MODULE -I

LINEAR PROGRAMMING PROBLEM [L.P.P]

Formulation: Convertion of descriptive type problem into mathematical expression is called Formulation.

Formulation to L.P.P.

L.P.P :- The mathematical expressions are linear in nature. We use some particular step to solve (i.e programming). It contain objective function, constraints and non-negativity restriction.

problem NO.1: A marketing manager wishes to allocate his ~~annual~~ advertising budget of Rs. 2000 in two media A and B. The unit cost of message in media A is Rs 100 and in B is Rs.150. Media A is monthly magazine and not more than one insertion is desired in one issue. At least 5 messages should appear in media B. The expected effective audience for unit message for media A is 4000 and for media B is 5000. Formulate as L.P.P.

Solution:- [The problem is in descriptive type. To convert this into ~~expr~~ mathematical expressions, is called Formulation]

Read the problem carefully and identify how many variables are there in the problem.

Let x_1 be the number of messages in media A

x_2 be the number of messages in media B.

The object is to maximize effective audience for unit message. Thus the objective function is

$$\text{maximize } Z = 4000x_1 + 5000x_2 \text{ [objective function]}$$

Constraints

Subject to $100x_1 + 150x_2 \leq 2000$ (Budget Constraint)

$x_1 \leq 1$ (Issue constraint)

$x_2 \geq 5$ (Message constraint)

& $x_1, x_2 \geq 0$ [non-negativity restriction]

2. The manager of an oil refinery has to decide upon the optimal mix of two possible blending processes of which the inputs and outputs per production run are as follows;

Input		
process	Crude A	Crude B
1	5	3
2	4	5

Output	
Gasoline X	Gasoline Y
5	8
4	4

The maximum amounts available of crude A and B are 200 and 150 units respectively. Market requirements show that atleast 100 units of gasoline X and 80 units of gasoline Y must be produced. The profits per production run from process 1 and process 2 are Rs. 3 and Rs. 4 respectively. Formulate the problem as a L.P. model.

Solution:- Let x_1 be the number of production run of process 1

x_2 be the number of production run of process 2

Since the profit is involved, it should be maximized. Thus the objective function is

Maximize $Z = 3x_1 + 4x_2$

Subject to

$5x_1 + 4x_2 \leq 200$ (crude A constraint)

$3x_1 + 5x_2 \leq 150$ (crude B constraint)

$5x_1 + 4x_2 \geq 100$ (Gasoline X constraint)

$8x_1 + 4x_2 \geq 80$ (Gasoline Y constraint)

[Hint; atleast means $\geq$
atmost means $\leq$]

$x_1, x_2 \geq 0$

↳ [non-negativity restriction]

3. A farmer has to plant two kinds of trees A and B in a land of 4400 m² area. Each A tree requires atleast 25m² and B tree atleast 40 m² of Land. The annual water requirement of A is 30 units and B is 15 units per tree, while atmost 3300 units of water is available. It is also estimated that the ratio of number of B trees to the number of A trees should not be less than 6/19 and not more than 17/8. The return per tree from A tree is expected to be one and half times as much as from B tree. Formulate as L.P.P.

Solution:- Let x_1 be the number of A trees to be planted

x_2 be the number of B trees to be planted

If the return from B tree is one unit then that from A tree is 1.5 unit. The objective function is to

Maximize $Z = 1.5x_1 + x_2$

Subject to

$25x_1 + 40x_2 \leq 4400$ (Land constraint)

$30x_1 + 15x_2 \leq 3300$ (water constraint)

$\frac{x_2}{x_1} \geq \frac{6}{19}$ (proportion constraint)

$\frac{x_2}{x_1} \leq \frac{17}{8}$ (proportion constraint)

and $x_1, x_2 \geq 0$ [non-negativity constraint]

Hint

$\frac{x_2}{x_1} \geq 6/19 \rightarrow [6x_1 - 19x_2 \geq 0]$

or $19x_2 \geq 6x_1$

$-6x_1 - 19x_2 \leq 0$

$\frac{x_2}{x_1} \leq 17/8$ $17x_1 - 8x_2 \geq 0$

4. A dairy feed company may purchase and mix one or more of the three types of grains containing different amounts of nutritional elements. The data are given below. The production manager specifies that any feed mix for his livestock must meet atleast minimal ~~[illegible]~~ nutritional requirements and seeks the least costly among all such mixes.

	Item	One Unit Weight of			Minimal requirement
		Grain I	Grain II	Grain III	
Nutritional ingredients	A	2	3	7	1250
	B	1	1	0	250
	C	5	3	0	900
	D	6	25	1	1232.5
Cost Rs / Unit Weight		41	35	96	

Formulate LPP model.

Solution :- Let x_1 be the weight of grain I in unit weight of the mix

x_2 be the weight of grain II in unit weight of the mix

x_3 be the weight of grain III in unit weight of the mix.

The objective function is to

$$\text{Minimize } Z = 41x_1 + 35x_2 + 96x_3$$

Subject to,

$$2x_1 + 3x_2 + 7x_2 \geq 1250 \text{ (Item A Constraint)}$$

$$x_1 + x_2 \geq 250 \text{ (Item B constraint)}$$

$$5x_1 + 3x_2 \geq 900 \text{ (Item C constraint)}$$

$$6x_1 + 25x_2 + x_3 \geq 1232.5 \text{ (Item D. constraint)}$$

& $x_1, x_2, x_3 \geq 0$ [non-negativily constraints].

5. Old hens can be bought for Rs. 2 each and young ones cost Rs. 5 each. The old hens lay 3 eggs per week and young ones 5 eggs per week. Each egg is sold for 30 paise. The feeding cost per week for each hen is Rs. 1. If a person has only Rs. 80 to spend on the hens, how many of each kind should he buy to give a profit more than Rs. 6 per week assuming that he cannot house more than 20 hens? Formulate this as L.P.P (do not solve).

Solution:- Let x_1 be the number of old hens
x_2 be the number of young hens

Sales income from the eggs laid by old hens per week

= Number of eggs laid by each hen × number of hens × selling price of each egg

$= 3 \times x_1 \times 0.3 = 0.9x_1$ rupees

Feeding cost of old hens per week $= x_1$ rupees

Hence profit from old hens per week $= 0.9x_1 - x_1 = -0.1x_1$

|||ly profit from young hens per week $= 5 \times x_2 \times 0.3 - x_2$
$= 0.5x_2$

The objective function is

Maximize $Z = -0.1x_1 + 0.5x_2$

Subject to $2x_1 + 5x_2 \leq 80$ (Budget constraint)

$-0.1x_1 + 0.5x_2 \geq 6$ (profit constraint)

$x_1 + x_2 \leq 20$ (Housing constraint)

& $x_1, x_2 \geq 0$ [non-negativity restriction]

Hint:- [In this problem, the objective function is to maximize the profit. Hence profit constraint is not mandatory to write. Even if you leave, it not affect the problem]

6. A farmer has a 100 acre farm. He can sell all the tomatoes, lettuce and radishes he can grow. The price he can obtain is Rs.1 per kg for tomatoes, Rs.0.75 a head for lettuce and Rs 2 per kg for radishes. The average yield per acre is 2000 kg. of tomatoes, 3000 heads of lettuce and 1000 kg of radishes. Fertilizer is available at Rs 0.5 per kg. and the quantity required per acre is 100 kg each for tomatoes, and lettuce and 50 kg for radishes. Labour required for sowing, cultivating and harvesting per acre is 5 man-days for tomatoes and radishes and 6 man-days for lettuce. A total of 400 man-days of labour is available at Rs 20 per man day. Formulate a LP model for this problem.

Solution:- Let x_1 be the acre of Land for growing tomatoes
x_2 be the acre of Land for growing lettuce
x_3 be the acre of Land for growing radishes

Sales income = product per acre × Sale price × acre

$$= 2000 \times 1 \times x_1 + 3000 \times 0.75 \times x_2 + 1000 \times 2 \times x_3$$

$$= 2000x_1 + 2250x_2 + 2000x_3$$

Total cost of fertilizer = price of fertilizer per kg × quantity of fertilizer per acre × acre

$$= 0.5\left[100(x_1+x_2) + 50x_3\right]$$

$$= 50x_1 + 50x_2 + 25x_3$$

Total cost of Labour = cost per man day × man-days per acre × acre

$$= 20(5x_1 + 6x_2 + 5x_3)$$

$$= 100x_1 + 120x_2 + 100x_3$$

profit = Sales income − Total cost of fertilizer − Total cost of Labour

$= 2000x_1 + 2250x_2 + 2000x_1 - (50x_1 + 50x_2 + 25x_3)$

$- (100x_1 + 120x_2 + 100x_3)$

$= 1850x_1 + 2080x_2 + 1875x_3$

Subject to $x_1 + x_2 + x_3 \leq 100$ (Land Constraint)

$5x_1 + 6x_2 + 5x_3 \leq 400$ (Labour Constraint)

& $x_1, x_2, x_3 \geq 0$ [non-negativity restriction]

7. A firm manufactures two components A and B. It purchases the casting for the components and then processes the castings through machining, boring and polishing. The casting for the components A and B cost Rs. 20 and Rs. 30 per component respectively. The finished components are sold at Rs. 50 and Rs. 60 per component respectively. The running cost of the processes and the machine capacities for only one type of component are given below:

process	process Capacity		Running cost Per hour
	for only component A	for only component B	
Machining	25 components/hr	40 components/hr	Rs. 200
Boring	28 components/hr	35 components/hr	Rs. 140
polishing	35 components/hr	25 components/hr	Rs. 175

Formulate the above problem as a L.P.P.

Solution:- Let x_1 be the number of components A

x_2 be the number of components B

Sales income = Sales price × number of components

$= 50x_1 + 60x_2$

Total cost = cost of casting + cost of processing cost of processing a component for particular process

$$= \frac{\text{Running cost per hour}}{\text{Components per hour undergoing particular processing}}$$

$$\text{Total cost} = \left[20x_1 + \left[\frac{200}{25}x_1 + \frac{140}{28}x_1 + \frac{175}{35}x_1\right] + 30x_2 + \frac{200}{40}x_2 + \frac{140}{35}x_2 + \frac{175}{25}x_2\right]$$

[Hint :- $200 \text{ cost}/\text{hr} \times \frac{1}{25} \text{ hr}/\text{compont} = 8 \text{ cost}/\text{component}$]

Total cost = $38x_1 + 46x_2$

profit = Sales income - Total cost

Hence the objective function is

$$\text{Maximize } Z = 50x_1 + 60x_2 - (38x_1 + 46x_2)$$

$$= 12x_1 + 14x_2$$

Constraints are on the process capacity for each hours:

Hence $\frac{1}{25}x_1 + \frac{1}{40}x_2 \leq 1$ or $8x_1 + 5x_2 \leq 200$

(Machining constraint)

$\frac{1}{28}x_1 + \frac{1}{35}x_2 \leq 1$ or $5x_1 + 4x_2 \leq 140$

(Boring constraint)

$\frac{1}{35\cancel{0}}x_1 + \frac{1}{25}x_2 \leq 1$ or $5x_1 + 7x_2 \leq 175$

(polishing constraints)

and $x_1, x_2 \geq 0$ [non-negativity restriction]

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8. A firm manufactures two products A and B. It can sell an unlimited amount of each product and wishes to determine the optimal mix for maximum profit. Resources : Raw material $x = 1600$ kg, $y = 750$ kg. Equipment time = 60 hours, Labour time = 150 hours.

Bill of Materials	product A	product B
Materials : x (kg)	2	–
y (kg)	0.5	0.5
Equipment time (hour) ;	0.06	0.04
Labour time (hour) :	0.10	0.15
Cost and price information ;		
Selling price (Rs)	6.0	5.0
Cost of materials Rs/Kg : x	1.0	–
: y	0.5	0.5
Equipment cost Rs/hour	3.0	2.0
Labour cost Rs/hour.	0.40	0.60

Formulate LPP !

Solution: The objective is to determine the optional product mix for maximum profit.

$$\therefore \text{Maximize } Z = \text{profit} = 6x_1 + 5x_2 - 2\times1\times x_1$$

$$- (0.5\times0.5\, x_1 + 0.5\times0.5\, x_2)$$

$$- (0.06\times x_1\times3 + 0.04\, x_2\times2)$$

$$- (0.1\, x_1\times0.4 + 0.15\, x_2\times0.6)$$

[profit = Selling price − Material cost − Equipment cost − Labour cost]

$$\text{Max } Z = \text{profit} = 3.53x_1 + 4.58x_2$$

Subjected to

$2x_1 \leq 1600$ (Raw material Constraints)

$0.5x_1 + 0.5x_2 \leq 750$ (Raw material y)

$0.06x_1 + 0.04x_2 \leq 60$ (Equipment constraints)

$0.1x_1 + 0.15x_2 \leq 150$ (Labour constraints)

& $x_1, x_2 \geq 0$ [non-negativity restriction]

9. A transport company with Rs 40,00,000 to spend, is contemplating to purchase three types of vehicles. Vehicle A has 10 ton pay load and expected to average 35 km. per hour. It costs Rs. 80,000. Vehicle B has 20 ton pay load, expected to average 30 km-per hour. It costs 100000, vehicle C is modified form of B. It is having provision for sleeping for one driver and its capacity is 18 tons and averages 28 km. per hour. A and B with one driver can run 12 hours per day, C requires two drivers and run 20 hours a day. Company has one hundred drivers available. Maintenance facilities restrict the total vehicle to 30. Formulate this as L.P.P to maximise ton-km per. day.

Soln: - Here the objective is to maximize ton-km per. day.

let x_1 be the number of vehicle of type A
x_2 be the number of vehicle of type B
x_3 be the number of vehicle of type C.

Ton-km/day = Ton pay load × average km/hour × number of running per day × no: of vehicles

For type A vehicle, ton-km/day $= 10 \times 35 \times 12 \times x_1 = 4200x_1$

For Type B vehicle ton-km/day $= 20 \times 30 \times 12 \times x_2 = 7200x_2$

For type C vehicle ton-km/day $= 18 \times 28 \times 20 \times x_3 = 10080x_3$

Maximize $Z = 4200x_1 + 7200x_2 + 10080x_3$

Subject to $80{,}000x_1 + 100000x_2 + 100000x_3 \leq 40{,}00000$ [Finance constraint]

$x_1 + x_2 + 2x_3 \leq 100$ (Driver constraint)

$x_1 + x_2 + x_3 \leq 300$ (Maintenance constraint)

& $x_1, x_2, x_3 \geq 0$ [non-negativity restriction]

10. A farmer owns 200 pigs that consume 90 kg. of Special feed daily. The feed is prepared as a mixture of corn and Soybean meal with the following compositions.

Kg per kg of feed stuff

Feed stuff	Calcium	protein	Fiber	Cost Rs / kg
Corn	0.001	0.09	0.02	0.2
Soybean meal	0.002	0.6	0.06	0.6

The dietary requirement of the pigs are as follows:
(i) At most 1% calcium (ii) Atleast 30% protein
(iii) Atmost 5% Fiber Formulate the problem as L.P.P.

Soln':- There are two kinds feed stuff corn and Soybean meal. The quantity of each in the feed mix is to be determined. Hence the decision variables Corresponds to feed stuff.

Let x_1 = kg of corn
x_2 = kg of Soybean meal.

The objective function Z will be the sum of cost of each feed stuff and the cost is naturally should be minimized. Hence Z may be defined as

Minimize $Z = 0.2x_1 + 0.6x_2$

There are certain constraints Subject to which Z is to be minimized. For example the daily minimum feed mix required is 90 kg. This may be stated by the inequality $x_1 + x_2 \geq 90$

The maximum percentage of calcium is restricted to 1, Hence in 90kg, the Calcium will be $\frac{1}{100} \times 90 = 0.9$ kg

In one kg of mix, the Calcium due to corn is 0.001 kg and due to Soy bean meal is 0.002 kg.

Hence this constraint may be stated as

$$0.001x_1 + 0.002x_2 \leq 0.9$$

[At most 1% Calcium means ≤]

$|||^{ly}$ the constraint for protein and fiber content may be stated as

$$0.09x_1 + 0.6x_2 \geq \frac{30}{100} \times 90$$

[Atleast 30% protein ie ≥]

protein $= 0.09x_1 + 0.6x_2 \geq 27$

Fiber $0.02x_1 + 0.06x_2 \leq \frac{5}{100} \times 90$ [At most 5% fiber]

ie $= 0.02x_1 + 0.06x_2 \leq 4.5$.

The variables should be non-negative. These constraints may be stated as $x_1 \geq 0$, $x_2 \geq 0$.

Thus the mathematical model of the problem may be stated as follows:

$$\text{Minimize } Z = 0.2x_1 + 0.6x_2$$

$$\text{Subject to } \quad x_1 + x_2 \geq 90$$

$$0.001x_1 + 0.002x_2 \leq 0.9$$

$$0.09x_1 + 0.6x_2 \geq 27$$

$$0.02x_1 + 0.06x_2 \leq 4.5$$

$$\& \quad x_1, x_2 \geq 0.$$

Graphical Solution to two variables L.P.P.

A two variable problem can be solved by graphical method. This method is impractical or impossible for more than two variables.

In this method, a solution space also called Feasible region, Satisfying all the constraints Simultaneously, is determined. The non-negativity Constraints ($x_1, x_2 \geq 0$) confine all feasible values to the first quadrant.

This quadrant is defined by the space above the horizontal reference axis x_1 and to the right of the The space enclosed by the vertical reference axis x_2. The space enclosed by the remaining constraints is determined by first replacing $\leq$ or $\geq$ by $=$ for each constraint. thus yielding a straight line equation. Each Straight line is then plotted on the (x_1, x_2) plane. The region in which each constraint holds when the inequality is actuated is indicated by the direction of the arrow on the associated Straight line.

Each point within or on the boundary of the Solution Space represents a feasible point. The optimum Solution is determined by Observing the direction in which the objective function increases or decreases (ie, Max Z or Min Z)

Plot the objective line passing through the origin.

Move this line as far away from the origin as possible and yet within or touching the boundary of the solution space. The optimum solution occurs at that point. The co-ordinate of the point gives the optimum values of x_1 and x_2.

* [Important Note]:· Clearly read the above procedure before solving graphical method. Use graph sheet. Indication of arrows for the constraints [$\geq$ or $\leq$] are vely important in defining Feasible region]

Example: 1. Solve graphically the following L.P.P.

Maximize $Z = 3x_1 + 2x_2$

Subject to $x_1 + 2x_2 \leq 6$ ------(1)

$2x_1 + x_2 \leq 8$ -----(2)

$x_2 - x_1 \leq 1$ ------(3)

$x_2 \leq 2$ -------(4)

$x_1, x_2 \geq 0$

Solution:- Step 1:· Convert inequality into equality

Take (1) constraint $x_1 + 2x_2 = 6$

When $x_1 = 0$, $x_2 = 3$
when $x_2 = 0$, $x_1 = 6$
} → represent these values on the graph to get a St. Line]

|||^{ly} (2) constraint $2x_1 + x_2 = 8$

When $x_1 = 0$, $x_2 = 8$
When $x_2 = 0$, $x_1 = 4$
} draw st. line on the graph.

(3) constraint $x_2 - x_1 \leq 1$

when $x_1 = 0$, $x_2 = 1$
when $x_2 = 0$, $x_1 = -1$
] represent st. line on the graph

(4) constraint $x_2 = 2$ → it represent a line.

Draw two axis horizontal and vertical.

Here $x_1, x_2 \geq 0$ means all feasible values confine to first quadrant.

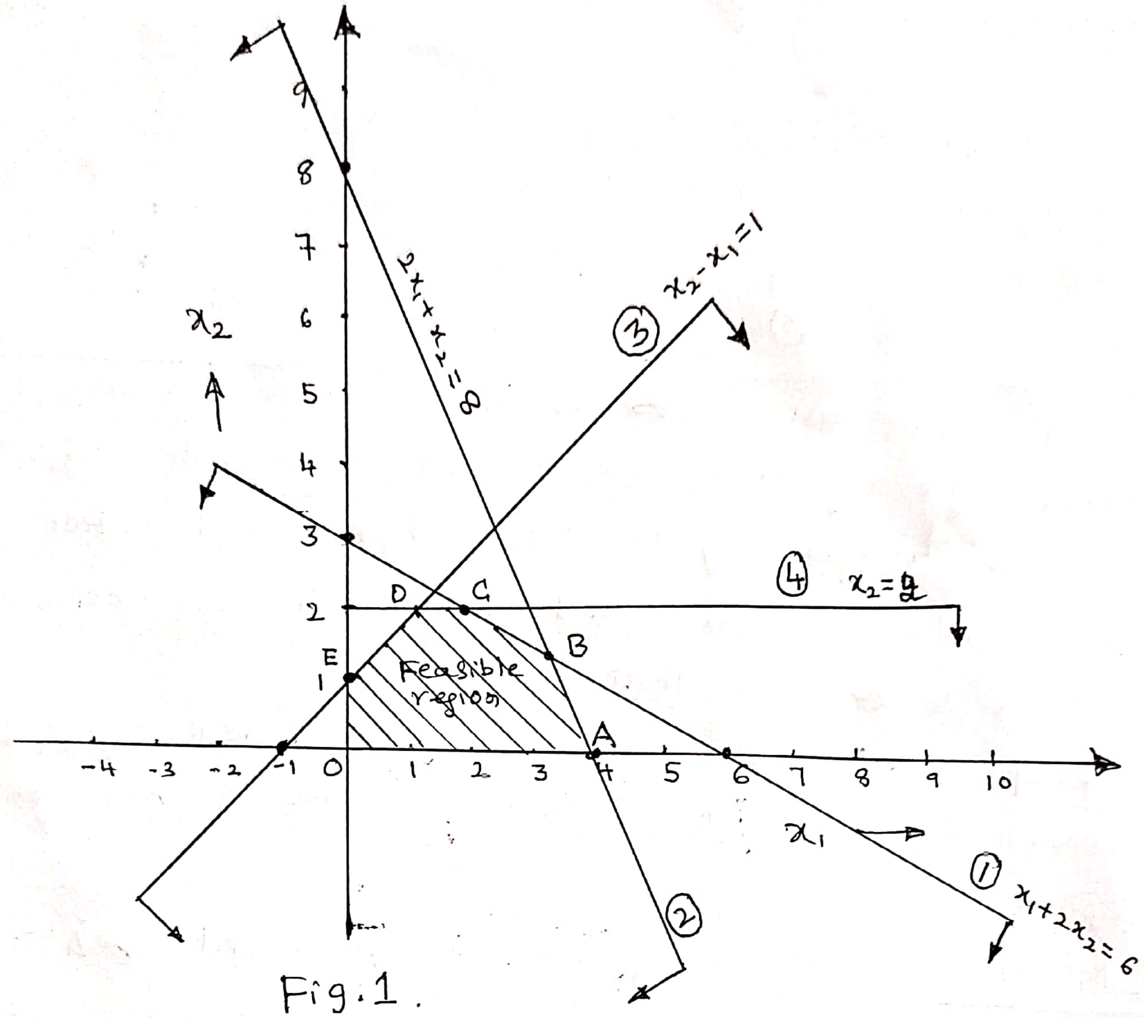


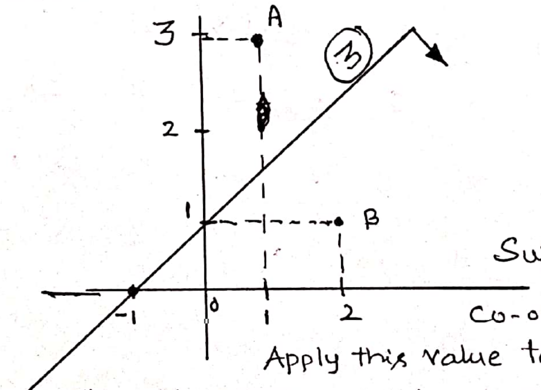
Fig. 1.

Note:- St. line indicate equality sign. ie $x_2 - x_1 = 1$

Arrows along with St. line indicate inequality sign.

For example

Indication of arrows for the constraint ③ $x_2 - x_1 \leq 1$



Verification:-

Take Co-ordinate points one above and another below the line ③

Suppose First point is above the line ③

Co-ordinate of A (1, 3)

Apply this value to $x_2 - x_1 = 3 - 1 = 2$

The value obtained is 2, But The constraint is $x_2 - x_1 \leq 1$, ie value is ≤ 1. If you

Show arrow above the line, the constraints becomes

$x_2 - x_1 \geq 1$.

Suppose take point B below the line ③

The co-ordinate points for B is (2, 1)

Apply this value to constraints $x_2 - x_1 \leq 1$

≤ 1

$1 - 2 \leq 1$

$-1 \leq 1$

ie $-1 \leq 1$, Hence it satisfy the condition $x_2 - x_1 \leq 1$.

∴ The direction of the arrow for the constraint $x_2 - x_1 \leq 1$ is below the line ③.

Fig. 1. Shows all the four constraints plotted as straight lines. The region in which each constraints holds when the inequality is actuated by the direction of the arrow on the associated st. line. The solution space or Feasible region is thus determined.

The optimum solution can be identified with one of the feasible corner points A, B, C, D and E of the Feasible region.

<u>I METHOD</u> :- Considering co-ordinate points of O, A, B, C D and E, which co-ordinate point gives maximum value of Z, that is the optional solution.

At O (origin) $x_1 = 0, x_2 = 0$ $Z = 3x_1 + 2x_2 = 0$

At A (4, 0) $x_1 = 4, x_2 = 0$ $Z = 3 \times 4 + 2 \times 0 = 12$

At B (3.3, 1.3) $x_1 = 3.3$ $x_2 = 1.3$ $Z = 3 \times 3.3 + 2 \times 1.3 = \underline{12.67}$

At C (2, 2) $x_1 = 2, x_2 = 2,$ $Z = 3 \times 2 + 2 \times 2 = 10$

At D (1, 2) $x_1 = 1, x_2 = 2$ $Z = 3 \times 1 + 2 \times 2 = 7$

At E (0, 1) $x_1 = 0, x_2 = 1,$ $Z = 3 \times 0 + 2 \times 1 = 2$

The maximum value of Z occurs at B (3.3, 1.3), that is the optimum solution. Max Z = 12.67.

Redundant constraints

These constraints do not bind the solution space. The solution space is not affected even if these constraints are imposed.

2. Determine the Solution Space graphically for the following inequalities.

$$x_1 + x_2 \leq 4 \longrightarrow ①$$

$$4x_1 + 3x_2 \leq 12 \longrightarrow ②$$

$$-x_1 + x_2 \geq 1 \longrightarrow ③$$

$$x_1 + x_2 \leq 6 \longrightarrow ④$$

$$x_1, x_2 \geq 0$$

Which constraint are redundant?

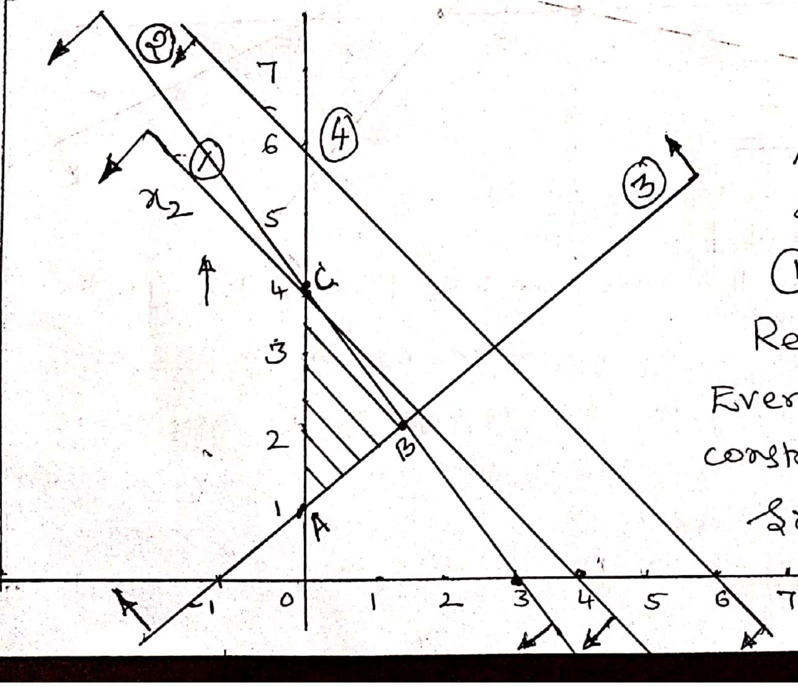
Sol^n:- Convert inequality into equality.

Constraint ① $x_1 + x_2 = 4$, when $x_1 = 0, x_2 = 4$; $x_2 = 0, x_1 = 4$

Constraint ② $4x_1 + 3x_2 = 12$ when $x_1 = 0, x_2 = 4$; $x_2 = 0$ $x_1 = 3$

Constraint ③ $-x_1 + x_2 \geqq 1$ when $x_1 = 0, x_2 = 1$; $x_2 = 0, x_1 = -1$

Constraint ④ $x_1 + x_2 = 6$, when $x_1 = 0, x_2 = 6$; $x_2 = 0, x_1 = 6$



By seeing the graph ABC is the solution space. The constraints ① and ④ are called Redundant constraints. Even if you remove these constraints it not affect the solution space.

