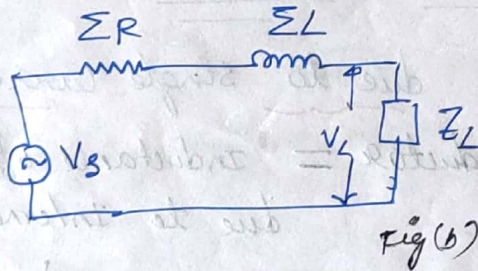
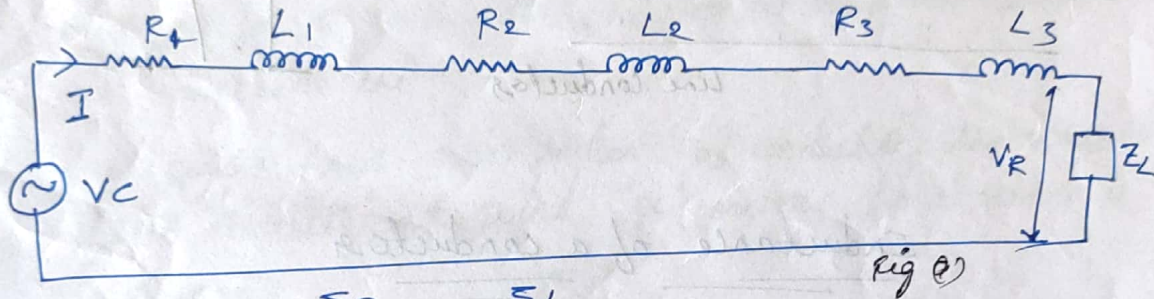


Line parameters

Transmission line constants:

A Transmission line has resistance, inductance and capacitance uniformly distributed along the length of line these are called as constants of a Transmission line.

Efficiency and voltage regulation is decided by these constants.



(i) Resistance:

The resistance is distributed uniformly along the length of line as shown in Fig (a) However the performance of a Transmission line is analysed conveniently if distributed resistance is considered as lump as shown in Fig (b)

$$R = \frac{\rho l}{a}$$

(ii) Inductance:

When an alternating current flows through the conductor, a changing flux is set up which links the conductor. Due to this flux linkages the conductor possesses inductance.

$$L = \frac{\Psi}{I} \text{ Henry}$$

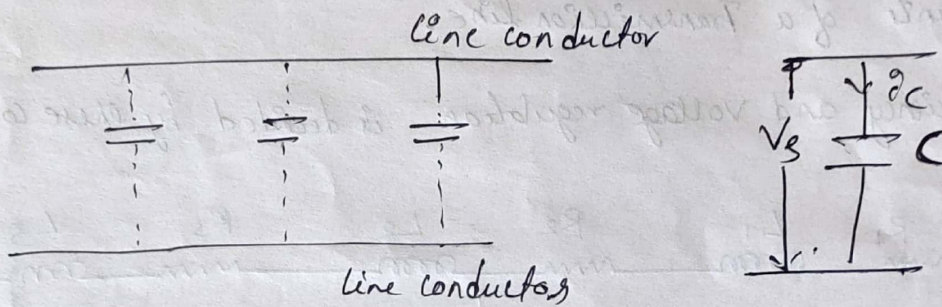
Ψ = Flux linkages in weber-turn
 I = current in amp

For analysis, Inductance is taken as lumped as sh.

(iii) capacitance:

As any two conductors of an overhead transmission line is separated by air which acts as an insulation. So capacitance exists between any two overhead line conductors.

$$C = \frac{Q}{V} \text{ farad}$$

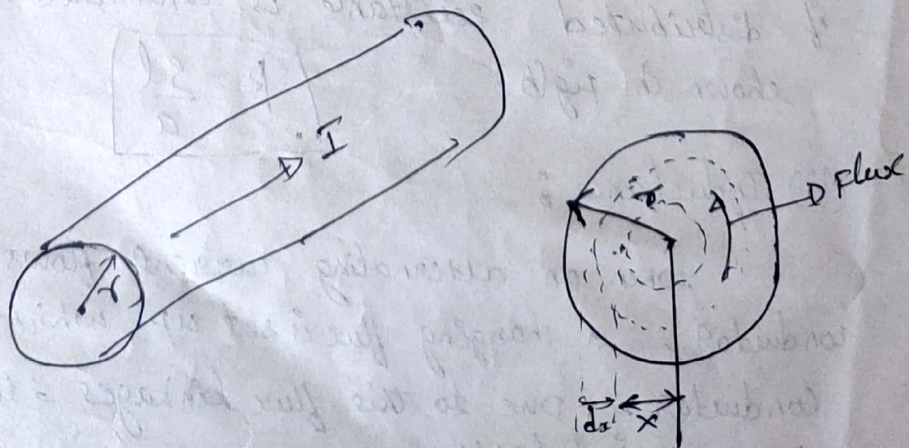


Inductance of a conductor

(i) Flux linkage due to single current carrying conductor:
 Inductance of a conductor = Inductance of a conductor due to Internal flux

+

Inductance of a conductor due to External flux.



Consider a long straight conductor of radius r m and carrying a current I (rms) as shown in fig.

This current sets up magnetic field. The magnetic line of forces will exist inside the conductor as well as outside the conductor both these flux contribute to inductance of conductor.

(i) Flux linkages due to internal flux:

Figure shows cross section of conductor, the magnetic line of forces will exist inside the conductor as well as outside the

Figure shows cross section of conductor, the magnetic field intensity at point 'x' meters from centre is given by

$$H_x = \frac{I_x}{2\pi x}$$

Assuming uniform current density

$$\frac{I_x}{\pi x^2} = \frac{I}{\pi r^2}$$

$$I_x = \frac{x^2}{r^2} I$$

Substitute I_x in H_x

$$H_x = \frac{x \cdot I}{2\pi r^2} \quad \text{AT/m}$$

If $\mu = \mu_0 \mu_r$ is the permeability of conductor the flux density is

$$B_x = \mu_0 \mu_r H_x$$

$$B_x = \frac{\mu_0 \mu_r x I}{2\pi r^2} \quad \text{wb/m}^2$$

As $\mu_r = 1$ for non-magnetic material

$$B_x = \frac{\mu_0 x I}{2\pi r^2} \quad \text{wb/m}^2$$

Now flux $d\phi$ to a cylindrical shell of radial thickness dx and axial length of 1m is given by,

$$d\phi = B_x \times 1 \times dx.$$

$$d\phi = \frac{\mu_0 x I}{2\pi r^2} dx$$

This flux linkage with current I_x only.
 \therefore Flux linkage per meter length of conductor.

$$d\psi = \frac{\pi x^2}{\pi r^2} d\phi \quad \text{--- (3)}$$

\therefore Flux linkage per meter length of the conductor

$$d\psi = \frac{\mu_0 x^3 I}{2\pi r^4} dx \quad \text{wb/m}$$

\therefore Total flux linkage from centre upto conductor surface r

$$\psi_{\text{int}} = \int_0^r d\psi = \int_0^r \frac{\mu_0 I x^3}{2\pi r^4} dx$$

$$= \frac{\mu_0 I}{2\pi r^4} \int_0^r x^3 dx$$

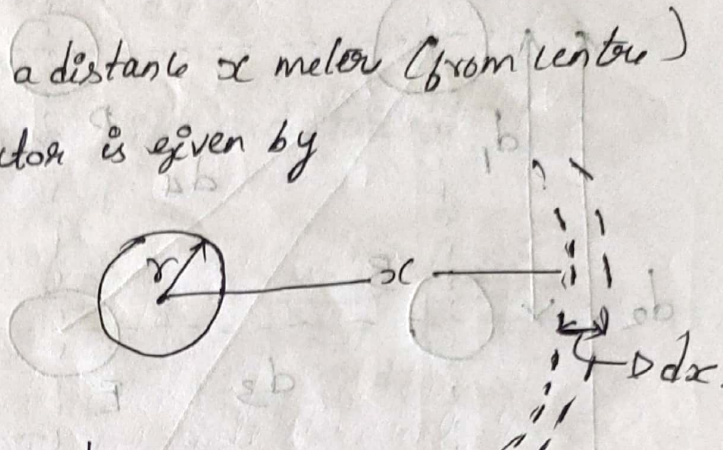
$$= \frac{\mu_0 I}{2\pi r^4} \left[\frac{x^4}{4} \right]_0^r = \frac{\mu_0 I r^4}{8\pi r^4}$$

$$\boxed{\psi_{\text{int}} = \frac{\mu_0 I}{8\pi}}$$

(ii) Flux linkage due to external flux:

External flux extends from the surface of the conductor to infinity.

Field intensity at a distance x meter (from center) outside the conductor is given by



$$H_x = \frac{I}{2\pi x} \text{ AT/m}$$

Flux density

$$B_x = \mu_0 H_x = \frac{\mu_0 I}{2\pi x} \text{ wb/m}^2$$

Now $d\phi$ through a cylindrical shell of thickness dx and axial length 1 meter is

$$d\phi = B_x dx = \frac{\mu_0 I}{2\pi x} dx \text{ webers.}$$

Flux linkages

$$\psi_{ext} = \int_r^\infty \frac{\mu_0 I}{2\pi x} dx \text{ weber-turns}$$

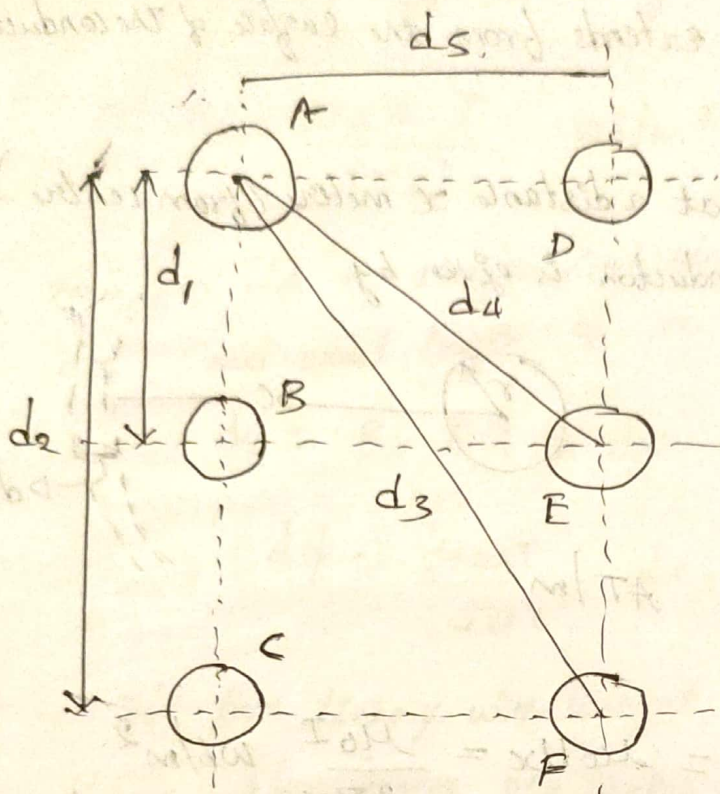
Total flux linkage

$$\psi = \psi_{int} + \psi_{ext}$$

$$= \frac{\mu_0 I}{8\pi} + \int_r^\infty \frac{\mu_0 I}{2\pi x} dx$$

$$\psi = \frac{\mu_0 I}{2\pi} \left[\frac{1}{4} + \int_r^\infty \frac{dx}{x} \right] \text{ wb-turns/m.}$$

