

[A] Calculation of Double Pipe Heat Exchanger :

The calculation procedure consists simply of computing  $h_o$  and  $h_i$  to obtain  $U_c$ .  $U_D$  is obtained taking into account fouling resistances. Knowing  $U_D$ , the surface area can be found with the help of the following equation

$$Q = U_D \cdot A \cdot \Delta T_{lm}$$

In the stepwise procedure of calculation given below, the hot and cold fluid temperatures are represented by the upper and lower case letters respectively. All the fluid properties are indicated by the lower case letters.

**Process conditions required :**

Hot fluid :  $T_1, T_2, \dot{m}_h, C_p, \mu, k, \Delta P, R_{do}$  or  $R_{di}$

Cold fluid :  $t_1, t_2, \dot{m}_c, C_p, \mu, k, \Delta P, R_{di}$  or  $R_{do}$

The diameter of each pipe must be given or assumed. A convenient order of the calculation is as given below :

1. Check the heat balance,  $Q$ , from  $T_1, T_2, t_1, t_2$  using  $C_p$  at  $T_{mean}$  ( $T_m$ ) and  $t_{mean}$  ( $t_m$ ).

Calculate  $Q$  using the following equation :

$$Q = \dot{m}_h C_{p_h} (T_1 - T_2) = \dot{m}_c C_{p_c} (t_2 - t_1)$$

$$T_1 > T_2 \text{ and } t_2 > t_1$$

$$T_m = T_1 + T_2/2, \quad t_m = t_1 + t_2/2$$

2. Calculate the log mean temperature difference LMTD ( $\Delta T_{lm}$ ) for a counterflow arrangement.

$$T_1 \longrightarrow T_2$$

$$t_2 \longleftarrow t_1$$

$$\Delta T_1 = T_1 - t_2, \quad \Delta T_2 = T_2 - t_1$$

$$L.M.T.D. = \Delta T_{lm} = \frac{\Delta T_1 - \Delta T_2}{\ln \frac{\Delta T_1}{\Delta T_2}}, \text{ K}$$

3. Evaluate the physical properties of the hot and cold fluid at the arithmetic mean of  $T_1$  and  $T_2$  and  $t_1$  and  $t_2$ . For non-viscous fluids,  $\left(\frac{\mu}{\mu_w}\right)^{0.14}$  may be taken as 1.0 (as assumed below).

**[B] Calculation of Shell and Tube Heat Exchanger :**

**1. Shell side coefficient :**

The shell side coefficient is given by the following equation :

$$\frac{h_o D_e}{k} = 0.36 \left( \frac{D_e G_s}{\mu} \right)^{0.55} \left( \frac{C_p \mu}{k} \right)^{1/3} \left( \frac{\mu}{\mu_w} \right)^{0.14}$$

where  $D_e$  is the shell equivalent diameter and  $G_s$  is the shell side mass velocity.

All properties of the shell side fluid are evaluated at an average/mean bulk temperature which is the average of inlet and outlet temperatures of the shell side fluid.

Above equation is valid for  $N_{Re}$  in the range of 2000 to 1000000.

**2. Shell side mass velocity :**

The shell side or bundle cross flow area ' $a_s$ ' is calculated as

$$a_s = \frac{I.D. \times C' \times B}{P_T} = \frac{\text{Shell I.D.} \times \text{Clearance} \times \text{Baffle spacing}}{\text{Pitch}}, \text{ m}^2$$

Mass velocity  $G_s$  is given as

$$G_s = \dot{m} / a_s, \quad \text{kg}/(\text{m}^2 \cdot \text{s})$$

### 3. Shell side equivalent diameter, $D_e$ :

$$D_e = 4 \times \text{Free area} / \text{Wetted perimeter, m}^2$$

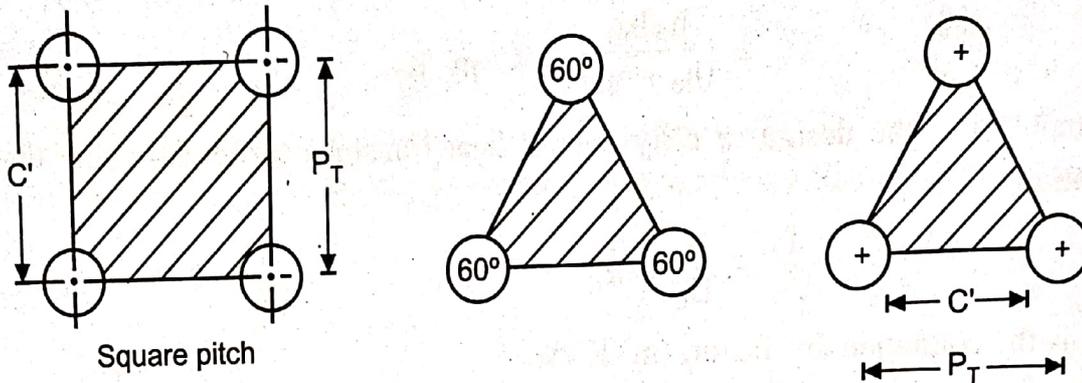


Fig. 5.19 : Equivalent diameter

For a square pitch :

$$D_e = \frac{4 \times (P_T^2 - \pi d_o^2 / 4)}{\pi d_o}$$

For a triangular pitch :

$$D_e = \frac{4 \times \left( \frac{1}{2} P_T \times 0.86 P_T - \frac{1}{2} \pi d_o^2 / 4 \right)}{1/2 \times \pi d_o}$$

where,  $d_o$  = outer diameter of the tube and  $P_T$  = Pitch of the tube

### 4. Tube side coefficient :

It can be evaluated either by using Sieder-Tate equation or Dittus-Boelter equation.

5. For the remaining part, the procedure as outlined for the double pipe heat exchanger can be used.

Area of heat transfer,  $A_o = n \times a \times L$

where  $n$  is number of tubes,  $L$  is length of tube and ' $a$ ' is the outside area (external surface) in  $m^2$  per linear m of tube [ $a$  is given in  $m^2/m$ ].

$A_o$  can also be given by

$$A_o = n \cdot \pi d_o \cdot L$$

where,  $n$  is number of tubes,  $d_o$  is outside diameter of the tube, and  $L$  is length of the tube.

### 6. Check for suitability of heat exchanger :

$$R_d = \frac{U_c - U_D}{U_c \cdot U_D} \text{ (m}^2 \cdot \text{K) / W}$$

If  $R_d$  calculated from the above equation is equal to or greater than  $R_d$  required/provided (given in the problem statement) then the exchanger of a given specifications (e.g., i.d. of shell, o.d. of tube, length of tube and number of tubes, etc.) is suitable.

**Example 5.8 :** 14500 kg/h of nitrobenzene are to be cooled from 400 K (127°C) to 317 K (44°C) by heating up 40000 kg/h of benzene from 305 K (32°C) to 345 K (72°C). There are two heat exchangers available and these are to be operated in parallel, each with a shell diameter of 45 cm I.D. fitted with 166 tubes of 19 mm O.D. and 15 mm I.D. and 5 m long. The exchangers are of 2-2 shell and tube type. The tubes are arranged on a 25 mm square pitch with 15 cm of baffle spacing. There are two passes on the shell side. Counter-current operation is used. Assuming that benzene is flowing through tubes and heat transfer coefficient on the tube side to be 1050 W/(m<sup>2</sup>·K), find the order of scale resistance that could be allowed if the heat exchangers are used ?

**Data :** For nitrobenzene :

$$C_p = 2.387 \text{ kJ}/(\text{kg}\cdot\text{K})$$

$$\mu = 7.0 \times 10^{-4} \text{ Pa}\cdot\text{s}, \quad k = 0.151 \text{ W}/(\text{m}\cdot\text{K})$$

Use the following correlation for calculating the shell side heat transfer coefficient :

$$N_{Nu} = 0.36 \left( \frac{D_e G_s}{\mu} \right)^{0.55} (C_p \mu/k)^{1/3}$$

**Solution :** As the heat exchangers are in parallel, each of the heat exchangers will handle 20000 kg/h of benzene through tubes and 7250 kg/h of nitrobenzene in shell.

For counter-current flow :

$$400 \text{ K} \xrightarrow{\text{nitrobenzene}} 317 \text{ K}$$

$$345 \text{ K} \xleftarrow{\text{benzene}} 305 \text{ K}$$

$$\Delta T_1 = 400 - 345 = 55 \text{ K}$$

$$\Delta T_2 = 317 - 305 = 12 \text{ K}$$

$$\Delta T_{lm} = \frac{55 - 12}{\ln(55/12)} = 28.24 \text{ K}$$

For nitrobenzene :

$$Q = \dot{m} C_p (T_1 - T_2)$$

$$= 7250 \times 2.387 \times (400 - 317)$$

$$= 1436377 \text{ kJ/h} = 398994 \text{ W}$$

$$n = \text{Number of tubes} = 166$$

$$L = \text{Length of tube} = 5 \text{ m}$$

$$D_o = \text{O.D. of tube} = 19 \text{ mm} = 0.019 \text{ m}$$

$$A_o = \text{Area of heat transfer}$$

$$= n \cdot \pi D_o L = 166 \times \pi \times 0.019 \times 5 = 49.54 \text{ m}^2$$

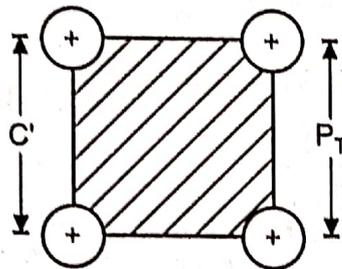
$$Q = U_o A_o \Delta T_{lm}$$

$$U_o = Q / (A_o \Delta T_{lm})$$

$$= \frac{398994}{49.54 \times 28.24} = 285.2 \text{ W}/(\text{m}^2 \cdot \text{K})$$

This is a design ( $U_D$ ) or an actual overall heat transfer coefficient.

