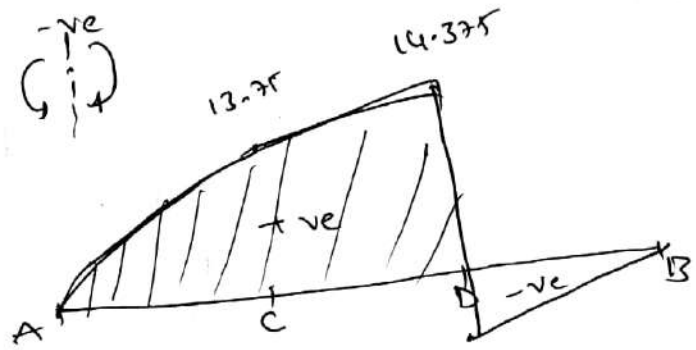
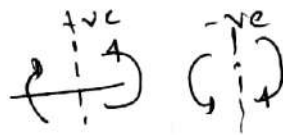
Bm) C = $+RA \times 5 - 5 \times 2.5$
 $= 5.25 \times 5 - 5 \times 2.5 =$

Bm) C = 13.75 kNm

Bm) D = $RA \times 7.5 - 5 \times 5 = 14.375$ kNm

Bm) E D = Bm) D - 15
 $= 14.375 - 15$

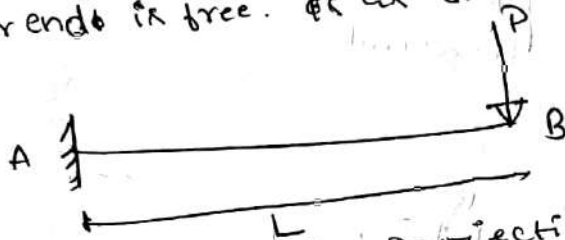
Bm) E D = -0.625 kNm



BMD

Cantilever beam:

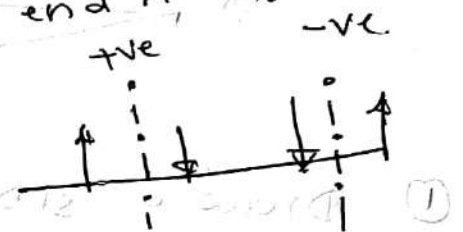
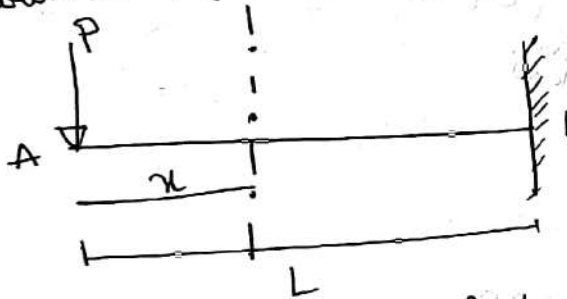
It is a type of beam in which one end is fixed & other end is free. As shown in fig



Example: (1) Chajja projection above the window, flag hoisting pole, cycle stand, & Transmission tower.

Cantilever beam subjected to concentrated beam load at the free end

Consider a cantilever of beam AB, fixed at B and subjected to a concentrated load at free end A, as shown in fig.



Shear force at any point B section a distance x from A

$$SF]_A = -P$$

Hence S.F.D will be rectangle of height P as shown in fig. (9) [S.F. at support is zero]

Bending moment at x , i.e.

$$B.M]_x = -P(x)$$

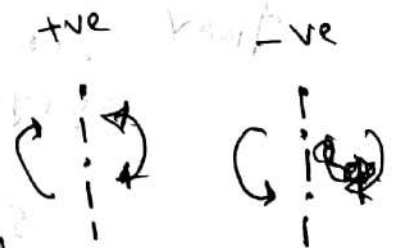
$$B.M]_{x=0} = 0$$

$$B.M]_B = -PL$$

$x=L$

Hence BM varies linearly from zero at the free end to PL at the fixed end.

hence fig (6) show the B.M



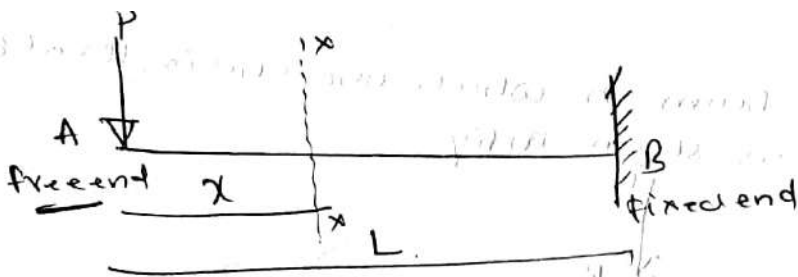


Fig 9 S.F.D.

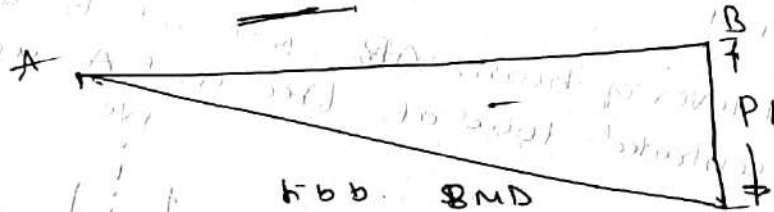
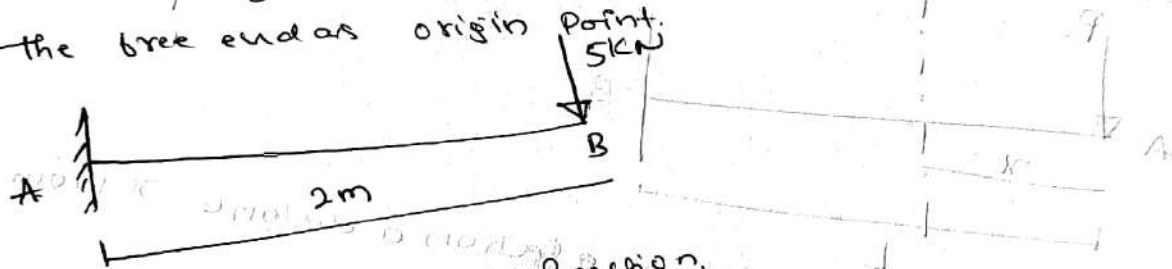
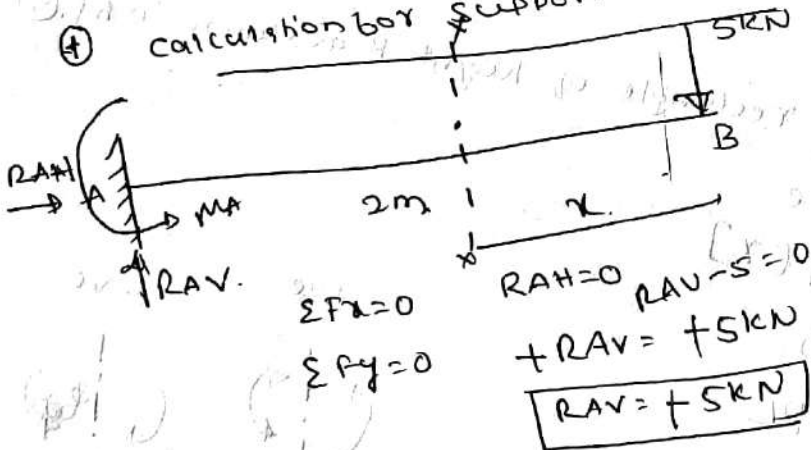


Fig 10 BMD

① Draw a SFD and BMD for the cantilever beam, selecting the free end as origin point.



② Calculation for support Reaction.



$$\sum F_x = 0$$

$$\sum F_y = 0$$

$$\sum M_A = 0$$

$$R_{AH} = 0$$

$$R_{AV} - 5 = 0$$

$$+R_{AV} = +5 \text{ kN}$$

$$\boxed{R_{AV} = +5 \text{ kN}}$$

$$+5 \times 2 = 0 = 10 \text{ kNm}$$

selecting free end as origin the shear force is uniform through out the length with +5 kN as magnitude

Bending moment at any section is $[-5 \times x]$ from the free end

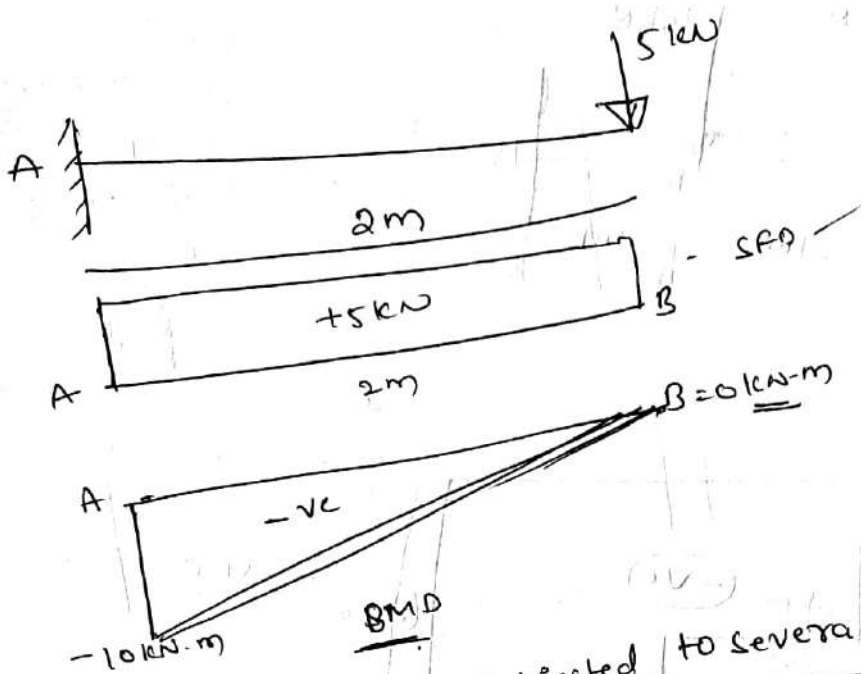
$$BM|_{x-x} = -5 \times x$$

$$x=0$$

$$BM|_B = 0$$

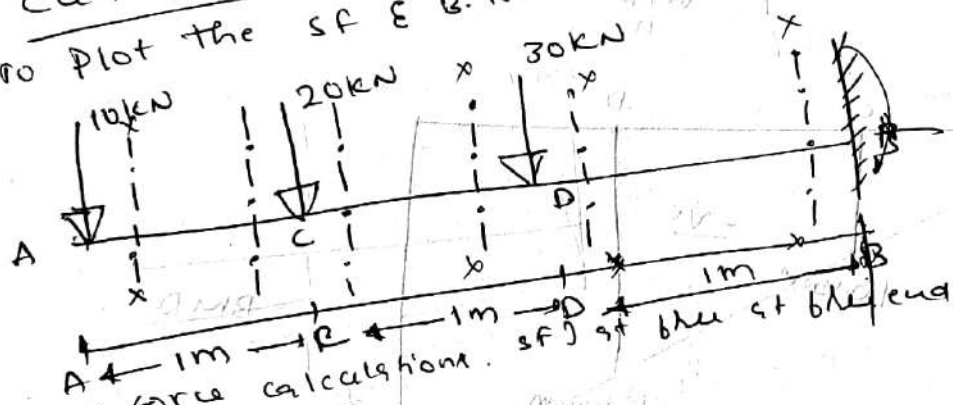
$$x=2$$

$$BM|_A = -5(2) = -10 \text{ kNm}$$



$SF]_A = +5kN$
 $SF]_B = 5kN$
 Upr

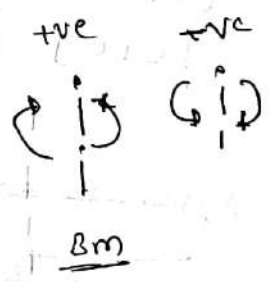
② Cantilever beam subjected to several concentrated loads.
 To plot the SF & B.M. Diagram



$SF_2 = 0$
 $SF_4 = 0$
 $R_{BV} = 0$
 $R_{BV} = -10 - 20 - 30 + R_{BV} = 0$
 $R_{BV} = 60kN$

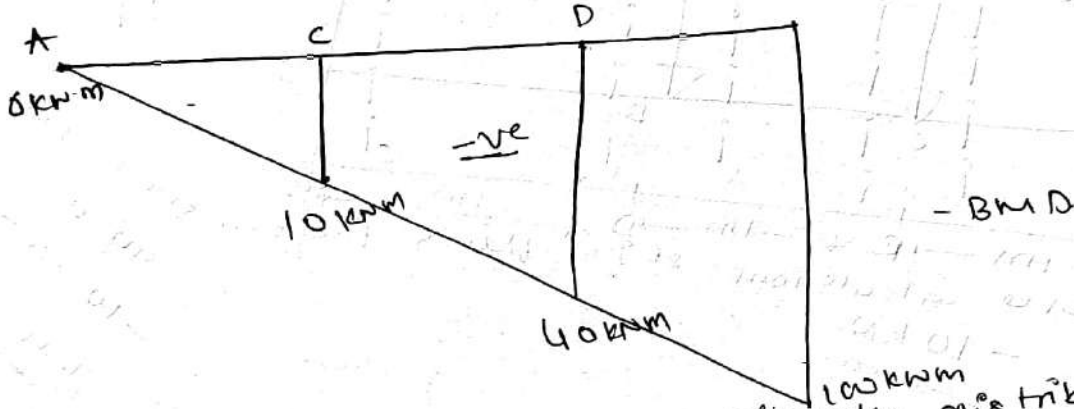
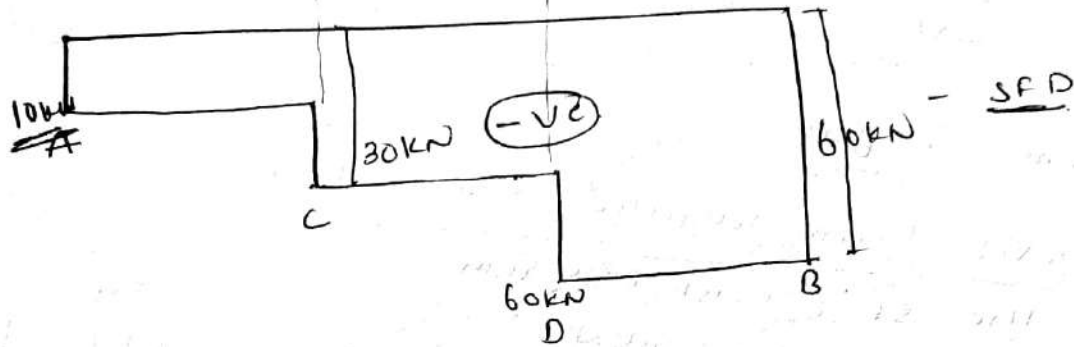
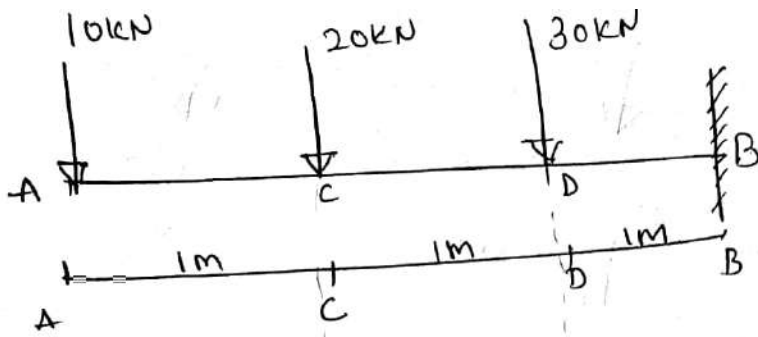
Shear force calculations:
 $SF]_A = -10kN$
 $SF]_{C \text{ left}} = -10kN$
 $SF]_{C \text{ right}} = -10 - 20 = -30kN$
 $SF]_{D \text{ right}} = -10 - 20 = -30kN$
 $SF]_{D \text{ left}} = -10 - 20 - 30 = -60kN$
 $SF]_B \text{ right} = -60kN$

fig 9

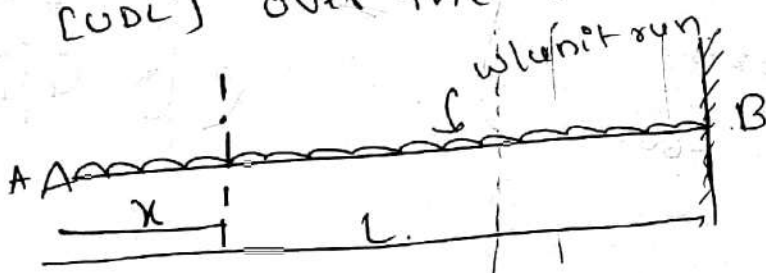


To plot the bending diagram.

$B.M]_A = 0$ [It is free end]
 $B.M]_C = -10 \times 1 = -10kNm$
 $B.M]_D = -10 \times 2 - 20 \times 1 = -40kNm$
 $B.M]_B = -10 \times 3 - 20 \times 2 - 30 \times 1 = -70kNm$



* Cantilever beam subjected to a uniformly distributed load [UDL] over the whole span.



S.F at any section from A.

$$SF]_A = 0$$

$$SF]_x = -w \cdot x$$

$$SF]_{x=L} = -w \cdot L$$

S.F. varies linearly from zero at free end to wL at fixed end.

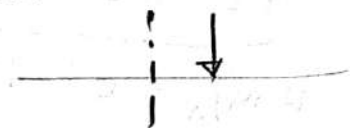
Bending moment at any section x from A.

$$B.M]_x = -(w \cdot x) \cdot \frac{x}{2}$$

$$B.M]_{x=0} = 0$$

$$B.M]_{x=L} = -\frac{wL^2}{2}$$

[Variation of Bending moment is parabolic]



② Shear force calculations

$SF]_A = 0$

$SF]_C = 0$

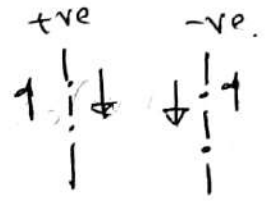
$SF]_D = -3 \text{ kN}$

$SF]_E = -7 \text{ kN}$

$SF]_B = -7 \text{ kN}$

[There is to no loading from A to C. There will not be any shear force & hence the shear force originating from A to C is zero.]

[In B/w D & E, shear force is constant 3 kN, & In B/w shear E & B. Shear force is $3+4=7 \text{ kN}$ is constant magnitude.]



③ Bending moment calculations

$B.M]_A = 0$

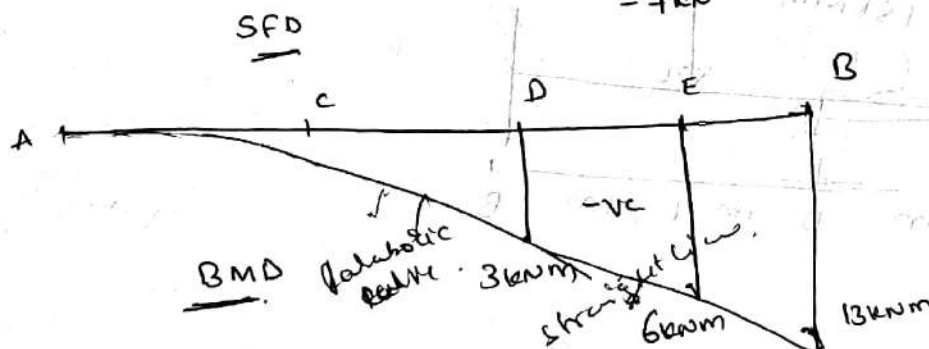
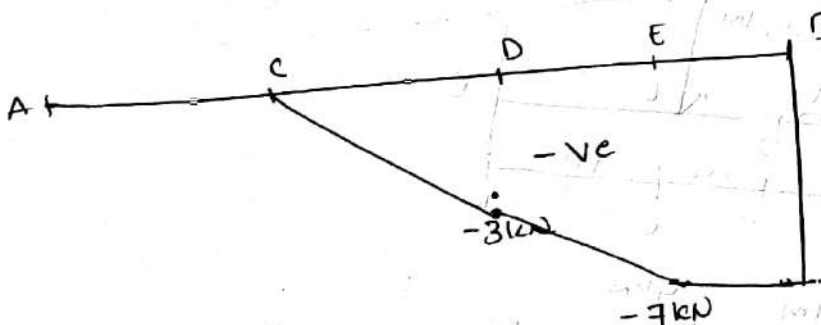
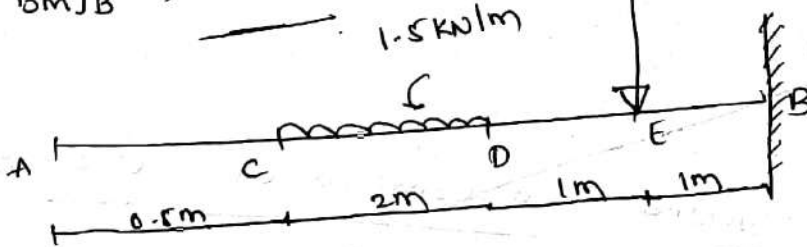
$B.M]_C = 0$

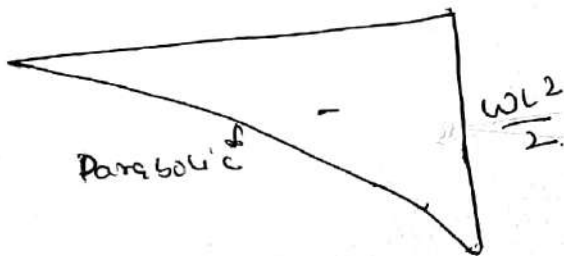
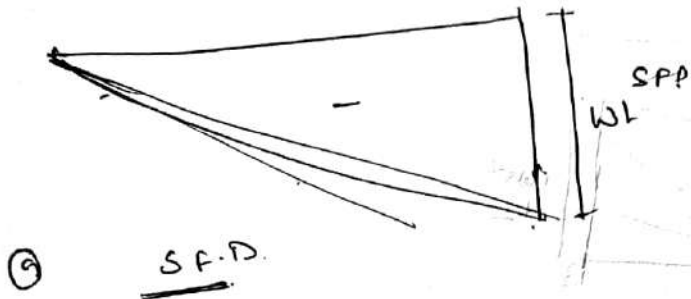
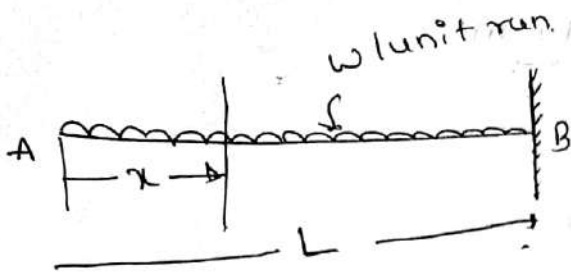
$B.M]_D = [-1.5 \times 2 \times \frac{2}{2}] = -3 \text{ kNm}$

$B.M]_E = -[1.5 \times 2] [1+1] = -6 \text{ kNm}$

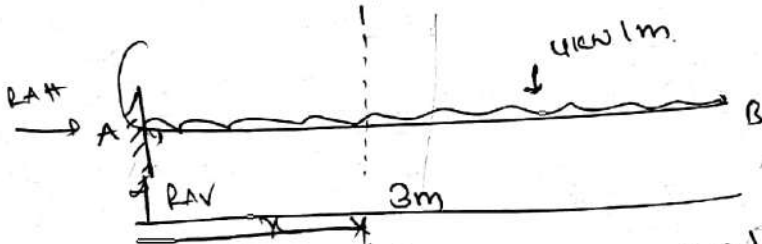
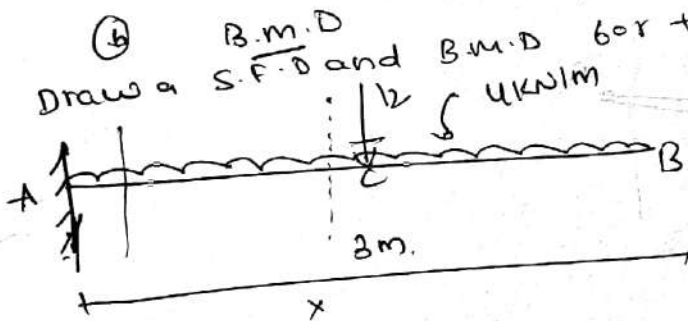
$B.M]_B = -[1.5 \times 2] [1+1+1] = -9 - 4 = -13 \text{ kNm}$

$B.M]_B = -13 \text{ kNm}$





Draw a S.F.D and B.M.D for the given cantilever beam with ud.



Reaction calculations
 $\sum F_x = 0$
 $R_{AH} = 0$
 $\sum F_y = 0$

S.F] $x-x = 4(x)$

SF] $x = +R_{AV} = 12 \text{ kN}$
 SF] $x = R_{AV} - 12 = 0$
 $\Rightarrow R_{AV} = 12 \text{ kN}$

SF] $x=0 = 4 \times 0 = 0$
 SF] $x=3 = +4 \times 3 = 12 \text{ kN}$

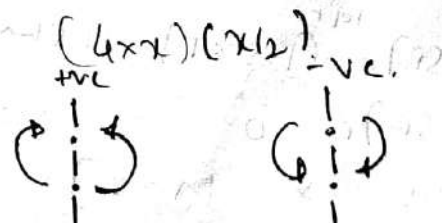
SF] $x = +R_{AV}$



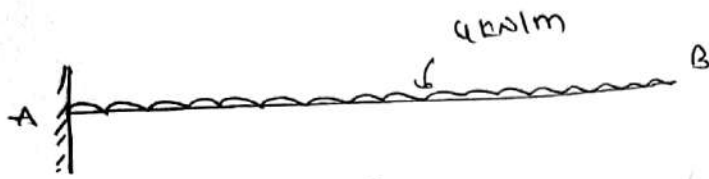
SF calculations, \rightarrow (k)

Bm] $x-x = -4(x)(x/2) = -2x^2$

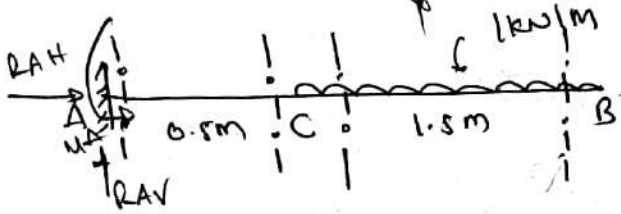
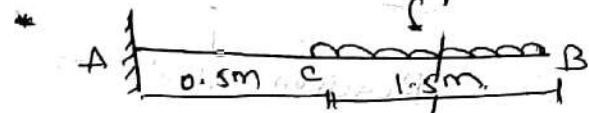
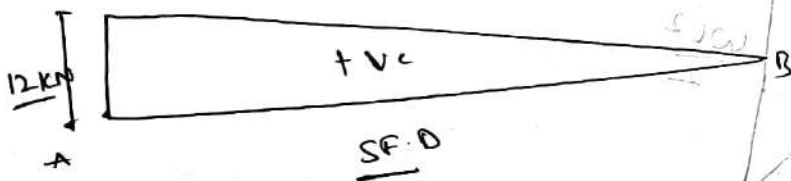
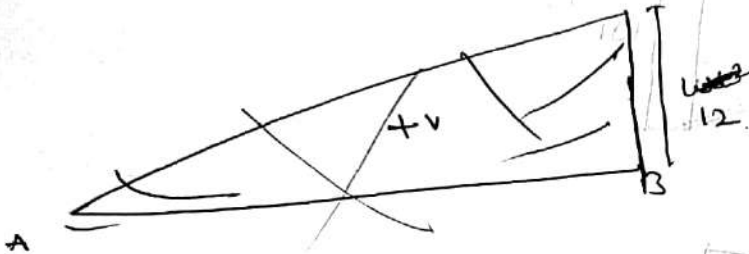
$x=0$ Bm] $x = 0$



$$x=3 \text{ BM})_A = -2(3)^2 = -18 \text{ kNm}$$



3m



$$\begin{aligned} \sum F_x &= 0 & \sum F_y &= 0 \\ R_{AH} &= 0 & R_{AV} - 1.5 &= 0 \\ \boxed{R_{AV} &= 1.5 \text{ kN}} \end{aligned}$$

$$SF)_A = 1.5 \text{ kN} \quad SF)_A = +R_{AV} = +1.5 \text{ kN}$$

$$SF)_C = 1.5 \text{ kN}$$

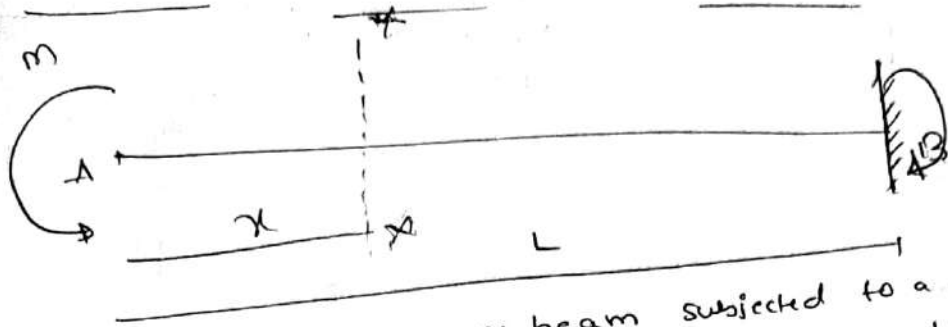
$$SF)_C = (1.5 \times 1) = 1.5 \text{ kN}$$

$$SF)_B = 0 = +R_{AV} - 1.5 = +1.5 - 1.5 = 0 \quad \checkmark$$

$$SF)_x-x = [1 \times x]$$

As there is no UDL load b/w A & C SFD remains constant

cantilever beam with a couple acting at the free end.



$$SF]_x = 0$$

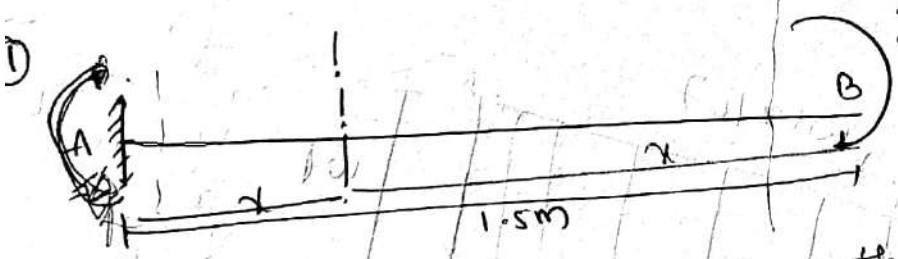
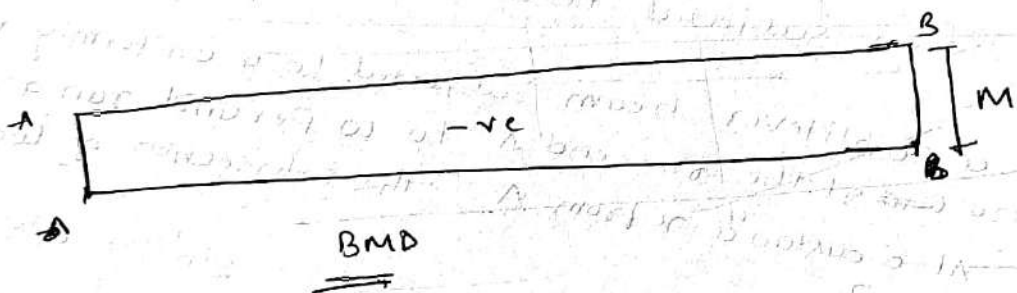
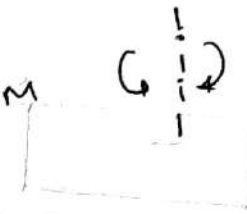
$$SF]_A = 0$$

$$SF]_B = 0$$

Figure shows a cantilever beam subjected to a couple M at the free end A . Consider a section at a distance x from the free end A . Since there is no lateral load acting on the beam, the shearing force is zero throughout the length of the beam.

