

# Algebraic Approach to Targeting Direct Recycle.

## Algebraic Targeting procedure.

Algebraic procedure can be summarized as follows.

1. Rank the sinks in ascending order of maximum admissible composition  
$$z_1^{\max} \leq z_2^{\max} \leq \dots \leq z_s^{\max} \leq \dots \leq z_{N_{\text{sinks}}}^{\max}$$
2. Rank sources in ascending order of pollutant composition  
$$y_1 \leq y_2 \leq \dots \leq y_i \leq \dots \leq y_{N_{\text{sources}}}$$
3. Calculate the load of each sink ( $M_i^{\text{sink}, \max} = G_i z_i^{\max}$ ) and source ( $M_j^{\text{source}} = W_j y_j$ )
4. Compute the cumulative loads for the sinks and for the sources (by summing up their individual loads)
5. Rank the cumulative loads in ascending order,
6. Develop the load interval diagram (LID) shown in Fig. First, the loads are represented in ascending order starting with zero load. The scale is irrelevant. Next, each source and each sink is represented as an arrow whose tail corresponds to its starting load. The following equations are used to calculate interval load, source flow rate, and sink flow rate

The load within interval  $k$

$$\Delta M_k = M_k - M_{k-1}$$

The source flow rate for  $k^{\text{th}}$  interval

$$\Delta W_k = \frac{\Delta M_k}{y_{\text{source in interval } k}}$$

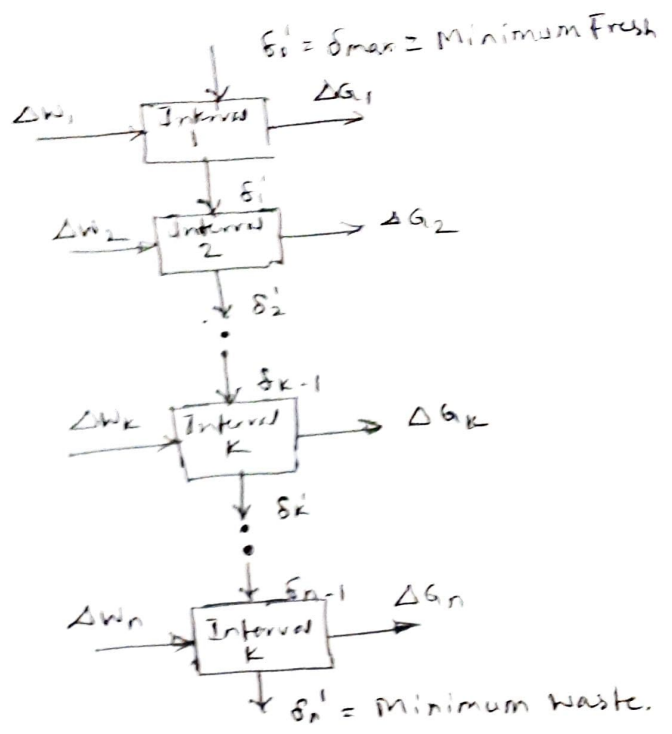
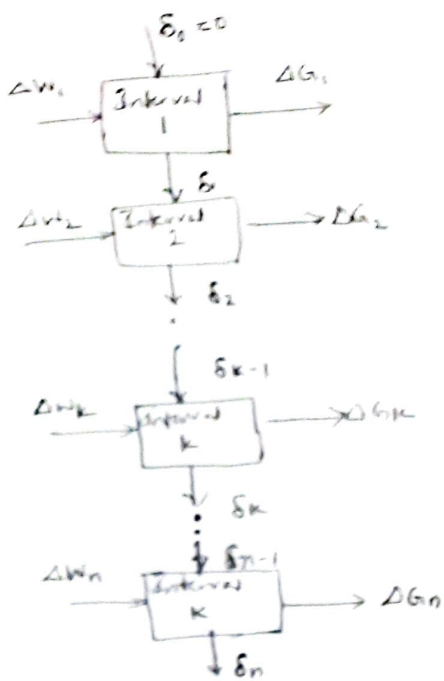
The sink flow rate for  $k^{\text{th}}$  interval