**Shell side heat transfer coefficient :**



**Fig. 5.20 : Square pitch of tubes**

$$P_T = 25 \text{ mm} = 0.025 \text{ m}$$

$$C' = P_T - \left( \frac{1}{2} \text{ O.D.} + \frac{1}{2} \text{ O.D.} \right) = P_T - \text{O.D.}$$

$$= 25 - 19 = 6 \text{ mm} = 0.006 \text{ m}$$

**Shell side cross flow area :**

$$a_s = \frac{1 \text{ D (shell)} \times C' \times B}{P_T}$$

where

$$B = 15 \text{ cm} = 0.15 \text{ m}$$

$$\text{I.D. (shell)} = 45 \text{ cm} = 0.45 \text{ m}$$

$$a_s = \frac{0.45 \times 0.006 \times 0.15}{0.025} = 0.0162 \text{ m}^2$$

As there are two shell passes, the flow area per pass is :

$$a'_s = 0.0162 / 2 = 0.0081 \text{ m}^2$$

Equivalent diameter of shell :

$$D_e = \frac{4 \times (P_T^2 - \pi/4 D_o^2)}{\pi D_o} \quad \text{where } D_o \text{ is O.D. of tube.}$$

$$= \frac{4 \times [(0.025)^2 - \pi/4 (0.019)^2]}{\pi \times 0.019} = 0.0229 \text{ m}$$

Mass velocity on shell side :

$$G_s = \dot{m} / a_s'$$

$$= \frac{7250}{0.0081} = 895062 \text{ kg}/(\text{m}^2 \cdot \text{h}) = 248.63 \text{ kg}/(\text{m}^2 \cdot \text{s})$$

$$\mu = 7.0 \times 10^{-4} \text{ Pa} \cdot \text{s}, \quad C_p = 2.387 \times 10^3 \text{ J}/(\text{kg} \cdot \text{K}), \quad k = 0.151 \text{ W}/(\text{m} \cdot \text{K})$$

Calculation of  $h_o$  :

$$N_{Re} = \frac{D_e G_s}{\mu}$$

$$= \frac{0.0229 \times 248.63}{7.0 \times 10^{-4}} = 8133.7$$

$$N_{Pr} = C_p \mu / k$$

$$= \frac{2.387 \times 10^3 \times 7.0 \times 10^{-4}}{0.151} = 11.06$$

$$N_{Nu} = 0.36 (N_{Re})^{0.55} (N_{Pr})^{1/3}$$

$$= 0.36 \times (8133.7)^{0.55} \times (11.06)^{1/3} = 113.5$$

$$h_o = 113.5 \times \frac{k}{D_e} = 113.5 \times \frac{0.151}{0.0229} = 748.4 \text{ W}/(\text{m}^2 \cdot \text{K})$$

Calculation of  $U_o$  :

Neglecting the thermal resistance of the pipe wall,

$$\frac{1}{U_o} = \frac{1}{h_o} + \frac{1}{h_i} \times \frac{D_o}{D_i}$$

$$\frac{1}{U_o} = \frac{1}{748.4} + \frac{1}{1050} \times \frac{0.019}{0.015}$$

$$U_o = 393.3 \text{ W}/(\text{m}^2 \cdot \text{K})$$

This is a clean overall heat transfer coefficient ( $U_c$ ) for the heat exchanger.

Calculation of  $R_d$  :

$$R_d = \frac{U_c - U_D}{U_c U_D}$$

$$= \frac{393.3 - 285.2}{393.3 \times 285.2} = 9.637 \times 10^{-4} \text{ (m}^2 \cdot \text{K)}/\text{W}$$

$$\text{Fouling factor} = \text{Scale resistance} = 9.637 \times 10^{-4} \text{ (m}^2 \cdot \text{K)}/\text{W}$$

... Ans.

**Example 5.9 :** Water at 303 K (30°C) enters a 25 mm I.D. tube at a rate of 1200 l/h. Steam condenses on the outside surface of tube (28 mm O.D.) at a temperature of 393 K (120°C) and its film heat transfer coefficient may be taken as 6000 W/(m<sup>2</sup>·K). Determine the length of the tube required to heat water to 343 K (70°C).

**Data :** Thermal conductivity of tube wall material = 348.9 W/(m·K).

Properties of water at mean temperature of 323 K (50°C) are :

$k = 0.628$  W/(m·K),  $\rho = 980$  kg/m<sup>3</sup> and  $\mu = 6 \times 10^{-4}$  kg/(m·s),  $C_p = 4.187$  kJ/(kg·K)

**Solution : Basis :** 1200 l/h of water flow.

$$\begin{aligned}\text{Mass flow rate of water} &= 1200 \times 10^{-3} \times 980 = 1176 \text{ kg/h} \\ &= 0.3267 \text{ kg/s}\end{aligned}$$

$$\text{Inside area of tube} = \pi/4 D_i^2 = \frac{\pi}{4} \times (0.025)^2 = 4.9087 \times 10^{-4} \text{ m}^2$$

$$\text{Mass velocity of water} = G = \frac{m_w}{A_i} = \frac{0.3267}{4.9087 \times 10^{-4}} = 665.55 \text{ kg/(m}^2 \cdot \text{s)}$$

$$\mu = 6 \times 10^{-4} \text{ kg/(m} \cdot \text{s)}$$

$$\rho = 980 \text{ kg/m}^3, D_i = 25 \text{ mm} = 0.025 \text{ m}$$

$$N_{Re} = \frac{D_i G}{\mu} = \frac{0.025 \times 665.55}{6.0 \times 10^{-4}} = 27731$$

Since  $N_{Re} > 10000$ , the flow is turbulent.

$$k \text{ for water} = 0.628 \text{ W/(m} \cdot \text{K)}$$

$$\mu \text{ for water} = 6.0 \times 10^{-4} \text{ kg/(m} \cdot \text{s)}$$

$$C_p \text{ for water} = 4.187 \text{ kJ/(kg} \cdot \text{K)} = 4.187 \times 10^3 \text{ J/(kg} \cdot \text{K)}$$

$$N_{Pr} = \frac{C_p \mu}{k} = \frac{4.187 \times 10^3 \times 6.0 \times 10^{-4}}{0.628} = 4.0$$

**Inside heat transfer coefficient :**

Since water is getting heated, the Dittus-Boelter equation for heating is :

$$\begin{aligned}N_{Nu} &= 0.023 (N_{Re})^{0.8} (N_{Pr})^{0.4} \\ &= 0.023 \times (27731)^{0.8} \times (4.0)^{0.4} \\ &= 143.52\end{aligned}$$

$$h_i = 143.52 \times \frac{k}{D_i} = \frac{143.52 \times 0.628}{0.025}$$

$$h_i = 3605 \text{ W/(m}^2 \cdot \text{K)}$$

$$h_o \text{ (steam side coefficient)} = 6000 \text{ W/(m}^2 \cdot \text{K)}$$

$$D_o = 28 \text{ mm} = 0.028 \text{ m}$$

$$D_w = \frac{0.028 - 0.025}{\ln(0.028/0.025)} = 0.0265 \text{ m}$$

$$x = (D_o - D_i)/2 = (0.028 - 0.025)/2 = 0.0015 \text{ m}$$

$$k \text{ for metal} = 348.9 \text{ W/(m}\cdot\text{K)}$$

Calculation of  $U_o$  :

$$\frac{1}{U_o} = \frac{1}{h_o} + \frac{1}{h_i} \times \frac{D_o}{D_i} + \frac{x}{k} \times \frac{D_o}{D_w}$$

$$\frac{1}{U_o} = \frac{1}{6000} + \frac{1}{3605} \times \frac{0.028}{0.025} + \frac{0.0015}{348.9} \times \frac{0.028}{0.0265}$$

$$U_o = 2075 \text{ W/(m}^2\cdot\text{K)}$$

Calculation of  $Q$  :

$$Q = \dot{m}_w C_{pw} (t_2 - t_1)$$

$$= 1176 \times 4.187 \times (343 - 303)$$

$$= 196956.48 \text{ kJ/h} = 54710.13 \text{ W}$$

Calculation of  $\Delta T_{lm}$  :

$$T_s = \text{Temperature of condensing steam} = 393 \text{ K}$$

$$t_2 = 343 \text{ K} \quad \text{and} \quad t_1 = 303 \text{ K}$$

$$\Delta T_1 = 393 - 303 = 90 \text{ K}, \quad \Delta T_2 = 393 - 343 = 50 \text{ K}$$

$$\Delta T_{lm} = (90 - 50)/\ln(90/50) = 68.05 \text{ K}$$

$$Q = UA \Delta T_{lm} = U_o A_o \Delta T_{lm}$$

Calculation of  $A$  and  $L$  :

$$A_o = \frac{Q}{U_o \cdot \Delta T_{lm}}$$

$$= \frac{54710.13}{2075 \times 68.05}$$

$$= 0.3874 \text{ m}^2$$

$$\text{Area of heat transfer} = A_o = \pi D_o L$$

$$L = A_o / (\pi D_o)$$

$$= 0.3874 / (\pi \times 0.028)$$

$$= 4.40 \text{ m}$$

$$\text{Length of tube required} = 4.40 \text{ m}$$

... Ans.

**Example 5.10 :** 1-2 shell and tube heat exchanger is to be used to cool nitrobenzene from 400 K (127°C) to 317 K (44°C) with the help of benzene, entering at 300 K (27°C) and leaving at 333 K (60°C). Benzene is flowing at a rate of 20000 kg/h through tubes and the tube side coefficient is 1050 W/(m<sup>2</sup>·K). Nitrobenzene is flowing through the shell at a rate of 7250 kg/h. The shell inside diameter is 450 mm and is fitted with 170 tubes of 19 mm o.d. and 15 mm i.d. and 5 m long. The tubes are arranged on a 25 mm square pitch. Baffle spacing is 150 mm. Fouling factor to be provided is  $9 \times 10^{-4}$  m<sup>2</sup>·K/W.

Check the suitability of this exchanger.

**Data :** For nitrobenzene :

$$C_p = 2.387 \text{ kJ/(kg}\cdot\text{K)}, \quad \mu = 7.0 \times 10^{-4} \text{ kg/(m}\cdot\text{s)}, \quad k = 0.151 \text{ W/(m}\cdot\text{K)}.$$

Viscosity correction factor = 1.0 and LMTD correction factor = 0.90 for the calculation purpose.

