Relization of IIR digital Systems

The digital filters are basically discrete time LTI systems. The discrete time systems can be FIR or IIR type. These systems can be described by difference Eqⁿ. There are various configurations or structures for the realizations. These structures are derived on the basis of computational complexity, ease of implementation, finite word length effect etc.

Describing Equations

WKT the LTI systems are described by general difference equation as,

$$y(n) = -\sum_{k=1}^{N} a_k y(n-k) + \sum_{k=0}^{M} b_k x(n-k) \quad \text{--- (1)}$$

Taking z transform of above Equations

$$Y(z) = -\sum_{k=1}^{N} a_k z^{-k} Y(z) + \sum_{k=0}^{M} b_k z^{-k} X(z).$$

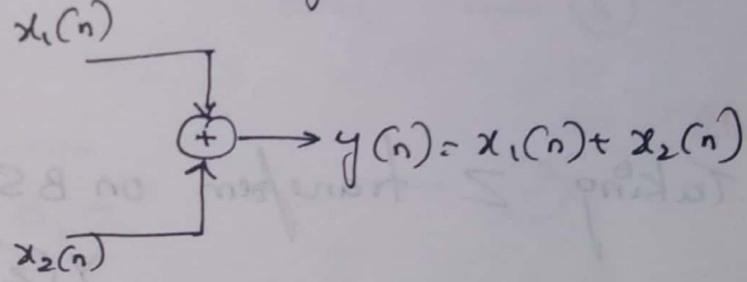
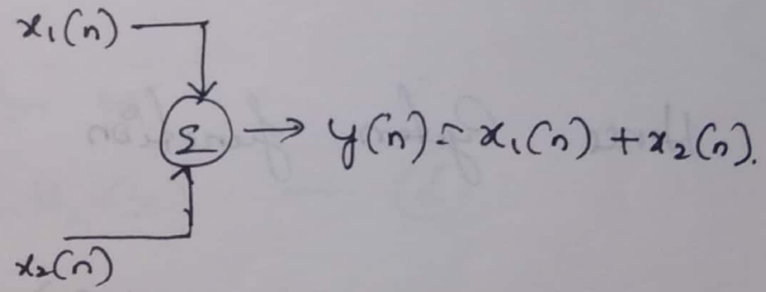
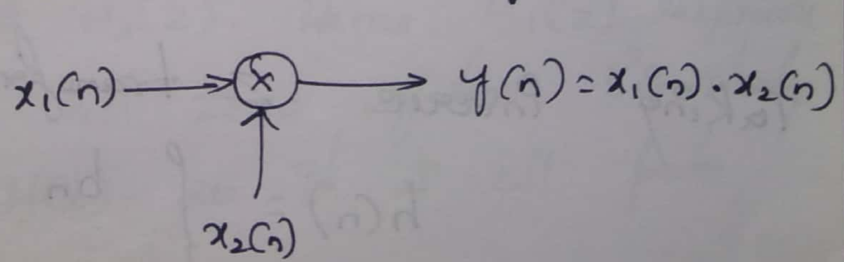
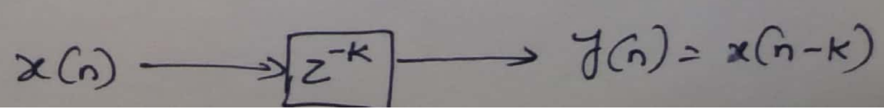
$$Y(z)\left[1 + \sum_{k=1}^{N} a_k z^{-k}\right] = \sum_{k=0}^{M} b_k z^{-k} X(z)$$

WKT the system functions

$$H(z) = \frac{Y(z)}{X(z)}$$

$$H(z) = \frac{\sum_{k=0}^{M} b_k z^{-k}}{1 + \sum_{k=1}^{N} a_k z^{-k}} \quad \text{——} (2)$$

This is the rational form of system function which is expressed as the ratio of two polynomial in z^{-1}. The discrete time system is implemented with the help of delay elements, multiplexers and adders as elementary blocks.

Name of the block	Symbol
1. Adder	 
2. Constant Multiplier	
3. Signal Multiplier	
4. Delay Elements	 

Advance Elements $x(n) \longrightarrow \boxed{z} \longrightarrow y(n) = x(n+1)$

$$x(n) \longrightarrow \boxed{z^{k}} \longrightarrow y(n) = x(n+k)$$

Basic FIR filter structure :-

An FIR system does not have feedback. Hence the past output term $y(n-k)$ will be absent in Eq ①

Hence output is given by $y(n) = \sum_{k=0}^{M} b_k x(n-k)$

if there are M Co-efficients then

$$y(n) = \sum_{k=0}^{M-1} b_k x(n-k)$$

Taking Z transform on BS

$$Y(z) = \sum_{k=0}^{M-1} b_k z^{-1} X(z)$$

Hence System function $H(z) = \dfrac{Y(z)}{X(z)}$ becomes

$$H(z) = \sum_{k=0}^{M-1} b_k z^{-k}$$

Taking inverse Z transform on both side

$$h(n) = \begin{cases} b_n & \text{for } 0 \le n \le M-1 \\ 0 & \text{otherwise.} \end{cases}$$

Basic IIR filter structures

* **Direct form structures of IIR systems**

WKT IIR system can be described by a generalized difference equation given by ① & Eq ②

Let us consider
$$H(z) = \frac{\sum_{k=0}^{M} b_k z^{-k}}{1 + \sum_{k=1}^{N} a_k z^{-k}} \quad \text{— (a)}$$

Let
$$H_1(z) = \sum_{k=0}^{M} b_k z^{-k} \quad \text{and.} \quad \text{— (b)}$$

$$H_2(z) = \frac{1}{1 + \sum_{k=1}^{N} a_k z^{-k}} \quad \text{— (c)}$$

Hence Eq (a) can be written as
$$H(z) = H_1(z) \cdot H_2(z) \quad \text{— (d)}$$

The overall IIR system can be realized as cascade of two functions $H_1(z)$ & $H_2(z)$. Hence $H_1(z)$ represents zero of $H(z)$. so it is all zero system. Similarly $H_2(z)$ represents poles of $H(z)$. so it is all pole system.

Form I structure of IIR system

Let us first compare the direct form structures of $H_1(z)$ and $H_2(z)$. So we can write $H_1(z)$ using Eq(

$$H_1(z) = b_0 + b_1 z^{-1} + b_2 z^{-2} + \ldots\ldots + b_M z^{-M}$$

Since $H_1(z) = \dfrac{Y_1(z)}{X_1(z)}$

so $Y_1(z) = b_0 X_1(z) + b_1 X_1(z) z^{-1} + b_2 X_1(z) z^{-2} + \ldots + b_M X_1$

Taking inverse z-transform of above Equation

$$y_1(n) = b_0 x_1(n) + b_1 x_1(n-1) + b_2 x_1(n-2) + \ldots + b_M x_1(n-M) \quad \text{— (c)}$$

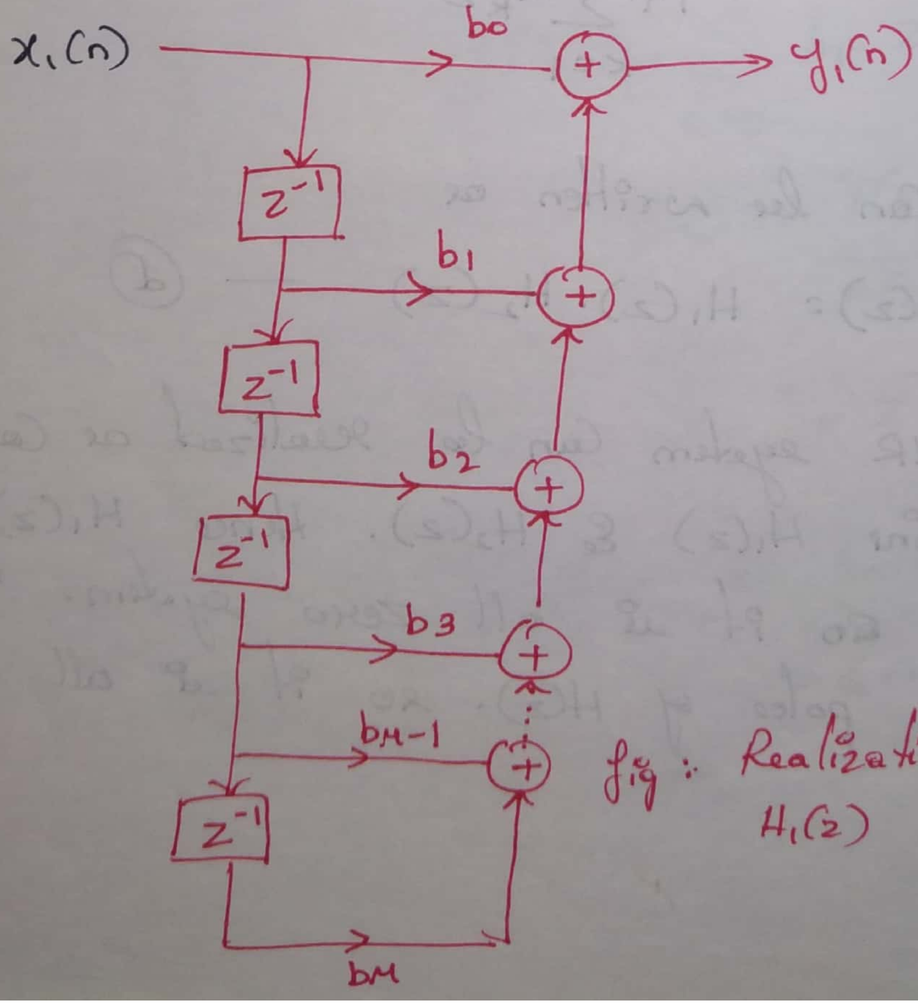


fig: Realization of System function $H_1(z)$

Consider $H_2(z)$ given by Equation

$$H_2(z) = \frac{1}{1+\sum_{k=1}^{N} a_k z^{-k}} \quad \text{— (f)}$$

(n-M)

