

Module :
7
**Centrifugal Separation processes and
their calculations**

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Centrifugal Separation Processes

In centrifugal separation processes, centrifugal force is used to induce separation.

Fig. 7.1 demonstrates stages during centrifugal separation.

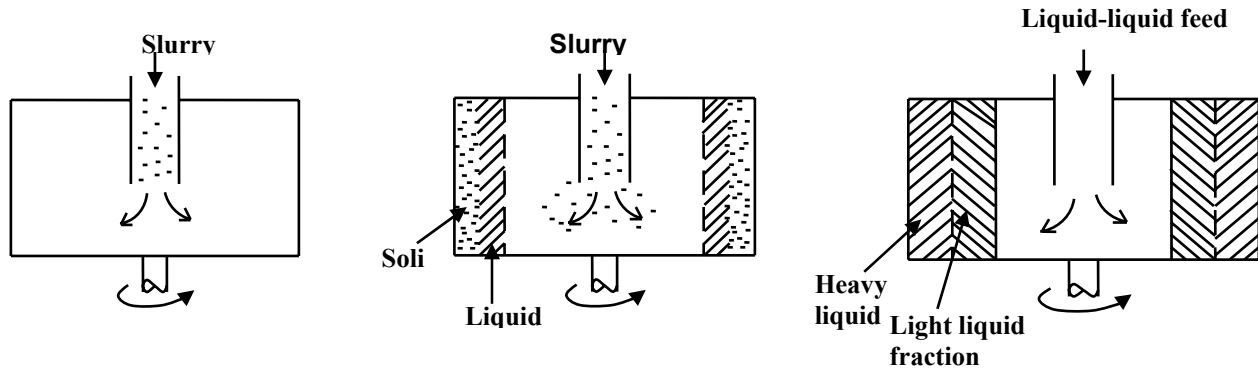


Fig. 7.1: Centrifugal separation: (a) the solid particles experience a force directed towards the wall; (b) In case of solid-liquid slurry, solid particles are pushed towards the wall and get collected; (c) In case of liquid-liquid system, heavier liquid having higher density are pushed towards the wall and can be collected.

Principle:

Centrifugal force acting on particle of mass m at a radial location r is

$$F_c = m\omega^2 r \quad (7.1)$$

The angular velocity is related to the linear velocity as, $\omega = \frac{v}{r}$ (7.2)

If rotating speed is N rev/min, then $\omega = \frac{2\pi N}{60}$ and the centrifugal force in Newton is calculated as,

$$F_c = m \frac{4\pi^2 N^2}{3600} r = 0.01097 m r N^2 \quad (7.3)$$

On the other hand, it is interesting to estimate the gravitational force on a particle of mass m ,

$$F_g = mg \quad (7.4)$$

The ratio of centrifugal to gravitational force can be computed as,

$$\frac{F_c}{F_g} = \frac{r\omega^2}{g} = \frac{r}{g} \left(\frac{2\pi N}{60} \right)^2 = 0.001118rN^2 \quad (7.5)$$

Where, $g \approx 10 \text{ m/s}^2$. Thus, force developed in centrifuge is $\frac{r\omega^2}{g}$ times that of gravity. This is after expressed as times 'g' forces. For example, if radius of a centrifuge bowl is 0.1016 m, and rotating at $N=1000 \text{ rev/min}$, the ratio of centrifugal to gravitational force becomes,

$$\frac{F_c}{F_g} = 0.001118rN^2 = 113.6 \text{ times gravity or } \underline{\underline{\text{"113.6g"}}}$$

If $r=0.2032$ then, $\frac{F_c}{F_g} = 227.2$ times gravity or **"227.2g"** centrifugal force is developed.

Settling rates in centrifuge:

If a centrifuge is used for sedimentation (removal of particles by settling), a particle of a given size can be removed from the liquid in the bowl if sufficient residence time of particle in the bowl is available for the particle to reach the wall. For a particle moving radially at its terminal velocity, the diameter of smallest particle removed can be calculated. The detailed geometry of a typical solid-liquid centrifuge is shown in Fig. 7.2.

