

Module - 3.

Performance of transmission line

Introduction:

The performance of transmission line decided by factors like voltage drop, line loss, efficiency of transmission line etc and these factors depends upon line parameters such as Resistance, Inductance and capacitance.

• Classification of overhead transmission line

depending upon the manner in which capacitance is taken into account, the overhead transmission lines are classified as

(i) short transmission line:

The length of transmission line is upto 50km and line voltage is less than 20kV due to small length.

As the line length is small and the voltage is also less, the capacitive effects are small. Due to this effect of capacitance is neglected. Thus only Resistance and Inductance is to be taken into account while analysing short transmission line.

(ii) medium transmission line:

Length of overhead line is 50km to 150km and line voltage is 20kV - 100kV.

Due to sufficient length and voltage of line, the capacitance effects taken into account. For the purpose of calculation, distributed capacitance of line and lumped in the form of conductance, shunted across line at one more points.

(iii) long transmission line:

length of line is more than 150km and line voltage is more than 100kV. For accurate calculation line constants are considered uniformly distributed over pole length of line.

Terms related to performance of transmission line

(i) Voltage Regulation:

The difference in voltage at the receiving end of transmission line at no load and full load is termed as voltage regulation.

$$\% \text{ Voltage Regulation} = \frac{V_{\text{No load}} - V_{\text{Full load}}}{V_{\text{Full load}}} \times 100$$

$$\% \text{ voltage regulation} = \frac{V_S - V_R}{V_R} \times 100$$

V_R = voltage at receiving end

V_S = voltage at sending end

(ii) Transmission Efficiency:

The ratio of receiving end power to the sending end power of a transmission line is known as the transmission efficiency of the line

$$\eta = \frac{\text{Receiving end power}}{\text{Sending end power}} \times 100$$

$$\eta = \frac{V_R I_R \cos \phi_R}{V_S I_S \cos \phi_S} \times 100$$

power obtained at the receiving end of a transmission line is generally less than sending end power due to losses

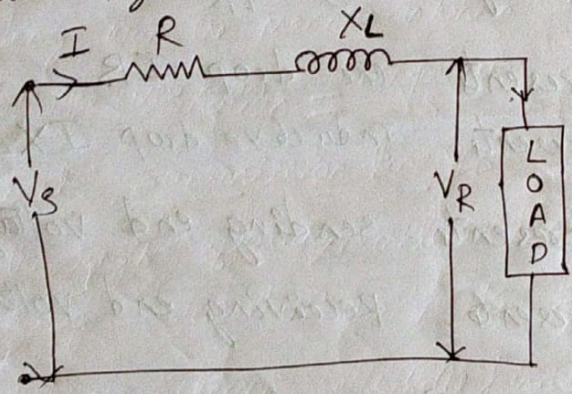
where V_R , I_R and $\cos \phi_R$ are the receiving end voltage, current and power factor

V_S , I_S and $\cos \phi_S$ are the sending end voltage, current and power factor.

performance of single phase short transmission lines

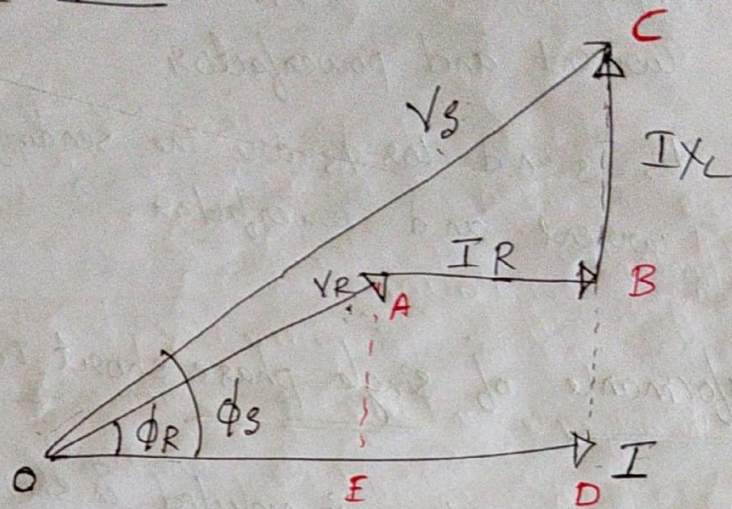
Effect of capacitance is neglected in short transmission line. So only resistance and inductance is considered.