Bending moment $]_x = -M$

Hence Bending moment is constant magnitude M for the entire length of the beam.



As there are no vertical loads on the cantilever beam the shear force at each & every section is zero.

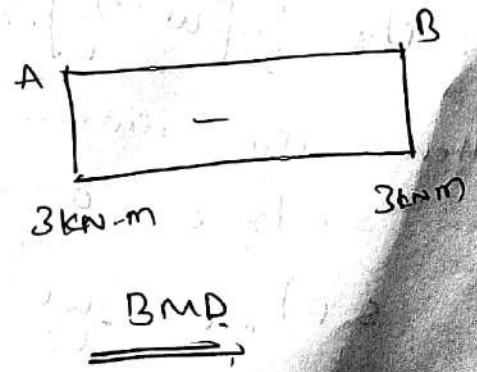
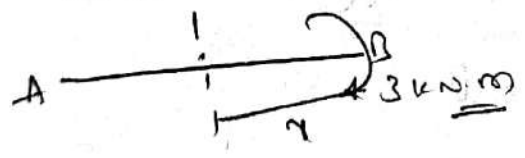
$$SF]_A = 0, SF]_B = 0$$

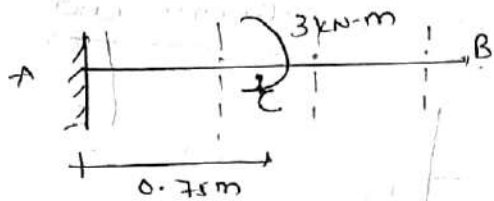
Bending moment calculations:

$$Bm]_A = -3 \text{ kN-m}$$

$$Bm]_B = -3 \text{ kN-m}$$

$$Bm]_{x-x} = -(-3)$$





$$S.F.]_A = 0$$

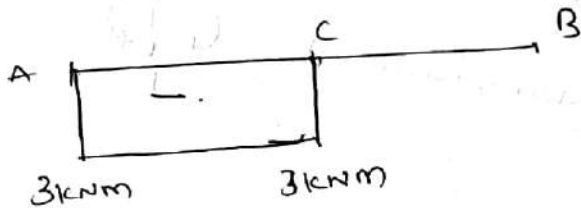
$$S.F.]_B = 0$$

$$S.F.]_C = 0$$

$$B.M.]_B = 0 \text{ kN-m}$$

$$B.M.]_C = -3 \text{ kN-m}$$

$$B.M.]_A = -3 \text{ kN-m}$$



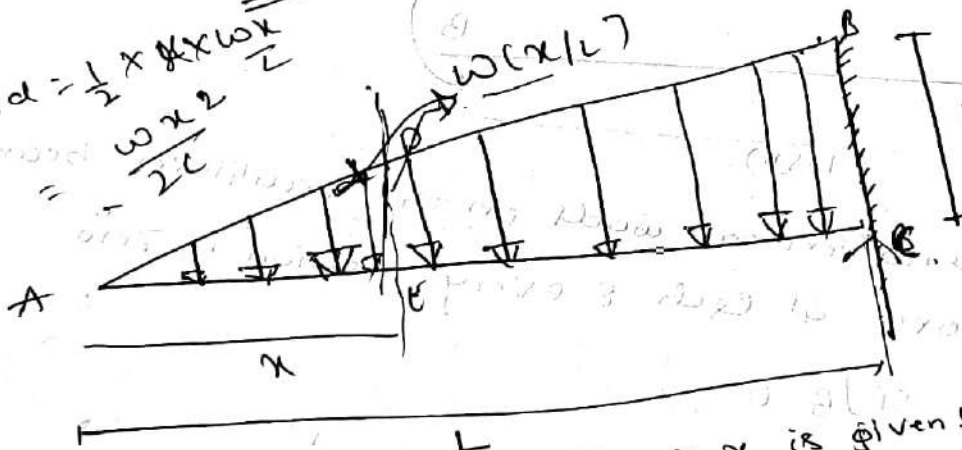
BMD

Cantilever beam subjected to a uniformly varying load.

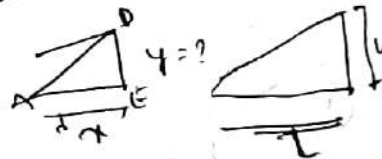
Consider a cantilever beam subjected to a uniformly varying load from zero at the free end A to w per unit run at B as shown in fig. At a distance x from A, the intensity of loading is given by $w \left[\frac{x}{L} \right]$.

$$\text{Total load} = \frac{1}{2} \times L \times w$$

$$= \frac{wL^2}{2}$$



Similar Δ 's



$$\frac{y}{x} = \frac{w}{L}$$

$$y = \frac{wx}{L}$$

Shearing force at any distance x is given by

$$S.F.]_x = \frac{1}{2} \left[w \cdot \frac{x}{L} \right] \cdot x = \frac{wx^2}{2L}$$

Hence the variation is parabolic.

$$S.F.]_{x=0} = 0$$

$$S.F.]_{x=L} = \frac{wL^2}{2L}$$

$$\frac{2wL^2}{2L} = wL$$

$$\frac{w \cdot L^2}{2L} = \frac{wL}{2}$$

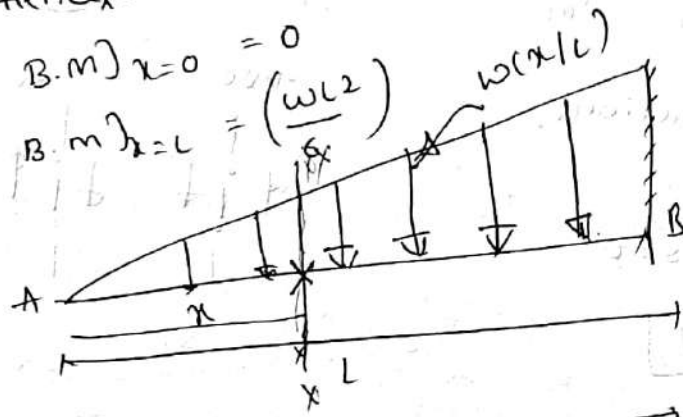
To plot the BMD.

$$B.M]_x = \int \frac{1}{2} \left[w \cdot \frac{x}{L} \right] \cdot x \cdot \left[\frac{1}{2} x \right] = \frac{wx^3}{6L} = \frac{wx^2}{2L} \cdot \frac{x}{3}$$

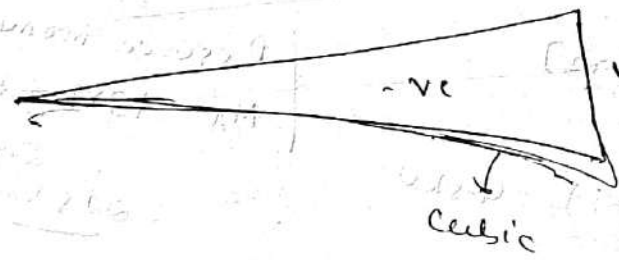
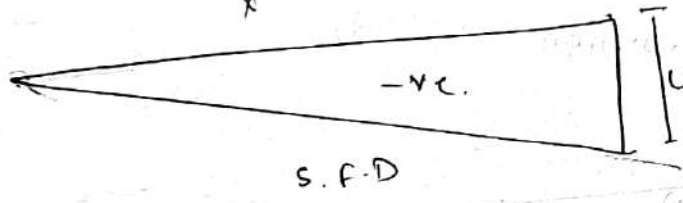
Hence, the variation is cubic

B.M] $x=0 = 0$

B.M] $x=L = \left(\frac{wL^2}{6} \right) \cdot L = \frac{wL^3}{6}$



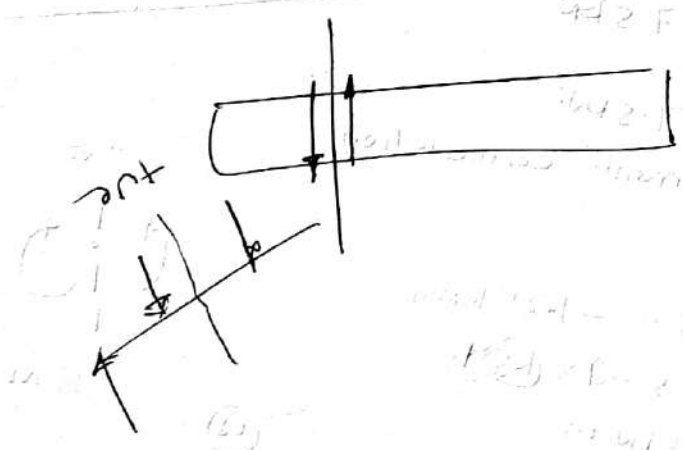
w/unit run
1, 2, 3,



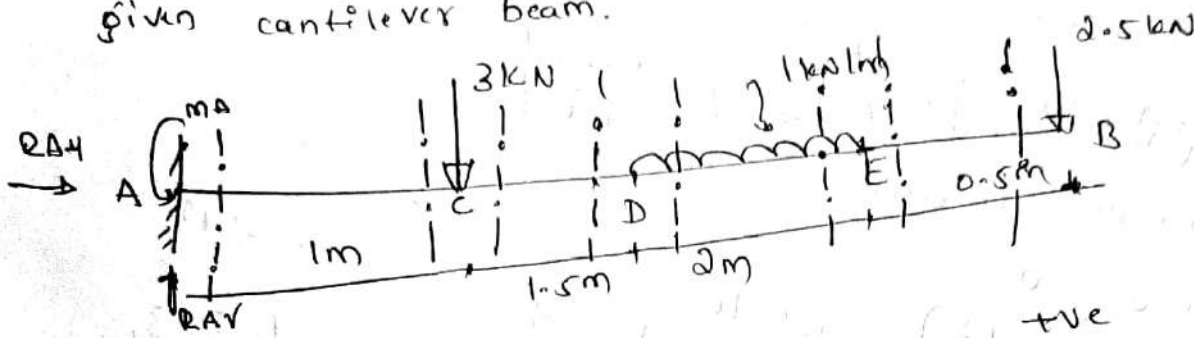
$$\frac{wx^2}{2L} \times \frac{1}{3} x = \frac{wx^3}{6L}$$

where $x=L$

$$= \frac{wL^3}{6}$$



1. Draw the SFD and Bending moment Diagram for the given cantilever beam.

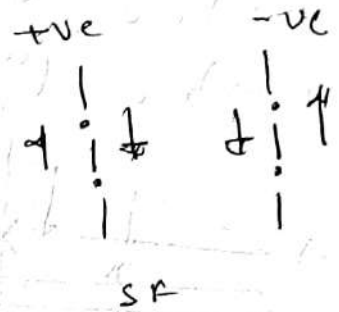


Step 1 Calculation for the Reactions.

$$\sum F_y = 0$$

$$R_{AV} - 3 - 2 - 2.5 = 0$$

$$R_{AV} = 7.5 \text{ kN}$$



Step 2 Shear force calculation

$$SF]_B = +2.5 \text{ kN}$$

$$SF]_E = +2.5 \text{ kN}$$

$$SF]_D = +2.5 + (1 \times 2)$$

$$SF]_D = 4.5 \text{ kN}$$

$$SF]_C = +2.5 + (1 \times 2) = 4.5 \text{ kN}$$

$$SF]_{LC} = +2.5 + (1 \times 2) + 3$$

$$SF]_{LC} = 7.5 \text{ kN}$$

$$SF]_A = +7.5 \text{ kN}$$

Respective moment about MA = $+3 \times 1 + 2 \times 3.5 + 2.5 \times 5$
 $= 22.5 \text{ kN-m}$ Clockwise - clockwise

Right

Step 3 Bending moment calculation

$$BM]_B = 0$$

$$BM]_E = -2.5 \times 0.5 = -1.25 \text{ kNm}$$

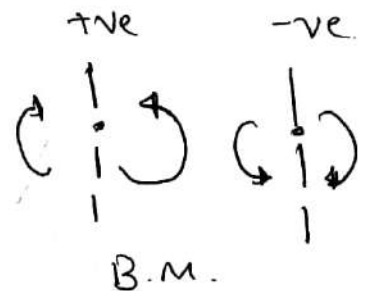
$$BM]_D = -2.5 \times 2.5 - 2 \times \frac{1}{2} \times 2 \times 2 = -8.25 \text{ kNm}$$

alternatively

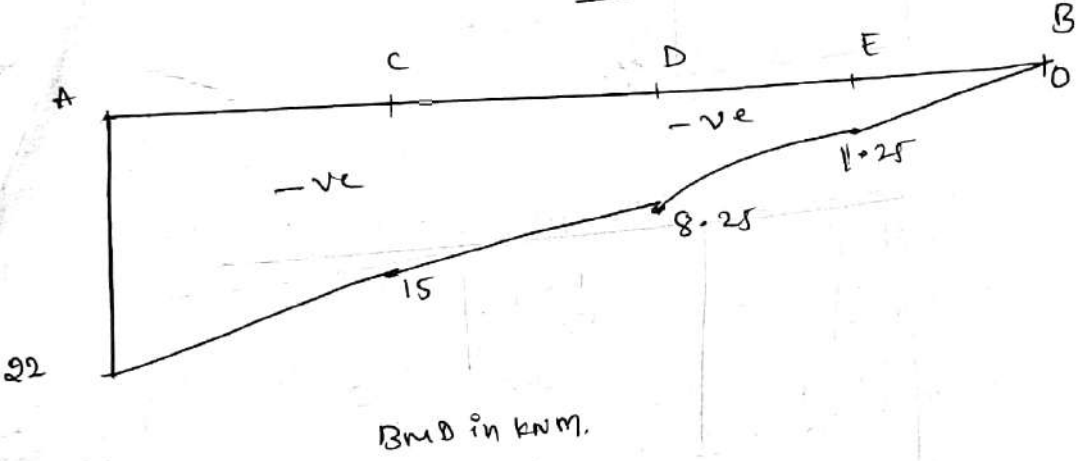
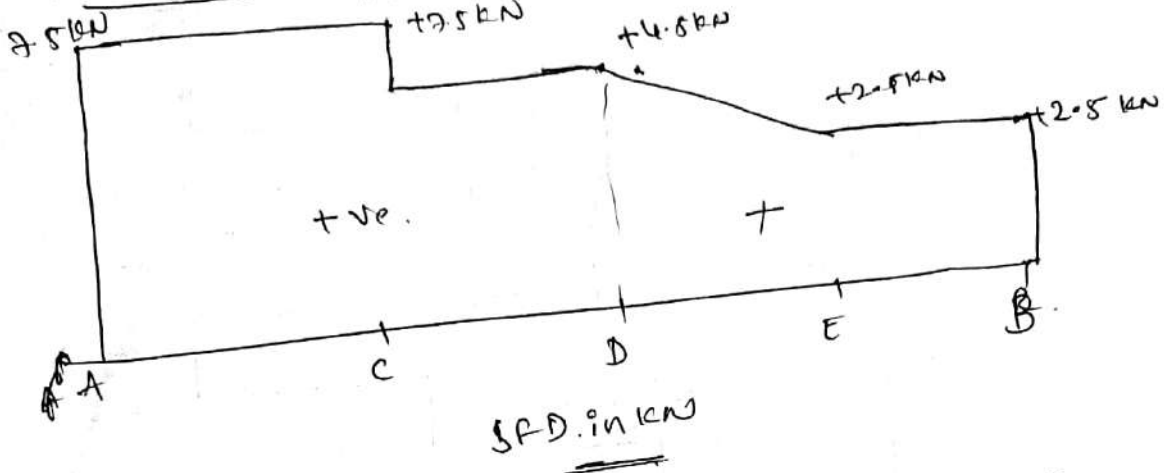
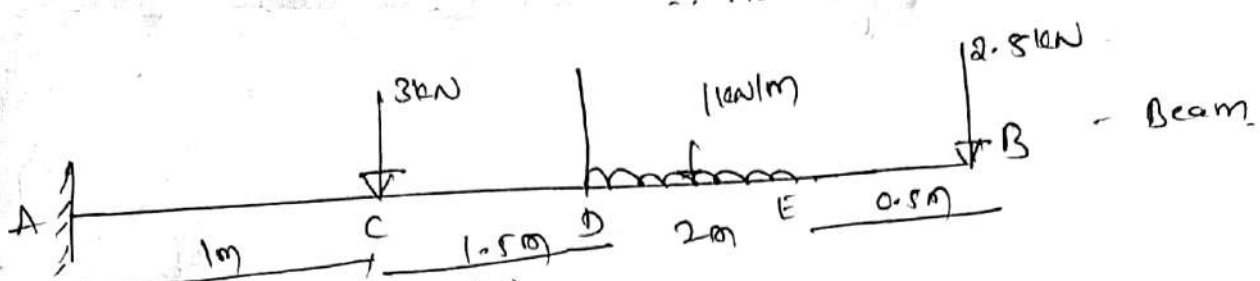
$$BM]_D = -22.5 + 7.5 \times 2.5 - 3 \times 1.5 = -8.25 \text{ kNm}$$

$$BM]_C = +7.5 \times 1 - 2.5 \times 1 - 2 \times 2.5 = -15 \text{ kNm}$$

$$BM]_A = -2.5 \times 5 - 2 \times 3.5 - 3 \times 1 = -22.5 \text{ kNm}$$



(5)



Draw the shear force and bending moment diagrams for the cantilever beam shown in fig

$\rightarrow \sum F_x = 0$

$R_{DH} = 0$

$\sum F_y = 0$

$R_{DV} - 8 - 20 - 10 = 0$

$R_{DV} = 38 \text{ kN}$

Considering the forces from right, the SF calculations section

$SF)_D = +R_{DV} = +38 \text{ kN}$

$SF)_C = +R_{DV} - 8 = +38 - 8 = 30 \text{ kN}$
Left

$SF)_C = +R_{DV} - 8 = 30 \text{ kN}$
Right

$SF)_B = 30 \text{ kN}$
Left

$SF)_B = 30 - 20 = 10 \text{ kN}$
Right

$SF)_A = 10 \text{ kN}$
Left

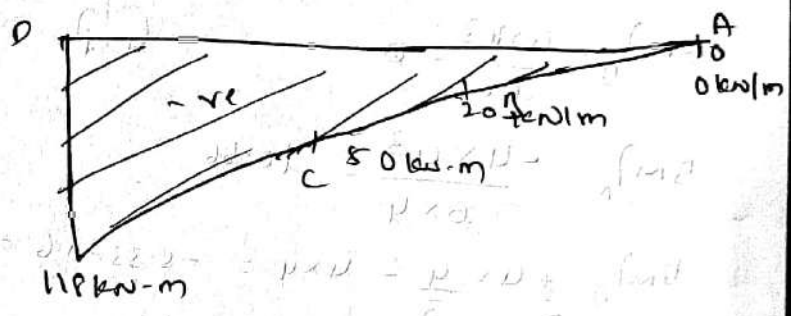
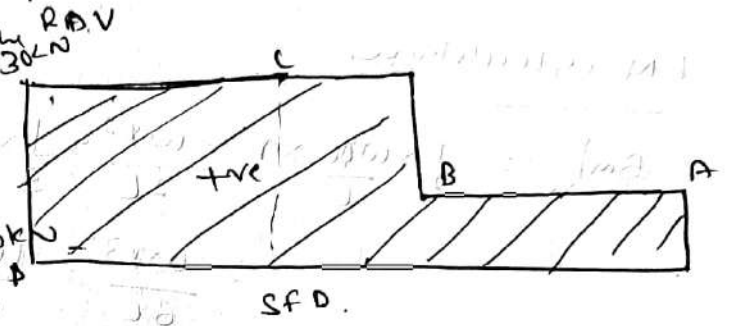
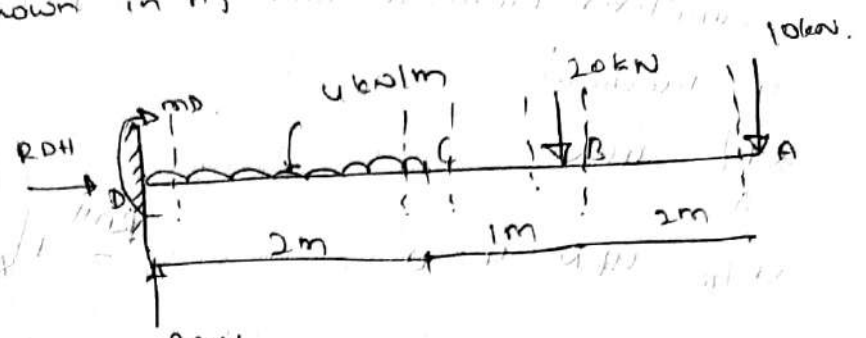
BM calculations

$BM)_A = 0$

$BM)_B = +10 \times 2 = 20 \text{ kN-m}$

$BM)_C = 10 \times 3 + 20 \times 1 = 50 \text{ kN-m}$

$BM)_D = 10 \times 5 + 20 \times 3 + 8 \times 1 = 118 \text{ kN-m}$



* Draw the shear force and Bending moment diagrams for the cantilever beam as shown in fig.

SF calculations

$$SF|_A = 4 \text{ kN}$$

$$SF|_B = 4 \text{ kN} - 4 = 0$$

BM calculations

$$BM|_x = \frac{1}{2} \times w \times x \times x = \frac{w \times x^2}{2L} \times \frac{1}{3} \times x$$

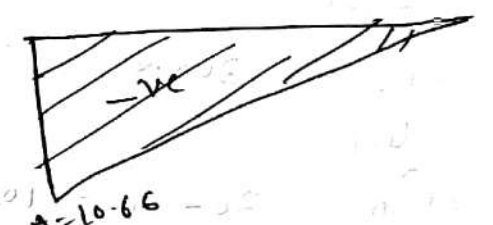
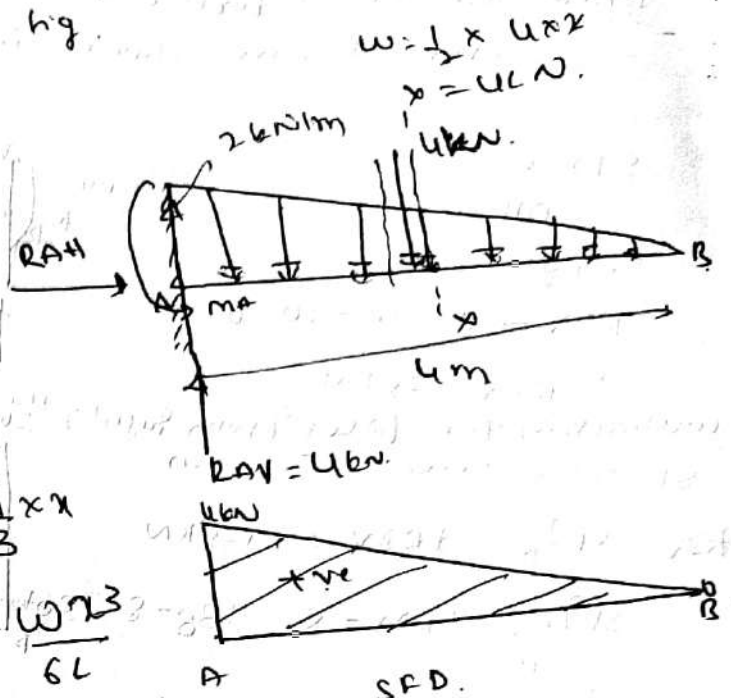
$$= \frac{w \times x^3}{6L} = \frac{w \times 3}{6L}$$

$$= \frac{wL^2}{6}$$

$$BM|_B = \frac{w \times 3}{6L} = 0$$

$$BM|_A = \frac{-4 \times 4^3}{6 \times 4} = -10.66$$

$$BM|_A = -4 \times \frac{4}{3} - 4 \times 4 = -5.33 - 16 = -10.66$$



* Simply supported beam shown in fig. Draw Bending moment Diagram.

$$\sum F_x = 0$$

$$\sum F_y = 0$$

$$+R_{AV} + R_{BV} - 60 - 80 = 0$$

$$R_{AV} + R_{BV} = 140 \text{ kN}$$

$$\sum M_A = 0$$

$$+60 \times \frac{2}{3} \times 3 + 80 \times 4.5 - R_{BV} \times 6 = 0$$

$$R_{BV} = 80 \text{ kN}$$

$$R_{AV} + R_{BV} = 140$$

$$R_{AV} = 140 - 80$$

$$R_{AV} = 60 \text{ kN}$$

Shear force calculation from portion AC: Measuring x from A. the load intensity is $\frac{40x}{3}$

SF] x

$$\text{Load is } \frac{1}{2} \times x \times \frac{40x}{3} = \frac{20x^2}{3}$$

$$F = R_{AV} - \frac{20x^2}{3}$$

$$\text{At } x=0$$

$$SF]_{x=0} = +R_{AV} = 60 \text{ kN}$$

$$SF]_C = +R_{AV} - \frac{20 \times 3^2}{3}$$

$$x=3 = +60 - \frac{20 \times 3^2}{3} = 0$$

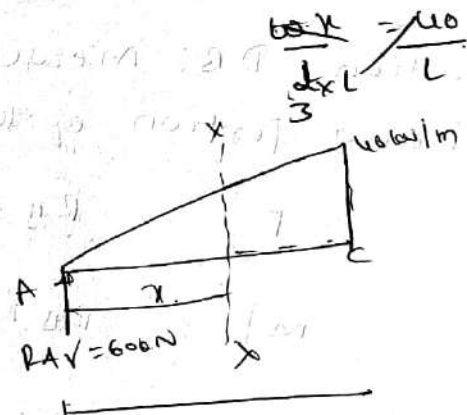
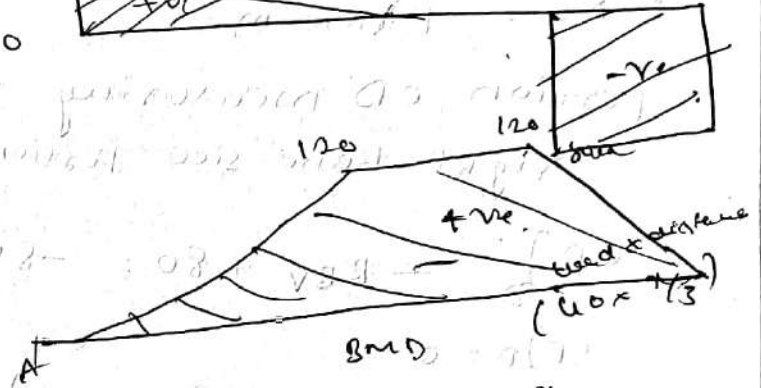
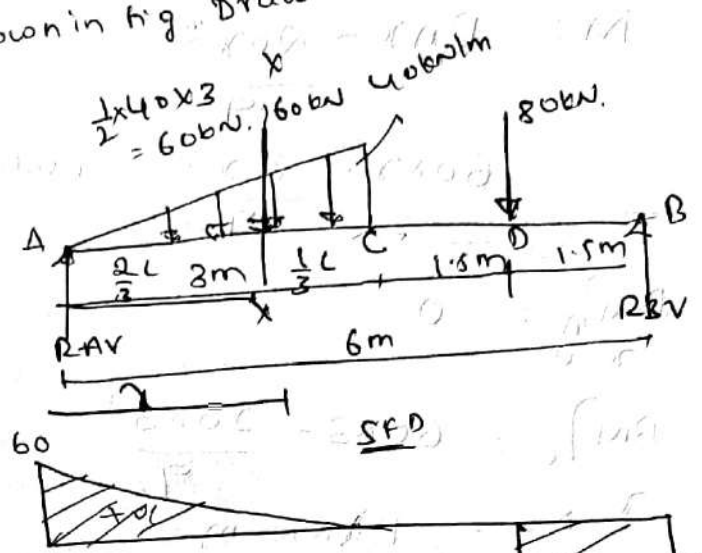
$$SF]_C = 0$$

Moment Bending moment calculation

$$M = R_{AV} \cdot x - \frac{1}{2} \times x \cdot \frac{40 \cdot x}{3} \cdot \frac{x}{3}$$

$$= R_{AV} \cdot x - \frac{20x^3}{9}$$

Head of C.E.D



similar concept

$$\frac{40}{3} = \frac{2}{x}$$

$$9 = \frac{40x}{3}$$

$$M = R_A x - \frac{20x^3}{9}$$

$$= 60x - \frac{20x^3}{9} \quad \text{(cubic variation)}$$

$$BM|_A = 0 \quad x=0$$

$$BM|_C = 60 \times 3 - \frac{20 \times 3^3}{9}$$

$$x=3 = 120 \text{ kN-m}$$

Portion CD: measuring x from B and considering right hand side portion of beam

$$SF|_D = -R_B V + 80 = -80 + 80 = 0 \quad \text{(constant)}$$

$$SF|_D = 0$$

$$M = R_B x - 80(x - 1.5)$$

$$= 80x - 80(x - 1.5) = 120 \text{ kN-m} \quad \text{(constant)}$$

Portion DB: measuring x from B considering right hand portion of the beam.

$$SF = -R_B = -80 \text{ kN} \quad \text{(constant)}$$

$$M = R_B x = 80x \quad \text{(linear variation)}$$

$$AFx = 0$$

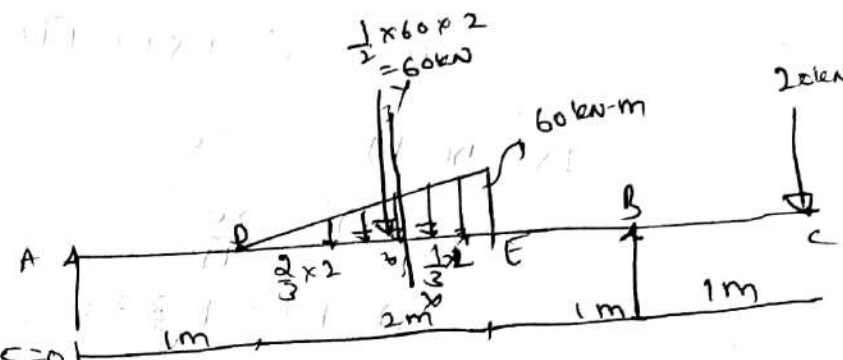
$$AFx = 1.5$$

$$M = 0$$

$$M = 80 \times 1.5 = 120 \text{ kN-m}$$

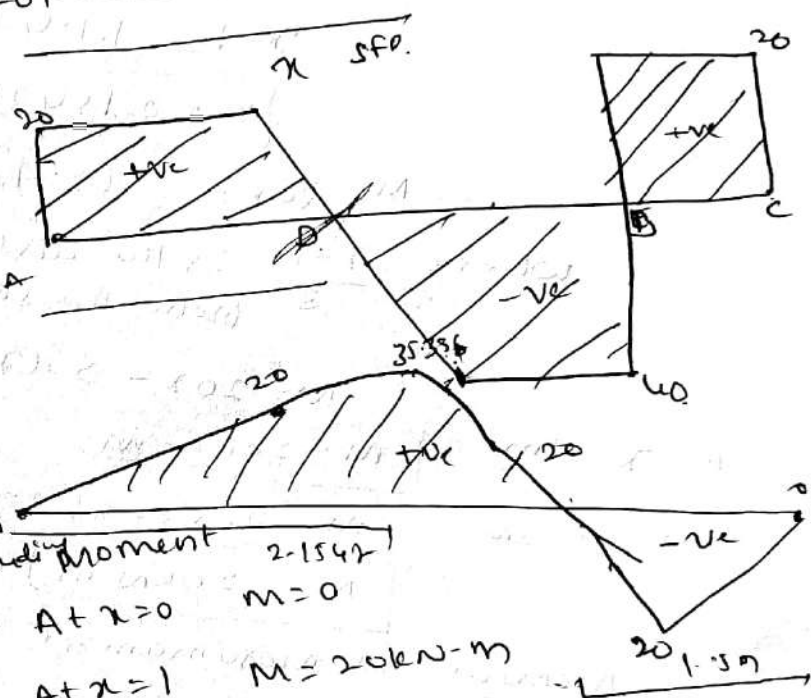
Draw the SFD and BMD for the overhanging beam shown in fig. Indicate all significant values including the point of contraflexure.