WKT $\quad H_2(z) = \frac{Y_2(z)}{X_2(z)}$

$$\frac{Y_2(k)}{X_2(z)} = \frac{1}{1+\sum_{k=1}^{N} a_k z^{-k}}$$

$$\therefore \; Y_2(k)\left[1+\sum_{k=1}^{N} a_k z^{-k}\right] = X_2(z)$$

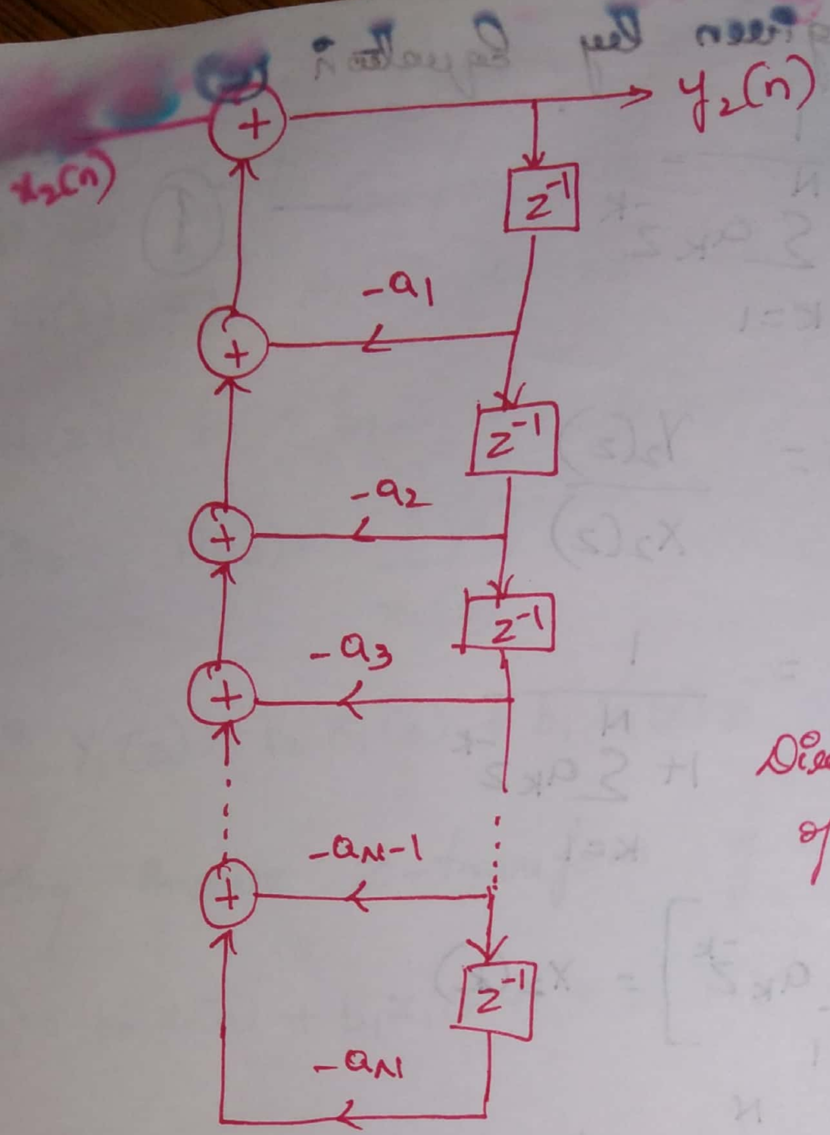
$$Y_2(z) = -\sum_{k=1}^{N} a_k z^{-k} Y_2(z) + X_2(z)$$

Expanding summation of this Equation of above Equation

$$Y_2(z) = -a_1 z^{-1} Y_2(z) - a_2 z^{-2} Y_2(z) - a_3 z^{-3} Y_2(z) - \cdots$$
$$\cdots - a_N z^{-N} Y_2(z) + X_2(z)$$

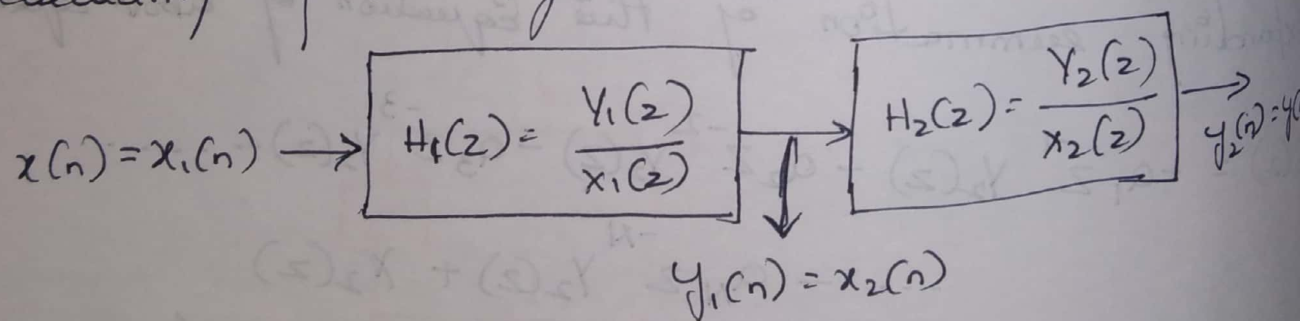
Taking Inverse Z-transform of above Equation

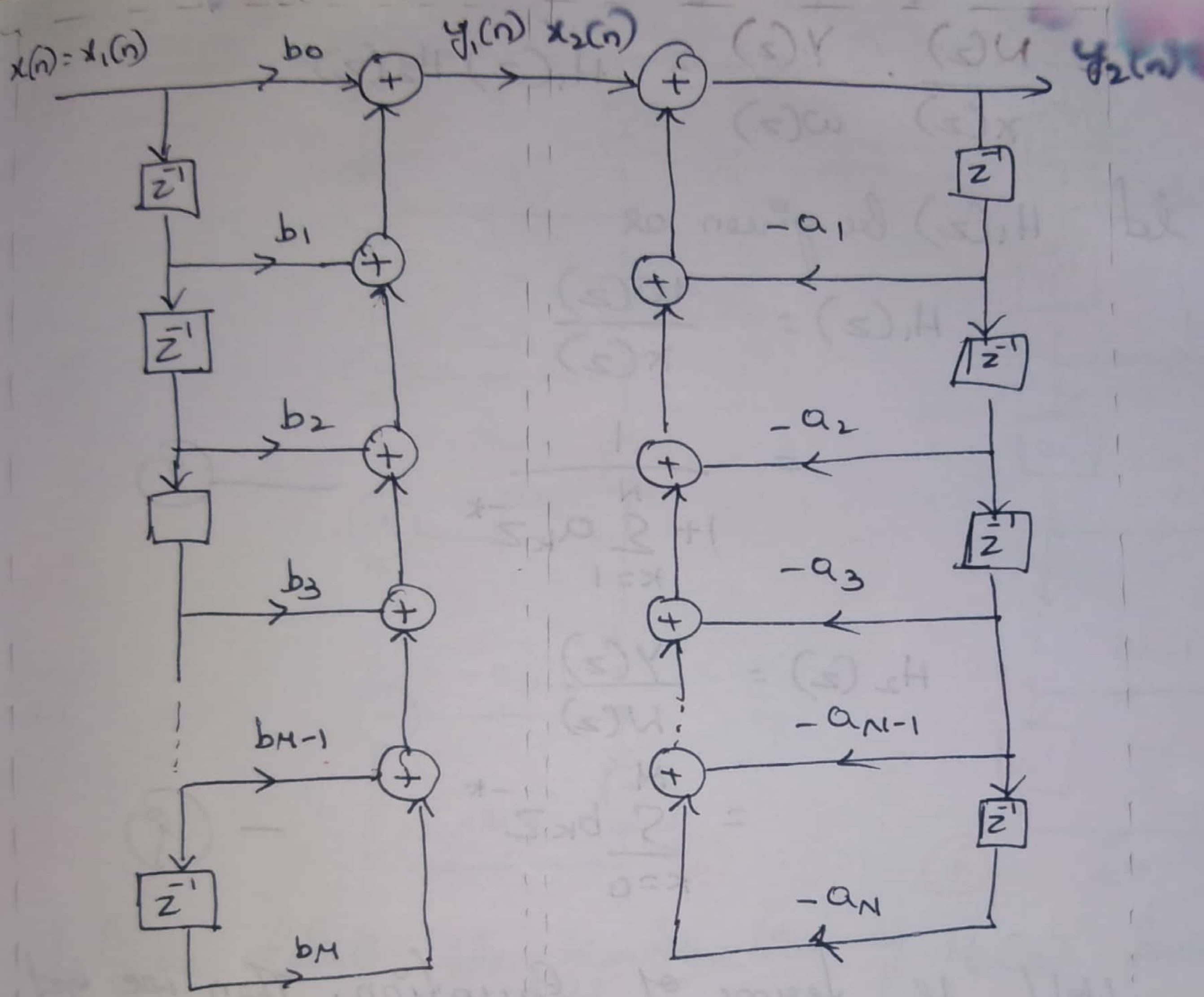
$$y_2(n) = -a_1 y_2(n-1) - a_2 y_2(n-2) - a_3 y_2(n-3) - \cdots$$
$$\cdots - a_N y_2(n-N) + x_2(n) \quad \text{— (g)}$$



Direct form realization of system function $H_2(z)$

From Equation KIKT $H(z) = H_1(z)\, H_2(z)$. This eq
Cascading of two systems $H_1(z)$ and $H_2(z)$





Direct Form II structure for IIR system

$$H(z) = \frac{\sum_{k=0}^{m} b_k z^{-k}}{1+\sum_{k=1}^{N} a_k z^{-k}} \quad \text{— (h)}$$

Let $H(z)$ be written as

$$H(z) = \frac{Y(z)}{X(z)}$$

$$= \frac{Y(z)}{W(z)} \cdot \frac{W(z)}{X(z)}$$ By rearranging terms.

$$\frac{W(z)}{X(z)} \cdot \frac{Y(z)}{W(z)} = H_1(z)\,H_2(z)$$

Now let $H_1(z)$ be given as

$$H_1(z) = \frac{W(z)}{X(z)}$$

$$= \frac{1}{1 + \sum_{k=1}^{N} a_k z^{-k}} \quad \text{—— (i)}$$

$$H_2(z) = \frac{Y(z)}{W(z)}$$

$$= \sum_{k=0}^{M} b_k z^{-k} \quad \text{— (j)}$$

Cross multiply the terms of Equation, Then we get

$$W(z)\left[1 + \sum_{k=1}^{N} a_k z^{-k}\right] = X(z)$$

$$W(z) = X(z) - \sum_{k=1}^{N} a_k\ W(z)$$

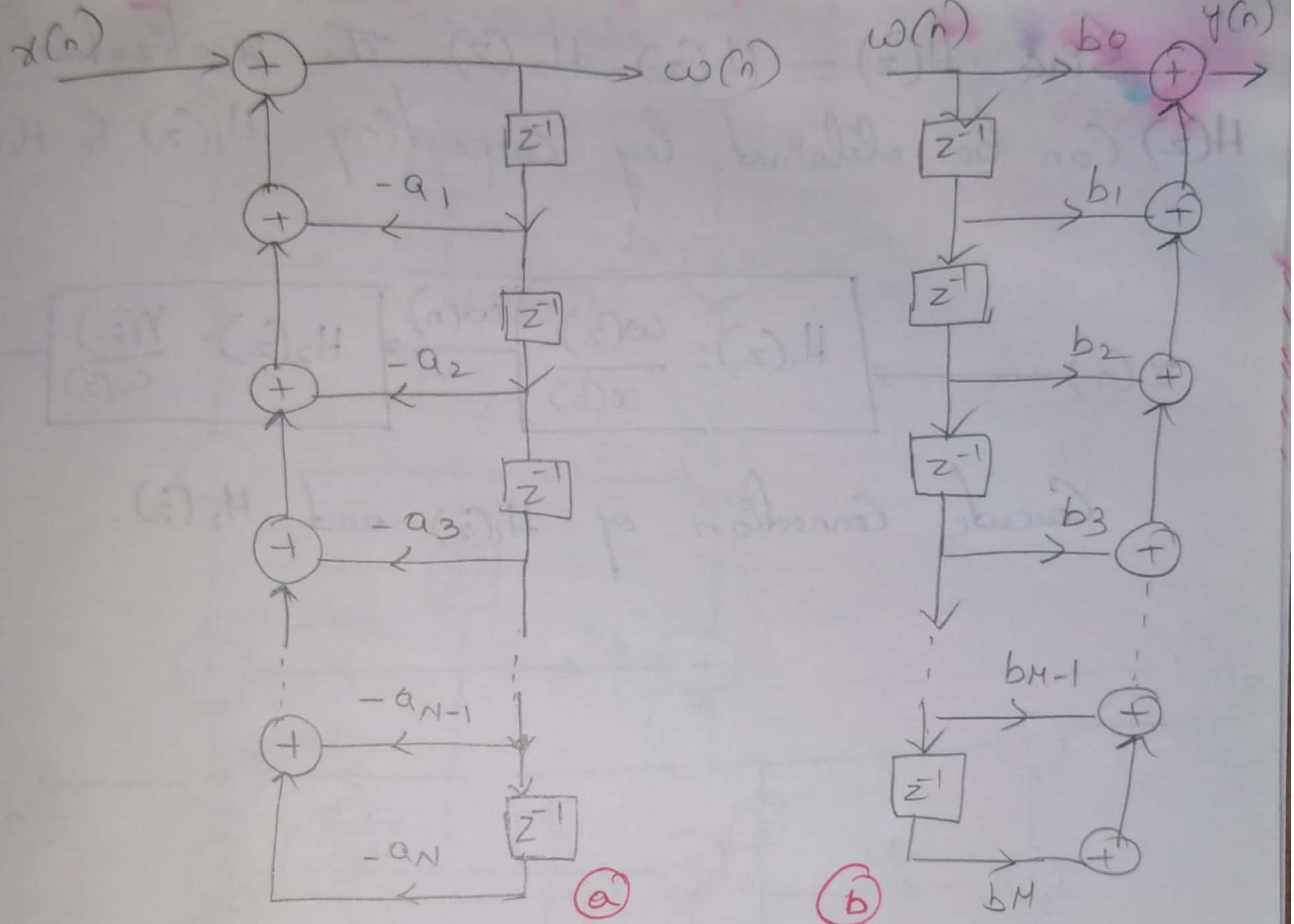
$$= X(z) - a_1 z^{-1} W(z) - a_2 z^{-2} W(z) - a_3 z^{-3} W(z)$$