2] Flux linkages in parallel wires carrying conductors:



Here conductors A, B, C, D, E, F carrying current I_A, I_B, I_C, I_D, I_E and I_F

consider conductor A,

There will be flux linkages with conductor A due to its own current (I_A)

Flux linkages with conductor A due to its own current I_A

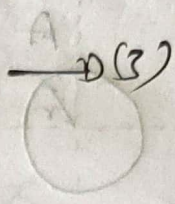
$$\Psi_A = \frac{\mu_0 I_A}{2\pi} \left[\frac{1}{4} + \int_y^{\infty} \frac{dx}{x} \right] \rightarrow (1)$$

Flux linkages with conductor A due to current I_B

~~$$= \frac{\mu_0 I_B}{2\pi} \left[\frac{1}{4} + \int_y^{\infty} \frac{dx}{x} \right]$$~~

$$= \frac{\mu_0 I_B}{2\pi} \int_{d_1}^{\infty} \frac{dx}{x} \rightarrow (2)$$

flux linkages with conductor A due to current I_c .

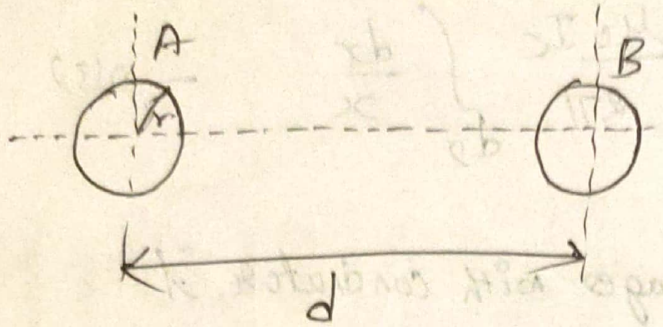
$$= \frac{\mu_0 I_c}{2\pi} \int_{d_2}^{\infty} \frac{dx}{x}$$


Total flux linkage with conductor A

$$= \frac{\mu_0 I_A}{2\pi} \left[\frac{1}{4} + \int_a^{\infty} \frac{dx}{x} \right] + \frac{\mu_0 I_B}{2\pi} \int_{d_1}^{\infty} \frac{dx}{x} + \frac{\mu_0 I_c}{2\pi} \int_{d_2}^{\infty} \frac{dx}{x}$$

~~Above equation~~

Inductance of single phase two wire line



consider a single phase overhead line consisting of two parallel conductors A and B spaced d meters as shown in figure.

The conductors A and B carry the same amount of current (i.e. $I_A = I_B$) but in opposite direction as one forms the return circuit.

$$I_A + I_B = 0$$

There will be flux linkages with conductor A due to its own current I_A and also due to mutual inductance effect of current I_B in the conductor B.

Flux linkages with conductor A due to its own current

$$= \frac{\mu_0 I_A}{2\pi} \left[\frac{1}{4} + \int_r^\infty \frac{dx}{x} \right] \quad \text{--- (i)}$$

Flux linkages with conductor A due to current I_B

$$= \frac{\mu_0 I_B}{2\pi} \int_d^\infty \frac{dx}{x} \quad \text{--- (ii)}$$

Total flux linkages with conductor A is

$$\Psi_A = \cancel{il} + il'$$

$$= \frac{\mu_0 I_A}{2\pi} \left[\frac{1}{4} + \int_r^\infty \frac{dx}{x} \right] + \frac{\mu_0 I_B}{2\pi} \int_x^\infty \frac{dx}{x}$$

$$= \frac{\mu_0}{2\pi} \left[\left(\frac{1}{4} + \int_r^\infty \frac{dx}{x} \right) I_A + I_B \int_d^\infty \frac{dx}{x} \right]$$

$$= \frac{\mu_0}{2\pi} \left[\left(\frac{1}{4} + \log_e \infty - \log_e r \right) I_A + \left(\log_e \infty - \log_e d \right) I_B \right]$$

$$= \frac{\mu_0}{2\pi} \left[\frac{I_A}{4} + \log_e \infty (I_A + I_B) - I_A \log_e r - I_B \log_e d \right]$$

$$= \frac{\mu_0}{2\pi} \left[\frac{I_A}{4} - I_A \log_e r - I_B \log_e d \right]$$

$$\boxed{I_A = -I_B}$$

$$\frac{\mu_0}{2\pi} \left[\frac{I_A}{4} + I_A \log_e \frac{d}{r} \right]$$

$$= \frac{\mu_0 I_A}{2\pi} \left[\frac{1}{4} + \log_e \frac{d}{r} \right]$$

Inductance of conductor A, $L_A = \frac{\Psi_A}{IA}$

$$= \frac{\mu_0}{2\pi} \left[\frac{1}{4} + \log_e \frac{d}{r} \right] \text{ H/m}$$

$$= \frac{4\pi \times 10^{-7}}{2\pi} \left[\frac{1}{4} + \log_e \frac{d}{r} \right]$$

$$L_A = 10^{-7} \left[\frac{1}{2} + 2 \log_e \frac{d}{r} \right]$$

Loop Inductance = $2 L_A$ H/m

$$\text{Loop Inductance} = 10^{-7} \left[1 + 4 \log_e \frac{d}{r} \right] \text{ H/m.}$$

Expression in Alternate form

Expression for the inductance of a conductor can be put in concise form.

$$L_A = 10^{-7} \left[\frac{1}{2} + 2 \log_e \frac{d}{r} \right] \text{ H/m}$$

$$= 2 \times 10^{-7} \left[\frac{1}{4} + \log_e \frac{d}{r} \right]$$

$$L_A = 2 \times 10^{-7} \log_e \frac{d}{r e^{-1/4}}$$

$$r' = r e^{-1/4}$$

$$L_A = 2 \times 10^{-7} \log \frac{d}{r'}, \text{ H/m}$$

$$r' = r e^{-1/4} = 0.7788 r$$

The term r' is called as Geometric mean radius of wire.

mutual GMD :

It is the geometrical mean distance from one conductor to other.

GMD b/w two conductors is equal to distance b/w their centers i.e. $D_m = d$.

Inductance of 3 ϕ overhead line:

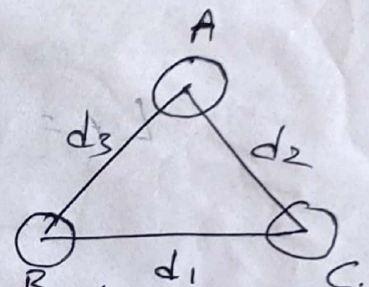
(i) unsymmetrical and untransposed

consider 3 conductors A, B and C of 3 ϕ line carrying current I_A, I_B and I_C respectively.

let d_1, d_2 and d_3 are the spacing b/w the conductors

Assume that loads are balanced i.e.,

$$I_A + I_B + I_C = 0$$



flux linkage with conductor A due to its own current (I_A)

$$= \frac{\mu_0 I_A}{2\pi} \left[\frac{1}{4} + \int_{r}^{\infty} \frac{dx}{x} \right] \quad \text{--- } \textcircled{1}$$

Flux linkage with conductor A due to current I_B

$$= \frac{\mu_0 I_B}{2\pi} \int_{d_3}^{\infty} \frac{1}{x} dx \quad \text{--- } \textcircled{2}$$

Flux linkage with conductor A due to current I_C

$$= \frac{\mu_0 I_C}{2\pi} \int_{d_2}^{\infty} \frac{1}{x} dx \quad \text{--- } \textcircled{3}$$

Total flux linkage with conductor A

$$\psi_A = \textcircled{1} + \textcircled{2} + \textcircled{3}$$

$$= \frac{\mu_0 I_A}{2\pi} \left[\frac{1}{4} + \int_r^\infty \frac{1}{x} dx \right] + \frac{\mu_0 I_B}{2\pi} \int_{d_3}^\infty \frac{1}{x} dx$$

$$+ \frac{\mu_0 I_C}{2\pi} \int_{d_2}^\infty \frac{1}{x} dx$$

$$= \frac{\mu_0}{2\pi} \left[\left(\frac{1}{4} + \int_r^\infty \frac{dx}{x} \right) I_A + I_B \int_{d_3}^\infty \frac{dx}{x} \right.$$

$$\left. + I_C \int_{d_2}^\infty \frac{dx}{x} \right]$$

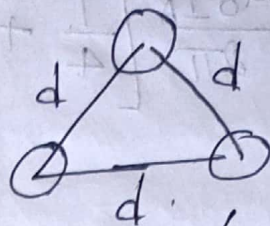
$$= \frac{\mu_0}{2\pi} \left[\left(\frac{1}{4} + \log \infty - \log r \right) I_A + I_B (\log \infty - \log d_3) \right.$$

$$\left. + I_C (\log \infty - \log d_2) \right]$$

$$= \frac{\mu_0}{2\pi} \left[\frac{1}{4} + \log \infty (I_A + I_B + I_C) - \log r I_A - \log d_3 I_B - \log d_2 I_C \right]$$

$$\Psi_A = \frac{\mu_0}{2\pi} \left[\frac{I_A}{4} - \log_e r I_A - \log d_3 I_B - \log d_2 I_C \right]$$

(ii) symmetrical spacing



If all the 3 conductors A, B and C are placed symmetrically at the corners of equilateral triangle of side d then $d_1 = d_2 = d_3$

under this condition flux linkage with conductor A

$$\Psi_A = \frac{\mu_0}{2\pi} \left[\left(\frac{1}{4} - \log_e r \right) I_A - \log_e d I_B - \log_e d I_C \right]$$

$$= \frac{\mu_0}{2\pi} \left[\left(\frac{1}{4} - \log_e r \right) I_A - \log_e d (I_B + I_C) \right]$$

$$= \frac{\mu_0}{2\pi} \left[\left(\frac{1}{4} - \log_e r \right) I_A + \log_e d I_A \right]$$