3. Find the optimal solution to the formulation problem of example. 5.

Maximize $Z = -0.1x_1 + 0.5x_2$

Subject to $2x_1 + 5x_2 \le 80 \longrightarrow ①$

$-0.1x_1 + 0.5x_2 \ge 6 \longrightarrow ②$

$x_1 + x_2 \le 20 \longrightarrow ③$

$x_1, x_2 \ge 0$

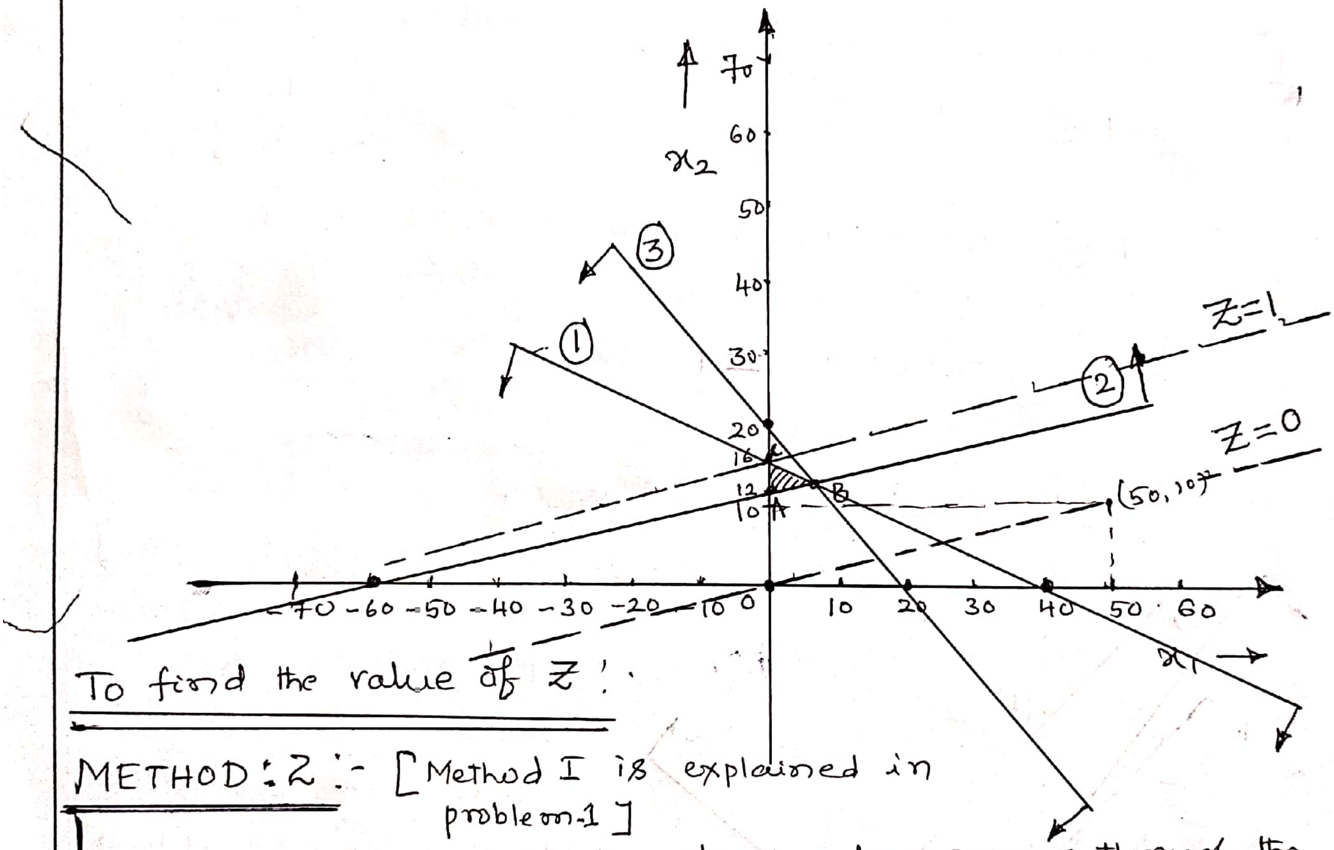
Soln:-

Convert inequality into equality.

Constraint ① $2x_1 + 5x_2 = 80$ when $\left.\begin{matrix} x_1 = 0, & x_2 = 16 \\ x_2 = 0 & x_1 = 40 \end{matrix}\right\}$

② $-0.1x_1 + 0.5x_2 = 6$ when $\left.\begin{matrix} x_1 = 0, & x_2 = 12 \\ x_2 = 0 & x_1 = -60 \end{matrix}\right\}$

③ $x_1 + x_2 = 20$ when $\left.\begin{matrix} x_1 = 0, & x_2 = 20 \\ x_2 = 0, & x_1 = 20 \end{matrix}\right\}$



To find the value of Z :

METHOD : 2 :- [Method I is explained in problem-1]

put Z = 0, ie plot the objective line passing through the origin. More this line parallel as far away from the origin as possible and yet within or touching the boundary of the solution space. [ie Draw iso-lines passing through the extreme corner of the feasible region. For Max.Z, iso-line is far away from the origin. [iso-line means parallel line to Z=0]

The isoline $Z = 1$ passes through the extreme corner (0, 16) and is the required point.

[Note: Donot consider the iso-line passing inside the Feasible region, it only pass through the extreme corner of the feasible region]

put $Z = 0 = -0.1x_1 + 0.5x_2$

$0.1x_1 = 0.5x_2$

$\frac{x_1}{x_2} = \frac{0.5}{0.1} = \frac{50}{10}$

At C (0, 16) $\quad \text{Max } Z = -0.1x_1 + 0.5x_2$

$\text{max } Z = -0.1 \times 0 + 0.5 \times 16$

$\underline{\text{Max } Z = 8}$: This value justify the formulation problem (ie old hen and young hen problem) i.e profit is more than 6]

Alternate optimum Solution

4. Determine graphical Solution for the following LPP.

$\text{Maximize } Z = 5x_2 - x_1$

Subject to $\quad 2x_1 - x_2 \geq -2$ or $-2x_1 + x_2 \leq 2$ Also $\rightarrow$ ①

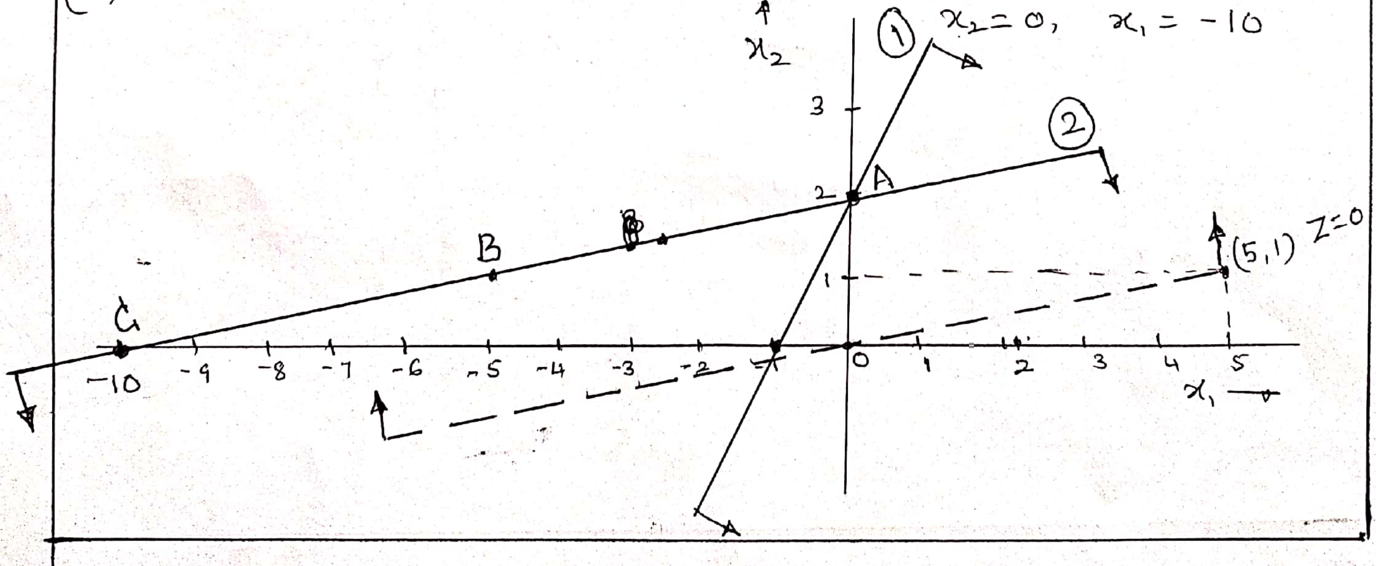
$-0.2x_1 + x_2 \leq 2 \rightarrow$ ②

and $x_1, x_2 \geq 0$

Solution: Convert inequality into equality.

(1) Constraint $\quad -2x_1 + x_2 = 2$, $\quad$ when $x_1 = 0$, $x_2 = 2$; $x_2 = 0$, $x_1 = -1$

(2) Constraint $\quad -0.2x_1 + x_2 = 2$ $\quad$ when $x_1 = 0$, $x_2 = 2$; $x_2 = 0$, $x_1 = -10$



put $Z = 0 = 5x_2 - x_1$

$$5x_2 = x_1$$

$$\frac{5}{1} = \frac{x_1}{x_2}$$

The objective function $Z=0$ is parallel to the Constraint ②. Hence all points on the lin 2 represent optimum solution. For example

at A $(0,2)$ $Z_{max} = 5x_2 - x_1 = 10$

at B $(-5,1)$ $Z_{max} = 5\times1-(-5) = 10$

at C $(-10,0)$ $Z_{max} = 5\times0-(-10) = 10$

Such solutions are called alternative optima. Even if the values of x_1 and x_2 are arbitrarily large, the value of objective function will be same.

Infeasible Solution:-

5. Solve graphically the following LPP.

$$\text{Max } Z = x_1 + 0.5x_2$$

Subject to $3x_1 + 2x_2 \le 12 \longrightarrow$ ①

$5x_1 \le 10 \longrightarrow$ ②

$x_1 + x_2 \ge 8 \longrightarrow$ ③

$-x_1 + x_2 \ge 4 \longrightarrow$ ④

Soln:- & $x_1, x_2 \ge 0$

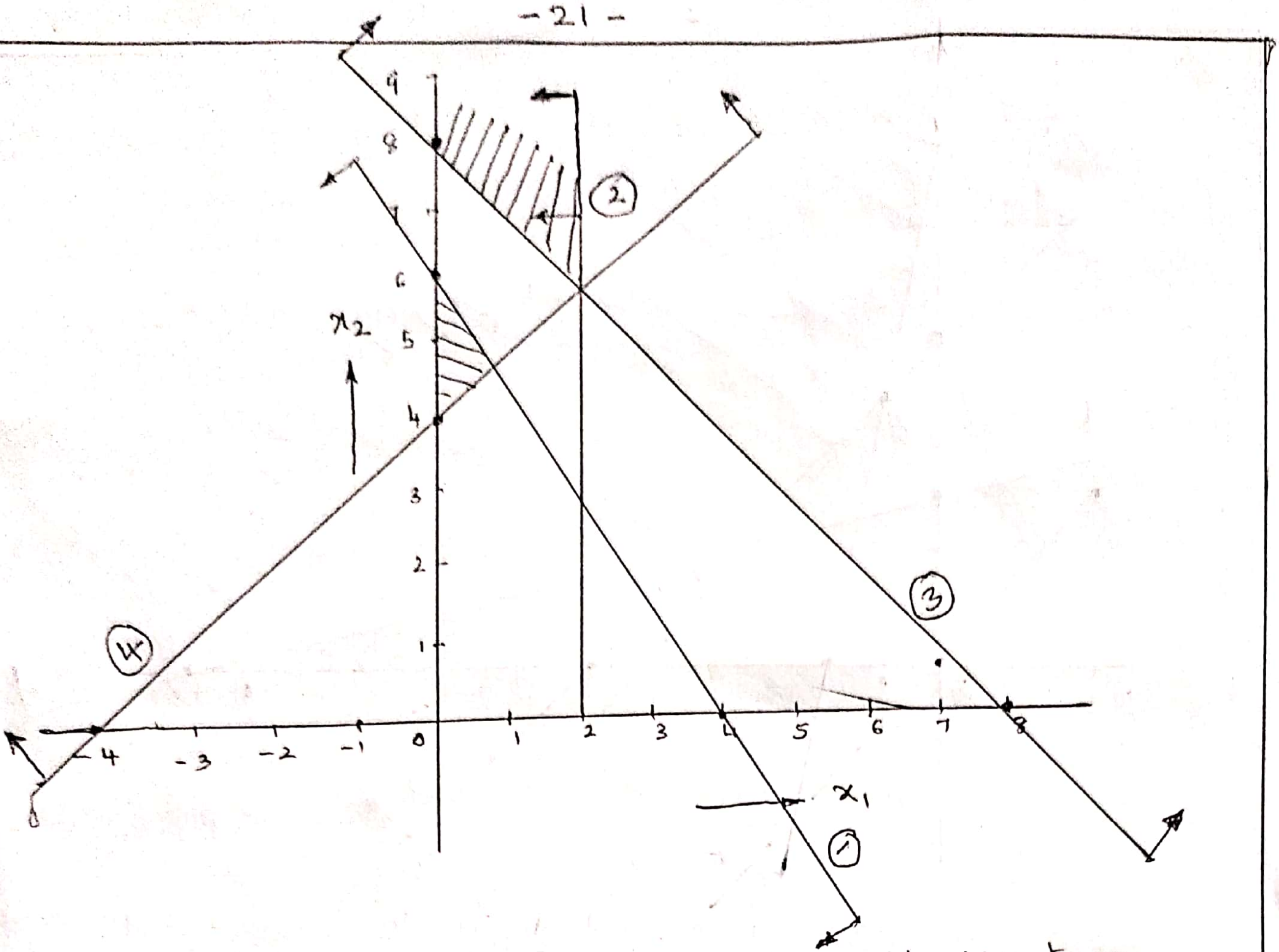
Convert inequality into equality.

① $3x_1 + 2x_2 = 12$ when $\left.\begin{array}{l} x_1=0, \ x_2=6 \\ x_2=0 \ \ x_1=4 \end{array}\right\}$

② $5x_1 = 10$ $\therefore$ $x_1 = 2\}$

③ $x_1 + x_2 = 8$ when $\left.\begin{array}{l} x_1=0, \ x_2=8 \\ x_2=0, \ x_1=8 \end{array}\right\}$

④ $-x_1 + x_2 = 4$ when $\left.\begin{array}{l} x_1=0, \ x_2=4 \\ x_2=0 \ \ x_1=-4 \end{array}\right\}$



The two shaded areas shown above indicate non-overlapping regions that can be considered as feasible solutions areas in the sense that they satisfy some subsets of the constraints.

There is no point lying in both the solution space. The problem cannot be solved by graphical or any other method. The problem has infeasible solution.

6. Use graphical method to solve the following L.P.P.

Maximize $Z = 3x_1 + 2x_2$

Subject to $5x_1 + x_2 \geq 10 \rightarrow ①$

$x_1 + x_2 \geq 6 \rightarrow ②$

$x_1 + 4x_2 \geq 12 \rightarrow ③$

& $x_1, x_2 \geq 0$

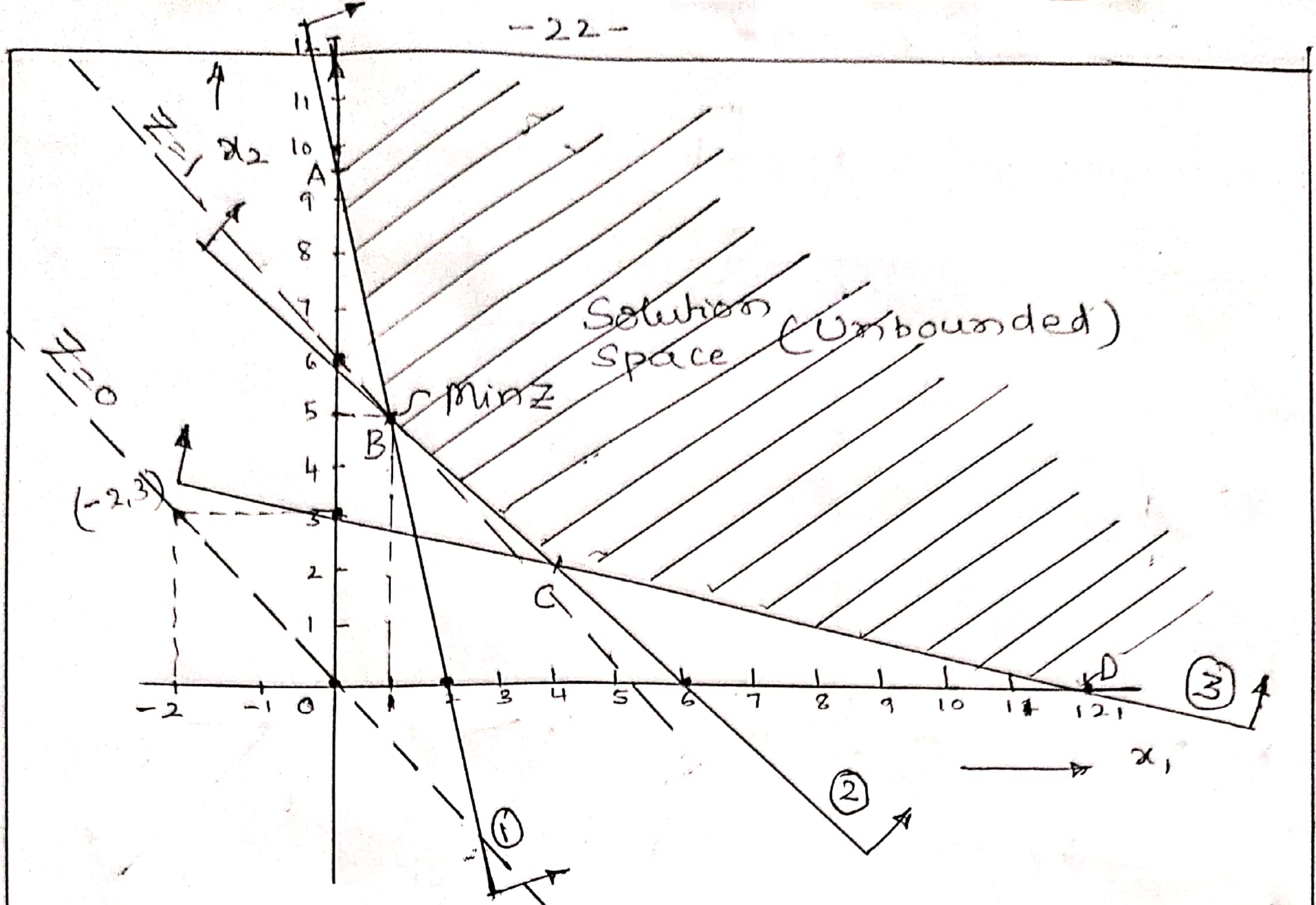
UNBOUNDED SOLUTION

Soln:- Convert inequality into equality.

① $5x_1 + x_2 = 10$
when $x_1 = 0, \quad x_2 = 10$
$x_2 = 0 \quad x_1 = 2$

② $x_1 + x_2 = 6$
when $x_1 = 0, x_2 = 6$
$x_2 = 0, x_1 = 6$

③ $x_1 + 4x_2 = 12$
when $x_1 = 0, \quad x_2 = 3$
$x_2 = 0 \quad x_1 = 12$



The solution space is bounded on the lower side, but not on the upper side as shown above. The objective line (i.e Z is maximum) can be moved indefinitely. Hence there is no finite value of Z. The problem is said to have Unbounded solution but not infeasible solution

Note:- The solution may be found if the objective function is to be minimized. The point B definitely the optimum solution [ie Z is minimum]

put $Z = 0 = 3x_1 + 2x_2$

$3x_1 = -2x_2$ $\quad \frac{x_1}{x_2} = -\frac{2}{3}$

The Minimum value of Z occur at B [The extreme point (B) nearest to the origin. Z=1 line is parallel to Z=0]

The Co-ordinate points of B (1,5)

$\therefore \min Z = 3x_1 + 2x_2$

$= 3\times1 + 2\times5 = 13$

$\boxed{\text{Min } Z = 13}$

7. Solve the following LPP graphically

Maximize $Z = 6x_1 - 2x_2$

Subject to $2x_1 - x_2 \leq 2$

$x_1 \leq 3$

$x_1, x_2 \geq 0$

ANS:- Solution space is unbounded. the finite maximum exists. Thus an unbounded solution space does not always means an unbounded solution.

$Z_{max} = 10$ at co-ordinate point $(3,4)$.

8. Determine graphically the minimum and maximum value of the objective function

$Z = 4x_1 + 5x_2$

Subject to $2x_1 + x_2 \leq 6$

$x_1 + 2x_2 \leq 5$

$x_1 - 2x_2 \leq 2$

$-x_1 + x_2 \leq 2$

$x_1 + x_2 \geq 1$

& $x_1, x_2 \geq 0$

ANS:- The extreme point of the solution space nearest to the origin is the minimum value of Z. and the point farthest from the origin is the maximum value of Z.

$Z_{min} = 4$ at $(1,0)$

$Z_{max} = 16$ at $(2.66, 1.33)$

— x — x — x —

MODULE- TRANSPORTATION PROBLEM

If there are more than one centre called origins or sources from where the goods needs to be transported. to more than one place called destinations. cost of Shipping or cost of transportation from each of the origin to each of the destination being different and known. The ~~[illegible]~~ problem is to transport the goods from varies origin to the different destination in such a way that the cost of transportation is minimum.

Tabular Form :-

O \ D	Destination D_1	D_2	D_3	……	D_j	……	D_n	Supply or Capacilg
O_1	c_{11}	c_{12}	c_{13}	----	c_{1j}	----	c_{1n}	a_1
O_2	c_{21}	c_{22}	c_{23}	----	c_{2j}	----	c_{2n}	a_2
Origins O_3	c_{31}	c_{32}	c_{33}	----	c_{3j}	----	c_{3n}	a_3
⋮	⋮	⋮	⋮		⋮		⋮	⋮
O_i	c_{i1}	c_{i2}	c_{i3}	----	c_{ij}	----	c_{ij}	a_j
⋮	⋮	⋮	⋮					⋮
O_m	c_{m1}	c_{m2}	c_{m3}	----	c_{mj}	----	c_{mj}	a_m
Demand OR Requirement	b_1	b_2	b_3	……	b_j	……	b_n	$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$

There are 'm' origins $O_1, O_2 \ldots O_m$ and n destinations $D_1, D_2 \ldots D_n$. Let $a_1, a_2, a_3 \ldots a_m$ be the quantity of goods available at the origins, $O_1, O_2, O_3 \ldots O_m$ and $b_1, b_2, b_3 \ldots b_n$ be the quantity of goods received at the destinations $D_1, D_2, \ldots D_n$.

Let C_{ij} be the cost of transporting one unit from i^{th} origin to j^{th} destination, the objective is to determine the quantity x_{ij}, so that the total transportation cost is minimum

Mathematically the problem can be stated as to find x_{ij} to minimize the transportion cost.

$$Min\ Z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij}$$

Subject to the constraints

$$\sum_{j=1}^{n} x_{ij} = a_i \quad \text{where } a_i = 1, 2, 3 \ldots m$$

(Row sum)

$$\sum_{i=1}^{m} x_{ij} = b_j \quad j = 1, 2, 3 \ldots n$$

(column Sum)

& $x_{ij} \geq 0$ for all i and j.

The given transportation problem is said to be balanced

$$\text{if } \sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$$

i.e, if the total supply is equal to the total demand.

DEFINITIONS:

Feasible Solution:- Any set of non-negative individual allocations (ie $x_{ij} \geq 0$) which satisfies the row and column sum is called a feasible solution.

BAsic Feasible Solution: A feasible solution to 'm' origins to 'n' destinations are said to be basic, if the no: of +ve allocations are $(m+n-1)$ one less than the number of rows and column.

Non-degenerate Basic feasible solution

Any feasible solution to a transportation problem containing m origins and n destinations is said to be non-degenerative, if it contains $m+n-1$ occupied cells and each allocation is in independent positions.

The allocations are said to be in independent positions, if it is impossible to form a closed path. (i.e it should not form any loop).

Degenerate Basic feasible Solution:- If a basic feasible solution contains less than $(m+n-1)$ non negative allocations, it is said to be degenerate.

Optimal Solution: A feasible solution is said to be optimal, if it minimize the total transportation cost.

Initial Basic feasible solution:

The initial basic feasible solution can be obtained by any one of the following methods.

(i) North-West Corner rule (N-W C R)
(ii) Row-minima method
(iii) Column-minima method
(iv) Least cost method or Matrix-minima method
(v) Vogel's approximation method (VAM)

Obtain initial basic feasible solution for the following problems using the above methods.

i) North-West Corner method:

D O	D_1	D_2	D_3	D_4	Supply
O_1	$\boxed{20}$ 1	$\boxed{10}$ 2	1	4	30
O_2	4	$\boxed{30}$ 2	$\boxed{20}$ 5	9	50
O_3	20	40	$\boxed{10}$ 30	$\boxed{10}$ 10	20
Demand	20	40	30	10	100 100

IBFS = 20×1 + 10×2 + 30×2 + 20×5 + 10×30 + 10×10

= Rs. 600

Row-minima method (Identify minimum cost cell in each row and allocate the quantity)

O \ D	D_1	D_2	D_3	D_4	Supply
O_1	$\boxed{20}$ 1	2	$\boxed{10}$ 1	4	30
O_2	4	$\boxed{40}$ 2	$\boxed{10}$ 5	9	50
O_3	20	40	$\boxed{10}$ 30	$\boxed{10}$ 10	20
Demand	20	40	30	10	100 / 100

$m+n-1$

$3+4-1 = 06$

IBFS = $20\times1 + 10\times1 + 40\times2$
$+ 10\times5 + 10\times30 + 10\times10$

$=$ Rs. 560/-

Column-minima method: (Identify minimum cost cell in each column and allocate the quantity).

O \ D	D_1	D_2	D_3	D_4	Supply
O_1	$\boxed{20}$ 1	$\boxed{10}$ 2	1	4	30
O_2	4	$\boxed{30}$ 2	$\boxed{20}$ 5	9	50
O_3	20	40	$\boxed{10}$ 30	$\boxed{10}$ 10	20
Demand	20	40	30	10	100 / 100

IBFS = $20\times1 + 10\times2 + 30\times2 + 20\times5 + 10\times30 + 10\times10$

$=$ Rs. 600/-

Least cost method or Matrix Minima method

[Identify minimum cost cell in a matrix and allocate the quantity] Minimum cost is 1 in the matrix, either allocate quantity to cell (1,1) or (1,3)

Suppose cell (1,1)

Here no: of allocations obtained are (m+n-1) ie m=row n=columns

3+4-1 = ⑥

allocations obtained are ⑥. Hence it is a non-degenerate basic feasible sol^n.

O \ D	D_1	D_2	D_3	D_4	Supply
O_1	$\boxed{20}$ 1	2	$\boxed{10}$ 1	4	30
O_2	4	$\boxed{40}$ 2	$\boxed{10}$ 5	9	50
O_3	20	40	$\boxed{10}$ 30	$\boxed{10}$ 10	20
Demand	20	40	30	10	100 \ 100

IBFS = 20×1 + 10×1 + 40×2 + 10×5 + 10×30 + 10×10

= 20 + 10 + 80 + 50 + 300 + 100

IBFS = Rs 560/-

OR.

Suppose cell (1,3)

Here Allocations obtained are 05, which is less than (m+n-1) = ie 06 Hence it is a denegerate basic feasible soln.

O \ D	D_1	D_2	D_3	D_4	Supply
O_1	1	2	$\boxed{30}$ 1	4	30
O_2	$\boxed{10}$ 4	$\boxed{40}$ 2	5	9	50
O_3	$\boxed{10}$ 20	40	30	$\boxed{10}$ 10	20
Demand	20	40	30	10	100 \ 100

IBBS = 30×1 + 10×4 + 40×2 + 10×20 + 10×10
= Rs 450/-

(V) Vogel's Approximation method (unit-cost penalty method)

Soln: (VAM)

Step1:

[Find the penalty cost, namely the difference between the smallest and next smallest cost in each & column.]

O \ D	D1	D2	D3	D4	supply	penalties
O1	1	2	$\boxed{30}$ 1	4	30	(0) (0) (0)
O2	$\boxed{10}$ 4	$\boxed{40}$ 2	5	9	50	(2) (2) (2)
O3	$\boxed{10}$ 20	40	30	$\boxed{10}$ 10	20	(10) ~~(10)~~
Demand	20	40	30	10	100/100	
penalties	(3) (3) (3)	(0) (0) (0)	(4) (4) (4)	(5)		

IBFS = 30×1 + 10×4 + 40×2 + 10×20 + 10×10 = Rs 450.
and is degenerate Basic feasible solution.

Obtain an initial basic solution using North-West corner method and VAM.

(i) North-West corner method :

O \ D	A₁	B₁	C₁	D₁	E₁	Supply
O₁	$\boxed{3}$ 2	$\boxed{1}$ 11	10	3	7	4
O₂	1	$\boxed{2}$ 4	$\boxed{4}$ 7	$\boxed{2}$ 2	1	8
O₃	3	9	4	$\boxed{3}$ 8	$\boxed{6}$ 12	9
Demand	3	3	4	5	6	21 \ 21

IBFS = 3×2 + 1×11 + 2×4 + 4×7 + 2×2 + 3×8 + 6×12

= Rs 153/-

(ii) Vogel's Approximation method (VAM)

O \ D	A₁	B₁	C₁	D₁	E₁	Supply	Penalties
O₁	2	11	10	3	7	4	(1) (1) [illegible] (1)
O₂	1	$\boxed{3}$ 4	7	2	$\boxed{6}$ 1	8	~~(0) (1)~~ [illegible]
O₃	3	9	4	8	12	9	(1) (1) [illegible]
Demand	3	3	4	5	6	21 \ 21	

Penalties

(1) (5) (3) (1) (6)

(1) (5) (3) (1) *

(1) [illegible] (3) (1)

Vogel's Approximation method (VAM)

O \ D	A₁	B₁	C₁	D₁	E	Supply	Penalties
O₁	2	11	10	$\boxed{4}$ 3	7	4	(1) (1) (1) (1) (8)←
O₂	1	$\boxed{2}$ 4	7	2	$\boxed{6}$ 1	8	(0) (1) *
O₃	$\boxed{3}$ 3	$\boxed{1}$ 9	$\boxed{4}$ 4	$\boxed{1}$ 8	12	9	(1) (1) (1) (5)←(1)
Demand	3	3	4	5	6	21 / 21	

Penalties	(1)	(5)	(3)	(1)	(6)↑
	(1)	(5)↑	(3)	(1)	*
	(1)	(2)	(6)↑	(5)	
		(2)	*	(5)	
	(1)	(2)		(5)	

(See there is a tie between row 3 and column 4, Hence select the penalty which is having minimum cost cell)

IBFS = 4×3 + 2×4 + 6×1 + 3×3 + 1×9 + 4×4 + 1×8 = 12 + 8 + 6 + 9 + 9 + 16 + 8

IBFS = Rs 68/-

OPTIMALITY TEST by MODI METHOD OR (U-V method (MODI - Modified distribution)

- The optimality test can be performed on feasible solution, in which

(i) the no: of allocation is (m+n-1) where m - no: of rows and n - no. of columns.

(ii) These (m+n-1) allocations are in independent position. i.e it should not form any loop.

EXAMPLE:

0	1	2	3	4	Supply	Penalties
1	$\boxed{1}$ 2	$\boxed{5}$ 3	11	7	6	(1) (1)(5)←
2	1	0	6	$\boxed{1}$ 1	1	(1) *
3	$\boxed{6}$ 5	8	$\boxed{3}$ 15	$\boxed{1}$ 9	10	(3)(3)(4)
Demand	7	5	3	2	17 / 17	
Penalties	(1) (3) (3)	(3) (5)↑ *	(5) (4) (4)	(6)↑ (2) (2)		

IBFS = 1×2 + 5×3 + 1×1 + 6×5 + 3×15 + 1×9 = Rs 102/-

Now apply optimality test.