$$\ldots\ldots - a_N z^{-N} W(z).$$

Taking inverse Z-transform of this Equation

$$w(n) = x(n) - a_1 w(n-1) - a_2 w(n-2) - \ldots - a_N w(n-$$

$$\text{— (k)}$$



(a) (b)

Let us obtain the realization of $H_2(z)$ from Eq (j)

$$Y(z) = \sum_{K=0}^{M} b_K z^{-k} \omega(z)$$

$$= b_0 W(z) + b_1 z^{-1} \omega(z) + b_2 z^{-2} \omega(z) + \dots + b_M z^{-M} W(z).$$

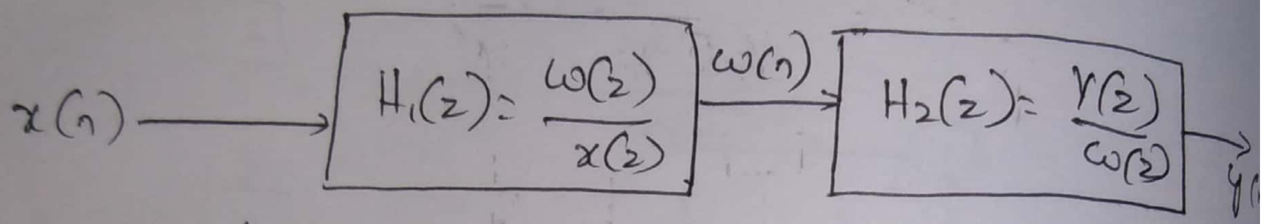
Taking inverse z transform of this Equation

$$y(n) = b_0 \omega(n) + b_1 \omega(n-1) + b_2 \omega(n-2) + \dots + b_M \omega(n-M). \quad \text{— (l)}$$

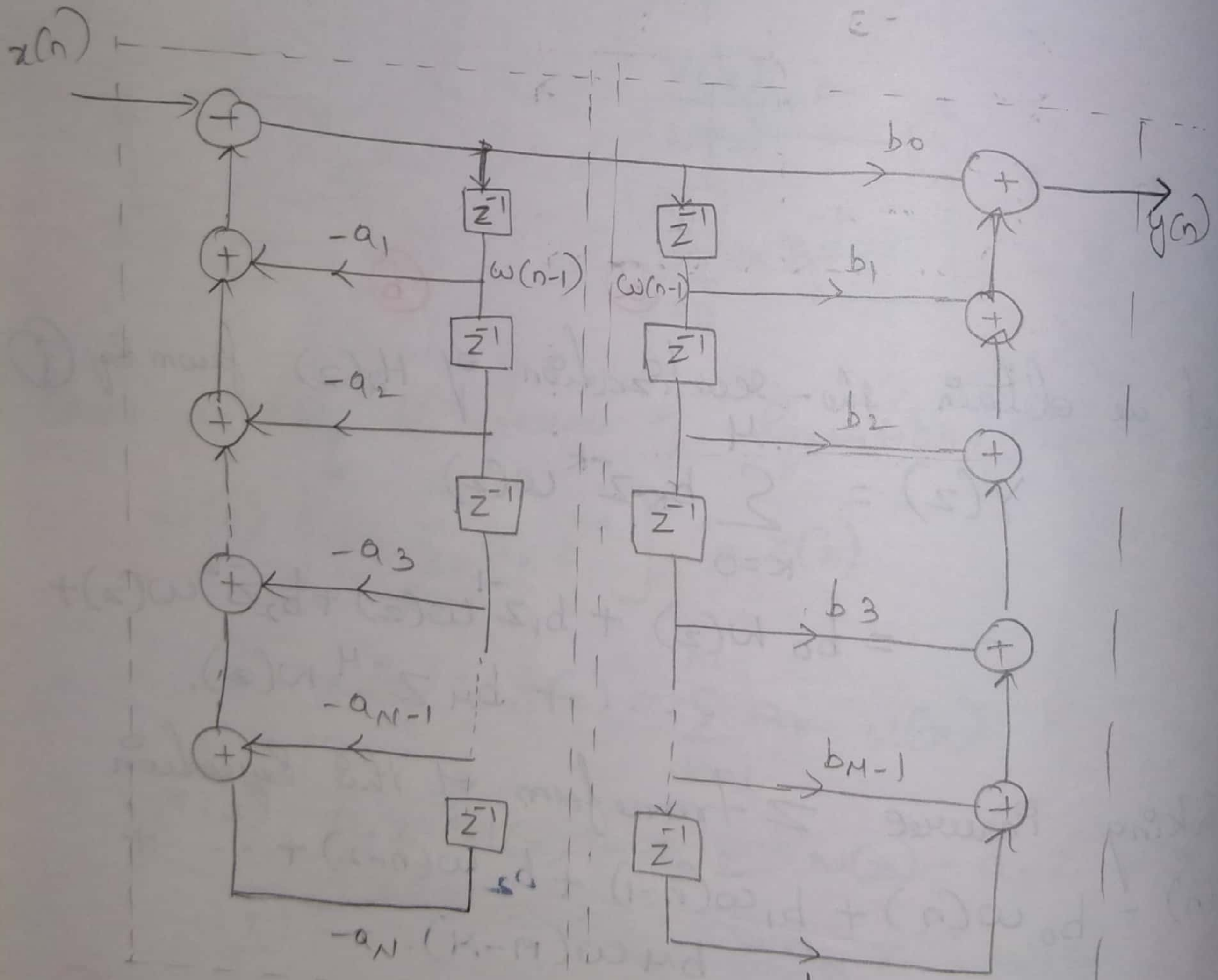
fig (a) represents Direct form implem of. Eq (k) ie All pole system

fig (b) represents Direct form implem of Eq (l) ie All zero system.

Since $H(z) = H_1(z) \cdot H_2(z)$. The realization of $H(z)$ can be obtained by cascading $H_1(z)$ & $H_2(z)$



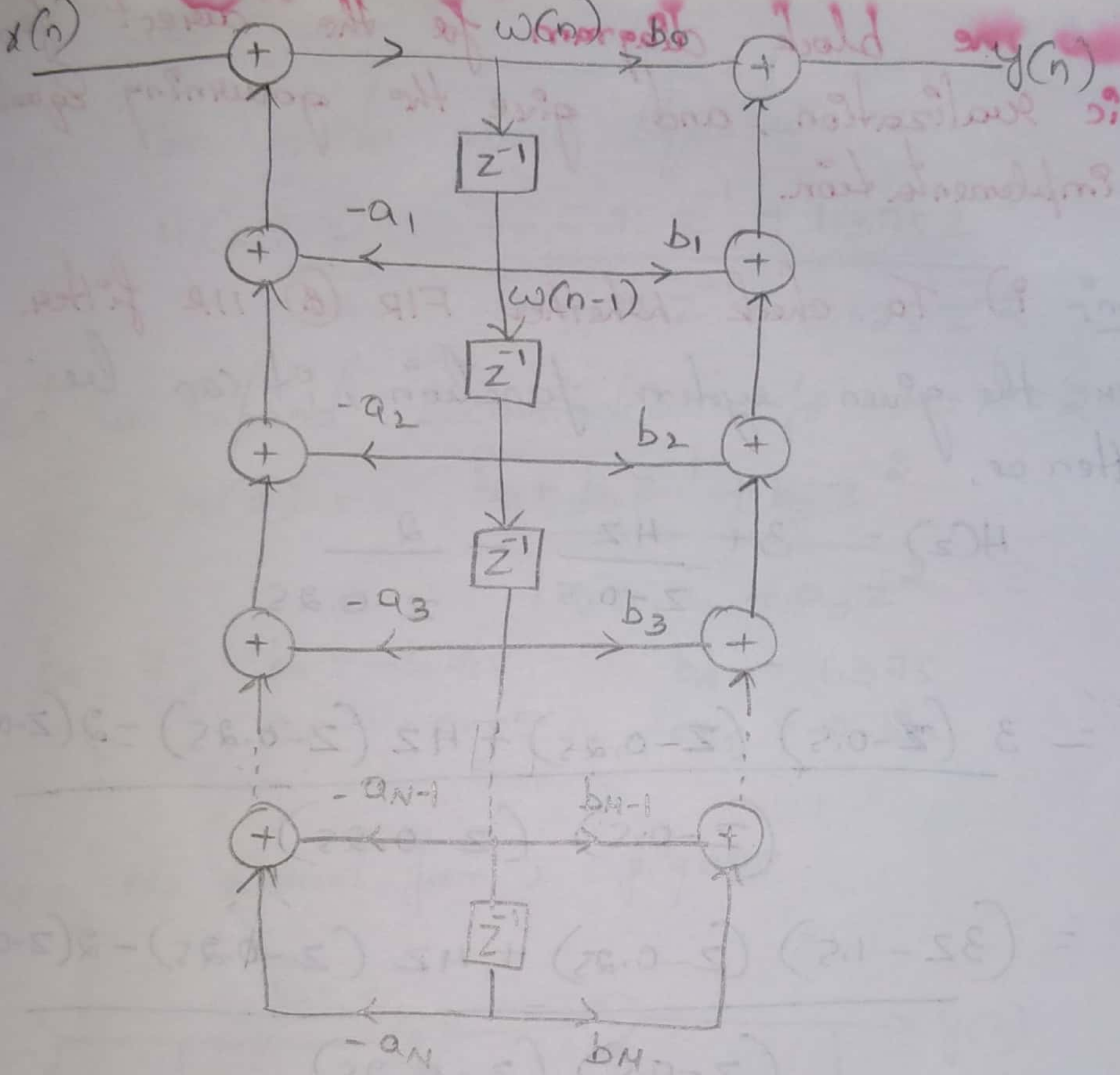
Cascade Connection of $H_1(z)$ and $H_2(z)$.



All pole System

All Zero system

In this above fig two delay elements of all pole and zero system can be merged into single delay element.



In this realization shown in figure, we have assumed $N=4$. This is called direct form-II realization of IIR system. Since direct form-II reduces memory location so it is called **Canonic form**.