**Solution : Basis :** 7250 kg/h of nitrobenzene flowing through the shell.

**For counter-current flow :**

$$\begin{array}{ccc} 400 \text{ K} & \xrightarrow{\text{nitrobenzene}} & 317 \text{ K} \\ & & \xleftarrow{\text{benzene}} \\ 333 \text{ K} & & 300 \text{ K} \end{array}$$

$$\Delta T_1 = 400 - 333 = 67 \text{ K}$$

$$\Delta T_2 = 317 - 300 = 17 \text{ K}$$

$$\Delta T_{lm} = (67 - 17) / \ln(67/17) = 36.46 \text{ K}$$

**For nitrobenzene :**

$$\begin{aligned} Q &= \dot{m} C_p (T_1 - T_2) \\ Q &= 7250 \times 2.387 (400 - 317) \\ &= 1436377 \text{ kJ/h} = 398994 \text{ W} \end{aligned}$$

$$\text{Number of tubes} = n = 170$$

$$\text{Length of tubes} = L = 5 \text{ m}$$

$$\text{o.d. of the tube} = D_o = 19 \text{ mm} = 0.019 \text{ m}$$

$$\text{i.d. of the tube} = D_i = 15 \text{ mm} = 0.015 \text{ m}$$

$$\begin{aligned} \text{Area of heat transfer} &= A_o = n \pi D_o L \\ &= 170 \times \pi \times 0.019 \times 5 \\ &= 50.74 \text{ m}^2 \end{aligned}$$

$$Q = U_o A_o F_T \Delta T_{lm}$$

$$\begin{aligned} U_o &= Q / (A_o F_T \Delta T_{lm}) \\ &= 398994 / (50.74 \times 0.90 \times 36.46) \\ &= 239.64 \text{ W/(m}^2\cdot\text{K)} \end{aligned}$$

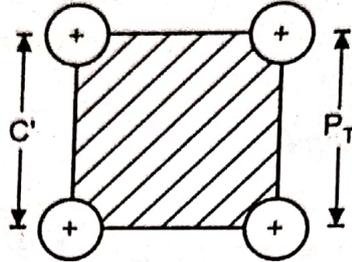
$$U_o = U_D = 239.64 \text{ W/(m}^2\cdot\text{K)}$$

**Shell side heat transfer coefficient :**

$$\text{Baffle spacing} = B = 150 \text{ mm} = 0.15 \text{ m}$$

$$\text{Tube pitch} = P_T = 25 \text{ mm} = 0.025 \text{ m}$$

**For square pitch of tubes :**



**Fig. 5.21**

$$\begin{aligned} \text{Clearance} = C' &= P_T - 2 \times \frac{1}{2} \text{ O.D.} = 25 - 2 \times \frac{1}{2} \times 19 \\ &= 6 \text{ mm} = 0.006 \text{ m} \end{aligned}$$

$$\text{Shell inside diameter} = \text{I.D.} = 450 \text{ mm} = 0.45 \text{ m}$$

**Shell side flow area :**

$$a_s = \frac{\text{I.D.} \times C' \times B}{P_T} = \frac{0.45 \times 0.006 \times 0.15}{0.025} = 0.0162 \text{ m}^2$$

**Equivalent diameter of shell :**

$$D_e = \frac{4 \times (P_T^2 - \pi/4 D_o^2)}{\pi D_o}$$

$$D_e = \frac{4 \times [(0.025)^2 - \pi/4 (0.019)^2]}{\pi \times 0.019} = 0.0229 \text{ m}$$

**Mass velocity on shell side :**

$$\begin{aligned} G_s &= \dot{m} / a_s \\ &= \frac{7250}{0.0162} = 447531 \text{ kg}/(\text{m} \cdot \text{h}) \\ &= 124.3 \text{ kg}/(\text{m}^2 \cdot \text{s}) \end{aligned}$$

$$\mu = 7.0 \times 10^{-4} \text{ kg}/(\text{m} \cdot \text{s}), \quad C_p = 2.387 \times 10^3 \text{ J}/(\text{kg} \cdot \text{K})$$

$$k = 0.151 \text{ W}/(\text{m} \cdot \text{K})$$

$$\begin{aligned} N_{Re} &= \frac{D_e G_s}{\mu} \\ &= \frac{0.0229 \times 124.3}{7.0 \times 10^{-4}} = 4066.4 \end{aligned}$$

$$\begin{aligned} N_{Pr} &= C_p \mu / k \\ &= \frac{2.387 \times 10^3 \times 7.0 \times 10^{-4}}{0.151} = 11.06 \end{aligned}$$

The empirical equation for the calculation of heat transfer coefficient is

$$N_{Nu} = 0.36 (N_{Re})^{0.55} (N_{Pr})^{1/3} (\mu/\mu_w)^{0.14}$$

Given :  $(\mu/\mu_w)^{0.14} = 1.0$

$$\begin{aligned} \therefore N_{Nu} &= 0.36 (N_{Re})^{0.55} (N_{Pr})^{1/3} \\ &= 0.36 (4066.4)^{0.55} \times (11.06)^{1/3} = 77.5 \end{aligned}$$

$$\frac{h_o D_c}{k} = 77.5$$

$$\therefore h_o = 77.5 \times \frac{0.151}{0.0229} = 511 \text{ W/(m}^2 \cdot \text{K)}$$

$$h_i \text{ (given)} = 1050 \text{ W/(m}^2 \cdot \text{K)}$$

Neglecting the thermal resistance of the pipe wall :

$$\frac{1}{U_o} = \frac{1}{h_o} + \frac{1}{h_i} \times \frac{D_o}{D_i}$$

$$\therefore \frac{1}{U_o} = \frac{1}{511} + \frac{1}{1050} \times \frac{0.019}{0.015}$$

$$\therefore U_o = 316.1 \text{ W/(m}^2 \cdot \text{K)}$$

This is a clean overall heat transfer coefficient.

$$\therefore U_D = 239.64 \text{ W/(m}^2 \cdot \text{K)}$$

$$U_C = 316.10 \text{ W/(m}^2 \cdot \text{K)}$$

**Suitability of the heat exchanger :**

If the  $R_d$  calculated is equal to or more than the  $R_d$  given, then the heat exchanger is suitable.

$$R_d \text{ given} = 9.0 \times 10^{-4} \text{ W/(m}^2 \cdot \text{K)}$$

Let us calculate  $R_d$ . It is given by

$$\frac{1}{U_D} = \frac{1}{U_C} + R_d$$

$$R_d = \frac{U_C - U_D}{U_C \cdot U_D}$$

$$R_d = \frac{316.1 - 239.64}{316.1 \times 239.64}$$

$$= 1.01 \times 10^{-3} \text{ W/(m}^2 \cdot \text{K)}$$

$R_d$  calculated is the maximum allowable scale resistance.

$$R_d \text{ calculated} = 1.01 \times 10^{-3} \text{ W/(m}^2 \cdot \text{K)}$$

As  $R_d$  calculated is more than  $R_d$  given or provided, the given heat exchanger is suitable. ... Ans.

**Example 5.11 :** A heat exchanger is to be designed to heat 1720 kg/h of water from 293 K (20°C) to 318 K (45°C) with saturated steam condensing on the outside surface of the brass tubes of 25 mm O.D. and 22.5 mm I.D. Tube length is 4 m. Assuming water velocity is being constant at 1.2 m/s, determine the number of tubes required in the heat exchanger.

**Data :** Thermal conductivity of brass = 460 kJ/(h·m·K)

Latent heat of vaporisation of steam = 2230 kJ/kg

Steam side coefficient = 19200 kJ/(h·m<sup>2</sup>·K)

Physical properties of water at mean fluid temperature are as follows :

Density = 995.7 kg/m<sup>3</sup>, Specific heat = 4.28 kJ/(kg·K)

Thermal conductivity = 2.54 kJ/(h·m·K)

Kinematic viscosity = 0.659 × 10<sup>-6</sup> m<sup>2</sup>/s

**Solution : Basis :** 1720 kg/h of water flow rate

Inlet temperature of water =  $t_1 = 293$  K

Outlet temperature of water =  $t_2 = 318$  K

Specific heat of water =  $C_{pw} = 4.28$  kJ/(kg·K)

**Calculation of Q :**

$$\begin{aligned} Q &= m_w C_{pw} \times (t_2 - t_1) \\ &= 1720 \times 4.28 \times (318 - 293) \\ &= 184040 \text{ kJ/h} \equiv 51122.2 \text{ W} \end{aligned}$$

**Calculation of  $\Delta T_{lm}$  :**

Latent heat of vaporisation of steam =  $\lambda = 2230$  kJ/kg

Saturation temperature of condensing steam corresponding to the latent heat from steam table is 383 K (110°C).

$$\Delta T_1 = 383 - 293 = 90 \text{ K}$$

$$\Delta T_2 = 383 - 318 = 65 \text{ K}$$

$$\Delta T_{lm} = \text{LMTD} = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1 / \Delta T_2)} = \frac{90 - 65}{\ln(90/65)} = 77 \text{ K}$$

**Calculation of inside heat transfer coefficient :**

$$D_i = \text{I.D. of tube} = 22.5 \text{ mm} = 0.0225 \text{ m}$$

$$u = 1.2 \text{ m/s}$$

$$\rho = 995.7 \text{ kg/m}^3$$

$$\nu = 0.659 \times 10^{-6} \text{ m}^2/\text{s}$$

$$\mu = \nu \times \rho = 0.659 \times 10^{-6} \times 995.7 = 6.56 \times 10^{-4} \text{ kg/(m}\cdot\text{s)}$$

$$N_{Re} = \frac{D_i u \rho}{\mu} = \frac{0.0225 \times 1.2 \times 995.7}{6.56 \times 10^{-4}} = 40981$$

Since  $N_{Re} > 10000$ , the flow is turbulent.

$$C_p = 4.28 \text{ kJ/(kg}\cdot\text{K)} = 4.28 \times 10^3 \text{ J/(kg}\cdot\text{K)}$$

$$\mu = 6.56 \times 10^{-4} \text{ kg/(m}\cdot\text{s)}$$

$$k = 2.54 \text{ kJ/(h}\cdot\text{m}\cdot\text{K)} = 0.706 \text{ W/(m}\cdot\text{K)}$$

$$N_{Pr} = \frac{C_p \mu}{k} = \frac{4.28 \times 10^3 \times 6.56 \times 10^{-4}}{0.706} = 3.98$$

The Dittus-Boelter equation for heating (as water is heated) is

$$\begin{aligned} N_{Nu} &= 0.023 (N_{Re})^{0.8} (N_{Pr})^{0.4} \\ &= 0.023 \times (40981)^{0.8} \times (3.98)^{0.4} \\ &= 195.77 \end{aligned}$$

$$\frac{h_i D_i}{k} = 195.77$$

$$\therefore h_i = 195.77 \times \frac{0.706}{0.0225} = 6143 \text{ W/(m}^2\cdot\text{K)}$$

$$\begin{aligned} \text{Steam side coefficient} &= h_o \\ &= 19200 \text{ kJ/(h}\cdot\text{m}^2\cdot\text{K)} \\ &= 5333 \text{ W/(m}^2\cdot\text{K)} \end{aligned}$$

$$D_o = 25 \text{ mm} = 0.025 \text{ m}, \quad D_i = 22.5 \text{ mm} = 0.0225 \text{ m}$$

$$D_w = \text{log mean diameter} = \frac{0.025 - 0.0225}{\ln(0.025/0.0225)} = 0.0237 \text{ m}$$

$$\text{Thickness of tube, } x = (0.025 - 0.0225)/2 = 0.00125 \text{ m}$$

$$k \text{ for tube wall material} = 460 \text{ kJ/(h}\cdot\text{m}\cdot\text{K)} = 127.8 \text{ W/(m}\cdot\text{K)}$$

**Calculation of  $U_o$ :**

$$\frac{1}{U_o} = \frac{1}{h_o} + \frac{1}{h_i} \times \frac{D_o}{D_i} + \frac{x}{k} \times \frac{D_o}{D_w}$$

$$\frac{1}{U_o} = \frac{1}{5333} + \frac{1}{6143} \times \frac{0.025}{0.0225} + \frac{0.00125}{127.8} \times \frac{0.025}{0.0237}$$

$$\therefore U_o = 2640.6 \text{ W/(m}^2\cdot\text{K)}$$

**Calculation of A and n:**

$$Q = U_o A_o \Delta T_{lm}$$

$$A_o = Q/(U_o \cdot \Delta T_{lm})$$

$$= \frac{51122.2}{2640.6 \times 77} = 0.2514 \text{ m}^2$$

$$A_o = n\pi D_o L$$

$$n = A_o/(\pi D_o L) = 0.2514/(\pi \times 0.025 \times 4) = 0.8 \approx 1.0$$

Number of tubes required = 1.0

... Ans.