Here (i) No: of allocations obtained = 6

and No: of allocations required = $(m+n-1)$

$= 3+4-1$

$= 06$

and (ii) These allocations are in independent position. (i.e it should not form any loop).

Hence it is eligible for optimality test.

BY MODI METHOD

(i) Determine the set of arbitrary numbers u_i and v_j for all occupied cells,

[ie Find out a set of numbers u_i and v_j for each row and column satisfying $c_{ij} = u_i + v_j$ for each occupied cell. To start with, we assign a number '0' to any row (say $u_1 = 0$). and entering successively the values of u_i and v_j on the transportation problem.

(ii) Determine the net cell evaluation ie $c_{ij} - (u_i + v_j)$ for all unoccupied cells.

(iii) If all $c_{ij} - (u_i + v_j) \geq 0$ Then the solution is optimal, otherwise we are proceed to find leaving and entry variables.

Let $u_1=0$	$\boxed{1}$ 2	$\boxed{5}$ 3	11 -1	7 1
$u_2=-5$	1 4	0 2	6 -1	$\boxed{1}$ 1
$u_3=3$	$\boxed{6}$ 5	8 2	$\boxed{3}$ 15	$\boxed{1}$ 9
	(2) v_1	(3) v_2	(12) v_3	(6) v_4

Let $u_1=0$, for cell (1, 1) (occupied cell)

$$\underset{uv}{C_{11}} = u_1 + v_i$$

$$2 = 0 + V_1 \qquad \boxed{V_1 = 2}$$

$$C_{\cancel{1}2} = u_{\cancel{1}} + V_2$$

$$\boxed{3 = 0 + V_2} \qquad \boxed{V_2 = 3}$$

$$C_{24} = u_2 + V_4$$

$$\boxed{1 = u_2 + V_4}$$

$$C_{31} = u_3 + V_1$$

$$\boxed{5 = u_3 + 2} \qquad \boxed{u_3 = 3}$$

$$C_{33} = u_3 + v_3$$

$$15 = 3 + v_3 \qquad \boxed{v_3 = 12}$$

$$C_{34} = u_3 + v_4$$

$$9 = 3 + v_4 \qquad \boxed{v_4 = 6}$$

∴ $1 = u_2 + V_4 \longrightarrow 1 = u_2 + 6$ $\boxed{u_2 = -5}$

~~$3 = u_2 + V_2 \longrightarrow 3 = -5 + V_2$~~

Now determine $(c_{ij} - (u_i + v_j))$ for all unoccupied cells.

i.e $C_{13} - (u_1 + v_3) = 11 - (0 + 12) = \boxed{-1}$

$C_{14} - (u_1 + v_4) = 7 - (0 + 6) = 1$

$C_{21} - (u_2 + v_1) = 1 - (-5 + 2) = 4$

$C_{22} - (u_2 + v_2) = 0 - (-5 + 3) = 2$

$C_{23} - (u_2 + v_3) = 6 - (-5 + 12) = \boxed{-1}$

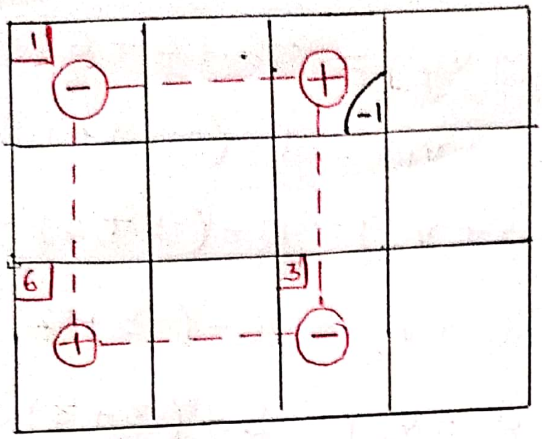
$C_{32} - (u_3 + v_2) = 8 - (3 + 3) = 2$

Here All $c_{ij} - (u_i + v_j) \ngeq 0$. Hence solution is not optimal. Then we have to proceed to find entry and leaving variables.

Here the two cells C_{13} and C_{23} having negative values. Select the minimum negative value but here two cells C_{13} and C_{23} have same negative value, [~~Choose assig cell, so that it~~ [illegible] ~~formed~~ a [illegible] ~~loop~~] Suppose choose the cell (C_{13}). From this cell we draw a closed path by drawing horizontal and vertical lines with the corner cells occupied. Assign sign + and − alternately and find the minimum allocation from the cell having negative sign. This allocation should be added to the allocation

$\boxed{1}$ 2	$\boxed{5}$ 3	11 -1	7 1
1 4	0 2	6 -1	$\boxed{1}$ 1
$\boxed{6}$ 5	8 2	$\boxed{3}$ 15	$\boxed{1}$ 9

having positive Sign and subtracted from the allocation having negative sign.



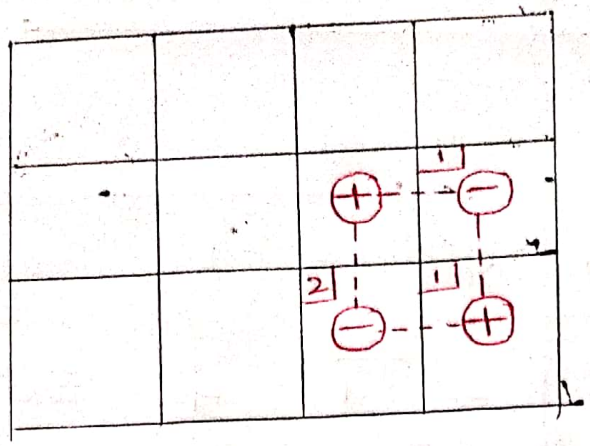
$\ominus$ min (1, 3) = 1 Now we get new basic & feasible solⁿ. Again apply MODI method (ie conduct optimality test

	v_1 (1)	v_2 (3)	v_3 (11)	v_4 (5)
$u_1 = 0$	2 1	$\boxed{5}$ 3	$\boxed{1}$ 11	7 2
u_2(−4)	1	0 1	6 -1	$\boxed{1}$ 1
u_3 =(4)	$\boxed{7}$ 5	8 1	$\boxed{2}$ 15	$\boxed{1}$ 9

(i) No: of allocation Obtained = No: of allocation required
06 = (m+n−1) = 06
(ii) and these allocations are in independent position.

Let $u_1=0$, and find remaining arbitrary numbers Determine $C_{ij} - (u_i+v_j)$ for all unoccupied cells.

Again in new matrix, The only cell (C_{23}) is negative, Hence solution is not optimal. Again find leaving & entry variables.



⊖ min (1, 2)
= 1

New basic feasible soln (New matrix becomes)

Apply optimality test. (i) No: of allocations obtained = No: of allocations required

06 = 06

& these allocations are in independent position.

$u_1(0)$	2 1	$\boxed{5}$ 3	$\boxed{1}$ 11	7 2
$u_2(-5)$	1 1	0 2	$\boxed{1}$ 6	1 1
$u_3(4)$	$\boxed{7}$ 5	8 1	$\boxed{1}$ 15	$\boxed{2}$ 9
	(1) v_1	(3) v_2	(11) v_3	(5) v_4

Determine $C_{ij} = u_i + v_j$ for all occupied cells

Determine $C_{ij} - (u_i + v_j)$ for all unoccupied cells

Here in the above cell All $c_{ij} - (u_i + v_j) \geq 0$.

Hence solution is optimal.

∴ Optimal solution = $5 \times 3 + 1 \times 11 + 1 \times 6 + 7 \times 5$
$+ 1 \times 15 + 2 \times 9$

Optimal solution = $15 + 11 + 6 + 35 + 15 + 18$ = Rs 100/-

IBFS = Rs 102, Hence the solution is improved.
ie Save Rs 2.

DEGENERACY IN TRANSPORTATION

Example:

D / O	D_1	D_2	D_3	D_4	Supply
O_1	1	2	$\boxed{30}$ 1	4	30
O_2	$\boxed{10}$ 4	$\boxed{40}$ 2	5	9	50
O_3 ~~Demand~~	$\boxed{10}$ 20	40	30	$\boxed{10}$ 10	20
Demand	20	40	30	10	100 / 100

(After Applying VAM, The allocations obtained as shown above)

$$\text{IBFS} = 30\times1 + 10\times4 + 40\times2 + 10\times20 + 10\times10$$

$$= \text{Rs } 450/-$$

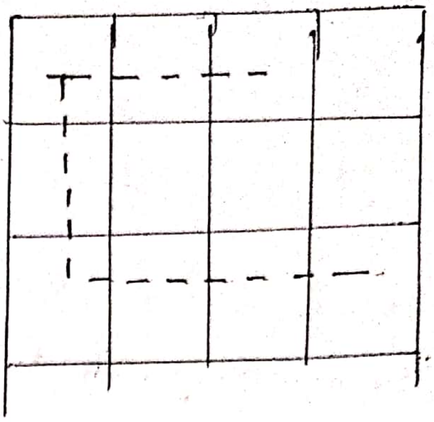
Optimality Test:-

(i) There should be $(m+n-1)$ allocations
i.e $(3+4-1) = \boxed{06}$.

Here Number of allocations obtained is 05
Hence the problem is degenerate transportation problem:

HOW TO RESOLVE IT: Allot (E) to a least cost cell, so that new allocation does not form a closed loop. Apply now optimality test.

Let $u_1=0$	$\boxed{\epsilon}$ 1	2 3	$\boxed{30}$ 1	4 13
$u_2=3$	$\boxed{10}$ 4	$\boxed{40}$ 2	5 1	9 15
$u_3=19$	$\boxed{10}$ 20	40 22	30 10	$\boxed{10}$ 10
	(1)	(-1)	(1)	(-9)
	v_1	v_2	v_3	v_4



No:- of allocation now obtained = 06

No! of allocation required $= (m+n-1)$

$= 3+4-1 = \underline{\underline{06}}$

and these allocations are in independent position.

Determine $c_{ij} = (u_i+v_j)$ for all occupied cells.

Determine $c_{ij} - (u_i+v_j)$ for all unoccupied cells.

All $c_{ij} - (u_i+v_j) \geq 0$ Hence soln is optimal

Optimal cost $= \epsilon \times 1 + 30 \times 1 + 10 \times 4 + 40 \times 2 + 10 \times 20 + 10 \times 10$.

Tending $\epsilon \to 0$ (ϵ is a very small quantity):

optimal cost = Rs 450/-

A company has 4 warehouses and 6 stores. The warehouses all together have 22 units of commodities, divided among themselves as follows:

Warehouse	Commodity
1	5
2	6
3	2
4	9
	22 units

The 6 stores together needs 22 units of this commodity. Individual requirements are

Store	Commodity
1	4
2	4
3	6
4	2
5	4
6	2
	22 units

The cost of shipping one unit from warehouse i to store j as shown in below. Find the shipping schedule which minimizes the cost.

Table.1

	O \ D	1	2	3	4	5	6	Supply
Let $u_1(0)$	1	9 3	12 12	$\boxed{5}$ 9	6 4	9 7	10 8	5
$u_2(3)$	2	7 -2	$\boxed{4}$ 3	7 -5	7 2	$\boxed{6}$ 5	$\boxed{2}$ 5	6
$u_3(0)$	3	$\boxed{1}$ 6	5 5	$\boxed{1}$ 9	11 9	3 1	11 9	2
$u_4(0)$	4	$\boxed{3}$ 6	8 8	11 2	$\boxed{2}$ 2	$\boxed{4}$ 2	10 8	9
	Requirement	4	4	6	2	4	2	22 22
		(6) v_1	(0) v_2	(9) ~~Table.1~~ v_3	(2) v_4	(2) v_5	(2) v_6	

Solution:- Here Demand = Supply Hence it is a balanced transportation problem.

[Apply YAM to get the allocation, it is shown in the Table.1]

No: of allocations obtained = 08

No: of allocations requirement $= (m+n-1)$

$= (4+6-1)$

$= ⑨$

Hence degeneracy exists.

Allot ∈ to a least cost cell, So That it should not formed any loop. The least cost cell in this table is C_{35}. (ie 3). If you allot ∈ to this cell, it forms a loop. Hence it not satisfied The optimality test. Then allot ∈ to a next least cost cell ie C_{25} or C_{32} So That it not form any closed cell. Now choose C_{25}, (cost is 5).

Now No: of allocations obtained = 9

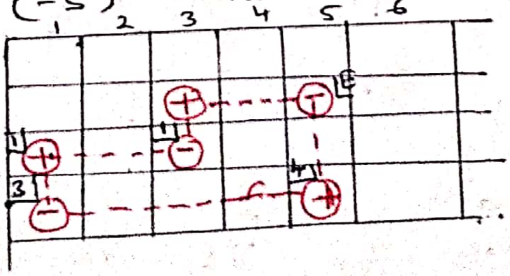
Hence it is eligible for optimality test.

Determine $c_{ij} = (u_i + v_j)$ for all occupied cells

Determine $c_{ij} - (u_i + v_j)$ for all unoccupied cells

All $c_{ij} - (u_i + v_j) \ngeq 0$, Hence sol^n is not optimal.

Now Identify minimum negative cost cell, So That it form a loop. Here (-5) is the minimum negative.



Θ min (∈, 1, 3)

∈

New Allocated matrix as shown below:

$u_1(0)$	9 3	12 7	$\boxed{5}$ 9	6 4	9 7	10 3
$u_2(-2)$	7 3	$\boxed{4}$ 3	$\boxed{E}$ 7	7 7	5 5	$\boxed{2}$ 5
$u_3(0)$	$\boxed{1+E}$ 6	5 0	$\boxed{1-E}$ 9	11 9	3 1	11 4
$u_4(0)$	$\boxed{3-E}$ 6	8 3	11 2	$\boxed{2}$ 2	$\boxed{4+E}$ 2	10 3
	(6) v_1	(5) v_2	(9) v_3	(2) v_4	(2) v_5	(7) v_6.

optimal cost
= 5×9+ 4×3
+ E×7 + 2×5
+ (1+E)6 +
(1-E) 9 +
(3-E) 6 + 2×2
(4+E) 2
= Rs 112 ✓

Again apply optimality test.

Determine $c_{ij}=(u_i+v_j)$ for all occupied cells

Determine $c_{ij}-(u_i+v_j)$ for all unoccupied cells

All $c_{ij}-(u_i+v_j) \geq 0$, Hence solution is optimal.

and also Alternate solution exists in This problem.

Alternate Solution: - When $c_{ij}-(u_i+v_j)=0$, Then Alternate solution exists. In The above problem The cell C_{32} (is 0). Again find out entering and leaving variable from This cell. only allocations positions change & optimal soln is same.

$\boxed{4}$ 3	$\boxed{E}$ 7
5 0	$\boxed{1-E}$ 9

$u_1(0)$	9 3	12 7	$\boxed{5}$ 9	6 4	9 7	10 3
$u_2(-2)$	7 3	$\boxed{3+E}$ 3	$\boxed{1}$ 7	7 7	5 5	$\boxed{2}$ 5
$u_3(0)$	$\boxed{1+E}$ 6	$\boxed{1-E}$ 5	9 0	11 9	3 1	11 4
$u_4(0)$	$\boxed{3-E}$ 6	8 3	11 2	$\boxed{2}$ 2	$\boxed{4+E}$ 2	10 3
	(6) v_1	(5) v_2	(9) v_3	(2) v_4	(2) v_5	(7) v_6

⊖	⊕
⊕	⊖

⊖ min (4, 1-E)
(1-E)

$\boxed{3+E}$ 3	$\boxed{1}$ 7
$\boxed{1-E}$ 5	9

All $c_{ij} - (u_i + v_j) \geq 0$, Hence solution is optimal.

Tending $\epsilon \to 0$

Optimal cost $= 5\times9 + (3+\epsilon)\times3 + 1\times7 + 2\times5$

$(1+\epsilon)\times6 + (1-\epsilon)\times5 + (3-\epsilon)\times6 + 2\times2 + (4+\epsilon)2$

$= 45 + 9 + \cancel{3\epsilon}^{0} + 7 + 10 + 6 + \cancel{6\epsilon}^{0} + 5 - \cancel{5\epsilon}^{0} + 18 - \cancel{6\epsilon}^{0}$

$+ 4 + 8 + \cancel{2\epsilon}^{0}$

Rs 112/-

UNBALANCED Transportation problem :-

When demand ≠ supply then the problem is said to be unbalanced.

Example : 1 : A textile firm has 3 factories F_1, F_2 F_3 and four warehouses W_1, W_2, W_3 & W_4. The factory capacity and transportation cost, the warehouse requirement are given in the following table. Determine the shipping schedule to minimize the cost.

F \ W	W_1	W_2	W_3	W_4	Capacity
F_1	15	24	11	12	500
F_2	25	20	14	16	400
F_3	12	16	22	13	700
Requirement	300	250	250	400	

Solution :- Σ Capacity = 1600 units

Σ requirement = 1300 Units

Excess capacity = 1600 - 1300 = 300 units

Introduce a dummy requirement in the transportation table with cost of transportation is zero.

The matrix becomes Apply VAM to get IBFS.

		F \ W	W1	W2	W3	W4	WD	Capacity	Penalties
				2	3		1		
5		F1	15	24	350 11	150 12	0	~~500~~	(11) (1) (1)(3)(3) ←
4		F2	25	20	14	100 16	300 0	~~400~~	(14)←(2) (2)(9) * ←
		F3	300 12	250 16	22	150 13	0	700	(12) (1) (1) (1)(1)
		Requirement	300	250	350	400	300	1600 1600	
		Penalties	(3)	(4)	(3)	(1)	(0)		
			(3)	(4)	(3)	(1)	*		
			(3)	↑ *	(3)↑	(1)			
			(3)		*	(1)			
			(3)			(1)			

Now it becomes balanced

IBFS = 350×11 + 150×12 + 100×16 + 300×0 + 300×12

~~Option~~. + 250×16 + 150×13 = Rs 16,800/-

Optimality test :- No: of allocations obtained = 07

No: of allocations required = (m+n-1)

= 3+5-1 = 07

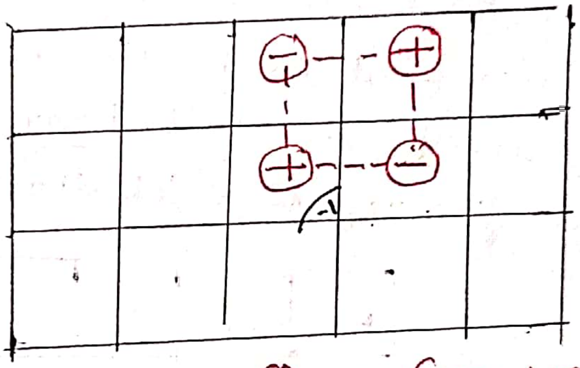
and these allocations are in independent position. Hence it is eligible for optimality test.

Determine $C_{ij} = (u_i + v_j)$ for all occupied cells

Determine $C_{ij} - (u_i + v_j)$ for all unoccupied cells.

Let $u_1(0)$	15 4	24 9	350 11	150 12	0 4
$u_2(4)$	25 10	20 1	14 -1	100 16	300 0
$u_3(1)$	300 12	250 16	22 10	150 13	0 3
	(11) v_1	(15) v_2	(11) v_3	(12) v_4	(-4) v_5

All $c_{ij} - (u_i + v_j) \ngtr 0$ Hence solution is not optimal. Here only cell C_{23} is -1. Try to find entry and leaving variables.



− min (350, 100)
= 100

New matrix table

			(1)	(2)	
$U_1=(0)$	15 4	24 9	250 11	250 12	0 3
(0) $U_2=(3)$	25 11	20 2	100 14	16 1	300 0
(3) U_3 (7)	300 12	250 16	22 10	150 13	0 2
	(14) (4) v_1	(15) (5) v_2	(11) v_3	(12) v_4	(-3) v_5 (7)

Again apply optimality test.

No: of allocation obtained = 07

No: of allocation required = 07 and these allocation are in independent positions.

Determine $c_{ij} = (u_i + v_j)$ for all occupied cells

Determine $c_{ij} - (u_i + v_j)$ for all unoccupied cells.

All $c_{ij} - (u_i + v_j) \geq 0$ Hence solution is optimal.

$\therefore$ Optimal cost $= 250 \times 11 + 250 \times 12 + 140 \times 14 + 300 \times 0$
$+ 300 \times 12 + 250 \times 16 + 150 \times 13$
$=$ Rs 16700/-

Ex: 2:- A product is produced by 4 factories ABCD. The unit production cost in these are Rs 2, Rs 3, Rs 1 and Rs 5 respectively. This production capacities are A = 50, B = 70, C = 30, D = 50 unit. These factories supply the product to 4 stores. The demand of which are 25, 35, 105, 20 units respectively. The unit transportation cost in Rs from each factory to each store is given below:

F \ S	1	2	3	4
A	2	4	6	11
B	10	8	7	5
C	13	3	9	12
D	4	6	8	3

Determine the extent of delivery from each of the factory to each of the 4 stores, so that the total production and transportation cost is minimum.

Solution:- Total cost = Total production cost + Total transportation cost

Total cost A1 = 2+2 = 4 |||ly for B1, B2, B3, B4
A2 = 2+4 = 6   C1, C2, C3, C4
A3 = 2+6 = 8   D1, D2 D3 D4
A4 = 2+11 = 13

The final matrix becomes

F \ S	1	2	3	4	Capacity
A	4	6	8	13	50
B	13	11	10	8	70
C	14	4	10	13	30
D	9	11	13	8	50
Demand	25	35	105	20	200 185

Total capacity = Σ Capacity = 200 units ;
Σ Demand = 185 units Hence it is unbalanced.

∴ Excess capacity = 200 − 185 = 15 units.

Introduce a dummy requirement with associated cost as zero.

	F \ S	1	2	3	4	~~5~~ D	capacity
Let $u_1(0)$	A	$\boxed{25}$ 4	$\boxed{5}$ 6	$\boxed{20}$ 8	13 10	0 2	50
$u_2(2)$	B	13 7	11 3	$\boxed{55}$ 10 ⊕	8 3	$\boxed{15}$ ⊖ 0	70
$u_3(-2)$	C	14 12	$\boxed{30}$ 4	10 4	13 12	0 4	30
$u_4(5)$	D	9 0	11 0	$\boxed{30}$ 13 ⊖	$\boxed{20}$ 8	⊕ 0 -3	50
	Demand	25	35	105	20	15	200 200
		(4) (v_1)	(6) v_2	(8) v_3	(3) v_4	(−2) v_5	

Now it becomes Balanced. Apply VAM to get IBFS. and the following above allocations got by using VAM.

IBFS = 25×4 + 5×6 + 20×8 + 55×8 + 30×4 + 30×13 + 20×8 = Rs 1510/-

NO: of allocations obtained = 8

No: of allocations required = (m+n-1) = (4+5-1)
= 08.

and these allocations are in independent positions. Hence it is eligible for optimality test.

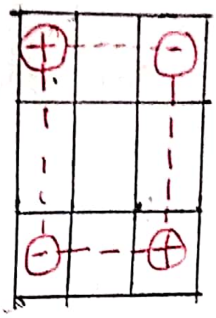
Determine $c_{ij} = (u_i + v_j)$ for all occupied cells

Determine $c_{ij} - (u_i + v_j)$ for all unoccupied cells.

All $c_{ij} - (u_i + v_j) \ngeq 0$. Hence solution is not optimal.

New matrix becomes

Let $u_1(0)$	25 4	5 6	20 8	13 11	0 5
$u_2(2)$	13 7	11 5	70 10	8 3	0 3
$u_3(-2)$	14	30 4	10 4	13 17	0 7
$u_4(5)$	9 0	11 0	15 13	20 8	15 0
	(4) v_1	(6) v_2	(8) v_3	(3) v_4	(-5) v_5



θ min (15, 30)
= 15

All $c_{ij} - (u_i + v_j) \geq 0$, Hence solution is optimal.

Optimal cost: 25×4 + 5×6 + 20×8 + 70×10 + 30×4
+ 15×13 + 20×8 + 0×15

= Rs 1465/-

Ex: 3 Consider the unbalanced transportation problem, since there is not enough supply. some of the demand at these destination may not be satisfied. Suppose there is a penalty cost for every unsatisfied demand unit which are given by 5, 3 & 2 for destination 1, 2, 3 respetively. Determine optimal transportation cost.

To / From	1	2	3	Supply
1	5	1	7	10
2	6	4	6	80
3	3	2	5	15
Demand	75	20	50	105 / 145

Solution: - (HINT) Σ supply = 105 Σ Demand = 145

Excess demand = 145 − 105 = 40 units.

Introduce a dummy capacity with penalty cost as 5, 3 and 2 respectively.

To / From	1	2	3	Supply
1	5	1	7	10
2	6	4	6	80
3	3	2	5	15
4	5	3	2	40
Demand	75	20	50	145 / 145

Hence it is balanced T.P

ANS: Optimal cost = Rs 595/-

Ex:4: The unit cost of transportation from site i to site j are given below. At site i = 1, 2, 3, the stocks are 150, 200 and 170 units respectively are available. 300 units are to be sent to site 4 and the rest to the site 5. Find the cheapest way of doing this.

Site j Site i	1	2	3	4	5	Stocks
1	-	3	4	10	7	150
2	1	-	2	16	6	200
3	7	4	-	12	13	170
4	8	3	9	-	5	
5	2	1	7	5	-	
Demand				300		

Soln: [HINT]: As per the restriction of demand and supply the above table get reduced to

Site j Site i	4	5	Stocks
1	10	7	150
2	16	6	200
3	12	13	170
Demand	300	220	520

ANS:

Optimal cost

Rs 4680/-

Ex: 5:- A company has factory A, B & C which supplies to warehouse D, E F & G. The factory capacities are 230, 280, 180 respectively for regular production. If overtime production is utilized. The capacity can be increases upto 300, 360 & 190 respectively. The current warehouse requirements are 165, 175, 205 and 165 respectively. Unit shipping cost in Rs. between the factories and warehouses are given in the table. Determine the optimum distribution for the company to minimize the cost, if the incremental unit over time cost are Rs 5, Rs 4 & Rs 6 respectively.

	D	E	F	G
A	7	8	9	11
B	5	11	8	7
C	4	23	3	12

Solⁿ:- Overtime production can be represented as additional factories, producing the items at their corresponding higher cost. The shipping cost for overtime shipment from factory A to warehouses D, E, F & G are 5+7, 8+5, 9+5, & 11+5 respectively. overtime shipping cost from factory B and C to the warehouses D, E F & G are calculated in the same manner

	D	E	F	G	Capacity
A	7	8	9	11	230
B	5	11	8	7	280
C	4	23	3	12	180
A'	12	13	14	16	70
B'	9	15	12	11	80
C'	10	29	9	18	10
Requirement	165	175	205	165	

Σcapacity = 850 Units

ΣRequirement = 710 Units

Excess Capacity = 850-710 = 140 units

Introduce a dummy requirement with transportation cost as zero.

	D	E	F	G	WD	Capacity
A	7	8	9	11	0	230
B	5	11	8	7	0	280
C	4	23	3	12	0	180
A_1	12	13	14	16	0	70
B_1	9	15	12	11	0	80
C_1	10	29	9	18	0	10
Requirement	165	175	205	165	140	850 850

Hence it is balanced

MAXIMIZATION PROBLEM.

A Company manufacturing air cooler has two plants located at Bombay and Calcutta with capacities of 200 units and 100 units per week respectively. Company supplies air cooler to its 4 showrooms situated at Ranchi, Delhi, Lucknow and Kanpur, which have max. demand of ~~100~~ 75, 100, 100 and 30 units respectively. Due to difference in raw material costs and transportation costs, the profit per unit in Rupees differs, which is shown in table below.

	Ranchi	Delhi	Lucknow	Kanpur
Bombay	90	90	100	110
Calcutta	50	70	130	85

plan the production program so as to maximize the profit.

Solution: Maximization problem is converted into minimization problem, all the elements can be subtracted from the highest element in the given table, The table becomes

40	40	30	20	200
80	60	0	45	100

[HINT, If it is a maximization problem, Firstly convert into ~~mini~~ minimization problem and then make it balance]

Total Supply = 300 units
Total demand = 305 units

Excess demand = 305 - 300 = 05 units.

Introduce a dummy source to absorb the excess demand. with cost as zero.

Apply VAM to get the allocations

	Ranchi	Delhi	Lucknow	Kanpur	Capacity
BOMBAY	$\boxed{70}$ 40	$\boxed{100}$ 40	30	$\boxed{30}$ 20	200
KOLKATA	80	60	$\boxed{100}$ 0	45	100
D_S	$\boxed{5}$ 0	0	$\boxed{\epsilon}$ 0	0	5
Demand	75	100	100	30	305 305

Optimality test:

① No: of allocation obtained ≠ No: of allocation required (m+n-1)

$$05 \neq (3+4-1) = 06$$

Hence degeneracy exists.

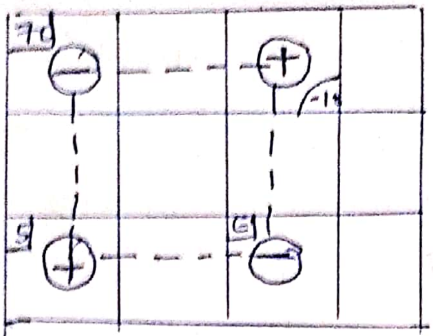
Allot ϵ to a least cost cell, so That it should not form any loop.

Let $u_1(0)$	$\boxed{70}$ 40	$\boxed{100}$ 40	30 -10	$\boxed{30}$ 20
$u_2(-40)$	80 80	60 60	$\boxed{100}$ 0	45 65
$u_3(-40)$	$\boxed{5}$ 0	0 0	$\boxed{\epsilon}$ 0	0 20
	(40) v_1	(40) v_2	(40)	(20) v_3

Determine $c_{ij} = (u_i + v_j)$ for all occupied cells.

Determine $c_{ij} - (u_i + v_j)$ for all unoccupied cells.

All $c_{ij} - (u_i + v_j) \neq 0$. Hence solution is not optimal,



$\ominus \min(70, \epsilon)$
$= \epsilon$.

Therefore new matrix becomes

Let $u_1(0)$	$\boxed{70-\epsilon}$ 40	$\boxed{100}$ 40	$\boxed{\epsilon}$ 30	$\boxed{30}$ 20
$u_2(-30)$	80 70	60 50	$\boxed{100}$ 0	45 55
$u_3(-40)$	$\boxed{5+\epsilon}$ 0	0 0	0 10	0 20
	(40) v_1	(40) v_2	(30) v_3	(20) v_4

Again apply optimality test.