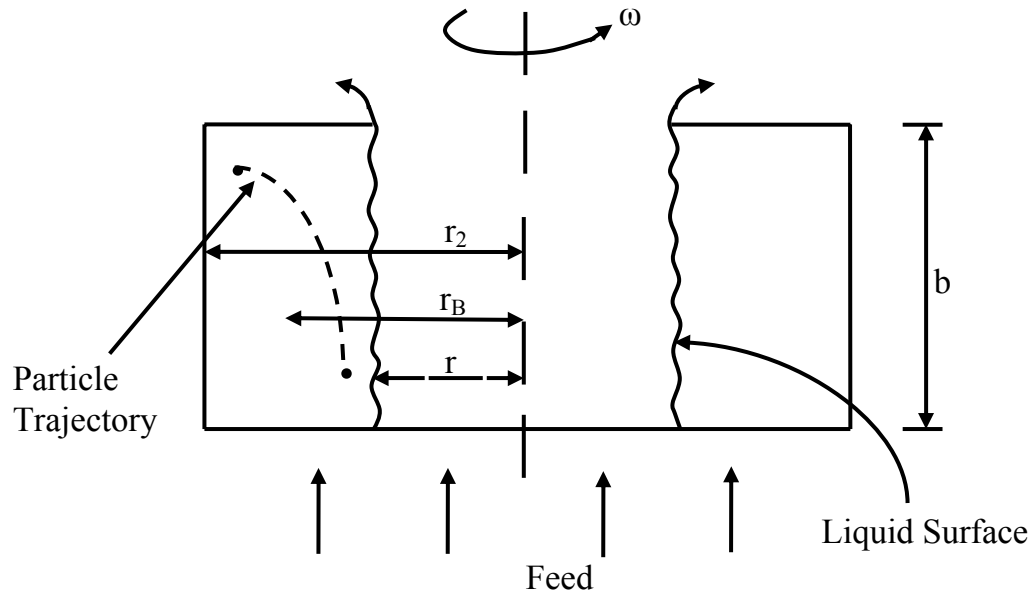


Fig. 7.2: Schematic of geometry in centrifuge

The feed enters from the bottom at uniform velocity. The length of bowl is b . A particle is removed if it has sufficient residence time to reach the wall. At the end of residence time, particle is at a distance r_B from the axis of rotation. If r_B is less than r_2 , the particle leaves the bowl with the fluid. If $r_B = r_2$, it is deposited on the wall and effectively removed from the liquid. In Stokes law regime ($Re < 0.1$), terminal velocity is expressed in gravitational field as,

$$v_t = \frac{gD_p^2}{18\mu}(\rho_p - \rho) \quad (7.6)$$

The terminal velocity in the centrifugal force field becomes,

$$v_t = \frac{\omega^2 r D_p^2}{18\mu}(\rho_p - \rho) \quad (7.7)$$

The terminal velocity in this field can be expressed as $v_t = \frac{dr}{dt}$. Therefore, by integrating

the above equation after equating $v_t = \frac{dr}{dt}$,

$$\int_0^{t_T} dt = \frac{18\mu}{\omega^2 (\rho_p - \rho) D_p^2} \int_{r_1}^{r_2} \frac{dr}{r} \quad (7.8)$$

$$t_T = \frac{18\mu}{\omega^2 (\rho_p - \rho) D_p^2} \ln \left(\frac{r_2}{r_1} \right) \quad (7.9)$$

If the volumetric flow rate at the steady state of the liquid is q and V is the volume of the liquid column in the centrifuge, then residence time can be expressed as,

$$t_T = \frac{q}{V} = \pi b (r_2^2 - r_1^2) \quad (7.10)$$

Combining Eqs. (7.9) and (7.10), the volumetric flow rate is obtained in terms of known variables.

$$q = \frac{\omega^2 (\rho_p - \rho) D_p^2}{18\mu \ln \left(\frac{r_2}{r_1} \right)} \left[\pi b (r_2^2 - r_1^2) \right] \quad (7.11)$$

Particles with diameter smaller than D_p calculated from the above equation, will not reach the wall of the bowl and will go out with the exit liquid. Larger particles reach the wall and are removed from the liquid. A cut point/ critical diameter D_{pc} is defined as the diameter of the particle that reaches half the distance between r_1 and r_2 within residence time, t_T .

$$\text{at } t=0, \quad r = \frac{r_1 + r_2}{2} \quad (7.12)$$

$$\text{at } t=t_T, \quad r=r_2 \quad (7.13)$$

Therefore, the critical flow rate with the diameter D_{pc} is given as,

$$q_c = \frac{\omega^2 (\rho_p - \rho) D_{pc}^2}{18 \mu \ln \left[\frac{2r_2}{r_1 + r_2} \right]} V$$

$$= \frac{\omega^2 (\rho_p - \rho) D_{pc}^2}{18 \mu \ln \left[\frac{2r_2}{r_1 + r_2} \right]} \{ \pi b (r_2^2 - r_1^2) \} \quad (7.14)$$

At this q_c , particles with diameter greater than D_{pc} will settle to the wall and most of the smaller particles ($D < D_{pc}$) will remain in the liquid.