Equivalent circuit of a single phase transmission line is shown in figure.



- let V_S = sending end voltage
- V_R = Receiving end voltage
- I = load current
- R = Resistance
- X_L = Reactance
- $\cos \phi_R$ = Receiving end power factor
- $\cos \phi_S$ = sending end power factor

Vector diagram



Taking current I as reference, vector diagram is plotted.

I lags V_R by an angle ϕ_R

AB represents the drop IR

BC represents Inductive drop IX_L

OC represents sending end voltage V_S .

OA represents Receiving end voltage V_R .

consider $\triangle O C D$

$$OC^2 = OD^2 + CD^2$$

$$= (OE + ED)^2 + (BD + CD)^2$$

$$V_S^2 = (V_R \cos \phi_R + IR)^2$$

$$OC^2 = OD^2 + DC^2$$

$$V_S^2 = (OE + ED)^2 + (DB + BC)^2$$

$$= (V_R \cos \phi_R + IR)^2 + (V_R \sin \phi_R + IX_L)^2$$

$$V_S = \sqrt{(V_R \cos \phi_R + IR)^2 + (V_R \sin \phi_R + IX_L)^2}$$

$$\% \text{ voltage regulation} = \frac{V_s - V_R}{V_R} \times 100$$

$$\text{sending end PF, } \cos \phi_s = \frac{OD}{OC} = \frac{V_R \cos \phi_R + IR}{V_s}$$

$$\text{power delivered} = V_R I_R \cos \phi_R$$

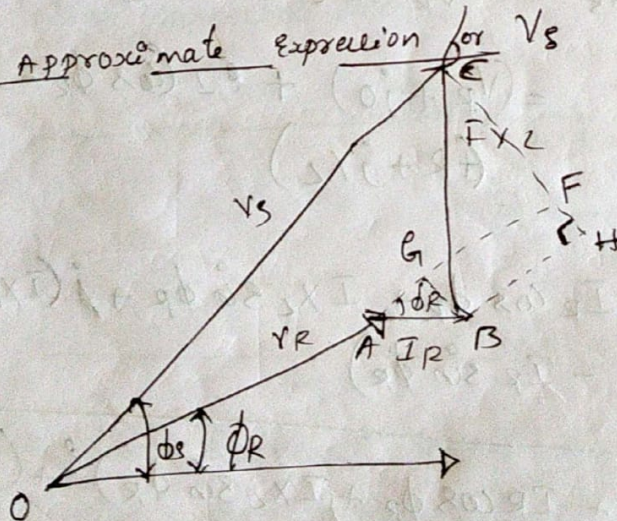
$$\text{line losses} = I^2 R$$

$$\text{power sent out} = V_R I_R \cos \phi_R + I^2 R$$

$$\% \text{ of Transmission efficiency} = \frac{\text{power delivered}}{\text{power sent out}} \times 100$$