A system represented by a transfer function $H(z)$ is given by $H(z) = 3 + \frac{4z}{z-\frac{1}{2}} - \frac{2}{z-\frac{1}{4}}$

i) Does this $H(z)$ represent a FIR or IIR. Why?

ii) Give a difference equation realization of this system using direct form - I.

the block diagram for the direct form canonic realization, and give the governing Equations for implementation.

Solution:- i) To check whether FIR (or) IIR filter.

Consider the given system function, it can be written as,

$$H(z) = 3 + \frac{4z}{z-0.5} - \frac{2}{z-0.25}$$

$$H(z) = \frac{3(z-0.5)(z-0.25) + 4z(z-0.25) - 2(z-0.\ldots}{(z-0.5)(z-0.25)}$$

Poles - Roots of denom

$$= \frac{(3z-1.5)(z-0.25) + 4z(z-0.25) - 2(z-0.\ldots}{(z-0.5)(z-0.25)}$$

Zeros - Roots of nominator

$$= \frac{3z^2 - 0.75z - 1.5z + 0.375 + 4z^2 - z - 2z\ldots}{z^2 - 0.75 + 0.125}$$

$$= \frac{7z^2 - 5.25z + 1.375}{z^2 - 0.75 + 0.125}$$

The system function has numerator polynomial of order 2 as well as deno poly of order 2.
The system function has poles as well as zeros. Hence it represents IIR filter.

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ii) Direct form - I.

we can write above Equation as.

$$H(z) = \frac{7 - 5.25 z^{-1} + 1.375 z^{-2}}{1 - 0.75 z^{-1} + 0.125 z^{-2}}$$

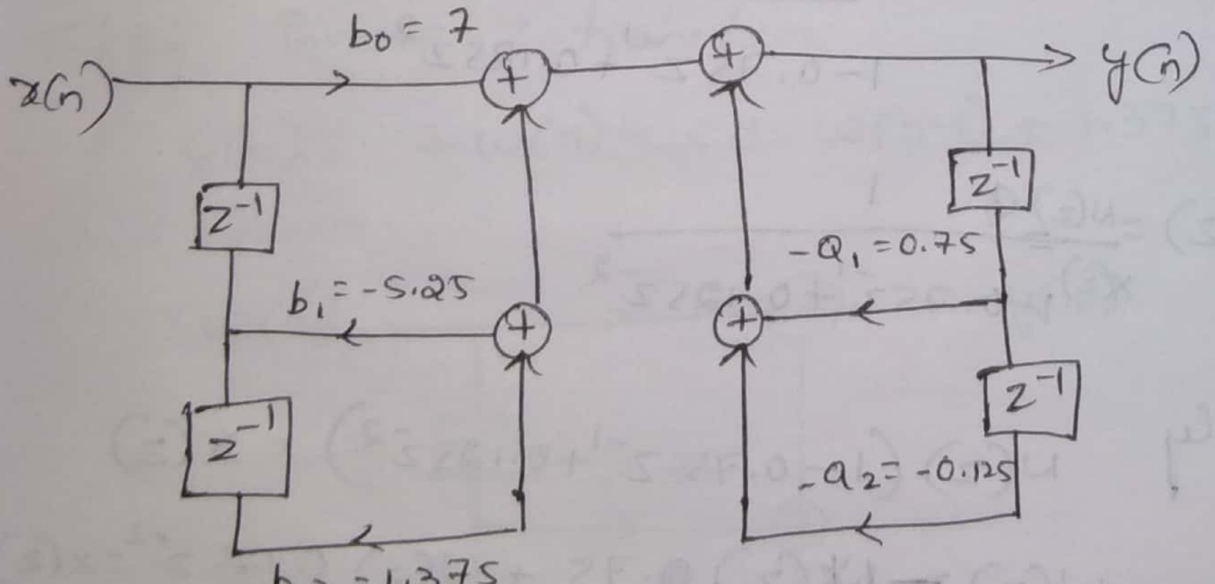
Let us Expand summation for $M = N = 2$ ie,

$$H(z) = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2}}{1 + a_1 z^{-1} + a_2 z^{-2}}$$

$b_0 = 7$, $b_1 = -5.25$ $b_2 = 1.375$

$a_1 = -0.75$ $a_2 = 0.125$

from the Direct form I figure



iii) Direct form II Cononic realization.

Consider $H(z) = \frac{7 - 5.25 z^{-1} + 1.375 z^{-2}}{1 - 0.75 z^{-1} + 0.125 z^{-2}}$

WKT $H(z) = \frac{Y(z)}{X(z)}$ $\therefore$ $H_1(z) = \frac{1}{1+\sum_{k=1}^{N} a_k}$

$$\frac{7-5.25z^{-1}+1.375z^{-2}}{1-0.75z^{-1}+0.125z^{-2}} = \frac{Y(z)}{X(z)}$$ $H_2(z) = \frac{Y(z)}{W(z)}$

Let us arrange above Equation as

$$\frac{Y(z)}{W(z)} \cdot \frac{W(z)}{X(z)} = \frac{7-5.25z^{-1}+1.375z^{-2}}{1-0.75z^{-1}+0.125z^{-2}}$$

ie $$\frac{W(z)}{X(z)} \cdot \frac{Y(z)}{W(z)} = \frac{7-5.25z^{-1}+1.375z^{-2}}{1-0.75z^{-1}+0.125z^{-2}}$$

$$H_1(z) \cdot H_2(z) = \frac{1}{1-0.75z^{-1}+0.125z^{-2}} \left(7-5.25z^{-1}+1.3\right.$$

Take $H_1(z) = \frac{W(z)}{X(z)} = \frac{1}{1-0.75z^{-1}+0.125z^{-2}}$

Cross $x^{l}y$ $W(z)\left(1-0.75z^{-1}+0.125z^{-2}\right) = X(z)$

$$W(z) - \cancel{W}(z)\,0.75 + W(z)\,0.125z^{-2} = X(z)$$

$$W(z) = X(z) + 0.75z^{-1}W(z) - 0.125z$$

Taking inverse z transf

$$w(n) = x(n) + 0.75\,w(n-1) - 0.125\,w(n-2)$$

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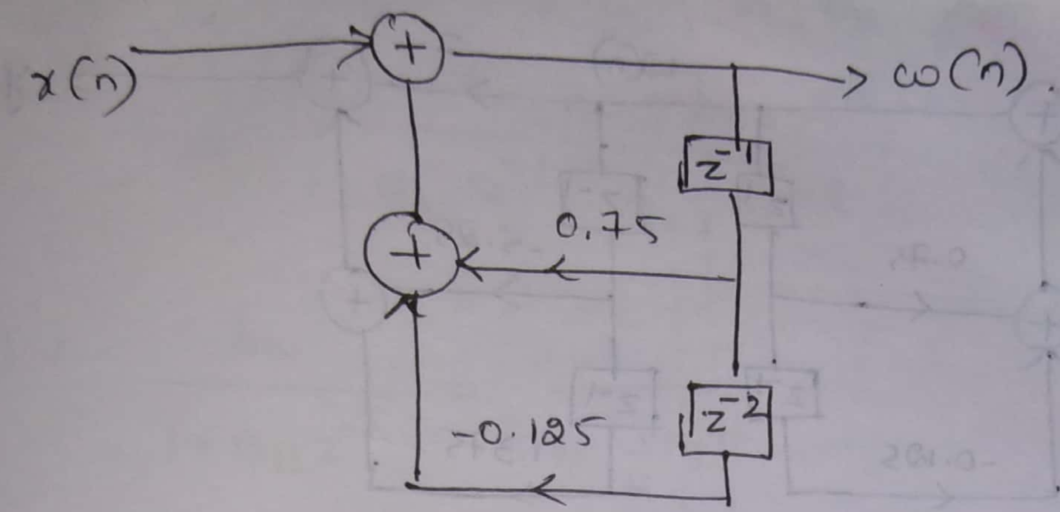


Take $H_2(z) = \dfrac{Y(z)}{W(z)} = 7 - 5.25z^{-1} + 1.375z^{-2}$

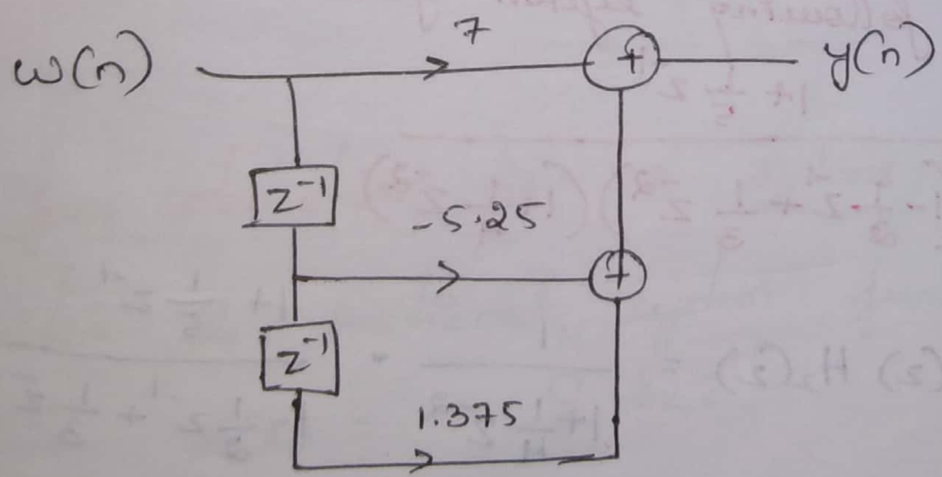
crossmultiply the terms in Equation

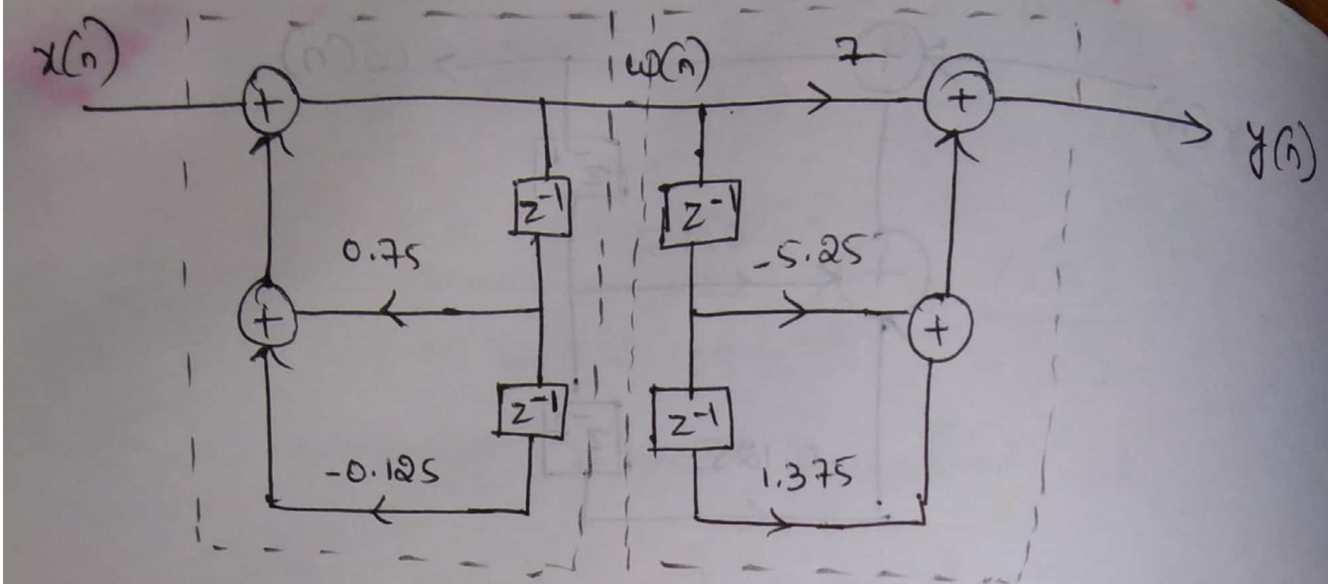
$$Y(z) = (7 - 5.25z^{-1} + 1.375z^{-2})\, W(z)$$