As $I_A + I_B + I_C = 0$
 $I_B + I_C = -I_A$

$$= \frac{\mu_0}{2\pi} I_A \left[\frac{1}{4} - \log_e r + \log_e d \right]$$

$$\Psi_A = \frac{\mu_0 I_A}{2\pi} \left[\frac{1}{4} + \log_e \frac{d}{r} \right] \text{ H/m}$$

$$= \frac{4 \times 10^{-7}}{2\pi} I_A \left[\frac{1}{4} + \log_e \frac{d}{r} \right] \text{ H/m}$$

$$L_A = \frac{\Psi_A}{I_A} \left[L_A = 10^{-7} \left[\frac{1}{2} + 2 \log_e \frac{d}{r} \right] \right] \text{ H/m}$$

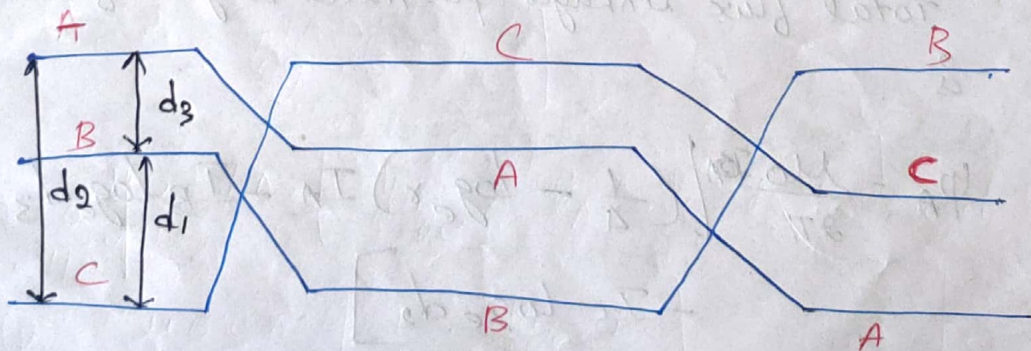
Only the expression for Inductance are same for conductor B and C.

Note: Transposition:

Due to unsymmetrical spacing,

- ⊗ Flux linkages and Inductance of each phase is Not same
- ⊗ A different inductance in each phase results in a unequal voltage drops even if currents are balanced.
- ⊗ So voltage at receiving end is not same in all phases.

To make voltage drops equal, interchange the positions of the conductors at regular intervals along the line so that each conductor occupies the original position. Such an exchange of positions is known as Transposition.



Inductance of 3 ϕ line conductors

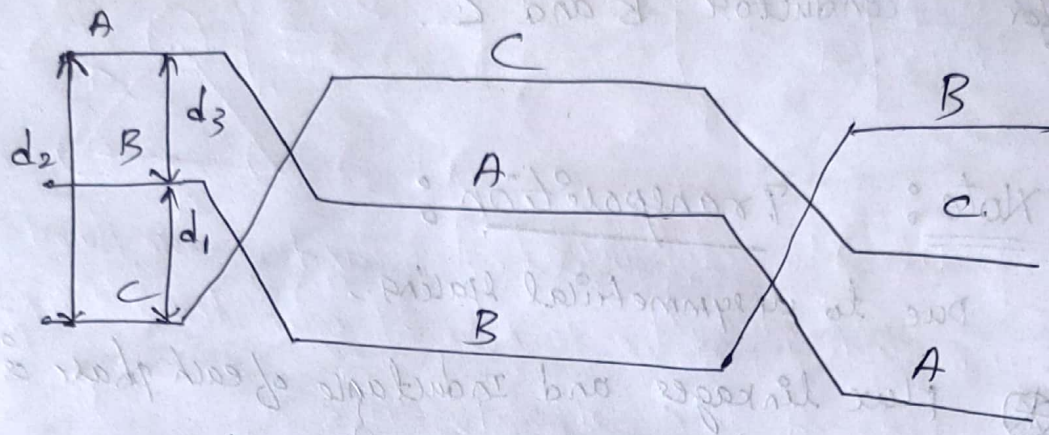


Figure shows 3 ϕ transposed line having unsymmetrical spacing.

Assume that each of the three sections is 1m in length.

Assume that load is balanced i.e.,

$$I_A + I_B + I_C = 0$$

Let line currents

$$I_A = I(1 + j0) = I \angle 0^\circ$$

$$I_B = I(-0.5 - j0.866) = I \angle -120^\circ$$

$$I_C = I(-0.5 + j0.866) = I \angle 120^\circ$$

Total flux linkages per meter length of conductor A is

$$\Psi_A = \frac{\mu_0 I_A}{2\pi} \left[\left(\frac{1}{4} - \log_e r \right) I_A - I_B \log_e d_3 - I_C \log_e d_2 \right]$$

put values of I_A , I_B and I_C

$$= \frac{\mu_0}{2\pi} \left[\left(\frac{1}{4} - \log_e r \right) I - I(-0.5 - j0.866) \log_e d_3 \right. \\ \left. - I(-0.5 + j0.866) \log_e d_2 \right]$$

$$= \frac{\mu_0}{2\pi} \left[\frac{1}{4} I - I \log_e r + 0.5 I \log_e d_3 + j0.866 I \log_e d_3 \right. \\ \left. + 0.5 I \log_e d_2 - j0.866 I \log_e d_2 \right]$$

$$= \frac{\mu_0}{2\pi} \left[\frac{1}{4} I - I \log_e r + 0.5 I (\log_e d_3 + \log_e d_2) \right. \\ \left. + j0.866 I (\log_e d_3 - \log_e d_2) \right]$$

$$= \frac{\mu_0}{2\pi} \left[\frac{1}{4} I - I \log_e r + I \log_e \sqrt{d_2 d_3} + j0.866 I \log_e \frac{d_3}{d_2} \right]$$

* *

$$= 0.5 I \log_e d_2 d_3$$

$$= I \log_e (d_2 d_3)^{0.5} = I \log_e (d_2 d_3)^{1/2}$$

$$= I \log_e \sqrt{d_2 d_3}$$

$$= \frac{\mu_0}{2\pi} \left[\frac{1}{4} I + I \log_e \frac{\sqrt{d_2 d_3}}{r} + j0.866 I \log_e \frac{d_3}{d_2} \right]$$

$$= \frac{\mu_0 I}{2\pi} \left[\frac{1}{4} + \log_e \frac{\sqrt{d_2 d_3}}{r} + j 0.866 \log_e \frac{d_3}{d_2} \right]$$

Inductance of conductor A is

$$L_A = \frac{\Psi_A}{I_A} = \frac{\Psi_A}{I}$$

$$L_A = \frac{\mu_0}{2\pi} \left[\frac{1}{4} + \log_e \frac{\sqrt{d_2 d_3}}{r} + j 0.866 \log_e \frac{d_3}{d_2} \right]$$

$$= \frac{4\pi \times 10^{-7}}{2\pi} \left[\frac{1}{4} + \log_e \frac{\sqrt{d_2 d_3}}{r} + j 0.866 \log_e \frac{d_3}{d_2} \right]$$

$$L_A = 10^{-7} \left[\frac{1}{2} + 2 \log_e \frac{\sqrt{d_2 d_3}}{r} + j 1.732 \log_e \frac{d_3}{d_2} \right]$$

Inductance of conductors B and C will be

$$L_B = 10^{-7} \left[\frac{1}{2} + 2 \log_e \frac{\sqrt{d_1 d_3}}{r} + j 1.732 \log_e \frac{d_3}{d_2} \right]$$

$$L_C = 10^{-7} \left[\frac{1}{2} + 2 \log_e \frac{\sqrt{d_1 d_2}}{r} + j 1.732 \log_e \frac{d_3}{d_2} \right]$$

Inductance of each line conductor

$$L = \frac{1}{3} (L_A + L_B + L_C)$$

$$L = \left[\frac{1}{2} + 2 \log_e \frac{\sqrt[3]{d_1 d_2 d_3}}{r} \right] \times 10^{-7} \text{ H/m}$$

$$L = 0.5 + 2 \log_e \frac{\sqrt[3]{d_1 d_2 d_3}}{r} \times 10^{-7} \text{ H/m}$$

If we compare the formula of inductance of an unsymmetrically spaced transposed line with that of symmetrically spaced line, we find that inductance of each line conductor in the two cases will be

$$d = \sqrt{d_1 d_2 d_3}$$

The distance d is known as equivalent equilateral spacing for unsymmetrically transposed line.

Inductance formulas in terms of GMD

(i) single phase line

$$\text{Inductance / conductor / m} = 2 \times 10^{-7} \log_e \frac{D_m}{D_s}$$

$$D_s = 0.7788 r$$

$$D_m = \text{spacing b/w conductors} = d$$

(ii) single circuit 3 ϕ line:

$$\text{Inductance / phase / m} = 2 \times 10^{-7} \log_e \frac{D_m}{D_s}$$

$$D_s = 0.7788 r$$

$$D_m = (d_1 d_2 d_3)^{1/3}$$

(iii) double circuit 3 ϕ line:

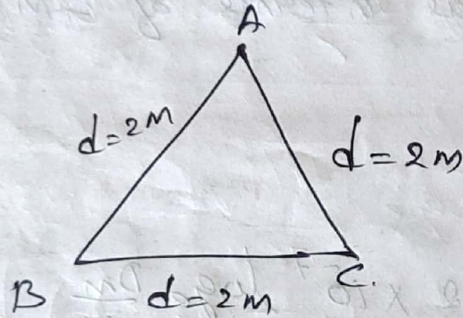
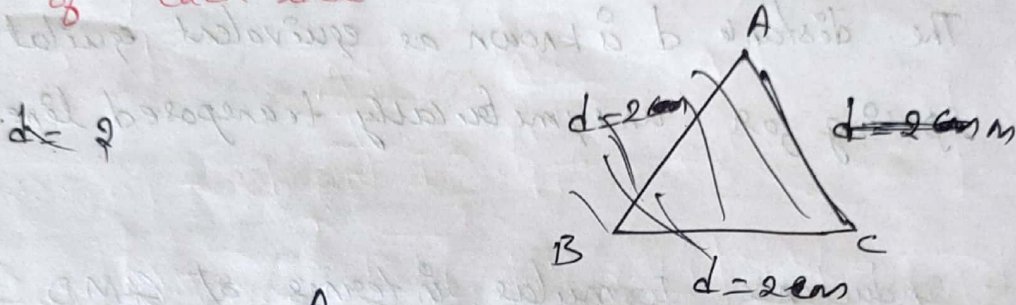
$$\text{Inductance / phase / m} = 2 \times 10^{-7} \log_e \frac{D_m}{D_s}$$

$$\text{where } D_s = (D_{s1} D_{s2} D_{s3})^{1/3}$$

$$D_m = (D_{AB} \times D_{BC} \times D_{CA})^{1/3}$$

problems

- 1] Find inductance per km of 3 phase transmission line using 1.24 cm diameter conductors when these are placed at the corners of equilateral of each side 2m.