$\sum F_y = 0$
 $+R_{AV} + R_{BV} - 60 - 20 = 0$
 $R_{AV} + R_{BV} = 80 \text{ kN}$
 $\sum M_A = 0$



$60 \times \left[1 + \frac{4}{3}\right] - R_{BV} \times 4 + 20 \times 5 = 0$
 $R_{BV} \times 4 = 720$
 $R_{BV} = \frac{720}{4}$
 $R_{BV} = 180 \text{ kN}$

$R_{AV} + 60 = 80$
 $R_{AV} = 80 - 60$
 $R_{AV} = 20 \text{ kN}$



SF calculation

$SF|_A = +R_{AV} = +20 \text{ kN}$
 $SF|_D = +R_{AV} = +20 \text{ kN}$
 $SF|_E = \dots$

Bending Moment
 At $x=0$ $M=0$
 At $x=1$ $M=20 \text{ kN-m}$
BMD

Measuring x from A & considering the forces on left hand side of the section.

$(\frac{1}{2} \times b \times h) \text{ load intensity at } x = \frac{x-1}{2} \times 60 = 30(x-1)$
 $F = 20 - \frac{1}{2}(x-1)30(x-1)$
 $= 20 - 15(x-1)^2$ [Parabolic variation.]
 At $x=1 \text{ m}$
 $F = 20 - 15(1-1)^2$
 $F = 20 \text{ kN}$
 At $x=3 \text{ m}$
 $F = 20 - 15(3-1)^2$
 $F = 20 - 15(2)^2$
 $F = 20 - 60$
 $F = -40 \text{ kN}$

SF is zero at x where x is given by

$$F = 20 - 15(x-1)^2$$

$$0 = 20 - 15(x-1)^2$$

$$15(x-1)^2 = 20$$

$$(x-1)^2 = \frac{20}{15}$$

$$(x-1)^2 = 1.333$$

$$x-1 = 1.1547$$

$$x = 2.1547 \text{ m}$$

$$M = 20x - \frac{1}{2}(x-1)30(x-1)(x-1)$$

where $\frac{x-1}{3}$ is the distance of c.g. of triangular load from the section.

$$M = 20x - 5(x-1)^3 \quad (\text{cubic variation})$$

At $x = 1 \text{ m}$

$$M = 20(1) - 0 = 20 \text{ kN-m}$$

At $x = 3 \text{ m}$

$$M = 20 \times 3 - 5(3-1)^3$$

$$M = 20 \text{ kN-m}$$

Moment is maximum at $x = 2.1547 \text{ m}$ where $SF = 0$.

$$M_{\text{max}} = 20 \times 2.1547 - 5(2.1547 - 1)^3$$

$$M_{\text{max}} = 35.396 \text{ kN-m}$$

Portion EB: Measuring x from blue end and considering the right hand side portion

$$F = 20 - R_B = 20 - 60 = -40 \text{ kN. (constant)}$$

$$M = -20x + R_B(x-1)$$

$$= -20x + 60(x-1)$$

$$M = +40x - 60 \quad (\text{Linear variation})$$

At $x = 1 \text{ m}$ $M = 40 - 60 = -20 \text{ kN-m}$

At $x = 2 \text{ m}$ $M = 4 \times 2 - 60 = 20 \text{ kN-m}$

Point of contra flexure is in this portion E is obtained by $0 = 40x - 60$ i.e. at $x = 1.5 \text{ m}$ from blue end

1 mitch
Portion BC. measuring x from blue end & considering
the right hand side force

$$F = 20 \text{ kN} \text{ (Constant)}$$

$$M = -20x \text{ (Linear variation)}$$

$$\text{At } x=0 \quad M=0$$

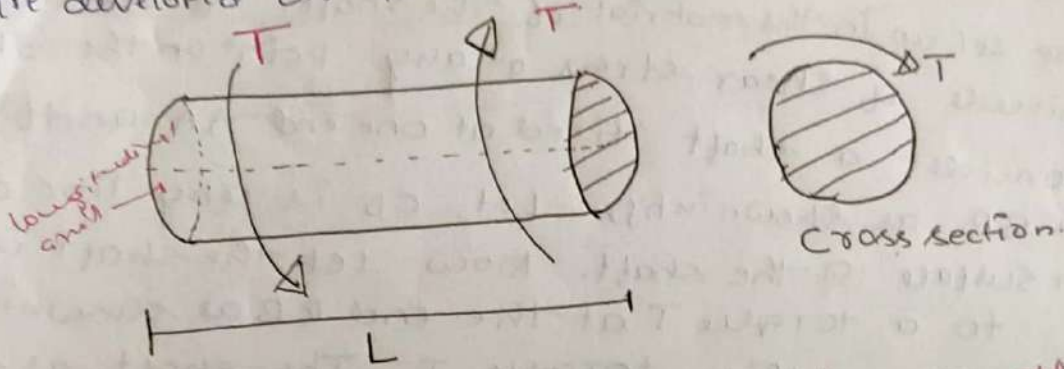
$$\text{At } x=1 \text{ m} \quad M = -20 \text{ kN-m.}$$



Module: 4

Torsion in circular shafts:

A shaft is said to be in "pure torsion" when it is subjected to equal and opposite moments which are acting parallel (or) tangential to the cross section. Hence shear stresses are developed on the cross section which is under pure shear.



Assumption made in pure torsion / torsion theory

1. Material is homogeneous and isotropic.
2. All the stresses are within the elastic limit.
3. Shaft is subjected to pure torsion only. [No shear force & Bending moment]
4. Shaft is uniform cross section.
5. The applied torque is uniform through out the length.
6. A radial line remains straight even after twisting.
7. A plane normal sections remain plane even after twisting. [No warping (or) bending) (or) distortions are seen]

Note:

We know that Young's modulus (or) modulus of elasticity.

$$E = \frac{\text{Linear Stress}}{\text{Linear Strain}} = \frac{f}{e} \quad \text{N/mm}^2$$

Shear modulus (or) Rigidity modulus is

$$C \text{ (or) } G = \frac{\text{Shear Stress}}{\text{Shear Strain}} = \frac{fs}{\phi} = \frac{\tau}{\phi} \quad \text{N/mm}^2$$

Derivation of Shear stress Produced in a circular

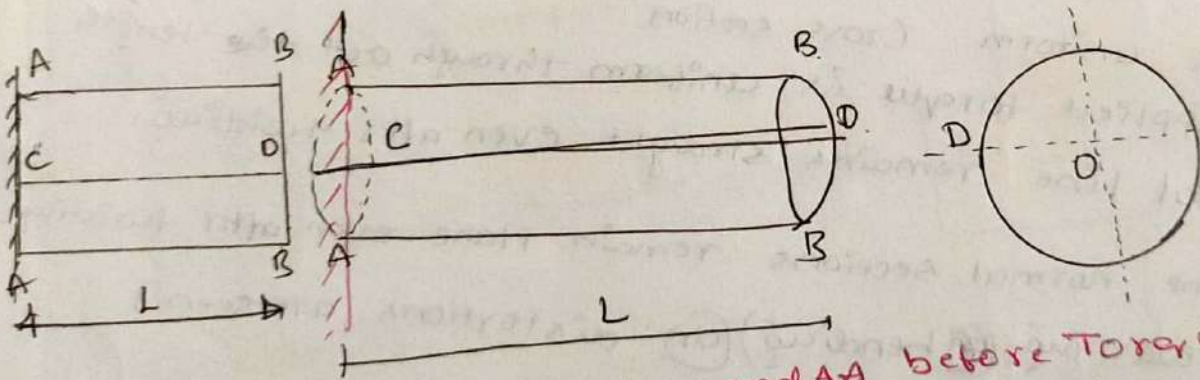
Shaft subjected to torsion

Derive the torsion equation with usual notations.

When a circular shaft is subjected to torsion, shear stresses are set up in the material of the shaft. To determine the magnitude of shear stress at any point on the shaft,

Consider a shaft fixed at one end AA and free at the other end BB as shown in fig. Let CD is any line on the outer surface of the shaft. Now let the shaft is subjected to a torque T at the end BB as shown in fig.

As a result of this torque T, the shaft at the end BB will rotate clockwise & every cross-section of the shaft will be subjected to shear stresses. The point D will shift to D' & hence line CD will be deflected to CD' as shown in fig. The line OD will be shifted to OD'



is shaft fixed at one end AA before Torque T is applied.

Let R = Radius of shaft, L = length of shaft,

T = Torque applied at the end BB.

τ = Shear stress induced at the surface of the shaft due to torque T. G = modulus of rigidity of the materials of the shaft. ϕ = $\angle DCD'$ also equal to shear strain.

θ = $\angle DOD'$ & ϕ is also called angle of twist.

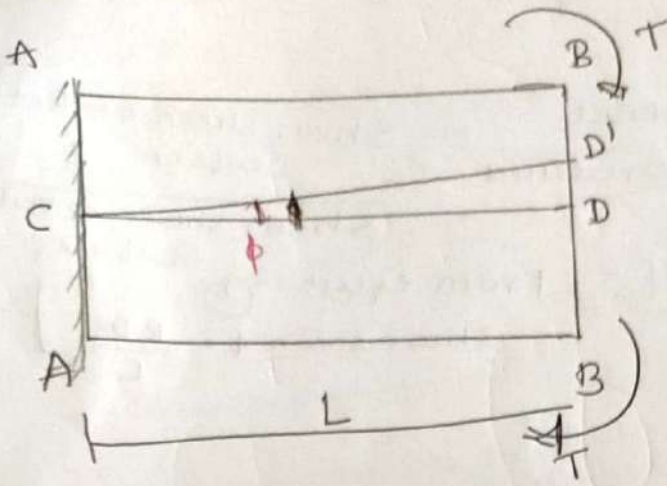


fig (a)

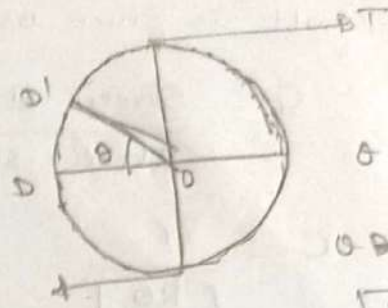


fig (b)

$$\theta = \frac{\text{ARC}}{R}$$

$$\theta = \frac{DD'}{R}$$

$$DD' = R\theta$$

fig shaft fixed at AA and subjected to torque T at BB

From fig (a)

Now

Shear strain at outer surface = Distortion per unit length

$$\tan \phi = \frac{DD'}{L}$$

where $[\tan \phi$ is very small $(\tan \phi = \phi)$]

$$\phi = \frac{DD'}{L}$$

Shear strain at outer surface

$$\phi = \frac{DD'}{L} \quad \text{--- (1)}$$

From fig (b)

$$\theta = \frac{\text{ARC}}{\text{Radius}}$$

$$\theta = \frac{DD'}{R}$$

$$DD' = R\theta \quad \text{--- (2)}$$

where

$$OD = \text{Radius of circle } R$$

$$OD = R \quad \text{--- (3)}$$

Substituting the value of DD' in eq (1) we get

Shear strain at the outer surface $\phi = \frac{DD'}{L}$

$$\phi = \frac{R\theta}{L} \quad \text{--- (3)}$$

Now the modulus of Rigidity (C) of the material of the shaft is given as.

$$C = \frac{\text{Shear stress induced}}{\text{Shear strain Produced}} = \frac{\text{Shear stress at the outer surface}}{\text{Shear strain at outer surface}}$$

$$C = \frac{\tau}{\left[\frac{R\theta}{L}\right]}$$

[∴ From equation (3) shear strain $\phi = \frac{R\theta}{L}$]

$$C = \frac{\tau L}{R\theta}$$

$$\boxed{\frac{C\theta}{L} = \frac{\tau}{R}} \quad \rightarrow (3)$$

$$\tau = \frac{C\theta R}{L} \quad \text{or} \quad \tau = \frac{R \times C \times \theta}{L}$$

Now for a given shaft subjected to a given torque (T) the values of C , θ , and L are constant, Hence shear stress produced is proportional to the radius R .

$$\tau \propto R \quad \text{or} \quad \frac{\tau}{R} = \text{constant} \quad \rightarrow (4)$$

If q is the shear stress induced at a radius r from the centre of the shaft then

$$\frac{\tau}{R} = \frac{q}{r} \quad \rightarrow (5)$$

But

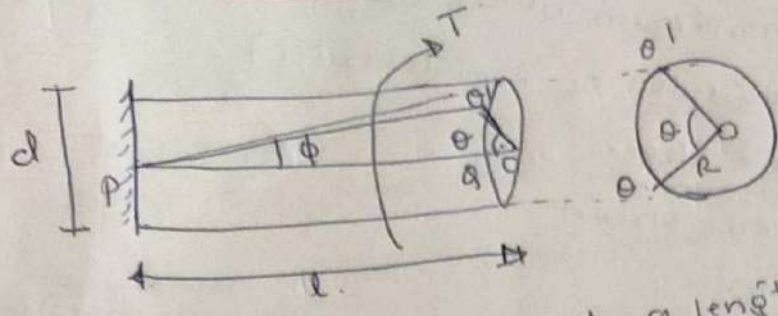
$$\frac{\tau}{R} = \frac{C\theta}{L} \quad \text{from equation (3)} \quad (a)$$

$$\frac{\tau}{R} = \frac{C\theta}{L} = \frac{q}{r} \quad \text{--- (E)}$$

from equation (4), it is clear that shear stress at any point on the shaft is proportional to the distance of the point from the axis of the shaft.

Hence the shear stress is maximum at the outer surface & shear stress is zero at the axis of the shaft.

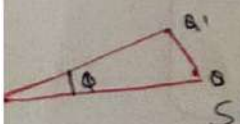
9) Shear stress Produced in a circular shaft subjected to torsion



Consider a shaft of length l diameter d fixed at one end & free at the other end as shown in fig. Take any line \underline{PA} on the outer surface of the shaft. The line PA is parallel to the longitudinal axis of the shaft. Now if torque T is applied to the shaft at the free end, the line \underline{PA} is shifted to the new position $\underline{PA'}$. The angle b/w PA and PA' i.e. $\angle APA'$ is ϕ which is the shear strain. The angle b/w OA & OA' in the end view i.e. $\angle OAA'$ is the angle of twist θ .

- Let l = length of the shaft, in mm
- R = Radius of shaft in mm
- θ = Angle of twist - radians
- ϕ = Shear strain.
- G = modulus of rigidity N/mm².
- T = Torque applied at the ends. [Torsional moment]
- Z = Shear stress induced at the outer surface. [Torsional moment]
- J = Polar moment of inertia in mm⁴ [Twisting moment]

from fig $\tan \phi = \frac{AA'}{PA}$ $\tan \phi = \theta$ [ϕ is very small]



Shear strain at the outer surface $\phi = \frac{AA'}{l}$

$\theta = \frac{AA'}{l}$ → (a)

Also from the end view Arc $AA' = OA \cdot \theta$

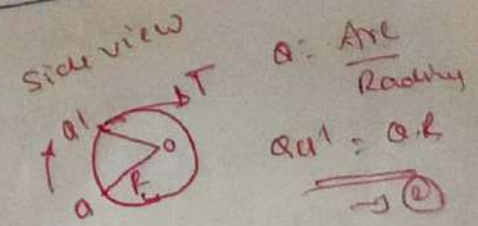
$AA' = R \cdot \theta$ → (b)

From eq (a) & (b)

$\theta = \frac{AA'}{l}$

$l \theta = R \cdot \theta$

$\phi = \frac{R \cdot \theta}{l}$ → (1)



$\theta = \frac{\text{Arc}}{\text{Radius}}$

$\theta = \frac{AA'}{R}$ → (2)

For a given torque T & length of shaft, θ is constant & therefore shear strain is directly proportional to its radial distance from the centre of the section. Hence shear strain is zero at the centre & maximum at the outer surface.

From Hook's law, Modulus of Rigidity (C)

$$i.e. \quad C = \frac{\text{Shear stress}}{\text{Shear strain}}$$

$$C = \frac{Z(\theta)}{\phi}$$

Substituting the value of ϕ in above equation, we get

$$C = \frac{Z @ R}{\frac{R\theta}{L}}$$

$$\frac{bs}{R} = \frac{C\theta}{L}$$

where θ is in radians

$$\frac{Z(R)}{R} = \frac{C\theta}{L} \quad \rightarrow (2)$$

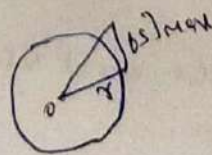
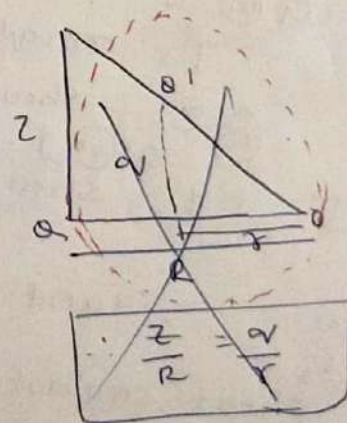
For a shaft subjected to a given torque T , the value of C , θ & L are constants. Hence shear stress induced is directly proportional to its radial R , i.e. radial distance from the centre of the section.

Hence shear stress is zero at the centre & maximum at the outer surface, τ is the shear stress at the radius r from the centre of the shaft.

The.

$$\frac{\tau}{R} = \frac{C\theta}{L} = \frac{\tau}{R} \quad \rightarrow (3)$$

$$\frac{I}{J} \cdot C\theta = \frac{bs}{R}$$



$$\frac{bs}{R} = \frac{C\theta}{L}$$

$$bs = \left(\frac{C\theta}{L}\right) R$$

$C\theta = \text{constant}$

$$bs \propto R$$

For equilibrium of the shaft the applied torque is being balanced T by resisting torque developed on the cross section by shear stress

Applied torque = Resisting torque

Consider an elementary area da at a distance r on which the shear stresses are acting as shown

$$\text{Resisting torque} = (\text{Shear stress}) (\text{Area}) (\text{distance})$$

$$= \left(\frac{N}{m^2}\right) (m^2) (m)$$

$$\text{Resisting torque} = \frac{N \cdot m}{L}$$

$$= \left(\frac{C\theta}{L}\right) (da) (r^2)$$

$$d.T = \left[\frac{C\theta}{L}\right] (da) (r^2) \quad (N \cdot mm^4)$$

In the above equation $(da) (r^2)$ is denoted by J @ IP. J = Polar moment of inertia

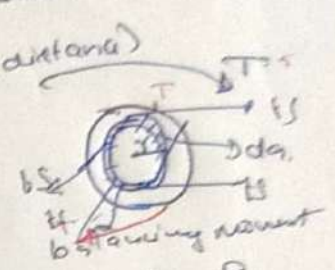
Thus total resisting torque equal to applied torque T for equilibrium

$$T = \left[\frac{C\theta}{L}\right] \int_0^R da \cdot r^2$$

$$T = \frac{C\theta}{L} [J]$$

$$\frac{T}{J} = \frac{C\theta}{L} \quad \text{--- (2)}$$

$$\frac{T}{J} = \frac{C\theta}{L} = \frac{\tau_s}{R}$$



$$\tau = \frac{P}{A}$$

$$P = \tau \cdot A$$

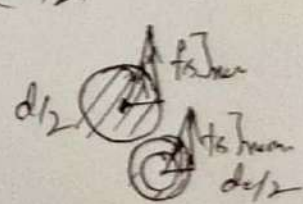
$$P = (\tau \cdot da) (r^2)$$

$$\frac{T}{J} = \frac{\tau_s}{R}$$

$$\tau_s = (R) \left(\frac{T}{J}\right)$$

$$\tau_{s_{max}} = \left(\frac{d}{2}\right) \left(\frac{T}{J}\right)$$

$$= \left(\frac{d}{2}\right) \left(\frac{T}{J}\right)$$



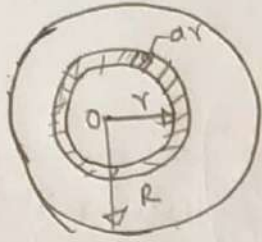
The above equation is the expression for pure torsion.

C = Shear Modulus

C @ C = modulus of shear stress
 L = length of the shaft

T = Torque
 J = Polar moment of Inertia
 R @ τ_s = Shear stress
 R = Radius of the circular shaft

Relationship b/w torque & shear stress in a solid circular shaft.



Consider an elementary strip at 'r' from the centre of the section. The thickness of the strip is dr as shown in fig.
 Let R = Radius of the shaft.

T = Torque applied

τ = Shear stress induced at radius 'r' from the centre.

Z = Maximum shear stress induced at the outer surface.

Area of the elementary circular ring/shaft $dA = 2\pi r \cdot dr$.

From eq (3) i.e. $\frac{Z}{R} = \frac{\tau}{r} = \frac{\tau}{R}$ $\frac{T}{J} = \frac{\tau}{r} = \frac{\tau}{R}$

Torsion equation

$$\frac{Z}{R} = \frac{\tau}{r}$$

Shear stress at radius 'r'

$$\tau = \frac{Z \cdot r}{R}$$

Shear force on the elementary circular ring
 δF = Shear stress in ring \times Area of the ring

$$= \left[\frac{Z}{R} r \right] \times [2\pi r \cdot dr]$$

$$\delta F = \frac{Z}{R} 2\pi r^2 \cdot dr$$

\therefore Torque on the elementary circular ring
 δT = $\delta F \times$ Distance of the ring from the centre.

$$\delta T = \left[\frac{Z}{R} \cdot 2\pi r^2 \cdot dr \right] \times r$$

$$\delta T = \frac{Z}{R} 2\pi r^3 \cdot dr$$

\therefore Total torque (δT) Total ^{resulting} torsional moment.

$$T = \int_0^R \frac{Z}{R} 2\pi r^3 \cdot dr$$

$$= \frac{Z}{R} \cdot 2\pi \int_0^R r^3 \cdot dr$$

$$= \frac{Z}{R} \cdot 2\pi \left[\frac{r^4}{4} \right]_0^R$$

$$= \frac{Z}{R} \cdot 2\pi \left[\frac{R^4}{4} \right] = \frac{Z \pi R^3}{2}$$

$$T = \frac{Z}{R} \left[\frac{\pi R^4}{2} \right] \dots \dots (R = d/2)$$

$$T = \frac{Z}{R} \left[\frac{\pi d^4}{32} \right]$$

$$T = \frac{Z}{R} \cdot J_P$$

Where $J_P = \frac{\pi d^4}{32}$ = Polar m.m. of solid circular shaft

$$\frac{T}{J_P} = \frac{Z}{R} \rightarrow (4)$$

from (1) & (4)

$$\frac{T}{J_P} = \frac{Z}{R} = \frac{C\theta}{L} \rightarrow (5)$$

The above equation (5) is known as torsional equation, where.

Polar modulus or section modulus

It is defined as the ratio of polar moment of inertia to the radius of the shaft is known as polar modulus. It is denoted by Z or Z_P .

w.k.t - $\frac{T}{J} = \frac{Z}{R}$

$$T = \left(\frac{J}{R} \right) \cdot Z$$

$$T = Z \cdot Z_P$$

where $Z_P = \frac{J}{R}$ = polar modulus.
 $Z_P = m^3$

$T = \frac{\tau \cdot R}{J}$
 $\tau = \left(\frac{J}{R} \right) \cdot \frac{\tau}{R}$
 $\tau = Z_P \cdot \tau$

Z or Z_P for solid shaft

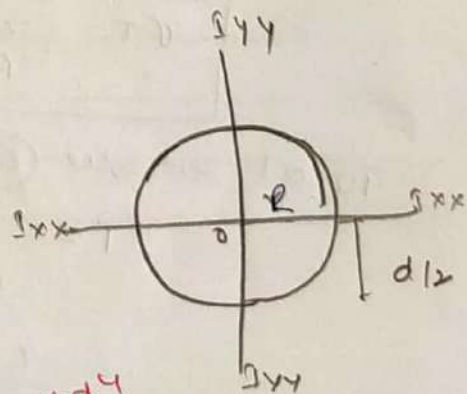
$$Z = J/R$$

where = $I_{xx} + I_{yy}$

$$J_P = \frac{\pi d^4}{64} + \frac{\pi d^4}{64}$$

$$J = \frac{\pi d^4}{32}$$

solid shaft $J_P = J = \frac{\pi d^4}{32}$



$$Z = \left(\frac{\pi d^4}{32} \right) / \left(\frac{d}{2} \right)$$

$$Z = \frac{\pi d^3}{16}$$

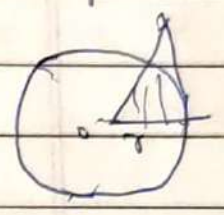
Polar modulus:

Let T be the torsional moment of resistance of the section of a shaft of radius R and I_p (J) the Polar moment of Inertia of the shaft section.

The shear stress intensity τ at any point on the section distant r from the axis of the shaft is given by

$$\tau = \frac{T \cdot r}{I_p}$$

$$\left[\frac{T}{J} = \frac{\tau}{R} \right]$$



$$\tau = \left(\frac{T}{J} \right) \cdot r$$

The maximum shear stress τ_{max} occurs at the greatest radius R .

$$\tau_{max} = \left(\frac{T}{J} \right) R$$

$$T = \tau_{max} \cdot Z_p$$

$$Z_p = \frac{I_p}{R}$$

$$\tau_{max} = \frac{I_p}{R} \left(\frac{T}{J} \right)$$

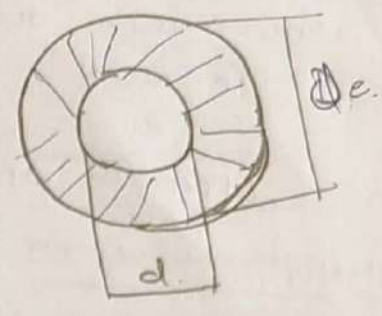
$Z_p =$ Polar moment of Inertia of the shaft
Maximum radius

Z (or) ZP for Hollow shaft

$$J = \frac{\pi}{32} [D_o^4 - D_i^4] \quad \text{--- } \epsilon \quad R = D/2$$

$$Z = \frac{J}{R} = \frac{\pi/32 (D_o^4 - D_i^4)}{D/2}$$

$$Z = \frac{\pi}{16} \left[\frac{D_o^4 - D_i^4}{D} \right]$$



Torsional Rigidity

It is defined as the ratio of torque to the angle of twist. It is denoted by C or G .

(or) N

Torsional Rigidity = Torque / angle of twist

$$C = \frac{T}{\theta}$$

$$(or) \frac{T}{J \cdot P} = \frac{C \cdot \theta}{L}$$

$$\theta = \frac{T \cdot L}{J \cdot P \cdot C}$$

Torsional flexibility

It is defined as the ratio of angle of twist to the torque. In another word, it is the reciprocal of torsional rigidity.

$$\text{Torsional flexibility} = \frac{\theta}{T} = \frac{1}{C}$$

Torsional strength (or) efficiency of a shaft

It is defined as the ratio of torque to the shear stress. It is called torsional strength or twisting moment.

$$\text{Torsional strength} = \frac{\text{Torque}}{\text{Shear stress}} = \frac{T}{\tau}$$

Power transmitted by solid & hollow circular shaft

Consider a shaft subjected to torque T & rotating at N revolutions per minute.

Angle through which the torque moves is $\frac{2\pi N}{60}$. Power is defined as the ratio of work done & it is denoted as

$$P = \frac{2\pi N}{60} \times T$$

$$P = \frac{2\pi N T}{60 \times 1000} \quad \text{in k.w}$$

$$P = \frac{2\pi N T}{60} \quad \text{in watt}$$

$$P = \frac{2\pi N T}{60}$$

Note: When power P is (substituted) in watts, n is in rpm, then the torque T obtained will be always in N-m.

Strength and stiffness conditions

Strength condition:

$$\frac{T}{J} = \frac{\tau}{R}$$

The above condition shows, that the shaft should be strong enough to resist shear stress.

Stiffness condition:

$$\frac{T}{J} = \frac{C \cdot \theta}{L}$$

The above condition shows that the shaft should be stiff enough to resist twisting.

Power Transmitted by Shaft:

If a shaft transmits a Torque 'T' @ 'N' rpm then work done is given by $P = (\text{Torque})(\text{Angular displacement})$

$$P = (T) \left(\frac{2\pi N}{60} \right) \therefore P = \frac{2\pi NT}{60}$$

When P is in watts & 'N' is in rpm; then Torque obtained will be in N-m.

① A shaft is subjected to a Torque 16,000 Nm. If the maximum permissible stress in the material of the shaft is 65 N/mm², find. i) the diameter of the solid shaft & ii) the dimensions of a hollow circular shaft if the thickness is 10% of the internal diameter.

- Given: Torque $T = 16,000 \text{ Nm} = 16 \times 10^6 \text{ N-mm}$
Permissible stress $f_s = 65 \text{ N/mm}^2$

i) hollow shaft

$$\frac{I}{J} = \frac{f_s}{R} = \frac{C\theta}{L}$$

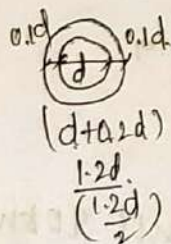
Strength Equations $\frac{I}{J} = \frac{f_s}{R} \quad J = \frac{\pi d^4}{32} \quad \frac{16 \times 10^6}{\left(\frac{\pi d^4}{32}\right)} = \frac{65}{(d/2)} \quad d = 107.82 \text{ mm}$
 $L = 53.91 \text{ mm}$

ii) Solid Shaft

Let d be the inner diameter

\therefore External diameter be 1.1d.

$$J = \frac{\pi}{32} (1.2d^4 - d^4) = \frac{\pi}{32} (0.2d^4)$$



$$\frac{16 \times 10^6}{\left(\frac{\pi \times 0.2d^4}{32}\right)} = \frac{65}{(1.2d/2)}$$

T is taken in N-m, then unit of power is N-m/sec i.e. watt.

One horse power = 736 watts.

$$\text{1HP} = 736 \text{ N-m}$$

$$\text{1HP} = 736 \times 10^3 \text{ N-mm}$$

$$\theta = 1^\circ = \frac{\pi}{180} \text{ radian}$$

② A solid shaft is required to transmit a max torque of 170 kNm. If shear stress does not exceed 50 MPa & the angle of twist limited to 0.17°. Find the dia of shaft. $C = 8.4 \times 10^4$ MPa length of shaft is 1m

Pr $\frac{T}{J} = \frac{fs}{R} = \frac{C\theta}{L}$

$J = \frac{\pi d^4}{32}$

$\theta = 0.17^\circ \times \frac{\pi}{180} = 2.96 \times 10^{-3}$

Case i) $\frac{T}{J} = \frac{C\theta}{L}$

$T = \frac{8.4 \times 10^4 \times 2.96 \times 10^{-3}}{1000} \times \frac{\pi d^4}{32} = 170 \times 10^6$

$d = 288.8 \text{ mm} \quad R = 144.4 \text{ mm}$

Case ii) $\frac{T}{J} = \frac{fs}{R}$

$\frac{170 \times 10^6}{\frac{\pi d^4}{32}} = \frac{50}{d/2}$

$d = 258.71 \text{ mm} \quad R = 129.35 \text{ mm}$

Adopt always the higher value of diameter to satisfy both strength & stiffness conditions.

