

6.2 DEFINITIONS

Humidification involves contact between a pure liquid (A) and a gas phase consisting of an insoluble gas (B) and the vapours of the liquid (A). Certain special terms and definitions are used in the study of humidification operations, particularly those involving water (A)-air (B) interactions.

Humidity

The term *humidity* is used to indicate the concentration of the vapour in a gas-vapour mixture and is defined as the amount of vapour present per unit quantity of vapour-free gas. For air-water vapour system, humidity refers to the amount of water vapour present per unit quantity of dry air. Humidity can be expressed on mole basis or on mass basis. The *molal humidity* is defined as the moles of vapour present per mole of vapour-free gas. Let there be n_A moles of vapour and n_B moles of dry gas in a given sample of the gas-vapour system. Then,

$$\text{Molal humidity, } Y = \frac{\text{moles of vapour}}{\text{moles of dry gas}} = \frac{n_A}{n_B} \quad \dots(6.2)$$

Assuming that the gas phase behaves as an ideal gas, the following equations can be used to determine the number of moles of the constituents.

$$\begin{aligned} n_A &= \frac{\bar{p}_A V}{RT} \\ n_B &= \frac{\bar{p}_B V}{RT} \end{aligned} \quad \dots(6.3)$$

where V is the volume of the mixture and T is the temperature. Combining Eqs. (6.2) and (6.3), we get

$$\text{Molal humidity, } Y = \frac{n_A}{n_B} = \frac{\bar{p}_A}{\bar{p}_B} = \frac{\bar{p}_A}{P - \bar{p}_A} \quad \dots(6.4)$$

Equation (6.4) means that the molal humidity is equal to the ratio of the partial pressure of the vapour to the partial pressure of vapour-free gas.

If the gas were saturated with the vapours, the molal humidity is the *molal humidity at saturation* denoted as Y_S . It is equal to the moles of vapour per mole of dry gas under saturation. Since, at saturation, the partial pressure of the vapour is equal to the vapour pressure, the molal saturation humidity can be written as

$$Y_S = \frac{P_A^S}{P - P_A^S} \quad \dots(6.5)$$

where P_A^S is the vapour pressure of the liquid A.

The *absolute humidity* is defined as the mass of vapour present per unit mass of vapour-free gas. For the air-water vapour system it is the kilograms of water present per kg of dry air. If m_A and m_B are the weights of the vapour and vapour-free gas, then

$$\text{Absolute humidity, } Y' = \frac{m_A}{m_B} \quad \dots(6.6)$$

The absolute humidity is related to the molal humidity as:

$$Y' = \frac{m_A}{m_B} = \frac{n_A}{n_B} \times \frac{M_A}{M_B} = Y \times \frac{M_A}{M_B} \quad \dots(6.7)$$

where M_A and M_B are the molecular weights of A and B, respectively. Combining with Eq. (6.4), the absolute humidity can be expressed in terms of the partial pressures of A and B in the gas phase.

$$Y' = \frac{\bar{p}_A}{P - \bar{p}_A} \times \frac{M_A}{M_B} \quad \dots(6.8)$$

Noting that for air, the average molecular weight is approximately 29 and the molecular weight of water is 18, the absolute humidity of the air-water vapour system can be related to the partial pressure as

$$Y' = \frac{\bar{p}_A}{P - \bar{p}_A} \times \frac{18}{29} \quad \dots(6.9)$$

The *absolute saturation humidity* (Y'_S) denotes the kilograms of water vapour that one kg dry air can hold at saturated conditions. Analogous to Eq. (6.5), we can write

$$Y'_S = \frac{P_A^S}{P - P_A^S} \times \frac{M_A}{M_B} \quad \dots(6.10)$$

At the boiling point of the liquid, the vapour pressure becomes equal to the total pressure so that the denominator in Eq. (6.10) reduces to zero. It means that as the boiling point of the liquid is approached, the saturation humidity tends to infinity. We can see from Eqs. (6.5) and (6.10) that at a given temperature, the molal humidity at saturation is independent of the nature of the gas and depends only upon the pressure, whereas the absolute saturation humidity depends upon the characteristics of the gas as well.