$$Y(z) = 7W(z) - 5.25z^{-1}W(z) + 1.375z^{-2}W(z)$$

Taking inverse Z-transform

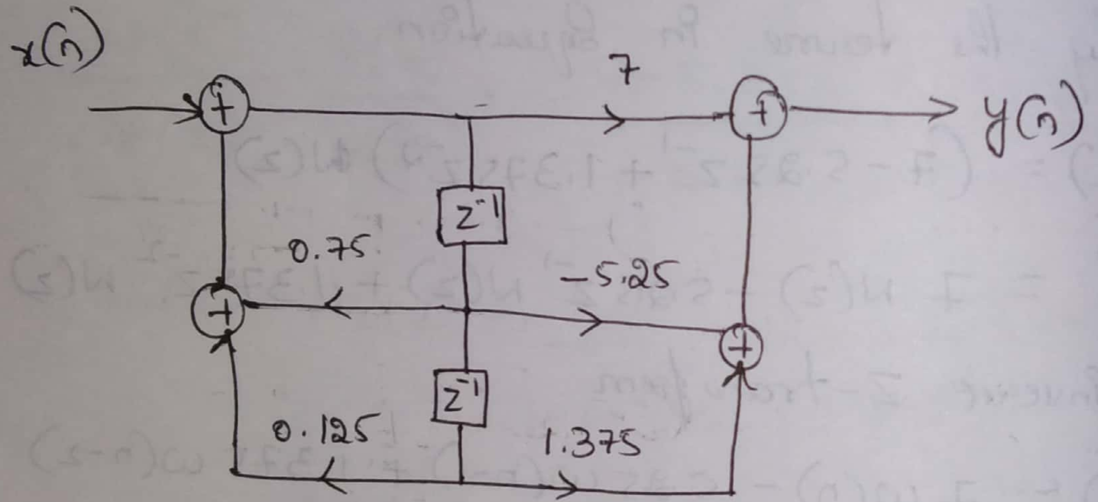
$$y(n) = 7w(n) - 5.25w(n-1) + 1.375w(n-2)$$





All pole System   All zero system.

The delays can be combined into two.



Realize the following system functions in Cascade form

$$H(z) = \frac{1+\frac{1}{5}z^{-1}}{\left(1-\frac{1}{2}z^{-1}+\frac{1}{3}z^{-2}\right)\left(1+\frac{1}{4}z^{-2}\right)}$$

$$H(z) = H_1(z)\,H_2(z) = \frac{1}{1+\frac{1}{4}z^{-2}} \cdot \frac{1+\frac{1}{5}z^{-1}}{1-\frac{1}{2}z^{-1}+\frac{1}{3}z^{-2}}$$

$$H_1(z) = \frac{1}{1+\frac{1}{4}z^{-2}}$$

$$H_2(z) = \frac{1+\frac{1}{5}z^{-1}}{1-\frac{1}{2}z^{-1}+\frac{1}{3}z^{-2}}$$

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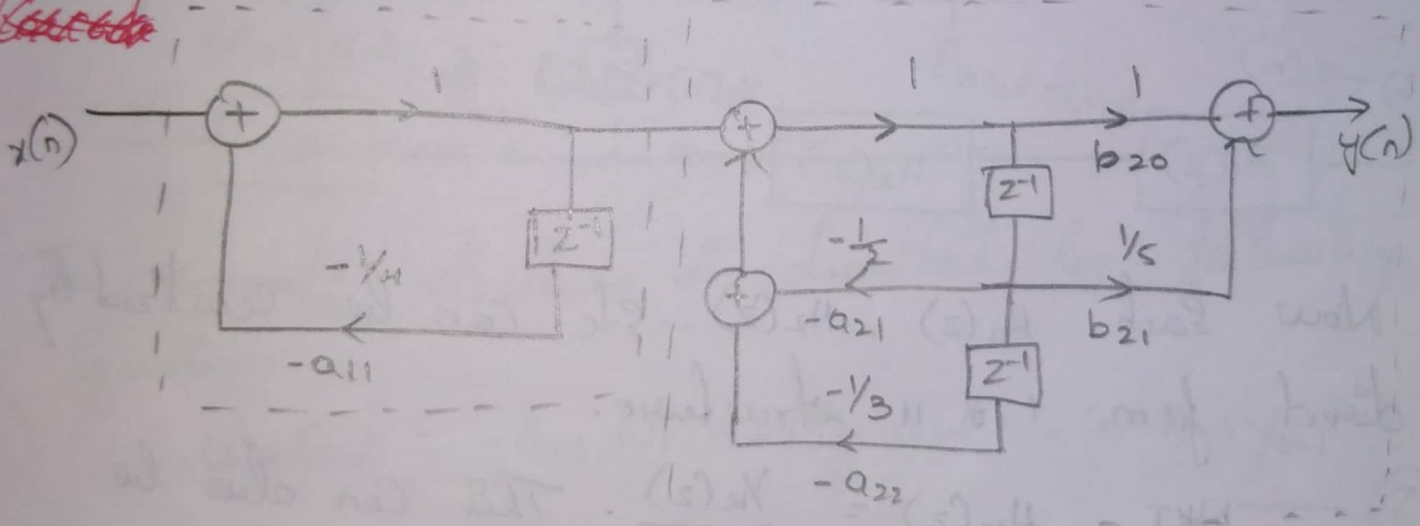
The above two Equations are in the form

$$H_o(z) = \frac{b_{k0} + b_{k1} z^{-1} + b_{k2} z^{-2}}{1 + a_{k1} z^{-1} + a_{k2} z^{-2}}$$

$$H_1(z) = \frac{b_{10}}{1 + a_{11} z^{-1}} = \frac{1}{1 + \frac{1}{4} z^{-1}}$$

$$H_2(z) = \frac{b_{20} + b_{21} z^{-1}}{1 + a_{21} z^{-1} + a_{22} z^{-2}} = \frac{1 + \frac{1}{5} z^{-1}}{1 - \frac{1}{2} z^{-1} + \frac{1}{3} z^{-2}}$$

~~Cascade~~



Cascade form structure for IIR systems

Consider the rational system function of Equations

$$H(z) = \frac{\sum_{k=0}^{M} b_k z^{-k}}{1 + \sum_{k=1}^{N} a_k z^{-k}}$$

$$= \frac{b_0 + b_1 z^{-1} + b_2 z^{-2} + \dots + b_M z^{-M}}{1 + a_1 z^{-1} + a_2 z^{-2} + \dots + a_N z^{-N}} \quad \text{--- (1)}$$

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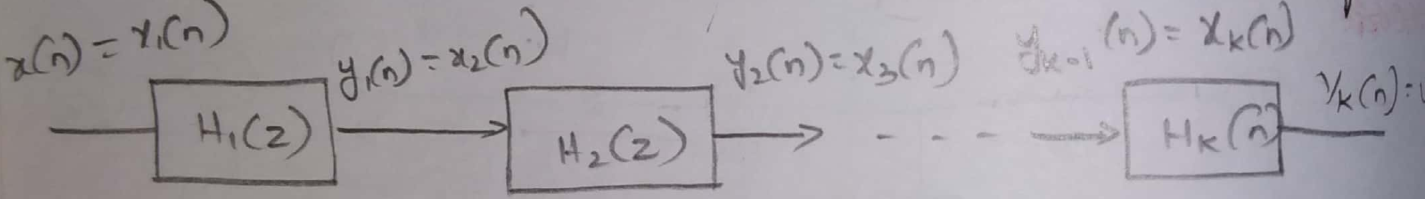
The num and deno polynomials of above Equation can be expressed as multiplication of second order polynomials i.e.

$$H(z) = H_1(z) \times H_2(z) \times H_3(z) \times \cdots \times H_K(z) \quad \text{— (2)}$$

$$H_K(z) = \frac{b_{K0} + b_{K1} z^{-1} + b_{K2} z^{-2}}{1 + a_{K1} z^{-1} + a_{K2} z^{-2}} \quad K = 1, 2 \ldots \quad \text{— (3)}$$

WKT function $H_1(z)$, $H_2(z)$ etc of Eq (2) can be connected in cascade to obtain realization of $H(z)$



Now each $H_1(z)$, $H_2(z)$ .. etc can be realized by direct form I & II structure.

WKT $H_K(z) = \dfrac{Y_K(z)}{X_K(z)}$. This can also be written as,

$$H_K(z) = \frac{W_K(z)}{X_K(z)} \cdot \frac{Y_K(z)}{W_K(z)}$$

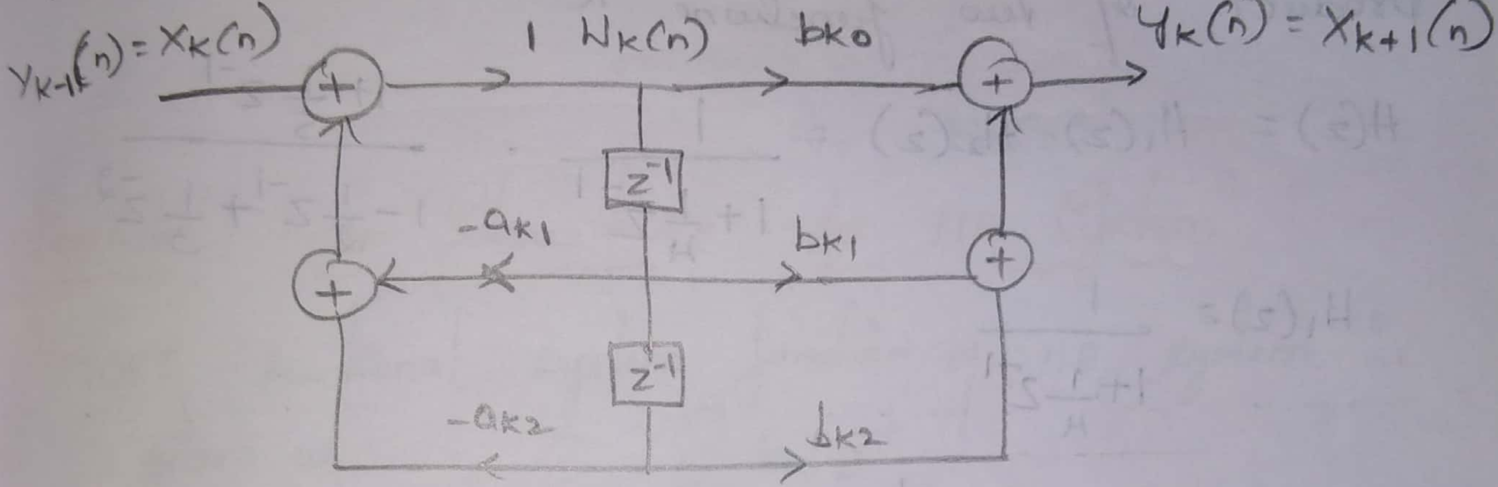
$$= H_{K1}(z) \cdot H_{K2}(z)$$

Let $H_{K1}(z) = \dfrac{W_K(z)}{X_K(z)} = \dfrac{1}{1 + a_{K1} z^{-1} + a_{K2} z^{-2}}$