**Example 5.12 :** A shell-and-tube heat exchanger is used as a single pass condenser. There is a condensation of benzene vapour in its shell at 350 K (77°C). The condenser consists of 120 tubes of 22 mm internal diameter and length 2500 mm each. Through the tubes, cooling water flows with a velocity of 75 cm/s, ensuring a condensation rate of benzene vapours equal to 14.4 t/h initially when there is no scaling. But after a long-standing operation, a scale resistance of  $2.5 \times 10^{-4}$  ( $\text{m}^2 \cdot \text{K}/\text{W}$ ) develops on the water side on the tube surfaces. To what extent the velocity of water must be increased so as to regain the initial rate of condensation of benzene on the assumption that the heat transfer coefficient on the water side is proportional to the flow velocity raised to 0.8 in the power.

**Data :** Inlet temperature of cooling water = 290 K (17°C)

Heat transfer coefficient for condensing vapour based on inside area = 2250 W/( $\text{m}^2 \cdot \text{K}$ )

Latent heat of vaporisation of benzene = 400 kJ/kg.

**Solution : Basis :** 14.4 t/h condensation rate of benzene vapour.

**I. With no scale :**

$$\begin{aligned} \text{Heat duty of condenser} = Q &= \dot{m}_b \times \lambda \\ &= 14.4 \times 10^3 \times 400 \\ &= 5760000 \text{ kJ/h} = 16.0 \times 10^5 \text{ W} \end{aligned}$$

Shell and tube heat exchanger is used as a surface condenser. The condenser consists of tubes with :

$$D_i = \text{I.D. of tube} = 22 \text{ mm} = 0.022 \text{ m}$$

$$L = \text{Length of each tube} = 2500 \text{ mm} = 2.5 \text{ m}$$

$$\text{Number of tubes} = 120$$

$$\text{Area for heat transfer based on I.D. per 1 m length} = \pi D_i L = \pi \times 0.022 \times 1 = 0.0691 \text{ m}^2/\text{m}$$

$$\begin{aligned} \text{Total area of heat transfer} &= n \times \pi D_i L \\ &= 120 \times 0.0691 \times 2.5 = 20.74 \text{ m}^2 \end{aligned}$$

$$\begin{aligned} \text{Cross-sectional area of each tube} &= \pi/4 D_i^2 \\ A_i &= \pi/4 (0.022)^2 = 0.00038 \text{ m}^2 \end{aligned}$$

$$\begin{aligned} \text{Total area of flow for single pass} &= \text{Number of tubes} \times \text{Area of each tube for flow} \\ &= 120 \times 0.00038 = 0.0456 \text{ m}^2 \end{aligned}$$

$$\text{Velocity of water} = 75 \text{ cm/s} = 0.75 \text{ m/s}$$

$$\begin{aligned} \text{Volumetric flow of water} &= \text{Velocity} \times \text{Area of flow} \\ &= 0.75 \times 0.0456 \\ &= 0.0342 \text{ m}^3/\text{s} \end{aligned}$$

$$\text{Density of water} = 1000 \text{ kg/m}^3$$

$$\text{Mass flow rate of water} = \dot{m}_w = 0.0342 \times 1000 = 34.2 \text{ kg/s}$$

Heat balance :

$$Q = m_w C_{pw} (t_2 - t_1)$$

$$16 \times 10^5 = 34.2 \times 4.187 \times 10^3 \times (t_2 - 290)$$

$$t_2 = 301.2 \text{ K (28.2}^\circ\text{C)}$$

Condensing benzene temperature = 350 K

$$\Delta T_1 = 350 - 290 = 60 \text{ K}$$

$$\Delta T_2 = 350 - 301.2 = 48.8 \text{ K}$$

$$\Delta T_{lm} = \frac{(60 - 48.8)}{\ln(60/48.8)} = 54.2 \text{ K}$$

$$Q = UA \Delta T_{lm}$$

$$U = Q / (A \cdot \Delta T_{lm})$$

$$= \frac{16 \times 10^5}{20.74 \times 54.2} = 1423 \text{ W/(m}^2 \cdot \text{K)}$$

Neglecting the wall resistance, we have

$$\frac{1}{U} = \frac{1}{h_o} + \frac{1}{h_i}$$

where

$$h_o = h_{oi} = 2250 \text{ W/(m}^2 \cdot \text{K)}$$

$$\frac{1}{1423} = \frac{1}{2250} + \frac{1}{h_i}$$

$$\therefore h_i = 3871.5 \text{ W/(m}^2 \cdot \text{K)}$$

Given :  $h_i$  is proportional to  $u^{0.8}$ .

$$h_i = C (u)^{0.8}$$

$$\therefore 3871.5 = C (0.75)^{0.8}$$

$$\therefore C = 4873.4$$

II. With scale :

$$h_i = 4873.4 u^{0.8}$$

$$R_d = 2.5 \times 10^{-4} \text{ m}^2 \cdot \text{K/W}$$

$$\therefore \frac{1}{U} = \frac{1}{h_i} + \frac{1}{h_o} + R_d$$

$$\frac{1}{U} = \frac{1}{4873.4 u^{0.8}} + \frac{1}{2250} + 2.5 \times 10^{-4}$$

$$\frac{1}{U} = \frac{1}{4873.4 u^{0.8}} + 6.944 \times 10^{-4}$$

$$\therefore U = \frac{4873.4 u^{0.8}}{[1 + 3.38 u^{0.8}]}, \text{ W/(m}^2 \cdot \text{K)}$$

$$\text{Mass flow of water} = \rho u A_i$$

$$= 1000 \times u \times 0.0456 = 45.6 u \text{ kg/s}$$

Let  $t_2$  be the outlet temperature of water in the second case.

$$\begin{aligned}
 Q &= \dot{m}_w C_{pw} (t_2 - t_1) \\
 16 \times 10^5 &= 45.6 u \times 4.187 \times 10^3 (t_2 - 290) \\
 \therefore t_2 &= 290 + (8.373/u) \\
 \Delta T_1 &= 350 - 290 = 60 \text{ K} \\
 \Delta T_2 &= 350 - (290 + 8.373/u) = 60 - (8.373/u) \\
 \Delta T_{lm} &= \frac{60 - [60 - (8.373/u)]}{\ln [60/(60) - (8.373/u)]} \\
 &= \frac{(8.373/u)}{\ln [60/(60) - (8.373/u)]} \\
 &= \frac{8.373}{u \cdot \ln \left[ \frac{60 u}{60 u - 8.373} \right]}
 \end{aligned}$$

$$\begin{aligned}
 Q &= UA \Delta T_{lm} \\
 16 \times 10^5 &= \frac{4873.4 u^{0.8}}{[1 + 3.38 u^{0.8}]} \times 20.74 \times \frac{8.373}{u \ln \left[ \frac{60 u}{60 u - 8.373} \right]} \\
 1.89 &= \frac{u^{-0.2}}{[1 + 3.38 u^{0.8}]} \times \frac{1}{\ln \left[ \frac{60 u}{60 u - 8.373} \right]}
 \end{aligned}$$

To find  $u$ , we have to adopt a trial and error procedure.

Assume a value of  $u$  more than 0.75 m/s and see whether LHS of the above equation is equal to RHS or not. If not, assume a new value of  $u$ . Repeat the procedure till we do not get LHS  $\approx$  RHS.

For	$u = 2 \text{ m/s,}$	RHS = 1.748
	$u = 2.5 \text{ m/s,}$	RHS = 1.8
	$u = 3 \text{ m/s,}$	RHS = 1.84
	$u = 4 \text{ m/s}$	RHS = 1.8976
	$u = 3.8 \text{ m/s,}$	RHS = 1.888 $\approx$ LHS (= 1.89)

$\therefore$  Water velocity must be **3.88 m/s**

... Ans.

**Example 5.13 :** A condenser consists of a number of metal pipes of outer diameter 25 mm and thickness 2.5 mm. Water flowing at 0.6 m/s, enters the pipe at 290 K (17°C) and it is not permissible that it should be discharged at a temperature higher than 310 K (37°C). 1.25 kg/s of hydrocarbon vapour is to be condensed at 345 K (72°C) on the outside of the pipes. Determine the length of each pipe and the number of pipes required. Take the heat transfer coefficient on the water side as 2.5 kW/(m<sup>2</sup>·K) and on the vapour side as 0.8 kW/(m<sup>2</sup>·K) and assume that the overall heat transfer coefficient, from vapour to water, based on these figures, is reduced to 20 % by the effects of pipe walls, dirt, and scale.