All $c_{ij} - (u_i + v_j) \geq 0$, Hence solution is optimal.

Tending $\epsilon \to 0$, (Hint corresponding profit table ~~is~~ allocation is taken)

$$\text{profit} = (70-\epsilon) \times 90 + 100 \times 90 + \overset{0}{\cancel{\epsilon \times 100}}$$

$$+ 30 \times 110 + 100 \times 130$$

$$= 6300 + 9000 + 3300 + 13000$$

profit = Rs 31,600

2. A Firm has 3 factories located at Cities A, B, C respectively and supply goods to 4 dealers 1, 2, 3 & 4, spread all over the Country. production capacities are 1000, 700 and 900 units per month respectively. Monthly order from dealers are 900, 800, 500 and 400 respectively. The per unit return excluding transportation cost are Rs 8, Rs 7, Rs 9 at 3 factories Unit production cost from factories to dealers are given below.

Dealers

		1	2	3	4
Factories	City A	2	2	2	4
	City B	3	5	3	2
	City C	4	3	2	1

Determine the optimum distribution system to maximize the total return.

Solution: From the given data, the matrix return is completed as follows

Return = profit − transportation cost:

	1	2	3	4
Rs 8	6	6	6	4
Rs 7	4	2	4	5
Rs, 9	5	6	7	8

To convert into minimization problem. Subtract all the elements in the table from the highest element, so that matrix becomes

F \ D	1	2	3	4	Supply
City A	[200] 2	[800] 2	2	4	1000
City B	[700] 4	6	4	3	700
City C	3	2	[500] 1	[400] 0	900
Demand	900	800	500	400	2600 2600

Total supply = Total demand Hence balanced T.P

Apply VAM to get the allocation.

Here No. of allocations obtained ≠ No. of allocations required

05 ≠ (m+n-1) = (3+4-1) = 06.

Hence degeneracy exists.

Allot ε to a least cost cell so that it may [illegible] form a closed loop.

	1	2	3	4
$(u_1 = 0)$ A	[200] 2	[800] 2	[ε] 2	4 [illegible]
(u_2) B	[100] 4	6 [illegible]	4 [illegible]	3 [illegible]
(u_3) C	[illegible]	2 [illegible]	[500] 1	[400] 0
	(v_1)	(v_2)	(v_3)	(v_4)

All d_{ij} (cells) ≥ 0

Hence soln is optimal

Putting ε = 0

Profit = [illegible]

[illegible]

[illegible]

= [illegible]

[illegible] = Rs [illegible]/-

Here, Allocations obtained = 06

Required = (m+n-1) = (3+4-1) = (06)

Hence degeneracy [illegible]

3. A Company has factories F_1 F_2, F_3 and F_4 manufacturing the Same product. production and raw material costs differ from factory to factory are given in the following table in the first two rows. The transportation costs from the factories to Sale depots S_1 S_2, S_3 are also given. The last columns in The table gives the sale price and the total requirement at each depot. The production capacity of each factory is given in the last row.

	F_1	F_2	F_3	F_4	Sales price/unit	Requirement Units
production cost/unit	15	18	14	13		
Raw material cost/unit	10	9	12	9		
S_1	3	9	5	4	34	80
S_2	1	7	4	5	32	120
S_3	5	8	3	6	31	150
production Capacity Units	10	150	50	100		

Determine the most profitable production and distribution Schedule and the Corresponding profit. The surplus production should be taken to yield Zero net profit.

Solution:-

Hint. profit = Selling price - (production cost + Raw material cost + Transportation cost]

profit Table becomes.

profit table

	F1	F2	F3	F4
S1	34-(15+10+3) = 6	34-(18+9+9) = -2	34-(14+12+5) = 3	34-(13+9+4) = 8
S2	32-(15+10+1) = 6	32-(18+9+7) -2	32-(14+12+4) 2	32-(13+9+5) = 5
S3	31-(15+10+5) = 1	31-(18+9+8) = -4	31-(14+12+3) = 2	31-(13+9+6) = 3

It is a maximization problem, To convert into a minimization problem, subtract all the elements from the highest element in the cell, So that minimization table becomes

(profit table)

	F1	F2	F3	F4
S1	6	-2	3	8
S2	6	-2	2	5
S3	1	-4	2	3

→ (Minimization table)

	F1	F2	F3	F4
S1	2	10	5	0
S2	2	10	6	3
S3	7	12	6	5

Rearrange the matrix

	S1	S2	S3	Capacity
F1	2	2	7	10
F2	10	10	12	150
F3	5	6	6	50
F4	0	3	5	100
Requirements	80	120	150	

Total capacity

= 10+150+50+100

= 310 units

Total requirement

= 80+120+150 = 350 units

Excess requirement

= 350-310 = 40 units

Introduce a dummy row with cost cell as zero.

Now Apply VAM to get the allocation.

		2 S1	S2	S3	Capacity	Penalties
3	F1	2	$\boxed{10}$ 2	7	10	(0) (0) (5) 4
	F2	10	$\boxed{90}$ 10	$\boxed{60}$ 12	150	(0) (0) (2) (2) (2)
5	F3	5	6	$\boxed{50}$ 6	50	(1) (1) (0) (0) (0)
4	F4	$\boxed{80}$ 0	$\boxed{20}$ 3	5	100	(3) (3) 4 (2) (2)
1	FD	0	0	$\boxed{40}$ 0	40	(0) *
		80	120	150	350 350	
		(0)	(2)	(5) ①		
		(2)	(1)	(1)		
		*	(1)	(1)		
			(3) ↑	(1)		
			(4)	(6) ↑		

Apply optimality test:.

No: of allocations obtained = No: of allocations required

7 = (m+n-1) = (5+3-1) = 7

Hence eligible for optimality test.

Determine $c_{ij} = (u_i + v_j)$ for all occupied cells

Determine $c_{ij} - (u_i + v_j)$ for all unoccupied cells.

Let $u_1(0)$	2 / 3	[10] 2	7 / 3
$u_2(8)$	10 / 3	[30] 10	[60] 12
$u_3(2)$	5 / 4	6 / 2	[50] 6
$u_4(1)$	[20] 0	[20] 3	5 / 0
$u_5(-4)$	0 / 5	0 / 2	[40] 0
	(-1) v_1	(2) v_2	(4) v_3

All $c_{ij}-(u_i+v_j) \geq 0$ Hence Sol^n is optimal.

profit = Max Z = 10×6 + 90×-2 + 60×-4 + 50×2 + 80×8 + 20×5

= 60 - 180 - 240 + 100 + 640 + 100

Profit = Max Z = 480/-

---

4. A multiplant Company has 3 manufacturing plants A, B & C and two markets X and Y. The product cost at A, B, C are Rs 1500, Rs 1600 Rs 1700 respectively. Selling price in X and Y are 4400 and 4700 respectively. Demand in X and Y are 3500 and 3600 pieces respectively. The transportation cost are as shown in the matrix. Built the mathematical model for this.

	To / From	X	Y	Capacity
Rs 1500	A	1000	1500	2000
Rs 1600	B	2000	3000	3000
Rs 1700	C	1500	2500	4000

Solⁿ. - We know profit = Selling price - Transportation cost - Production cost

For AX profit = 4400 - 1000 - 1500 = 1900

AY profit = 4700 - 1500 - 1500 = 1700

III^ly for BX, BY, CX & CY.

profit table becomes

To / From	X	Y	Capacity
A	1900	1700	2000
B	800	100	3000
C	1200	500	4000
Demand	3500	3600	9000 7100

Total capacity = 9000

Total demand = 7100

Excess capacity = 9000 - 7100 = 1900 units

Introduce a dummy column with cost as zero

The mathematical expression is as follows

$$Z_{max} = 1900x_{11} + 1700x_{12} + 0x_{13} + 800x_{21} + 100x_{22} + 0x_{23} + 1200x_{31} + 500x_{32} + 0x_{33}$$

Subjected to

$x_{11} + x_{12} + x_{13} = 2000$

$x_{21} + x_{22} + x_{23} = 3000$

$x_{31} + x_{32} + x_{33} = 4000$

$x_{11} + x_{21} + x_{31} = 3500$

$x_{12} + x_{22} + x_{32} = 3500$

$x_{13} + x_{23} + x_{33} = 2000$

where $x_{ij} \geq 0$ where $i = 1, 2, 3$

$j = 1, 2, 3$

To convert into minimization problem, Subtract all the elements in the table from the highest element, so that matrix becomes

F \ D	1	2	3	4	Supply
City A	$\boxed{200}$ 2	$\boxed{800}$ 2	2	4	1000
City B	$\boxed{700}$ 4	6	4	3	700
City C	3	2	$\boxed{500}$ 1	$\boxed{400}$ 0	900
Demand	900	800	500	400	2600 \ 2600

Total supply = Total demand Hence balanced T.P.

Apply VAM to get The allocation.

Here No: of allocation obtained = No: of allocations required

$05 \neq (m+n-1) = (03+4-1) = 06.$

Hence degeneracy exists.

Allot ∈ to a least cost cell so That it not form a closed loop.

	1	2	3	4
Let u_1(0) A	$\boxed{200}$ 2	$\boxed{800}$ 2	$\boxed{\epsilon}$ 2	4 3
u_2(2) B	$\boxed{700}$ 4	6 2	4 0	3 0
u_3(−1) C	3 2	2 1	$\boxed{500}$ 1	$\boxed{400}$ 0
	(2) v_1	(2) v_2	(2) v_3	(1) v_4

All $c_{ij} - (u_i + v_j) \geq 0$

Hence solⁿ is optimal.

Tending ∈ → 0

profit = 6×200 + 6×800 + 700×4 + 500×7 + 400×8

= 1200 + 4800 + 2800 + 3500 + 3200 = Rs 15,500/-

Now, Allocations obtained = 06

Required = (m+n−1) = (03+04−1) = ⓪⑥

Hence eligible for optimal test.

NETWORK TECHNIQUES

PERT AND CPM

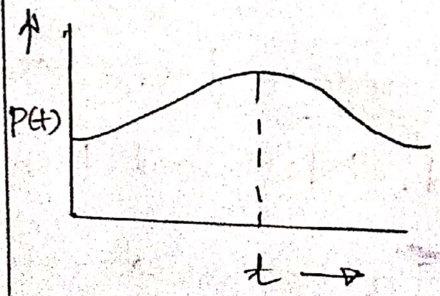
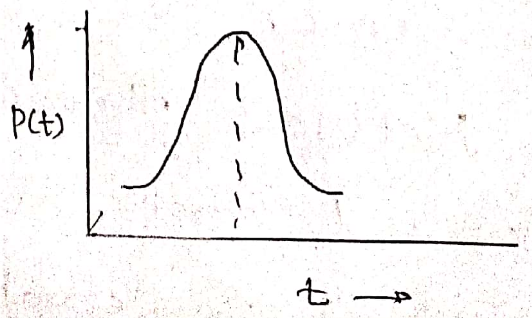
Introduction: Nowadays the term" Network technique is very extensively used in a the field of project planning and control.

The Completed date for a project is most often a part of the contract. Heavy penalties are imposed for not completing the project within contracted time.

The project of national importance, such as (i) irrigation projects (ii) power plants (iii) construction of dams (iv) Fertilizer plants etc. have a large impact on national economy. The delay in completion of these projects may affect the production and industrilization of a very large region and may in turn the economy of the nation as a whole and hence the early completion of such project is of importance.

Network planning techniques have been developed to meet this need. Network Techniques represent a systematic approach to developing information for decision making, proper planning techniques help to minimize the change of Scheduled Slippage, cost over reconstruction and provide any easy method to take appropriat corrective measures at the proper time to achieve the Company objective.

Difference between PERT AND CPM

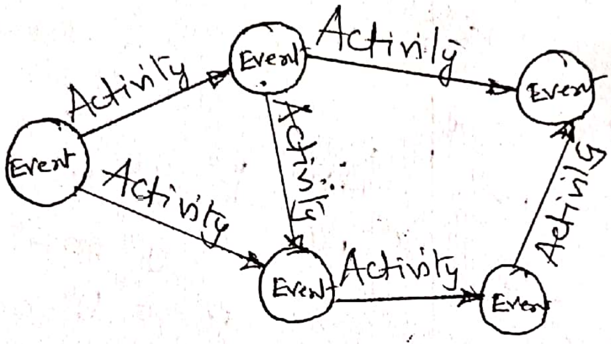
1	PERT Approach is Even based / oriented So it is builtup of event oriented diagram	1.	CPM approach is activity based / oriented, So it is built up of activity oriented diagram.
2.	PERT adopts a probabalistic approach towards the problem	2.	CPM adopts a deterministic approach towards the problem
3.	Dispersion of the Curve is more and hence more is in the Uncertainily 	3.	Dispersion of the curve is less and hence more in the certainily. 

4.	In PERT, there may not be any relation between the cost and the execution time of an activity, ie PERT Costs are not related to time.	4.	In CPM, there may be direct relation between the cost and the execution time of an activity. i.e CPM costs are related to time.
5.	PERT is used more in larger project such as (i) R and D projects (ii) product development and other similar projects involving factor of uncertainity	5.	CPM is used more in smaller projects such as (i) Constructional activities (ii) Maintenance /overhauling repair iii) production control, planning & scheduling are done through CPM
6.	The use of dummy activity is not required for representing the proper sequencing	6	The use of dummy activity is necessary.

CPM AND PERT NETWORKS:

There are two basic elements in a network plan. These are the activities and the event. The activity stands for time consuming part of a project. It represents the job. The event also called a node, on the other end is either the beginning or end of the job. The activity are denoted by arrows and the events by circles or rectangles. When all activities and events in a project are connected logically and sequentially, they form a network, such a network is a basic document in a network based management system.

Fig. shows how the event are connected by activities.



Some jobs can be taken up concurrently, In some cases a job cannot be undertaken until another job is over.

For ex: if concrete pouring requires that the foundation digging is complete, then job A representing digging will have to precede job B. which represent the pouring of concrete

Fig. below shows represents this

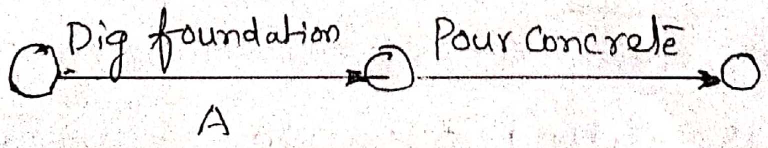
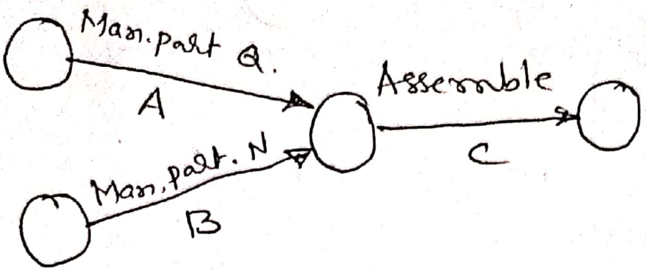


Fig. below might represent
A - manufacturing part Q
B - manufacturing part N
C - Assemble Q and N.



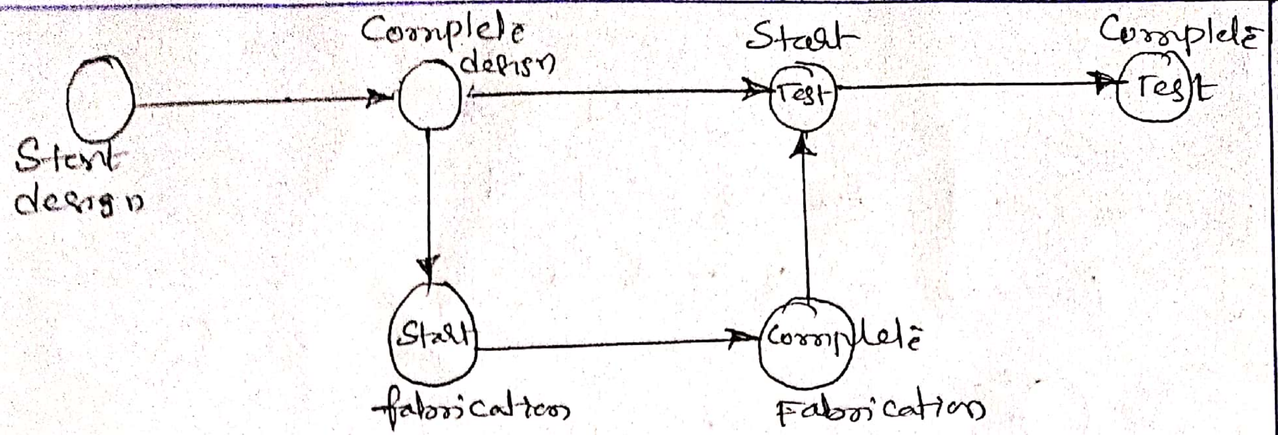
In a network based management system, the stress could be laid either on the event or on the activity. One of the difference between PERT and CPM network is that

PERT - Event oriented
CPM - Activity oriented.

CPM Analysis is activity oriented as shown above.

PERT (Program Evaluation and Review Technique) is event oriented. Fig below gives an example of a network that is event oriented

Here the interest is focussed upon the start or completion of events rather than on the activities themselves. The activities that takes place between the events are not specified.



PERT NETWORK is a event based.

Event may be defined

(i) It must indicate a note worthing or significant point in the project.

(ii) It is a start or Completion of a job.

(iii) It doesnot Consume time or resources.

Examples of what an event and what it is not are:-

Foundation digging started : is a pert event

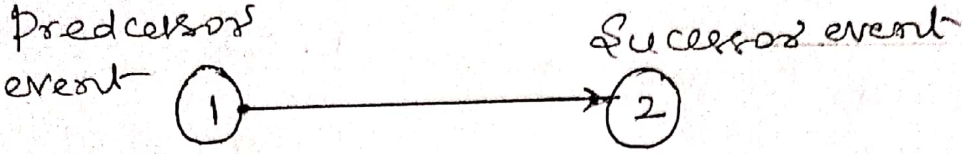
Foundation is being dug : is not a pert event

Assemble parts A and B : is not a pert event

Electrical design Completed : is a pert event.

Event or Events That immediately Come before another event without any interviewing events are called predecessor event to That event.

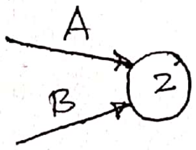
Event or Events that immediately follow another event without any intervening events are called Successor event to that event.



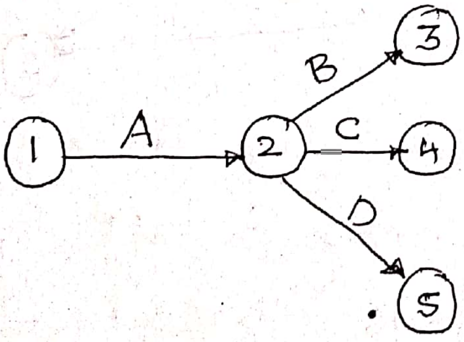
HINTS FOR DRAWING NETWORK.

The general rules of Network Construction are

① An event is achieved only when all the activities leading into it are completed.



② No activity Can begin till The preceding event of The activity is achieved.



③ All the restraints and inter dependencies must be shown in the network.

④ No activity or event should be shown twice.

⑤ Time flows from left to right.

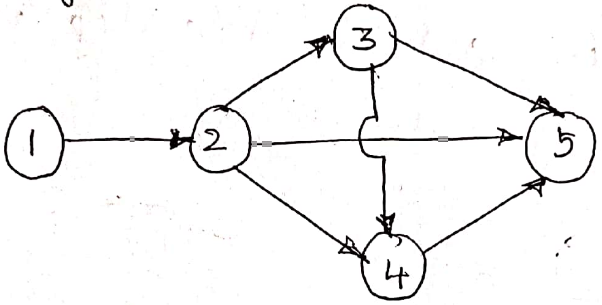
Scanned with CamScanner

⑥ To show the multiple dependency of activity /event, dummy activity is used.

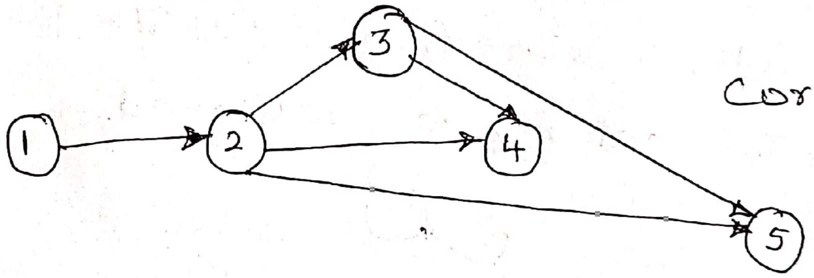
PRECAUTIONS:-

In addition to The general rules Quoted above, the following precautions must be taken while drawing a network.

① Arrows representing activities should not usually cross each other.

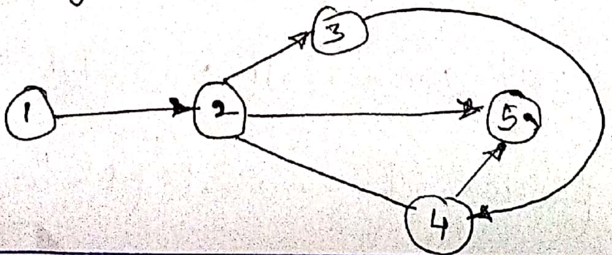


Wrong method



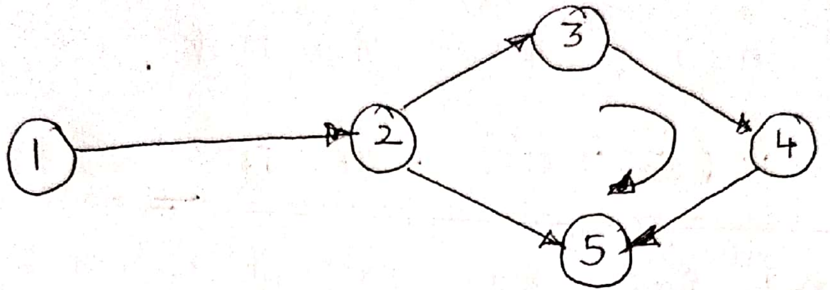
Correct method

② Activities should usually be represented by straight arrows only & not by curved arrows. For this events should be so arranged (without disturbing logical sequence) that the activity do not intersect



Wrong method

③ : The loop formation must be avoided. Thus may occur due to the duplication of events members or repetition of a particular activity or inaccurate collection of date. i.e. it occurs in a complicated network.

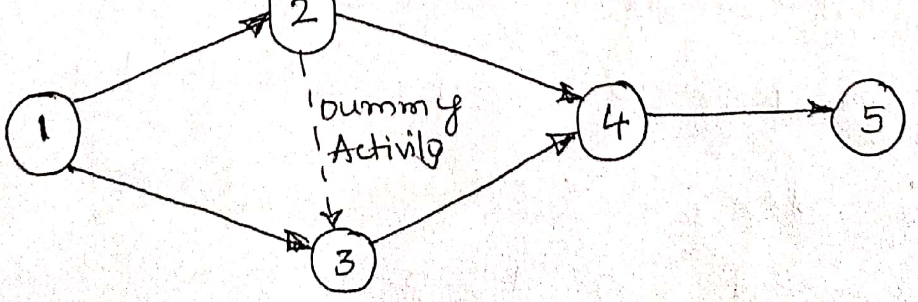


Looping.

④ Dummy or Redundant Activity.

Some times an event j cannot occur unless the other event i has occured, although no speific job/task occurs between them.

In such a case, a dummy arrow is inserted. The function of which is simply to indicate the sequence of events. Dummy activity does not consume any time or resources. It is represented by broken or dotted arrows



Network Showing The dummy activity 2-3

Dummy activities serves the following purposes.

(i) To maintain logic in the network diagram

(ii) To show the relationships between events i.e when an activity has to be completed before the other can be started.

NUMBERING THE EVENTS :

A logical sequence must be reflected by event numbers in a network. This is achieved by making use of D.R. FULKERSON RULE, which consists of the following steps.

(i) An initial events is one which has arrows coming out of it and none entering it. In any network, there will be one such event number it as 1.

(ii) Delete all arrows emerging from event 1. This will create atleast one more initial events.

(iii) Number these new initial events as 2,3...

(iv) Delete all emerging from these numbered events which will create new initial events.

(V) Follow step (iii)

(VI) Continue until the last event, which has no arrows emerging from it is obtained.

In Large networks, where modification may have to be as the project progress, freedom must be there to add and number new event without causing inconsistency or loops. This is achieved by "**Skip Numbering**" In this, every tenth number is used for the initial event Numbering. Any event added later may be assigned a number, which lies between the number of predecessor and successor events.

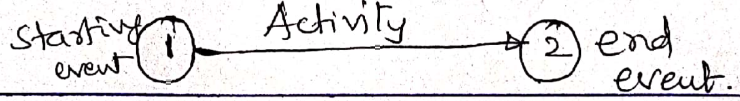
NETWORK REPRESENTATION

There are two systems

① AOA System (Activity on Arrow System)

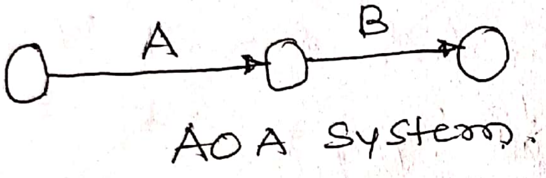
② AON System (Activity on Node system)

AOA System: This method of representation is called Arrow diagram method, Here the activity is represented by an arrow



So the tail of the activity represent the start and head represents the finish activity.

AON System :- In this method, activities are represented by the circles & nodes and arrows shows only the dependency relationship between the activity nodes. In AON System dummy activities are eliminated



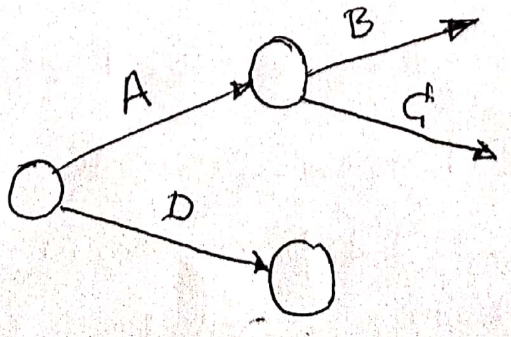
AOA System.

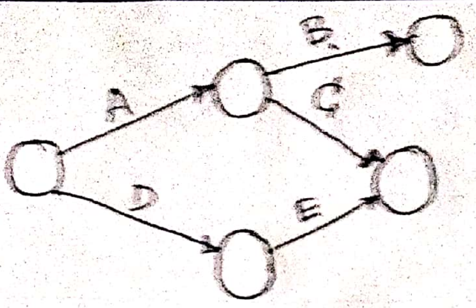
AON System

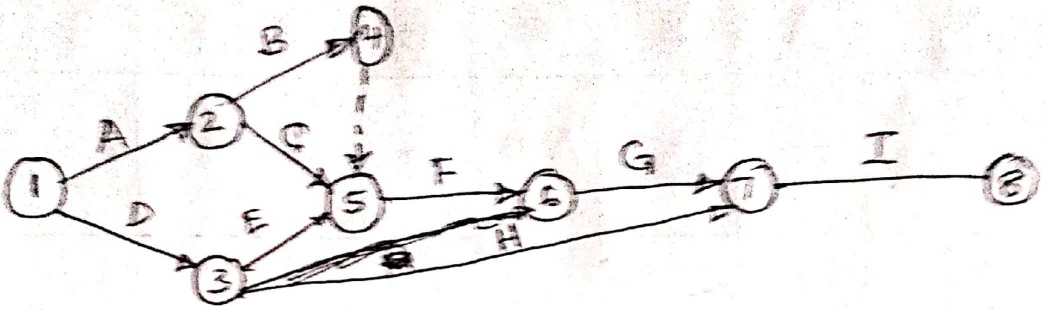
Ex.1. Construct a network for the project whose activities and their precedence relation-ships are as given below:

Activities	A	B	C	D	E	F	G	H	I
Immediate predecessors	—	A	A	—	D	B, C, E	F	D	G, H

Solution :-



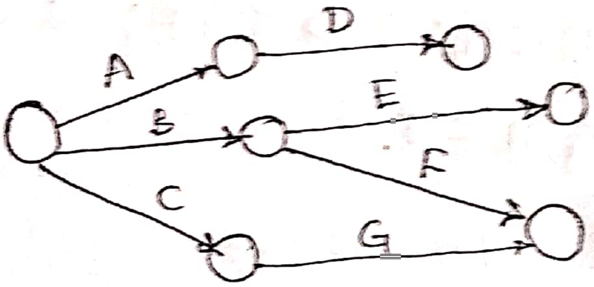


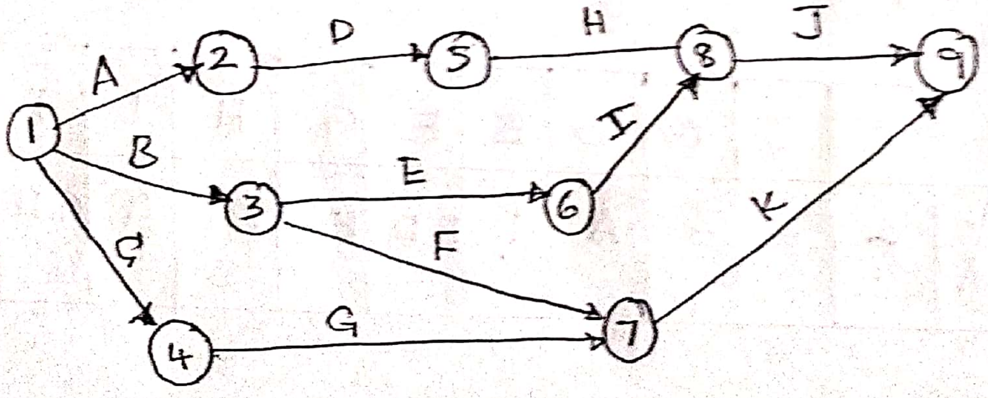


Ex: 2 :- Construct a network for each of the projects, whose activities and their precedence relationships are given below.