$$r = \frac{1.24}{2} = 0.62 \text{ cm} \quad d = 2 \text{ m}$$

$$\text{Inductance / phase / m} = 10^{-7} \left[0.5 + 2 \log_e \frac{d}{r} \right] \text{ H/m}$$

$$= 10^{-7} \left[0.5 + 2 \log_e \frac{200}{0.62} \right] \text{ H}$$

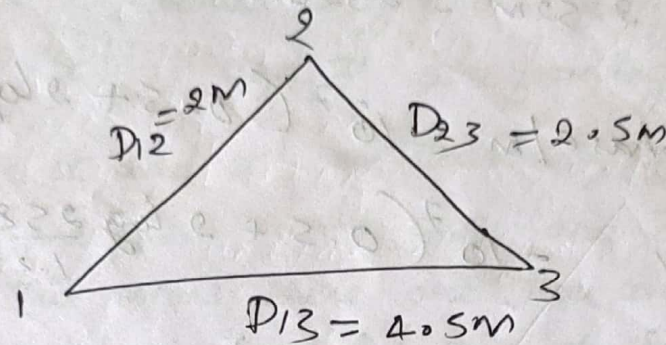
$$= 12 \times 10^{-7} \text{ H}$$

$$\text{Inductance / phase / km} = 12 \times 10^{-7} \times 1000$$

$$= 1.2 \times 10^{-3}$$

$$= 1.2 \text{ mH}$$

- ② The three conductors of a 3 ϕ line are arranged at the corners of a triangle of side 2m, 2.5m and 4.5m. Calculate the inductance per km of line when the conductors are regularly transposed. The diameter of each conductor is 1.24 cm.



$$r = \frac{1.24}{2} = 0.62 \text{ cm}$$

$$D_{eq} = \sqrt[3]{D_{12} D_{23} D_{31}} = \sqrt[3]{2 \times 2.5 \times 4.5}$$

$$= 2.82 \text{ m} = 282 \text{ cm}$$

$$\text{Inductance/phase} = 10^{-7} \left(0.5 + 2 \log_e \frac{D_{eq}}{r} \right)$$

$$= 12.74 \times 10^{-7} \text{ H}$$

$$\text{Inductance/phase/km} = 12.74 \times 10^{-7} \times 1000$$

$$= 1.274 \text{ mH}$$

- ③ Calculate inductance of each conductor in a 3 ϕ , 3 wire system when the conductors are arranged in a horizontal plane with a spacing such that $D_{31} = 4\text{m}$; $D_{12} = D_{23} = 2\text{m}$. The conductors are transposed and have diameter of 2.5 cm.

$$\gamma = \frac{2.5}{2} = 1.25 \text{ cm}$$

$$\text{Deg} = \sqrt[3]{D_{12} \times D_{23} \times D_{31}} = \sqrt[3]{2 \times 2 \times 4}$$

$$= 2.52 \text{ m} = 252 \text{ cm}$$

$$\text{Inductance/phase/m} = 10^{-7} \left(0.5 + 2 \log \frac{D}{\gamma} \right)$$

$$= 10^{-7} \left(0.5 + 2 \log \frac{252}{1.25} \right)$$

$$= 11.1 \times 10^{-2} \text{ H}$$

$$\text{Inductance/phase/km} = 11.1 \times 10^{-2} \times 1000$$

$$= 1.11 \times 10^3 \text{ H}$$

$$= 1.11 \text{ mH}$$

Inductance of double circuit of 3 ϕ line with symmetrical spacing

Double circuit line consists of 3 conductors in each circuit. Consider x, y and z forming one circuit where as x', y' and z' are forming other circuit. Conductors x and x' are electrically parallel and constitute one phase, similarly y, y' and z, z' for other phase. This means there are two conductors per phase. In order to achieve low inductance, the individual conductors are separated as widely as possible & distance b/w the phase is kept small.

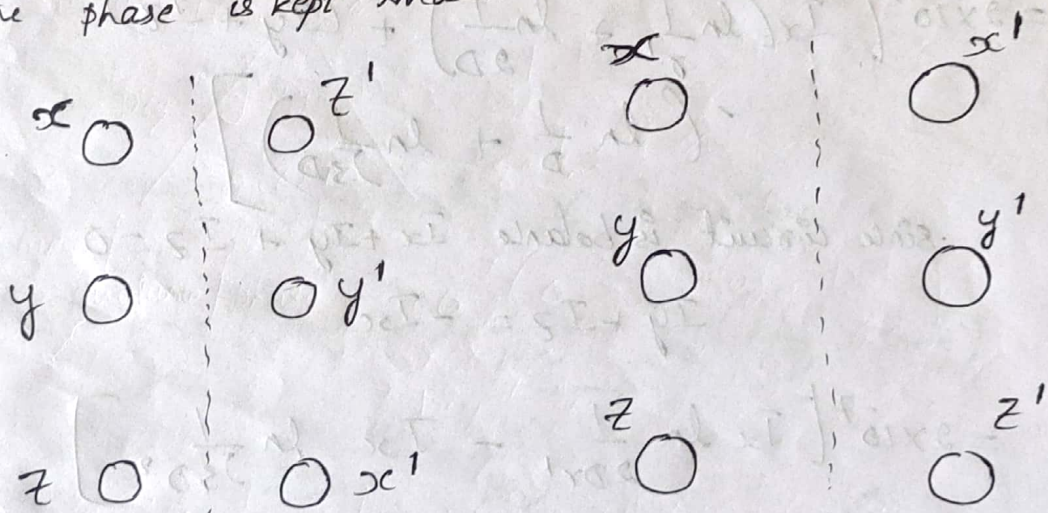
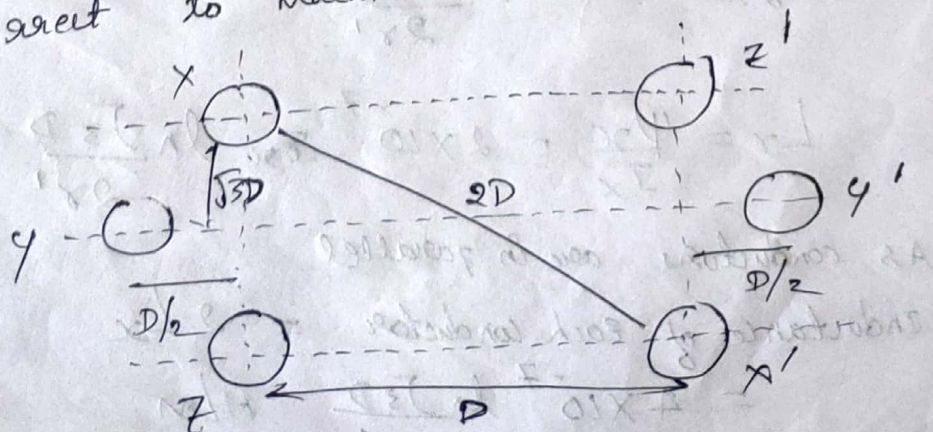


Fig (1)

Fig (2)

The arrangement of conductors in case of fig 2 is correct to maintain low value of inductance.



These conductors are placed at the vertex of regular hexagon.

Flux linkages of conductor X due to current in other phases

$$\Psi_X = 2 \times 10^{-7} \left[I_x \left(\ln \frac{1}{r'} + \ln \frac{1}{2D} \right) \right.$$

$$+ I_y \left(\ln \frac{1}{D} + \ln \frac{1}{\sqrt{3}D} \right)$$

$$\left. + I_z \left(\ln \frac{1}{\sqrt{3}D} + \ln \frac{1}{D} \right) \right]$$

$$= 2 \times 10^{-7} \left[I_x \left(\ln \frac{1}{r'} + \ln \frac{1}{2D} \right) + (I_y + I_z) \right.$$

$$\left. - \left(\ln \frac{1}{D} + \ln \frac{1}{\sqrt{3}D} \right) \right]$$

Since circuit is balanced $I_x + I_y + I_z = 0$

$$I_y + I_z = -I_x$$

$$= 2 \times 10^{-7} \left[I_x \ln \frac{1}{2Dr'} - I_x \ln \frac{1}{\sqrt{3}D} \right]$$

$$= 2 \times 10^{-7} I_x \left[\ln \frac{\sqrt{3}D^2}{2Dr'} \right]$$

$$= 2 \times 10^{-7} I_x \frac{\ln \sqrt{3}D}{2r'}$$

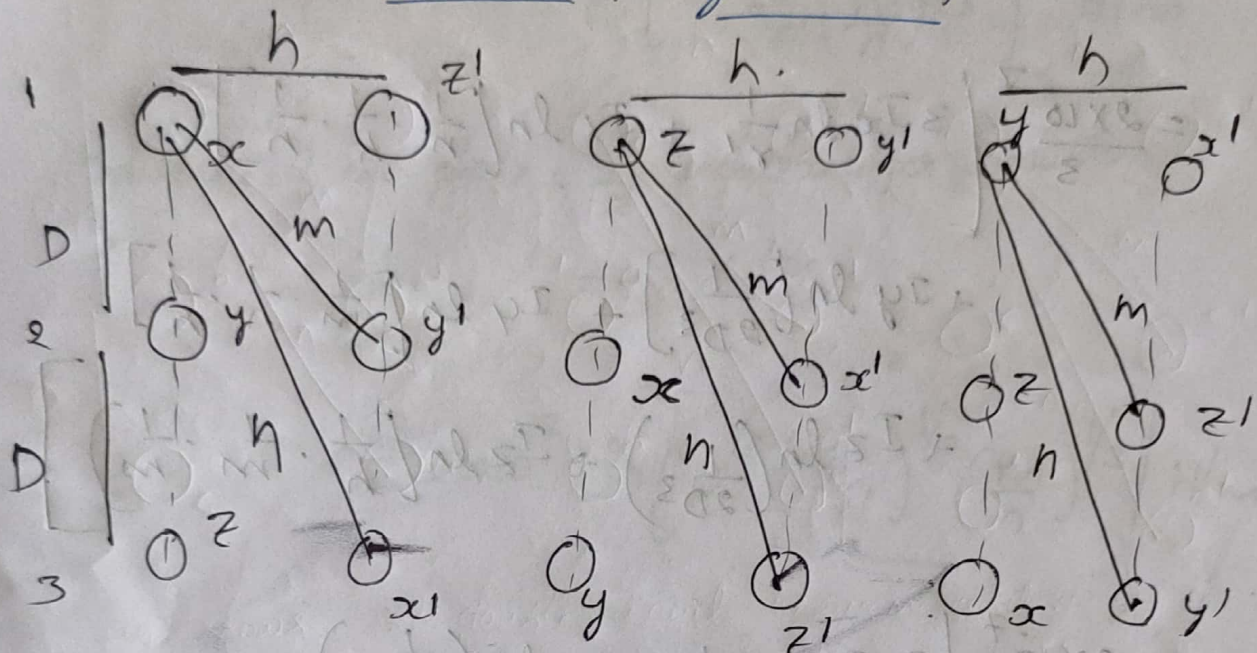
$$L_x = \frac{\Psi_x}{I_x} = 2 \times 10^{-7} \frac{\ln \sqrt{3}D}{2r'} \text{ H/m}$$