Latent heat of hydrocarbon at 345 K (72°C) = 315 kJ/kg

Heat capacity of water = 4.187 kJ/(kg·K)

(Pune University, Dec. 97)

**Solution : Basis :** 1.25 kg/s rate of condensation of hydrocarbon vapour.

$\lambda$  for hydrocarbon vapour = 315 kJ/kg

$\therefore$  Rate of heat transfer from vapour =  $\dot{m}_h \cdot \lambda$

$$Q = 1.25 \times 315$$

$$Q = 393.75 \text{ kJ/s} = 393.75 \times 10^3 \text{ W}$$

Temperature of condensing vapour = 345 K

Inlet temperature of water = 290 K

Outlet temperature of water = 310 K

$$\Delta T_1 = 345 - 290 = 55 \text{ K}, \quad \Delta T_2 = 345 - 310 = 35 \text{ K}$$

$$\Delta T_{lm} = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1/\Delta T_2)} = \frac{55 - 35}{\ln(55/35)} = 44.3 \text{ K}$$

Heat removed from vapour = Heat gained by water

$$Q = \dot{m}_w C_{pw} (t_2 - t_1)$$

$$393.75 \times 10^3 = \dot{m}_w \times 4.187 \times 10^3 \times (310 - 290)$$

$$\dot{m}_w = 4.7 \text{ kg/s}$$

Mass flow rate of water = 4.7 kg/s

Inside heat transfer coefficient =  $h_i = 2.5 \text{ kW}/(\text{m}^2 \cdot \text{K}) = 2500 \text{ W}/(\text{m}^2 \cdot \text{K})$

$$D_o = 25 \text{ mm} = 0.025 \text{ m}, \quad D_i = 25 - 2 \times 2.5 = 20 \text{ mm} = 0.020 \text{ m}$$

$h_{io}$  = Inside coefficient referred to outside diameter

$$h_{io} = h_i \times \frac{D_i}{D_o} = 2500 \times \frac{0.02}{0.025} = 2000 \text{ W}/(\text{m}^2 \cdot \text{K})$$

Outside heat transfer coefficient =  $h_o = 0.8 \text{ kW}/(\text{m}^2 \cdot \text{K}) = 800 \text{ W}/(\text{m}^2 \cdot \text{K})$

$$\frac{1}{U_o} = \frac{1}{h_o} + \frac{1}{h_{io}}$$

$$= \frac{1}{800} + \frac{1}{2000}$$

$$U_o = 572 \text{ W}/(\text{m}^2 \cdot \text{K})$$

$U_o = U_c$  = Clean overall coefficient

By providing allowance for the effects of the scale, dirt and wall, we have

$U_D$  is 80% of  $U_c$  (given).

$$U_D = \text{Dirty overall coefficient} = \left( \frac{100 - 20}{100} \right) \times 572 = 457 \text{ W}/(\text{m}^2 \cdot \text{K})$$

$$Q = U_D \cdot A_o \Delta T_{lm}$$

$$A_o = Q / (U_D \Delta T_{lm})$$

$$A_o = 393.75 \times 10^3 / (457 \times 44.3)$$

$$= 19.45 \text{ m}^2$$

$$\text{Outside area of pipe per m length of pipe} = \pi D_o L$$

$$= \pi \times 0.025 \times 1 = 0.0785 \text{ m}^2/\text{m}$$

$$\text{Total length of piping required} = 19.45 / 0.0785 = 247.6 \text{ m}$$

$$\text{Mass flow rate of water} = 4.7 \text{ kg/s}$$

$$\text{Density of water} = 1000 \text{ kg/m}^3$$

$$\text{Volumetric flow rate of water} = 4.7 / 1000 = 0.0047 \text{ m}^3/\text{s}$$

$$\text{Velocity of water (given)} = 0.6 \text{ m/s}$$

Cross-sectional area for flow per pass to give a velocity of 0.6 m/s is calculated as :

$$a = 0.0047 / 0.60 = 0.007833 \text{ m}^2$$

(As Velocity = Volumetric flow rate / Cross-sectional area of flow)

$$\text{Cross-sectional area of single tube} = \pi/4 \cdot D_i^2$$

$$= \pi/4 (0.02)^2 = 0.000314 \text{ m}^2$$

$$\text{Total cross-sectional area of tube per pass} = a = 0.00783 \text{ m}^2$$

$$\text{No. of tubes} \times \text{C.S. area of single tube} = \text{C.S. area of all tubes}$$

$$\therefore \text{No. of tubes per pass} = 0.00783 / 0.000314 = 24.94 \approx 25$$

For single pass on tube side fluid (water) :

$$\text{Total length of tubes} = 247.6 \text{ m}$$

$$\text{Number of tubes} = 25$$

$$\text{Length of each tube} = 247.6 / 25 = 9.90 \text{ m}$$

For two passes on water side / tube side :

$$\text{Total number of tubes} = 2 \times 25 = 50$$

$$\text{Total length of all tubes} = 247.6 \text{ m}$$

$$\therefore \text{Length of each tube} = 247.6 / 50 = 4.95 \text{ m}$$

For four passes on water side / tube side :

$$\text{Total number of tubes} = 4 \times 25 = 100$$

$$\text{Total length of all tubes} = 247.6 \text{ m}$$

$$\therefore \text{Length of each tube} = 247.6 / 100$$

$$= 2.476 \text{ m} \approx 2.48 \text{ m}$$

Out of these three proposals, the last one seems to be most realistic.

$$\therefore \text{No. of tubes} = 100, \text{ Length of tube} = 2.48 \text{ m}$$

**Example 5.14 :** 1-2 shell and tube heat exchanger is to be used to heat a crude oil from 295 K (22°C) to 330 K (57°C) with the help of the bottom product of a distillation unit that is to be cooled from 420 K (147°C) to 380 K (107°C). Crude oil flows through the tubes at a rate of 135000 kg/h and the bottom product flows through the shell at a rate of 10600 kg/h. The shell of an inside diameter 600 mm, consists of 324 tubes of 19 mm o.d., of wall thickness 2.1 mm, each 4.88 m long arranged on a 25 mm square pitch and supported by the segmental baffles spaced 0.23 m apart. A combined dirt factor of 0.001 m<sup>2</sup>·K/W is to be provided. Is this heat exchanger is suitable (i.e., what is the dirt factor) ?

Assume viscosity correction factor to be one and LMTD correction factor to be one for the calculation purpose. Use Dittus-Boelter equation for the tube side fluid.

**Data :** Properties of crude oil :

$$C_p = 1.986 \text{ kJ/(kg}\cdot\text{K)}, \quad \mu = 2.9 \times 10^{-3} \text{ N}\cdot\text{s/m}^2$$

$$k = 0.136 \text{ W/(m}\cdot\text{K)}, \quad \text{and } \rho = 824 \text{ kg/m}^3$$

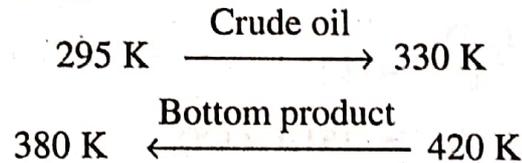
Properties of bottom product :

$$C_p = 2.202 \text{ kJ/(kg}\cdot\text{K)}, \quad \rho = 867 \text{ kg/m}^3$$

$$\mu = 5.2 \times 10^{-3} \text{ N}\cdot\text{s/m}^2, \quad k = 0.119 \text{ W/(m}^2\cdot\text{K)}$$

**Basis :** 135000 kg/h of crude oil flow rate.

**Counter-current flow :**



$$\Delta T_1 = 420 - 330 = 90 \text{ K}, \quad \Delta T_2 = 380 - 295 = 85 \text{ K}$$

$$\Delta T_{lm} = (90 - 85) / \ln(90/85) = 87.5 \text{ K}$$

$$\begin{aligned}
 \text{Heat load of exchanger} = Q &= \dot{m}_c C_{pc} (t_2 - t_1) \\
 &= 135000 \times 1.986 \times (330 - 295) \\
 &= 938385 \text{ kJ/h} = 2607 \times 10^3 \text{ W}
 \end{aligned}$$

**Shell side calculation :**

$$P_T = 25 \text{ mm} = 0.025 \text{ m}$$

$$B = 0.23 \text{ m}$$

$$\begin{aligned}
 C' = \text{Clearance} &= P_T - d_o \text{ (for square pitch)} \\
 &= 25 - 19 = 6 \text{ mm} = 0.006 \text{ m}
 \end{aligned}$$

$$\text{I.D. of shell} = 600 \text{ mm} = 0.6 \text{ m}$$

$$a_s = \frac{\text{I.D.} \times C' \times B}{P_T} = \frac{0.6 \times 0.006 \times 0.23}{0.025} = 0.0353 \text{ m}^2$$

$$\text{Cross flow area of shell} = a_s = 0.0353 \text{ m}^2$$

$$\begin{aligned} \text{Shell side mass velocity} = G_s &= \dot{m}/a_s \\ &= \frac{106000}{0.0353} \\ &= 3002833 \text{ kg}/(\text{m}^2 \cdot \text{h}) \\ &= 834.1 \text{ kg}/(\text{m}^2 \cdot \text{s}) \end{aligned}$$

$$\text{Equivalent diameter of shell} = D_e = \frac{4 [P_T^2 - \pi/4 d_o^2]}{\pi d_o}$$

$$D_e = \frac{4 [(0.025)^2 - \pi/4 (0.019)^2]}{\pi \times 0.019} = 0.023 \text{ m}$$

$$N_{Re} = \frac{D_e G_s}{\mu} = \frac{0.023 \times 834.1}{5.2 \times 10^{-3}} = 3689$$

$$N_{Pr} = \frac{C_p \mu}{k} = \frac{2.202 \times 10^3 \times 5.2 \times 10^{-3}}{0.119} = 96.22$$

$$N_{Nu} = 0.36 (N_{Re})^{0.55} (N_{Pr})^{1/3} (\mu/\mu_w)^{0.14}$$

$$(\mu/\mu_w)^{0.14} = 1.0$$

$$\begin{aligned} N_{Nu} &= 0.36 (N_{Re})^{0.55} (N_{Pr})^{1/3} \\ &= 0.36 (3689)^{0.55} (96.22)^{1/3} \end{aligned}$$

$$\frac{h_o D_e}{k} = 151.1$$

$$h_o = 151.1 \times k/D_e$$

$$= 151.1 \times 0.119/0.023 = 782 \text{ W}/(\text{m}^2 \cdot \text{K})$$

#### Tube side heat transfer coefficient :

Tube side fluid = Crude oil

No. of tubes = 324

No. of tubes per pass =  $324/2 = 162$

$D_o = \text{o.d. of tube} = 19 \text{ mm} = 0.019 \text{ m}$

Thickness = 2.1 mm

$D_i = \text{i.d. of tube} = 19 - 2 \times 2.1 = 14.8 \text{ mm} = 0.0148 \text{ m}$

$$\begin{aligned} \text{Cross-sectional area of one tube} &= \pi/4 \times D_i^2 \\ &= \pi/4 \times (0.0148)^2 = 1.72 \times 10^{-4} \text{ m}^2 \end{aligned}$$

$$\text{Total cross-sectional area for flow per pass} = 162 \times 1.72 \times 10^{-4} = 0.02786 \text{ m}^2$$

$$\text{Mass velocity through tube} = G = 135000/0.02786$$

$$= 4845657 \text{ kg}/(\text{m}^2 \cdot \text{h})$$

$$= 1346 \text{ kg}/(\text{m}^2 \cdot \text{s})$$

$$N_{Re} = \frac{D_i G}{\mu} = \frac{0.0148 \times 1346}{2.9 \times 10^{-3}} = 6869$$

$$N_{Nu} = 0.023 (N_{Re})^{0.8} (N_{Pr})^{0.4}$$

$$N_{Pr} = \frac{C_p \mu}{k} = \frac{1.986 \times 10^3 \times 2.9 \times 10^{-3}}{0.136} = 42.35$$

$$N_{Nu} = 0.023 \times (6869)^{0.8} \times (42.35)^{0.4}$$

$$\frac{h_i D_i}{k} = 120.8$$

$$h_i = 120.8 \times \frac{k}{D_i} = 120.8 \times \frac{0.136}{0.0148} = 1110 \text{ W/(m}^2 \cdot \text{K)}$$

$$h_{io} = h_i \times \frac{D_i}{D_o} = 1110 \times \frac{0.0148}{0.019} = 865 \text{ W/(m}^2 \cdot \text{K)}$$