Activity	A	B	C	D	E	F	G	H	I	J	K
predcessor	–	–	–	A	B	B	C	D	E	H,I	F,G

Sol^n :-

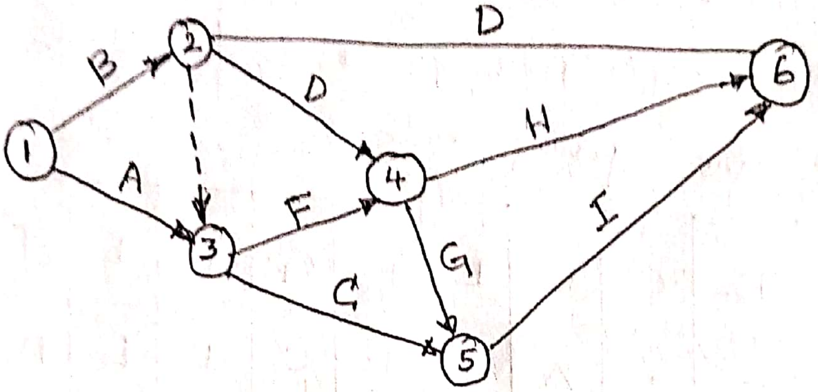




Ex. No: 3: Construct the network for the following and numbering the events using D.R. Fulkerson's rule.

Activity	A	B	C	D	E	F	G	H	I
Immediate predecessor	–	–	A, B	B,	B	A, B	F, D	F, D	C, G

Soln:-



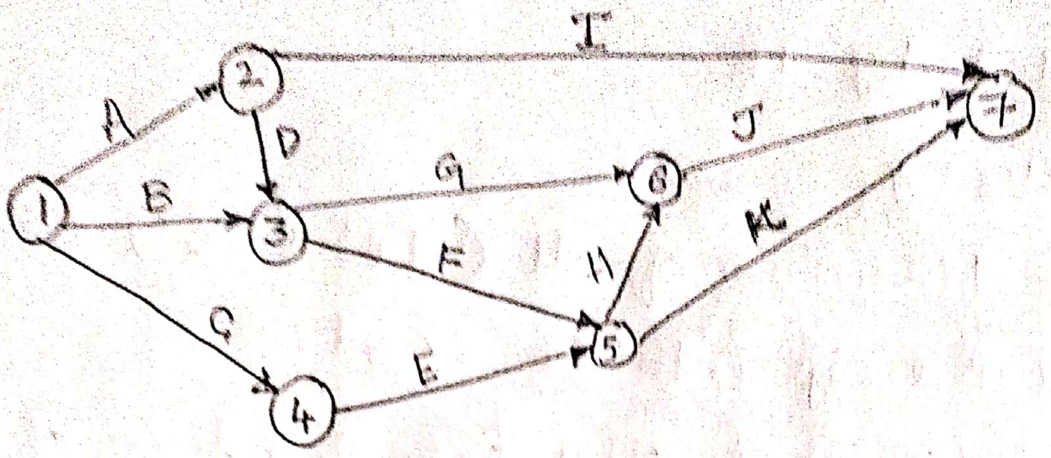
Ex.No: 4 :· A, B, C can start simultaneously

A < D, I ; B < G, F ; D < G, F ; C < E ; E < H, K ;

F < H, K ; G, H < J.

Solution :- The above constraints can be formatted into a table.

Activity	A	B	C	D	E	F	G	H	I	J	K	
Immediate predecessor	–	–	–	A	C	B, D	B, D	E, F	A	G, H	E, F	



CRITICAL PATH :

It is the longest path in the network from the starting event to the end event, and it takes the maximum of time is called Critical path. and the activities on the Critical path are called critical activities. The procedure for identifying the critical path both the PERT and CPM network is similar. The critical path calculation consist of two phases, the forward pass computations & Backward pass computation.

FORWARD PASS COMPUTATIONS:

Calculation begin at The initial event and move towards the end event. Initial event is assigned Zero time and then proceeding to The next event in sequence, the time at which That event is expected to occur at the earliest is calculated. This is called

Earliest expected time for that event and is denoted by TE.

Generalizing

T_E^j = Maximum of all $(T_E^i + t_E^{ij})$ for all ij leading into the event.

Where T_E^j → Earliest expected time of the successor event.

T_E^i → Earliest expected time of the predecessor event.

BACKWARD PASS COMPUTATION

Calculations start from the last node of the project and proceeds towards the start node. To start the calculations, the time of Occurance for the last node is decided. This is the time at which the project must be completed. This is called "contractual obligation time and is denoted by Ts. If not known, the contractual obligation time is taken to be equal to the earliest expected time for the end point. The objective of the "backward pass" is to calculate the "latest allowable occurance time", the time at which a particular event must occur at the earliest. This is denoted by TL.

Generalising

T_L^i = Min of all $[T_L^j - t_e^{ij}]$ for all ij emerging from i

where T_L^i = Latest allowable occurance time for event i

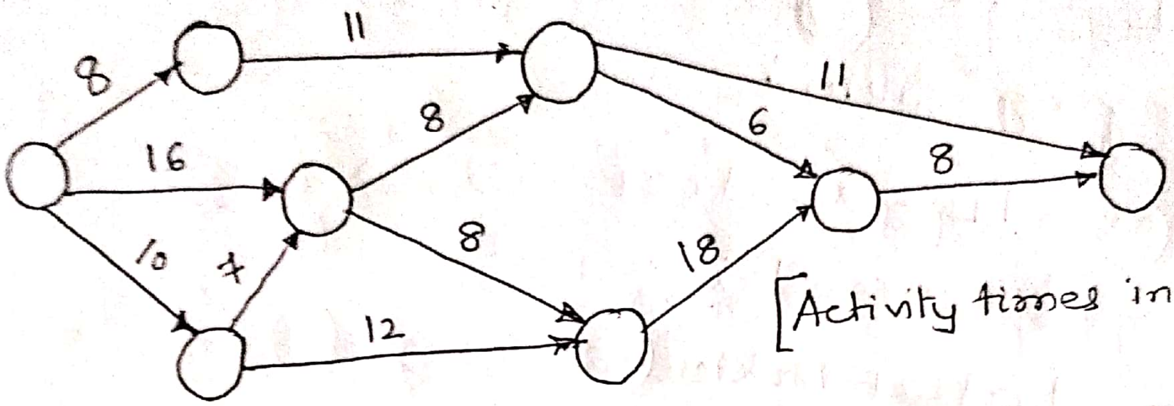
T_L^j = Latest allowable occurance time for event j.

t_e^{ij} = expected time for activity ij.

Slack or Float: Slack and float both refer to the amount of time by which a particular event or activity can be delayed without affecting the time schedule of the network. The term slack refers to the event and is used in the PERT network and the term Float refers to the activity and is used in CPM network.

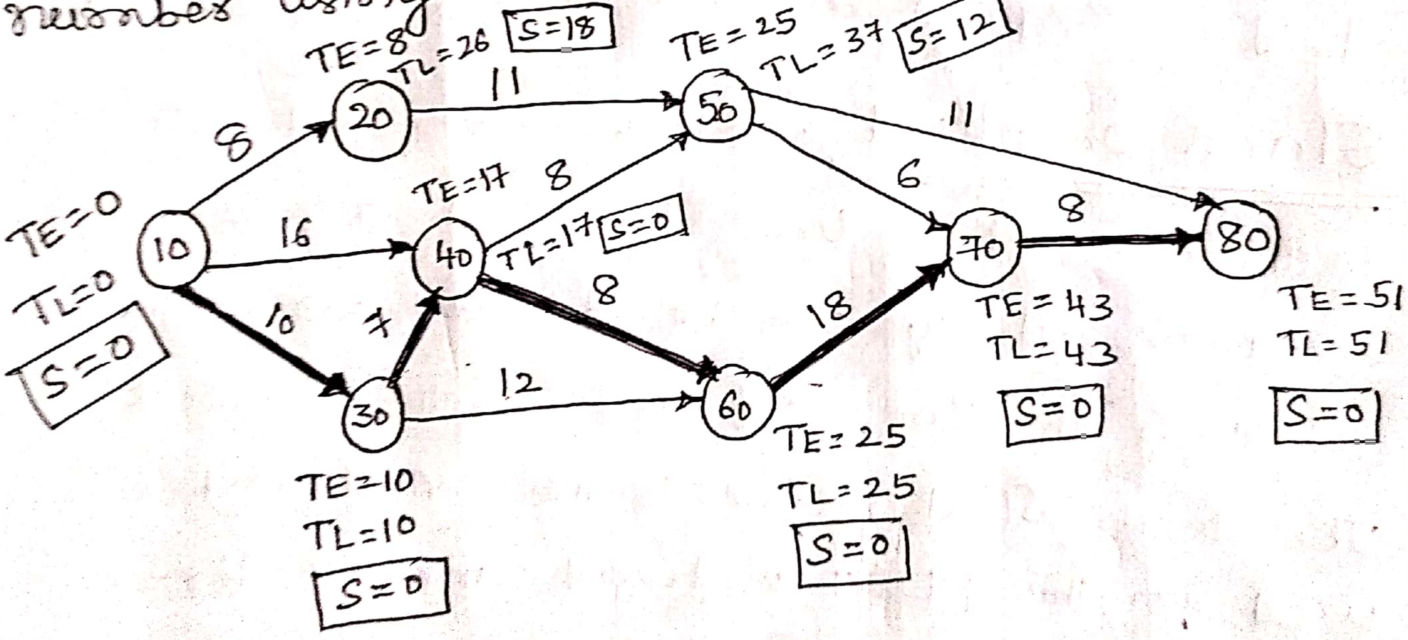
The path forming an unbroken chain of critical activities from start event to the end event is called the critical path. On the critical path, all event have zero slacks. In a network the critical path is shown by thick lines

EX NO: 1: Calculate the slacks for the events and critical path for the following network, put the calculation in tabular form as well as on the network itself.



[Activity times in days]

Solution:- The event of the network are first number using D.R. Fulkerson's rule.



The critical path

10 - 30 - 40 - 60 - 70 - 80.

Tabular form

From network ↓ From table ↓

Activity i-j		t_e^{ij}	T_E^i	T_E^j	T_L^i	T_L^j	Slack S_j $T_L^j - T_E^j$
Predecessor event (i)	Successor event (j)						
10	20	8	0	8	18	26	18
10	30	10	0	10	0	10	0
10	40	16	0	16	1	17	1
20	50	11	8	19	26	37	18
30	40	7	10	17	10	17	0
30	60	12	10	22	13	25	3
40	50	8	17	25	29	37	12
40	60	8	17	25	17	25	0
50	70	6	25	31	37	43	12
50	80	11	25	36	40	51	15
60	70	18	25	43	25	43	0
70	80	8	43	50	43	51	0

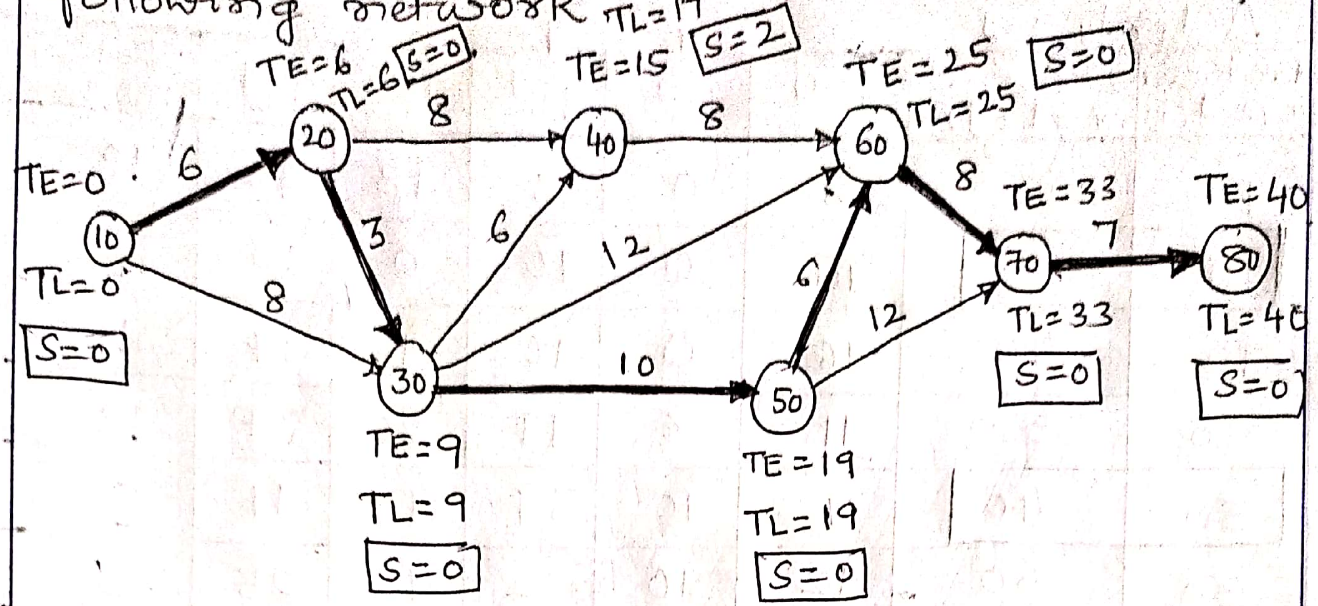
↑ From Network

The critical path

10 - 30 - 40 - 60 - 70 - 80

$\xrightarrow{\hspace{8em}}$

EX.NO: 2 Find the critical path for the following network

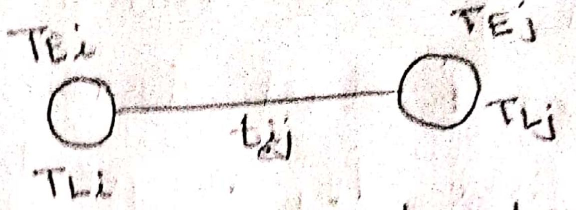


The critical path : 10 - 20 - 30 - 50 - 60 - 70 - 80

Activity i-j		Activity duration t_e^{ij}	EST or TE_i	EFT or TE_j	LST or TL_i	LFT or TL_j	Slack $TL_j - TE_j$
Predecessor Event i	Successor event j						
10	20	6	0	6	0	6	0
10	30	8	0	8	1	9	1
20	30	3	6	9	6	9	0
20	40	8	6	14	9	17	3
30	40	6	9	15	11	17	2
30	50	10	9	19	9	19	0
30	60	12	9	21	13	25	4
40	60	8	15	23	17	25	2
50	60	6	19	25	19	25	0
50	70	12	19	31	21	33	2
60	70	8	25	33	25	33	0
70	80	7	33	40	33	40	0

Float: There are three types of floats

Total float:



It is the maximum time available for the job and the actual time it takes.

Total float for i-j $= (T_{Lj} - T_{Ei}) - t_{ij}$

$= (T_{Lj} - t_{ij}) - T_{Ei}$

$= [T_{Li} - T_{Ei}]$

This is equal to Latest Start time for the activity minus the earliest start time.

FREE FLOAT:- This is based on the possibility that all events occur at their earliest times, i.e, all activities start as early as possible. Consider two activities i-j and j-k, where j-k is successor activity to i-j.

Let the earliest occurance time for event i to be T_E^i and for event j to be T_{Ej}. This means that earliest ~~possib~~ possible start time for activity i-j is T_{Ei} and for activity j-k is T_{Ej}. Let the duration for activity i-j be t_{ij}.

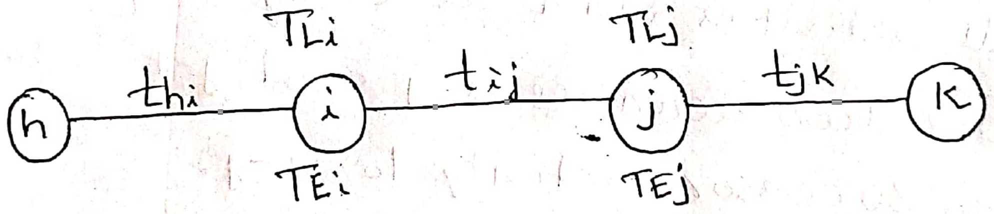
Assume that i-j start as TE^i and takes te^{ij} unit of time and that the next activity j-k cannot start, because its earliest possible time TE_j is greater than $(TE_i + te_{ij})$

Then $TE_j - (TE_i + te_{ij})$ is called free float for the activity i-j. i.e

Free Float for i-j = $TE_j - (TE_i + te_{ij})$

We can restate it as follows.
The free float for activity (i-j) is the difference between its earliest finish time and earliest start time for its sucessor activity.

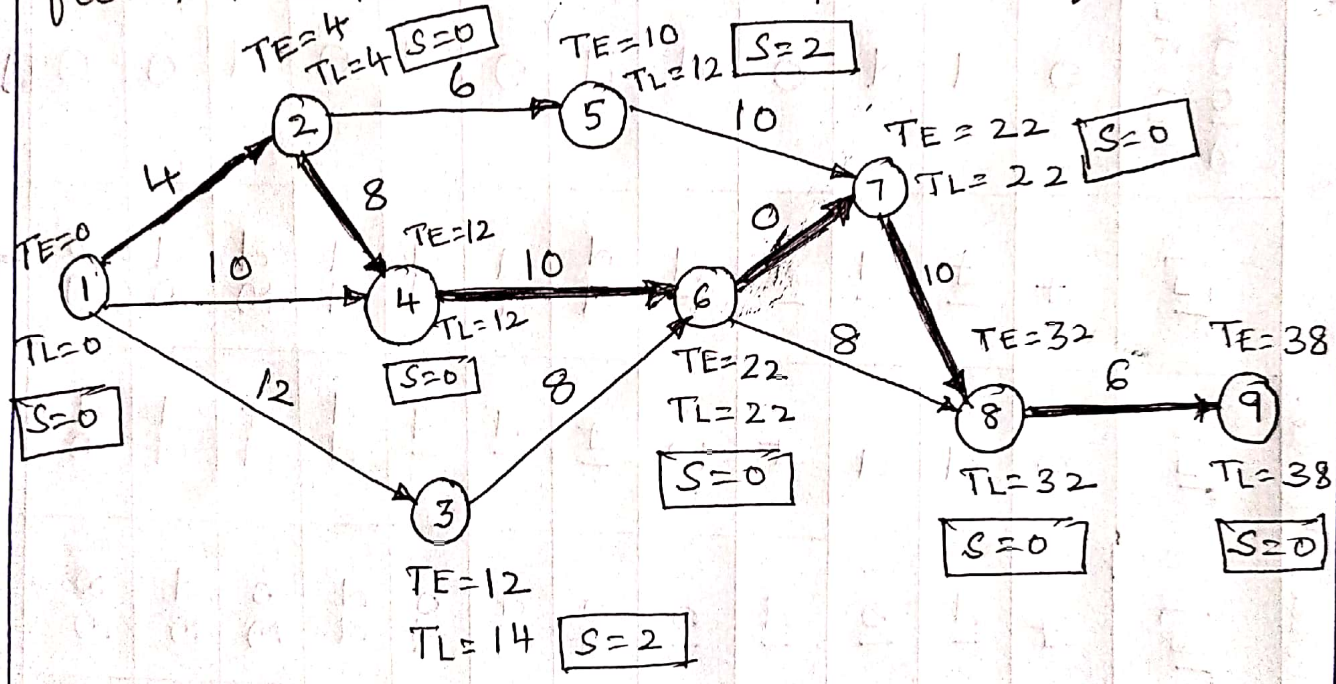
Independent Float :



Let i-j be the activity under interest. and h-i and j-k respectively be its predecessor and successor activities. Let the predecessor job h-i finish at its latest possible moment TLi and successor job j-k start at its earliest possible moment TEj. The activity i-j can take up any duration from teij to (TEj - TLi) without in anyway affecting network. The difference between $(TE_j - TL^i)$ and t_{ij} is called The Independent float. i.e

Independent Float $= (TE_j - TL_i) - t_{ij}$

Ex: Find The Critical path for The following network and also find its floats i.e Total float, Free Float and Independent float.



Critical path: 1 - 2 - 4 - 6 - 7 - 8 - 9.

Tabular Form:

Activity				From table	From table				$T_{Ej}-(T_{Ei}+t^{ij})$	$(T_{Ej}-T_{Li})-t^{ij}$
Predecessor Event i	Sucessor event j	t^{ij}	T_{Ei}	T_{Ej}	T_{Li}	T_{Lj}	Slack $T_{Lj}-T_{Ej}$	Total Float TF	Free Float FF	Independent Float IF
✓ 1	2	4	0	4	0	4	0	0	0	0
1	3	12	0	12	2	14	2	2	0	0
1	4	10	0	10	2	12	2	2	2	2
✓ 2	4	8	4	12	4	12	0	0	0	0
2	5	6	4	10	6	12	2	2	0	0
3	6	8	12	20	14	22	2	2	2	0
✓ 4	6	10	12	22	12	22	0	0	0	0
5	7	10	10	20	12	22	2	2	2	0
✓ 6	7	0	22	22	22	22	0	0	0	0
6	8	8	22	30	24	32	2	2	2	2
✓ 7	8	10	22	32	22	32	0	0	0	0
✓ 8	9	6	32	38	32	38	0	0	0	0

Critical path 1 - 2 - 4 - 6 - 7 - 8 - 9

From network

Ex.No. 2: Find the critical path from the following network and its floats.

or

A project schedule has the following characteristics

Activity	1-2	1-3	2-4	3-4	3-5	4-9	5-6
Time (days)	4	1	1	1	6	5	4
Activity	5-7	6-8	7-8	8-10	9-10		
Time (days)	8	1	2	5	7		

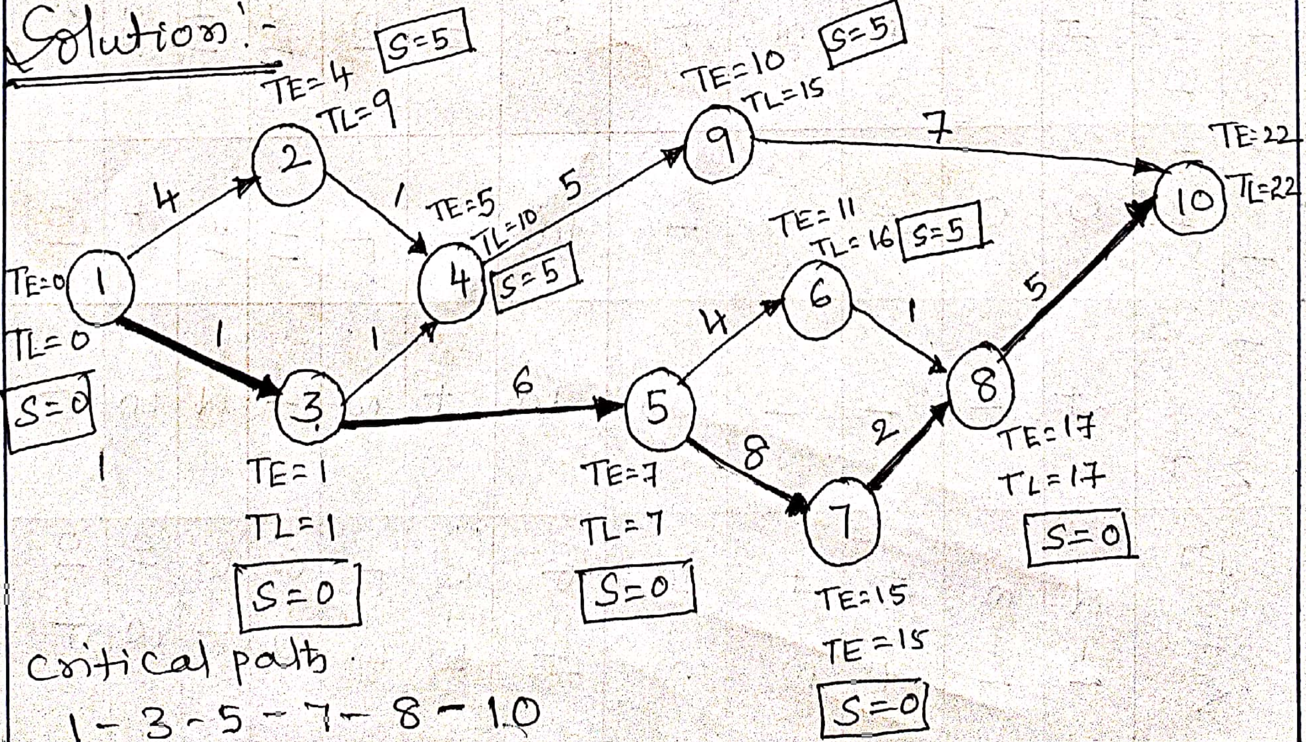
(i) construct a network diagram

(ii) Compute the earliest event time and latest event time

(iii) Determine the critical path and total project duration

(iv) Compute total, free float, Independent float for each activity.

Solution:-



critical path

1 - 3 - 5 - 7 - 8 - 10

Activity	Normal time t_{ij} t_e	Earliest		Latest		Total Float $(TL_j - TE_j)$	Independent Float $TE_j - (TL_i + t_{ij})$	Free Float $TE_j - (TE_i + t_e^{ij})$
		Start TE_i	Finish TE_j	Start TL_i	Finish TL_j			
1 - 2	4	0	4	5	9	5	0	0
1 - 3	1	0	1	0	1	0	0	0
2 - 4	1	4	5	9	10	5	0 (-ve)	0
3 - 4	1	1	2	9	10	8	3	3
3 - 5	6	1	7	1	7	0	0	0
4 - 9	5	5	10	10	15	5	0 (-ve)	0
5 - 6	4	7	11	12	16	5	0 ([illegible])	0
5 - 7	8	7	15	7	15	0	0	0
6 - 8	1	11	12	16	17	5	0	5
7 - 8	2	15	17	15	17	0	0	0
8 - 10	5	17	22	17	22	0	0	0
9 - 10	7	10	17	15	22	5	0	5

critical path 1 - 3 - 5 - 7 - 8 - 10

PERT : TIME ESTIMATES

PERT Stands for programme (or project or performance) evaluation and Review Techniques. which can be applied to any field requiring

- planning
- Controlled &
- integrated / Scheduled work efforts

to accomplish established goals.

The PERT System uses a network diagram consists of events, which must be established to reach project activities. The commencement or completion of an activity is called an event. It indicates a point in time and does not require any resources.

Time is the most essential basic variable in PERT. It is assumed that there is always some factor of Uncertainity in estimating time. (or some other measure of performances) of any operation, which had not been done before the time required to complete any job also varies.

In PERT We try to find out the best estimate of time using appropriate statistical method. PERT also provides the confidence limit for the expected project duration.

Thus to Take the Uncertainity into account, PERT planners make three kinds of time estimates.

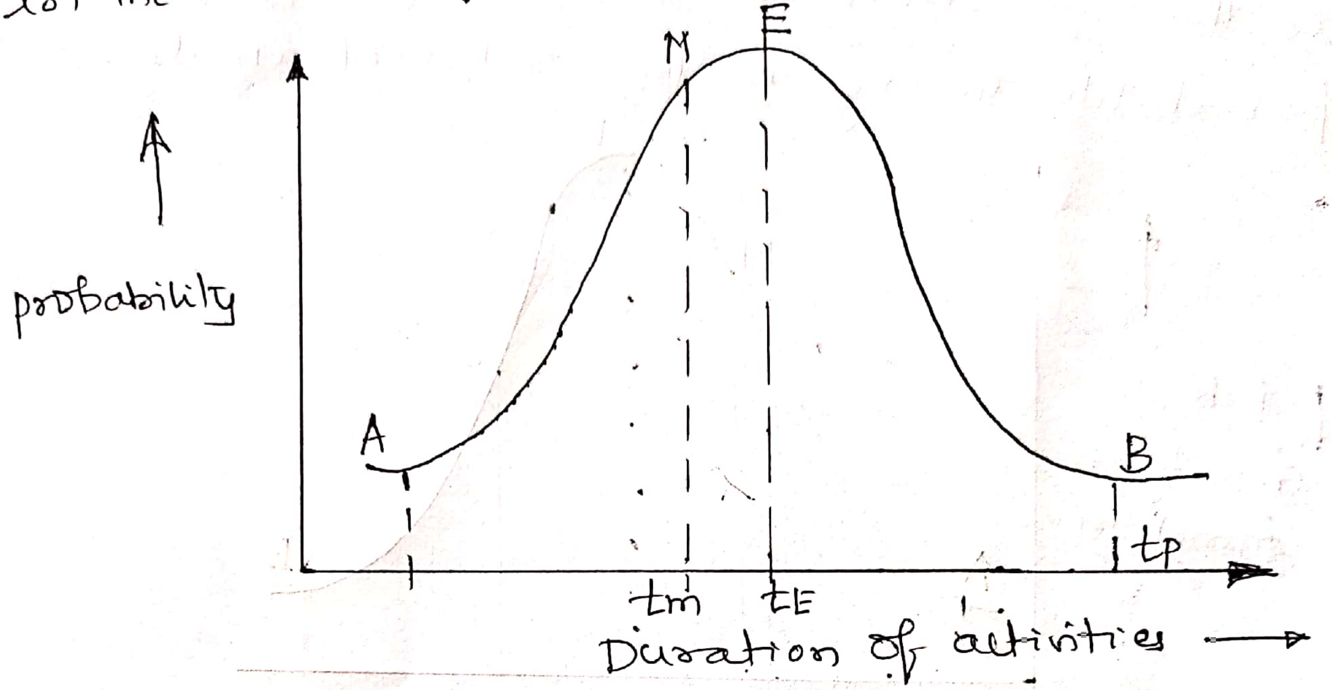
① **Optimistic time (to)** It is The shortest possible time to Complete the activity if all goes well.

② **Most likely time (tm)** Most probable time. The time which most often is required, if the activity is repeated a number of times. Most likely time is the time that, in the mind of The estimator, represents the time The activity would most often require if normal conditions prevail.

tm lies between the Optimistic time and pessimistic time estimates.

③ **Pessimistic time (tp)**: It is The longest time for the execution of any activity Under adverse conditions, excluding the acts of nature such as labour strikes or unrests etc. This time is most difficult to estimate.