As conductors are in parallel

$$\text{Inductance of each conductor} = 2 L_x$$

$$= 4 \times 10^{-7} \frac{\ln \sqrt{3}D}{2r'} \text{ H/m}$$

Inductance of 3 φ double circuit with unymmetrical spacing but transposed



flux linkages of conductor x in position 1

$$\Phi_{x1} = 2 \times 10^{-7} \left[I_x \left(\ln \frac{1}{r'} + \ln \frac{1}{h} \right) + I_y \left(\ln \frac{1}{D} + \ln \frac{1}{m} \right) + I_z \left(\ln \frac{1}{2D} + \ln \frac{1}{h} \right) \right]$$

Similarly

$$\Phi_{x2} = 2 \times 10^{-7} \left[I_x \left(\ln \frac{1}{r'} + \ln \frac{1}{h} \right) + I_y \left(\ln \frac{1}{D} + \ln \frac{1}{m} \right) + I_z \left(\ln \frac{1}{2D} + \ln \frac{1}{h} \right) \right]$$

$$\Phi_{x3} = 2 \times 10^{-7} \left[I_x \left(\ln \frac{1}{r'} + \ln \frac{1}{h} \right) + I_y \left(\ln \frac{1}{2D} + \ln \frac{1}{h} \right) + I_z \left(\ln \frac{1}{D} + \ln \frac{1}{m} \right) \right]$$

$$\Psi_x = \frac{1}{3} (\Psi_{x1} + \Psi_{x2} + \Psi_{x3})$$

$$= \frac{2 \times 10^{-7}}{3} \left[3 I_x \ln \frac{1}{r_1} + I_x \ln \left[\frac{1}{n} \cdot \frac{1}{h} \cdot \frac{1}{n} \right] \right. \\ \left. + I_y \ln \left[\frac{1}{2D^3} \right] + I_y \ln \left[\frac{1}{m} \cdot \frac{1}{h} \cdot \frac{1}{n} \right] \right. \\ \left. + I_z \ln \left(\frac{1}{2D^3} \right) + I_z \ln \left[\frac{1}{h} \cdot \frac{1}{m} \cdot \frac{1}{n} \right] \right]$$

$$= \frac{2 \times 10^{-7}}{3} \left[3 I_x \ln \frac{1}{r_1} + I_x \ln \left(\frac{1}{n^2 h} \right) \right.$$

$$\left. + (I_y + I_z) \ln \left(\frac{1}{2D^3} \right) \right]$$

$$\left[\left(\frac{1}{m} + \frac{1}{n} \right) + (I_y + I_z) \ln \frac{1}{m^2 h} \right]$$

$$I_x + I_y + I_z = 0 \quad I_x = -(I_y + I_z)$$

$$= \frac{2 \times 10^{-7}}{3} \left[3 I_x \ln \frac{1}{r_1} + I_x \ln \left(\frac{1}{n^2 h} \right) - I_x \ln \left(\frac{1}{2D^3} \right) \right. \\ \left. - I_x \ln \left(\frac{1}{m^2 h} \right) \right]$$

$$= \frac{2 \times 10^{-7}}{3} I_x \left[\ln \frac{2D^3 m^2 h}{r_1^3 n^2 h} \right]$$

$$= 2 \times 10^{-7} I_x \left\{ \frac{\ln 2^{1/3} D m^{2/3} h^{1/3}}{(r_1^3 n^{2/3} h^{1/3})} \right\}$$

$$L_x = \frac{4\pi \times 10^{-7}}{I_x} = 2 \times 10^{-7} \ln \left[2^{1/3} \frac{D}{r'} \left(\frac{m}{n} \right)^{1/3} \right] \text{ H/m}$$

Inductance of each phase = $\frac{1}{2}$ Inductance per conductor
 $= \frac{1}{2} \times L_{oc}$

$$= 2 \times 10^{-7} \ln \left[2^{1/6} \left(\frac{D}{r'} \right)^{1/2} \left(\frac{m}{n} \right)^{1/3} \right] \text{ H/m.}$$

problems

Inductance formulae in terms of GMD (double circuit)
 we have,

Self GMD of conductor

$$r' = 0.7788r$$

Self GMD of combination of aa' is $^{1/4}$

$$D_{S1} = (D_{aa} \times D_{aa'} \times D_{a'a} \times D_{a'a'})^{1/4}$$

where D_{aa} or $D_{a'a'}$ = self GMD of conductors a & a'

$$D_{aa'} = D_{a'a} = \text{Distance b/w } a \text{ and } a'$$

Self GMD combination of bb' is

$$D_{S2} = (D_{bb} \times D_{bb'} \times D_{bb'} \times D_{b'b})^{1/4}$$

$$D_{S3} = (D_{cc} \times D_{c'c'} \times D_{cc'} \times D_{c'c})^{1/4}$$

Equivalent self GMD of an phase.

$$D_S = (D_{S1} \times D_{S2} \times D_{S3})^{1/3}$$

mutual GMD between phases A and B is

$$D_{AB} = (D_{ab} \times D_{ab'} \times D_{a'b'} \times D_{a'b})^{1/4}$$

mutual GMD between phases B and C is

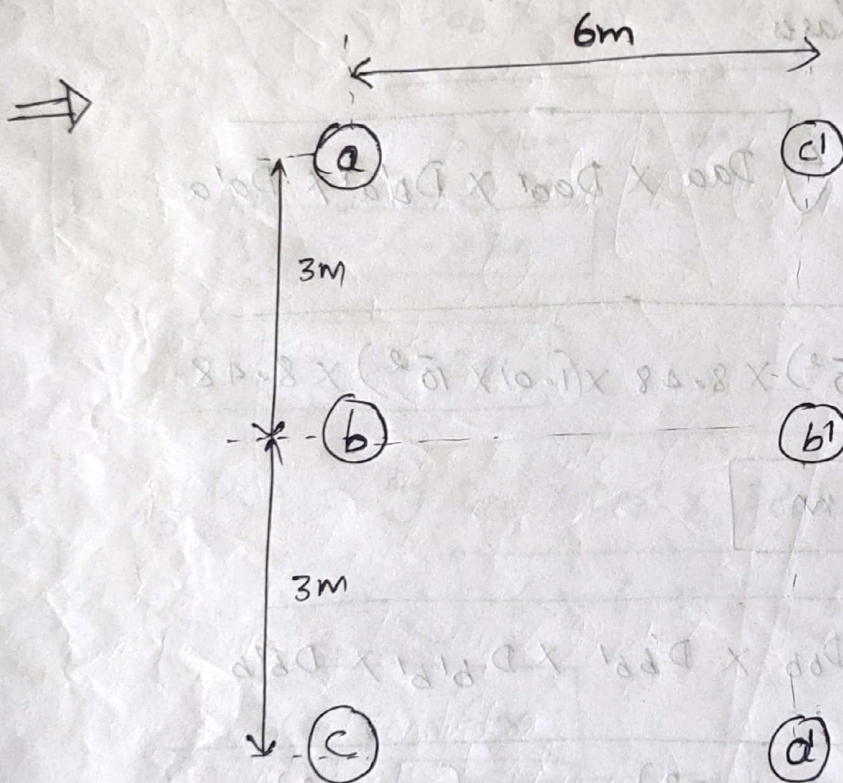
$$D_{BC} = (D_{bc} \times D_{b'c} \times D_{b'c'} \times D_{bc'})^{1/4}$$

$$D_{CA} = (D_{ca} \times D_{ca'} \times D_{c'a} \times D_{c'a'})^{1/4}$$

Equivalent mutual

$$D_M = (D_{AB} \times D_{BC} \times D_{CA})^{1/3}$$

Figure shows the spacing of a double circuit 3 phase overhead line. The phase sequence is ABC and line is completely transposed. The conductor radius is 1.3 cm. Find the inductance per phase per kilometre.



$$GMR \text{ of conductor} = 0.7788 r$$

$$= 0.7788 \times 1.3$$

$$= 1.01 \text{ cm}$$

$$\text{Distance } a \text{ to } b' = \sqrt{6^2 + 3^2} = 6.7 \text{ m}$$

$$\text{Distance } a \text{ to } a' = \sqrt{6^2 + 6^2} = 8.48 \text{ m}$$

Equivalent D_s of self GMD of one phase

$$D_s = \sqrt[3]{D_{S1} \times D_{S2} \times D_{S3}}$$

D_{S1} , D_{S2} and D_{S3} represent self GMD in positions 1, 2, 3 respectively. Also D_s is same for all the phases

Now $D_{S1} = \sqrt[4]{D_{aa} \times D_{aa'} \times D_{a'a'} \times D_{a'a}}$

$$= \sqrt[4]{(1.01 \times 10^{-2}) \times 8.48 \times (1.01 \times 10^{-2}) \times 8.48}$$

$$D_{S1} = 0.292 \text{ m}$$

$$D_{S2} = \sqrt[4]{D_{bb} \times D_{bb'} \times D_{b'b'} \times D_{b'b}}$$

$$= \sqrt[4]{(1.01 \times 10^{-2}) \times 6 \times (1.01 \times 10^{-2}) \times 6}$$

$$D_{S2} = 0.246 \text{ m}$$

$$D_{S3} = \sqrt[4]{D_{cc} \times D_{cc'} \times D_{c'c'} \times D_{c'c}}$$

$$D_{S3} = 0.292 \text{ m}$$

$$D_s = \sqrt[3]{D_{S1} \times D_{S2} \times D_{S3}}$$

$$= \sqrt[3]{0.292 \times 0.246 \times 0.292}$$

$$D_s = 0.275 \text{ m}$$

$$\text{mutual GMD } D_m = \sqrt[3]{D_{AB} \times D_{BC} \times D_{CA}}$$

D_{AB} - D_{BC} and D_{CA} represent mutual GMD b/w phases A and B, B and C & C and A respectively.