③ Determine the diameter of solid shaft which transmit 440 kW at 280 rpm. The angle of twist must not exceed one degree per metre length and the maximum torsional shear stress is to be limited to 40 N/mm². Assume $C = 84 \text{ kN/mm}^2$.

$P = 440 \text{ kW} = 440 \times 10^3 \text{ W} = 440 \times 10^3 \text{ N-m/sec} = 440 \times 10^6 \text{ N-mm/sec}$

$N = 280 \text{ rpm} \quad \theta = 1^\circ = \frac{1 \times \pi}{180} = 0.0174 \quad L = 1 \text{ m}$

$P = 2\pi NT \quad T = 15.00 \times 10^6 \text{ N-mm}$

$\frac{T}{J} = \frac{fs}{R} = \frac{C\theta}{L}$

$\frac{T}{J} = \frac{fs}{R}$

$\frac{15 \times 10^6}{\left(\frac{\pi d^4}{32}\right)} = \frac{84 \times 10^3 \times 0.0174}{1000} \quad d = 101.11 \text{ mm}$

$\frac{T}{J} = \frac{fs}{R}$

$\frac{15 \times 10^6}{\left(\frac{\pi d^4}{32}\right)} = \frac{40}{(d/2)}$

$d = 124.087 \text{ mm}$

$\therefore d_{\text{min}} = 124.087 \text{ mm}$

④ Determine dia of the solid shaft which will transmit 90 kW @ 160 rpm if the shear stress in the shaft is limited to 60 MPa. Find also the length of the shaft if twist is limited to 1°

Take $C = 8 \times 10^4$ MPa

$P = \frac{2\pi NT}{60}$

$T = 5.371 \times 10^6 \text{ N-mm}$

$\frac{T}{J} = \frac{fs}{R}$

$\frac{T}{J} = \frac{C\theta}{L}$

$\frac{5.371 \times 10^6}{\left(\frac{\pi \times 76.96^4}{32}\right)} = \frac{8 \times 10^4 \times 0.0174}{L}$

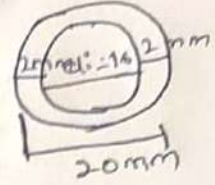
$\frac{5.371 \times 10^6}{\left(\frac{\pi d^4}{32}\right)} = \frac{60}{d/2}$

$d = 76.96 \text{ mm}$

$L = 892.57 \text{ mm}$

06 A hollow shaft of 20mm outer diameter & 2mm thick is subjected to a Torque of 40 N-m. Find shear stress at inner and outer diameter of shaft. Also find maximum shear stress.

→ $d_e = 20\text{mm}$ $d_i = 16\text{mm}$ $T = 40\text{N-m}$
 $T = 40 \times 10^3\text{ N-mm}$



$$\frac{T}{J} = \frac{bs}{R}$$

$$R = d_e/2$$

$$J = \frac{\pi}{32} [20^4 - 16^4]$$

$$J = 9273.98\text{ mm}^4$$

$$\frac{40 \times 10^3}{9273.98} = \frac{bs_e}{d_e/2} = \frac{bs_e}{20/2}$$

$$bs_e = 43.13\text{ N/mm}^2$$

$$\frac{40 \times 10^3}{9273.98} = \frac{bs_i}{d_i/2} = \frac{bs_i}{16/2}$$

$$bs_i = 34.50\text{ N/mm}^2$$

bs_{max} at circumference at outer diameter = 43.13 N/mm^2

07 A hollow shaft with outer diameter d_o & inner diameter d_i is transmitting 375 kW at 105 rpm with a working stress 40 MPa. Determine dimension of shaft & twist in a length equal to 10 times external diameter. $C = 8 \times 10^4\text{ MPa}$.

→ $d_o = d$ $d_i = d/2$ $P = 375 \times 10^6\text{ N-mm}$ $n = 105\text{ rpm}$

$bs = 40$

$$P = \frac{2\pi NT}{60}$$

$$375 \times 10^6 = \frac{2 \times \pi \times 105 \times T}{60}$$

$$T = 34.16 \times 10^6\text{ N-mm}$$

$$\frac{T}{J} = \frac{bs}{R}$$

$$\frac{34 \cdot 10 \times 10^6}{\frac{\pi}{32} [d^4 - (\frac{d}{3})^4]} = \frac{40}{d/2}$$

$$d_e = 166.68 \text{ mm}$$

$$d_i = 83.84 \text{ mm}$$

$$d_i = d/2 = \frac{166.68}{2} = 83.84 \text{ mm}$$

$$\frac{T}{J} = \frac{C\theta}{L}$$

$$L = 10 \times d_e = 10 \times 166.68$$

$$\frac{34 \cdot 10 \times 10^6}{70.92 \times 10^6} = \frac{8 \times 10^4 \times \theta}{10 \times 166.68}$$

$$\theta = 0.101 \text{ rad}$$

$$\theta = 0.57^\circ$$

2) A 150mm dia solid steel shaft is transmitting 450kW Power at 90 rpm. Compute the maximum shear stress. Find the change that would occur in the shearing stress. If the speed increased to 360 rpm.

$$d = 150 \text{ mm} \quad P = 450 \text{ kW} \quad n = 90 \text{ rpm} \quad bs = ?$$

$$\text{case i)}: P = \frac{2\pi NT}{60}$$

$$450 \times 10^6 = \frac{2 \times \pi \times 90 \times T}{60}$$

$$T = 47.74 \times 10^6 \text{ N}\cdot\text{mm}$$

$$\frac{\pi}{32} d^4 = \frac{T}{bs} [150]^4$$

$$= 49.7 \times 10^6 \text{ mm}^4$$

$$\frac{T}{J} = \frac{bs}{R}$$

$$\frac{47.74 \times 10^6}{49.7 \times 10^6} = \frac{bs}{150/2}$$

$$bs = 72.04 \text{ N/mm}^2$$

$$bs = 72.04 \text{ N/mm}^2$$

11) A shaft is required to transmit 245 kW power

case ii) when speed increased at 360 rpm

$$P = \frac{2\pi NT}{60}$$

$$450 \times 10^6 = \frac{2 \times \pi \times 360 \times T}{60}$$

$$T = 11.936 \times 10^6 \text{ N}\cdot\text{mm}$$

$$\frac{T}{J} = \frac{\delta s}{R}$$

$$\frac{11.936 \times 10^6}{45.7 \times 10^6} = \frac{\delta s}{150/2}$$

$$\delta s = 18.012 \text{ N/mm}^2$$

As speed of shaft increases the shear stress and Torque applied decreases.

09) During tests on a sample of steel bar 25mm in diameter, it is found that the pull of 50kN produces an extension of 0.095mm on a length of 200mm and a torque 200N-m produces an angular twist of 0.9 degree on a length 250mm. Find the Poisson's ratio of the steel.

u) $d = 25\text{mm}$ $P = 50 \times 10^3\text{N}$ $\delta L = 0.095$ $L = 200\text{mm}$
when under Tension test.

$$\sigma = \frac{P}{A}$$

$$\sigma = \frac{50 \times 10^3}{\frac{\pi}{4} (25)^2}$$

$$\sigma = 101.85 \text{ N/mm}^2$$

$$E = \frac{\sigma}{e}$$

$$e = \frac{0.095}{200}$$

$$E = 4.75 \times 10^4$$

$$E = \frac{\sigma}{e} = 214$$

$$E = \frac{101.85}{4.75 \times 10^{-4}} = 214.42 \times 10^3 \text{ N/mm}^2$$

Now under Torsion.

$$d = 25\text{mm}$$

$$T = 200 \times 10^3 \text{ N}\cdot\text{mm}$$

$$\theta = 0.9^\circ = 0.015 \text{ rad.}$$

$$L = 250\text{mm}$$

$$\frac{T}{J} = \frac{C\theta}{L}$$

$$= \frac{200 \times 10^3}{38.34 \times 10^3} = C \times \frac{0.015}{250}$$

$$J = \frac{\pi}{32 \times 25^4} = 38.34 \times 10^3 \text{ mm}^4$$

$$C = 117.61 \times 10^3 \text{ N/mm}^2$$

A shaft is required to transmit 245 kW power

WKT $E = 2C(1+M)$

$2.14 \times 10^5 = 2 \times 1.17 \times 10^5 (1+M)$

$M = 0.2918$

10) A hollow shaft of a steam ship is to transmit 3750 kW at 240 rpm. If the internal diameter is 0.8 times the external diameter & if the maximum shear stress developed is to be limited to 160 N/mm², determine the size of the shaft.

$P = 3750 \text{ kW} = 3750 \times 10^3 \text{ N-mm}$
 $n = 240 \text{ rpm}$
 $\tau_s = 160 \text{ N/mm}^2$
 $D_i = 0.8 D_e$

$P = \frac{2\pi NT}{60}$

$3750 \times 10^3 \times 60 = 2 \times \pi \times 240 \times T$
 $T = 1.492 \times 10^8 \text{ N-mm}$

$J = \frac{\pi}{32} [d^4 - (0.8d)^4]$

$J = 0.05796 d^4$

$\frac{T}{J} = \frac{\tau_s}{R}$

$\frac{1.492 \times 10^8}{0.05796 d^4} = \frac{160}{d/2}$

$d = 200.37 \text{ mm}$

$D_i = 0.8 d_e$
 $= 0.8 \times 200.37$

$D_i = 160.30 \text{ mm}$

① A shaft is required to transmit 245 kW power at 240 rpm. The maximum torque may be 1.5 times the mean torque. The shear stress in the shaft should not exceed 40 N/mm^2 and twist 1° per meter length. Determine the diameter required.

i) a) The shaft is solid.

b) The shaft is hollow with external diameter twice the internal diameter. $\tau = 80 \text{ N/mm}^2$.

$\rightarrow P = 245 \text{ kW} = 245 \times 10^3 \text{ N-m} \quad n = 240 \text{ rpm} \quad \theta = 1^\circ$
 $L = 1 \text{ m}$

$T_{\text{max}} = 1.5 T =$

$\tau_s = 40 \text{ N/mm}^2$

$P = \frac{2\pi NT}{60}$

$245 \times 10^3 = \frac{2 \times \pi \times 240 \times T}{60}$

$T = 9.748 \times 10^6 \text{ N-mm}$

$T_{\text{max}} = 1.5 \times 9.748 \times 10^6$

$T_{\text{max}} = 14.622 \times 10^6 \text{ N-mm}$

a) Solid shaft

$\frac{T}{J} = \frac{\tau_s}{R}$

$\frac{T}{J} = \frac{C\theta}{L}$

$\frac{14.622 \times 10^6}{\frac{\pi}{32} d^4} = \frac{80 \times 10^3 \times 0.017}{1000}$

$14.622 \times 10^6 \times 1000 \times 32 = 80 \times 10^3 \times 0.017 \times \pi \times d^4$

$d = 109.5358 \text{ mm}$

$d = 102.29 \text{ mm}$

$\theta = 0.017$

$$\frac{T}{J} = \frac{bs}{R}$$

$$\frac{14.622 \times 10^6}{\frac{\pi}{32} \times d^4} = \frac{40}{d/2}$$

~~$$\frac{14.622 \times 10^6 \times 2}{\pi \times d^4} = \frac{40}{d/2}$$~~

$$d = 123.02 \text{ mm}$$

adopt 123.02 mm ϕ

Hollow shaft.

d = outer dia d_i = Inner dia. mm

$$J = \frac{\pi}{32} [d^4 - 0.5d_i^4]$$

$$= 0.0920d^4 = T$$

$$\frac{T}{J} = \frac{bs}{R}$$

$$\frac{14.622 \times 10^6}{0.0920d^4} = \frac{40}{d/2}$$

$$d = 125.896 \text{ mm}$$

$$\frac{T}{J} = \frac{C\theta}{L}$$

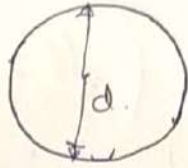
$$d = 103.281 \text{ mm}$$

Adopt $d_e = 125.896 \text{ mm}$

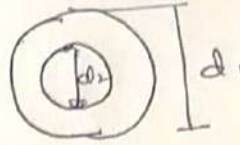
$$d_i = 62.841 \text{ mm}$$

Dec 2011

Prove that a hollow shaft is stronger & stiffer than the solid shaft of the same material, length & weight.



Solid shaft



Hollow shaft

Let,

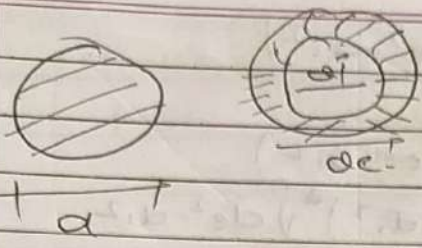
d = diameter of solid shaft.

d_1 = outer dia of hollow shaft.

d_2 = inner diameter of hollow shaft.

The two shafts have equal weight & length & are of same material. Hence equating the weight of solid shaft to that of hollow shaft.

weight



$$\rho_s = \rho_H$$

$$L_s = L_H$$

$$W_s = W_H$$

$$W_s = W_H$$

$$(\text{Density}) (\text{Volume})_s = (\text{Density}) (\text{Volume})_H$$

$$(\text{Volume})_s = (\text{Volume})_H$$

$$(\text{Area}) (\text{length})_s = (\text{Area}) (\text{length})_H$$

$$(\text{Area})_s = (\text{Area})_H$$

$$\frac{\pi d^2}{4} = \frac{\pi (d_e^2 - d_i^2)}{4}$$

$$d^2 = (d_e^2 - d_i^2)$$

$$d = \sqrt{d_e^2 - d_i^2}$$

Case: 2: The hollow shaft is stronger than solid shaft

$$W = T \cdot L \quad \frac{T}{J} = \frac{bs}{R} = \frac{C\theta}{L}$$

$$\frac{T}{J} = \frac{bs}{R} = \frac{T}{bs} = \frac{J}{R}$$

$$\left(\frac{T}{bs}\right)_H = \left(\frac{T}{R}\right)_H = \left(\frac{\frac{\pi}{32} (d_e^4 - d_i^4)}{\left(\frac{1}{d_e/2}\right)}\right)_H$$

$$\frac{T}{bs}_s = \left(\frac{T}{R}\right)_s = \frac{\frac{\pi}{32} (d^4)}{\left(\frac{1}{d/2}\right)}$$

$$= \frac{(d_e^4 - d_i^4) \frac{1}{d_e}}{d^4 \left(\frac{1}{d}\right)} = \frac{(d_e^4 - d_i^4)}{d_e \cdot d^3}$$

$$= \frac{(d_e^2 + d_i^2)(d_e^2 - d_i^2)}{d_e \cdot d_e \cdot d_e}$$

$$= \frac{(d_e^2 + d_i^2)(d_e^2 - d_i^2)}{d_e \cdot (d_e^2 - d_i^2) \sqrt{d_e^2 - d_i^2}}$$

$$\frac{T_s}{b_s} \Big|_H = \frac{(d_e^2 + d_i^2)}{d_e \cdot \sqrt{d_e^2 - d_i^2}} \quad \left(d_i \sqrt{\left(1 - \frac{d_i^2}{d_e^2}\right)} d_e \right)$$

multiplying numerator & denominator by $(1/d_e)$
we get

$$\frac{T_s}{b_s} \Big|_H = \left(\frac{d_e^2 + d_i^2}{d_e^2} \right) \frac{d_e^2 \sqrt{\frac{d_e^2}{d_e^2} - \frac{d_i^2}{d_e^2}}}{d_e^2}$$

$$\frac{T_s}{b_s} \Big|_H = \frac{\left(1 + \frac{d_i^2}{d_e^2}\right)}{\sqrt{\left(1 - \frac{d_i^2}{d_e^2}\right)}} > 1$$

hence hollow shaft is stronger than solid shaft

Case: 2

Hollow shaft is stiffer.

From stiffness comparison

$$\frac{T}{J} = \frac{C\theta}{L}$$

$$\frac{T}{\theta} = \frac{C \cdot J}{L}$$

$$\frac{T/\theta}{H} = \frac{\frac{C \cdot J}{L}}{H} = \frac{T/\theta}{H} = \frac{J}{L}$$

$$\frac{T/\theta}{H} = \frac{\frac{\pi}{32} (d_o^4 - d_i^4)}{\frac{\pi}{32} (d_o^4)} = \frac{(d_o^4 - d_i^4)}{d_o^4}$$

$$= \frac{(d_o^2 + d_i^2)(d_o^2 - d_i^2)}{d_o^4}$$

$$\frac{T/\theta}{H} = \frac{(d_o^2 + d_i^2)(d_o^2 - d_i^2)}{(d_o^2 - d_i^2)(d_o^2 + d_i^2)}$$

$$= \frac{d_o^2 + d_i^2}{d_o^2 + d_i^2} > 1$$

Hence hollow shaft is stiffer than solid shaft.

13) Compare the weight of solid shaft with that of a hollow one having the same length to transmit a given power at a given speed, if the material used for both the shaft is the same. Take the inside dia of the hollow shaft as 0.6 times the outer diameter.

- Let d be the diameter of solid shaft.

Let d_1 be the outer dia of hollow shaft.

\therefore Inner diameter of hollow shaft = $0.6d_1$

Let P be the power transmitted by both shaft

N be their rpm

T be the corresponding shaft.

From Strength consideration

$$\frac{T}{J} = \frac{f_s}{R} \quad \text{Solid shaft} \quad J = \frac{\pi}{32} d^4$$

$$T = \frac{f_s}{\frac{d}{2}} \cdot \frac{\pi}{32} d^4 = \frac{\pi}{16} d^3 f_s \quad \text{--- (1)}$$

$$\text{Hollow shaft} \quad J = \frac{\pi}{32} [d_1^4 - (0.6d_1)^4] = \frac{\pi}{32} 0.8704 d_1^4$$

$$T = \frac{\pi}{16} 0.8704 d_1^3 f_s \quad \text{--- (2)}$$

Equating (1) & (2)

$$d^3 = 0.8704 d_1^3$$

$$d = 0.9547 d_1$$

$$\text{Area of Solid Shaft} \quad A_s = \frac{\pi d^2}{4} = \frac{\pi (0.9547 d_1)^2}{4} = 0.7158 d_1^2$$

$$\text{Hollow shaft} \quad A_H = \frac{\pi [d_1^2 - (0.6d_1)^2]}{4} = 0.5026 d_1^2$$

$$\frac{A_s}{A_H} = 1.423$$

Area of Solid shaft is 42% more than Hollow shaft. ~~with~~ when strength is considered.

From Stiffness consideration

$$\frac{T}{J} = \frac{G\theta}{L} \quad \text{SS} \quad J = \frac{\pi d^4}{32}$$

$$\text{HS} \quad J = \frac{\pi}{32} [d_1^4 - (0.6d_1)^4] = \frac{\pi}{32} (0.8704 d_1^4)$$

$$\text{SS} \quad T = \frac{G\theta}{L} \frac{\pi d^4}{32}$$

$$\text{HS} \quad T = \frac{G\theta}{L} \frac{\pi (0.8704 d_1^4)}{32}$$

$$d^4 = 0.8704 d_1^4 =$$

$$d^2 = 0.9329 d_1^2$$

$$A_S = \frac{\pi}{4} 0.9329 d_1^2$$

$$A_H = \frac{\pi}{4} [d_1^2 - (0.6d_1)^2]$$

$$\frac{A_S}{A_H} = \frac{0.9329}{0.64} = 1.45$$

Area of Solid shaft is ~~more~~ 45% more than hollow shaft when stiffness is considered.

- (14) A solid shaft transmits 250 kW at 1000 rpm. If the shear stress is not to exceed 75 N/mm², what should be the diameter of the shaft? If this shaft is to be replaced by a hollow one whose internal dia is equal to 0.6 times outer dia, determine the size & the % saving in weight, the maximum shearing stress being same.

$$P = 250 \text{ kW} = 250 \times 10^3 \text{ N-m/sec}$$

$$P = \frac{2\pi NT}{60} \quad T = 23.8732 \times 10^6 \text{ N-m}$$

Solid shaft: $\frac{T}{J} = \frac{fs}{R} \quad \therefore d = 117.473 \text{ mm}$

Hollow shaft: $d_i = 0.6 d_o$

$$d_o = 123.036 \text{ mm} \quad d_i = 73.822 \text{ mm}$$

$$A_S = 10838.42 \text{ mm}^2 \quad A_H = 7609.164 \text{ mm}^2$$

$$\therefore \text{Saving in weight} = \frac{W_S - W_H}{W_S} \times 100$$

$$= \frac{(A_S - A_H) \rho L}{A_S \rho L} \times 100 = 29.75\%$$

A solid shaft of 60mm dia rotates at 1600rpm. Find the power that the shaft can transmit if permissible shear stress is 80MPa & max Torque likely to exceed 30% Avg T. Also find the angle of twist in degrees for a length of 2000mm if $G = 80GPa$.

$$T_{max} = T_{avg} \times 1.3$$

$$\frac{T_{max}}{J} = \frac{f_s}{R}$$

$$T_{max} = 3.3937 \times 10^5 \text{ N-m}$$

$$T_{avg} = \frac{T_{max}}{1.3} = 2.6099 \times 10^5 \text{ N-m}$$

$$\frac{T_{max}}{J} = \frac{C\theta}{L} \Rightarrow \theta = 0.2669 \text{ rad } (15^\circ 17'')$$

$$P = \frac{2\pi NT}{60} = 43.73 \text{ kW}$$

Compare torsional strength and weight of shafts of same material, length & weight & having same max. shear stress. One is solid & other is hollow having with I.D of diameter is 0.5. Take same rigidity modulus.

$$L_s = L_H, W_s = W_H, f_s J_H = f_s J_s$$

$$d_i = 0.5 d_e, d.$$

$$W_s = W_H = W$$

$$\rho(\text{Vol})_H = \rho(\text{Vol})_s$$

$$d_e^2 - d_i^2 = d^2$$

$$\frac{T}{J} = \frac{f_s}{R}$$

$$T_H = f_s \left[\frac{\pi/32 (d_e^4 - d_i^4)}{(d_e/2)} \right]$$

$$T_s = \left[\frac{f_s \pi/32 d^4}{(d/2)} \right]$$

$$\frac{T_H}{T_s} = \frac{(d_e^2 - d_i^2)(d_e^2 + d_i^2)}{d_e d^2}$$

$$\frac{T_H}{T_s} = \frac{d_e^2 + d_i^2}{d_e d} = \frac{d_e^2 + 0.25 d_e^2}{d_e \sqrt{d_e^2 - 0.25 d_e^2}} = \frac{1.25 d_e^2}{0.866 d_e^2} = 1.453$$

$$\frac{T_H}{T_s} = 1.453$$

17) What % of strength of a solid circular shaft 100mm diameter is lost by boring 50mm axial hole in it? Compare the strength & weight ratio of two cases.

$$T_s = \frac{\pi}{32} \times 100^3 \times \frac{f_s}{50} = 19.63 f_s$$

$$T_H = \frac{\pi}{32} (100^3 - 50^3) \frac{f_s}{50} = 14.72 f_s$$

$$\frac{T_H}{T_s} = \frac{14.72}{19.63} = 0.749$$

Loss in strength = $\frac{T_s - T_H}{T_s} \times 100 = 25\%$

Weight ratio = $\frac{W_H}{W_s} = \frac{\rho \pi (100^2 - 50^2) L}{\rho \pi 100^2 L} = 0.75$

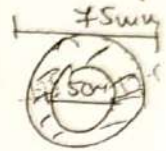
18) P.T. Shear stress due to the pure torsion is proportional to the radius of circular shaft.
(Refer derivation)

$$\frac{\tau}{r} = \frac{T}{J}$$

$$\tau = \frac{T}{J} \times r$$

$$\tau \propto r$$

14. What power can be transmitted at 300 rpm by a hollow shaft of 75 mm external dia & 12.5 mm thickness. When the permissible shear stress is 70 N/mm^2 & the maximum torque is 1.30 times the mean. Compare the strength of this hollow shaft with that of a solid shaft of the same material, weight & length.



$\Rightarrow P = ?$
 $n = 300 \text{ rpm}$
 $d_e = 75 \text{ mm}$
 $d_i = 50 \text{ mm}$
 $\tau_b = 70 \text{ N/mm}^2$
 $T_{\text{max}} = 1.3 T_{\text{mean}}$

$$J = \frac{\pi}{32} (75^4 - 50^4)$$

$$J = 2.0492 \times 10^6 \text{ mm}^4$$

$$P = \frac{2\pi n T_{\text{avg}}}{60} = \frac{(2)(300)(\pi) T_{\text{avg}}}{60}$$

$$P = 31.41 T_{\text{avg}} \rightarrow (*)$$

$$R = d_e/2 = 37.5 \text{ mm}$$

$$\frac{T_{\text{max}}}{J} = \frac{\tau_b}{R} \Rightarrow \frac{T_{\text{max}}}{2.0492 \times 10^6} = \frac{70}{37.5}$$

$$T_{\text{max}} = 4.65 \times 10^6 \text{ N-mm}$$

$$T_{\text{max}} = T_{\text{avg}} + (1.3) T_{\text{avg}}$$

$$T_{\text{mean}} = \frac{T_{\text{max}}}{1.3} = 3.57 \times 10^6 \text{ N-mm}$$

$$T_{mean} = 3.57 \times 10^3 \text{ Nm}$$

$$P = 31.41 T_{mean}$$

$$P = 31.41 \times 3.57 \times 10^3$$

$$P = \underline{\underline{112.13 \times 10^3 \text{ W}}}$$

Consider a solid shaft of dia 'd' as shown in figure.

$$W_H = W_S$$

$$l_H = l_S$$

$$W_H = W_S$$

$$P_H = P_S$$



$$[(\rho)(v)]_H = [(\rho)(v)]_S$$

$$[(A)(R)]_H = [(A)(R)]_S$$

$$A_H = A_S$$

$$\frac{\pi}{4} [75^2 - 50^2] = \frac{\pi}{4} [d^2]$$

$$2454.36$$

$$d = \underline{\underline{55.9 \text{ mm}}}$$

WKT

Strength equation $\frac{T}{J} = \frac{f_s}{R}$

$$\frac{T_H}{J_H} = \frac{[f_s \times J/R]_H}{[f_s \times J/R]_S}$$

$$= \frac{2049 \times 10^6}{\frac{d^{1/2} = 75/2}}{\frac{158.6 \times 10^3}{55.9/2}}$$

$$\frac{T_H}{J_H} = \frac{66400}{34296.95} = 1.93$$

f_s - material same.

$$J = \frac{\pi}{32} (55.9)^4$$

$$J = 958.6 \times 10^3 \text{ mm}^4$$

$$27.95$$

15. A hollow shaft of dia. of ratio $\frac{3}{5}$ is required to transmit 700 kW at 110 rpm, the max torque being 12% greater than the mean. the shear stress is not to exceed 60 MPa & twist in a length of 3m is not to exceed $(1)^\circ$. calculate the cross sectional dimension. Given $G = 0.8 \times 10^5$ MPa.

$$\Rightarrow P = 700 \text{ kW} = 700 \times 10^3 \text{ W}$$

$$n = 110 \text{ rpm}$$

$$T_{\text{max}} = 12\% T_{\text{avg}}$$

$$f_s = 60 \text{ N/mm}^2$$

$$L = 3000 \text{ m}$$

$$\theta = (1)^\circ = 0.01745$$

$$G = 0.8 \times 10^5 \text{ N/mm}^2$$

$$\frac{d_i}{d_o} = \frac{3}{5}$$

$$d_i = 0.6 d_o$$

$$\frac{T_{\text{max}}}{J} = \frac{f_s}{R}$$

$$\frac{68.05 \times 10^6}{0.085 d_o^4} = \frac{60}{d_o/2}$$

$$d_o = 188.95 \text{ mm}$$

$$d_i = 113.37 \text{ mm}$$

$$P = \frac{2\pi n T_{\text{avg}}}{60}$$

$$T_{\text{avg}} = \frac{(P)(60)}{2\pi n}$$

$$= \frac{(700 \times 10^3 \times 60)}{2 \times 110 \times \pi}$$

$$T_{\text{avg}} = 60.76 \times 10^3 \text{ N-m}$$

$$T_{\text{avg}} = 60.76 \times 10^6 \text{ N-mm}$$

$$T_{\text{max}} = T_{\text{avg}} + 0.12 T_{\text{avg}}$$

$$T_{\text{max}} = (0.12)(60.76 \times 10^6) + 60.76 \times 10^6$$

$$T_{\text{max}} = 70.2912 \times 10^6 \text{ N-mm}$$

$$J = \frac{\pi}{32} [d_o^4 - 0.6^4 d_o^4] = 0.085 d_o^4$$

$$R = \frac{d_o}{2}$$

$$\frac{T_{\text{max}}}{J} = \frac{60}{L}$$

$$\frac{70.2912 \times 10^6}{0.085 d_o^4} = \frac{(0.8 \times 10^5)(0.01745)}{3000}$$

$$d_o = 208.66$$

$$d_i = 122.19$$

16. A solid circular shaft has to transmit a power of 1000 kW at 120 rpm. find the diameter of shaft if shear stress limited to 80 MPa. the max torque loss is 1.25 times mean. what % of saving in material is, would be obtained if the shaft is replaced by hollow shaft, whose internal dia is 0.6 times external dia, the length, material & maximum shear stress being same.