This is all pole second order subsystem and

$$H_{K2}(z) = \frac{Y_K(z)}{W_K(z)} = b_{K0} + b_{K1}z^{-1} + b_{K2}z^{-2} \quad \text{--- (5)}$$

we can obtain the direct-form II structure for $H_K(z)$ which is splitted into two functions given by Eq (4) and Eq (5)



The cascade structure is described by following equations

$$w_K(n) = -a_{K1} w_K(n-1) - a_{K2} w_K(n-2) + Y_{K-1}(n) \quad \text{--- (6)}$$

$$Y_K(n) = b_{K0} w_K(n) + b_{K1} w_K(n-1) + b_{K2} w_K(n-2) \quad \text{--- (7)}$$

This equation represents the second order subsystem and cascading is represented by following equations

$$\left.\begin{aligned} y_{K-1}(n) &= x_K(n) \\ y_K(n) &= x_{K+1}(n) \\ y_0(n) &= x(n) \\ y(n) &= x_K(n) \end{aligned}\right\} \quad \text{--- (8)}$$

Realize the following system function in Cascade form

$$H(z) = \frac{1+\frac{1}{5}z^{-1}}{\left(1-\frac{1}{2}z^{-1}+\frac{1}{3}z^{-2}\right)\left(1+\frac{1}{4}z^{-1}\right)}$$

The given transfer function can be written as the product of two functions ie

$$H(z) = H_1(z) \cdot H_2(z) = \frac{1}{1+\frac{1}{4}z^{-1}} \cdot \frac{1+\frac{1}{5}z^{-1}}{1-\frac{1}{2}z^{-1}+\frac{1}{3}z^{-2}}$$

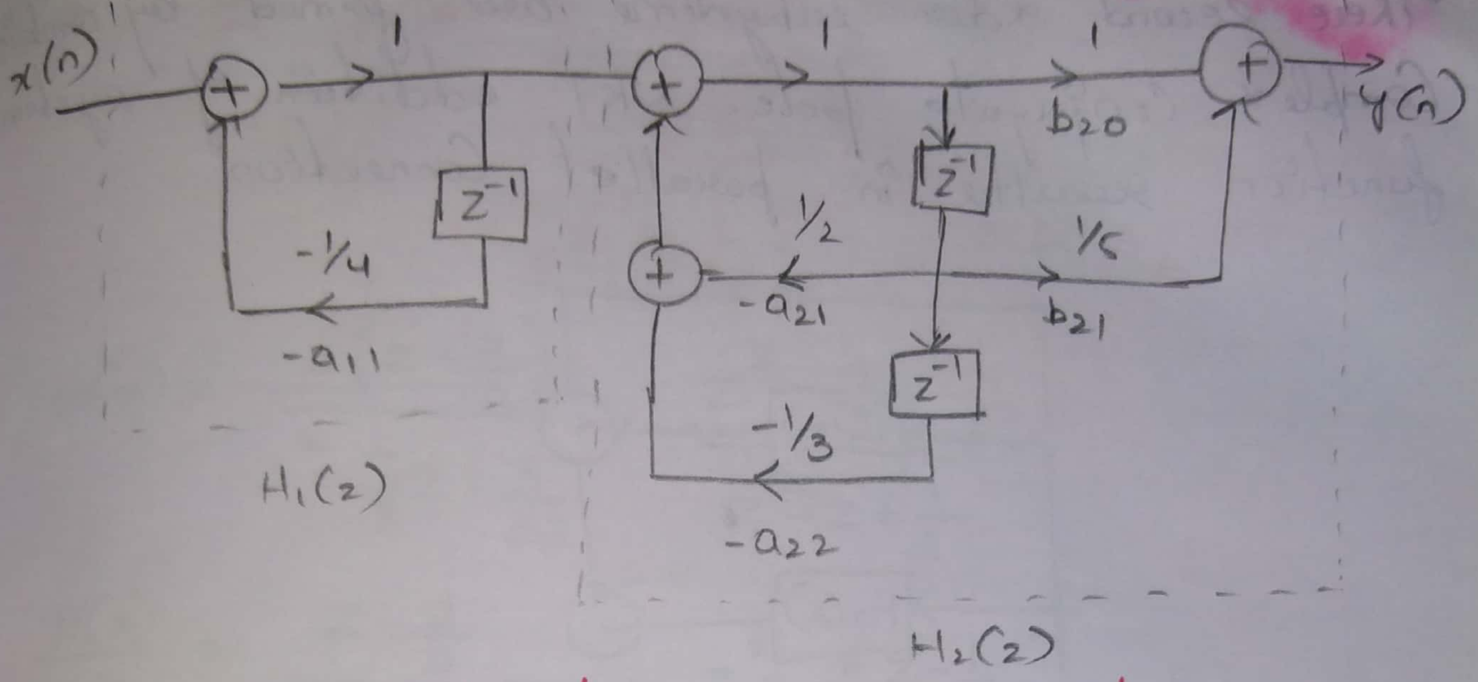
$$H_1(z) = \frac{1}{1+\frac{1}{4}z^{-1}}$$

$$H_2(z) = \frac{1+\frac{1}{5}z^{-1}}{1-\frac{1}{2}z^{-1}+\frac{1}{3}z^{-2}}$$

The above two Equations are in the form

$$H_1(z) = \frac{b_{10}}{1+a_{11}z^{-1}} = \frac{1}{1+\frac{1}{4}z^{-1}}$$

$$H_2(z) = \frac{b_{20}+b_{21}z^{-1}}{1+a_{21}z^{-1}+a_{22}z^{-2}} = \frac{1+\frac{1}{5}z^{-1}}{1-\frac{1}{2}z^{-1}+\frac{1}{3}z^{-2}}$$



Parallel Form Structure for IIR System

WKT rational system function of IIR system is given as

$$H(z) = \frac{\sum_{k=0}^{M} b_k z^{-k}}{1+\sum_{k=1}^{N} a_k z^{-k}} \quad \text{--- (1)}$$

$$H(z) = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2} + \cdots + b_M z^{-M}}{1 + a_1 z^{-1} + a_2 z^{-2} + \cdots + a_N z^{-N}} \quad \text{--- (2)}$$

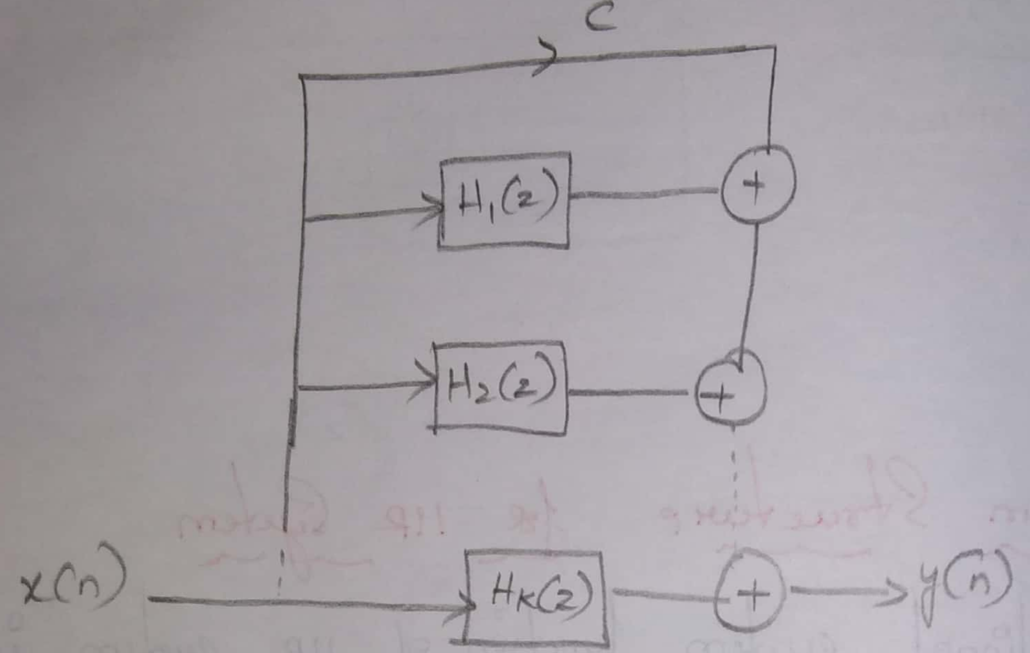
The above system function can be partial fractions as follows.

$$H(z) = C + H_1(z) + H_2(z) + \cdots + H_K(z) \quad \text{--- (3)}$$

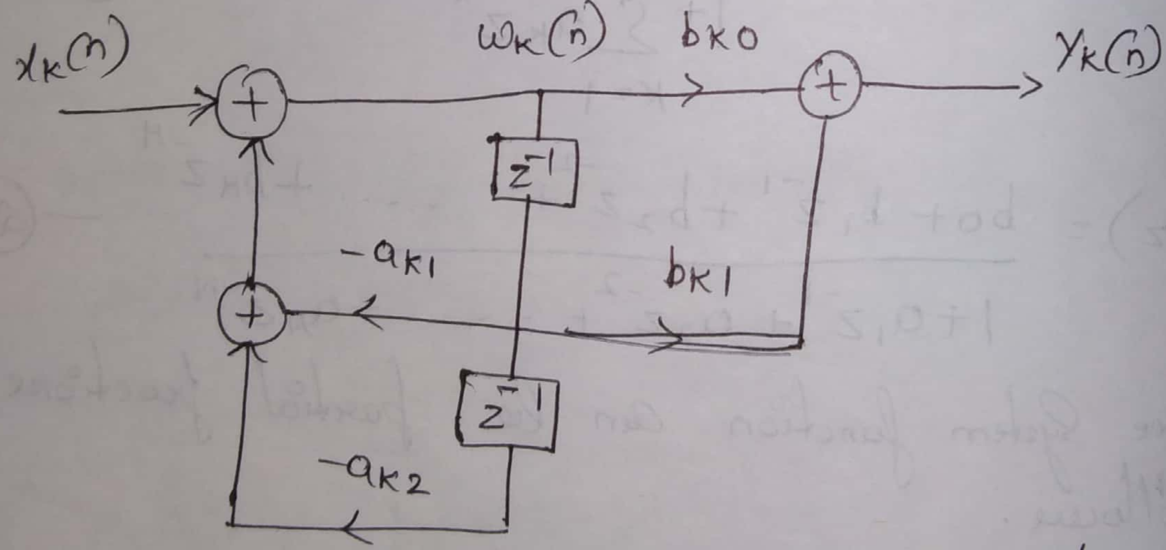
Here C is constant and each $H_1(z)$, $H_2(z)$ .. etc is the second order subsystem which is given as

$$H_K(z) = \frac{b_{K0} + b_{K1} z^{-1}}{1 + a_{K1} z^{-1} + a_{K2} z^{-2}} \quad \text{--- (4)}$$

These second order subsystems are formed by combining complex conjugate pole. NKT addition of system function results in parallel connection.