$$D_{AB} = \sqrt[4]{D_{ab} \times D_{ab'} \times D_{a'b} \times D_{a'b'}}$$

$$= \sqrt[4]{3 \times 6.7 \times 6.7 \times 3}$$

$$D_{AB} = 4.48 \text{ m}$$

$$D_{BC} = 4.48 \text{ m}$$

$$D_{CA} = \sqrt[4]{D_{ca} \times D_{ca'} \times D_{c'a} \times D_{c'a'}}$$

$$= \sqrt[4]{6 \times 6 \times 6 \times 6}$$

$$D_{CA} = 6 \text{ m}$$

$$D_m = \sqrt[3]{4.48 \times 4.48 \times 6} = 4.94 \text{ m}$$

$$D_m = 4.94 \text{ m}$$

Inductance / phase / meter length

$$= 10^{-7} \cdot 2 \log_e \frac{D_m}{D_s}$$

$$= 10^{-7} \cdot 2 \log_e \frac{4.94}{0.275}$$

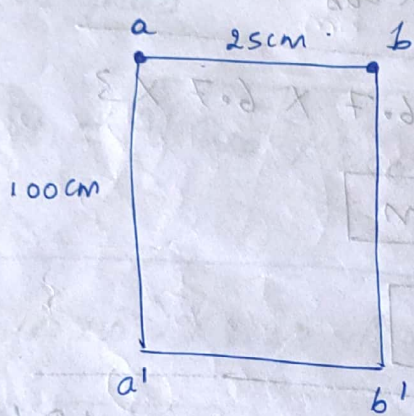
$$= 5.7 \times 10^{-7} \text{ H}$$

Inductance / phase / km = $5.7 \times 10^{-7} \times 1000 \text{ H}$

$$L = 0.57 \text{ mH}$$

② Two conductors of a single phase line, each of 1 cm diameter, are arranged in a vertical plane with one conductor mounted 1 m above the other. A second identical line is mounted at the same height as the first and spaced horizontally 0.25 m apart from it. The two upper and two lower conductors are connected in parallel. Determine inductance per km of resulting double circuit line

⇒



$$r = \frac{1 \text{ cm}}{2} = 0.5 \text{ cm}$$

$$\begin{aligned} \text{GMR of conductor} &= 0.7788 r \\ &= 0.7788 \times 0.5 \\ &= 0.389 \text{ cm} \end{aligned}$$

Self GMD of aa' combination is

$$\begin{aligned} D_{S1} &= \sqrt[4]{D_{aa} \times D_{aa'} \times D_{a'a} \times D_{a'a'}} \\ &= \sqrt[4]{(0.389 \times 100)^2} = 6.23 \text{ cm} \end{aligned}$$

mutual GMD b/w a and b is

$$\begin{aligned} D_m &= \sqrt[4]{D_{ab} \times D_{ab'} \times D_{a'b} \times D_{a'b'}} \\ &= \sqrt[4]{25 \times 105 \times 105 \times 25} \\ &= 50.74 \text{ cm} \end{aligned}$$

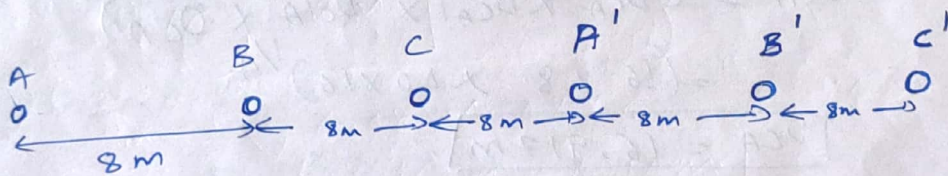
$$[D_{ab'} = D_{a'b} = \sqrt{25^2 \times 100^2} = 103 \text{ cm}]$$

$$\begin{aligned} \text{Inductance / phase / meter} &= 2 \times 10^{-7} \log_e \frac{D_m}{D_s} \\ &= 2 \times 10^{-7} \log_e \frac{50.74}{6.23} \end{aligned}$$

$$L_A = 0.42 \times 10^{-6} \text{ H}$$

$$\begin{aligned} \text{loop Inductance per km of line} &= 2 \times L_A \\ &= 2 \times 0.42 \times 10^{-6} \times 1000 \\ &= 0.84 \text{ mH} \end{aligned}$$

3] calculate the inductance per phase per meter for 3 ϕ double circuit line whose phase conductors have radius of 5.3 cm with the horizontal conductor arrangement as shown in figure.



$$\begin{aligned} \text{GMR of conductor } r' &= 0.7788 r \\ &= 0.7788 \times 5.3 \times 10^{-2} \\ &= 0.0413 \text{ m} \end{aligned}$$

Equivalent self GMD of one phase D_s

$$D_s = (D_{s1} \times D_{s2} \times D_{s3})^{1/3}$$

$$\begin{aligned} D_{s1} &= \sqrt[4]{D_{aa} \times D_{aa'} \times D_{a'a} \times D_{a'a}} \\ &= \sqrt[4]{(0.0413)^2 \times 24 \times (0.0413)^2 \times 24} \\ D_{s1} &= 0.995 \text{ m} \end{aligned}$$

$$D_{S2} = \sqrt[4]{D_{bb} \times D_{bb'} \times D_{b'b'} \times D_{b'b}}$$

$$D_{S2} = 0.995 \text{ m}$$

$$D_{S3} = 0.995 \text{ m}$$

$$D_S = \sqrt[3]{0.995 \times 0.995 \times 0.995} = 0.995 \text{ m}$$

$$D_S = 0.995 \text{ m}$$

Equivalent mutual GMD is

$$D_m = (D_{AB} \times D_{BC} \times D_{CA})^{1/3}$$

$$D_{AB} = (D_{AB} \times D_{AB'} \times D_{A'B} \times D_{A'B'})^{1/4}$$

$$= (8 \times 32 \times 16 \times 8)^{1/4}$$

$$D_{AB} = 13.45 = D_{BC}$$

$$D_{CA} = (D_{CA} \times D_{CA'} \times D_{C'A} \times D_{C'A'})^{1/4}$$

$$= (16 \times 8 \times 40 \times 16)^{1/4}$$

$$D_{CA} = 16.917 \text{ m}$$

$$D_m = (13.45 \times 13.45 \times 16.917)^{1/3}$$

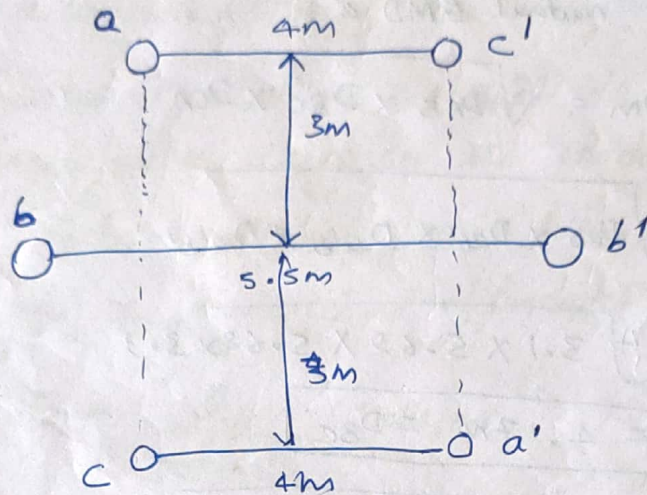
$$D_m = 14.518 \text{ m}$$

$$\text{Inductance / phase / m} = 10^{-7} \times 2 \log_e \frac{D_m}{D_S}$$

$$= 10^{-7} \times 2 \log_e \frac{14.518}{0.995} \text{ H/m}$$

$$L_A = 5.36 \times 10^{-7} \text{ H/m}$$

- ④ Find the inductance per phase per km of double circuit 3 ϕ line shown in fig. The conductors are transposed and are of radius 0.75 cm each. The phase sequence is ABC.



$$GMR \text{ of conductor} = 0.75 \times 0.7788 = 0.584 \text{ cm}$$

$$\text{Distance } a \text{ to } b = \sqrt{3^2 + (0.75)^2} = 3.1 \text{ m}$$

$$\text{Distance } a \text{ to } b' = \sqrt{3^2 + (4.75)^2} = 5.62 \text{ m}$$

$$\text{Distance } 'a \text{ to } a' = \sqrt{6^2 + 4^2} = 7.21 \text{ m}$$

Equivalent self GMD of one phase \bar{D}

$$D_S = \sqrt[3]{D_{S1} \times D_{S2} \times D_{S3}}$$

$$\text{where } D_{S1} = \sqrt[4]{D_{aa} \times D_{aa'} \times D_{a'a'} \times D_{a'a}}$$

$$= \sqrt{(0.584 \times 10^{-2}) \times 7.21 \times (0.584 \times 10^{-2}) \times 7.21}$$

$$D_{S1} = 0.205 \text{ m} = D_{S3}$$

$$D_{S2} = \sqrt[4]{D_{bb} \times D_{bb'} \times D_{b'b'} \times D_{b'b}}$$

$$= \sqrt{(0.584 \times 10^{-2}) \times 5.5 \times (0.584 \times 10^{-2}) \times 5.5}$$

$$D_{S2} = 0.18 \text{ m}$$

$$D_S = \sqrt[3]{0.205 \times 0.18 \times 0.205} = 0.195 \text{ m}$$

$$D_S = 0.195 \text{ m}$$

Equivalent mutual GMD is

$$D_M = \sqrt[3]{D_{AB} \times D_{BC} \times D_{CA}}$$

$$D_{AB} = \sqrt[4]{D_{ab} \times D_{a'b'} \times D_{a'b} \times D_{a'b'}}$$

$$= \sqrt[4]{3.1 \times 5.62 \times 5.62 \times 3.1}$$

$$D_{AB} = 4.17 \text{ m} = D_{BC}$$

$$D_{CA} = \sqrt[4]{D_{ca} \times D_{c'a'} \times D_{c'a} \times D_{c'a'}}$$

$$= \sqrt[4]{6 \times 4 \times 6 \times 4} = 4.9 \text{ m}$$

$$D_{CA} = 4.9 \text{ m}$$

$$D_M = \sqrt[3]{4.17 \times 4.17 \times 4.9}$$

$$D_M = 4.4 \text{ m}$$

$$\text{Inductance/phase/m} = 10^{-7} \times 2 \log_e \frac{D_M}{D_S}$$

$$= 10^{-7} \times 2 \log_e \frac{4.4}{0.195}$$

$$= 0.623 \times 10^{-7} \text{ H}$$

$$\text{Inductance/phase/km} = 0.623 \text{ mH}$$

Capacitance

Electric potential:

potential at charged single conductor:

consider a conductor A of radius r meters.

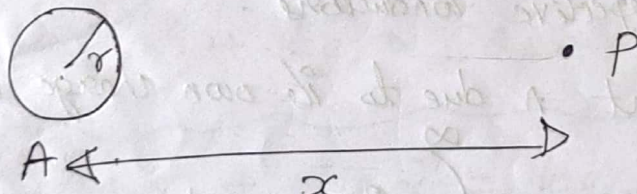
let conductor operates as potential V_A that charge Q_A coulombs/meter exists on the conductor.

let the electric potential field in intensity E at a distance x from centre of conductor in air.

$$E = \frac{Q_A}{2\pi x \epsilon_0} \text{ Volts/m}$$

where $Q_A =$ charge per metre length

$\epsilon_0 =$ permittivity of free space.