$\Rightarrow P = 1000 \times 10^3 \text{ W}$
 $n = 120 \text{ rpm}$
 $\tau_s = 80 \text{ N/mm}^2$

$\Rightarrow P = 1000 \times 10^3 \text{ W}$
 $n = 120 \text{ rpm}$
 $\tau_s = 80 \text{ N/mm}^2$

$T_{max} = 1.25 T_{avg}$

$d_i = 0.6 d_e$

$\frac{T_{max}}{J} = \frac{\tau_s}{R}$

$\frac{99.47 \times 10^6}{\frac{\pi}{32} (d^4)} = \frac{80}{d/2}$

$d = (99.47 \times 10^6 \times 32)^{1/4} = (80 \times \pi \times 2)^{1/4} d^4$

$d = \underline{\underline{185 \text{ mm}}}$

for given condition dia of the solid shaft is

185 mm



considers a hollow shaft of

external dia = d_e

internal dia = $d_i = (0.6) d_e$

given

$l_s = l_H$

$\tau_s = \tau_H$

$[\tau_s]_s = [\tau_s]_H = \text{maximum}$



given $f_s = f_H$

$$\frac{W}{V} = \frac{W}{A \cdot l}$$

$$f_s]_s = f_s]_H$$

$$\frac{W}{\frac{\pi}{4}(d^4)l} = \frac{W}{\frac{\pi}{4}(d_c^4)l}$$

$$\left[\frac{I}{J}\right]_s = \left[\frac{I}{J}\right]_H$$

$$\frac{I}{J} = \frac{f_s}{R}$$

$$R = \frac{d/2}{\frac{\pi}{32}(d^4)} = \frac{d_c/2}{\frac{\pi}{32}(d_c^4 - d_i^4)}$$

$$d_i = 185 \text{ mm}$$

$$d_o = 111 \text{ mm}$$

$$\frac{d}{2d^4} = \frac{92.5}{[185^4 - 111^4]}$$

101954358

$$\frac{1}{d^3} = \frac{2 \times 92.5}{(101954358)}$$

193

$$d_o =$$

$$\frac{d^4}{d_c^4} (92.5 \times 2 \times 0.8704) = (92.5)^4 d_c$$

$$\frac{92.5}{(92.5)^4} = \frac{d_c/2}{(d_c^4 - 0.6d_c^4)} = (0.8704) d_c$$

$$d_c = 193 \text{ mm}, d_i = 116.29 \text{ mm}$$

% saving in material = $\frac{W_s - W_H}{W_s} \times 100$

$$= \frac{\int (\rho)(V) \}_s - \int (\rho)(V) \}_H}{\int (\rho)(V) \}_s} \times 100$$

$$= \frac{\int (A \cdot l) \}_s - \int (A \cdot l) \}_H}{\int (A \cdot l) \}_s} \times 100$$

$$= \frac{\pi \frac{d^2}{4} - \pi/4 (d_e^2 - d_i^2)}{\pi/4 (d^2)} \times 100$$

$$= \frac{d^2 - (d_e^2 - d_i^2)}{d^2} \times 100$$

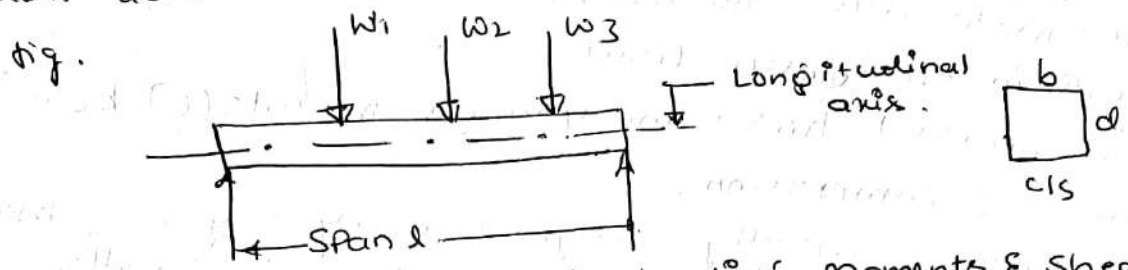
$$= \frac{185^2 - (193^2 - 116.29^2)}{(185)^2} \times 100$$

23725.62

$$\therefore = \underline{\underline{29.76\%}}$$

Module - 4. Bending & Shear Stresses in Beams

A Beam is a structural member on which a system of external loads acts at right angles to its longitudinal axis, as shown in



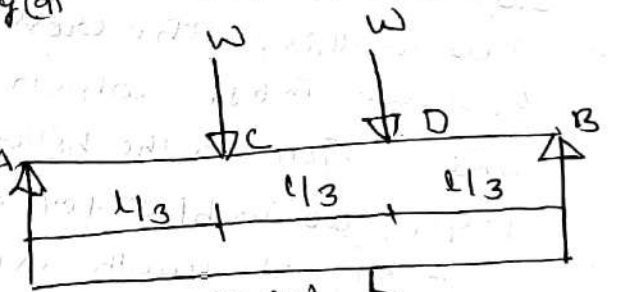
Due to these lateral loads, bending moments & shear forces are setup developed along the length of the beam & longitudinal stress is developed at any section. To bending is known as bending stress or flexure.

Theory of simple Bending

When a beam is bent due to the application of a constant bending moment, without any shear, it is said to be in a state of simple bending or pure bending.

Consider a simply supported beam AB acted upon by two equal loads as shown in fig (a)

In the simply supported beam, it is seen that in b/w C and D the bending is constant & there is no shear force,



Hence the beam portion CD is subjected only to constant bending moment, & free from shear force. This condition of the beam in b/w CD is called pure bending.

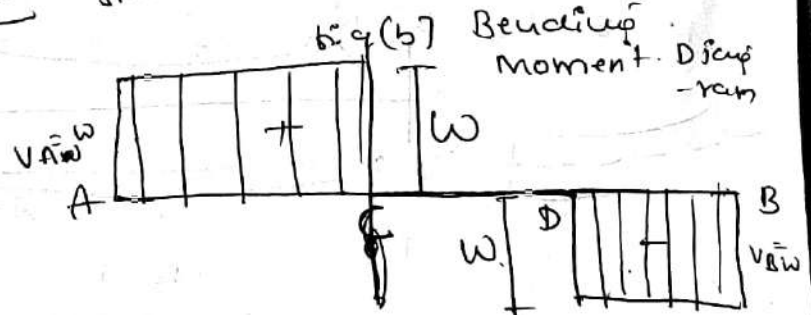
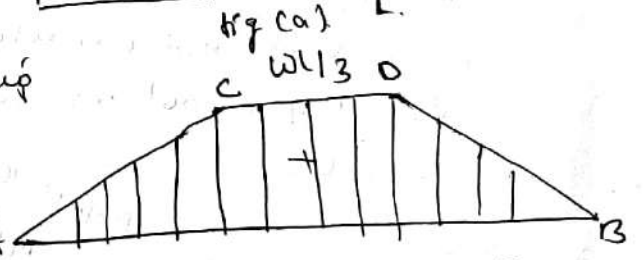


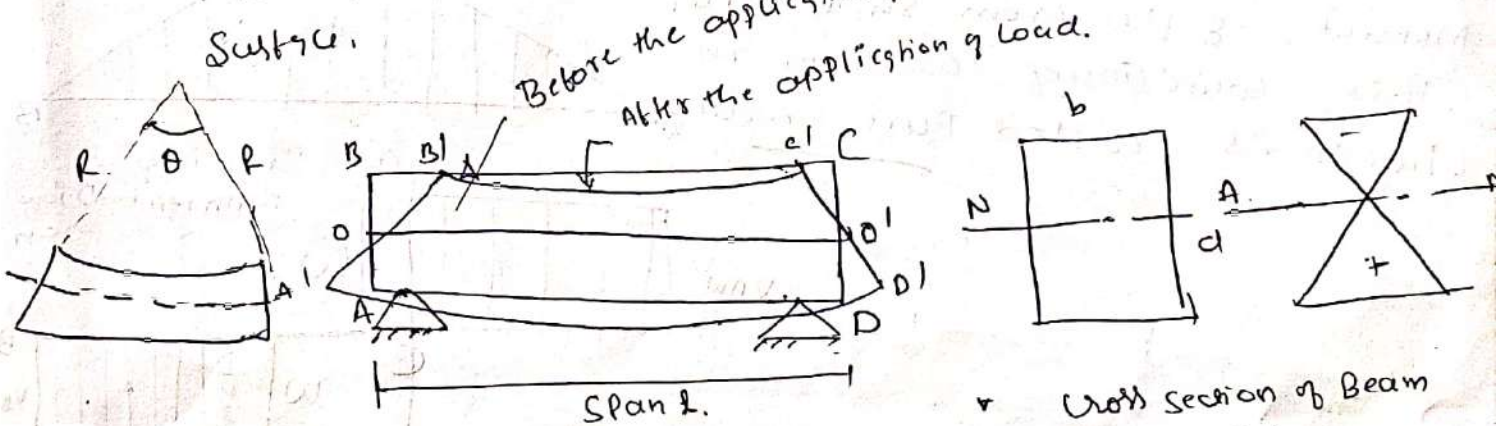
Fig (c) Shear force Diagram

Assumptions made in theory of pure bending:

- The following are some of the assumptions for theory of pure bending as follows.
1. The material is homogenous, isotropic, & obeys Hooke's Law within the elastic limit.
 2. The material has same Young's modulus (E) both in tension & compression.
 3. The beam is initially straight & every layer of material is free to expand \odot or contract \ominus longitudinally.
 4. The stress is purely longitudinal.
 5. Radius of curvature (R) of beam is very large compared to dimensions of beam.
 6. The cross section of beam which is plane before bending will remain plane even after bending it, no warping \odot or no distortion.

contin Theory of simplified Bending:

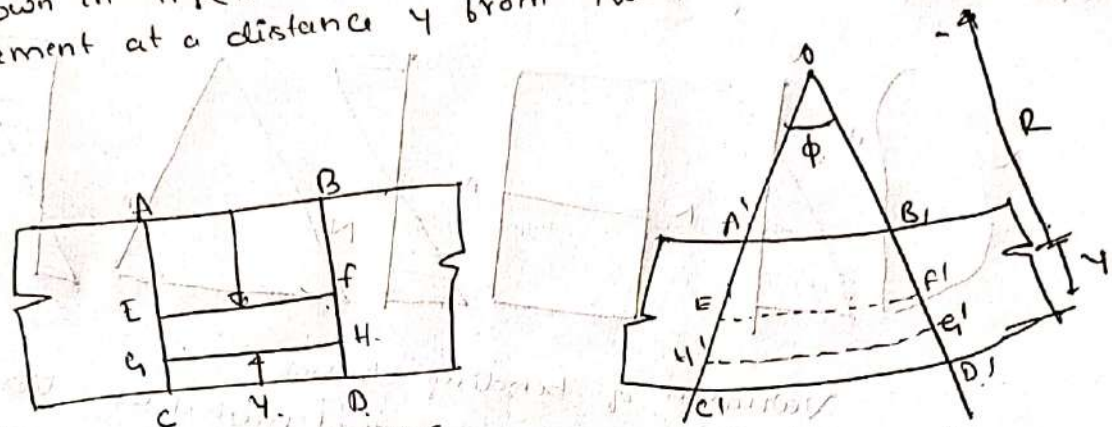
When a beam is subjected to external loads, it undergoes deformation and beam attains a curved shape. Due to this, the cross section AB rotates through an angle θ as shown in fig. which causes compression in the top fibres and tension in the bottom fibres. However there exists a layer oo' in b/w top and bottom fibres, which will retain its original length even after deformation. This layer oo' is called the neutral layer \odot neutral axis \ominus oo' .
Substn.



Neutral axis [N-A] always lies on the centroid of the cross section.

Relationship b/w Bending Stress and Radius of Curvature

Consider a portion of beam b/w sections AC and BD as shown in fig (a). Let EF be the neutral axis and let an element at a distance y from neutral axis.



(a) Before Bending. (b) After Bending.

Fig (b) shows the same portion after bending. Let R be the radius of curvature and ϕ be the angle subtended by $C'A'$ and $B'D'$ at the centre of curvature. Since EF is the neutral axis, there is no change in its length [at neutral axis stresses are zero]

$$EF = E'F' = R\phi$$

Now Strain in the layer $G'H' = \frac{\text{Final length} - \text{original length}}{\text{original length}}$

$$= \frac{G'H' - GH}{GH}$$

$$GH = EF = R\phi$$

$$G'H' = (R+y)\phi$$

$$\text{Strain in the layer } G'H' = \frac{(R+y)\phi - R\phi}{R\phi} = \frac{y}{R}$$

$$\epsilon = \frac{y}{R}$$

We know that from Hooke's law.

$$E = \frac{\sigma}{\epsilon}$$

Young's modulus = $\frac{\text{stress}}{\text{strain}}$

$$E = \frac{\sigma}{\epsilon}$$

$$E = \frac{\sigma}{\epsilon}$$

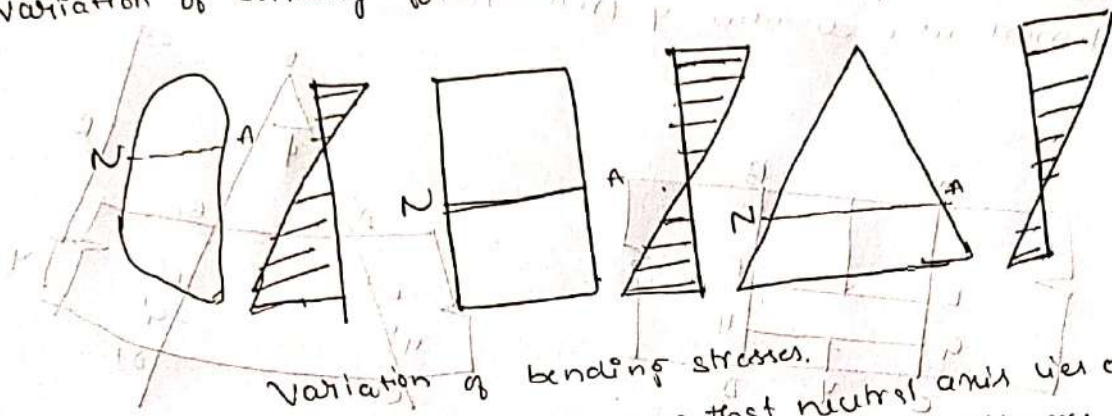
$$\sigma = \frac{E y}{R}$$

$$\sigma = \frac{E y}{R} \quad \text{--- (1)}$$

$$\sigma = \frac{E y}{R} \quad \text{--- (2)}$$

Head of C.E.D

Thus, bending stress varies linearly across the depth. The typical variation of bending for various sections, are shown in fig.



Variation of bending stresses.

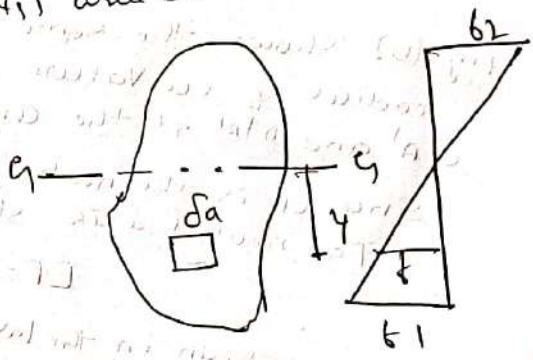
In all these cases it can be seen that neutral axis lies on the centroid of the section. Consider a beam with arbitrary cross-section as shown in fig. Consider an elemental area δa at distance y from the neutral axis.

Let f be the stress on this area.

Force on elemental area = $f \cdot \delta a$

Total force on cross section of

beam = $\sum f \cdot \delta a$



but from eq ①, $f = \frac{E \cdot y}{R}$

Total force on cross section = $\sum \frac{E \cdot y \cdot \delta a}{R} = \frac{E}{R} \sum y \cdot \delta a$

Since there is no axial force on the beam and the above force is in axial direction, from equilibrium condition, we get.

$$\frac{E}{R} \sum y \cdot \delta a = 0$$

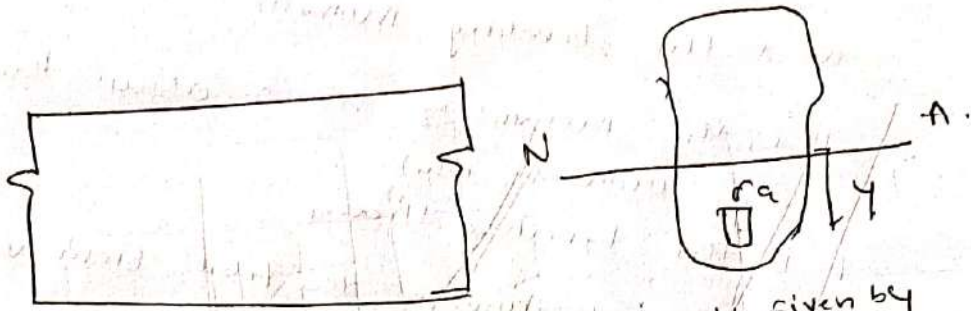
$$\sum y \cdot \delta a = 0$$

$$\sum y \cdot \delta a = 0$$

where A is the total area.

Since $\sum y \cdot \delta a$ is the first moment of area about the neutral axis, $\frac{\sum y \cdot \delta a}{A}$ is the distance of the centroid from the neutral axis. Thus neutral axis coincides with the centroid of the cross section.

Relationship b/w moment and Radius of curvature:
 Consider an elemental area δa at a distance y from the neutral axis in the beam, the cross section of which is shown in fig



Now stress f on this element is given by

$$f = \frac{E}{R} \cdot y$$

$$\begin{aligned} \text{Force on this element} &= f \cdot \delta a \\ &= \frac{E}{R} \cdot y \cdot \delta a \end{aligned}$$

Moment of this resisting force about the neutral axis

$$= \frac{E}{R} \cdot y \cdot \delta a \cdot y$$

$$= \frac{E}{R} \cdot y^2 \cdot \delta a$$

Total moment of resistance (M') of the cross section at any

$$M' = \sum \frac{E}{R} \cdot y^2 \cdot \delta a$$

From definition of moment of inertia, which is second moment of area about centroid, we can write

$$I = \sum y^2 \delta a$$

Where I is the centroidal moment of inertia.

$$M' = \frac{E}{R} I$$

For equilibrium of the moment of resistance (M') should be equal to the applied moment M

$$M' = M$$

$$M = \frac{E}{R} \cdot I$$

$$\frac{M}{I} = \frac{E}{R} \quad \text{--- (3)}$$

From eq (1) & (3) we get

$$\frac{M}{I} = \frac{E}{R} = \frac{f}{y}$$

$$\frac{M}{I} = \frac{b}{r} = \frac{E}{R}$$

Where = M is the bending moment
 I is the moment of Inertia about the centroidal axis
 b is the bending stress
 r is the distance of the fibre from neutral axis
 E = Young's modulus
 R = the radius of curvature

Section modulus

w.k.T. $\frac{M}{I} = \frac{f}{y} = \frac{E}{R}$ Bending moment equation.

hence $\frac{M}{I} = \frac{f}{y} \Rightarrow M = \left(\frac{I}{y}\right) f = Z f = M = \left(\frac{I}{y}\right) b = Z$

where Z is referred as section modulus. The ratio of Moment of Inertia to the distance of extreme fibre (or) layer in tension (or) compression is known as section modulus.

$$Z = \frac{I}{yt} = \frac{I}{yc} = \frac{I}{y_{max}}$$

$$= \frac{mm^4}{mm} \quad Z = mm^3$$

Denoted by Z and unit as mm^3

To Design a beam section if the value of bending moment M is known then the equation $M = [Z] [f]$ is used to calculate the required section modulus (or) moment of Inertia.

$$M = \left(\frac{I}{y}\right) (f)$$

Flexural rigidity.

From the bending equation we have

$$\frac{M}{I} = \frac{E}{R}$$

$$M = \frac{EI}{R}$$

$$\textcircled{2} \quad \frac{1}{R} = \frac{M}{EI}$$

In the above equation, bending moment M , Young's modulus E and moment of Inertia I are constant. Hence R will also be constant.

Radius of curvature R at any point of the elastic curve of a beam is directly proportional to (EI) & inversely proportional to (M) . The product EI is referred as flexural rigidity. (sudden failure).

Modulus of Rupture: When a beam is loaded upto failure the stresses in the beam section cannot be calculated by using bending equation

$$\therefore f = M/Z$$

But sometimes to compare strength of beams of different materials the same bending equation is used.

The stress calculated at rupture using bending equation is known as modulus of rupture.

The term modulus of rupture is limited to cast iron, timber and rectangular section.

Section modulus for various shapes (or) Beam sections

1. Rectangular section:

Moment of inertia of a rectangular section about an axis through its C.G. (or neutral axis NA) is given by

$$I = \frac{bd^3}{12}$$

Distance of outermost layer from NA is given by

$$y_{max} = \frac{d}{2}$$

∴ section modulus is given by

$$Z = \frac{I}{y_{max}} = \frac{bd^3}{12 \times \left[\frac{d}{2}\right]} = \frac{2bd^3}{12d} = \frac{bd^2}{6}$$

for Rectangular

$$Z = \frac{bd^2}{6}$$

For square b=d

$$Z = \frac{d^3}{6} \text{ (or) } \frac{b^3}{6}$$

② Hollow Rectangular section.

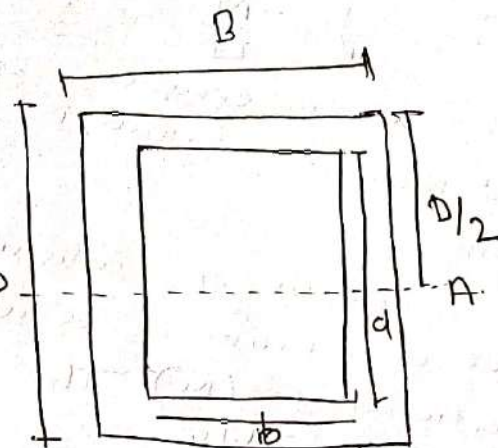
Here $I = \frac{BD^3}{12} - \frac{bd^3}{12}$

$$= \frac{1}{12} [BD^3 - bd^3]$$

$$y_{max} = D/2$$

$$Z = \frac{\frac{1}{12} [BD^3 - bd^3]}{D/2}$$

$$= \frac{1}{6D} [BD^3 - bd^3]$$



$$Z = \frac{BD^3 - bd^3}{6D}$$

③ For circular section:

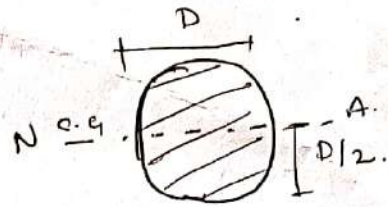
$$I = \frac{\pi D^4}{64} \quad y_{\max} = D/2$$

$$Z = \frac{I}{y_{\max}} \quad Z = \frac{\pi D^4}{64} \frac{1}{D/2}$$

$$Z = \frac{\pi D^4}{64 \times D/2}$$

$$= \frac{\pi D^3}{32}$$

$$Z = \frac{\pi D^3}{32}$$



④ Hollow circular section.

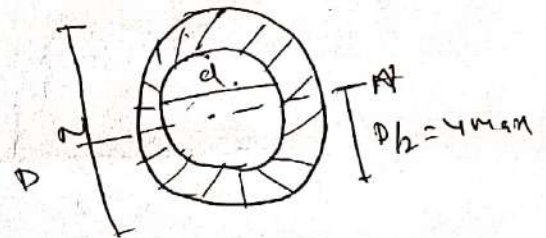
Here

$$I = \frac{\pi}{64} (D^4 - d^4)$$

$$y_{\max} = D/2$$

$$Z = \frac{\frac{\pi}{64} (D^4 - d^4)}{D/2}$$

$$Z = \frac{\pi}{32D} (D^4 - d^4)$$



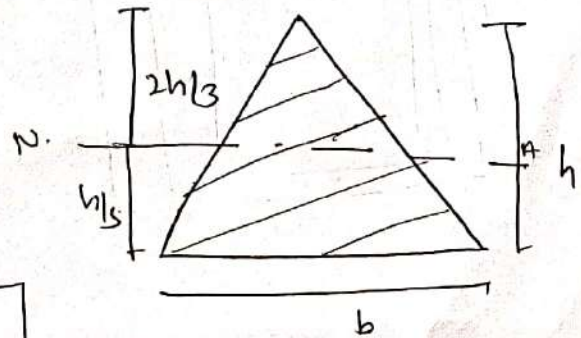
⑤ Triangular section:

$$Z = \frac{I}{y_{\max}}$$

$$I = \frac{bh^3}{36} \quad y_{\max} = \frac{2}{3}h$$

$$Z = \frac{\frac{bh^3}{36}}{\frac{2}{3}h}$$

$$Z = \frac{bh^2}{54}$$



Problems on Bending Stress

1. A steel plate of width 120mm and of thickness 20mm is bent into a circular arc of radius 10m. Determine the maximum stress induced and the bending moment which will produce the maximum stress. Take $E = 2 \times 10^5 \text{ N/mm}^2$.

→ width = 120mm, $t = 20\text{mm}$, $R = 10\text{m}$
 $E = 2 \times 10^5 \text{ N/mm}^2$.

$f_{\text{max}} = \sigma_{\text{max}} = \text{max stress induced}$
 $\therefore \text{moment of inertia} = \frac{bd^3}{12} = \frac{120 \times 20^3}{12} = 8 \times 10^4 \text{ mm}^4$

using equation $\frac{\sigma_{\text{max}} @ b_{\text{max}}}{y_{\text{max}}} = \frac{E}{R}$ eq (1)

$y_{\text{max}} = \frac{t}{2} = \frac{20}{2} = 10\text{mm}$.

eq (1) can be written as

$\sigma_{\text{max}} = \frac{E}{R} \cdot y_{\text{max}}$

$\sigma_{\text{max}} = \frac{2 \times 10^5}{10 \times 10^3} \times 10$

$\sigma_{\text{max}} = 200 \text{ N/mm}^2$

$\frac{M}{I} = \frac{E}{R}$

$M = \frac{E \times I}{R}$

$M = \frac{2 \times 10^5}{10 \times 10^3} \times 8 \times 10^4$

$M = 1.6 \times 10^6 \text{ N-mm}$

$M = 1.6 \text{ kN-m}$

2) Calculate the maximum stress induced in a cast iron pipe of external diameter 40mm, of internal diameter 20mm and of length 4 metre when the pipe is supported at its ends and carries a point load of 80N at its centre.

Given

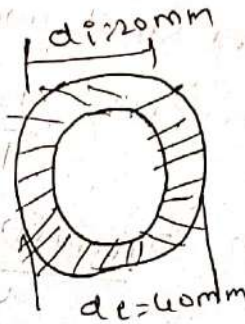
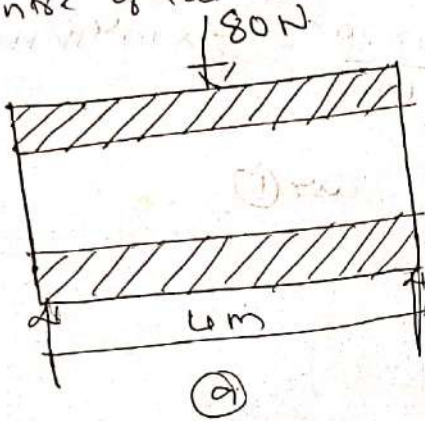
$$D_e = 40 \text{ mm},$$

$$d_i = 20 \text{ mm}$$

$$L = 4 \text{ m} = 4 \times 1000 = 4000 \text{ mm}.$$

$$W = 80 \text{ N}.$$

In case of simply supported beam carrying a point load at the centre, the maximum bending moment is at the centre of the beam.



(b) Cross section

(4) And maximum B.m = $\frac{W \times L}{4}$

$$\text{maximum B.m} = \frac{80 \times 4000}{4} = 8 \times 10^4 \text{ Nmm}$$

$$M = 8 \times 10^4 \text{ Nmm}$$

(5) The cross-section of the pipe
Moment of Inertia of hollow pipe

$$I = \frac{\pi}{64} [D^4 - d^4]$$

$$I = \frac{\pi}{64} [40^4 - 20^4]$$

$$I = 117809.72 \text{ mm}^4$$

When y is maximum, stress will be maximum.
But y is maximum at the top layer from the N.A.

$$y_{\text{max}} = \frac{D}{2} = \frac{40}{2} = 20 \text{ mm}$$

now equation $\frac{M}{I} = \frac{\sigma}{y}$

$$\sigma_{\text{max}} = \frac{M}{I} \cdot y_{\text{max}}$$

$$\sigma_{\text{max}} = \frac{8 \times 10^4}{117809.72} \times 20$$

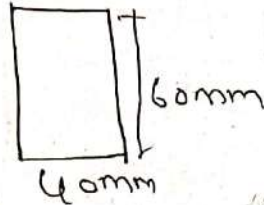
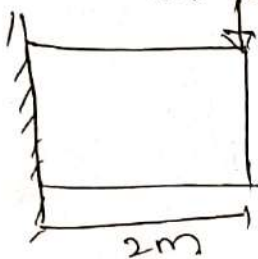
$$\sigma_{\text{max}} = 13.58 \text{ N/mm}^2$$

③ A cantilever of length 2m fails when a load of 2kN is applied at the free end. If the section of the beam is 40mm x 60mm. Find the stress at the failure.

↳ Length $L = 2\text{m}$ $W = 2\text{kN} = 2000\text{N}$.

$L = 2 \times 10^3\text{mm}$

$b = 40\text{mm}$ $d = 60\text{mm}$



Section modulus of Rectan^g C

$$Z = \frac{bd^2}{6}$$

$$= \frac{40 \times 60^2}{6}$$

$$Z = 24000\text{mm}^3$$

$$I = \frac{bd^3}{12}$$

$$Z = \frac{bd^3}{12} \times \frac{12}{d}$$

$$Z = \frac{bd^2}{12} \times \frac{12}{d}$$

$$Z = \frac{bd^2}{6}$$

Maximum bending moment for a cantilever is shown as

$$M = WL = 2000 \times 2 \times 10^3$$

$$= 4000 \times 10^3\text{N-mm}$$

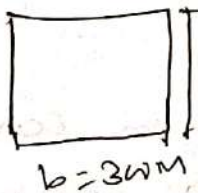
$\sigma_{\text{max}} = \text{Stress at the failure}$

$$M = \sigma_{\text{max}} \cdot Z$$

$$\sigma_{\text{max}} = \frac{M}{Z} = \frac{4 \times 10^6}{24000} = 166.66\text{N/mm}^2$$

$$\sigma_{\text{max}} = 166.66\text{N/mm}^2$$

④ A rectangular beam 200mm deep and 300mm wide is simply supported over a span of 8m. When uniformly distributed load per metre the beam may carry. If the bending stress is not to exceed 120 N/mm².



$b = 300\text{mm}$

$d = 200\text{mm}$

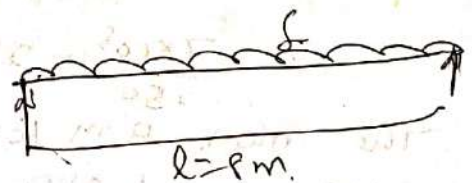
$L = 8\text{m}$

$\sigma_{\text{max}} = 120\text{N/mm}^2$

$W =$ uniformly distributed load per metre length over the beam.

Section modulus of rectangular beam

$$Z = \frac{bd^2}{6} = \frac{300 \times 200^2}{6} = 2 \times 10^6\text{mm}^3$$



Max B.M for a simply supported beam carrying uniformly distributed load as shown in fig. is at the centre of the beam. It is given by

$$M = \frac{w \times l^2}{8} = \frac{w \times l^2}{8} \quad (l = 10000 \text{ mm})$$

$$= 8w \text{ Nm}$$

$$M = 8 \times 1000 = 8000w \text{ Nm}$$

using eq

$$\frac{M}{I} = \frac{\sigma}{y}$$



$$M = \sigma_{\text{max}} \cdot Z$$

$$8000w = 120 \times 2 \times 10^6$$

$$w = \frac{120 \times 2 \times 10^6}{8000} = 30000 \text{ N/mm}$$

$$= 30 \text{ kN/m}$$

- 5) A beam is simply supported and carries a uniformly distributed load of 40 kN/m run over the whole span. The section of the beam is rectangular having depth as 500 mm. If the maximum stress in the material of the beam is 120 N/mm² and moment of inertia of the section is $7 \times 10^8 \text{ mm}^4$. find the span of beam.
- us UDL $w = 40 \text{ kN/m} = 40 \times 1000 \text{ N/m}$
 depth $d = 500 \text{ mm}$. $\text{Max } \sigma_{\text{max}} = 120 \text{ N/mm}^2$ $I = 7 \times 10^8 \text{ mm}^4$

$$L = ?$$

section modulus of the section is given by equation

$$Z = \frac{I}{y_{\text{max}}}$$

$$y_{\text{max}} = \frac{d}{2} = \frac{500}{2} = 250 \text{ mm}$$

$$Z = \frac{7 \times 10^8}{250} = 28 \times 10^5 \text{ N/mm}^3$$

The max. B.M for a simply supported beam carrying a u.d.l over the whole span is at the centre of the beam & is equal to $\frac{w \cdot l^2}{8}$

$$M = \frac{40000 \times l^2}{8}$$

$$M = 5000 l^2 \text{ Nm} = 5000 \times 1000^2 \times 1000 = \text{N-mm}$$

now equation. $M = \sigma_{\text{max}} \cdot Z$

$$M = \sigma_{max} \cdot Z$$

$$5000 \times 1000 \times L^2 = 120 \times 28 \times 10^6$$

$$L^2 = \frac{120 \times 28 \times 10^6}{5000 \times 1000}$$

$$L^2 = 672$$

$$L = \sqrt{672}$$

$$L = 8.19 \text{ m}$$

$$L = 8.20 \text{ m}$$

Q. A rolled steel joist of I section has the dimensions as shown in fig. This beam of I section carries a U.d.l of 40 kN/m over a span of 10m. Calculate the maximum stress produced due to bending.