Here each $H_1(z)$, $H_2(z)$.. etc can be realized by direct form I or direct form II



The partial form structure discussed here by following equations.

$$w_K(n) = a_{K1}\, w_K(n-1) - a_{K2}\, w_K(n-2) + x(n).$$

$$y_K(n) = b_{K0}\, w_K(n) + b_{K1}\, w_K(n-1)$$

$$y(n) = C\, x(n) + \sum_{k}^{K} y_K(n)$$

Realize the following system function in parallel form

$$H(z) = \frac{1-\frac{2}{3}z^{-1}}{1-\frac{7}{8}z^{-1}+\frac{3}{32}z^{-2}} \cdot \frac{1+\frac{7}{2}z^{-1}-\frac{1}{2}z^{-2}}{1-z^{-1}+\frac{1}{2}z^{-2}}$$

$$H(z) = \frac{z\left(z-\frac{2}{3}\right)}{z^2-\frac{7}{8}z+\frac{3}{32}} \cdot \frac{z^2-\frac{7}{2}z-\frac{1}{2}}{z^2-z+\frac{1}{2}}$$

$$\frac{H(z)}{z} = \frac{z-\frac{2}{3}}{z^2-\frac{7}{8}z+\frac{3}{32}} \cdot \frac{z^2-\frac{7}{4}z-\frac{1}{2}}{z^2-z+\frac{1}{2}}$$

$$= \frac{z-\frac{2}{3}}{\left(z-\frac{3}{4}\right)\left(z-\frac{1}{8}\right)} \cdot \frac{z^2-\frac{7}{4}z-\frac{1}{2}}{\left(z-\frac{1}{2}-j\frac{1}{2}\right)\left(z-\frac{1}{2}+j\frac{1}{2}\right)}$$

$$= \frac{A_1}{z-\frac{3}{4}} + \frac{A_2}{z-\frac{1}{8}} + \frac{A_3}{z-\frac{1}{2}-j\frac{1}{2}} + \frac{A_4}{z-\frac{1}{2}+j\frac{1}{2}}$$

Calculating the values of A_1, A_2, A_3 and A_4 we get

$$\frac{H(z)}{z} = \frac{2.933}{z-\frac{3}{4}} - \frac{2.947}{z-\frac{1}{8}} + \frac{2.507-j10.45}{z-\frac{1}{2}-j\frac{1}{2}} + \frac{2.507+j10.45}{z-\frac{1}{2}+j\frac{1}{2}}$$

Let us combine the first two term and last two terms.

$$\frac{H(z)}{z} = \frac{-0.014z+1.843}{z^2-\frac{7}{8}z+\frac{3}{32}} + \frac{5.022+7.743}{z^2-z+\frac{1}{2}}$$

$$H(z) = \frac{-0.014z^2 + 1.843z}{z^2 - \frac{7}{8}z + \frac{3}{32}} + \frac{5.02z^2 + 7.743z}{z^2 - z + \frac{1}{2}}$$

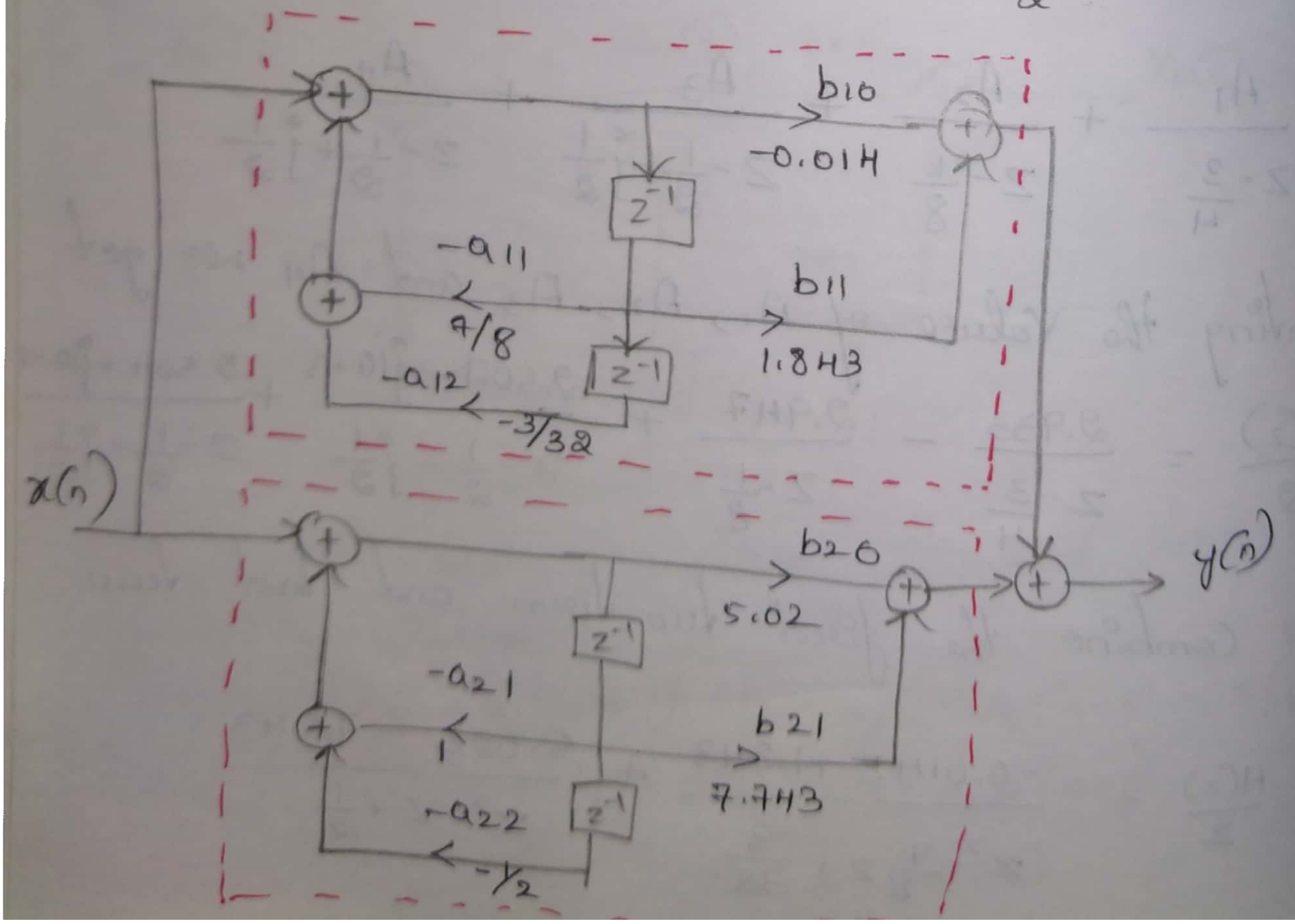
The above can also be written as.

$$H(z) = \frac{-0.014 + 1.843z^{-1}}{1 - \frac{7}{8}z^{-1} + \frac{3}{32}z^{-2}} + \frac{5.02 + 7.743z^{-1}}{1 - z^{-1} + \frac{1}{2}z^{-2}}$$

The above Equation has two terms, they are called as

$$H_1(z) = \frac{b_{10} + b_{11}z^{-1}}{1 + a_{11}z^{-1} + a_{12}z^{-2}} = \frac{-0.014 + 1.843z^{-1}}{1 - \frac{7}{8}z^{-1} + \frac{3}{32}z^{-2}}$$

$$H_2(z) = \frac{b_{20} + b_{21}z^{-1}}{1 + a_{21}z^{-1} + a_{22}z^{-2}} = \frac{5.02 + 7.743z^{-1}}{1 - z^{-1} + \frac{1}{2}z^{-2}}$$



Draw the block diagram of direct form I and direct form II realization for a digital IIR filter described by the system function:

$$H(z) = \frac{8z^3 - 4z^2 + 11z - 2}{\left(z - \frac{1}{4}\right)\left(z^2 - z + \frac{1}{2}\right)}$$

Direct form I

$$H(z) = \frac{Y(z)}{X(z)}$$

$$\frac{Y(z)}{X(z)} = \frac{8 - 4z^{-1} + 11z^{-2} - 2z^{-3}}{1 - \frac{5}{4}z^{-1} + \frac{3}{4}z^{-2} - \frac{1}{8}z^{-3}}$$

$$Y(z)\left(1 - \frac{5}{4}z^{-1} + \frac{3}{4}z^{-2} - \frac{1}{8}z^{-3}\right) = X(z)\left(8 - 4z^{-1} + 11z^{-2} - 2z^{-3}\right)$$

$$Y(z) - \frac{5}{4}z^{-1}Y(z) + \frac{3}{4}z^{-2}Y(z) - \frac{1}{8}z^{-3}Y(z) = X(z)8 - X(z)4z^{-1} + X(z)11z^{-2} - X(z)2z^{-3}$$

Taking inverse z transform on both sides of the above equation

$$y(n) - \frac{5}{4}y(n-1) + \frac{3}{4}y(n-2) - \frac{1}{8}y(n-3) = 8x(n) - 4x(n-1) + 11x(n-2) - 2x(n-3)$$

$$\Rightarrow y(n) = \frac{5}{4}y(n-1) - \frac{3}{4}y(n-2) + \frac{1}{8}y(n-3) + 8x(n) - 4(n-1) + 11(n-2) - 2x(n-3).$$

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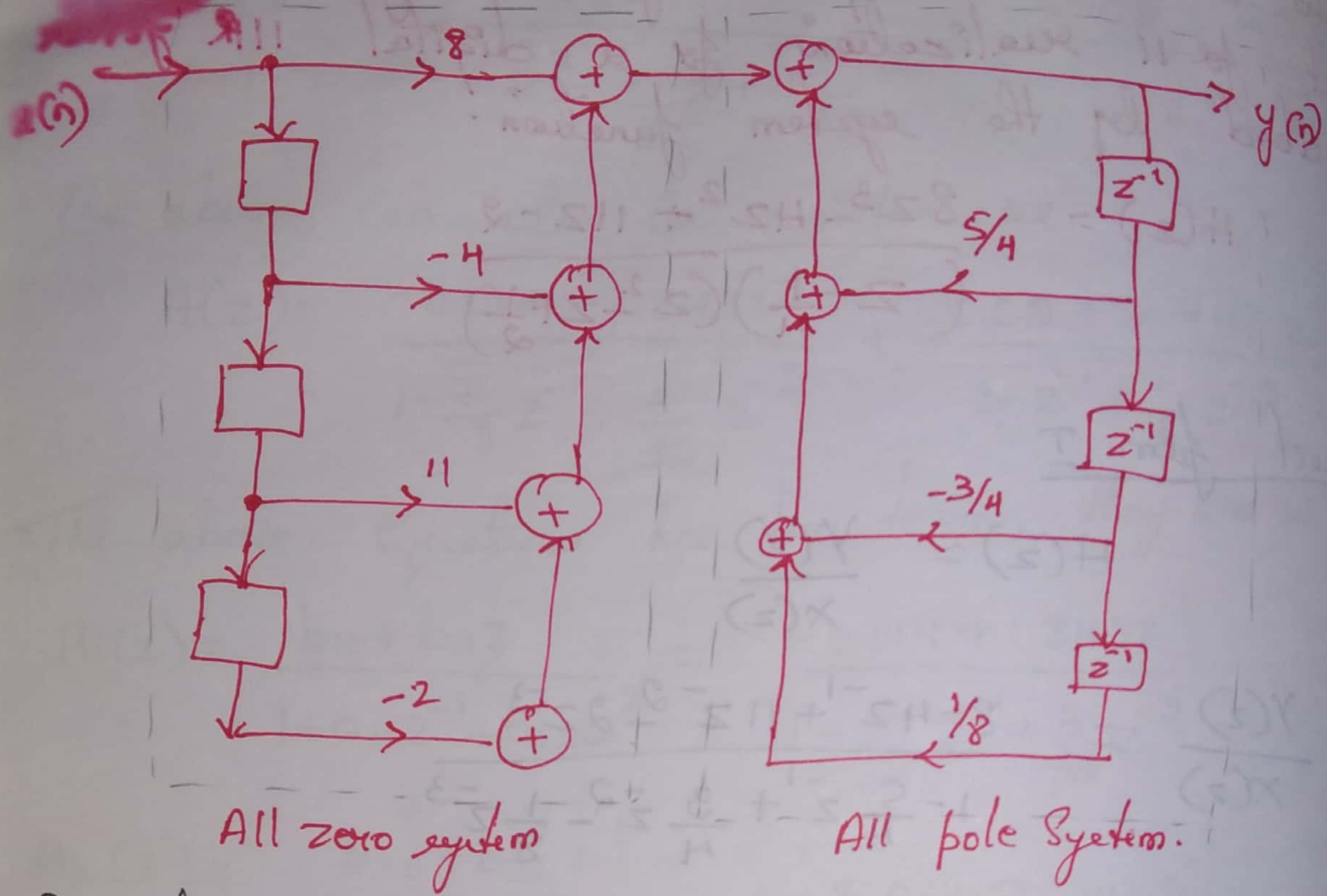