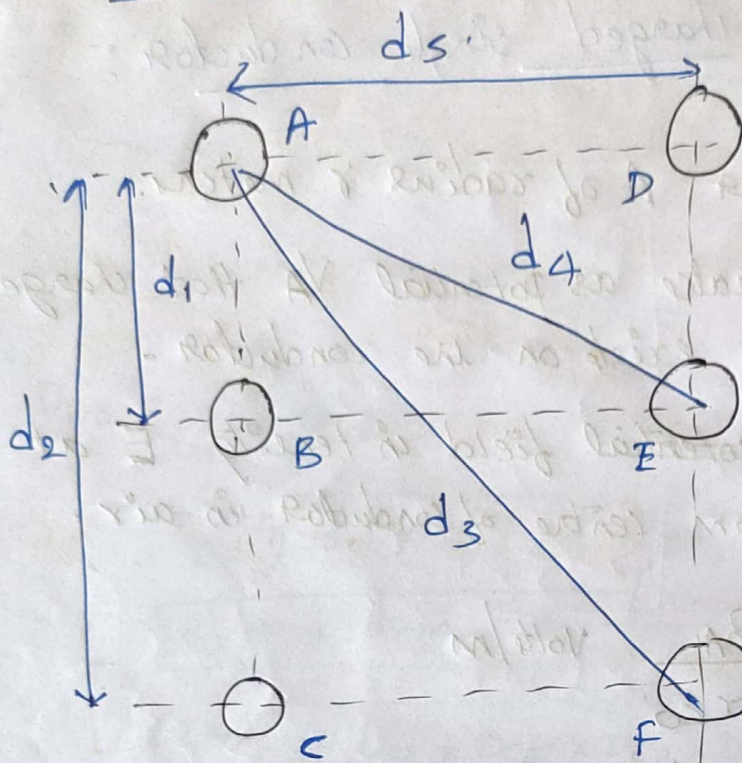
As x approaches ∞ , the value of E approaches zero.

\therefore potential difference b/w conductor and neutral plane at infinite distance is given by,

$$V_A = \int_0^{\infty} \frac{Q_A}{2\pi x \epsilon_0} dx$$

$$V_A = \frac{Q}{2\pi \epsilon_0} \int_0^{\infty} \frac{1}{x} dx$$

potential at a conductor in a group of charged conductors.



consider a group of long straight conductors A, B, C... operating at potentials such that charges Q_A, Q_B, Q_C, \dots exist on respective conductors.

potential at A due to its own charge Q_A

$$V_A = \int_{\infty}^A \frac{Q_A}{2\pi r \epsilon_0} dx \quad \text{--- (1)}$$

potential at A due to charge Q_B

$$= \int_{d_1}^{\infty} \frac{Q_B}{2\pi r \epsilon_0} dx \quad \text{--- (2)}$$

potential at A due to charge Q_C

$$= \int_{d_2}^{\infty} \frac{Q_C}{2\pi r \epsilon_0} dx \quad \text{--- (3)}$$

overall potential difference

$$V_A = (i) + (ii) + (iii) + \dots$$

$$= \int_{\infty}^{\infty} \frac{Q_A}{2\pi x \epsilon_0} dx + \int_{d_1}^{\infty} \frac{Q_B}{2\pi x \epsilon_0} dx + \int_{d_2}^{\infty} \frac{Q_C}{2\pi x \epsilon_0} dx + \dots$$

$$= \frac{1}{2\pi \epsilon_0} \left[Q_A (\log_e \infty - \log_e r) + Q_B (\log_e \infty - \log_e d_1) + Q_C (\log_e \infty - \log_e d_2) + \dots \right]$$

$$= \frac{1}{2\pi \epsilon_0} \left[Q_A \log_e \frac{1}{r} + Q_B \log_e \frac{1}{d_1} + Q_C \log_e \frac{1}{d_2} + \dots + \log_e \infty (Q_A + Q_B + Q_C) \dots \right]$$

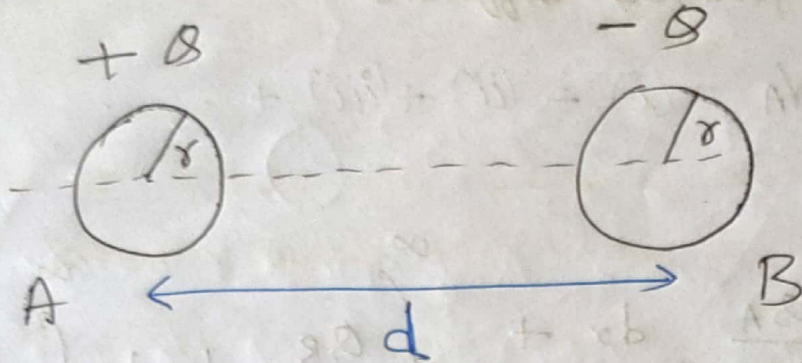
$$\rightarrow \left\{ \log_e r = \log_e r^{-1} = \log_e \frac{1}{r} \right\}$$

assuming balanced conditions i.e.

$$Q_A + Q_B + Q_C = 0$$

$$V_A = \frac{1}{2\pi \epsilon_0} \left[Q_A \log_e \frac{1}{r} + Q_B \log_e \frac{1}{d_1} + Q_C \log_e \frac{1}{d_2} + \dots \right]$$

Capacitance of a single phase two wire line



consider a single phase overhead transmission line consisting of two parallel conductors A and B spaced d metres apart in air.

let their respective charge be $+Q$ and $-Q$ Coulombs per metre length.

The total potential difference b/w conductor A and neutral infinite plane.

$$V_A = \int_r^{\infty} \frac{Q}{2\pi x \epsilon_0} dx + \int_d^{\infty} \frac{-Q}{2\pi x \epsilon_0} dx$$

$$= \frac{Q}{2\pi \epsilon_0} \left[\log_e \frac{\infty}{r} - \log_e \frac{\infty}{d} \right]$$

$$= \frac{Q}{2\pi \epsilon_0} \log_e \frac{d}{r}$$

Similarly potential difference b/w conductor B and neutral infinite plane is

$$V_B = \int_r^{\infty} \frac{-Q}{2\pi x \epsilon_0} dx + \int_d^{\infty} \frac{Q}{2\pi x \epsilon_0} dx$$

$$= \frac{-Q}{2\pi \epsilon_0} \left[\log_e \frac{\infty}{r} - \log_e \frac{\infty}{d} \right]$$

$$= \frac{-Q}{2\pi\epsilon_0} \log_e \frac{d}{r} \text{ Volts}$$

Both these potentials are w.r.t the same neutral plane. Since the unlike charges attract each other potential difference b/w conductors is

$$V_{AB} = 2V_A = \frac{2Q}{2\pi\epsilon_0} \log_e \frac{d}{r} \text{ Volts}$$

Capacitance

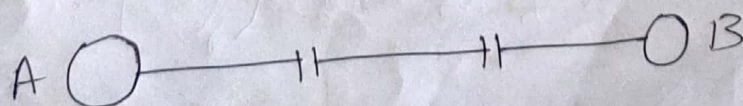
$$C_{AB} = \frac{Q}{V_{AB}} = \frac{Q}{\frac{2Q}{2\pi\epsilon_0} \log_e \frac{d}{r}} \text{ F/m}$$

$$C_{AB} = \frac{\pi\epsilon_0}{\log_e \frac{d}{r}} \text{ F/m}$$

Capacitance to Neutral:

Capacitance b/w conductors of 2 wire line is C_{AB} since potential of midpoint b/w conductors is zero, the potential difference b/w each conductor and ground or neutral is half the potential b/w the conductors.

Thus capacitance to neutral for 2 line is twice the line.



$$C_N = C_{AN} = 2C_{AB}$$

$$C_{BN} = 2C_{AB}$$

$$C_N = \frac{2\pi\epsilon_0}{\log_e \frac{d}{r}}$$

① A single phase transmission line has 2 parallel conductors, 3m apart, radius of each conductor being 1cm. Calculate the capacitance of the line per km given that

$$\epsilon_0 = 8.854 \times 10^{-12}$$

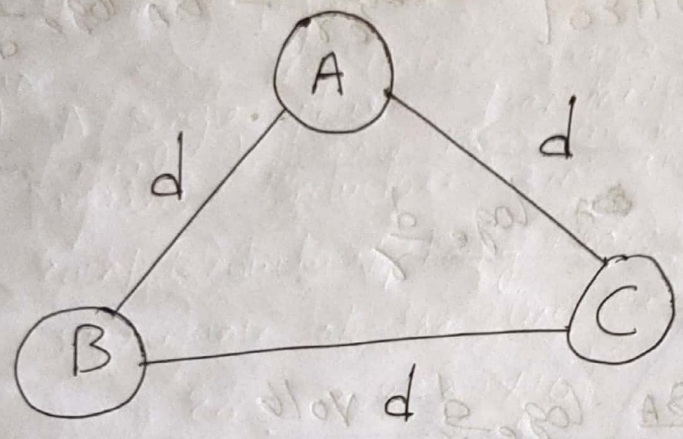
$$\Rightarrow \begin{aligned} d &= 3\text{m} \\ r &= 1\text{cm} \end{aligned}$$

$$\begin{aligned} C_{AB} &= \frac{\pi\epsilon_0}{\log_e \frac{d}{r}} \\ &= \frac{\pi \times 8.854 \times 10^{-12}}{\log_e \frac{3}{1 \times 10^{-2}}} \end{aligned}$$

$$C_{AB} = 4.87 \times 10^{-12} \text{ F/m}$$

$$C_{AB} = 4.87 \times 10^{-9} \text{ F/km}$$

capacitance of 3φ overhead line having symmetrical spacing



Consider 3 conductors A, B, C of 3φ overhead transmission line having charges Q_A , Q_B and Q_C . Let 'd' metres distance b/w conductors.

overall potential difference b/w conductor A and infinite neutral plane

$$V_A = \int_{\gamma}^{\infty} \frac{Q_A}{2\pi\epsilon_0 x} dx + \frac{1}{d} \int_{\gamma}^{\infty} \frac{Q_B}{2\pi\epsilon_0 x} dx + \frac{1}{d} \int_{\gamma}^{\infty} \frac{Q_C}{2\pi\epsilon_0 x} dx$$

$$= \frac{1}{2\pi\epsilon_0} \left[Q_A \log_e \frac{1}{\gamma} + Q_B \log_e \frac{1}{d} + Q_C \log_e \frac{1}{d} \right]$$

$$= \frac{1}{2\pi\epsilon_0} \left[Q_A \log_e \frac{1}{\gamma} + (Q_B + Q_C) \log_e \frac{1}{d} \right]$$