Neglecting the wall and scale resistance, the clean overall coefficient based on outside area is

$$\frac{1}{U_o} = \frac{1}{h_o} + \frac{1}{h_{io}}$$

$$= \frac{1}{782} + \frac{1}{865}$$

$$U_o = 411 \text{ W/(m}^2 \cdot \text{K)}$$

$$\therefore U_c = U_o = 411 \text{ W/(m}^2 \cdot \text{K)}$$

$$\text{Heat transfer surface} = A_o = n \pi D_o L$$

$$= 324 \times \pi \times 0.019 \times 4.88 = 94.4 \text{ m}^2$$

$$Q = U_o A_o \Delta T_{lm} = U_D A_o \Delta T_{lm}$$

$$U_D = \frac{Q}{A_o \Delta T_{lm}} = \frac{2607 \times 10^3}{94.4 \times 87.5} = 315.6 \text{ W/(m}^2 \cdot \text{K)}$$

$$\text{We have: } R_d = \frac{U_c - U_D}{U_c U_D} = \frac{411 - 315.6}{411 \times 315.6}$$

$$= 7.35 \times 10^{-4} \text{ m}^2 \cdot \text{K/W}$$

... Ans.

As  $R_d$  calculated is less than  $R_d$  provided, this heat exchanger if installed, will not give the required terminal temperatures without frequent cleaning.

... Ans.

**Example 5.15 :** A shell and tube heat exchanger is used to heat a liquid of specific heat  $4.0 \text{ kJ/(kg} \cdot \text{K)}$  and specific gravity of 1.10 by steam condensing at  $395 \text{ K}$  ( $122^\circ\text{C}$ ) on the outside of tubes. The exchanger heats the liquid from  $295 \text{ K}$  ( $22^\circ\text{C}$ ) to  $375 \text{ K}$  ( $102^\circ\text{C}$ ) when a flow rate is  $1.75 \times 10^{-4} \text{ m}^3/\text{s}$  and to  $370 \text{ K}$  ( $97^\circ\text{C}$ ) when the flow rate of the liquid is  $3.25 \times 10^{-4} \text{ m}^3/\text{s}$ . Estimate the heat transfer area and overall heat transfer coefficient when the flow rate is  $1.75 \times 10^{-4} \text{ m}^3/\text{s}$ . Neglect the thermal resistance of tube wall and scale and assume that the heat transfer coefficient on the tube side/liquid side is proportional to the 0.8 power of the velocity.

## Heat Transfer

Data : Heat transfer coefficient for condensing steam =  $3400 \text{ W}/(\text{m}^2 \cdot \text{K})$

Solution :

Case-I : Volumetric flow rate of liquid =  $1.75 \times 10^{-4} \text{ m}^3/\text{s}$

Sp. gr. of the liquid = 1.10

Density of the liquid =  $1.10 \times 1000 = 1100 \text{ kg}/\text{m}^3$

Mass flow rate of the liquid =  $1.75 \times 10^{-4} \times 1100 = 0.1925 \text{ kg}/\text{s}$

Heat load or Duty of the heat exchanger =  $Q = \dot{m} C_p (t_2 - t_1)$   
 $= 0.1925 \times 4 \times 10^3 \times (375 - 295)$   
 $= 61600 \text{ W}$

$T =$  Temperature of condensing steam =  $395 \text{ K}$

$t_1 = 295 \text{ K}$  and  $t_2 = 375 \text{ K}$  ... temperatures of liquid

Calculation of  $\Delta T_{lm}$  :

$$\Delta T_1 = 395 - 295 = 100 \text{ K}$$

$$\Delta T_2 = 395 - 375 = 20 \text{ K}$$

$$\Delta T_{lm} = (100 - 20) / \ln(100/20) = 49.7 \text{ K}$$

The rate of heat transfer is given by,

$$Q = U_1 A \Delta T_{lm}$$

$$U_1 A = Q / \Delta T_{lm}$$

$\therefore$

$$U_1 A = 61600 / 49.7 = 1239 \text{ W}/\text{K}$$

Case-II : Volumetric flow rate of the liquid =  $3.25 \times 10^{-4} \text{ m}^3/\text{s}$

Mass flow rate of the liquid =  $\dot{m} = 3.25 \times 10^{-4} \times 1100 = 0.3575 \text{ kg}/\text{s}$

Heat load of the heat exchanger =  $Q = 0.3575 \times 4.0 \times 10^3 \times (370 - 295)$

$$Q = 107250 \text{ W}$$

Calculation of  $\Delta T_{lm}$  :

$$\Delta T_1 = 395 - 295 = 100 \text{ K}$$

$$\Delta T_2 = 395 - 370 = 25 \text{ K}$$

$$\Delta T_{lm} = (100 - 25) / \ln(100/25) = 54.1 \text{ K}$$

The rate of heat transfer is given by

$$Q = U_2 A \Delta T_{lm}$$

$$U_2 A = Q / \Delta T_{lm}$$

$\therefore$

$$U_2 A = 107250 / 54.1 = 1982 \text{ W}/\text{K}$$

The velocity in the tubes is proportional to the volumetric flow rate.

[As Velocity = Volumetric flow rate / Area for flow]

$\therefore$

$$u \propto v$$

Heat Transfer  
It is given that :  $h_i \propto u^{0.8}$

$$\therefore h_i \propto v^{0.8}$$
$$h_i = C \cdot v^{0.8}$$

$$U_1 A = 1239$$

and

$$U_2 A = 1982$$

$$\therefore U_2/U_1 = 1982/1239 = 1.60$$

$$U_2 = 1.60 U_1$$

$$h_o = 3400 \text{ W}/(\text{m}^2 \cdot \text{K})$$

Neglecting the scale and wall resistances, U is given by

$$\frac{1}{U} = \frac{1}{h_o} + \frac{1}{h_i}$$

$$U = \frac{h_o h_i}{h_i + h_o} = \frac{3400 \times C \cdot v^{0.8}}{3400 + C \cdot v^{0.8}}$$

$\therefore$  When

$$v = v_1 = 1.75 \times 10^{-4} \text{ m}^3/\text{s}$$

$$U_1 = \frac{3400 \times C \times (1.75 \times 10^{-4})^{0.8}}{3400 + C \times (1.75 \times 10^{-4})^{0.8}}$$

$$U_1 = \frac{3.357 C}{3400 + 9.87 \times 10^{-4} C}$$

When

$$v = v_2 = 3.25 \times 10^{-4} \text{ m}^3/\text{s}$$

$$U_2 = \frac{3400 \times C \times (3.25 \times 10^{-4})^{0.8}}{3400 + C \times (3.25 \times 10^{-4})^{0.8}}$$

$$U_2 = \frac{5.508 C}{3400 + 1.62 \times 10^{-3} C}$$

We have,

$$U_2 = 1.60 U_1$$

Substituting the values of  $U_1$  and  $U_2$ , the above equation becomes

$$\frac{5.508 C}{3400 + 1.62 \times 10^{-3} C} = \frac{1.6 \times 3.357 C}{3400 + 9.87 \times 10^{-4} C}$$

$$5.508 \times (3400 + 9.87 \times 10^{-4} C) = 5.371 \times (3400 + 1.62 \times 10^{-3} C)$$

Solving, we get

$$C = 142497$$

$$\therefore U_1 = \frac{3.357 \times 142497}{3400 + 9.87 \times 10^{-4} \times 142497} = 135.1 \text{ W}/(\text{m}^2 \cdot \text{K})$$

Overall heat transfer coefficient when  $v = 1.75 \times 10^{-4} \text{ m}^3/\text{s} = 135.1 \text{ W}/(\text{m}^2 \cdot \text{K})$  ... Ans.

$$U_1 A = 1239$$

$$\therefore \text{Heat transfer area} = A = 1239/U_1 = 1239/135.1 = 9.17 \text{ m}^2$$
 ... Ans.

**Example 5.16 :** In an oil cooler, 60 g/s of hot oil enters a thin metal pipe of diameter 25 mm. An equal mass of cooling water flows through the annular space between the pipe and a large concentric pipe, the oil and water are flowing in opposite direction to each other. The oil enters at 420 K (147°C) and is to be cooled to 320 K (47°C). If water enters at 290 K (17°C), find the length of the pipe required. The heat transfer coefficient of 1.6 kW/(m<sup>2</sup>·K) on the oil side and 3.6 kW/(m<sup>2</sup>·K) on the water side. Specific heat of oil is 2.0 kJ/(kg·K) and that of water is 4.18 kJ/(kg·K).

**Solution :** Basis : 60 g/s oil flow rate.

$$\dot{m}_o = \text{Mass flow rate of oil} = 60 \text{ g/s} = 6.0 \times 10^{-2} \text{ kg/s}$$

The heat load of the cooler is

$$\begin{aligned} Q &= \dot{m}_o C_{po} (T_1 - T_2) \\ &= 6.0 \times 10^{-2} \times 2.0 \times 10^3 \times (420 - 320) \\ &= 12000 \text{ J/s} \equiv 12000 \text{ W} \end{aligned}$$

**Calculation of  $t_2$  (the outlet water temperature) :**

The heat balance is :

$$\text{Heat given out by oil} = \text{Heat gained by water}$$

$$\therefore 12000 = 6.0 \times 10^{-2} \times 4.18 \times 10^3 \times (t_2 - 290)$$

$$t_2 = 338 \text{ K (65°C)}$$

**Calculation of  $\Delta T_{lm}$  :**

$$420 \text{ K} \xrightarrow{\text{oil}} 320 \text{ K}$$

$$338 \text{ K} \xleftarrow{\text{water}} 290 \text{ K}$$

$$\Delta T_1 = 420 - 338 = 82 \text{ K}$$

$$\Delta T_2 = 320 - 290 = 30 \text{ K}$$

**Calculation of  $U$  :**

$$\Delta T_{lm} = (82 - 30) / \ln (82/30) = 51.7 \text{ K}$$

$$h_i \text{ for oil} = 1.6 \text{ kW/(m}^2 \cdot \text{K)} = 1600 \text{ W/(m}^2 \cdot \text{K)}$$

$$h_o \text{ for water} = 3.6 \text{ kW/(m}^2 \cdot \text{K)} = 3600 \text{ W/(m}^2 \cdot \text{K)}$$

As the pipe being thin, we have

$$\frac{1}{U} = \frac{1}{h_o} + \frac{1}{h_i}$$

$$1/U = 1/3600 + 1/1600$$

$$U = 1108 \text{ W/(m}^2 \cdot \text{K)}$$

Calculations of A and L :

$$Q = UA \Delta T_{lm}$$

$$A = Q/(U\Delta T_{lm})$$

$$= \frac{12000}{1108 \times 51.7} = 0.210 \text{ m}^2$$

$$\text{Tube diameter} = D = 25 \text{ mm} = 0.025 \text{ m}$$

We have :  $A = \pi DL$

$$\therefore L = A/\pi D = 0.210/(\pi \times 0.025) = 2.67 \text{ m}$$

$$\text{Tube length required} = 2.67 \text{ m}$$

... Ans.