Example 6.2: A mixture of acetone vapour and nitrogen gas at 101.3 kPa and 300 K contains acetone vapour to the extent that it exerts a partial pressure of 20 kPa. The vapour pressure of acetone is given by the Antoine equation

$$\ln P^S = 14.39155 - \frac{2795.817}{T - 43.198}$$

where the pressure is in kPa and temperature is in K. Determine the following:

- The molal humidity
- The absolute humidity
- The molal saturation humidity
- Absolute saturation humidity

Solution: (a) Molal humidity is the ratio of moles of acetone to moles of dry nitrogen. It is given by Eq. (6.4).

$$Y = \frac{\bar{p}_A}{P - \bar{p}_A} = \frac{20}{101.3 - 20} = 0.2460 \frac{\text{moles acetone}}{\text{mole nitrogen}} \quad (\text{Answer})$$

(b) Absolute humidity is the ratio of kilograms of acetone to kg of nitrogen. It is obtained by multiplying the molal humidity by the ratio of the molecular weights of acetone and nitrogen.

$$Y' = 0.2460 \times \frac{58}{28} = 0.5096 \frac{\text{kg acetone}}{\text{kg nitrogen}} \quad (\text{Answer})$$

- (c) Vapour pressure of acetone at 300 K, from Antoine equation

$$\ln P^S = 14.39155 - \frac{2795.817}{300 - 43.198}$$

$$P^S = 33.265 \text{ kPa}$$

$$\text{Saturation humidity, } Y_S = \frac{P^S}{P - P^S} = \frac{33.265}{101.3 - 33.265} = 0.4889 \frac{\text{moles of acetone}}{\text{mole of nitrogen}} \quad (\text{Answer})$$

$$(d) \text{ Absolute saturation humidity } Y'_S = 0.4889 \times \frac{58}{28} = 1.0128 \frac{\text{kg acetone}}{\text{kg nitrogen}} \quad (\text{Answer})$$

Relative Humidity and Percent Humidity

The degree of unsaturation of a gas-vapour mixture can be expressed either as the percent relative saturation or as the percent saturation. The *percent relative saturation* or the *percent relative humidity*

is the ratio of the actual partial pressure exerted by the vapour to its vapour pressure at the same temperature, expressed as percentage. Thus

$$\text{Relative saturation (relative humidity)} = \frac{\bar{p}_A}{P_A^S} \times 100\% \quad \dots(6.11)$$

The *percent saturation* or the *percent humidity* is the ratio of the moles of vapour present per mole of dry gas in the existing conditions to the moles of vapour per mole of dry gas if the gas were saturated, expressed as percentage. It is the same as the ratio of the kg of vapour present per kg of the dry gas at the given temperature to the kg of the vapour associated with one kg dry gas under saturated conditions at the same temperature.

$$\text{Percent saturation (percent humidity)} = \frac{(n_A/n_B)}{(n_A/n_B)_{\text{saturation}}} \times 100\% \quad \dots(6.12)$$

The numerator in Eq. (6.12) is the molal humidity and the denominator is the molal humidity at saturation. Therefore, the percent humidity can be defined as the ratio of the humidity of the mixture to the humidity in the saturated state.

$$\text{Percent saturation} = \frac{Y}{Y_S} \times 100\% = \frac{Y'}{Y'_S} \times 100\% \quad \dots(6.13)$$

Suppose that the percent saturation is 60%. It means that the vapour content of the mixture per one kg of vapour-free gas is only 60% of the vapour that will be present with one kg dry gas if the gas were saturated with the vapours at the same temperature. In contrast, the relative saturation of 60 percent means that the partial pressure of the vapour in the gas phase is sixty percent of the vapour pressure. The relationship between percent saturation and percent relative saturation can be derived by combining Eqs. (6.4), (6.5) and (6.13).

$$P.S. = \frac{\bar{p}_A}{P_A^S} \times \frac{P - P_A^S}{P - \bar{p}_A} \times 100\% \quad \dots(6.14)$$

Equation (6.14) can be rewritten using Eq. (6.11) as

$$P.S. = R.S. \times \frac{P - P_A^S}{P - \bar{p}_A} \quad \dots(6.15)$$

where P.S. and R.S. are the abbreviations for percent saturation and relative saturation, respectively. Since the ratio $(P - P_A^S)/(P - \bar{p}_A)$ in Eq. (6.15) is always less than 1, percent saturation is less than relative saturation. That is, $P.S. < R.S.$

Example 6.3: A mixture of acetone vapour and nitrogen gas at 101.3 kPa and 300 K contains acetone vapour to the extent that it exerts a partial pressure of 20 kPa. The vapour pressure of acetone at 300 K is 33.26 kPa. Determine the relative saturation and the percent saturation of the mixture.