$$\Delta G_k = \frac{\Delta M_k}{z_{\text{sink in interval } k}^{\max}}$$

7. Based on the interval source and sink flow rates, develop the cascade diagram and carry out flow balances around the intervals to calculate the values of the flow residuals ( $\delta_k$ ). The most negative  $\delta_k$  is the target for minimum fresh consumption

i.e.  $\delta_{\max} = \text{Target for minimum fresh usage.}$

8. Revise the cascade diagram by adding the maximum  $\delta_k$  to the first interval and calculate the revised residuals. The interval with the first zero residual is the material recycle/reuse global pinch point. The residual flow leaving the last interval is the target for minimum waste discharge i.e.  
$$\delta'_n = \text{Target for minimum waste discharge.}$$



Interval	Load	Interval Load ( $\Delta M_k$ )	Source	Source Flow per Interval ( $\Delta W_k$ )	Sink	Sink Flow per interval ( $\Delta G_k$ )
1	$M_1$	$\Delta M_1$	Source 1	$\frac{\Delta M_1}{y_1}$	Sink 1	$\frac{\Delta M_1}{z_{1, \max}}$
2			Source 2	$\frac{\Delta M_2}{y_1}$	Sink 2	$\frac{\Delta M_2}{z_{1, \max}}$
			Source 3	$\frac{\Delta M_3}{y_2}$		$\frac{\Delta M_3}{z_{2, \max}}$
$k$	$M_k$	$\Delta M_k$		$\frac{\Delta M_k}{y_{\text{sink in interval } k}}$	Sink 3	$\frac{\Delta M_k}{z_{\text{sink in interval } k}}$
	$M_{n-1}$		Source			
$n$	$M_n$	$\Delta M_n$		$\frac{\Delta M_n}{y_{\text{sink in interval } n}}$	Sink $N_{\text{sink}}$	$\frac{\Delta M_n}{z_{\text{sink in interval } n}}$

Load interval diagram.

# Case study : Targeting for Acetic Acid usage in a vinyl acetate plant.

Table 1. Source data for vinyl acetate example.

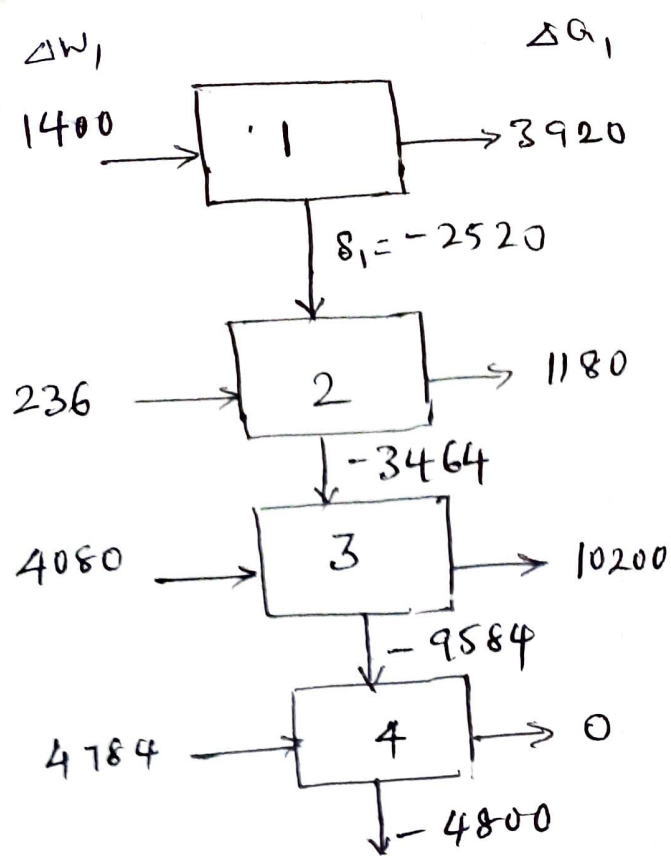
Source	Flowrate kg/h	Inlet mass fraction	Inlet load kg/h	Cumulative load (kg/h)
Bottoms of absorber II	1400	0.14	196	196
Bottoms of primary tower	9100	0.25	2275	2471