$$= \frac{V_R I_R \cos \phi_R}{V_R I_R \cos \phi_R + I^2 R} \times 100$$

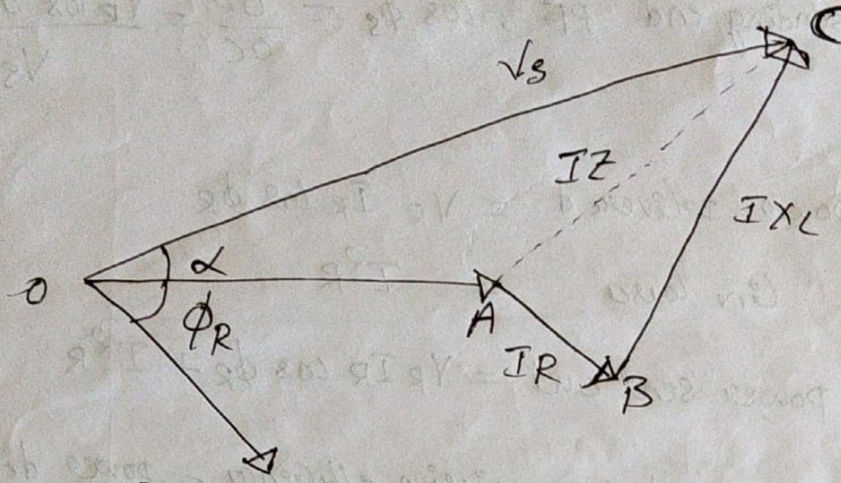
An approximate expression for V_s



$$OF = OC = OA + AG + GF$$

$$V_s = V_R + IR \cos \phi_R + IX_L \sin \phi_R$$

solution in complex notation



$$\vec{V}_R = V_R + j0$$

$$\vec{I} = I \angle -\theta_R$$

$$\vec{Z} = R + jX_L$$

$$\vec{V}_s = \vec{V}_R + \vec{I}\vec{Z} \quad \text{--- (1)}$$

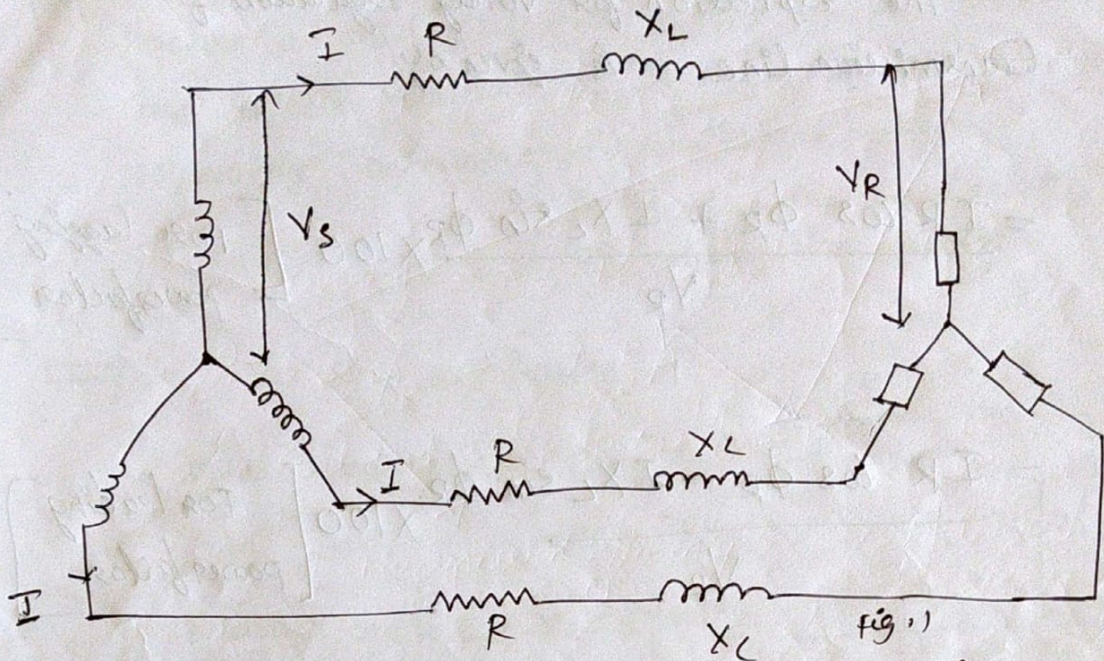
$$= (V_R + j0) + I(\cos \phi_R - j \sin \phi_R)(R + jX_L)$$

$$V_s = V_R + I R \cos \phi_R + I X_L \sin \phi_R + j(I X_L \cos \phi_R - I R \sin \phi_R)$$

$$V_s = \sqrt{(V_R + I R \cos \phi_R + I X_L \sin \phi_R)^2 + (I X_L \cos \phi_R - I R \sin \phi_R)^2}$$

$$V_s = V_R + I R \cos \phi_R + I X_L \sin \phi_R$$

3 ϕ short Transmission line



The Expression for regulation, η etc are derived for 3 ϕ system by drawing equivalent circuit diagram/phase. Figure.1 shows 3 ϕ systems with star connected load.