Solution: The relative saturation is the ratio of the partial pressure of acetone to the vapour pressure expressed as percentage, i.e.,

$$R.S. = \frac{\bar{p}_A}{P_A^S} \times 100 = \frac{20}{33.26} \times 100 = 60.13\% \quad (\text{Answer})$$

The percent saturation is given by Eq. (6.13).

$$Y = \frac{\bar{p}_A}{P - \bar{p}_A} = \frac{20}{101.3 - 20} = 0.2460 \frac{\text{kmol acetone}}{\text{kmol nitrogen}}$$

Here, the molal humidity,

$$Y_S = \frac{P_A^S}{P - P_A^S} = \frac{33.26}{101.3 - 33.26} = 0.4888 \frac{\text{kmol acetone}}{\text{kmol nitrogen}}$$

The molal humidity at saturation,

$$\text{The percent saturation P.S.} = \frac{Y}{Y_s} \times 100 = \frac{0.2460}{0.4888} \times 100 = 50.33\%$$

(Answer)

Example 6.4: Air contains 0.015 kg water vapour per cubic metre of the mixture at 303 K and 101.3 kPa. Determine:

- The partial pressure of water vapour
- The relative saturation
- The absolute humidity of the air
- The percent saturation
- The temperature to which the mixture be heated so that its percent saturation becomes 10%

The vapour pressure of water (in kPa) is approximated by the Antoine equation as

$$\ln P^s = 16.26205 - \frac{3799.887}{T - 46.854}$$

Solution: The total number of moles that one cubic metre of air at 303 K and 101.3 kPa contains can be calculated using the ideal gas equation:

$$N = \frac{V}{22.414} \times \frac{P}{P_0} \times \frac{T_0}{T} = \frac{1}{22.414} \times \frac{101.3}{101.3} \times \frac{273.15}{303} = 0.0402 \text{ kmol}$$

$$0.015 \text{ kg water} = 0.015/18 = 8.33 \times 10^{-4} \text{ kmol}$$

$$\text{(a) Mole fraction of water vapour} = \frac{n_w}{N} = \frac{8.33 \times 10^{-4}}{0.0402} = 0.0207$$

Partial pressure of water vapour = mole fraction \times total pressure

$$= 0.0207 \times 101.3 = 2.0969 \text{ kPa}$$

(Answer)

- (b) The vapour pressure at 303 K is calculated using Antoine equation.

$$\ln P^s = 16.26205 - \frac{3799.887}{303 - 46.854}$$

Therefore, $P^s = 4.167 \text{ kPa}$

Relative saturation is the ratio of partial pressure to vapour pressure.

$$\text{R.S.} = \frac{2.0969}{4.167} \times 100 = 50.32\% = \frac{\bar{p}_A}{P^s}$$

(Answer)

- (c) Absolute humidity can be evaluated by Eq. (6.9)

$$Y' = \frac{\bar{p}_A}{P - \bar{p}_A} \times \frac{18}{29} = \frac{2.0969}{101.3 - 2.0969} \times \frac{18}{29}$$

avg mol. wt of air = 29

$$= 0.01312 \text{ kg water/kg dry air}$$

(Answer)

- (d) Saturation absolute humidity is

$$Y'_s = \frac{P_A^s}{P - P_A^s} \times \frac{18}{29} = \frac{4.167}{101.3 - 4.167} \times \frac{18}{29} = 0.0266 \text{ kg water/kg dry air}$$

$$\text{Percent saturation} = \text{P.S.} = \frac{Y'}{Y'_s} \times 100 = \frac{0.01312}{0.0266} \times 100 = 49.32\%$$

(Answer)

- (e) When the mixture is heated at constant pressure, its humidity (Y') will not change. However, saturation humidity at the new temperature will be different from the earlier value. Since the percent saturation at the new temperature is given to be 10%, we have

$$\text{P.S.} = \frac{Y'}{Y'_S} \times 100 = 10.0\% \quad Y'_S = \frac{Y'}{0.1} = \frac{0.01312}{0.1} = 0.1312 \text{ kg water/kg dry air.}$$

$$\text{But, by Eq. (6.10), } Y'_S = \frac{P_A^S}{101.3 - P_A^S} \times \frac{18}{29} = 0.1312 \text{ kg water/kg dry air.}$$

Therefore, $P_A^S = 17.68$ kPa. The temperature corresponding to this vapour pressure is calculated using the Antoine equation. This temperature is 330.6 K. **(Answer)**

Dew Point

Dew point is the lowest temperature to which a vapour-gas mixture can be cooled at constant pressure without condensation, out of contact with the liquid. On cooling a gas to its dew point, the gas gets saturated with the vapours, without any additional vapour input. Stated in another way, the dew point is that temperature at which the vapour pressure of the liquid and the partial pressure of the vapour in the vapour-gas mixture are the same.