Table 2. Sink data for vinyl acetate example

Sink	Flow rate kg/h	Maximum inlet mass fraction	Maximum inlet load (kg/h)	Cumulative maximum load kg/h
Absorber I	5100	0.05	255	255
Acid Tower	10200	0.10	1020	1275

Interval	Lead kg/h	Interval Load ( $\Delta M_k$ ) kg/h	Sources	Source Flow per interval ( $\Delta W_k$ )	Sources Sink	Sink Flow per interval ( $\Delta G_k$ )
1	196	196	Source 1 $y = 0.14$	$\frac{196}{0.14} = 1400$	Sink 1 $x_{\max} = 0.05$	$\frac{196}{0.05} = 3920$
2	255	59		$\frac{59}{0.25} = 236$		$\frac{59}{0.05} = 1180$
3	1275	1020	Source 2 $z = 0.25$	$\frac{1020}{0.25} = 4080$	Sink 2 $x_{\max} = 0.1$	$\frac{1020}{0.1} = 10200$
4	2471	1196		$\frac{1196}{0.25} = 4784$		0



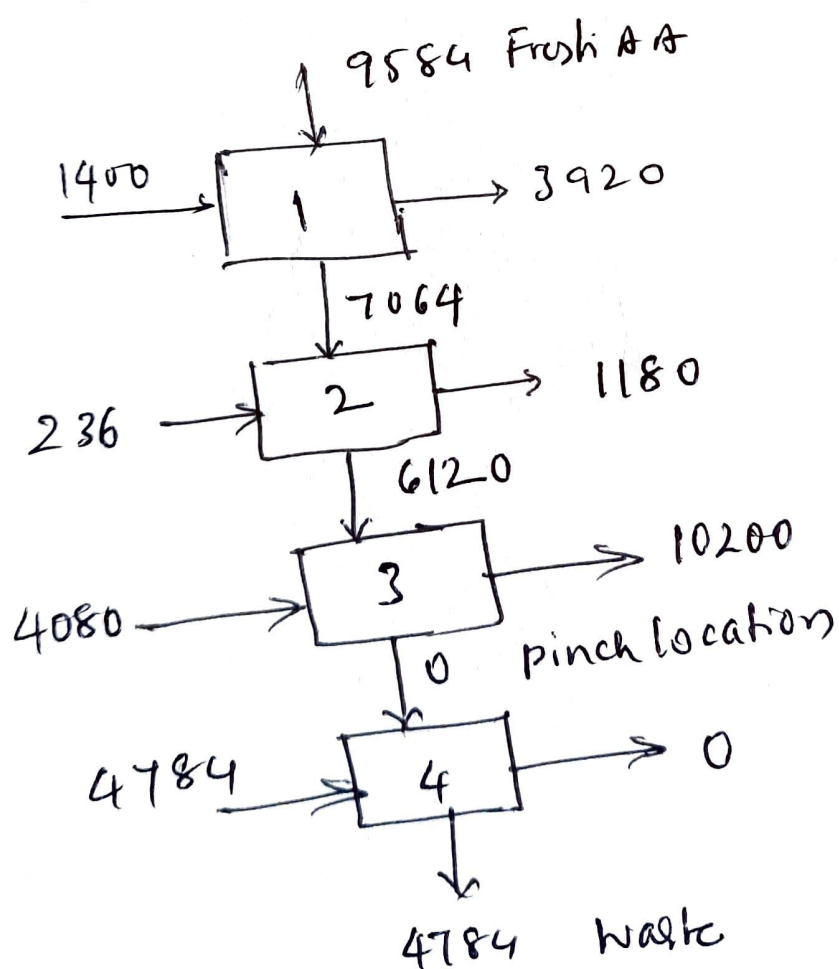


$$\delta_1 = \Delta W_1 - \Delta G_1$$

$$\delta_2 = \Delta W_1 + \Delta W_2 - \Delta G_1 - \Delta G_2$$

$$\delta_2 = \delta_1 + \Delta W_2 - \Delta G_2$$

Cascade diagram for the VAM case study with infeasibilities



Cascade diagram for the VAM case revised.