**Example 5.17 :** A counter-current flow heat exchanger is to be installed to cool 1.25 kg/s of benzene from 350 K (77°C) to 300 K (27°C) using water which is available at 290 K (17°C). The exchanger employs tubes of 25 mm O.D. and 22 mm I.D. through which water flows. Neglecting a scale resistance, determine the total length of the tubing required if the minimum quantity of water is used and its temperature is not to be permitted to rise above 320 K (47°C).

**Data :**

- k for tube material = 45 W/(m·K)
- h on water side = 850 W/(m<sup>2</sup>·K)
- h on benzene side = 1700 W/(m<sup>2</sup>·K)
- C<sub>p</sub> for benzene = 1.9 kJ/(kg·K)

**Solution : Basis :** 1.25 kg/s of benzene flow.

$$Q = \dot{m}_b C_{pb} (T_1 - T_2)$$

$$Q = 1.25 \times 1.9 \times 10^3 (350 - 300)$$

$$= 118750 \text{ J/s} \equiv 118750 \text{ W}$$

$$350 \text{ K} \xrightarrow{\text{benzene}} 300 \text{ K}$$

$$320 \text{ K} \xleftarrow{\text{water}} 290 \text{ K}$$

$$\Delta T_1 = 350 - 320 = 30 \text{ K}$$

$$\Delta T_2 = 300 - 290 = 10 \text{ K}$$

$$\Delta T_{lm} = (30 - 10)/\ln(30/10) = 18.2 \text{ K}$$

The heat balance is

Heat gained by water = Heat removed from benzene

$$\dot{m}_w C_{pw} (t_2 - t_1) = 118750$$

$$\dot{m}_w = 118750/[4.187 \times 10^3 (320 - 290)] = 0.945 \text{ kg/s}$$

Therefore,

Minimum flow rate of water = 0.945 kg/s

It is assumed to keep minimum flow rate of water so water leaves exchanger at 320 K.

$$h_i = h \text{ on water side} = 850 \text{ W}/(\text{m}^2 \cdot \text{K})$$

$$h_o = h \text{ on benzene side} = 1700 \text{ W}/(\text{m}^2 \cdot \text{K})$$

$$D_o = 25 \text{ mm} = 0.025 \text{ m}$$

$$D_i = 22 \text{ mm} = 0.022 \text{ m}$$

$$\text{Thickness of the tube} = x = (25 - 22)/2 = 1.5 \text{ mm} = 0.0015 \text{ m}$$

$$h_{io} = h_i \frac{D_i}{D_o} = 850 \times \frac{0.022}{0.025} = 748 \text{ W}/(\text{m}^2 \cdot \text{K})$$

$$D_w = (0.025 - 0.022) / \ln(0.025/0.022) = 0.0235 \text{ m}$$

$$k \text{ for the tube material} = 45 \text{ W}/(\text{m} \cdot \text{K})$$

$$\frac{1}{U_o} = \frac{1}{h_o} + \frac{1}{h_{io}} + \frac{x}{k} \cdot \frac{D_o}{D_w}$$

$$\frac{1}{U_o} = \frac{1}{1700} + \frac{1}{748} + \frac{0.0015}{45} \times \frac{0.025}{0.0235}$$

$$\therefore U_o = 510 \text{ W}/(\text{m}^2 \cdot \text{K})$$

We have :

$$Q = U_o A_o \Delta T \ln m$$

$\therefore$

$$A_o = Q / (U_o \Delta T \ln m)$$

$$= \frac{118750}{510 \times 18.2} = 12.79 \text{ m}^2$$

$$\text{Outside surface area of tube per 1.0 m length} = \pi d_o L = \pi \times 0.025 \times 1.0 = 0.0785 \text{ m}^2/\text{m}$$

$$\text{Total length of the tubing required} = \frac{\text{Total heat transfer area}}{\text{Heat transfer area per m length}} = \frac{12.79}{0.0785} = 163 \text{ m}$$

... Ans.

**Example 5.18 :** Benzene is to be condensed at a rate of 4500 kg/h in a vertical shell and tube heat exchanger on outside of the tubes. The heat exchanger is fitted with tubes of 25 mm outside diameter, 1.6 mm thick and 2.5 m long. Cooling water passes through the tubes at 1.05 m/s. Estimate the number of tubes required if the heat exchanger is arranged for a single pass on the water side.

**Data :** Heat capacity of water = 4.18 kJ/(kg·K)

Benzene condenses at 353 K (80°C) at 101.325 kPa

Latent heat of condensation of benzene = 394 kJ/kg

Inlet water temperature = 295 K (22°C)

Water outlet temperature = 300 K (27°C)

Thermal conductivity of tube material = 45 W/(m·K)

Properties of benzene at film temperature of 339 K are :

$$k = 0.15 \text{ W}/(\text{m} \cdot \text{K}), \quad \rho = 880 \text{ kg}/\text{m}^3, \quad \mu = 0.35 \times 10^{-3} \text{ N} \cdot \text{s}/\text{m}^2$$

**Solution :** Mass flow rate of benzene condensation =  $\dot{m} = 4500 \text{ kg/h}$

$$\begin{aligned} \text{Heat duty of the condenser} = Q &= \dot{m}\lambda \\ &= 4500 \times 394 \\ &= 1773000 \text{ kJ/h} = 492500 \text{ W} \end{aligned}$$

For water :

The heat gained by water is given by

$$\begin{aligned} Q &= \dot{m}_w C_{pw} (t_2 - t_1) \\ 1773000 &= \dot{m}_w \times 4.18 \times (300 - 295) \\ \dot{m}_w &= 84832.5 \text{ kg/h} = 23.60 \text{ kg/s} \end{aligned}$$

$$\text{Volumetric flow rate of water} = v = 23.60/1000 = 0.0236 \text{ m}^3/\text{s}$$

$$\text{(as } \rho = m/v \text{ and } \rho = 1000 \text{ kg/m}^3\text{)}$$

$$\text{Velocity of water through tubes} = 1.05 \text{ m/s}$$

Area required :

$$\begin{aligned} \text{Cross-sectional flow area required for the velocity of } 1.05 \text{ m/s} &= v/u \\ &= 0.0236/1.05 = 0.0225 \text{ m}^2 \end{aligned}$$

For tube :

$$D_o = 25 \text{ mm} = 0.025 \text{ m}$$

$$D_i = 25 - 2 \times 1.6 = 21.8 \text{ mm} = 0.0218 \text{ m}$$

$$\begin{aligned} \text{Cross-sectional area of tube} &= \pi/4 D_i^2 \\ &= \pi/4 \times (0.0218)^2 \\ &= 0.000373 \text{ m}^2 \end{aligned}$$

$$\begin{aligned} \text{No. of tubes required} &= \frac{\text{Total cross-sectional area of tubes}}{\text{Cross-sectional area of one tube}} \\ &= 0.0225/0.000373 \\ &= 60 \end{aligned}$$

$$\begin{aligned} \text{Surface area for heat transfer} = A_o &= n \pi D_o L \\ &= 60 \times \pi \times 0.025 \times 2.5 = 11.78 \text{ m}^2 \end{aligned}$$

$$\text{Condensing temperature of benzene} = 353 \text{ K}$$

$$\text{Inlet temperature of water} = 295 \text{ K}$$

$$\text{Outlet temperature of water} = 300 \text{ K}$$

$$\Delta T_1 = 353 - 295 = 58 \text{ K}, \quad \Delta T_2 = 353 - 300 = 53 \text{ K}$$

$$\Delta T_{lm} = (58 - 53) / \ln(58/53) = 55.5 \text{ K}$$

$$Q = U_o A_o \Delta T_{lm}$$

$$U_o = Q / (A_o \Delta T_{lm})$$

$$= 492500 / (11.78 \times 55.5)$$

$$= 754 \text{ W}/(\text{m}^2 \cdot \text{K})$$

This is a design overall heat transfer coefficient.

$$U_D = 754 \text{ W}/(\text{m}^2 \cdot \text{K})$$

**Overall heat transfer coefficient :**

**Inside/water side :**

For water in the tube, h can be calculated from :

$$h_i = 1063 \times (1 + 0.00293 T) u^{0.8} / D_i^{0.2}$$

where

$$u = 1.05 \text{ m/s}, \quad D_i = 0.0218 \text{ m}$$

and

$$T = (300 + 295) / 2 = 297.5 \text{ K}$$

$$h_i = 1063 (1 + 0.00293 \times 297.5) \times (1.05)^{0.8} / (0.0218)^{0.2}$$

$$= 4450 \text{ W}/(\text{m}^2 \cdot \text{K})$$

$$h_{io} = h_i \times \frac{D_i}{D_o}$$

$$= 4450 \times \frac{0.0218}{0.025} = 3880 \text{ W}/(\text{m}^2 \cdot \text{K})$$

$$x = \text{Thickness of tube wall} = 1.6 \text{ mm} = 0.0016 \text{ m}$$

$$D_w = (0.025 - 0.0218) / \ln(0.025/0.0218) = 0.0234 \text{ m}$$

$$k \text{ for the tube wall material} = 45 \text{ W}/(\text{m} \cdot \text{K})$$

**Outside of tube :**

For condensation on vertical tubes :

$$h_m (\mu^2 / k^3 \rho g)^{1/3} = 1.47 (4M/\mu)^{-1/3}$$

For vertical tubes,

$$M = \dot{m}' / \pi D_o$$

where,

$$\dot{m}' = \text{Mass flow rate of benzene per tube, kg/s}$$

$$\dot{m}' = 1.25 / 60 = 0.0208 \text{ kg/s}$$

$$M = \dot{m}' / \pi D_o = 0.0208 / (\pi \times 0.025) = 0.265 \text{ kg}/(\text{m} \cdot \text{s})$$

$\therefore$

$$k = 0.15 \text{ W}/(\text{m} \cdot \text{K}), \quad \rho = 880 \text{ kg}/\text{m}^3$$

$$\mu = 0.35 \times 10^{-3} \text{ N} \cdot \text{s}/\text{m}^2$$

$$h_m [(0.35 \times 10^{-3})^2 / (0.15)^3 \times (880)^2 \times 9.81]^{1/3} = 1.47 \left[ \frac{4 \times 0.0208}{0.35 \times 10^{-3}} \right]^{-1/3}$$

$$h_m = 1409 \text{ W}/(\text{m}^2 \cdot \text{K})$$

$$h_o = h_m = 1409 \text{ W}/(\text{m}^2 \cdot \text{K})$$

The overall heat transfer coefficient neglecting the scale resistance is :

$$\frac{1}{U_o} = \frac{1}{h_o} + \frac{1}{h_{io}} + \frac{x}{k} \frac{D_o}{D_w}$$

$$\frac{1}{U_o} = \frac{1}{1409} + \frac{1}{3880} + \frac{0.0016}{45} \times \frac{0.025}{0.0234}$$

$$U_o = 994.6 \text{ W}/(\text{m}^2 \cdot \text{K})$$

This is a clean overall heat transfer coefficient.

$$U_c = 994.6 \text{ W}/(\text{m}^2 \cdot \text{K})$$

$$U_D = 754 \text{ W}/(\text{m}^2 \cdot \text{K})$$

$U_c$  is in excess of  $U_D$  and therefore we would allow for a reasonable scale resistance.