Direct form II

H(z) be decomposed into a product of two transfer functions $H_1(z)$ and $H_2(z)$

$$x(n) \longrightarrow H_1(z) \xrightarrow{v(n)} H_2(z) \longrightarrow y(n)$$

All poles of $H(z)$ All zeros of $H(z)$

WKT $\quad V(z) = X(z)\, H_1(z)$

$$V(z) = X(z)\left(\frac{1}{1-\frac{5}{4}z^{-1}+\frac{3}{4}z^{-2}-\frac{1}{8}z^{-3}}\right)$$

$$V(z) - \frac{5}{4}z^{-1}V(z) + \frac{3}{4}z^{-2}V(z) - \frac{1}{8}z^{-3}V(z) = X(z)$$

$$v(n) = \frac{5}{4}v(n-1) - \frac{3}{4}v(n-2) + \frac{1}{8}v(n-3) + x(n)$$

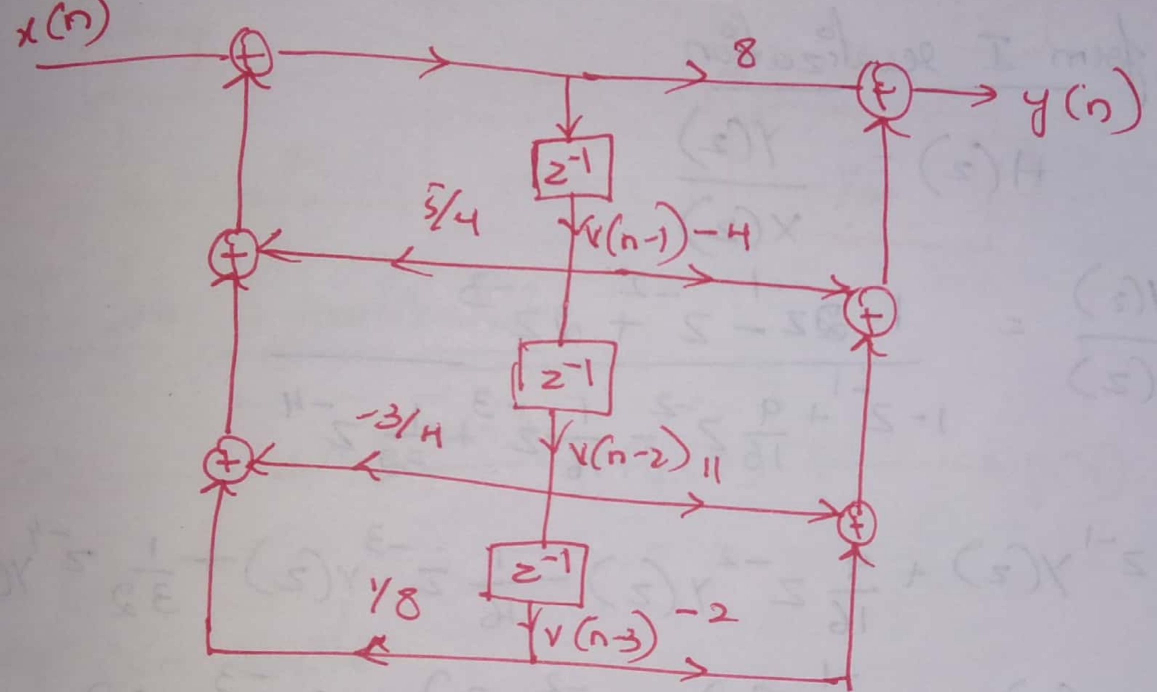
$$Y(z) = V(z) H_2(z)$$

$$Y(z) = V(z)(8 - 4z^{-1} + 11z^{-2} - 2z^{-3})$$

Taking inverse on both side.

$$Y(z) = V(z)8 - 4z^{-1}V(z) + 11z^{-2}V(z) - 2z^{-3}V(z)$$

$$y(n) = 8v(n) - 4v(n-1) + 11v(n-2) - 2v(n-3)$$



A linear time invariant digital IIR filter is specified by the following transfer function.

$$H(z) = \frac{(z-1)(z-2)(z+1)z}{\left[z-\left(\frac{1}{2}+j\frac{1}{2}\right)\right]\left[z-\left(\frac{1}{2}-j\frac{1}{2}\right)\right]\left[z-j\frac{1}{4}\right]\left[z+j\frac{1}{4}\right]}$$

$$H(z) = \frac{(z^2-1)(z^2-2z)}{\left(z^2+\frac{1}{16}\right)\left(z^2-z+\frac{1}{2}\right)}$$

$$= \frac{z^4 - 2z^3 - z^2 + 2z}{z^4 - z^3 + \frac{9}{16}z^2 - \frac{1}{16}z + \frac{1}{32}}$$

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For a Causal system, the num and deno of $H(z)$ must Contain only negative powers of Z. Hence divide the num and deno of $H(z)$ by z^4.

$$H(z) = \frac{1 - 2z^{-1} - z^{-2} + 2z^{-3}}{1 - z^{-1} + \frac{9}{16}z^{-2} - \frac{1}{16}z^{-3} + \frac{1}{32}z^{-4}}$$

Direct form I realization

$$H(z) = \frac{Y(z)}{X(z)}$$

then $\frac{Y(z)}{X(z)} = \frac{1 - 2z^{-1} - z^{-2} + 2z^{-3}}{1 - z^{-1} + \frac{9}{16}z^{-2} - \frac{1}{16}z^{-3} + \frac{1}{32}z^{-4}}$

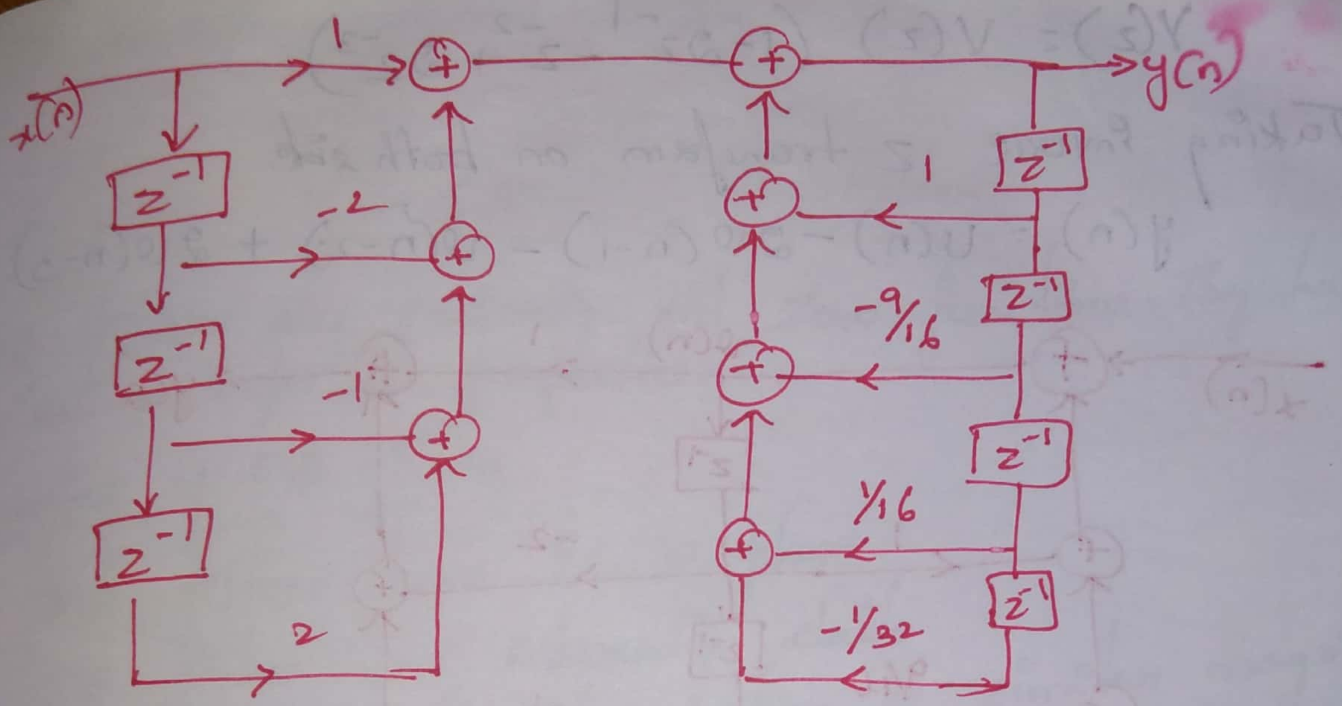
$$Y(z) - z^{-1}Y(z) + \frac{9}{16}z^{-2}Y(z) - \frac{1}{16}z^{-3}Y(z) + \frac{1}{32}z^{-4}Y(z)$$

$$= X(z) - 2z^{-1}X(z) - z^{-2}X(z) + 2z^{-3}X(z)$$

Taking inverse z transf on both side

$$y(n) - y(n-1) + \frac{9}{16}y(n-2) - \frac{1}{16}y(n-3) + \frac{1}{32}y(n-4) = x(n) - 2x(n-1) - x(n-2) + 2x(n-3)$$

$$y(n) = y(n-1) - \frac{9}{16}y(n-2) + \frac{1}{16}y(n-3) - \frac{1}{32}y(n-4) + x(n) - 2x(n-1) - x(n-2) + 2x(n-3)$$



Direct form II realization

$$x(n) \longrightarrow H_1(z) \xrightarrow{v(n)} H_2(z) \longrightarrow y(n)$$

We can write

$$V(z) = X(z)\,H_1(z)$$

$$V(z) = X(z)\left(\frac{1}{1 - z^{-1} + \frac{9}{16}z^{-2} - \frac{1}{16}z^{-3} + \frac{1}{32}z^{-4}}\right)$$

$$V(z) - z^{-1}V(z) + \frac{9}{16}z^{-2}V(z) - \frac{1}{16}z^{-3}V(z) + \frac{1}{32}z^{-4}V(z) = X(z)$$

Taking inverse z transform on both side.

$$v(n) = v(n-1) - \frac{9}{16}v(n-2) + \frac{1}{16}v(n-3) - \frac{1}{32}v(n-4) + x(n)$$

Again $Y(z) = V(z)\,H_2(z)$

$Y(z) = V(z)\left(1 - 2z^{-1} - z^{-2} + 2z^{-3}\right)$

Taking inverse z transform on both side.

$y(n) = v(n) - 2v(n-1) - v(n-2) + 2v(n-3)$