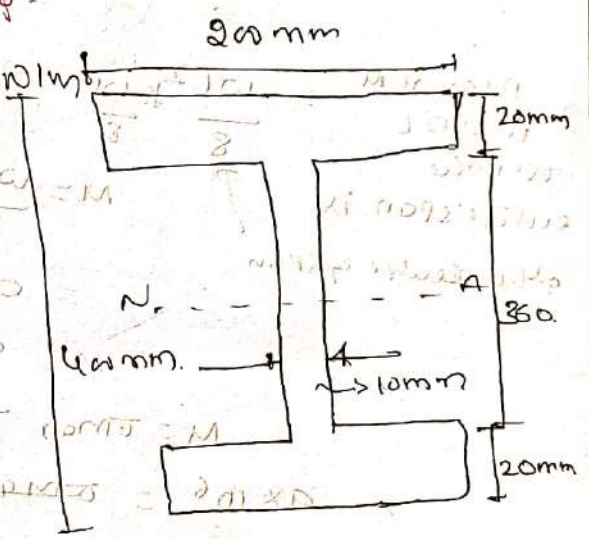
As UDL, $w = 40 \text{ kN/m} = 40 \times 1000 \text{ N/m}$
 Span $L = 10 \text{ m}$

Moment of Inertia about the neutral axis

$$I = \frac{200 \times 400^3}{12} - \frac{(200 - 10) \times 360^3}{12}$$

$$I = 1.066 \times 10^9 - 738.72 \times 10^6$$

$$I = 327.28 \times 10^6 \text{ mm}^4$$



Maximum B.M is given by

$$M = \frac{w \times L^2}{8} = \frac{40000 \times 10^2}{8} = 500000 \text{ N-m}$$

$$= 500000 \times 1000 \text{ N-mm}$$

$$= 5 \times 10^8 \text{ N-mm}$$

using relation

$$\frac{M}{I} = \frac{\sigma}{y}$$

$$\sigma = \frac{M}{I} \cdot y_{max}$$

$$\sigma_{max} = \frac{5 \times 10^8}{372.28 \times 10^6} \times 200$$

$$y_{max} = \frac{d}{2}$$

$$= \frac{400}{2}$$

$$= 200$$

$$\sigma_{max} = 304.92 \text{ N/mm}^2$$

① A timber beam of rectangular section is to support a load of 20 kN uniformly distributed over a span of 3.6 m. when beam is simply supported. If the depth of the section is to be twice the breadth, and the stress in the timber is not to exceed 7 N/mm², find the dimension of the cross section. How would you modify the cross section of the beam, if it carries a concentrated load of 20 kN placed at the centre with the ratio of breadth to depth?

$w = 20 \text{ kN/m} = 20 \times 1000 \text{ N/m}$, $L = 3.6 \text{ m}$, $d = 2b$

$\sigma_{\text{max}} = 7 \text{ N/mm}^2$

Section modulus of rectangular beam $Z = \frac{bd^2}{6} = \frac{b \times (2b)^2}{6}$

$Z = \frac{2b^3}{6}$

Max BM
 for UDL
 over the
 entire span is
 at the centre of span

$M = \frac{wL^2}{8} = \frac{20000 \times 3.6^2}{8} = 19000 \text{ N-m}$

$= 9000 \times 1000 \text{ N-mm}$

$M = 9 \times 10^6 \text{ N-mm}$

$M = \sigma_{\text{max}} \cdot Z$

$9 \times 10^6 = 7 \times \frac{2b^3}{6}$

$b^3 = \frac{3 \times 9 \times 10^6}{7 \times 2}$

$b^3 = (1.9285 \times 10^6)$

$b = 124.47 \text{ mm}$

$d = 2 \times b = 2 \times 124.47$

$d = 248.94 \text{ mm}$

Dimension of the section when the beam carries a point load at the centre

The B.M is maximum at the centre & it is equal to $\frac{w \times L}{4}$ when the beam carries a point load at the centre

$M = \frac{w \times L}{4} = \frac{20000 \times 3.6}{4} = 18000 \text{ N-m}$

$$18000 \times 1000 \text{ N mm}$$

$$\sigma_{\max} = 7 \text{ N/mm}^2$$

$$Z = \frac{2b^3}{3}$$



$$M = \sigma_{\max} Z$$

$$18 \times 10^6 = 7 \times \frac{2b^3}{3}$$

$$b^3 = \frac{18 \times 10^6 \times 3}{7 \times 2}$$

$$b^3 = 3857142.86$$

$$b = 156.82 \text{ mm}$$

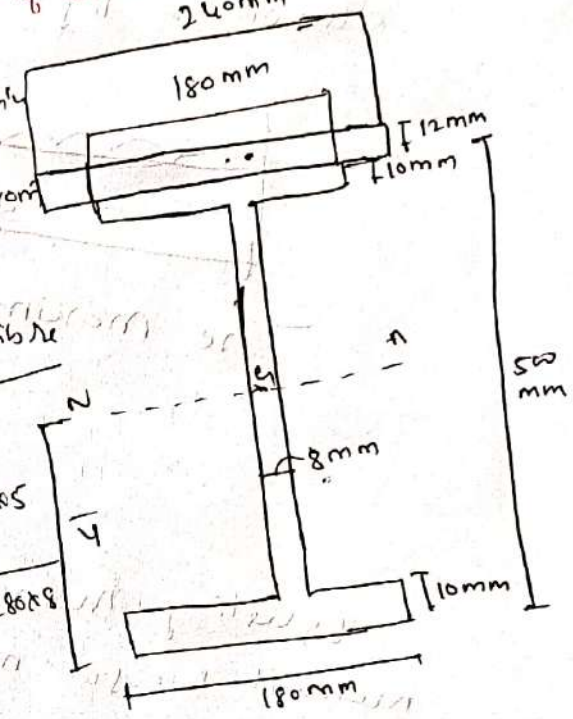
$$d = 2 \times b$$

$$d = 2 \times 156.82$$

$$d = 313.65 \text{ mm}$$

1. A Symmetric I-section has flanges of size 180mm x 10 mm and its overall depth is 500mm. Thickness of the web is 8mm. It is strengthened with a plate of size 240mm x 12mm on compression side. Distance of the section, to permissible stress is 150N/mm². How much uniformly distributed load it can carry, if it is used as a cantilever of span 3m?

w) The section of beam is as shown below.



Let \bar{y} be the distance of centroid from the bottom-most fiber.

$\bar{y} = \frac{\text{Moment of area about bottom fiber}}{\text{Total Area}}$

$$\bar{y} = \frac{240 \times 12 \times 506 + 180 \times 10 \times 495 + 180 \times 10 \times 5 + 480 \times 8 \times 250}{240 \times 12 + 180 \times 10 + 180 \times 10 + 480 \times 8}$$

$$\bar{y} = 321.442 \text{ mm}$$

$$I = \frac{1}{12} \times 240 \times 12^3 + 240 \times 12 [506 - 321.442]^2 + \frac{1}{2} \times 180 \times 10^3 + 180 \times 10 [495 - 321.442]^2 + \frac{1}{2} \times 180 \times 10^3 + 180 \times 10 [5 - 321.442]^2 + \frac{1}{12} \times 480^3 \times 8 + 8 \times 480 [250 - 321.442]^2$$

$$I = 4.25952 \times 10^8 \text{ mm}^4$$

$$y_{top} = 512 - 321.442 = 190.558 \text{ mm}$$

$y_{max} = \bar{y} = 321.442$
 Moment of Resistance (Carrying capacity)

$$M = f_{per} \times Z = 150 \times \frac{I}{y_{max}} = 150 \times \frac{4.25952 \times 10^8}{321.442} = 1.98769 \times 10^8 \text{ N-mm}$$

$$= 1.98769 \times 10^8 \text{ N-mm}$$

$$M = 198.769 \text{ kN-m}$$

Let the load on cantilever be w in length as shown in fig

w/m

3m

The maximum moment produced = $\frac{wl^2}{2} \text{ kN-m}$
 $= \frac{W \times 3^2}{2}$

$$= 4.5 W \text{ kN-m}$$

Equating moment of resistance to maximum moment we get maximum load w

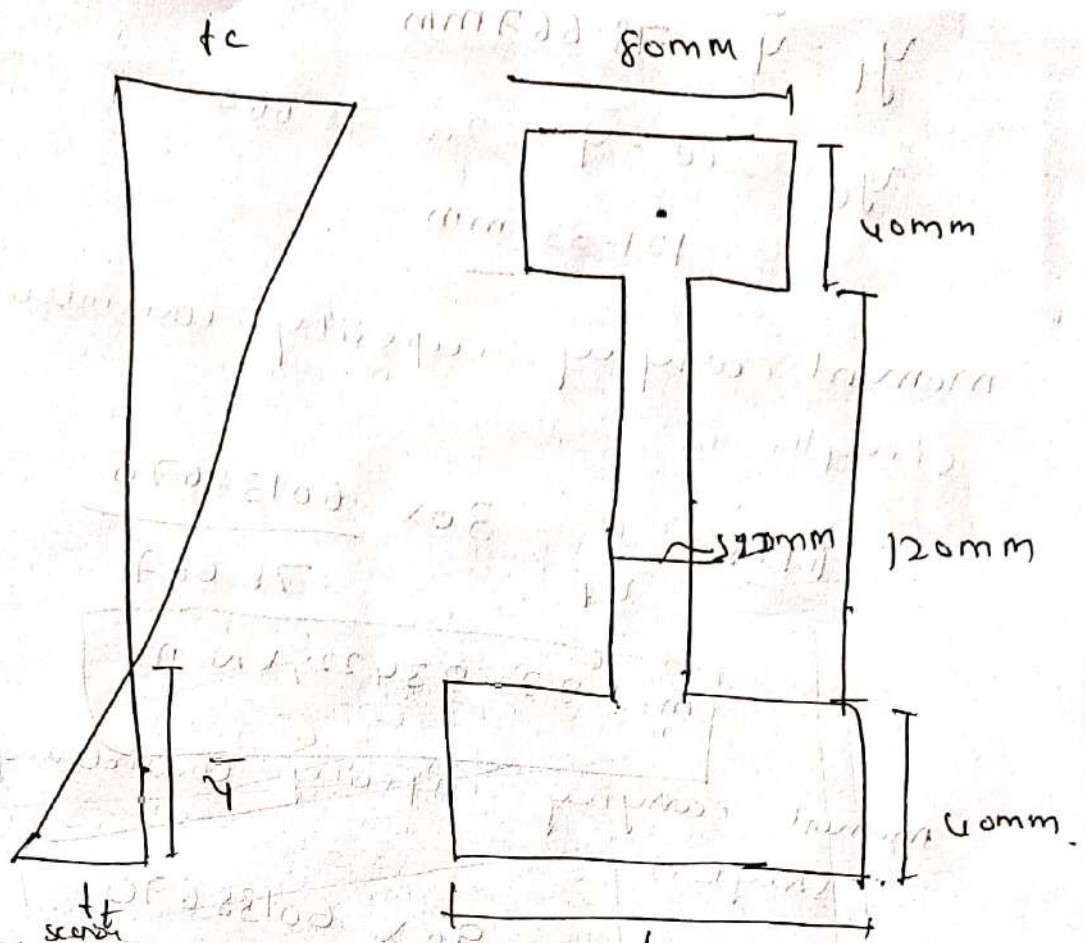
$$4.5 W = 198.769$$

$$W = \frac{198.769}{4.5}$$

$$W = 44.171 \text{ kN/m}$$

(02)

A cast iron beam, has a I-section with top flange $80\text{mm} \times 40\text{mm}$, web $120 \times 20\text{mm}$ and bottom flange $160\text{mm} \times 40\text{mm}$. If the tensile stress is not to exceed 30N/mm^2 and compressive stress 90N/mm^2 . What is the maximum UDL the beam can carry over a simply supported beam span of 6m. Is the larger span flange in tension?



The cross-section of beam is as shown in fig.

Let \bar{y} be distance of centroid (hence from the neutral axis) from the bottom fiber. Then

$$\bar{y} = \frac{\sum a y}{A}$$

$$\bar{y} = \frac{80 \times 40 \times 180 + 120 \times 20 \times 100 + 160 \times 40 \times 20}{80 \times 40 + 120 \times 20 + 160 \times 40}$$

$$\bar{y} = 78.6667 \text{ mm}$$

$$I = \frac{1}{12} \times 80 \times 40^3 + 80 \times 40 (180 - 78.6667)^2 + \frac{1}{12} \times 20 \times 120^3 + 20 \times 120 (100 - 78.6667)^2 + \frac{1}{12} \times 160 \times 40^3 + 160 \times 40 (20 - 78.6667)^2$$

$$I = 60138670 \text{ mm}^4$$

Tension occurs at the bottom and compression at the top. Extreme fiber distances of top and bottom fibers are

$$y_t = \bar{y} = 78.667 \text{ mm}$$

$$\bar{y}_c = 200 - \bar{y} = 200 - 78.667 = 121.333 \text{ mm}$$

moment carrying capacity considering tensile strength is

$$b_{per} \times \frac{f_t}{y_t} = 30 \times \frac{6038670}{78.667}$$

$$m_t = 22.934229 \text{ kN-m}$$

moment carrying capacity considering compression strength is

$$b_{per} \times z = 90 \times \frac{6038670}{121.333}$$

$$= 44.608 \text{ kN-m}$$

Actual moment carrying capacity of the section is smaller of the above two. i.e. moment carrying of the section = 22.9342 kN-m

max moment in a simply supported beam of span l due to the UDL of w kN/m

$$= \frac{wL^2}{8} = \frac{w \times 6^2}{8}$$

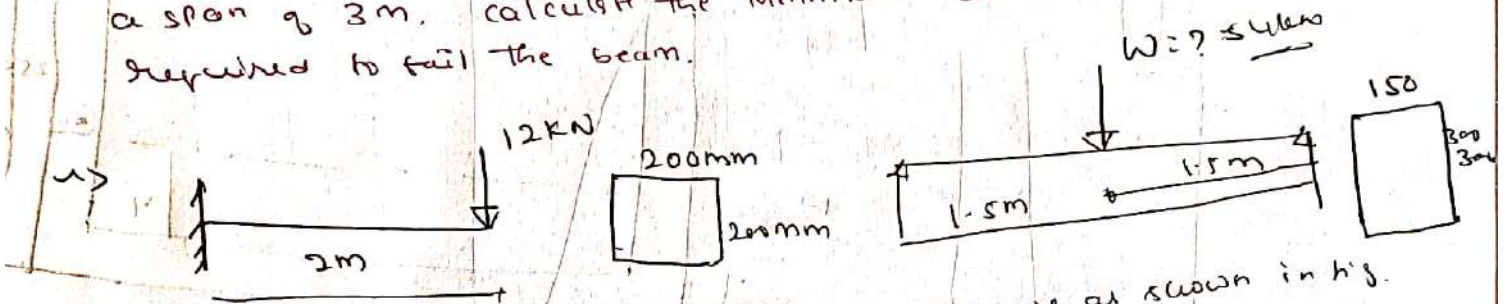
$$= 4.5w$$

By setting it to the moment carrying capacity we get load carrying

$$4.5w = 22.934$$

$$w = 5.0967 \text{ kN/m}$$

* A cantilever of square section $200\text{ mm} \times 200\text{ mm}$, 2 m long, just fails in flexure when a load of 12 kN is placed at its free end. A beam of the same material and having a rectangular cross section 150 mm wide and 300 mm deep is simply supported over a span of 3 m . Calculate the minimum central concentrated load required to fail the beam.



The two beams with their loading case are as shown in fig.

In cantilever beam the moment is

$$= 12 \times 2 = 24 \text{ kN-m}$$

$$= 24 \times 10^6 \text{ N-mm}$$

If f is the stress at which the beam fails, then

$$M = bZ = \frac{1}{2} b d^2 \times f$$

$$24 \times 10^6 = \frac{200 \times 200^2 \times f}{2}$$

$f = 18 \text{ N/mm}^2$

Let W kN be the central concentrated load in simply supported beam of span 3 m . Then the maximum moment is

$$M = \frac{WL}{4} = \frac{W \times 3}{4} = 0.75 W \text{ kN-m}$$

$$= 0.75 \times 10^6 W \text{ N-mm}$$

The section has a moment of resistance = Z

$$= 150 \times \frac{1}{6} \times 150 \times 300^2$$

Equating bending moment to the moment of resistance we get

Maximum load W

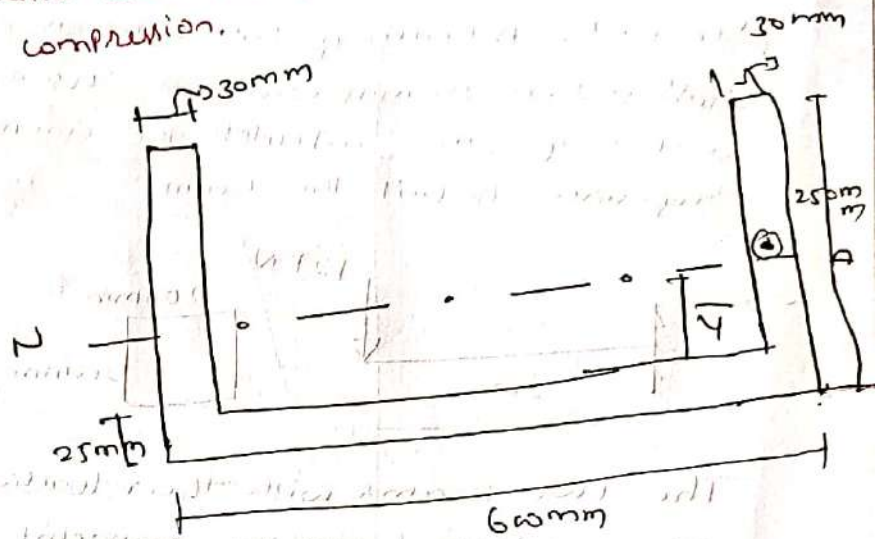
$$0.75 \times 10^6 W = 150 \times \frac{1}{6} \times 150 \times 300^2$$

Thus

$W = 54 \text{ kN}$

A cast iron channel section in fig. is available to be used as a simply supported beam over a span of 3m. If the permissible stress in Tension is 35 N/mm^2 and in compression is 90 N/mm^2 . Find its uniformly distributed load carrying capacity - if web is

- (a) In Tension and (b) In compression.



Soln

Let \bar{y} be the distance of neutral axis from extreme fibre.

$$\text{Then } \bar{y} = \frac{\sum a\bar{y}}{A}$$

$$= \frac{30 \times 250 \times 1.25 \times 2 + 540 \times 25 \times \frac{275}{2}}{30 \times 250 \times 2 + 540 \times 25}$$

$$\bar{y} = 71.71 \text{ mm}$$

$$I = \left[\frac{1}{12} \times 30 \times 250^3 + 30 \times 250 \times (125 - 71.71)^2 \right] \times 2 + \frac{1}{12} \times 540 \times 25^3 + 540 \times 25 \times (12.5 - 71.71)^2$$

$$I = 1.6874 \times 10^8 \text{ mm}^4$$

(a) If web is in Tension $y + \bar{y} = 71.71 \text{ mm}$

$$y_c = 250 - 71.71 = 178.29 \text{ mm}$$

Moment carrying capacity from consideration of Tensile stress is

$$= f_t Z$$

$$= 35 \times \frac{1.6874 \times 10^8}{71.71}$$

$$= 82.365 \times 10^6 \text{ N-mm}$$

From consideration of compressive stress the moment carrying capacity is

$$\begin{aligned}
 &= b c \sigma_c \\
 &= b \times \frac{I}{y_c} \\
 &= \frac{90 \times 1.6875 \times 10^8}{177.29} \\
 &= 85.1863 \times 10^6 \text{ N-mm}
 \end{aligned}$$

Moment carrying capacity = $82.365 \times 10^6 \text{ N-mm}$

If $w \text{ kN/m}$ is UDL in the beam, the maximum bending moment

$$\begin{aligned}
 M &= \frac{w L^2}{8} \\
 &= \frac{w \times 32}{8} \\
 &= 1.25 w \text{ kN-m} \\
 &= 1.25 w \times 10^6 \text{ N-mm}
 \end{aligned}$$

Equating bending moment to the moment of resistance, we get permissible load

$$1.25 \times 10^6 w = 82.365 \times 10^6$$

$$w = 73.213 \text{ kN/m}$$

(B)

26 Web in compression.

$$y_t = 177.299 \text{ mm}$$

$$y_c = 71.71 \text{ mm}$$

Moment carrying capacity from the consideration of tension is

$$\begin{aligned}
 &= b t \times \frac{f}{y_t} \\
 &= \frac{35 \times 1.6875 \times 10^8}{177.29} \\
 &= 33.127994 \times 10^6 \text{ N-mm}
 \end{aligned}$$

This value from consideration of compression

$$\begin{aligned}
 &= b c \times \frac{f}{y_c} \\
 &= \frac{90 \times 1.6875 \times 10^8}{71.71} \\
 &= 211.795 \times 10^6 \text{ N-mm}
 \end{aligned}$$

Moment of resistance = $33.127994 \times 10^6 \text{ N-mm}$

Equating it to bending moment, we get

Head of C.E.D

$$1. 125 \times 10^6 \omega = 32 \cdot 1279948 \times 10^6$$

$$\omega = 27.44416 \text{ rad/m}$$

$$L = 1$$

$$L = 1$$

$$y$$

$$20 \times 10^6 \times 1 \times 10^6$$

$$20 \cdot 10^6$$

$$20 \cdot 10^6 \cdot 10^6$$

... ..

... ..

$$\frac{10^6}{2}$$

$$2$$

$$\frac{10^6}{2}$$

$$2$$

... ..

... ..

... ..

$$\left[\dots \right]$$

$$\dots$$

$$\dots$$

... ..

... ..

$$L = 1$$

$$\dots$$

$$\dots$$

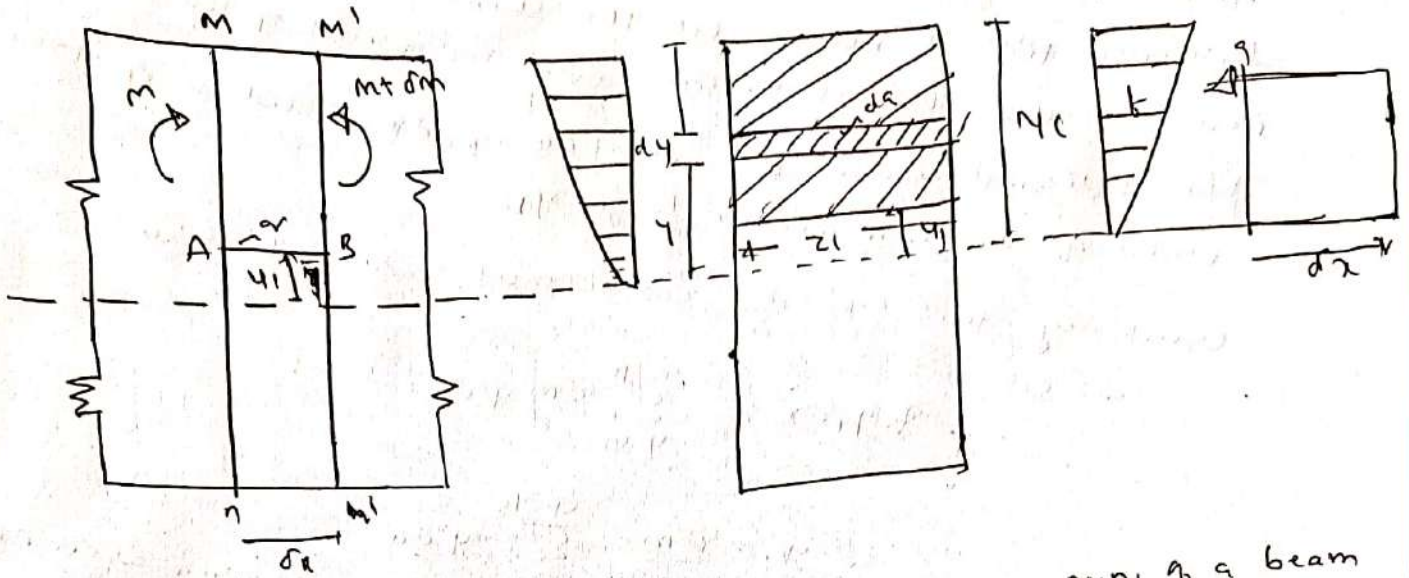
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$$L = 1$$

$$\dots$$

$$\dots$$

Distribution on Shearing Stresses



Let us consider two normal sections $m-n$ and m_1-n_1 of a beam at an infinitely small distance δx . Let the bending moment at the section $m-n$ be M and that at m_1-n_1 be $(M + \delta M)$. Let us investigate the amount of horizontal shear stress at a plane AB , distant y from N.A.

Due to the bending moment M at the section $m-n$, there will be bending stresses across the section and the value of the stress will depend on the position of the layer with respect to N.A. For any ~~analysis~~ at a distance y from N.A., the intensity of bending stress is $f = \frac{M}{I} y$. Hence the compressive thrust on the strip of δa area at this height will be equal to $f \delta a \cdot \delta a = \frac{M y \cdot \delta a}{I}$.

Similarly the elementary thrust on the strip of δa area at the height y from neutral axis at the section m_1-n_1 will be equal to $[\frac{M + \delta M}{I} y \cdot \delta a]$.

[Assuming that the section of the beam is constant from $m-n$ to m_1-n_1].

Hence elementary unbalanced horizontal thrust, between $m-n$ and m_1-n_1 is $\frac{M + \delta M}{I} y \delta a - \frac{M}{I} y \delta a = \frac{\delta M}{I} y \delta a$.

Total unbalanced horizontal thrust to the layer between $y = y_1$ and $y = y_2$ will be equal to $C' = \sum_{y=y_1}^{y=y_2} \frac{\delta M}{I} \cdot \delta y \cdot \delta a$.

For equilibrium of the portion ABM, a horizontal shear force at AB must act to counter-balance this as no other horizontal force is acting. Let the shear intensity at AB be τ . Assuming that this is uniform over the width of the section, the horizontal shear force at AB = $\tau \cdot \delta x \cdot z_1$ where z_1 is the width of the beam at the height y_1 .

Equating this to the unbalanced horizontal thrust, we set

$$\tau \cdot \delta x \cdot z_1 = \sum_{y=y_1}^{y=y_c} \frac{\sigma m}{I} y \cdot \delta a$$

$$\tau = \frac{\sigma m}{\delta x \cdot z_1} \sum_{y=y_1}^{y=y_c} y \cdot \delta a$$

But $\frac{\sigma m}{\delta x} = F$ and $\sum \delta a$ is the moment of the area above AB (i.e. the shaded area) about N.A.

Hence $\tau = \frac{F}{I z_1} (A \bar{y})$

(b) dropping the suffix, we set

$$\tau = \frac{F}{I z} (A \bar{y})$$

$$\tau = \frac{F}{I z} (A \bar{y})$$

Assumptions made:

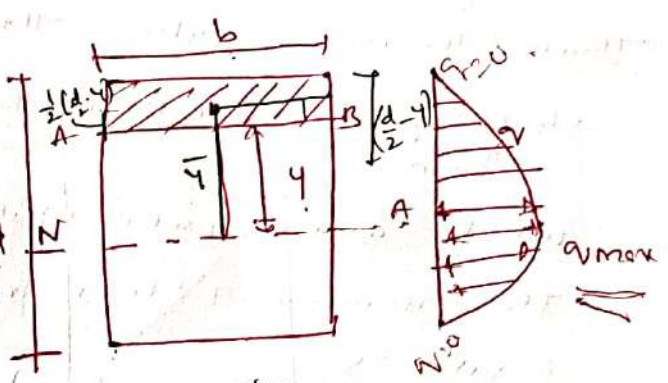
It is important to know the assumptions made they are:

1. The material is homogeneous and isotropic and has same value of E for tension and compression.
2. $F = \frac{dm}{dx}$ is derived from the assumption that the bending stress varies linearly across the section and is zero at the centroid. Actually, the stress curve is not a straight line passing through the centroid of the section.
3. For all values of y , τ is uniform across the width of section of whatever shape. This is not strictly correct because the tangential value must be zero at the boundaries of the section.

Shearing stress distribution over Rectangular Beam.

$$z = \frac{FA\bar{y}}{I}$$

Let us investigate the law of shearing stress distribution over the section of beam of rectangular cross section.



only layer AB, at distance y from neutral axis. the intensity of shear stress q from

$$q = \frac{F}{I} (A\bar{y})$$

where $I =$ moment of Inertia of whole section.
 $z =$ width of the section at $AB = b$

$A\bar{y} =$ moment of the area above.
 $AB =$ moment of the shaded area about the N.A.

Substituting above we get

$$q = \frac{F}{Ib} \left[b \left[\frac{d}{2} - y \right] \right] \left[\frac{1}{2} \left[\frac{d}{2} - y \right] + y \right]$$

$$q = \frac{F}{Ib} \left[b \left[\frac{d}{2} - y \right] \right] \left[\frac{1}{2} \left[\frac{d}{2} + y \right] \right]$$

$$q = \frac{F}{Ib} \times \frac{b}{2} \left[\frac{d^2 - y^2}{4} \right]$$

$$q = \frac{F}{2I} \left[\frac{d^2 - y^2}{4} \right]$$

This is the equation of a parabola \therefore hence the variation of q is parabolic

At $y = \pm d/2, q = 0$

At $y = 0, q = q_{max} = \frac{F}{2I} \frac{d^2}{4} = \frac{F d^2}{8I}$ at the N.A.

$$= \frac{F d^2}{8 \times \frac{bd^3}{12}} = \frac{3F}{2bd}$$

The mean shear stress on the section = $\frac{F}{\text{area}} = \frac{F}{bd}$

$$\frac{q_{max}}{q_{mean}} = \frac{\frac{3F}{2bd}}{\frac{F}{bd}} = \frac{3}{2} = 1.5$$

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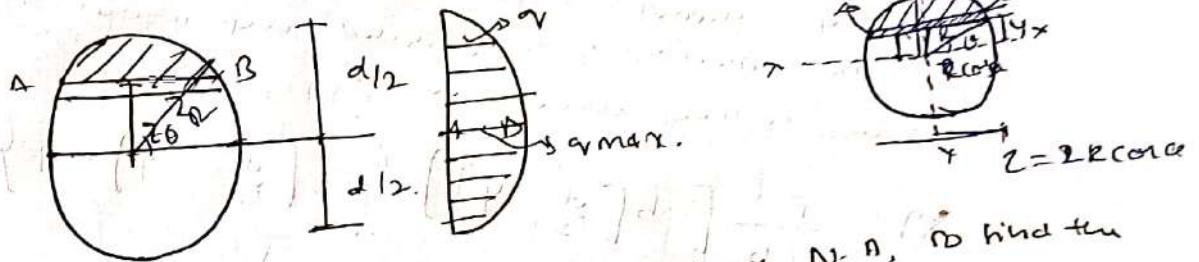
Thus the maximum shear stress at the N.A. is 50% more than the mean value. The

Shearing stress distribution over beam of solid circular section

Consider a beam of solid circular section of diameter d (radius r). The intensity of shear stress at level AB at height y from N.A. is given by

$$\tau = \frac{F}{Iz} (A\bar{y})$$

where $z =$ width of the section at the height $y = 2(r^2 - y^2)^{1/2}$



$A\bar{y} =$ moment of the shaded area, about N.A. To find the moment of the shaded area, consider a strip of thickness δy at the height y . The area of δy of the strip $= z \cdot \delta y$.