Consider an air-water vapour system at 101.3 kPa and 313.15 K with a partial pressure of water, say, 1.7 kPa. At this condition, the humidity of the air can be estimated to be $Y' = \frac{\bar{p}_A}{P - \bar{p}_A} \times \frac{18}{29}$

$$= \frac{1.7}{101.3 - 1.7} \times \frac{18}{29} = 0.0106 \text{ kg water per kg dry air.}$$

The vapour pressures of water at different temperatures are listed in Table 6.1.

Table 6.1: Vapour pressures of water at different temperatures

T, K	283.15	288.15	293.15	298.15	303.15	308.15	313.15
P_A^S , kPa	1.23	1.70	2.34	3.17	4.24	5.62	7.36

Since the vapour pressure of water at 313.15 K is 7.36 kPa, the relative humidity is $\frac{1.7}{7.36} \times 100 = 23.1\%$. If this mixture is cooled to say, 303.15 K, its humidity remains at 0.0106 kg water per kg dry air, since the partial pressure of water is still 1.7 kPa. However, since the vapour pressure at the new temperature is 4.24 kPa, the relative saturation becomes $\frac{1.7}{4.24} \times 100 = 40.1\%$. On cooling further to say, 293.15 K, the relative humidity increases to 72.6%. If the cooling is continued, eventually the air becomes saturated (R.S. = 100%) at a temperature of 288.15 K, which is the dew point of the given air-water vapour mixture. The vapour pressure of water at this temperature equals the partial pressure of the water in the original air. If cooling is continued to temperature below the dew-point, the water vapour condenses and the humidity decreases, air remaining saturated at all temperatures below the dew-point. Suppose that the air is cooled to, say, 283.15 K, the partial pressure reduces to 1.23 kPa, the vapour pressure at 283.15 K. This corresponds to a humidity of 0.0076 kg water vapour per kg dry air. Since the initial humidity was 0.0106 kg water per kg dry air, cooling the air by 5 K below the dew point, results in condensation of 0.003 kg water vapour per every kg of dry air.

Since the dew point is that temperature at which the saturation humidity will be equal to the humidity of the given air-water vapour system, knowing the dew point and the vapour pressure of water as a function of the temperature, the humidity of the air is readily evaluated. However, to establish the state of the given mixture uniquely one more specification such as the temperature, the relative saturation or the percent saturation also is necessary.

Dew point of air can be determined experimentally by cooling a well polished metal surface in the presence of air and measuring the temperature at which, the water vapour begins to condense and a fog

appears on the metallic surface. As the dew point is a measure of the humidity, this is one of the methods for measuring atmospheric humidity.

Example 6.5: Air-water vapour mixture at 325 K and 120 kPa has a dew point of 293 K. The vapour pressure of water is given by the Antoine equation (P in kPa, T in K)

$$\ln P^S = 16.26205 - \frac{3799.887}{T - 46.854}$$

Determine:

- The absolute humidity of air
- The saturation humidity
- The relative saturation and percent saturation
- The dew point if the total pressure is reduced to 100 kPa.

Solution: The partial pressure of water vapour is equal to the vapour pressure at the dew point of 293 K. The vapour pressure at 293 K is calculated from Antoine equation:

$$P^S = \exp\left(16.26205 - \frac{3799.887}{293 - 46.854}\right) = 2.28 \text{ kPa.}$$

- (a) Partial pressure of water \bar{p}_A = vapour pressure at the dew point = 2.28 kPa

Absolute humidity:

$$Y' = \frac{\bar{p}_A}{P - \bar{p}_A} \times \frac{M_A}{M_B} = \frac{2.28}{120 - 2.28} \times \frac{18}{29} = 0.012 \text{ kg water/kg dry air.} \quad (\text{Answer})$$

- (b) The vapour pressure of water at 325 K

$$= P_A^S = \exp\left(16.26205 - \frac{3799.887}{325 - 46.854}\right) = 13.47 \text{ kPa.}$$