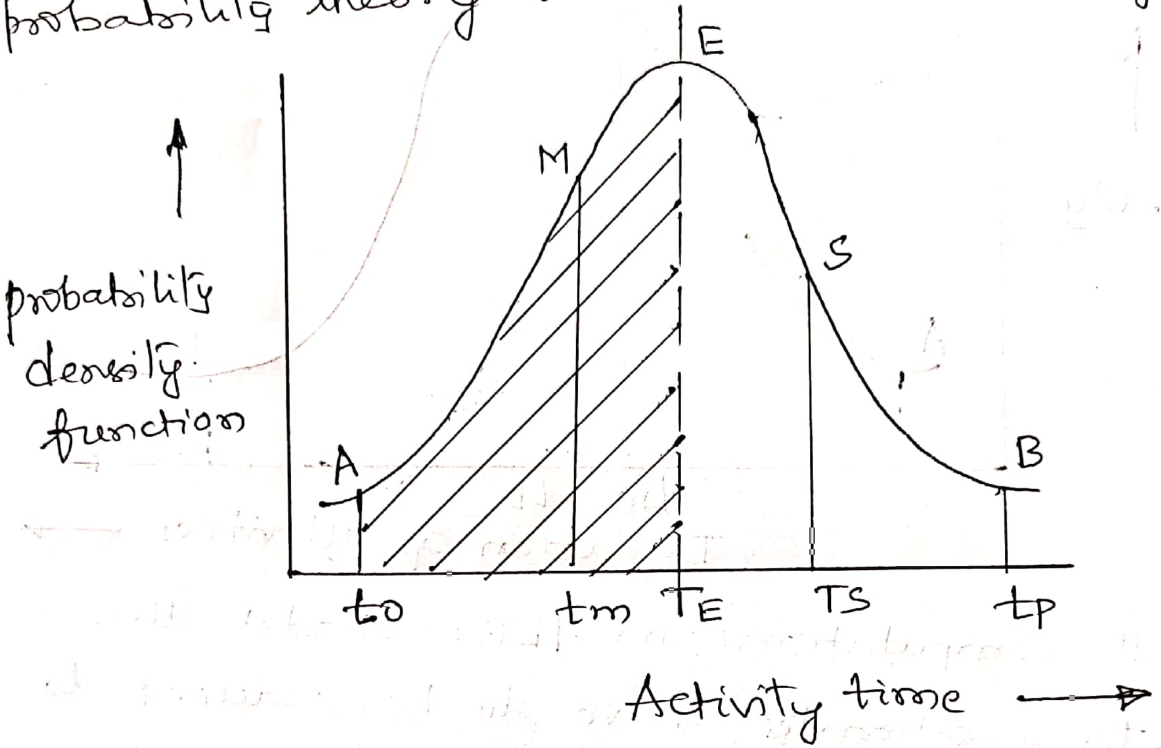
The three time estimates are shown in relation to activity completion time distribution in the below Fig.



For all computations in PERT model, these three time estimates have to be reduced to single time estimate and this accomplished with the help of B-distribution. The average time or the expected time denoted by 'te' indicates that there is a 50% of chance of completing the activity within this time.

PROBABILITY OF MEETING THE SCHEDULED DATES:

After identifying the critical path and latest allowable time with the help of assumption $(T_L^j = TE_j)$ or given Scheduled completion time, for the project, the next question that remaing to be answered is " What is the probability of meeting the scheduled time'. The answer to this question is sought by applying the probability theory to the network analysis.



Average expected time

$$\boxed{t_E^{ij} = \frac{to + 4tm + tp}{6}}$$

This represents the probability of completing the activity ij is 50%, which is shown by the shaded area.

Next we will calculate what is the probability of completing the project at time (Ts)

$$\text{probability } P(TS) = \frac{\text{Area under AES}}{\text{Area under AEB}}$$

So P(S) depends upon the location. of TS. Taking TE as a reference point, and distance TE, TS Can be expressed in terms of standard deviation The value of the std. deviation for a network is calculated.

Std. deviation for network

$$\sigma = \sqrt{\text{Sum of variance along the critical path}}$$

$$\sigma = \sqrt{(\sigma_{ij})^2}$$

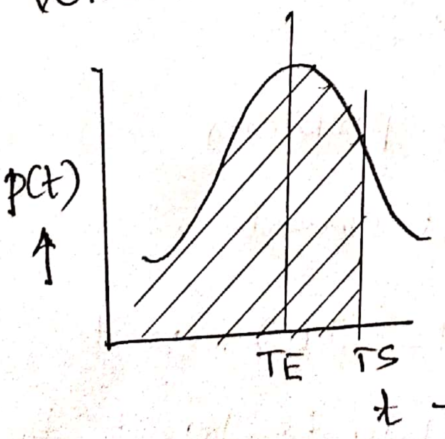
Since the std. deviation for a normal curve is 1. the σ calculated above is used as a scale factor for calculating the normal deviate.

Normal deviation $Z = \frac{T_S - T_E}{\sigma}$

probability factor

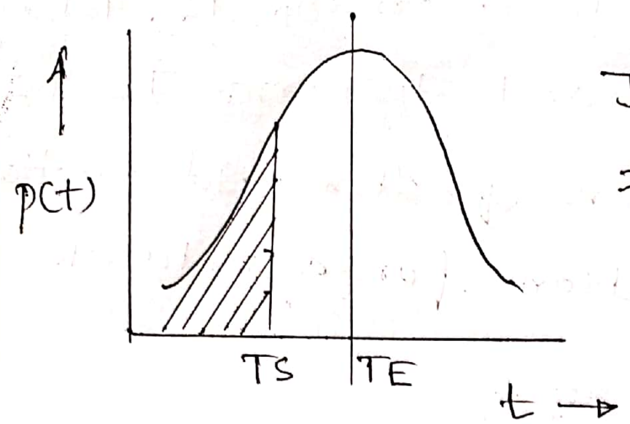
The probability factor (Z) can be +ve, -ve or zero.

① when $Z \rightarrow +ve$ (Ts to the right of TE)



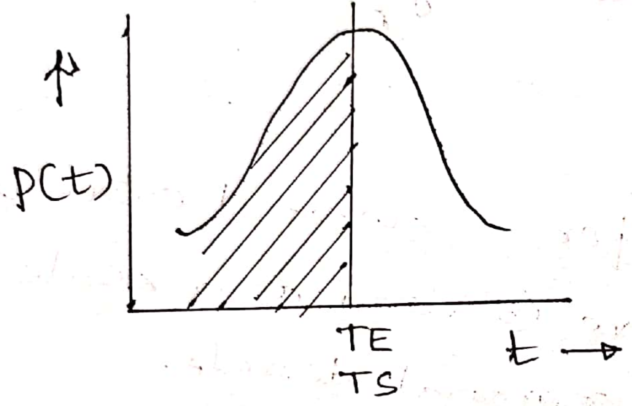
the change of completing the project in time are more than 50%.

② when $Z \rightarrow -ve$ (Ts to the left of TE)



The change of completing the project in time is less than 50%.

when Z → zero (TS coincide with TE)



The chance of completing the project in time is 50%.

Steps for finding the probability of meeting the scheduled time of completion.

① $\sigma = \sqrt{\text{Sum of variances along the critical path}}$

$$\sigma = \sqrt{(\sigma_{ij})^2} = \sqrt{\left(\frac{tp-to}{6}\right)^2}$$

② Ts is Known / Given in the problem

③ TE is Known for the last event

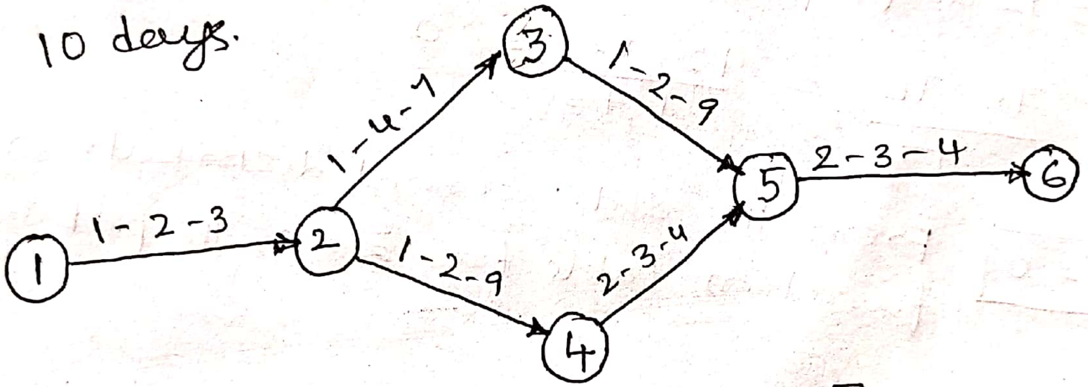
④ Find the time distance (TS − TE) and expressed in terms of probability factor Z by the relation $Z = \frac{TS - TE}{\sigma}$

5) Find % probability w.r.t the normal deviate from the table.

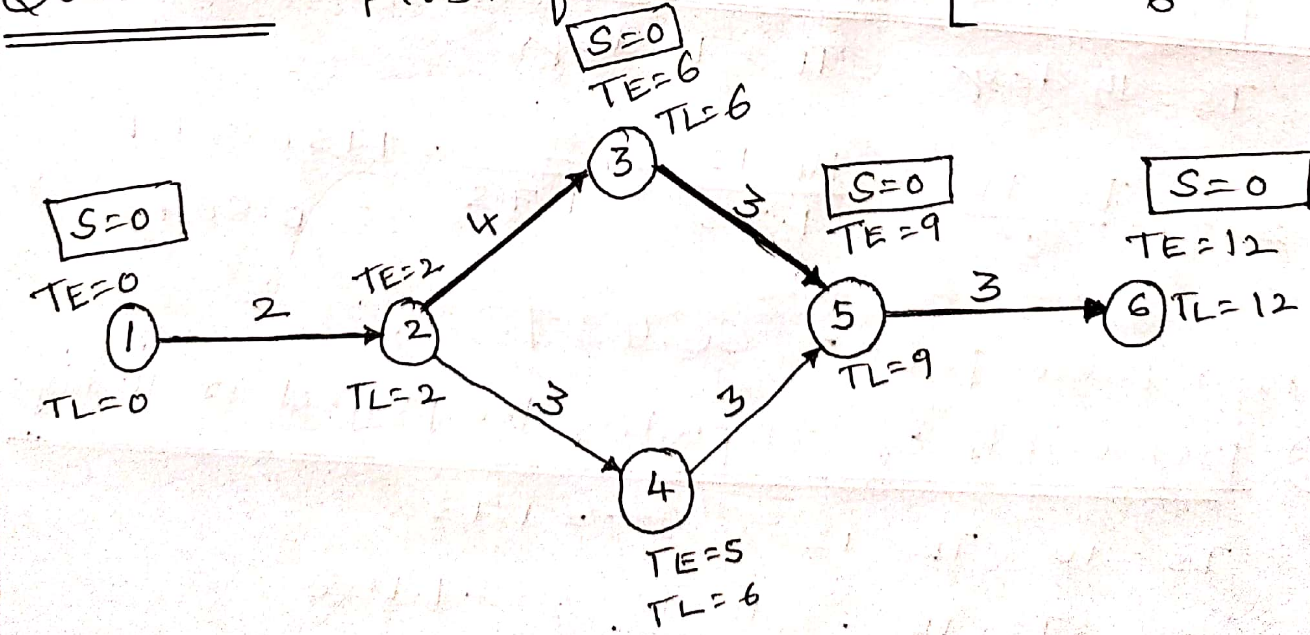
problems:- The three time estimates are given for the activities. Calculate the critical path & also ⓐ what is the probability of completing The project in 12 days

ⓑ what is the probability of completing The project in 14 days.

ⓒ what is the probability of completing The project in 10 days.



Solution:- First find out $t_e^{ij} = \left[\frac{t_o + 4t_m + t_p}{6}\right]$



Critical path : 1 - 2 - 3 - 5 - 6

$\sigma = \sqrt{(\sigma_{ij})^2 \text{ for the Critical path}}$

$\sigma = \sqrt{\left(\frac{3-1}{6}\right)^2 + \left(\frac{7-1}{6}\right)^2 + \left(\frac{9-1}{6}\right)^2 + \left(\frac{4-2}{6}\right)^2}$

$\boxed{\sigma = 1.73}$ is used as scale factor to deviate Z.

Calculate the normal deviate Z.

$Z = \frac{TS - TE}{\sigma}$

ⓐ probability of completing the project in 12 days

TS = 12 days TE = 12 days

$Z = \frac{TS - TE}{\sigma} = \frac{12-12}{1.73} = 0$

$\boxed{Z = 0}$ From the std. normal distribution function table for Z = 0, The probability

θ = 50%.

ⓑ probability of completing the project in 14 days.

TS = 14 days TE = 12 days

$Z = \frac{TS - TE}{\sigma} = \frac{14-12}{1.73} = \frac{2}{1.73} = 1.156 = 1.1$

$= 0.8643$

From table P(t) = 86.43%.

ⓒ probability of Completing the project in 10 days

TS = 10 TE = 12 $\sigma = 1.73$

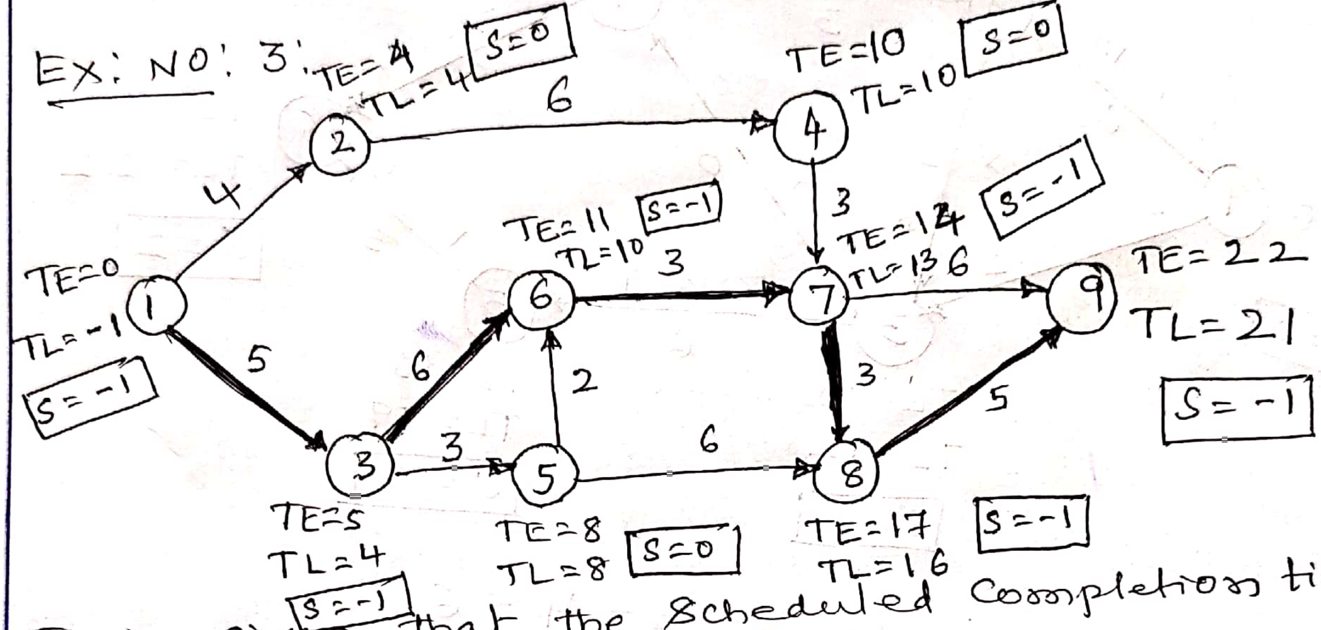
$Z = \frac{TS - TE}{\sigma} = \frac{10-12}{1.73} = -1.156$

$= -1.1$

$$\therefore \quad Z = \frac{TS - TE}{\sigma} = \frac{22-20}{2} = 2/2 = 1.0$$

From table, the probability of finishing the project in 22 days is <u>84.1%</u>.

Ex: NO: 3:



It is given that the scheduled completion time in 21 days. Also identify subcritical path if any.

Solution: The project works 1 day beyond the scheduled date.

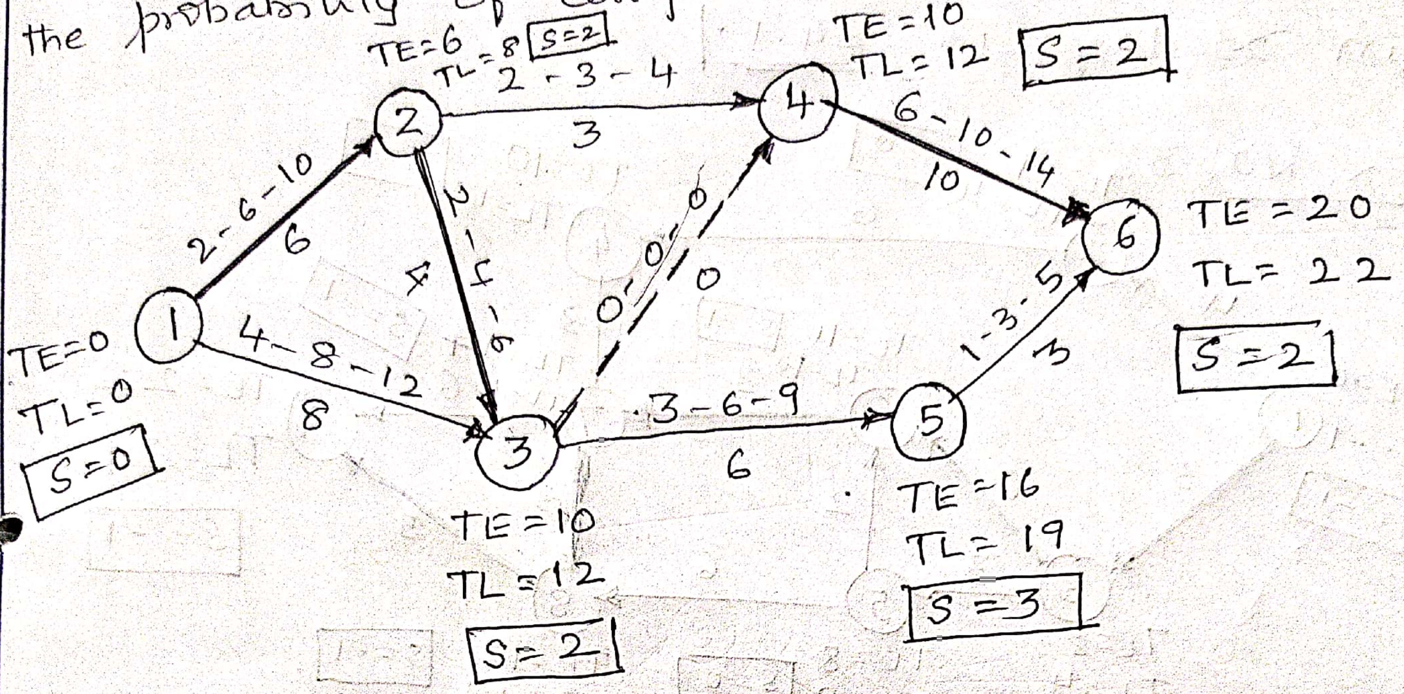
The critical path: 1 - 3 - 6 - 7 - 8 - 9

Sub critical path: 1 - 2 - 4 - 7 - 8 - 9

Sub critical path: 1 - 3 - 5 - 6 - 7 - 8 - 9

P(t) = 13.6%.

For the network shown in Fig. below. Calculate the probability of completion in 22 days.



Critical path: 1-2-3-4-6.

Earliest expected time for the project is 20 days

Scheduled completion time is 22 days

Along the critical path:

Activity	to	tp	$(\sigma_{ij})^2 = \sqrt{\left(\frac{tp-to}{6}\right)^2}$
1-2	2	10	1.78
2-3	2	6	0.44
3-4	0	0	0
4-6	6	14	1.78

$\Sigma \sigma_{ij}^2 = 4.00$

∴ σ for the network $= \sqrt{(\sigma_{ij})^2} = \sqrt{4} = 2.$

MODULE-5 GAMETHEORY

A game theory is a decision theory applicable to competative situations. Competative situations arrives frequently in the field of economic and military operations.

GAME : The competative situation is called a game if (i) there is a finite number of participants called players

(ii) A finite number of possible course of action. i.e (strategies available to each player)

(iii) If the play of the game result, when each player as choosen a course of action

(iv) Every play is associated with an out come (generally money). it determines set of game one to each player.

(v) A loss is consider as negative game.

When 'n' players are involved in a game, then it is a n-person game. A game in which the gain of one player and loss of another player is called Zero-sum game, i.e in a zero-sum game, the algebraic sum of gain of all the player after a game is zero. When two players are involved, the game is called two-person zero sum game or rectangular game. In this the resulting gain is represented in the form of pay-off matrix. The pay-off matrix shows how the payment should be made at the end of the play.

Definition:-

(i) Strategy: A decision rule by which the player determine his course of action is called a Strategy

Generally two types of Strategies are available.

(a) pure strategy: If a player knows exactly what the other player is going to do. A deterministic situation is obtained and the objective function is to maximize the expected gain. pure Strategy is a decision rule always to select a particular course of action.

(b) Mixed Strategy:- If the player is guessing which activity is to be selected by the other. A probabilistic situation is obtained and the objective function is to maximize the expected gain. Thus The mixed Strategy is a selection among pure Strategy with a fixed probability.

Two-person zero sum game or Rectangular game.

* Two players participates
* Each player has certain no! of strategies available to him
* Each strategy result in a pay-off (or) outcome
* The total pay of the two player at the end of the play is zero.

Pay-off Matrix:- It is the outcome of playing the game. The pay-off matrix is a table showing the amount received by the player made at the left hand of the table and the payment is made by the player is at the top of the table.

MAXIMIN AND MINIMAX PRINCIPLE

MAXIMIN PRINCIPLE:

For ex:-

$$\begin{array}{cc|cc|c} & & \multicolumn{2}{c}{\text{player B}} & \\ & & 4 & 5 & \\ \hline & 1 & -2 & -3 & -3 \\ \text{player A} & 2 & -1 & 2 & -1 \\ & 3 & \textcircled{1} \rightarrow \text{Key element} & 3 & \overset{\text{value of game}}{\textcircled{1}} \\ \hline & & \textcircled{1} & 3 & \end{array}$$

In this example, the player A will get atleast -3, -1 and 1 when he play the strategies 1, 2 and 3 respectively. These are the worst gain of player A. out of these maximum is ① which correspond to the Strategy 3 of the player A. Thus according to maximum principle, The player A should use third Strategy, in this guaranted gain is maximum.

MINIMAX PRINCIPLE: From player B point of view the maximum losses are to be 1 and 3, when he use the strategies 4 and 5. The player B is interested to minimize his losses. The minimum of these losses is 1 which correspond to Strategy 4 of the player B. Thus according to minimax principle, the player B should use strategy 4 by which he assure that he will not loose more than one. The Value of the game is 1. Some times the maximin is called

Here MAXIMIN = MINIMAX = 0

Hence saddle point exists.

∴ Saddle point is (2,5) and the value of the game is zero. Hence it is a fair game.

(ii) Solve the game whose pay-off matrix is given below:

B

player A

$$\begin{array}{c|cccc|c} & B_1 & B_2 & B_3 & B_4 & \\ \hline A_1 & -5 & 2 & 0 & 7 & -5 \\ A_2 & 5 & 6 & \boxed{4} & 8 & \boxed{4} \\ A_3 & 4 & 0 & 2 & -3 & -3 \\ \hline & 5 & 6 & \boxed{4} & 8 & \end{array}$$

MAXIMIN

MINIMAX

Here MAXIMIN = MiniMax

4 = 4

Hence saddle point exist.

$A_2 B_3$ is a saddle point and valee of the game = 4.

(iii) Solve the game whose pay-off matrix is given below:-

Player B

player A

$$\begin{array}{c|ccc|cc} & B_1 & B_2 & B_3 & & \\ \hline A_1 & \boxed{-2} & 15 & \boxed{-2} & \to & \boxed{-2} \\ A_2 & -5 & -6 & -4 & \to & -6 \\ A_3 & -5 & 20 & -8 & \to & -8 \\ \hline & \downarrow & \downarrow & \downarrow & & \\ & \boxed{-2} & 20 & \boxed{-2} & & \end{array}$$

MaxiMIN

(First Take Minimum value and out of this take Maximum value)

MINIMAX

(First take Maximum value and out of this take minimum value)

MaxiMIN = MINIMAX

-2 = -2

Hence Saddle Point exists.

Saddle point :- $(A_1 B_1)$ and $(A_1 B_3)$ and value of game = -2

V = -2

(iv) For what value of μ, the game with the following pay-off matrix is strictly determinable.

$$\begin{array}{cc|ccc|cc} & & & \text{Player B} & & & \\ & & 4 & 5 & 6 & & \\ \hline & 1 & \textcircled{\mu} & 6 & 2 & \rightarrow \textcircled{2} & \\ \text{Player A} & 2 & -1 & \mu & -7 & \rightarrow -7 & \text{MAXIMIN} \\ & 3 & -2 & 4 & \mu & \rightarrow -2 & \\ \hline & & \downarrow & \downarrow & \downarrow & & \\ & & \textcircled{-1} & 6 & 2 & & \\ & & & \text{MINIMAX} & & & \end{array}$$

μ lies between -1 and 2 $\left.\right\}$ $-1 \leq \mu \leq 2$

MIXED STRATEGY PROBLEM BY USING DOMINANCE PRINCIPLE :-

DOMINANCE: When There is no saddle point, the pure Strategy cannot be used and mixed strategy will have too restore, when pure strategy are not available (ie MAXIMIN = MINIMAX). The next step is to eliminate certain strategy by Dominance.

GENERAL RULES OF DOMINANCE

1. If all the element of the column say (i^{th} column) are ≤ the corresponding element of the other column say (j^{th} column). Then i^{th} column dominates j^{th} column (i,e. j^{th} column can be deleted from the matrix)

For ex:

Consider columns I&II

$$\begin{array}{cc} I & \cancel{II} \\ 3 & \cancel{3} \\ 2 & \cancel{4} \\ 5 & \cancel{7} \end{array}$$

$$\left.\begin{array}{l} 3=3 \\ 2<4 \\ 5<7 \end{array}\right\}$$

Here column I elements are ≤ to the element of the column II. Therefore column I dominates column II

2. If all the elements of i^{th} row ≥ to the corresponding elements of other row say j^{th} row. Then ith row dominates j^{th} row (i.e j^{th} row can be deleted from the matrix.

For. ex, Consider row I and II

$$\begin{array}{c|cc} I & 4 & 8 \\ \cancel{II} & \cancel{4} & \cancel{4} \end{array}$$

$$\left.\begin{array}{l} 4=4 \\ 8>4 \end{array}\right\}$$

Here, the elements of row I ≥ the elements of row II Therefor row I dominate row II

3. A given strategy can be dominated if it is inferior to an average of two or more other pure stratagies.

For. ex: 1)

$$\begin{array}{c|cc|c} & \downarrow I & \downarrow II & \downarrow \cancel{III} \\ 1 & 1 & 3 & \cancel{2} \\ 2 & 7 & -5 & \cancel{1} \\ 3 & 4 & -1 & \cancel{2} \end{array}$$

Here in this matrix, The above two 1 and 2 rules are not satisfied. Hence apply rule 3. i.e Avg. of columns I and II ≤ the column III

Hence Avg. of columns I and II dominates Column III

$$\left.\begin{array}{l} \frac{1+3}{2} = 2 = 2 \\ \frac{7-5}{2} = 1 = 1 \\ \frac{4-1}{2} = 1.5 \le 2 \end{array}\right\}$$

Column III

The matrix reduces to →

Ex. 2.

$$\begin{array}{r|cc|} & I & II \\ \to 1 & 1 & 3 \\ \to 2 & 7 & -5 \\ \cancel{3} & \cancel{4} & \cancel{-1} \end{array}$$

Here Avg of rows 1 and 2 are ≥ to row 3. Hence Avg. of rows 1 and 2 dominates row 3

Here Avg. of rows 1 and 2 = [illegible] row 3. }

Avg. of rows I and 2

$\frac{1+7}{2} = 4 \quad \frac{3-5}{2} = -1$

$$\left.\begin{array}{l} 4=4 \\ -1=-1 \end{array}\right\}$$

problem 1: Solve the following game by using dominance principle:

player B

$$\begin{array}{c|ccccc} \text{player A} & I & II & III & IV & V \\ \hline 1 & 3 & 5 & 4 & \cancel{9} & \cancel{6} \\ 2 & 5 & 6 & 3 & \cancel{7} & \cancel{8} \\ 3 & 8 & 7 & 9 & \cancel{8} & \cancel{7} \\ 4 & 4 & 2 & 8 & \cancel{5} & \cancel{3} \end{array}$$

Solution:- consider 1 rule, ie column wise, Here by seeing columns, column II elements (ie 5, 6, 7, 2) are ≤ to the corresponding elements columns IV (ie, 9, 7, 8, 5) and column V (i.e 6,8,7,3) Hence column II dominates column IV and V. Hence matrix reduces to

player B

$$\begin{array}{c|ccc} \text{player A} & I & II & III \\ \hline \cancel{1} & \cancel{3} & \cancel{5} & \cancel{4} \\ \cancel{2} & \cancel{5} & \cancel{6} & \cancel{3} \\ 3 & 8 & 7 & 9 \\ \cancel{4} & \cancel{4} & \cancel{2} & \cancel{8} \end{array}$$

Again observe all the elements in rows and columns, Here rule I is not satisfied (i.e. Columnwise) Hence apply rule 2 (i.e row wise). By seeing rows, row 3 elements are ≥ to the elements rows 1, 2 and 4. Hence row 3 dominates row 1, 2 and 4. Hence matrix reduces to

$$\begin{array}{cc|ccc|} & & & \text{player B} & \\ & & \cancel{I} & II & \cancel{III} \\ \text{Player A} & 3 & \cancel{8} & 7 & \cancel{9} \end{array}$$

Here in the reduced matrix column II element 7 is ≤ to the element of column I and III (ie 8 and 9)

Hence Column II dominates column I and III.

Hence matrix reduces to 1X1 and the value of the game is 7 and Saddle point is (3, II).

$$\begin{array}{cc|c|} & & B \\ & & II \\ A & 3 & 7 \end{array}$$

* Imp Note:- By applying dominance rule, it is not always reduces to 1X1. In maximum majority it is reduced to 2X2 matrix. When it is reduced to 2X2 matrix and there is no saddle point. then we can apply either Arithmatic method or Algebraic method to find the value of the game.

I METHOD:- Arithmatic method for 2X2 game:-

Step 1:- Subtract the two digits in column 1 and Write them under column 2, ignoring the sign.

Step 2:- Subtract the two digits in column 2 and Write them under column 1, ignoring the sign.

Step 3:- Repeat the above steps for rows also. These values are called oddments. These ~~$~~ represents the frequency with which the players uses their course of action (ie Strategy).

For ex:- In a game of matching coins with two players, player A wins Rs 2 if there are two heads, win nothing, if There are two tails and looses Rs.1 when there are one head and one tail. Determine the pay of matrix, the best strategy for each player and the value of the game for players A.

Solution:- In this problem, Both players have two strategies $\overset{H}{\text{Head}}$ and $\overset{T}{\text{Tail}}$. Therefore it is a 2X2 matrix.