The moment of elementary area $= \delta y \cdot y = z \cdot \delta y \cdot y$.

Hence the moment of the shaded area about the N.A. is

$$A\bar{y} = \int_{y=y}^{y=r} y \cdot z \cdot dy$$

$$z = 2\sqrt{r^2 - y^2} \quad \text{or} \quad z^2 = 4(r^2 - y^2)$$

$$2z dz = 4(-2y) dy = -8y \cdot dy$$

$$y dy = -\frac{1}{4} z dz$$

When $y=r$, $z=0$ When $y=0$, $z=2r$.

$$z = f(y)$$

$$z = b(0)$$

$$z = b(\bar{y})$$

$$z = 2R \cos \theta$$

$$z = \frac{F \bar{A} \bar{y}}{z I}$$

$$dA = 2x dy$$

$$\bar{A} \bar{y} = \int 2x dy \cdot y$$

$$I = \frac{\pi R^4}{4}$$

$$z = \frac{F \times \int 2x dy \cdot y}{(2R \cos \theta) \left(\frac{\pi R^4}{4} \right)}$$

$$z = \frac{4F \int 2x dy \cdot y}{\pi R^5 \cos \theta}$$

$$x = \sqrt{R^2 - y^2}$$

$$z = \frac{4F}{\pi R^5 \cos \theta} \int_0^R \sqrt{R^2 - y^2} \cdot y \cdot dy$$

$$z = \frac{4F}{\pi R^5 \cos \theta} \left[\frac{1}{3} (R^2 - y^2)^{3/2} \right]_0^R$$

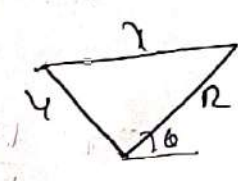
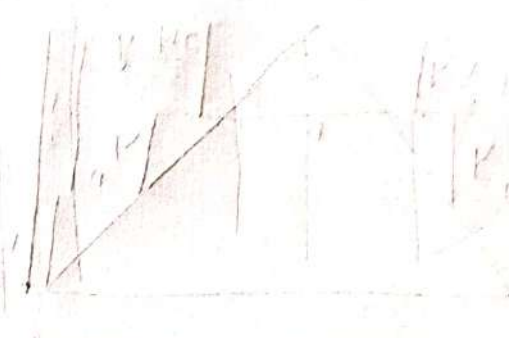
$$z = \frac{4F}{\pi R^5 \cos \theta} \left[\frac{1}{3} (1 - \sin^2 \theta)^{3/2} \right]$$

$$z = \frac{4}{3} \frac{F}{R^2 \pi} \cos^2 \theta$$

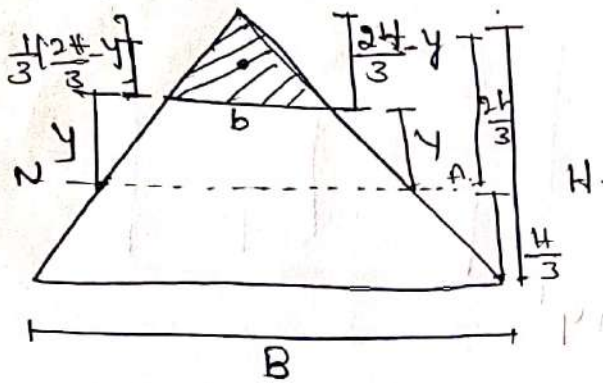
$$z = \frac{4}{3} \frac{F}{R^2 \pi} \cos^2 \theta$$

Neutral axis

$$z_{N.A.} = \frac{4}{3} \left[\frac{F}{\pi R^2} \right] - \text{Mean shear} = 1.33 \tau_{\text{mean}}$$



Shear stress distribution in triangular section



$$dA = \frac{1}{2} b \times \left(\frac{2H}{3} - y \right) \times \left(y + \frac{1}{3} \left(\frac{2H}{3} - y \right) \right)$$

$$= \frac{1}{2} b \left(\frac{2H}{3} - y \right) \left(\frac{2y}{3} + \frac{2H}{3} \right)$$

$$dA = \frac{1}{2} b \left(\frac{2H}{3} - y \right) \frac{2}{3} \left(y + \frac{H}{3} \right)$$

$$dA = \frac{1}{3} b \left(\frac{2H}{3} - y \right) \left(y + \frac{H}{3} \right)$$

$$dA = \frac{b}{3} \left[\frac{2H^2}{9} - y^2 + \frac{H}{3} y \right]$$

$$z \bar{y} = \frac{F A \bar{y}}{z I}$$

$$= \frac{F \left[\frac{b}{3} \left(\frac{2H^2}{9} - y^2 + \frac{H}{3} y \right) \right]}{\frac{B H^3}{36} \times b}$$

$$z \bar{y} = \frac{12F}{B H^3} \left[\frac{2H^2}{9} - y^2 + \frac{H}{3} y \right]$$

$$\text{at } y = \frac{2H}{3} \quad z = 0$$

$$\text{at } y = -\frac{H}{3} \quad z = 0$$

$$\frac{dz}{dy} = 0$$

$$\frac{12F}{B H^3} \left[-2y + \frac{H}{3} \right] = 0$$

$$y = \frac{H}{6}, \quad z = z_{max}$$

$$z_{max} = \frac{12F}{B H^3} \left(\frac{2H^2}{9} - \frac{H^2}{36} + \frac{H}{3} \times \frac{H}{6} \right)$$

$$= \frac{12F}{B H} \left(\frac{8-1+2}{36} \right) = \frac{3F}{B H}$$

$$z_{max} = \frac{3F}{B H} = \frac{3}{2} z_{avg}$$

$$\left(z_{max} = \frac{3}{2} z_{average} \right)$$

$$z = t(0)$$

$$z = t(\bar{y})$$

$$z = 2R \cos \theta$$

$$z = \frac{F \bar{y}}{z I}$$

$$dA = 2x dy$$

$$A \bar{y} = \int 2x dy \cdot y$$

$$I = \frac{\pi R^4}{4}$$

$$z = \frac{F \times \int 2x dy \cdot y}{(2R \cos \theta) \left[\frac{\pi R^4}{4} \right]}$$

$$z = \frac{4F \int 2x dy \cdot y}{\pi R^5 \cos \theta}$$

$$z = \sqrt{R^2 - y^2}$$

$$z = \frac{4F}{\pi R^5 \cos \theta} \int_{R \sin \theta}^R \sqrt{R^2 - y^2} \cdot y \cdot dy$$

$$z = \frac{4F}{\pi R^5 \cos \theta} \left[\frac{1}{3} (R^2 - y^2)^{3/2} \right]_{R \sin \theta}^R$$

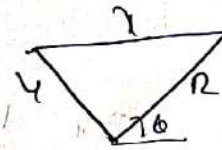
$$z = \frac{4F}{\pi R^5 \cos \theta} \left[\frac{1}{3} (1 - \sin^2 \theta)^{3/2} \right]$$

$$z = \frac{4F}{3} \frac{\cos^2 \theta}{R^2 \pi}$$

$$z = \frac{4}{3} \frac{F}{R^2 \pi} \cos^2 \theta$$

Neutral axis

$$z_{NA} = \frac{4}{3} \left[\frac{F}{\pi R^2} \right] - \text{Mean shear} = 1.33 \tau_{\text{mean}}$$



Module - 5

Columns and struts

Structural members subjected to compression, which are relatively long in comparison with cross sectional dimension and being vertical are called columns.

Ex: Rcc columns, Steel columns, timber columns, and Composite columns. if the member of the structure is vertical and both ends are fixed rigidly while subjected to axial compressive load, the member is known as column.

A strut is commonly used for compression member in a roof truss. it may either be in vertical position

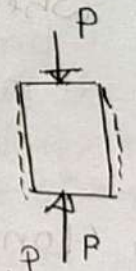
(or) in inclined position. The principal compression member in crane is called a boom. if the member of the structure is not vertical & one or both of its ends are hinged or pin joined, the member is known as a strut.

Thus, a compression member is divided into struts

Three categories. Failure of a column

① Short compression members: which fails primarily by crushing without buckling. if a short length of bar or block is subjected to a compressive force P, uniform compressive stress $\sigma = P/A$ is induced. Such a member fails by crushing on increasing the value of force P.

Crushing

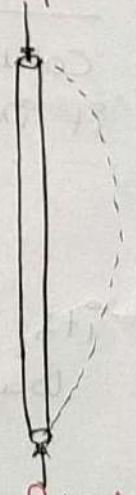


② Long columns: which fails by buckling excessive lateral bending. Fig 1

③ Intermediate columns: which fails by combination of crushing and buckling.

A compression member is classified by its length and least lateral dimension. Most of the practical cases of compression members fall under the second category. The critical load, which the members can carry before failure.

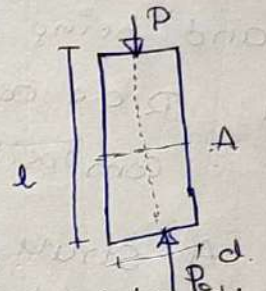
Fig 2 Buckling



Depends upon (1) member dimensions [area, shape and length] and (2) end conditions

* Short column: If the ratio of effective length of the column to its least lateral dimension does not exceed 15

i.e.
$$\frac{\text{length effective}}{\text{breadth or depth}} \Rightarrow \frac{l_{eff}}{b \text{ or } d} < 15$$



If the ratio of effective length of the column to its least radius of gyration does not exceed 50. Then it is called short column.

$$\frac{l_{eff}}{k_{min}} < 50$$

$$k = \sqrt{\frac{I}{A}}$$

k_{min} = minimum radius of gyration.

A short column fails by crushing or yielding

$$f_c = \left[\frac{P_c}{A} \right]$$

where f_c = crushing stress or ultimate stress.

P_c = crushing load

A = Area

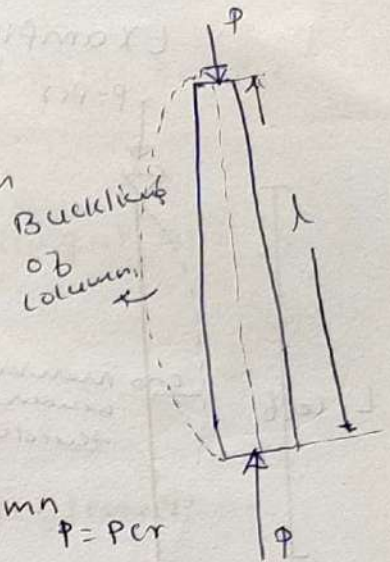
* Long column: If the ratio of effective length of the column to its least lateral dimension exceeds 15, then it is long column.

If the ratio of effective length of the column to its least radius of gyration exceeds 50 then it is long column.

$$\frac{l_{eff}}{b \text{ or } d} > 15 \quad \frac{l_{eff}}{k_{min}} > 50$$

A long column fails by buckling and stresses are well below the elastic limit

$$P = P_{cr}$$



③ Critical load: The maximum load which a column can support before becoming unstable is known as critical load (or) buckling load (or) crippling load.

④ Slenderness ratio: $[\lambda]$

The ratio of length of the column to its least radius of gyration.

$$\lambda = \frac{l}{k_{min}}$$

$$k_{min} = \sqrt{\frac{I_{min}}{A}}$$

where l = length of the column (mm)

k_{min} = min radius of gyration (mm)

λ = slenderness ratio (unit less)

Effective length of a column:

Effective length of a column with given end conditions is the length of an equivalent column of the same material of the section with hinged ends having the value of the crippling load equal to that of the given column.

① It is the distance b/w points of zero moments.

Examples are shown below.

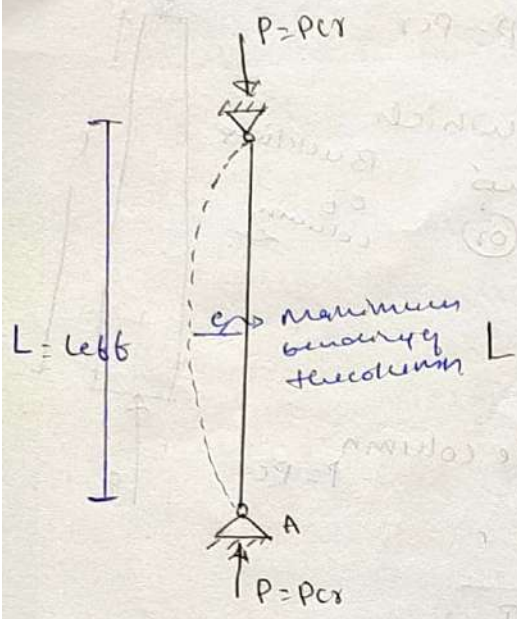


fig: 1

Both ends hinged.

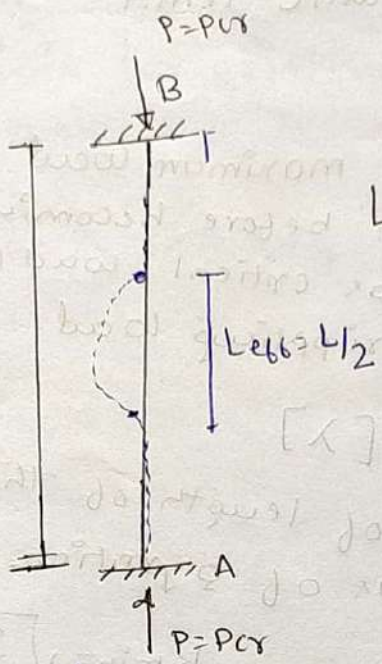


fig: 2

Both ends fixed.

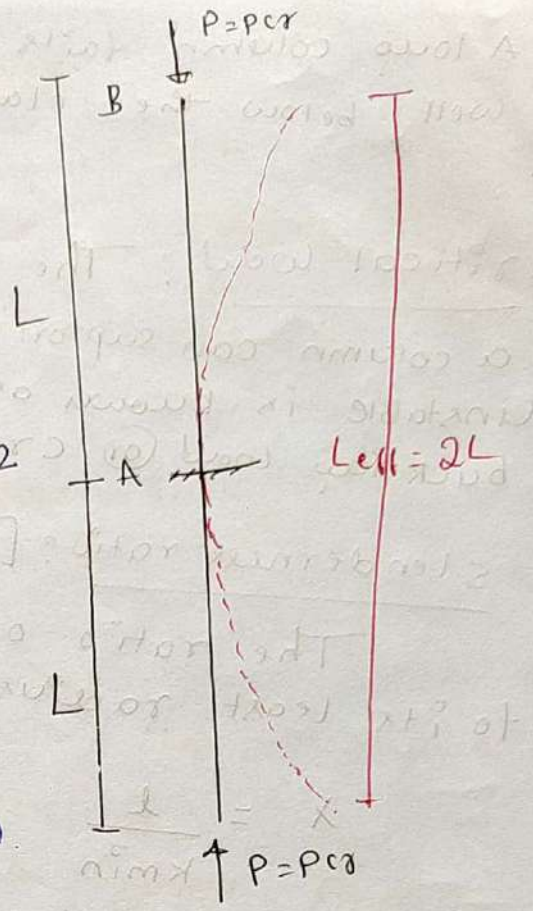


fig: 3

One end is free and other end is fixed.

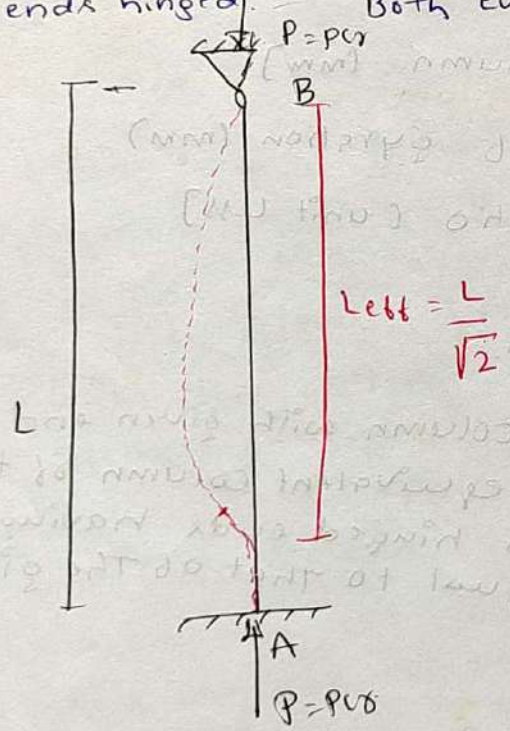


fig: 4 one end is hinged and other end is fixed.

Assumptions of Euler's theory [long column] (3)

Euler's formula for crippling load is based on the following assumptions:

1. The column is initially perfectly straight and is axially loaded.
2. The section of the column is uniform.
3. The self weight of the column is neglected.
4. The column material is perfectly elastic, homogeneous & isotropic.
5. Stresses are within elastic limit and hence column obeys Hooke's law.
6. The length is very large compared to the lateral dimensions (or) cross sectional dimension ($l \gg b$ & d).
7. The Direct stresses are very small compared to with the bending stresses corresponding to the buckling conditions.
8. The column fails due to buckling alone.

End conditions for long columns:

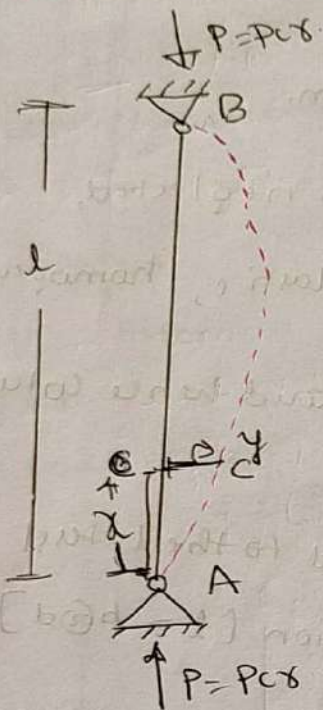
In case of long columns, the stresses due to direct load is very small in comparison with the stresses due to buckling. Hence the failure of long columns take place entirely due to buckling (or bending).

The following four types of end conditions of the columns are important.

1. Both ends of the column are hinged (or) [pinned].
 2. One end is fixed and other end is free.
 3. Both the ends of the column are fixed.
 4. One end is fixed & the other is pinned.
- For a hinged end, the deflection is zero, but at fixed end the deflection and slope is zero. For a free end the deflection is not zero.

Derive the expression for Euler's theory of buckling load when both ends are hinged. Consider a long column of length l subjected to gradually increasing load P with both ends hinged as shown in figure.

Case: 1 Both ends hinged (Pinned)



The load at which the column just buckles (or bends) is called crippling load. Consider a column AB of length l and uniform cross-sectional area, hinged at both its ends A and B. Let P be the crippling load at which the column has just buckled.

Due to the crippling load the column will deflect into curved form ACB as shown in fig.

Consider any section at a distance x from the end A.

Let y be the deflection (lateral displacement) at the section.

Consider a point C on the column where the deflection is y

Bending moment at the section

constant
 $B_m] = (-Pcr)(y) \text{ --- } -Pcr y$ (negative sign)

Also WKT.

$$B_m] = EI \left[\frac{d^2 y}{dx^2} \right] = -Pcr y$$

$$EI \left[\frac{d^2 y}{dx^2} \right] + Pcr (y) = 0$$

÷ by EI

$$\frac{d^2 y}{dx^2} + \frac{Pcr}{EI} y = 0 \rightarrow \frac{d^2 y}{dx^2} + \frac{Pcr y}{EI} = 0$$

$$y = c_1 \cos \left[x \sqrt{\frac{Pcr}{EI}} \right] + c_2 \sin \left[x \sqrt{\frac{Pcr}{EI}} \right]$$

Moment Curvature Relation

$$\frac{M}{EI} = \frac{d^2 y}{dx^2}$$

$$M = \frac{d^2 y}{dx^2} \cdot EI$$

$$2 = \sqrt{\frac{Pcr}{EI}}$$

$$\left[\frac{d^2 y}{dx^2} + \frac{Pcr}{EI} y = 0 \right]$$

$$y = c_1 \cos \alpha x + c_2 \sin \alpha x$$

The moment due to crippling load at the section
 $= -P \cdot y$

But moment $= EI \cdot \frac{d^2y}{dx^2}$

Moment curvature relationship

$$\frac{m}{EI} = \frac{d^2y}{dx^2}$$

Equating the two moments, we have

$$EI \cdot \frac{d^2y}{dx^2} = -P \cdot y$$

$$m = \frac{d^2y}{dx^2} \cdot EI$$

$$EI \cdot \frac{d^2y}{dx^2} + P \cdot y = 0$$

$$\div EI$$

$$\frac{d^2y}{dx^2} + \frac{P}{EI} \cdot y = 0$$

The equation

$$\frac{d^2y}{dx^2} + \frac{P}{EI} \cdot y = 0$$

can be written as

$$\frac{d^2y}{dx^2} + d^2 \cdot y = 0$$

The solution of the above differential equation is

$$y = c_1 \cos dx + c_2 \sin dx$$

$$d^2 = \frac{P}{EI}$$

$$y = c_1 \cos\left(\sqrt{\frac{P}{EI}} \cdot x\right) + c_2 \sin\left(\sqrt{\frac{P}{EI}} \cdot x\right) \quad \text{--- (1)}$$

$$d = \sqrt{\frac{P}{EI}}$$

Where c_1 and c_2 are the constants of integration. The values of c_1 and c_2 are as follows.

The solution of equation $y = c_1 \cos dx + c_2 \sin dx$

(1) At A $x=0, y=0$, substituting their values in eq (1) we get

$$y = c_1 \cos\left[\sqrt{\frac{P}{EI}} \cdot x\right] + c_2 \sin\left[\sqrt{\frac{P}{EI}} \cdot x\right]$$

$$0 = c_1 \cos\left[\sqrt{\frac{P}{EI}} \cdot 0\right] + c_2 \sin\left[\sqrt{\frac{P}{EI}} \cdot 0\right]$$

$$0 = c_1 \times 1 + c_2 \times 0$$

$$\therefore \begin{cases} \cos 0 = 1 \\ \sin 0 = 0 \end{cases}$$

$$\boxed{c_1 = 0} \rightarrow \text{(2)}$$

(2) At B $x=l, y=0$, substituting their values in eq (1) we get

$$0 = c_1 \cos\left[\sqrt{\frac{P}{EI}} \cdot l\right] + c_2 \sin\left[\sqrt{\frac{P}{EI}} \cdot l\right] \quad (c_1 = 0)$$

$$0 = 0 + c_2 \sin\left[\sqrt{\frac{P}{EI}} \cdot l\right]$$

$$= c_2 \sin\left[l\sqrt{\frac{P}{EI}}\right] \rightarrow \text{(3)}$$

From equation (3) it is clear that either $C_2 = 0$

$$\sin \left(l \sqrt{\frac{P}{EI}} \right) = 0$$

As $C_1 = 0$ then if $C_2 = 0$ is also equal to zero, then from equation (1) we will get $y = 0$, this means that the column will be zero @ the column will not bend at all. which is not true.

$$\therefore \sin \left(l \sqrt{\frac{P}{EI}} \right) = 0$$

$$= \sin 0 \text{ @ } \sin \pi \text{ @ } \sin 2\pi \text{ @ } \sin 3\pi \text{ or } \dots$$

$$l \sqrt{\frac{P}{EI}} = 0 \text{ @ } \pi \text{ @ } 2\pi \text{ @ } 3\pi$$

Taking the least practical values

$$l \sqrt{\frac{P}{EI}} = \pi$$

$$P = \frac{\pi^2 EI}{l^2}$$

- ① Derive the expression for crippling load when both the ends are fixed.
- ② Derive the expression for crippling load when one end is free and other end fixed.
- ③ Derive the Euler's equation for buckling load for an elastic column with one end fixed and other end hinged.

Where c_1 and c_2 are constants which are obtained by boundary conditions.

at $x=0$, $y=0$, $\therefore c_1=0$

at $x=l$, $y=0$, $c_2 \neq 0$

As c_2 can not be zero the derivation is observed at all other points hence.

$$\sin\left[l\sqrt{\frac{PCr}{EI}}\right] = 0$$

for minimum non-zero value of 0, we have solution has

$$\sin\left[l\sqrt{\frac{PCr}{EI}}\right] = 0, \pi, 2\pi, \dots$$

$$l\sqrt{\frac{PCr}{EI}} = \pi$$

$$\frac{l^2 PCr}{EI} = \pi^2$$

$$PCr = \frac{\pi^2 EI}{l^2}$$

As both ends are hinged (or) Pinned $PCr = \frac{\pi^2 EI}{(l/2)^2}$

When both ends are fixed,

$$PCr = \frac{\pi^2 EI}{(l/2)^2} = \frac{4\pi^2 EI}{l^2}$$

When one end is free and other end

$$PCr = \frac{\pi^2 EI}{(2l)^2} \text{ (or) } \frac{\pi^2 EI}{4l^2}$$

one end hinged and other end is fixed,

$$PCr = \frac{\pi^2 EI}{(4\sqrt{2})^2} \text{ (or) } \frac{\pi^2 EI}{32} = \frac{2\pi^2 EI}{l^2}$$

$$k^2 = \frac{P}{A}$$

Sign Conventions : The following sign conventions for the bending of the columns will be used.

1. A moment which will bend the column with its convexity towards its initial central line as shown in fig (a) is taken as positive.

Fig (a) AB represents the initial centre line of a column. Whether the column bends taking the shape AB' or AB'', the moment producing this type of curvature is positive.

(2) A moment which will bend the column with its concavity towards its initial centre line as shown in fig (b) is taken as negative.

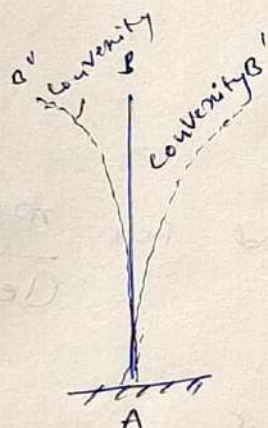


fig (a) positive

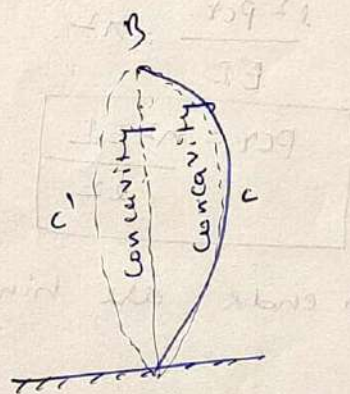


fig (b) negative

$$\frac{d^2y}{dx^2} + \frac{P}{EI} \cdot y = 0$$

$$y = c_1 \cos(\lambda x) + c_2 \sin(\lambda x)$$

$$y = c_1 \cos\left[\sqrt{\frac{P}{EI}} \cdot x\right] + c_2 \sin\left[\sqrt{\frac{P}{EI}} \cdot x\right] \quad \text{--- (1)}$$

$$\frac{d^2y}{dx^2} + \frac{P}{EI} \cdot y = 0$$

rewritten as

$$\frac{d^2y}{dx^2} + \lambda^2 \cdot y = 0$$

where

$$\lambda^2 = \frac{P}{EI}$$

(1) At A, $x=0, y=0$, in eq (1) we get

$$0 = c_1 \cdot \cos 0 + c_2 \sin 0$$

$$= c_1 \cdot 1 + c_2 \cdot 0$$

$$0 = c_1 \quad \text{--- (2)}$$

(2) At B, $x=l, y=0$, in eq (1) we get

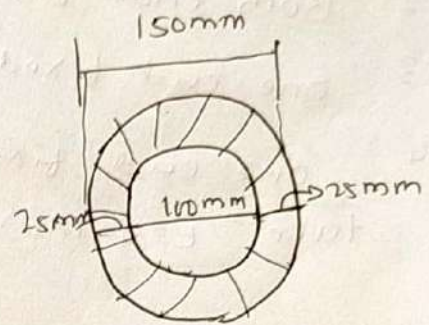
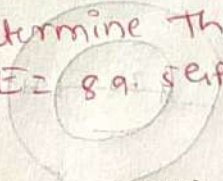
$$0 = c_1 \cos\left[\sqrt{\frac{P}{EI}} \cdot l\right] + c_2 \sin\left[\sqrt{\frac{P}{EI}} \cdot l\right]$$

$$y = c_1 \cos(\lambda x) + c_2 \sin(\lambda x)$$

Problems on Euler's Theory

1. A hollow cast iron cylindrical column 3m long is hinged at both ends. The external diameter is 150mm & thickness is 25mm using the factor of safety is 4. determine the same load that the column can support take

$E = 89.5 \text{ GPa}$



$l = 3000 \text{ mm}$
 $d_e = 150 \text{ mm}$
 $d_i = 100 \text{ mm}$
 $l = l_e = 3000 \text{ mm}$

$d_i = 150 - 25 - 25 = 100 \text{ mm}$

$E = 89.5 \times 10^3 \text{ MPa} = 89.5 \text{ N/mm}^2$
 $FOS = 4$

Both the end conditions are pinned or hinged.

$$P_{cr} = \frac{\pi^2 EI}{(l_e)^2}$$

$$I = \frac{\pi}{64} [d_e^4 - d_i^4]$$

$$I = \frac{\pi}{64} [150^4 - 100^4]$$

$$I = 19.94 \times 10^6 \text{ mm}^4$$

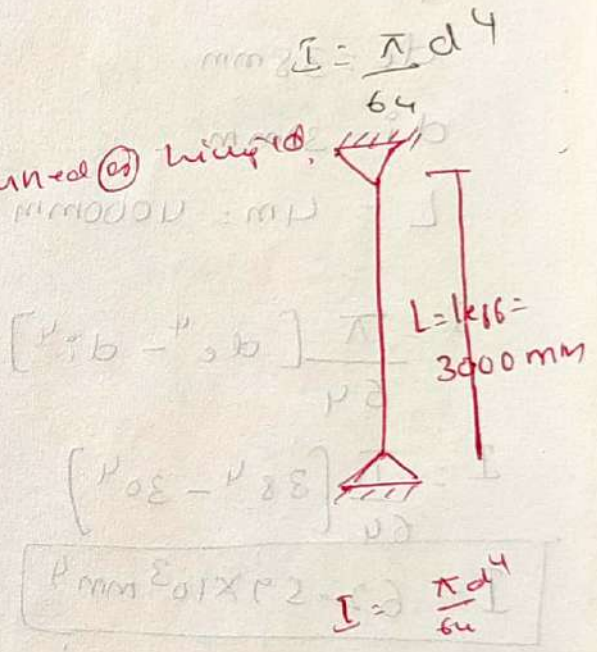
$$P_{cr} = \frac{\pi^2 \times 89.5 \times 10^3 \times 19.94 \times 10^6}{(3000)^2}$$

$$P_{cr} = 1.95 \times 10^6 \text{ N}$$

$$P_{cr} = 1.95 \times 10^3 \text{ kN}$$

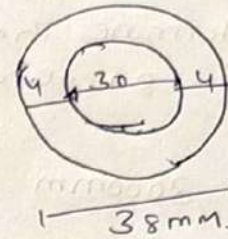
Safe load = $\frac{\text{cripping load}}{FOS} = \frac{1.95 \times 10^3}{4}$

$$\text{Safe load} = 0.4875 \times 10^3 \text{ kN}$$



Q. A mild steel tube 4m long 30mm internal diameter and 4mm thick is used as a strut. Find the collapse load in

1. Both ends hinged.
 2. Both ends fixed.
 3. One end fixed & other free.
 4. One end fixed and other hinged.
- take $E = 2.1 \times 10^5 \text{ MPa}$



Given

$$d_o = 38 \text{ mm}$$

$$d_i = 30 \text{ mm}$$

$$L = 4 \text{ m} = 4000 \text{ mm}$$

$$I = \frac{\pi}{64} [d_o^4 - d_i^4]$$

$$I = \frac{\pi}{64} [38^4 - 30^4]$$

$$I = 62.59 \times 10^3 \text{ mm}^4$$

$$\frac{\text{N/mm}^2 \times \text{mm}^4}{\text{mm}^4} = \frac{\text{N/mm}^2 \times \text{mm}^4}{\text{mm}^4}$$

① Both ends hinged

$$P_{cr} = \frac{\pi^2 EI}{(L)^2}$$

$$P_{cr} = \frac{\pi^2 \times 2.1 \times 10^5 \times 62.59 \times 10^3}{(4000)^2}$$

$$P_{cr} = 8.108 \times 10^3 \text{ N}$$

② Both ends fixed

$$P_{cr} = \frac{\pi^2 EI}{\left(\frac{L}{2}\right)^2}$$

$$= \frac{\pi^2 \times 2.1 \times 10^5 \times 62.59 \times 10^3}{\left(\frac{4000}{2}\right)^2}$$

$$P_{cr} = 32.43 \times 10^3 \text{ N}$$

③ one end is fixed and other free

$$P_{cr} = \frac{\pi^2 EI}{(2L)^2} = \frac{\pi^2 \times 2.1 \times 10^5 \times 62.59 \times 10^3}{(2 \times 4000)^2} = P_{cr} = 2.07 \times 10^3 \text{ N}$$

④ one end is fixed & other end hinged

$$P_{cr} = \frac{\pi^2 EI}{\left(\frac{L}{\sqrt{2}}\right)^2} = \frac{\pi^2 \times 2.1 \times 10^5 \times 62.59 \times 10^3}{\left(\frac{4000}{\sqrt{2}}\right)^2} =$$

$$P_{cr} = 16.21 \times 10^3 \text{ N}$$

3) Find the shortest length of box pinned ended steel column have cross section of 60mm x 100mm take $E_s = 2 \times 10^5 \text{ N/mm}^2$ and critical proportionality limit as 250MPa.