Saturation humidity:

$$Y'_S = \frac{P_A^S}{P - P_A^S} \times \frac{M_A}{M_B} = \frac{13.47}{120 - 13.47} \times \frac{18}{29} = 0.0785 \text{ kg water/kg dry air.} \quad (\text{Answer})$$

$$(c) \text{ Relative humidity} = \frac{\bar{p}_A}{P_A^S} \times 100 = \frac{2.28}{13.47} \times 100 = 16.93\%$$

$$\text{Percent humidity} = \frac{Y'}{Y'_S} \times 100 = \frac{0.012}{0.0785} \times 100 = 15.29\% \quad (\text{Answer})$$

→ (d) The mole fraction of water vapour in the air is equal to the ratio of partial pressure to total pressure.

$$y = \frac{\bar{p}_A}{P} = \frac{2.28}{120} = 0.019$$

Since the humidity remains the same, the partial pressure of water at 100 kPa = mole fraction \times total pressure = $0.019 \times 100 = 1.9$ kPa. Using the Antoine equation, the temperature at which the vapour pressure of water is 1.9 kPa is found out, which is the dew point at 100 kPa.

Dew point = 290.12 K.

Dry-bulb Temperature

In psychrometry, the temperature of the gas vapour-mixture that is measured by an ordinary thermometer or other temperature measuring devices placed in the gas is known as the dry-bulb temperature.

Humid Volume

The volume of one kg dry gas and the accompanying vapour is known as the humid volume of the vapour-gas mixture. Assuming the ideal gas law to be applicable,

$$V_H = V_0 \times \frac{P_0}{P} \times \frac{T}{T_0} \quad \dots(6.16)$$

where V_H is the humid volume and V_0 is the volume occupied by one kg of the dry gas and the accompanying vapour at the standard temperature and pressure.

$$V_0 = \left(\frac{1}{M_B} + \frac{Y'}{M_A} \right) \times 22.414 \quad \dots(6.17)$$

where V_0 is in m^3 . For the gas at the dry-bulb temperature T , pressure P (kPa) and having humidity Y' , the humid volume is calculated using Eqs. (6.16) and (6.17) as

$$V_H = \left(\frac{1}{M_B} + \frac{Y'}{M_A} \right) \times 22.414 \times \frac{101.325}{P} \times \frac{T}{273.15} \quad \dots(6.18)$$

V_H is the humid volume in m^3/kg dry air. M_A and M_B are the molecular weights of the vapour and the dry gas, respectively. For the air-water vapour system, Eq. (6.18) gives

$$V_H = 8.314 \left(\frac{1}{29} + \frac{Y'}{18} \right) \times \frac{T}{P} \quad \dots(6.19)$$

where P is in kPa and V_H is in m^3/kg dry air. Given the humidity of the air, the humid volume at any temperature can be calculated using this equation.

Humid Heat

Humid heat (C_S) is the amount of heat required to raise the temperature of one kg of dry gas and the accompanying vapour by one degree. Thus it represents the heat capacity of the gas-vapour system at constant pressure. If the specific heat of the vapour is C_A (kJ/kg K) and that of the dry gas is C_B (kJ/kg K), then the humid heat is

$$C_S = C_B + Y' C_A \quad \dots(6.20)$$

For the air-water vapour system, $C_A = 1.884$ kJ/kg K and $C_B = 1.005$ kJ/kg K.

Knowing the humid heat of the air-water system, the heat requirement in the operations such as drying and humidification can be easily calculated as

$$Q = G_S C_S \Delta T \quad \dots(6.21)$$

where Q is the quantity of sensible heat required for heating without phase change G_S kg of dry air and the accompanying vapour by (ΔT) K.

Example 6.6: Air at 101.3 kPa and 328 K has a volume of 100 m^3 and is 10% saturated with water vapour. The vapour pressure of water at 328 K is 15.7 kPa. Determine:

- The humid volume of the sample
- The humid heat
- The amount of heat to be supplied to heat the given quantity of air to 370 K.

Solution: Since the vapour pressure of water at 328 K is 15.7 kPa, the saturation humidity of the air is

$$Y'_S = \frac{p_A^S}{P - p_A^S} \times \frac{18}{29} = \frac{15.7}{101.3 - 15.7} \times \frac{18}{29} = 0.1138 \text{ kg water/kg dry air}$$

Since the air is 10% saturated, the humidity

$$Y' = 0.1 \times Y'_S = 0.10 \times 0.1138 = 0.0114 \text{ kg water/kg dry air}$$

- Eq. (6.19) is used to calculate the humid volume.