Sigma values and scale up issues

For scaling up of the centrifugal systems, the concept of sigma values is invoked. In this case, the volumetric flow rate is expressed in terms of terminal velocity under gravity and geometric factor. The volumetric flow rate can be recast from Eq.(7.14) as,

$$q_c = \frac{\omega^2 (\rho_p - \rho) D_{pc}^2}{18 \mu \ln \left[\frac{2r_2}{r_1 + r_2} \right]} V$$

$$= \frac{(\rho_p - \rho) D_{pc}^2}{18 \mu} \frac{\omega^2 V}{\ln \left[\frac{2r_2}{r_1 + r_2} \right]}$$

$$= \frac{2g (\rho_p - \rho) D_{pc}^2}{18 \mu} \frac{\omega^2 V}{2g \ln \left[\frac{2r_2}{r_1 + r_2} \right]}$$

$$= 2v_{t_{ug}} \Sigma \quad (7.15)$$

Where, v_{tug} is the terminal velocity under gravity

$$\Sigma = \frac{\omega^2 \pi b (r_2^2 - r_1^2)}{2g \ln \left[\frac{2r_2}{r_1 + r_2} \right]} \quad (\text{m}^2) \quad (7.16)$$

Σ is a physical characteristic of centrifuge, not the fluid-particle system.

Physical interpretation of Σ

Σ has the unit of area. It is area in m^2 of a gravitational settler that will leave same sedimentation characteristics as the centrifuge at the same feed rate. For scaling up, from lab to pilot scale,

$$v_{t1} = v_{t2} \Rightarrow \frac{q_1}{\Sigma_1} = \frac{q_2}{\Sigma_2} \quad (7.17)$$

This is dependable if centrifugal forces between the two are within a factor of 2 from each other. If different centrifuges are used efficiency factors have to be used as,

$$\frac{q_1}{E_1 \Sigma_1} = \frac{q_2}{E_2 \Sigma_2} \quad (7.18)$$

These efficiency factors are determined experimentally.

Separation of liquids

Separation of liquids are relevant in various applications like, emulsions in food product, fruit juice processing and relevant industries. This is also common in dairy industries for separation of skim milk and cream, etc. Tubular bowl centrifuge is quite common equipment in this regard. Fig. 7.3 shows the one half of this kind of centrifuge with various layers of fluids with geometrical details.

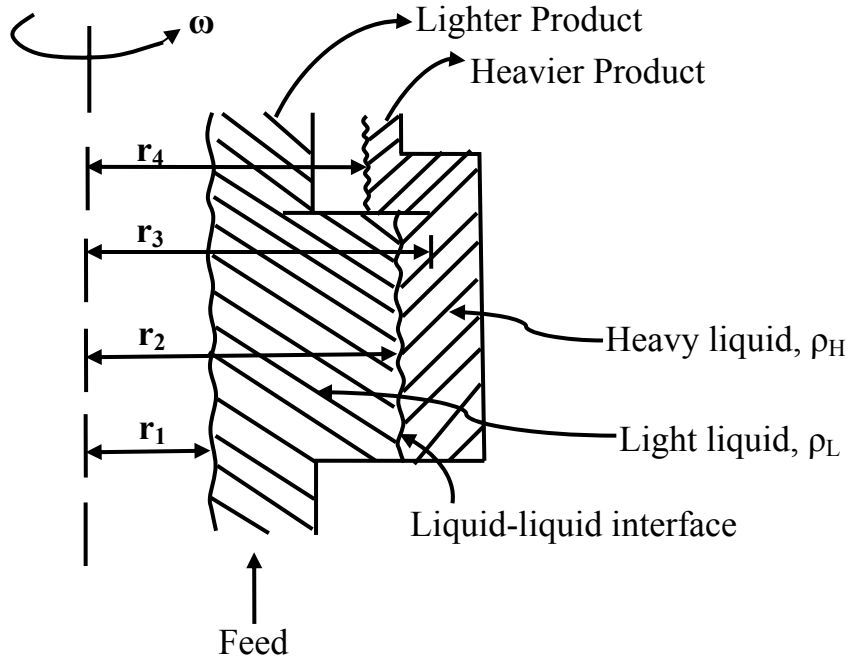


Fig. 7.3: Schematic of tubular centrifuge with geometrical details showing various liquid layers

In the above figure, following are the details of geometry. r_1 is the location of lighter liquid; r_2 is the location of liquid liquid interface; r_3 is the location of weir; r_4 is the location of heavy liquid downstream/product. Location of interface can be calculated by a balance of pressure in two layers. Force on the fluid located at a radial location r ,

$$F_c = mr\omega^2 \quad (7.19)$$

Differential force on a differential thickness dr is calculated as

$$dF_c = dm.r\omega^2 \quad (7.20)$$

Differential mass can be expressed as,

$$dm = (2\pi brdr) \rho \quad (7.21)$$

Where, $b \approx$ height of bowl. The pressure force can be expressed as,

$$dP = \frac{dF_c}{A}$$

$$= \omega^2 \rho r dr \quad (7.22)$$

Where, $A=2\pi r b$. Integrating Eq. (7.22) between r_1 and r_2 , the pressure difference between two points 1 and 2 is obtained.