$\Rightarrow L = ?$ $E_s = 2 \times 10^5 \text{ N/mm}^2$

Both ends are pinned $l_{eff} = l = ?$

$\sigma_{cr} = 250 \text{ MPa} = \left[\frac{P_{cr}}{A} \right]$

$$P_{cr} = \frac{\pi^2 E I (AK)^2}{(L_c)^2}$$

$$\frac{P_{cr}}{A} = \frac{\pi^2 E K^2}{L_c^2} = 250$$

$$K = \sqrt{\frac{I}{A}}$$

$$I_{xx} = \frac{bd^3}{12} = \frac{60 \times 100^3}{12} = 5 \times 10^6 \text{ mm}^4$$

$$I_{yy} = \frac{b^3d}{12} = \frac{100 \times 60^3}{12} = 1.8 \times 10^6 \text{ mm}^4$$

$$K_{yy} = \sqrt{\frac{I_{yy}}{A}} = \sqrt{\frac{1.8 \times 10^6}{(60 \times 100)}}$$

$$K_{yy} = 17.32 \text{ mm}$$

$$K_{xx} = \sqrt{\frac{I_{xx}}{A}} = \sqrt{\frac{5 \times 10^6}{(60 \times 100)}} = 29.8 \text{ mm}$$

$$\sigma_{cr} = \frac{P_{cr}}{A}$$

$$P_{cr} \times A = \frac{P_{cr}}{A}$$

$$P_{cr} \times A = \frac{\pi^2 E I}{L_c^2}$$

where $I = AK^2$

$$K = \sqrt{\frac{I}{A}} = \frac{L_c}{A} = L_c$$

$$P_{cr} \times A = \frac{\pi^2 E (AK^2)}{L_c^2}$$

$$P_{cr} = \frac{\pi^2 E K^2}{L_c^2}$$

$$250 = \frac{\pi^2 E K^2}{L_c^2}$$

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{\pi^2 E K^2}{L_c^2}$$

$$250 = \frac{\pi^2 \times 2 \times 10^5 \times (17.32)^2}{L_c^2}$$

$$L_c = 1.539 \times 10^3 \text{ mm}$$

$$L_c = \sqrt{\frac{I}{A}}$$

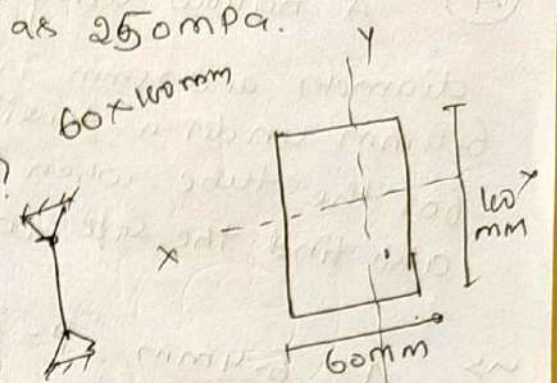
$$L_c^2 = \frac{I}{A}$$

$$P_{cr} \times A = P_{cr}$$

$$P_{cr} \times A = \frac{\pi^2 E I}{L_c^2}$$

$$P_{cr} \times A = \frac{\pi^2 E \times I}{L_c^2 \times A}$$

$$P_{cr} = \frac{\pi^2 E I}{L_c^2}$$

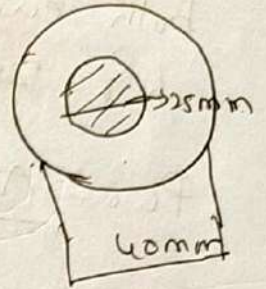


Determine K_{min} consider I_{min}

b → Parallel to the reference axis x-x
d → Parallel to the reference axis y-y.

④ A hollow alloy tube 5m long with 40mm external diameter and 25mm internal dia was found to extend by 6.4mm under a tensile load of 60kN. Find the buckling load for the tube when it is used as column with both ends pinned also find the safe compressive load for the tube if $FOS = 4$.

$\Delta = 6.4 \text{ mm}$
 $P = 60 \text{ kN} = 60 \times 10^3 \text{ N}$ [T]



$Leff = 5 \text{ m} = 5000 \text{ mm}$

$P_{cr} = ?$ $P_{safe} = ?$

$FOS = 4$

Case 1)

Young's modulus

$\Delta = \frac{PL}{AE}$

$6.4 = \frac{60 \times 10^3 \times 5000}{\frac{\pi}{4} [40^2 - 25^2] E}$

$E = 61.213 \times 10^3 \text{ MPa} \text{ (or) } \text{N/mm}^2$

Case 2)

To find P_{cr} [Pinned]

$P_{cr} = \frac{\pi^2 EI}{L_e^2} = \frac{\pi^2 [61.213 \times 10^3] \left[\frac{\pi}{64} [40^4 - 25^4] \right]}{(5000)^2}$

$P_{cr} = 2.57 \times 10^3 \text{ N}$

$FOS = \frac{P_{cr}}{P_{safe}}$

$P_{safe} = \frac{P_{cr}}{FOS} = \frac{2.57 \times 10^3}{4}$

$P_{safe} = 642.5 \text{ N}$

5) Determine ratio of buckling strength of 2 circular columns one solid hallow and other solid both are made of same material, same length, same cross sectional area & same end condition. The internal dia of hallow column is $\frac{1}{2}$ external diameter.

Hallow

$$d_i = \frac{1}{2} d_e$$

$$d_i = 0.5 d_e$$

Solid

$$E_H = E_S$$

$$A_H = A_S$$

$$L_H = L_S$$

$$L_{eff}]_{Hallow} = L_{eff}]_{Solid}$$

$$A_S = A_H$$

$$\frac{\pi}{4} [d]^2 = \frac{\pi}{4} [d_e^2 - d_i^2]$$

$$\frac{\pi}{4} [d]^2 = \frac{\pi}{4} [d_e^2 - 0.5^2 d_e^2]$$

$$\frac{\pi}{4} [d]^2 = \frac{\pi}{4} d_e^2 [1 - 0.5^2]$$

$$\frac{\pi}{4} [d]^2 = \frac{\pi}{4} d_e^2 \times 0.75$$

$$d^2 = \frac{0.58 d_e^2}{0.75}$$

$$d = 0.866 d_e$$

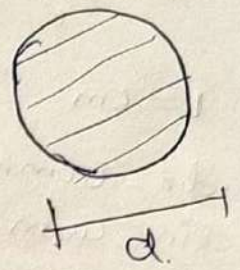
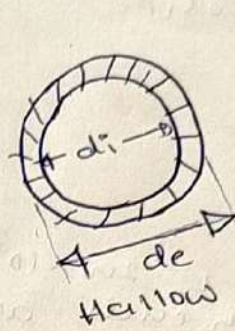
$$\frac{P_{cr}]_H}{P_{cr}]_S} = ? = \frac{\pi^2 EI}{L_e^2}$$

$$= \frac{I]_H}{I]_S}$$

$$\frac{P_{cr}]_H}{P_{cr}]_S} = \frac{[d_e^4 - d_i^4]}{d^4}$$

$$= \frac{d_e^4 - 0.5^4 d_e^4}{[0.866 d_e]^4}$$

$$= 1.667$$



Q6) A hollow tube of 6m length with external diameter 60mm & thickness 10mm is subjected to minimum crippling load. Find Euler's critical load for this column.

- When both ends are fixed.
 - When one end is fixed & other end hinged.
- $E = 200 \text{ GPa}$

$L = 6 \text{ m}$

$d_e = 60 \text{ mm}$

$d_i = 40 \text{ mm}$

$P_{cr} = ?$

When both ends are fixed.

$$P_{cr} = \frac{4\pi^2 EI}{L^2}$$

$$I = \frac{\pi}{64} [d_e^4 - d_i^4] = \frac{\pi}{64} [60^4 - 40^4] = 98.17 \text{ mm}^4$$

$$P_{cr} = \frac{4 \times \pi^2 \times 200 \times 10^3 \times 98.17}{6000^2} = 111.967 \times 10^3 \text{ N}$$

$P_{cr} = 2.153 \times 10^5 \text{ N}$

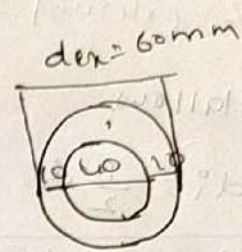
②

When one end is fixed & other end hinged.

$$P_{cr} = \frac{2\pi^2 EI}{L^2}$$

$$P_{cr} = \frac{2 \times \pi^2 \times 200 \times 10^3 \times 98.17}{6000^2} = 5.598 \times 10^4 \text{ N}$$

$P_{cr} = 16.76 \text{ N}$



S.No	Material	σ_c in N/mm^2	α
1.	wrought	250	$\frac{1}{900}$
2.	cast iron	550	$\frac{1}{1600}$
3.	Mild steel	320	$\frac{1}{750}$
4.	Timber	50	$\frac{1}{750}$

Any column.

Rankine - Gordon's formula [Equation]
 (suitable for any type of column)

Euler equation for long column

Empirical formula.

Rankine + Gordon has developed an empirical equation considering the failure by buckling & crushing as given below

$$\frac{1}{P_R} = \frac{1}{P_C} + \frac{1}{P_E}$$

Where P_R = Rankine's load

P_C or P_C = load carrying capacity of short column

P_E = Euler's load.

P_E or P_{cr} = Euler's critical load $\frac{\pi^2 EI}{(l_e)^2}$

(or) crushing load.

$$f_c = \frac{P_C}{A}$$

$$P_C = f_c \cdot A$$

l_e = effective length.

$$\frac{1}{P_R} = \frac{P_C + P_E}{P_C \times P_E}$$

Taking the reciprocal on both sides

$$P_R = \frac{P_C \times P_E}{P_C + P_E}$$

$$P_R = \frac{P_C}{\frac{P_C}{P_E} + 1}$$

$$P_R = \frac{P_C}{1 + \frac{P_C}{P_E}}$$

$$P_R = \frac{P_C}{1 + \frac{P_C \cdot A}{\pi^2 EI}}$$

$$P_R = \frac{P_C}{1 + \frac{P_C A l_e^2}{\pi^2 EI}}$$

radius of gyration
 It is the centre area which should be made it into a strip and should kept at a distance r from the centre of gravity.

$$r_{min} = \sqrt{\frac{I_{min}}{A}}$$

r = radius of gyration

$$r^2 = \frac{I}{A}$$

$$I = A r^2$$

$$P_R = \frac{P_C}{1 + \frac{\sigma_c A L^2}{\pi^2 E I}}$$

$$= \frac{P_C}{1 + a x^2}$$

Where,

$$\lambda = \frac{L}{r}$$

$x =$ Slenderness ratio
 Buckling factor

less

$$P_R = \frac{P_C}{1 + \frac{\sigma_c}{\pi^2 E} \left(\frac{L^2}{r^2} \right)}$$

where:

$$\frac{\sigma_c}{\pi^2 E} = a \text{ (or) } a$$

$$P_R = \frac{P_C}{1 + a x^2}$$

where $a = \frac{\sigma_c}{\pi^2 E}$ & is known as Rankine's constant.

For a given column material the crushing stress σ_c is a constant. Hence the crushing load P_C will also be constant for a given cross-sectional area of the column. In eq (1) P_C is constant & hence value of P depends upon the value of P_E . But for a given column material & given cross-sectional area, the value of P_E depends upon the effective length of the column.

(1) If the column is short, which means the value of L is small, then the value of P_E will be large. Hence the value of $\frac{1}{P_E}$ will be small enough & is negligible as compared to the value of $\frac{1}{P_C}$. Neglecting the value of $\frac{1}{P_E}$ in eq (1) we get

Hence the crippling load by Rankine's formula for a short column is approximately equal to crushing load.

(2) If the column is long, which means the value of L is large, then the value of P_E will be small & the value of $\frac{1}{P_E}$ will be large enough compared with $\frac{1}{P_C}$. Hence the value of $\frac{1}{P_C}$ may be neglected in eq (1).

$$\frac{1}{P} = \frac{1}{P_E} \text{ (or) } P = P_E$$

Problems on Rankine's Gordon equation

① A cast iron column 100mm dia external diameter & 10mm thick is 3m long. Calculate the safe load using Rankine's formula.

i/ (i) Both ends are hinged. (1)

(ii) Both ends are fixed (1/2)

if take $f_c = 60 \text{ MPa}$. $a = [1/1600]$ $\rho = 3$

$d_e = 100 \text{ mm}$

$d_i = 80 \text{ mm}$

$l = 3000 \text{ mm}$

$A = \pi/4 [d_e^2 - d_i^2]$

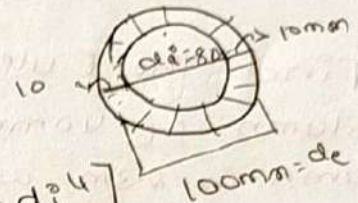
$A = \pi/4 [100^2 - 80^2]$

$A = 2.827 \times 10^3 \text{ mm}^2$

$I = \frac{\pi}{64} [d_e^4 - d_i^4]$

$I = \frac{\pi}{64} [100^4 - 80^4]$

$I = 2.898 \times 10^6 \text{ mm}^4$



radius $k = \sqrt{\frac{I}{A}}$

Rankine's equation $P_R = \frac{f_c \cdot A}{1 + a \cdot \lambda^2} = \frac{P_c}{1 + a \left(\frac{l}{r}\right)^2}$

$P_c = f_c \cdot A$

$\lambda = \frac{l}{r} = \frac{l}{R \cdot k}$

Case 1.

When both ends are hinged
effective $l = l = 3000 \text{ mm}$

$P_R = \frac{600 \times [2.827 \times 10^3]}{1 + \left[\frac{1}{1600}\right] \left[\frac{3000}{32.05}\right]^2}$

$P_R = 262.02 \times 10^3 \text{ N}$

$I = A k^2$

$k = \sqrt{\frac{I}{A}}$

$k = \sqrt{\frac{2.898 \times 10^6}{2.827 \times 10^3}}$

$k = 32.05 \text{ mm}$

When both ends are fixed $l_{eff} = \frac{l}{2} = \frac{3000}{2} = 1500 \text{ mm}$

$P_R = \frac{f_c \cdot A}{1 + a \left(\frac{l}{r}\right)^2} = \frac{(600 \times 2.827 \times 10^3)}{1 + \left[\frac{1}{1600}\right] \left[\frac{1500}{32.05}\right]^2}$

$P_R = 715.1 \times 10^3 \text{ N}$

$$\text{Factor of safety} = \frac{P_{cr}}{P} = \frac{\text{Critical load}}{\text{Safe load}}$$

Case 1 = $P = \frac{P_{cr}}{FOS}$

$$P_{safe} = \frac{262.02 \times 10^3}{3} = 8734 \times 10^3 \text{ N}$$

Case 2

$$P_{safe} = \frac{715.1 \times 10^3}{3} = 238.36 \times 10^3 \text{ N}$$

② Find the Euler's crippling load on a hollow steel column of 40mm external dia and 4mm thick. Length of column is 2.3m with pinned ends. Also determine crippling load by using Rankine's formula with $\tan a = \frac{1}{750}$ and $f_c = 335 \text{ MPa}$. $E = 205 \text{ GPa}$.

$$d_e = 40 \text{ mm}$$

$$d_i = 32 \text{ mm}$$

$$l = 2.3 \text{ m} = 2300 \text{ mm}$$

$$P_{cr} = ?$$

$$f_c = 335 \text{ MPa} \approx 335$$

$$a = \frac{1}{750}$$

$$E = 205 \times 10^3 \text{ MPa}$$

$$A = \frac{\pi}{4} [d_e^2 - d_i^2] = \frac{\pi}{4} [40^2 - 32^2]$$

$$A = 452.38 \text{ mm}^2$$

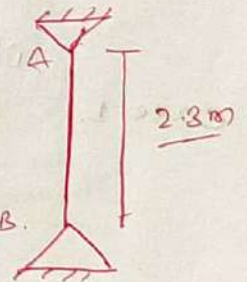
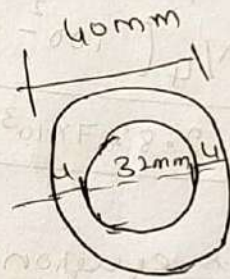
$$I = \frac{\pi}{64} [d_e^4 - d_i^4] = \frac{\pi}{64} [40^4 - 32^4] = 74.19 \times 10^3 \text{ mm}^4$$

$$I = 74.19 \times 10^3 \text{ mm}^4$$

$$k = \sqrt{\frac{I}{A}} = \sqrt{\frac{74.19 \times 10^3}{452.38}}$$

Radius of gyration.

$$k = 12.80 \text{ mm}$$



$$P_{cr} = \frac{\pi^2 EI}{l^2} = \frac{\pi^2 \times 205 \times 10^3 \times 74.19 \times 10^3}{(2300)^2}$$

$$P_{cr} = 28.37 \times 10^3 \text{ N}$$

$$P_R = \frac{(f_c) (A)}{1 + a(\lambda)^2} \quad \text{or} \quad \frac{P_c}{1 + a(\lambda)^2} \quad \lambda = \frac{l}{K}$$

$$P_R = \frac{335 \times 452.389}{1 + \left(\frac{1}{2500}\right) \left[\frac{2300}{12.806}\right]^2}$$

$$P_R = 28.58 \times 10^3 \text{ N}$$

Dec-2017

③ A hollow CI column whose outside diameter is 200mm as a thickness of 20mm. It is 4.5m long & fixed at

both the ends

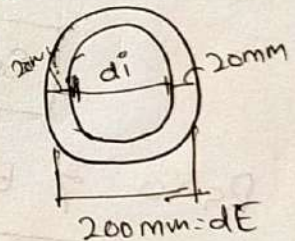
- ① Calculate safe load by Rankine's formula with $f_{oc} = 4$
- ② Calculate the ratio of Euler's and Rankine's formula using critical load and slenderness ratio.

Take $f_c = f_{oc} = 55 \text{ N/mm}^2$ and $E = 9.4 \times 10^4 \text{ N/mm}^2$ $a = \frac{1}{1600}$ in Rankine's form

$$d_e = 200 \text{ mm}$$

$$d_i = 160 \text{ mm}$$

$$l = 4.5 \text{ m}$$



$$A = \frac{\pi}{4} (d_e^2 - d_i^2) = \frac{\pi}{4} (200^2 - 160^2)$$

$$A = 11.30 \times 10^3 \text{ mm}^2$$

$$I = \frac{\pi}{64} [d_e^4 - d_i^4] = \frac{\pi}{64} [200^4 - 160^4]$$

$$I = 46.369 \times 10^6 \text{ mm}^4$$

$$K = \sqrt{\frac{I}{A}} = \sqrt{\frac{46.369 \times 10^6}{11.30 \times 10^3}}$$

$$K = 64.05 \text{ mm}$$

* End conditions are fixed

$$P_{cr} = \frac{\pi^2 EI}{L_e^2} = \frac{\pi^2 EI}{L^2}$$

length of column = 4m = 4000mm

* L_e = effective length

$$L_e = \frac{L}{2} = \frac{4000}{2} = 2000 \text{ mm}$$

* Factor of safety = 4 $\sigma_c = 550 \text{ N/mm}^2$, $E = 9.4 \times 10^4 \text{ N/mm}^2$

* Slenderness ratio $\lambda = \frac{L_e}{K} = \frac{2000}{64.05} = 31.22$

② Safe load by Rankine's formula

$$P_R = \frac{P_C}{1 + a\lambda^2} = \frac{\sigma_c \times A}{1 + a \left[\frac{L_e}{K} \right]^2} = \frac{550 \times 11.30 \times 10^3}{1 + \frac{1}{1600} \left[\frac{2000}{64.05} \right]^2}$$

$$P_R = 3.508 \times 10^6 \text{ N}$$

P_R = crippling load

Safe load = $\frac{\text{crippling load}}{\text{Factor of safety}} = \frac{3.508 \times 10^6}{4}$

$$\text{Safe load} = 877.19 \times 10^3 \text{ N}$$

Ratio of Euler's and Rankine's critical loads

③ Let P_E = Euler's critical load

$$P_E = \frac{\pi^2 EI}{L_e^2} = \frac{\pi^2 \times 9.4 \times 10^4 \times 46.39 \times 10^6}{2000^2}$$

$$P_E = 8.5013 \times 10^6 \text{ N}$$

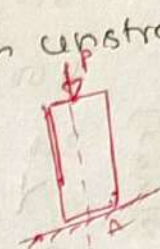
$$\frac{\text{Euler's critical load}}{\text{Rankine's critical load}} = \frac{P_E}{P_R} = \frac{8.5013 \times 10^6}{3.508 \times 10^6}$$

$$= 2.42$$

Limitations of Euler's Formula (Theory)

The Euler's theory is based on the following assumptions

- ① Ideal struts. The axis of the strut is perfectly straight when unloaded.
 - ② The line of thrust coincides exactly with upstrained axis of the strut.
 - ③ The flexural rigidity EI is uniform.
 - ④ The material is isotropic.
 - ⑤ The buckling value of $P = P_E$ is assumed to be obtained for all degrees of flexure.
- Usually the first two assumptions are not fully realized in practice. The column may have initial curvature, distortion or crookedness. The theory refers to ideal struts & not to a real one.



It is interesting to note that no strength property of the material appears in the Euler's formulae, yet they determine the carrying capacity of a column. The only material property involved in the elastic modulus E .

The critical stress P_E , which is defined as an average stress over the cross section is given by:

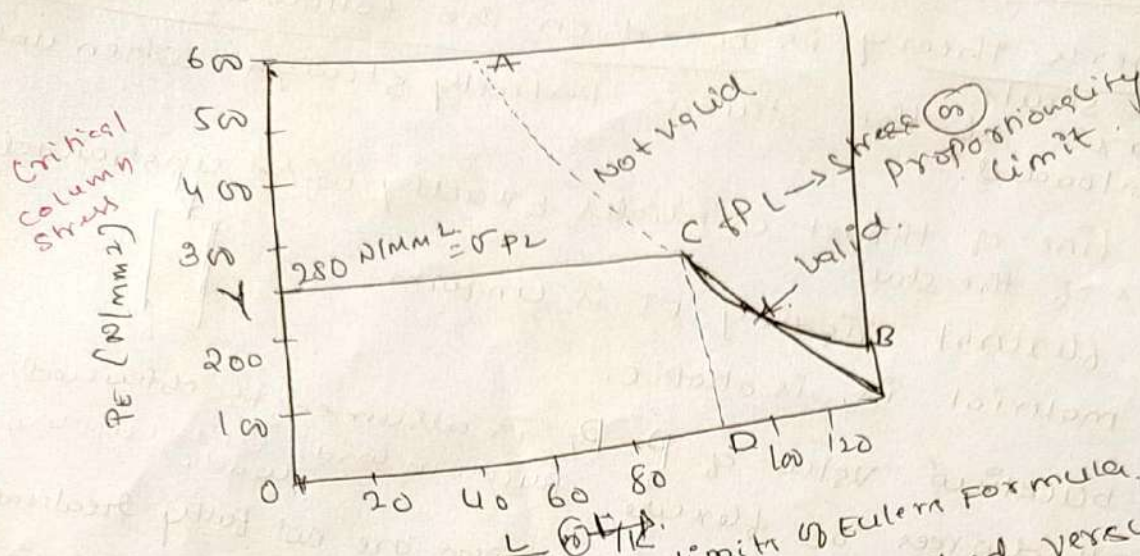
$$P_E = \frac{P_E}{A} = \frac{\pi^2 EI}{AL^2} = \frac{\pi^2 E A r^2}{AL^2}$$

where $r = \sqrt{\frac{I}{A}}$
 r = radius of gyration

$$P_E = \frac{\pi^2 E r^2}{L^2} = \frac{\pi^2 E}{\left(\frac{L}{r}\right)^2}$$

$$P_E = \frac{\pi^2 E}{(L/r)^2}$$

In the above expression L/r is known as slenderness ratio. The graphical interpretation of equation is shown by the curve ACB in fig



- ① Where the critical column stress is plotted versus the slenderness ratio for mild steel [$E = 2 \times 10^5 \text{ N/mm}^2$],
- ② The curve is entirely defined by the magnitude of E and is independent of its ultimate strength.
- ③ The ^{long} column has greater value of L/r & loses its strength at very small compressive strength stress.
- ④ This condition cannot be improved by taking a steel of higher strength, since the modulus of steel does not vary much with alloy and heat treatment, and remains practically constant.

In fig Let σ_y represents the Proportional Limit (σ_{PL}) of the material. It is clear that the Euler's formula can not possibly apply if L/r is less than $\frac{\sigma_D}{\sigma_y}$. Since the stress is greater than σ_y , the material become plastic & no longer follows Hooke's law. Taking the proportional limit (σ_{PL}) is equal to 250 N/mm^2 for mild steel.

The Euler's equation does not hold good @ ceizra for any value of (L/r) less than (B) That comes pond up to the value of σ_{PL} which is shown at point B on the curve. i.e in the curve only portion BC is valid. where as dotted line AC is not applicable to Euler theory.

① A steel bar of rectangular section $30\text{mm} \times 40\text{mm}$ pinned at each end is subjected to axial compression. The bar is 1.75m long. determine the buckling load & corresponding axial stress using Euler's formula.

Determine the minimum length for which Euler's equation is proportionality limit of material is 200MPa .

$$I_{xx} = \frac{bd^3}{12} = \frac{30 \times 40^3}{12} = 160 \times 10^3 \text{mm}^4$$

$$I_{yy} = \frac{b^3d}{12} = \frac{30^3 \times 40}{12} = 90 \times 10^3 \text{mm}^4$$

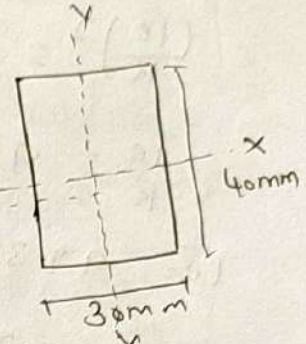
$$k_{\min} = \sqrt{\frac{I_{\min}}{A}}$$

$$k_{\min} = \sqrt{\frac{90 \times 10^3}{30 \times 40}}$$

$$k_{\min} = 8.66 \text{mm}$$

$$l = 1.75 \text{m} \quad l_e = l = 1.75 \text{m}$$

$$E = 2 \times 10^5 \text{MPa}$$



Case 1 : axial stress.

$$P_{cr} = \frac{\pi^2 E I_{\min}}{l_e^2} = \frac{\pi^2 \times 2 \times 10^5 \times (90 \times 10^3)}{(1.75)^2}$$

$$P_{cr} = 58.00 \times 10^3 \text{ N}$$

$$\text{Axial stress } f = \frac{\text{Load}}{\text{Area}} = \frac{P_{cr}}{A} = \frac{58 \times 10^3}{30 \times 40}$$

$$f = 48.33 \text{ MPa} \quad (00) \text{ N/mm}^2$$

$$k = \sqrt{\frac{I}{A}} \quad k^2 = \frac{I}{A}$$

Case 2 : $l_{\min} = ?$

$$P_{cr} = \frac{\pi^2 EI}{l_e^2} = \frac{\pi^2 E (k^2 A)}{l_e^2}$$

$$f_{cr} = \frac{P_{cr}}{A} = \frac{\pi^2 E k^2}{l_e^2}$$

$$l_e = \sqrt{\frac{\pi^2 E k^2}{f_{cr}}}$$

$$f_{cr} = \frac{\pi^2 E k^2}{l_e^2} \Rightarrow l_e = \sqrt{\frac{\pi^2 E k^2}{f_{cr}}}$$

$$P_{cr} = \frac{P}{A} \quad b \times a = A$$

$$b \times a = \frac{\pi^2 E}{(f_{cr})^2}$$

$$bcx = \frac{\pi^2 E}{\left(\frac{le}{k}\right)^2}$$

$$200 = \frac{\pi^2 \times 2 \times 10^5}{\left(\frac{le}{k}\right)^2}$$

$$\left(\frac{le}{k}\right)^2 = 9.86 \times 10^3$$

$$\frac{le}{k} = 99.34$$

$$le = 99.34 \times k_{min}$$

$$le = 99.34 \times 8.66$$

$$l_{min} = le = 860.33 \text{ mm}$$

$$bcx = \frac{\pi^2 E}{\left(\frac{le}{k}\right)^2}$$

$$\left(\frac{le}{k}\right)^2 = \frac{\pi^2 E}{bcx}$$

$$\frac{le}{k} = \sqrt{\frac{\pi^2 E}{bcx}}$$

$$le = k \sqrt{\frac{\pi^2 E}{bcx}}$$

$$le = 99.34 \times 8.66$$

$$l_{min} = 860.33 \text{ mm}$$

$$f = 28.00 \times 10^3 \text{ N}$$

$$f = 18.33 \times 10^3 \text{ N}$$