Player B

$$\text{Player A}\quad \begin{array}{c|cc|} & H & T \\ \hline H & 2 & -1 \\ T & -1 & 0 \\ \hline \end{array}$$

Before applying Arithmatic method first check whether it contain Saddlepoint. If there is a saddlepoint, we get a value of the game otherwise you apply Arithmatic method

player B

$$\text{player A}\quad \begin{array}{c|cc|c} & H & T & \\ \hline H & 2 & -1 & \boxed{-1} \\ T & -1 & 0 & \boxed{-1} \\ \hline & 2 & \boxed{0} & \end{array}$$

MAXIMIN

MINIMAX

MAXIMIN ≠ MINIMAX

Hence no saddle point.

Then by applying Arithmatic method

player B

$$\text{player A}\quad \begin{array}{c|cc|cc} & H & T & \text{oddment of A} & \\ \hline H & 2 & -1 & -1-0=1 \text{ (ignore sign)} & \tfrac{1}{4} \\ T & -1 & 0 & 2-(-1)=3 & \tfrac{3}{4} \\ \hline \text{oddment of B} & -1-0 & 2-(-1) & & \\ & 1 & 3 & & \\ & \tfrac{1}{4} & \tfrac{3}{4} & & \end{array}$$

Note

When B uses H strategy

$$\begin{array}{|c|c} H & \text{oddment} \\ \hline 2 & 1 \\ -1 & 3 \\ \hline \end{array}$$

$$V = \frac{2\times1 + -1\times3}{1+3}$$

$$V = -\tfrac{1}{4}$$

By using A's or B's oddment find the value of the game.

Suppose By using A's oddment when B uses H strategy

$$\text{Value of the Game} = V = \frac{2\times1 + -1\times3}{1+3} = -\tfrac{1}{4}.$$

OR

By using A's oddment when B uses T Strategy

$$V = \frac{-1\times1+0\times3}{1+3}$$

$$\begin{array}{c|c} T & \\ \hline -1 & 1 \\ 0 & 3 \end{array}$$

$$v = \frac{-1\times1+0\times3}{1+3}$$

$$v = -\frac{1}{4}$$

$$V = -\frac{1}{4}$$

OR

By using B's oddment when A uses H Strategy

$$V = \frac{2\times1+-1\times3}{1+3}$$

$$\begin{array}{ccc} A\text{'s H} & 2 & -1 \\ & 1 & 3 \end{array}$$

$$V = -\frac{1}{4}$$

OR

By using B's oddment when A uses T Strategy

$$V = \frac{-1\times1+0\times3}{1+3}$$

$$\begin{array}{ccc} A\text{'s T} & -1 & 0 \\ & 1 & 3 \end{array}$$

$$V = -\frac{1}{4}$$

[Note: use any one oddments to find value of the game.]

The complete Solution is player A uses H 25% ($\frac{1}{4}$) of the time and T 75% ($\frac{3}{4}$) of the time. and the player B uses H 25% ($\frac{1}{4}$) and T 75% ($\frac{3}{4}$) of the time and the value of the Game is $-\frac{1}{4}$

Note;- -ve value indicate player B is winning
+ve value indicate player A is winning

problem 1:. A and B play a game in which each has 3 coins a5P, a10P, a20p. Each select a coin without the Knowlege of the other choice. If the sum of The coins is an odd amount A wins B's coin. If the sum is even, B wins A's win. Find the best Strategy for each player and value of the game to player A.

Solution:-

Player B

$$\begin{array}{c|ccc} & a5P & a10P & a20P \\ \hline a5P & -5 & 10 & 20 \\ \text{player A } a10P & 5 & -10 & -10 \\ a20P & 5 & -20 & -20 \end{array}$$

[Note:- Take 1st cell (1,1)
5P + 5P = 10P
Even
B win
[-ve value indicate B win B win A coin]

[Take (1,2) cell
a5p + a10p
= a15p
odd
A wins B coins

Apply Dominance Rule:-

Row 2 dominate Row 3

and column 2 dominate column 3

Hence matrix redeuces to

player B

$$\begin{array}{c|cc|c} & a5P & a10P & \\ \hline a5P & -5 & 10 & \textcircled{-5} \\ \text{Player A } a10P & 5 & -10 & -10 \\ \hline & \textcircled{5} & 10 & \end{array}$$

MINIMAX

Apply maximin & minimax principle

MAXIMIN

$-5 \neq 5$

Hence no saddle point.

Then by applying Arithmatic method

B

$$\begin{array}{c|cc|c} & a5p & a10p & \text{oddment of } A \\ \hline A \quad a5p & -5 & 10 & 15 \longrightarrow \frac{15}{30} \rightarrow \frac{1}{2} \\ a10p & 5 & -10 & 15 \text{ (ignore sign)} \rightarrow \frac{15}{30} \rightarrow \frac{1}{2} \\ \hline \text{oddment of } B & 20 & 10 \text{ (ignore -ve sign)} & \\ & \frac{20}{30} & \frac{10}{30} & \\ & \downarrow & & \\ & \frac{2}{3} & \frac{1}{3} & \end{array}$$

By using A's oddment when B uses asp Strategy

Then value of the game $V = \dfrac{-5\times15+5\times15}{15+15}$

$$\boxed{V=0}$$

The complete solution is

optimal Strategy for A $\left(\frac{1}{2}, \frac{1}{2}, 0\right)$ [Here 3rd row & 3rd column dominated]

optimal Strategy for B $\left(\frac{2}{3}, \frac{1}{3}, 0\right)$

& value of the game = 0.

Problem 2:- Reduce the following game by dominance and find the value of the game.

player B

$$\text{player A}\quad\begin{array}{c|c|c|c|c}
 & \text{I} & \text{II} & \text{III} & \text{IV} \\
\hline
\text{I} & \cancel{3} & \cancel{2} & \cancel{4} & \cancel{0} \\
\hline
\text{II} & 3 & 4 & 2 & 4 \\
\hline
\text{III} & 4 & 2 & 4 & 0 \\
\hline
\text{IV} & 0 & 4 & 0 & 8
\end{array}$$

Solution: Here, observe the matrix, rule 1 is not satisfied (ie column wise). Then goto row wise. (ie rule 2).

Here Row III dominate Row I because Row III ≥ Row I

Hence matrix reduces to

player B

$$\text{player A}\quad\begin{array}{c|c|c|c|c}
 & \text{I} & \text{II} & \text{III} & \text{IV} \\
\hline
\text{II} & \cancel{3} & 4 & 2 & 4 \\
\hline
\text{III} & \cancel{4} & 2 & 4 & 0 \\
\hline
\text{IV} & \cancel{0} & 4 & 0 & 8
\end{array}$$

Here Column III dominate Column I because Column III ≤ Column I

Hence matrix reduces, to

$$\begin{array}{cc|c|c|c|} & & & B & \\ & & \text{II} & \text{III} & \text{IV} \\ \hline & \text{II} & 4 & 2 & 4 \\ \hline A & \text{III} & 2 & 4 & 0 \\ \hline & \text{IV} & 4 & 0 & 8 \\ \hline \end{array}$$

Avg. of III & IV Columns	Column II
$\frac{2+4}{2} = 3$	< 4
$\frac{4+0}{2} = 2$	$= 2$
$\frac{0+8}{2} = 4$	$= 4$

Here Avg. of columns III and IV dominate column II, because the average of all the elements of the column, III & IV are less than equal to all the elements in column II.

Hence matrix reduces to

$$\begin{array}{cc|c|c|} & & B & \\ & & \text{III} & \text{IV} \\ \hline & \text{II} & 2 & 4 \\ \hline A & \text{III} & 4 & 0 \\ \hline & \text{IV} & 0 & 8 \\ \hline \end{array}$$

$\frac{4+0}{2} = 2$ $\quad \frac{0+8}{2} \doteq 4$ $\quad [2=2,\ 4=4]$

Here Avg. of rows III and IV dominate row II, because the average of all the elements of rows, III and IV are greater than or equal to elements of row II. Hence matrix reduces to

$$\begin{array}{cc|c|c|c} & & B & & \\ & & \text{III} & \text{IV} & \\ \hline A & \text{III} & 4 & 0 & ⓪ \\ \hline & \text{IV} & 0 & 8 & ⓪ \\ \hline & & ④ & 8 & \\ & & \text{Minimax} & & \end{array}$$

Apply maximin and minimax principle

Maximin

$0 \neq 4$

Hence no saddle point.

By using Arithmatic method,

$$\begin{array}{cc|cc|l} & & B & & \\ & & \text{III} & \text{IV} & \text{oddment of } A \\ \hline & \text{III} & 4 & 0 & 8 \rightarrow \frac{8}{12} \rightarrow \frac{2}{3} \\ A & \text{IV} & 0 & 8 & 4 \rightarrow \frac{4}{12} \rightarrow \frac{1}{3} \\ \hline & \text{oddment of } B \rightarrow & 8 & 4 & \\ & & \frac{8}{12} & \frac{4}{12} & \\ & & \frac{2}{2} & \frac{1}{2} & \end{array}$$

→ By using A's oddment when B uses III strategy

$$V = \frac{4 \times 8 + 0 \times 4}{8+4}$$

$$V = \frac{32}{12} = \frac{8}{3}.$$

The complete solution is

Optimal Strategy for player A $(0, 0, 2/3, 1/3)$

optimal strategy for player B $(0, 0, 2/3, 1/3)$

& Value of Game $= 8/3$ (+ve value means player A wins)

Problem No: 3: Solve the following game by using principle of dominance.

player B

player A

	I	II	III	IV	V	VI
1	~~4~~	~~2~~	0	2	1	~~1~~
2	~~4~~	~~3~~	1	3	2	~~2~~
3	~~4~~	~~3~~	7	-5	1	~~2~~
4	~~4~~	~~3~~	4	-1	2	~~2~~
5	~~4~~	~~3~~	3	-2	2	~~2~~

Solution:- Column IV dominates column I, II and column VI dominate column VI

Because all the elements of column IV are ≤ the elements of Columns I, II ~~&VI~~. and elements of column V ≤ the element of column VI

Player B

player A

	III	IV	V
~~1~~	~~0~~	~~2~~	~~1~~
2	1	3	2
3	7	-5	1
4	4	-1	2
~~5~~	~~3~~	~~-2~~	~~2~~

Row 4 dominate Row 5

Row 2 dominate Row 1

Hence matrix reduces to

player B

$$\text{player A}\quad \begin{array}{c|ccc} & III & IV & V \\ \hline 2 & 1 & 3 & 2 \\ 3 & 7 & -5 & 1 \\ 4 & 4 & -1 & 2 \end{array}$$

Avg of III & IV — colum V

$\frac{1+3}{2} = 2 = 2$

$\frac{7-5}{2} = 1 = 1$

$\frac{4-1}{2} = 1.5 < 2$

Avg. of columns III and IV dominate column V. Hence matrix reduces to

player B

$$\text{player A}\quad \begin{array}{c|cc} & III & IV \\ \hline 2 & 1 & 3 \\ 3 & 7 & -5 \\ \cancel{4} & \cancel{4} & \cancel{-1} \end{array}$$

Avg. of rows 2 and 3 dominate row 4, Hence matrix reduces to

$$\begin{bmatrix} \frac{1+7}{2} & \frac{3-5}{2} \\ =4 & -1 \end{bmatrix}$$

B

$$A\quad \begin{array}{c|cc|c} & III & IV & \\ \hline 2 & 1 & 3 & ① \\ 3 & 7 & -5 & -5 \\ \hline & 7 & ③ & \end{array}$$

MINIMAX

MAXIMIN

$1 \neq 3$

No saddle point

By using Arithmatic method

B

$$A\quad \begin{array}{c|cc|c} & III & IV & \text{oddments of A} \\ \hline 2 & 1 & 3 & 12 \quad \frac{12}{14} = \frac{6}{7} \\ 3 & 7 & -5 & 2 \quad \frac{2}{12} = \frac{1}{6} \\ \hline \text{Oddments of B} & 8 & 6 & \\ & \frac{8}{14} & \frac{6}{14} & \\ & \frac{4}{7} & \frac{3}{7} & \end{array}$$

By using A's oddment when 'B' uses III strategy

$$V = \frac{1\times12 + 7\times2}{12+2} = \frac{26}{14}$$

$$V = \frac{13}{7}$$

The complete solution is

optimum strategy for player A $= (0, 6/7, 1/7, 0, 0)$

optimum strategy for player B $= (0, 0, 4/7, 3/7, 0)$

$$V = 13/7$$

GRAPHICAL SOLUTION To $2 \times n$ or $m \times 2$ game

1. Solve the game whose pay-off matrix is

$$\begin{array}{cc|cccc|ll} & & & & B & & & \\ & & \underline{I} & \underline{II} & \underline{III} & \underline{IV} & & \\ \hline & 1 & 1 & 4 & -2 & -3 & \xrightarrow{} -3 & x_1 \\ A & 2 & 2 & 1 & 4 & 5 & \xrightarrow{} \textcircled{1} & (1-x_1) \\ \hline & & 2 & 4 & 4 & \textcircled{5} & & \end{array}$$

MINIMAX

$\underline{\text{MAXIMIN}}$

MAXIMIN $\neq$ MINI MAX

$1 \neq 5$

<u>No saddle point</u>

<u>Solⁿ</u>:- It is a $2 \times n$ game. Here you have to find expected gain MAXIMIN point. (With respect to player A, or expected gain of player A) Use graph sheet. Reduce $2 \times n$ game into 2×2 game by using graphical method, when there is no saddle point.

Suppose a player A chooses 1 strategy with probability x_1, then the probability of choosing his 2 strategy is $(1-x_1)$

The expected gain of player A when player B uses his strategy I is $B_I = 1x_1 + 2(1-x_1)$

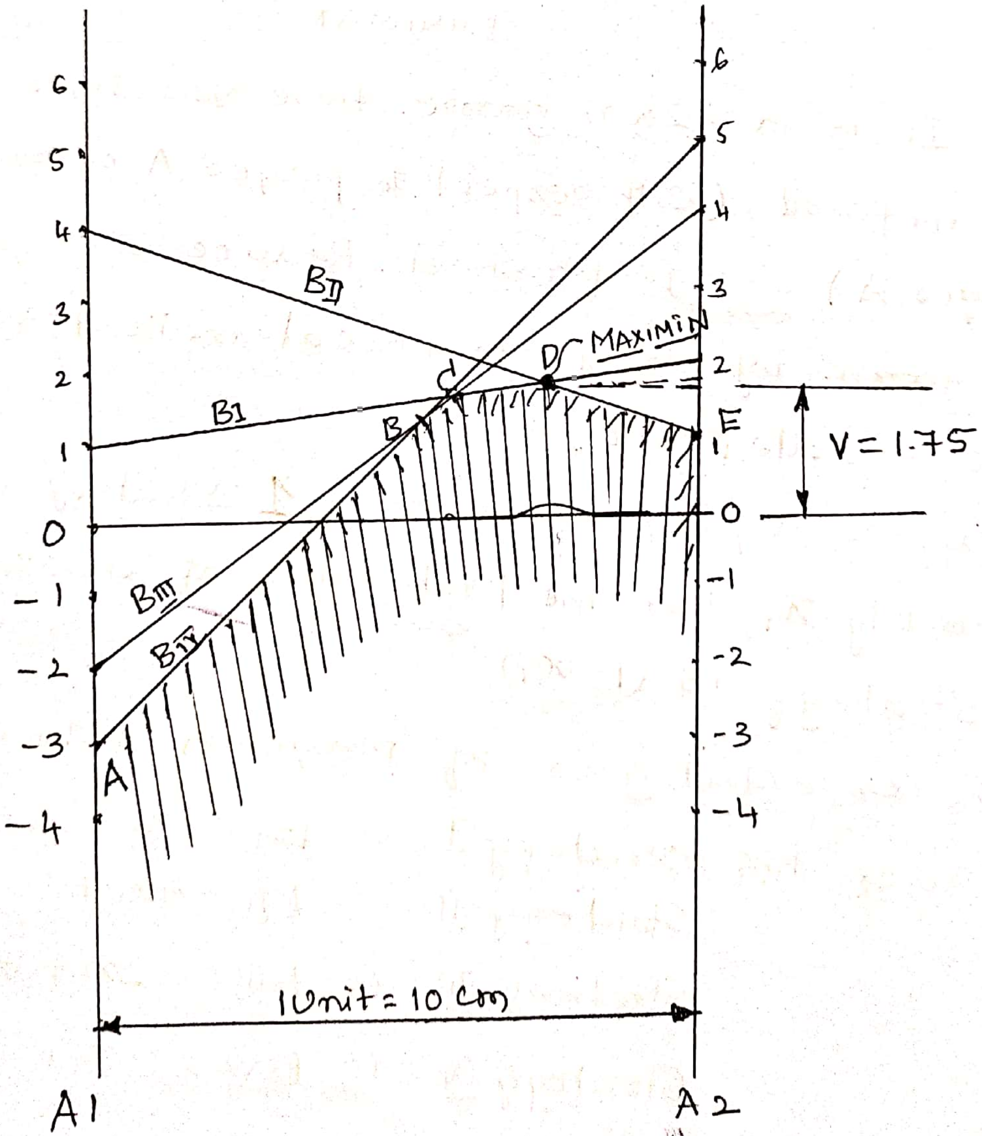
Strategy II is $B_{II} = 4x_1 + 1(1-x_1)$

Strategy III is $B_{III} = -2x_1 + 4(1-x_1)$

Strategy IV is $B_{IV} = -3x_1 + 5(1-x_1)$

Represent these B_I, B_{II}, B_{III} & B_{IV} by means of lines procedure :- Two vertical lines are drawn one unit apart Take 1Unit = 10 cm. (These vertical lines represent strategies of A A_1 and A_2).

To draw the gain line of player A, when player B uses strategy I, Join value of 1 on first line to value of 2 on line two. Draw other gain line of player A we have to find a point which maximize the minimum expected gain of player A (ie MAXIMIN)



From graph, point D'(MAXIMIN) is The highest point of the lowest boundry is the required point. There are two course of action (BI and BII) corresponding to This point are available. ie player B uses I and II Stralegy out of 4 Strategies. Hence 2X4 matrix reduces to 2X2 game.

$$\begin{array}{cc|c|c|c} & & \multicolumn{2}{c}{B} & \\ & & I & II & \\ \hline A & 1 & 1 & 4 & \textcircled{1} \\ \hline & 2 & 2 & 1 & \textcircled{1} \\ \hline & & \textcircled{2} & 4 & \\ & & \multicolumn{2}{c}{\text{MINIMAX}} & \end{array}$$

check saddle point.

MAXIMIN

$1 \neq 2$

MAXIMIN ≠ MINIMAX

No saddle point.

Then By using Arithmatic method,

$$\begin{array}{cc|c|c|cc} & & \multicolumn{2}{c}{B} & & \\ & & I & II & \multicolumn{2}{c}{\text{oddments of A}} \\ \hline A & 1 & 1 & 4 & 1 & 1/4 \\ \hline & 2 & 2 & 1 & 3 & 3/4 \\ \hline \multicolumn{2}{c}{\text{oddments of B}} & 3 & 1 & & \\ & & 3/4 & 1/3 & & \end{array}$$

To Find value of the game!

By using oddment's of A when 'B' uses strategy I

$$V = \frac{1 \times 1 + 2 \times 3}{1+3} = 7/4 = 1.75$$

$V = 1.75$

The value of the game is also shown on the graph.

2. Solve the game whose pay-off matrix is

$$\begin{array}{ccc|cc|} & & & B & \\ & & & I & II \\ \hline & & 1 & 3 & -2 \\ A & & 2 & -1 & 4 \\ & & 3 & 2 & 2 \\ \hline & & & y_1 & (1-y_1) \end{array}$$

Sol^n: It is a $\overset{(3\times 2)}{(m\times 2)}$ game. Here we have to find MINIMAX point C with respect to player B.

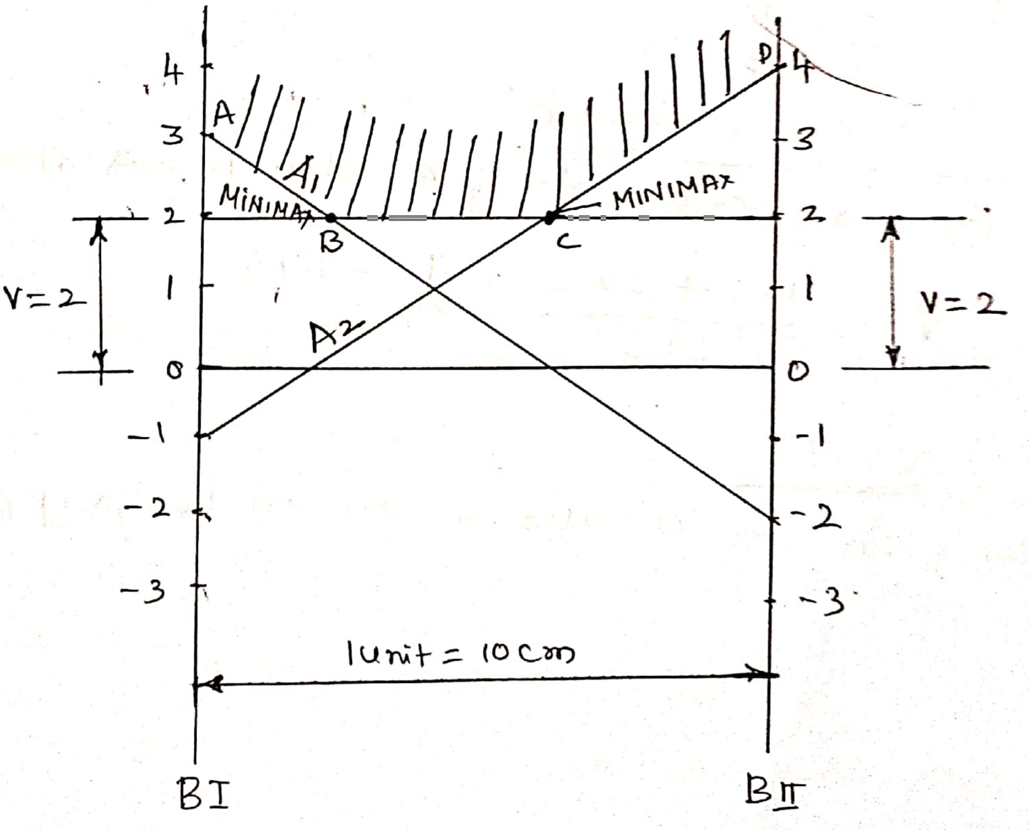
Suppose the player B chooses his I strategy with probability y_1, Then the probability of using his strategy is $(1-y_1)$

The expected pay-off of player B, when A uses

1 Strategy $= 3y_1 + -2(1-y_1) = A_1$

2 strategy $= -1y_1 + 4(1-y_1) = A_2$

3 Strategy $= 2y_1 + 2(1-y_1) = A_3$



ABCD is the upper boundry and B and C is the lowest point of the upper boundry i.e (Minimax point). Hence (3 x 2) game is reduced to (2 x 2) game.

Consider point B & point C

$$A \quad \begin{array}{c|cc} & \multicolumn{2}{c}{B} \\ & I & II \\ \hline 2 & -1 & 4 \\ 3 & 2 & 2 \end{array} \qquad\qquad A \quad \begin{array}{c|cc} & \multicolumn{2}{c}{B} \\ & I & II \\ \hline 1 & 3 & -2 \\ 3 & 2 & 2 \end{array}$$

↓

$$A \quad \begin{array}{c|cc|c} & \multicolumn{2}{c}{B} & \\ & I & II & \\ \hline 2 & -1 & 4 & -1 \\ 3 & \textcircled{2} & 2 & \textcircled{2} \\ \hline & \textcircled{2} & 4 & \end{array} \quad \text{MAXIMIN}$$

MINIMAX

MAXIMIN = MINIMAX

2 = 2

Hence Saddle point exists

$$A \quad \begin{array}{c|cc|c} & \multicolumn{2}{c}{B} & \\ & I & II & \\ \hline 1 & 3 & -2 & -2 \\ 3 & 2 & \textcircled{2} & \textcircled{2} \\ \hline & 3 & \textcircled{2} & \end{array} \quad \text{MAXIMIN}$$

MINIMAX

MAXIMIN = MINIMAX

2 = 2

Hence Saddle point exists

Hence value of Game = 2, is same from both the points B and C, only strategies changes.

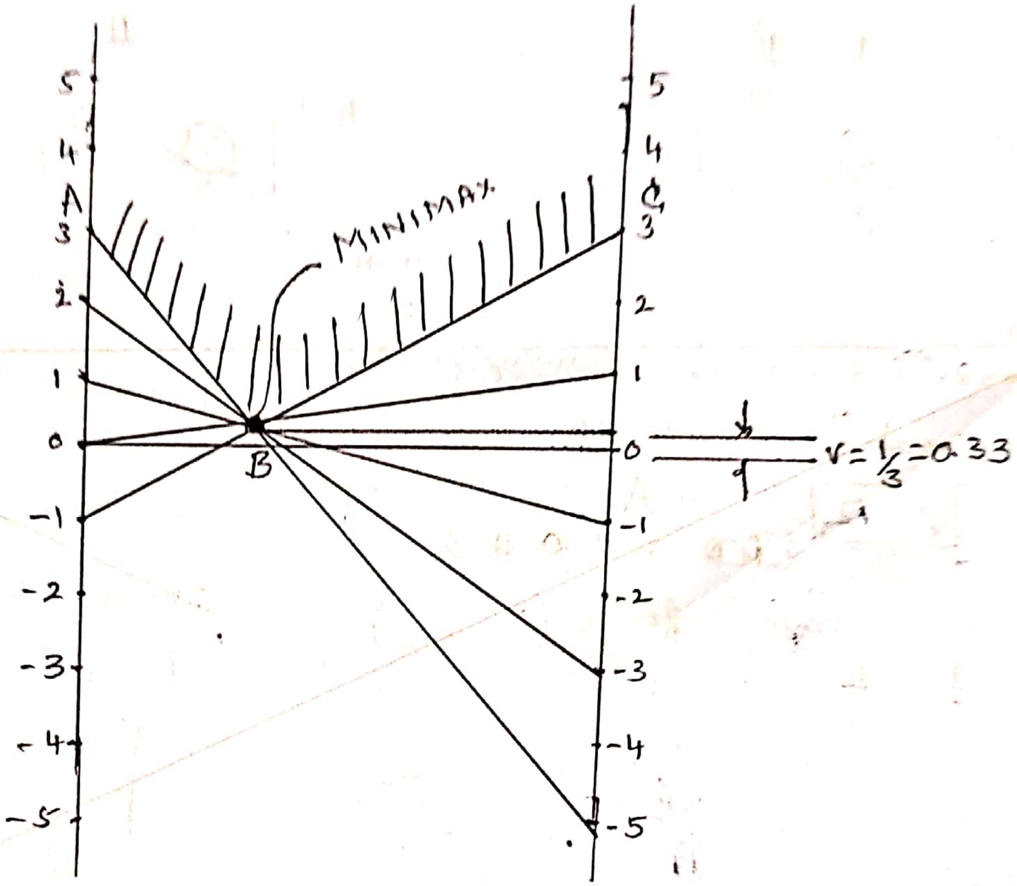
Hence Saddlepoint is (3, I) and V = 2 for point B,

Here Saddle point is (3, II) and V = 2 for point C problem 3 : Solve the following game graphically.

$$A\begin{array}{c|cc} & \multicolumn{2}{c}{B} \\ & I & II \\ \hline \checkmark I & 3 & -5 \\ \checkmark II & 1 & -1 \\ \checkmark \underline{III} & 2 & -3 \\ \underline{IV} & -1 & 3 \\ \underline{V} & 0 & 1 \end{array}$$

It is a (m×2) game. Reduce (m×2) to (2×2) by graphical method.

Solution :-



All the lines pass through the minimax point P. Line I, II, III have positive lines and lines IV & V have negative slopes. By the combination of positive and negative slop lines, the following six (2×2) reduced matrices are obtained.

$$A\begin{array}{c|cc} & \multicolumn{2}{c}{B} \\ & I & II \\ \hline \underline{I} & 3 & -5 \\ \underline{IV} & -1 & 3 \end{array} \quad A\begin{array}{c|cc} & \multicolumn{2}{c}{B} \\ \hline \underline{I} & 3 & -5 \\ \underline{V} & 0 & 1 \end{array} \quad A\begin{array}{c|cc} & \multicolumn{2}{c}{B} \\ & I & II \\ \hline \underline{II} & 1 & -1 \\ \underline{IV} & -1 & 3 \end{array} \quad A\begin{array}{c|cc} & \multicolumn{2}{c}{B} \\ & I & II \\ \hline \underline{II} & 1 & -1 \\ V & 0 & 1 \end{array}$$

$$A \quad \begin{array}{c|cc} & \multicolumn{2}{c}{B} \\ & I & II \\ \hline III & 2 & -3 \\ IV & -1 & 3 \end{array} \qquad\qquad A \quad \begin{array}{c|cc} & \multicolumn{2}{c}{B} \\ & I & II \\ \hline III & 2 & -3 \\ V & 0 & 1 \end{array}$$

Now for all six (2×2) game. Apply first MAXIMIN and MINIMAX principle, when saddle point exists find the value of Game, otherwise, by apply Arithmatic method to find the values of Game.

Solving the six games, we get the following results:

(1) $A\left(\frac{2}{3}, \frac{1}{3}\right)$ $B\left(\frac{1}{3}, 0, 0, \frac{2}{3}, 0\right)$

(2) $A\left(\frac{2}{3}, \frac{1}{3}\right)$ $B\left(\frac{1}{9}, 0, 0\ 0\ \frac{8}{9}\right)$ [Check, these values]

(3) $A\left(\frac{2}{3}, \frac{1}{3}\right)$ $B\left(0, \frac{2}{3}, 0, \frac{1}{3}, 0\right)$

(4) $A\left(\frac{2}{3}, \frac{1}{3}\right)$ $B\left(0, \frac{1}{3}\ 0, 0, \frac{2}{3}\right)$

(5) $A\left(\frac{2}{3}, \frac{1}{3}\right)$ $B\left(0, 0, \frac{4}{9}\ \frac{5}{9}, 0\right)$

(6) $A\left(\frac{2}{3}, \frac{1}{3}\right)$ $B\left(0, 0\ \frac{1}{6}, 0, \frac{5}{6}\right)$

& value of game $= v = \frac{1}{3}$.

- x - x - x -

lower value of the game and minimax is called upper value of the game, it can be shown That Maximin A ≤ Minimax B.

SADDLE POINT: A game in which the Maximin for A is equal to Minimax for B is called a game with Saddle point.