$$P_2 - P_1 = \frac{\rho \omega^2}{2} (r_2^2 - r_1^2) \quad (7.23)$$

Equating the pressure over r_2 - r_4 and r_2 - r_1 , the following expression is obtained.

$$\frac{\rho_L \omega^2}{2} (r_2^2 - r_1^2) = \frac{\rho_H \omega^2}{2} (r_2^2 - r_4^2) \quad (7.24)$$

The location of the interface is calculated now

$$r_2^2 = \frac{\rho_H r_4^2 - \rho_L r_1^2}{\rho_H - \rho_L} \quad (7.25)$$

The interface r_2 must be located such that $r_2 < r_3$; otherwise, no separation occurs between heavy and lighter fraction .

Solved Problems

1. A viscous solution containing particles with density $\rho_p=1461 \text{ Kg/m}^3$ is clarified by centrifugation. Solution density $\rho=801 \text{ Kg/m}^3$; Viscosity $\mu=100 \text{ cp}$. Centrifuge bowl with $r_2=0.02225 \text{ m}$; $r_1=0.00716 \text{ m}$; height= $b=0.197 \text{ m}$; Calculate critical particle diameter of the largest particles in the exit stream if $N=23000 \text{ rev/min}$, $q=0.002832 \text{ m}^3/\text{hr}$.

Solution:

$$\omega = \frac{2\pi N}{60} = \frac{2\pi(23000)}{60} = 2410 \text{ rad/s}$$

$$\begin{aligned} \text{Bowl volume } V, \quad V &= \pi b(r_2^2 - r_1^2) = \pi(0.197)[0.02225^2 - 0.00716^2] \\ &= 2.747 \times 10^{-4} \text{ m}^3 \end{aligned}$$

$$\mu = 10^{-3} \times 100 = 0.1 \text{ pa.s}$$

$$q_c = \frac{0.002832}{3600} = 7.887 \times 10^{-7} \text{ m}^3/\text{s}$$

$$D_{pc} = 0.746 \text{ } \mu\text{m}$$

Calculate v_t , check $N_{Re} < 1.0$

2. A viscous solution contains particles with a density $\rho_p = 1200 \text{ kg/m}^3$ is to be clarified by centrifugation. The solution density is $\rho = 850 \text{ kg/m}^3$ and its viscosity is 80 cp . The centrifuge has a bowl with $r_2 = 0.02 \text{ m}$ and $r_1 = 0.01 \text{ m}$ and height $b = 0.25 \text{ m}$. Calculate the critical particle diameter of the largest particles in the exit stream if $N = 15000 \text{ rpm}$ and flow rate $q = 0.002 \text{ m}^3/\text{hr}$?

Solution:

$$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 15000}{60} = 1570 \text{ rad/s}$$

$$\begin{aligned} \text{The bowl volume,} \quad V &= \pi b(r_2^2 - r_1^2) \\ &= \pi \times 0.25(0.02^2 - 0.01^2) \\ &= 2.355 \times 10^{-4} \text{ m}^3 \end{aligned}$$

$$q = \text{flowrate} = \frac{0.002}{3600} \text{ m}^3/\text{s} = 5.56 \times 10^{-7} \text{ m}^3/\text{s}$$

$$5.56 \times 10^{-7} = \frac{1570^2 (1200 - 850) D_{pc}^2}{18 \times 80 \times 10^{-3} \ln \left(\frac{2 \times 0.02}{0.01 + 0.02} \right)} \times 2.355 \times 10^{-4}$$

$$= 84.16 \times 10^3 D_{pc}^2$$

$$D_{pc} = 2.57 \mu m$$

3. In the above problem, we would like to scale up the centrifuge. We have to design a centrifuge such that it can handle 1.5 times q in Question (3). r_1 and r_2 remains same, find the length of the centrifuge. Both the centrifuges have same rotational speed?

Solution:

$$\frac{q_1}{\Sigma_1} = \frac{q_2}{\Sigma_2}$$

$$\frac{q_1}{q_2} = \frac{\Sigma_1}{\Sigma_2}$$

$$\Sigma = \frac{\omega^2 \pi b (r_2^2 - r_1^2)}{2g \ln \left(\frac{2r_2}{r_1 + r_2} \right)}$$

$$\frac{q_1}{1.5q_1} = \frac{b_1}{b_2}$$

$$b_2 = 1.5b_1 = 1.5 \times 0.25 \text{ m}$$

$$b_2 = 0.375 \text{ m}$$

References:

1. C. J. Geankoplis, Transport Processes and Unit Operations, Prentice Hall of India, New Delhi, 1997.