Assuming balanced supply

$$Q_A + Q_B + Q_C = 0$$

$$Q_B + Q_C = -Q_A$$

$$V_A = \frac{1}{2\pi\epsilon_0} \left[Q_A \log_e \frac{1}{r} - Q_A \log_e \frac{1}{d} \right]$$

$$= \frac{Q_A}{2\pi\epsilon_0} \log_e \frac{d}{r}$$

$$V_A = \frac{Q_A \log_e \frac{d}{r}}{2\pi\epsilon_0} \text{ Volt}$$

Capacitance of conductor A w.r.t neutral

$$C_A = \frac{Q_A}{V_A} = \frac{Q_A}{\frac{Q_A \log_e \frac{d}{r}}{2\pi\epsilon_0}}$$

$$C_A = \frac{2\pi\epsilon_0}{\log_e \frac{d}{r}} \text{ F/m}$$

Capacitance of 3 ϕ overhead line having unsymmetrical spacing and transposed

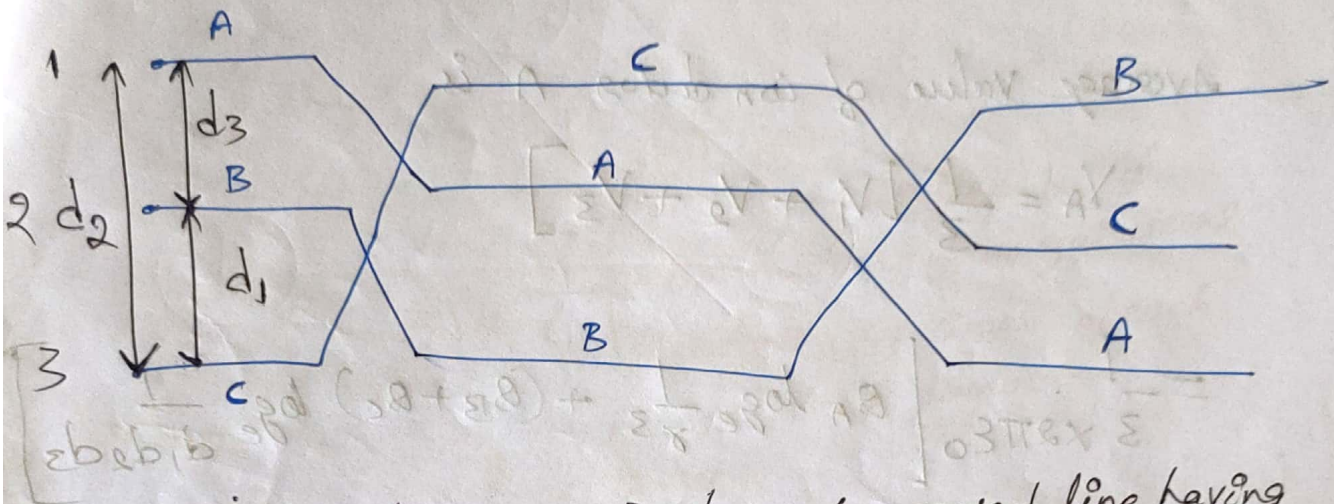


Figure shows a 3 phase transposed line having unsymmetrical spacing.

Let assume balanced condition

$$Q_A + Q_B + Q_C = 0$$

Considering all the 3 sections of the transposed line for phase 'A'

Potential of 1st position:

$$V_1 = \frac{1}{2\pi\epsilon_0} \left[Q_A \log_e \frac{1}{r} + Q_B \log_e \frac{1}{d_3} + Q_C \log_e \frac{1}{d_2} \right]$$

Potential of 2nd position:

$$V_2 = \frac{1}{2\pi\epsilon_0} \left[Q_A \log_e \frac{1}{r} + Q_B \log_e \frac{1}{d_1} + Q_C \log_e \frac{1}{d_3} \right]$$

potential of 3rd position:

$$V_3 = \frac{1}{2\pi\epsilon_0} \left[Q_A \log_e \frac{1}{r} + Q_B \log_e \frac{1}{d_2} + Q_C \log_e \frac{1}{d_1} \right]$$

Average value of conductor A is

$$V_A = \frac{1}{3} [V_1 + V_2 + V_3]$$

$$= \frac{1}{3 \times 2\pi\epsilon_0} \left[Q_A \log_e \frac{1}{r^3} + (Q_B + Q_C) \log_e \frac{1}{d_1 d_2 d_3} \right]$$

$$Q_A + Q_B + Q_C = 0$$

$$Q_B + Q_C = -Q_A$$

$$V_A = \frac{1}{3 \times 2\pi\epsilon_0} \left[Q_A \log_e \frac{1}{r^3} - Q_A \log_e \frac{1}{d_1 d_2 d_3} \right]$$

$$= \frac{Q_A}{3 \times 2\pi\epsilon_0} \left[\log_e \frac{d_1 d_2 d_3}{r^3} \right]$$

$$= \frac{1}{3} \times \frac{Q_A}{2\pi\epsilon_0} \log_e \frac{d_1 d_2 d_3}{r^3}$$

$$= \frac{Q_A}{2\pi\epsilon_0} \log_e \frac{(d_1 d_2 d_3)^{1/3}}{(r^3)^{1/3}}$$

$$= \frac{Q_A}{2\pi\epsilon_0} \log_e \frac{(d_1 d_2 d_3)^{1/3}}{r}$$

Capacitance from conductor to neutral is

$$C_A = \frac{Q_A}{V_A} = \frac{2\pi\epsilon_0}{\log_e \sqrt[3]{\frac{d_1 d_2 d_3}{r}}} \text{ F/m}$$

Problems

- ① A 3 ϕ overhead transmission line has its conductors arranged at the corners of an equilateral triangle of 2m side. Calculate capacitance of each line conductor per km given that diameter of each conductor is 1.25cm

$$\Rightarrow r = \frac{1.25}{2} = 0.625 \text{ cm}$$

$$d = 2 \text{ m} = 200 \text{ cm}$$

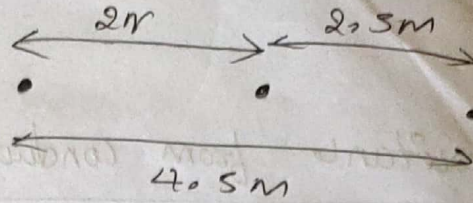
$$C = \frac{2\pi\epsilon_0}{\log_e \frac{d}{r}} \text{ F/m}$$

$$= \frac{2\pi \times 8.854 \times 10^{-12}}{\log_e \frac{200}{0.625}}$$

$$= 0.0096 \times 10^{-9} \text{ F/m}$$

$$C = 0.0096 \times 10^{-6} \text{ F/km}$$

- ② A 3 ϕ , 50 Hz, 66 kV overhead line conductors are placed in a horizontal plane as shown in Fig. Conductor diameter is 1.25 cm. If the line length is 100 km calculate (i) capacitance per phase, (ii) charging current per phase. Assuming the complete transposition of the line.



$$d_1 = 2m$$

$$d_2 = 2.5m$$

$$d_3 = 4.5m$$

$$r = \frac{1.25}{2} = 0.625m$$

$$d = \sqrt[3]{d_1 d_2 d_3}$$
$$= \sqrt[3]{2 \times 2.5 \times 4.5}$$

$$d = 2.82m = 282cm$$

(i) line to neutral capacitance = $\frac{2\pi\epsilon_0}{\log_e \frac{d}{r}}$ F/m

$$= \frac{2\pi \times 8.854 \times 10^{-12}}{\log_e \frac{282}{0.625}}$$

$$= 0.0091 \times 10^{-9} \text{ F/m}$$

$$= 0.0091 \times 10^{-6} \text{ F/km}$$

$$= 0.0091 \times 10^{-6} \text{ F/km}$$

line to neutral capacitance for 100km line is

$$C = 0.0091 \times 100 \times 10^{-6} \text{ F/km}$$

$$C = 0.91 \mu\text{F}$$

(ii) charging current per phase

$$I_c = \frac{V_{ph}}{X_c} = \frac{66000}{\sqrt{3}} \times 2\pi f C$$

$$= \frac{66000}{\sqrt{3}} \times 2\pi \times 50 \times 0.91 \times 10^{-6}$$

$$I_c = 10.9 \text{ A}$$

③ Calculate capacitance of a 100km long 3 ϕ 50 Hz overhead transmission line consisting of 3 conductors each of diameter 2cm and spaced 2.5m at the corners of an equilateral triangle.

⇒ Equilateral spacing $d = 2.5 = 250\text{cm}$
 Radius of conductor $r = \frac{2}{2} = 1\text{cm}$

Capacitance of each conductor to neutral

$$= \frac{2\pi\epsilon_0}{\log_e \frac{d}{r}} \text{ F/m}$$

$$= \frac{2\pi \times 8.85 \times 10^{-12}}{\log_e \frac{250}{1}} = 10.075 \times 10^{-12} \text{ F/m}$$

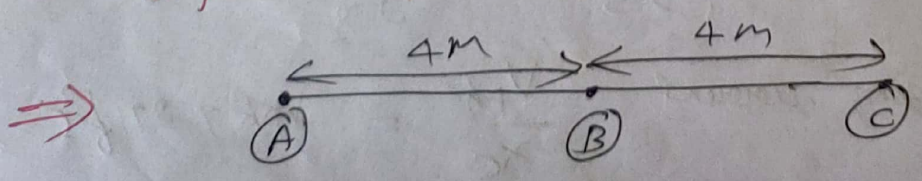
$$= 10.075 \times 10^{-9} \text{ F/km}$$

Capacitance of 100km line = $(10.075 \times 10^{-9}) \times 100$

$$= 1.0075 \times 10^{-6} \text{ F}$$

$$= 1.0075 \mu\text{F/phase.}$$

④ A 3 ϕ , 50 Hz, 132 kV overhead line has conductors placed in a horizontal plane 4m apart conductor diameter is 2cm If line length is 100km calculate the charging current per phase assuming complete transposition



$$r = \frac{2}{2} = 1 \text{ cm}$$

$$d_1 = AB = 4 \text{ m}$$

$$d_2 = BC = 4 \text{ m}$$

$$d_3 = AC = 8 \text{ m}$$

$$D = \sqrt[3]{d_1 d_2 d_3}$$
$$= \sqrt[3]{4 \times 4 \times 8}$$

$$D = 5.04 \text{ m}$$

Capacitance of each conductor to neutral

$$= \frac{2\pi\epsilon_0}{\log_e \frac{D}{r}} = \frac{2\pi\epsilon_0}{\log_e \frac{5.04}{1 \times 10^{-2}}}$$

$$C = 0.00885 \times 10^6 \text{ F/km}$$

Capacitance / phase for 100 km line is

$$C_n = 0.00885 \times 10^6 \times 100$$
$$= 0.885 \times 10^6 \text{ F}$$

$$V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{132 \times 10^3}{\sqrt{3}} = 76210 \text{ V}$$

$$I_C = \text{charging current} = \frac{V_{ph}}{X_C} = V_{ph} \times 2\pi f C$$

$$= 76210 \times 2\pi \times 50 \times 0.885 \times 10^6$$

$$I_C = 21.18 \text{ A}$$