VALUE OF THE GAME; The pay-off at the Saddle point is called a value of the game. and obviously the maximin for A is equal to minimax for B. Maximin = Minimax

FAIR GAME; A game in which Maximin = Minimax is equal to zero. Then the game is said to be Fair game

Strictly determinable Game:- A game is said to be strictly determinable if Maximin = Minimax = Value of the game.

PURE STRATEGY PROBLEM:-

(i) Solve the game whose pay-off matrix is given below

player B → strategies for B. are 4, 5 & 6

i.e. 1, 2 & 3 strategies for A →

$$\begin{array}{c|ccc|c} & 4 & 5 & 6 & \\ \hline 1 & -3 & -2 & 6 & -3 \\ \text{player A} \rightarrow 2 & 2 & \textcircled{0} & 2 & \textcircled{0} \\ 3 & 5 & -2 & -4 & -4 \\ \hline & 5 & \textcircled{0} & 6 & \end{array}$$

→ value of the game.

Note: Apply MaxiMin & Minimax principle.

MAXIMIN principle is for player A

MINIMAX principle is for player B.

Maximin

[Select minimum value of Strategies of players A ie -3, 0, -4] and select maximum of this, i.e 0, which correspond to Strategy 2 of the Player A.

MINIMAX

[Select Max value of the Strategies of player B. i.e 5, 0 and 6 when he play the strategies 4, 5 and 6] and select minimum value of this. i.e 0 which correspond

MODULE-5 :- SEQUENCING

The main objective of sequencing problem is to find a sequence among $(n!)^m$ (where n is the number of jobs and m - no: of m/cs) number of all possible sequences for processing the jobs (on the machine) so that the total elapsed time for all the job will be minimum

Terminology:-

(1) Number of machines: It means the service ~~facilities~~ through which a job must pass before it is completed.

(2) processing order: It refers to the order in which various machines are required for completing the job.

(3) processing time:- It means the time required by each job on each machine.

(4) Idle time on a machine:- This is the time for which a machine remains idle during ~~d~~ the total elapsed time.

(5) Total elapsed time:- This is the time between starting the first job and completing the last job which also include the idle time if exists

No passing rule:- It means passing is not allowed. i.e maintaining the same orders of jobs over each machine.

For ex:- There are two machines M_1 & M_2 and processed in the order $M_1 M_2$. Then this rule will means that each job will go to machin M_1 first and then to M_2 (i.e if a job is finished on M_1, it goes directly to machine M_2 if it is empty, otherwise it starts a waiting line or joins the end of the waiting line.

Principal Assumptions:-

(1) No machine can process more than one operation at a time.

(2) Each operation once started must be performed till completion.

(3) Each operation must be completed before any other operation which must precede is started.

(4) Time interval for processing are independent of the order in which operations are performed

(5) A job is processed as soon as possible subject to the ordering requirement.

(6) All jobs are known and are ready to start processing before the period under consideration begins.

(7) The time required to transfer jobs between machines is negligible.

TYPE I : PROBLEMS WITH N JOBS THROUGH TWO MACHINES :

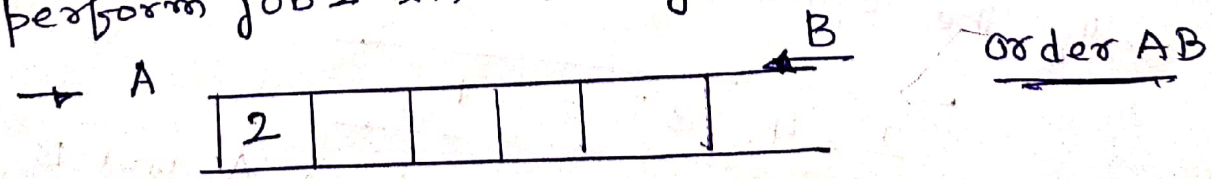
JOHNSON'S ALGORITHM :- The algorithm which is used to optimise The total elapsed time for processing n jobs through two machine is called Johnson's algorithm which has the following Steps,

For ex!- There are five jobs each of which must go through the two machines A and B in the order AB, processing times are given below.

JOB	1	2	3	4	5
Machine A	5	1	9	3	10
Machine B	2	6	7	8	4

Determine the sequence for five jobs that will minimise the total elapsed time.

Solution:- The smallest processing time in the given problem is 1 on machine A ~~on~~ for Job 2. So perform job 2 in the beginning as shown below



The reduced list of processing time becomes

JOB	1	3	4	5
M/C A	5	9	3	10
M/C B	2	7	8	4

Again the smallest processing time in the reduced list is 2 for job 1 on machine B.

So place the job 1 Last.

Machine A → Machine B ←

2				1

Continuing in the same manner, the next reduced list is obtained.

JOB	3	4	5
M|C A	9	3	10
M|C B	7	8	4

Leading to the sequence.

A → B ←

2	4			1

and the reduced list.

JOB	3	5
M|C A	9	10
M|C B	7	4

give rise to sequence

→ A B ←

2	4		5	1

Finally the optional sequence is obtained

2	4	3	5	1

Flow of jobs through machine A and B using the optional sequence (viz) 2 - 4 - 3 - 5 - 1

Computation of the total elapsed time and machine Idle time.

Job	Machine A		Machine B		Idle time	
	Time in	Time out	Time in	Time out	M/C A	M/C B
2	0	1	1	1+6=7	0	1
4	1	1+3=4	7	7+8=15	0	0
3	4	4+9=13	15	15+7=22	0	0
5	13	13+10=23	23	23+4=27	0	1
1	23	23+5=28	28	28+2=30	30-28=2	1 3

From the above table we find that total elapsed time is 30 hours and idle time on machine A is 2 hrs and for machine B is 3 hrs.

Example: 2: Find The Sequence that minimises the total elapsed time (in hours) required to Complete the following tasks on two machine.

Task	A ✓	B ✓	C ✓	D ✓	E ✓	F	G ✓	H ✓	I ✓
Machine I	2	5	4	9	6	8	7	5	4
Machine II	6	8	7	4	3	9	3	8	11

The Optimal Sequence

Machine I →

A							E	G

← Machine II

E & G

A	I	C					E	G

& C and I

The Final

A	I	C	B	H		D	E	G

B & H

The optimal Sequence

I → II ←

A								
A							G	E
A	C	I					G	E
A	C	I	B	H			G	E
A	C	I	B	H		D	G	E
A	C	I	B	H	F	D	G	E

JOB Sequence	Machine A		Machine B		Idle time	
	Time in	Time out	Time in	Time out	M/c A	M/c B
A	0	2	2	8	0	2
C	2	6	8	15	0	0
I	6	10	15	26	0	0
B	10	15	26	34	0	0
H	15	20	34	42	0	0
F	20	28	42	51	0	0
D	28	37	51	55	0	0
G	37	44	55	58	0	0
E	44	50	58	61	61-50 11 hrs	2 hrs.

TYPE II : PROCESSING n JOBS THROUGH THREE MACHINES A, B, C.

Consider n jobs (1, 2 ... n) processing on three machines A, B, C in the order ABC. The optimal Sequence can be obtained by converting the problem into two machine problem. From the so converted two machine problem, we get the optimum sequence using Johnson's algorithm.

To convert into two m/c problem, The following any one of the conditions must satisfy. For order <u>ABC</u>

(i) Find the minimum $A_i \geq \max B_i$

Or minimum $C_i \geq \max B_i$

If atleast one of the inequality (~~in step 2~~) is satisfied, we define two machines G and H such that the processing time on G and H are given by

$$G_i = A_i + B_i \qquad i = 1, 2 \ldots n$$

$$H_i = B_i + C_i \qquad i = 1, 2 \ldots n.$$

For the converted machines G and H, we obtain optimum sequence

Example 1: We have five jobs each of which must go through the machines A, B C in the order ABC. Determine the sequence that minimise the total elapsed time.

JOB NO	1	2	3	4	5
M/C A	5	7	6	9	5
M/C B	2	1	4	5	3
M/C C	3	7	5	6	7

Solution:- The optimum sequence can be obtained by converting the problem into two m/c by using the following steps..

Condition:- $\min(A_i\ C_i) = (5, 3)$

$\max(B_i) = 5$

$\min A_i = \max B_i$ ✓

$\therefore \min A_i \geq \max B_i$ is satisfied.

We convert the problem into two machine problem by defining two machines G and H, such that the processing time on G and H are given by

$G_i = A_i + B_i$

$H_i = B_i + C_i$

JOB	1	2	3	4	5
G	7	8	10	14	8
H	5	8	9	11	10

The optimal sequence is

G →

2	5	4	3	1

← H

Total elaped time and Idle time on Three machines

JOB	Machine A		Machine B		Machine C		Idle time		
	In	out	In	out	In	out	A	B	C
2	0	7	7	8	8	15	0	7	8
5	7	12	12	15	15	22	0	4	4
4	12	21	21	26	26	32	0	6	0
3	21	27	27	31	32	37	0	1	0
1	27	32	32	34	37	40	40-32 =08	1 + (40-34) =25	12

Ex: 2:- Given the following data:- (a)

Job	1	2	3	4	5	6
M/C A	12	10	9	14	7	9
M/C B	7	6	6	5	4	4
M/C C	6	5	6	4	2	4

(b) order of processing jobs : ACB.

(c) Sequence Suggested : Jobs 5-3-6-2-1-4

(i) Determine the total elapsed time for the sequence suggested

(ii) Is the given sequence optimal.

iii) If your answer to (ii) is NO, determine the optimal sequence and the total elapsed time associated with it.

Solution:- Arrange the data in the order of processing : ACB

JOB	1	2	3	4	5	6
M/C A	12	10	9	14	7	9
M/C C	6	5	6	4	2	4
M/C B	7	6	6	5	4	4

Verification of condition :

Minimum processing time for machine A = 7

Maximum processing time for machine C = 6

min Ai ≥ max Bi

∴ 7 ≥ 6.

Hence condition is satisfied.

Total elapsed time :

JOBS	M/C A		M/C C		M/C B		Idle time		
	IN	OUT	IN	OUT	IN	OUT	A	C	B
5	0	7	7	9	9	13			
3	7	16	16	22	22	28			
6	16	25	25	29	29	33			
2	25	35	35	40	40	45			
1	35	47	47	53	53	60			
4	47	61	61	65	65	70			

ii) The optimal sequence can be found by the method already described.

Considering two m/c's G_i and H_i and is given by

$G_i = A_i + C_i$ $H_i = C_i + B_i$

JOB	=	G_i	H_i
1	=	18	13
2	=	15	11
3	=	15	12
4	=	18	9
5	=	09	6
6	=	13	8

The optimal sequence is

1	3	2	4	6	5

(ii) Therefore the sequence suggested is not optimal

Total elapsed time:

JOB	M/C A		M/C C		M/C B		A	C	B
	IN	OUT	IN	OUT	IN	OUT			
1	0	12	12	18	18	25			
3	12	21	21	27	27	33			
2	21	31	31	36	36	42			
4	31	45	45	49	49	54			
6	45	54	54	58	58	62			
5	54	61	61	63	63	67			

TYPE III : PROCESSING OF n jobs Through m-machine. :

Consider n jobs (1, 2 ... n) processing through K machines $M_1, M_2 \dots M_K$ in the same order. The iterative procedure of obtaining an optimal Sequence is as follows.

Firstly check whether

Step: 2 :- $\begin{cases} \min M_{i1} \geq \max M_{ij} \text{ for } j = 2, 3 \dots K-1 \text{ or} \\ \min M_{iK} \geq \max M_{ij} \text{ for } j = 2, 3, \dots K-1. \end{cases}$

If the inequality in step 2 are not satisfied the method fails, otherwise, go to next step

In addition to step 2 if $M_{i2} + M_{i3} + \dots M_{iK-1} = C_i$, Where C is a positive fixed constant for all $i = 1, 2 \dots n$.

Ex :- Four jobs 1, 2 3 and 4 are to be processed on each of the five machines A, B, C, D and E in the order A B C D E. Find the total minimum elapsed time if no passing of jobs is permitted. Also find the idle time for each machine.

Machines	Jobs 1	2	3	4
A	7	6	5	8
B	5	6	4	3
C	2	4	5	3
D	3	5	6	2
E	9	10	8	6

Solution:- Convert the five machine problem into two machine problem, by adopting the following steps.

$$\min(A_i, E_i) = (5, 6)$$

$i = 1, 2, 3, 4$

$$\max(B_i, C_i, D_i) = (6, 5, 6)$$

The inequality

$$\min(E_i) = 6 \geq \max(B_i, C_i, D_i)$$

is satisfied. Therefore we can convert the problem into two machine problem. by considering two fictitious machines. as G_i and H_i.

Such that $G_i = A_i + B_i + C_i + D_i$ $i = 1, 2, 3, 4.$

$H_i = B_i + C_i + D_i + E_i$

Job	1	2	3	4
G	17	21	20	16
H	19	25	23	14

G → ← H

1	3	2	4

JOB	Machine A		Machine B		Machine C		Machine D		Machine E	
	In	out	In	out	In	out	In	out	In	out
1	0	7	7	12	12	14	14	17	17	26
3	7	12	12	16	16	21	21	27	27	35
2	12	18	18	24	24	28	28	33	35	45
4	18	26	26	29	29	32	33	35	45	51

JOB			Idle time		
	A	B	C	D	E
1	0	7	12	14	17
3	0	–	2	4	1
2	0	2	3	1	0
4	0	2	1	0	0
	51-26	51-29	51-32	51-35	–
	25	33	37	35	18

Ex: 2 When passing is not allowed. Solve the following problem giving an optimal solution

JOB	Machine			
	M_1	M_2	M_3	M_4
A	24	7	7	29
B	16	9	5	15
C	22	8	6	14
D	21	6	8	32

Solution :- The given problem is having four jobs on four machines. The Optimum Sequence can be obtained by converting into 2-machine problem. The following steps are adopted to find the optimum sequence.

$$\text{Min} (M_{i1}, M_{i4}) = (16, 14)$$

$$\max (M_{i2}, M_{i3}) = (9, 8)$$

Both the inequalities

$$\text{Min } M_{i1} = 16 \geq \text{Max} (M_{i12}, M_{i3})$$

$$16 \geq (9, 8)$$

and

$$\text{Min } M_{i4} = 14 \geq \text{Max} (M_{i2}, M_{i3}) \text{ are}$$

$$14 \geq (9, 8) \text{ satisfied.}$$

In addition to this inequality also we have $M_{i2} + M_{i3} = 14$ for $i = 2, 3$. We have two machines M_1 and M_4 in the order M_1, M_4.

JOB	A	B	C	D
M_1	24	16	22	21
M_4	29	15	14	32

The optimal sequence is

→ M_1 ← M_4

D	A	B	C

Total elapsed time :

JOB	Machine M₁		Machine M₂		Machine M₃		Machine M₄	
	Time in	Time out	In	out	In	out	In	out
D	0	21	21	27	27	35	35	67
A	21	45	45	52	52	59	67	96
B	45	61	61	70	70	75	96	111
C	61	83	83	91	91	97	111	125

	Idle time			
JOB	M₁	M₂	M₃	M₄
D	0	21	27	35
A	0	18	17	–
B	0	9	11	–
C	0	13	16	–
	125-83	125-91	125-97	–
	42 hrs	95 hrs	99 hrs	35 hrs

TYPE IV :- PROCESSING OF 2 JOBS ON M-Machines [By Graphical method]

Example 1:- Use graphical method to minimise the time needed to process the following jobs on the machines shown below. i.e for each machine find the job which should be done first. Also calculate the total time needed to complete both the jobs.

JOB 1	Sequence of m/c Time	A 2	B 3	C 4	D 6	E 2
JOB 2	Sequence of m/c Time	C 4	A 5	D 3	E 2	B 6

Solution :- Step 1:- First draw a set of axes where the horizontal axis represente processing time on job 1 and the vertical axis represents processing time on job 2.

Step 2:- Mark the processing time for job 1 and job 2 on the horizontal and vertical lines respectively according to the given order of the m/cs.

Step 3:- Construct various blocks starting from the origin (starting point) by pairing the same machines until the end point.

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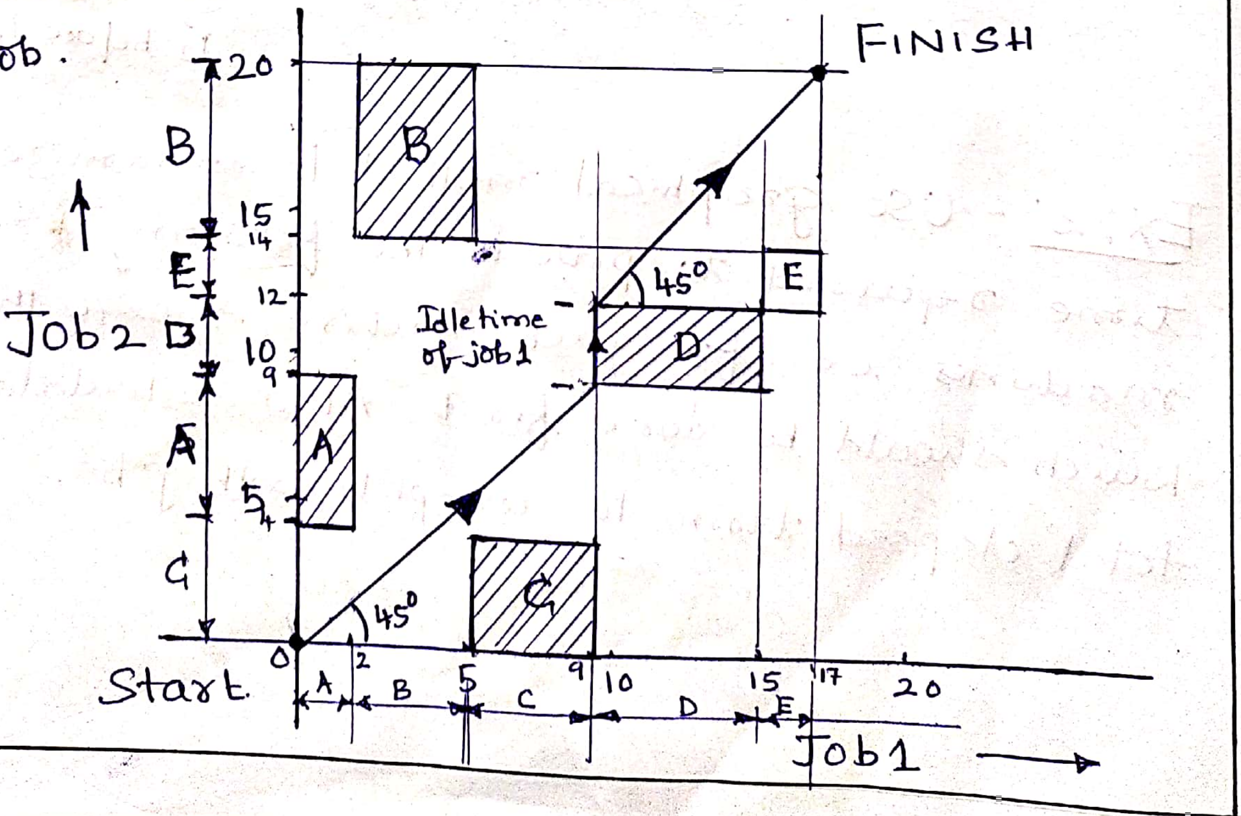
Step 4:- Draw the line starting from the origin to the end point by moving horizontally, vertically and diagonally along a line which makes an angle of 45° with the horizontal line (base). The horizontal segment of this line indicates that first job is under process while second job is idle. Similarly, the vertical line indicates that second job is under process while first job is idle. The diagonal segment of the line shows that the jobs are under process simultaneously.

Step 5:- An optimum path is one that minimises the idletime for both the jobs. Thus we must choose the path on which diagonal movement is maximum.

Step 6:- The total elapsed time is obtained by adding the idletime for either job to the processing time for that job.



Total processing time = processing time of job 1 + Idle time of job 1

= 17 + 3 = 20 hrs

or

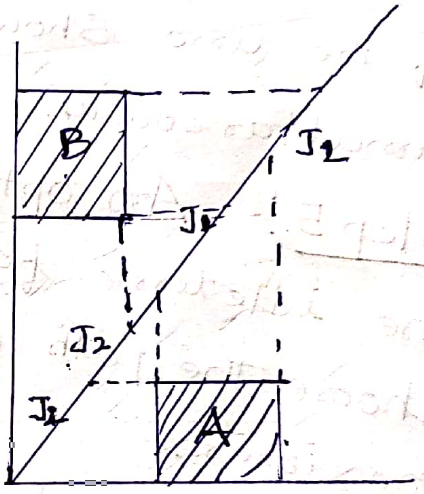
Total processing time = processing time of job 2 + Idle time of Job 2

= 20 + 0 = 20 hrs

The optimal sequence is

J_1 before J_2 on m/c's A & B

J_2 before J_1 on m/c's C, D, & E



For m/c A,

J_1 before J_2 on M/c B

J_2 before J_1 on m/c A

Ex: 2 :- Use graphical method to minimize the time required to process the following jobs on the machines i.e.. for each machine specify the job which should be done first. Also calculate the total elapsed time to complete both jobs.

JOB1	Sequence	A	B	C	D	E
	Time (in hrs)	7	9	5	13	5
JOB2	Sequence	B	C	A	D	E
	Time (in hrs)	11	9	7	5	13

Total processing time = processing time of Job1 + Idle time of Job1

= 39 + 4 + 7 = 50 hrs

OR

Total processing time = processing time of Job 2 + Idle time of Job2

= 45 + 5 = 50 hrs

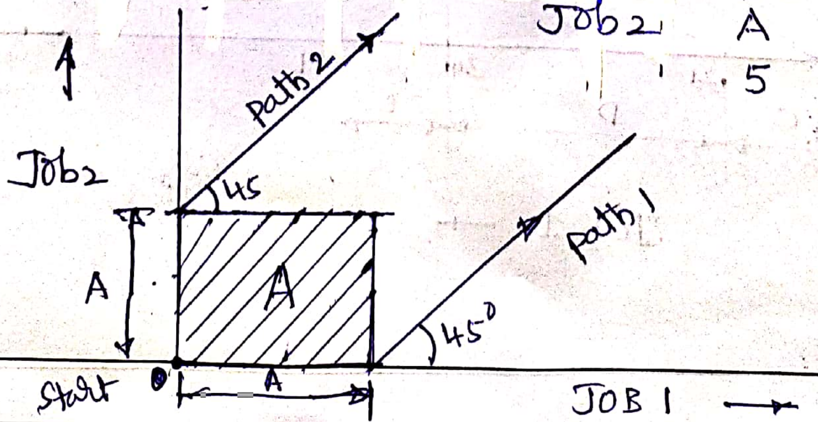
The optimal sequence is

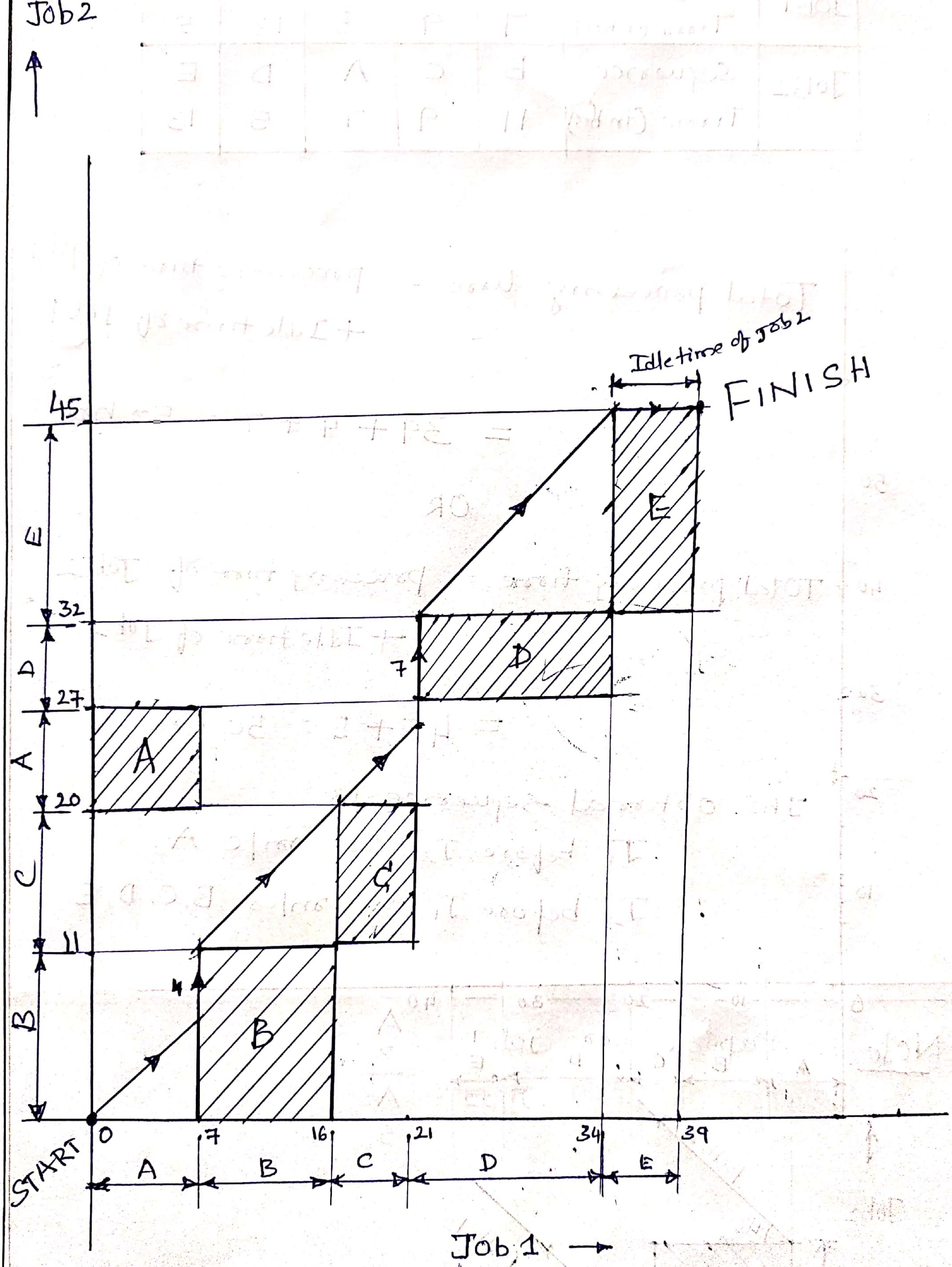
J_1 before J_2 on m/c A

J_2 before J_1 on m/cs B, C, D, E

Note: Suppose for job 1 A 3

Job2 A 5




Job2
Idle time of Job 2
FINISH
45
E
32
D
27
A
20
C
11
B
7
4
A
B
C
D
E
0
7
16
21
34
39
START
A
B
C
D
E
Job 1

Single Scheduling Rules or Single criterion Rule

These are the most simplest but effective way of assigning jobs. The decision is based on single criterion and selecting this criterion is a difficult job. This is because a rule which minimizes the processing time may not guarantee high labour or machine Utilization. In a similar way, There is no guarantee That the inprocess - inventory cost will be low. Sometimes it will be difficult to follow these rules strictly, as it happens with FCFS rule. Eventhough it is fair to give preference to a customer who comes first, Sometimes this rule has to be broken to give attention to another customer whose Urgency to provide a particular product has to be served at a faster rate.

The decision is based on any one of the following single criterion.

(i) **FCFS** - First Come First Served

priority is given to the job which arrives earliest. It is used in service industries egs: banks, post office.

(b) SPT - Shortest processing time

priority is given to the waiting job which has Shortest processing time or whose due date is earliest. The order of arrival and due dates are ignored.

(C) LPT - Longest processing time

priority is given to the job which has longest processing time.

(d) EDD - Earliest due date:

priority is given to the job which has its earliest due date, ignoring the processing period.

(e) Least Slack:-

priority is given to a job, which has least Slack period. Slack is the difference between delivery time and processing time

Slack = delivery time - processing period.

Ex: 1. Shown below are the due dates (number of days until due) and processing time remaining (number of days) for five jobs that were assigned as they arrived. Sequence the jobs by priority rule (a) FCFS (b) EDD (c) LS (d) SPT. (e) LPT

JOB	Due Date	process time
A	8	7
B	3	4
C	7	5
D	9	2
E	6	6

Find out (i) Average Completion (ii) Average job Lateness (iii) Average number of jobs at work Centre for FCFS and SPT.

The following table gives the priority to be given to the jobs, A, B, C, D and E, using different rules

S.L.NO.	FCFS	EDD	LS	SPT	LPT
1	A	B (3)	B (-1)	D (2)	A (7)
2	B	E (6)	E (0)	B (4)	E (6)
3	C	C (7)	A (1)	C (5)	C (5)
4	D	A (8)	C (2)	E (6)	B (4)
5	E	D (9)	(7)	A (7)	D (2)

performance of FCFS priority rule

JOB Sequence (1)	process time (2)	Flow time (3)	Duedate (4)	Days late (0 if negative) (5) (3-4)
A	7	7	8	0
B	4	11	3	8
C	5	16	7	9
D	2	18	9	9
E	6	24	6	18
	24	76		44

Performance of SpT rule:

D	2	2	9	0
B	4	6	3	3
C	5	11	7	4
E	6	17	6	11
A	7	24	8	16
	24	60		34

(a) Average Completion time

For FCFS : $76/5 = 15.2$ days

For SPT : $60/5 = 12$ days.

(b) Average Job Lateness :

For FCFS : 44/5 = 8.8 days

For SPT : 34/5 = 6.8 days

(C) Average number of jobs at WC

For FCFS : 76/24 = 3.2 jobs

For SPT : 60/24 = 2.5 jobs

Ex:2:- There are five jobs which are waiting to be processed at a shop. The jobs have arrived in the alphabetical order. Data on processing time delivery due in days from now onwards is tabulated.

Job	A	B	C	D	E
processing time-days	4	17	14	9	11
Due No: of days from now	6	20	18	12	12

Calculate how much delays is involved in delivering each job if jobs are processed.

(i) First come first served basis and

(ii) Based on Shortest processing time.

Solution :- (i) FCFS Basis

Job sequence	process time	Flow time	Due date	Days-late (2)-(3) (0 if negative)
A	4	4	6	0
B	17	21	20	1
C	14	35	18	17
D	9	44	12	32
E	11	55	12	43
				93

Delay in days = 93.

(ii) Shortest processing time :

Job Sequence	Process time	Flow time	Due date	Days late (2)-(3) (0 if negative)
A	4	4	6	0
D	9	13	12	1
E	11	24	12	12
C	14	38	18	20
B	17	55	20	35
				68

Delays in days = 68