Maximum allowable scale resistance =  $R_d$

$$R_d = \frac{U_c - U_D}{U_c \cdot U_D} = \frac{994.6 - 754}{994.6 \times 754} = 3.2 \times 10^{-4} \text{ m}^2 \cdot \text{K}/\text{W}$$

Number of tubes required = 60

... Ans.

**Example 5.19 :** A counter-current double pipe heat exchanger has oil in the tube and is being cooled from 413 K (140°C) to 373 K (100°C) with the help of water at 303 K (30°C). Because of this, water gets heated to 343 K (70°C). If the water side coefficient is 2.5 kW/(m<sup>2</sup>·K), oil side is 1.0 kW/(m<sup>2</sup>·K), and the fouling coefficient is 0.714 kW/(m<sup>2</sup>·K), calculate the heat transfer area for water flow rate of 5 kg/s,  $C_p$  of water is 4.18 kJ/(kg·K).

**Solution : Basis :** Mass flow rate of water =  $\dot{m}_w = 10 \text{ kg/s}$

$$\begin{aligned} Q &= \dot{m}_w C_{pw} (t_2 - t_1) \\ &= 5 \times 4.18 \times (343 - 303) \\ &= 836 \text{ kJ/s} = 836 \times 10^3 \text{ J/s} \equiv 836 \times 10^3 \text{ W} \end{aligned}$$

$$413 \text{ K} \xrightarrow{\text{oil}} 373 \text{ K}$$

$$343 \text{ K} \xleftarrow{\text{water}} 303 \text{ K}$$

$$\Delta T_1 = 413 - 343 = 70 \text{ K}$$

$$\Delta T_2 = 373 - 303 = 70 \text{ K}$$

$$\Delta T_1 = \Delta T_2$$

In this case,

Therefore, the logarithmic mean temperature difference becomes indeterminate [since  $\Delta T_1 - \Delta T_2 = 0$ ].

For such a case,

$$\Delta T_{lm} = \Delta T_1 = \Delta T_2 = \Delta T$$

$\therefore$

$$\Delta T_{lm} = 70 \text{ K}$$

$$h_i = 1.0 \text{ kW}/(\text{m}^2 \cdot \text{K}) = 1000 \text{ W}/(\text{m}^2 \cdot \text{K})$$

$$h_o = 2.5 \text{ kW}/(\text{m}^2 \cdot \text{K}) = 2500 \text{ W}/(\text{m}^2 \cdot \text{K})$$

$$1/R_d = 0.714 \text{ W}/(\text{m}^2 \cdot \text{K})$$

$$1/R_d = 714 \text{ kW}/(\text{m}^2 \cdot \text{K})$$

$\therefore$

$$R_d = 1.4 \times 10^{-3} (\text{m}^2 \cdot \text{K})/\text{W}$$

$$\frac{1}{U} = \frac{1}{h_i} + \frac{1}{h_o} + R_d$$

$$= \frac{1}{1000} + \frac{1}{2500} + 1.4 \times 10^{-3}$$

$$U = 357.14 \text{ W}/(\text{m}^2 \cdot \text{K})$$

$$Q = UA \Delta T_{lm}$$

$$A = Q/(U \Delta T_{lm})$$

$$= \frac{836 \times 10^3}{357.14 \times 70} = 33.44 \text{ m}^2$$

... Ans.

**Example 5.20 :** A heat exchanger is used to heat an oil in tubes from 288 K (15°C) to 358 K (85°C). Steam is blown continuously across the outside of tubes. It enters at 403 K (130°C) and leaves at 383 K (110°C) with a mass flow rate of 5.2 kg/s. Calculate the surface area of the heat exchanger.

**Data :**  $C_p$  for oil = 1.9 kJ/(kg·K),  $C_p$  for steam = 1.86 kJ/(kg·K)

Overall heat transfer coefficient = 275 W/(m<sup>2</sup>·K)

LMTD correction factor = 0.97

**Solution :** Mass flow rate of steam =  $\dot{m}_s = 5.2 \text{ kg/s}$

$$Q = \dot{m}_s C_{ps} (T_1 - T_2)$$

$$= 5.2 \times 1.86 \times (403 - 383) = 193.44 \text{ kJ/s} = 193440 \text{ W}$$

$$T_1 = 403 \text{ K}, T_2 = 383 \text{ K}, t_1 = 298 \text{ K}, t_2 = 358 \text{ K}$$

LMTD for cross flow =  $F_T$  LMTD from counter flow

$\therefore$  For counter-current flow :

$$\Delta T_2 = 403 - 358 = 45 \text{ K}, \Delta T_1 = 383 - 288 = 95 \text{ K}$$

$$\Delta T_{lm} = \text{LMTD} = (95 - 45)/\ln(95/45) = 66.91 \text{ K}$$

$$U = 275 \text{ W}/(\text{m}^2 \cdot \text{K})$$

$$Q = UA \Delta T_{lm} \text{ (cross flow)}$$

$$= UA F_T \Delta T_{lm} \text{ (counter flow)}$$

$$A = Q/[U F_T \Delta T_{lm} \text{ (counter flow)}]$$

$$= \frac{193440}{275 \times 0.97 \times 66.91} = 10.84 \text{ m}^2$$

... Ans.

**Example 5.21 :** Water is heated from 311 K (38°C) to 328 K (55°C) at a rate of 3.783 kg/s in a shell and tube heat exchanger. On the shell side, one pass is used with hot water as a heating medium. The hot water enters the shell at a rate of 1.892 kg/s and at 367 K (94°C), and the average water velocity in the 19 mm diameter tube is 0.366 m/s. Due to space limitations, the tube length must not be longer than 2.44 m. Calculate the number of tube side passes, the number of tubes per pass, and the length of the tubes, consistent with this restriction.

**Data :** Take  $C_p$  for hot and cold water = 4.18 kJ/(kg·K) and LMTD correction factor = 0.88. The overall heat transfer coefficient is 1450 W/(m<sup>2</sup>·K).

**Solution :** Cold water flow rate = 3.783 kg/s, hot water flow rate = 1.892 kg/s

$$Q = \dot{m}_c C_{pc} (t_2 - t_1) = \dot{m}_h C_{ph} (T_1 - T_2)$$

$$3.783 \times 4.18 \times (328 - 311) = 1.892 \times 4.18 \times (367 - T_2)$$

$$\therefore T_2 = 333 \text{ K (60°C)}$$

$$Q = 3.783 \times 4.18 (328 - 311) = 269 \text{ kJ/s} = 269 \times 10^3 \text{ W}$$

For counter-flow heat exchanger :

$$367 \text{ K} \xrightarrow{\text{H.W.}} 333 \text{ K}$$

$$328 \text{ K} \xleftarrow{\text{C.W.}} 311 \text{ K}$$

$$\Delta T_1 = 367 - 328 = 39 \text{ K}, \quad \Delta T_2 = 333 - 321 = 17 \text{ K}$$

$$\Delta T_{lm} = \text{LMTD} = (39 - 17) / \ln (39/17) = 26.50 \text{ K}$$

$$Q = UA \Delta T_{lm}$$

$$A = Q/U \cdot \Delta T_{lm} = 269 \times 10^3 / (1450 \times 26.50) = 7 \text{ m}^2$$

$$\text{Velocity through tubes} = 0.366 \text{ m/s}$$

$$\text{Cold water : Mass flow rate} = \rho A_i u$$

$$A_i = 3.783/1000 \times 0.366$$

$$= 0.01034 \text{ m}^2$$

$$\text{Total flow area on the tube side} = A_i = 0.01034 \text{ m}^2$$

This area is the product of the number of tubes and the flow area per tube.

$$\therefore A_i = 0.01034 = n \frac{\pi}{4} D^2 = n \cdot \frac{\pi}{4} (0.019)^2$$

$$\therefore n = 36.47 \text{ tubes}$$

$$\text{or } n = 36 \text{ tubes}$$

$$\text{The surface area per tube per 1 m length} = \pi DL$$

$$= \pi \times 0.019 \times 1$$

$$= 0.0597 \text{ m}^2/\text{m length}$$

$$\text{Area for heat transfer} = A = n\pi DL$$

$$L = A/n\pi D = 7/36 \times 0.0597 = 3.257 \text{ m}$$

$$\text{Length of tubes} = \frac{\text{Heat transfer area of all tubes}}{\text{Heat transfer area of one tube} \times \text{No. of tubes}} = \frac{7}{36 \times 0.0597} = 3.257 \text{ m}$$

This length is more than the allowable 2.44 m length, so we must use more than one tube side pass.

**For 2 passes on the tube side :**

$$Q = U A F_T \text{LMTD}$$

$$A = Q/(U F_T \text{LMTD}) = \frac{269 \times 10^3}{1450 \times 0.88 \times 26.50} = 7.955 \text{ m}^2$$

For two passes on the tube side with 36 tubes per pass, the total heat transfer area is now related to length by

$$A = 2 [n\pi DL]$$

$$\begin{aligned} L &= A/(2n\pi D) \\ &= 7.955/(2 \times 36 \times \pi \times 0.019) \\ &= 1.85 \text{ m} \end{aligned}$$

This length is within the 2.44 m requirement, so the design choice is :

**Type of heat exchanger : 1-2 shell-and-tube heat exchanger**

$$\text{Number of tubes per pass} = 36$$

$$\text{Length of tube per pass} = 1.85 \text{ m}$$

... Ans.