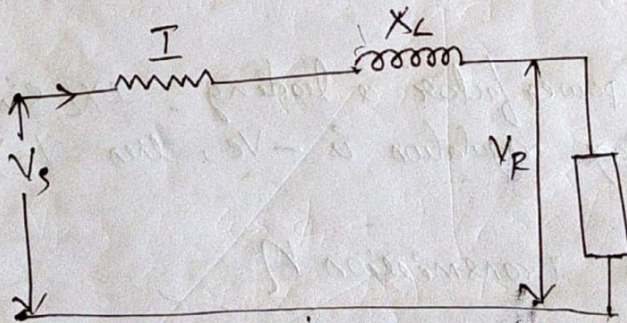


Fig. 2 shows equivalent circuit diagram/phase. Thus V_R, V_s are phase voltages, R and X_L are R and X_L are resistance and inductive reactance/phase

Regulation :

The expression for voltage regulation for short transmission line is given by

$$= \frac{I R \cos \phi_R + I X_L \sin \phi_R}{V_R} \times 100 \quad \left[\begin{array}{l} \text{For lagging} \\ \text{power factor} \end{array} \right]$$

$$= \frac{I R \cos \phi_R - I X_L \sin \phi_R}{V_R} \times 100 \quad \left[\begin{array}{l} \text{For leading} \\ \text{power factor} \end{array} \right]$$

Note :

* If load p.f. is lagging or unity then voltage regulation is +ve ~~or +ve~~ i.e., $V_S > V_R$

* If load power factor is leading, $I X_L \sin \phi_R > I R \cos \phi_R$ then voltage regulation is -ve, then $V_R > V_S$.

Effect on Transmission η

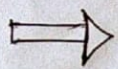
$$P = V_R I \cos \phi_R$$

$$I = \frac{P}{V_R \cos \phi_R}$$

$$\text{For } 3\phi, \quad P = 3 V_R I \cos \phi_R$$

(5)

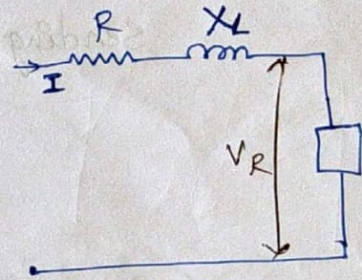
- ① A single phase overhead transmission line delivers 1100 kW at 33 kV at 0.8 pf lagging. The total resistance and inductive reactance of line are 10 Ω and 15 Ω respectively. Determine (i) sending end voltage (ii) sending end power factor (iii) transmission efficiency



$\cos \phi_R = 0.8$ lagging

total impedance $\bar{Z} = R + jX_L$
 $= 10 + j15$

Receiving end voltage $V_R = 33 \text{ kV}$



line current $I = \frac{P}{V_R \cos \phi_R}$

$= \frac{1100 \times 10^3}{33 \times 10^3 \times 0.8} = 41.67 \text{ A}$

$\cos \phi_R = 0.8$, $\sin \phi_R = 0.6$

$\bar{V}_R = V_R + j0$

$\bar{V}_R = 33000 \text{ Volt}$

$\bar{I} = I (\cos \phi_R - j \sin \phi_R)$

$= 41.67 (0.8 - j0.6)$

$\bar{I} = 33.33 - j25$

(i) sending end voltage $\bar{V}_S = \bar{V}_R + \bar{I} \bar{Z}$
 $= 33000 + (33.33 - j25)(10 + j15)$
 $= 33708.3 + j250$

$$\text{magnitude of } V_S = \sqrt{(33708.3)^2 + (250)^2} \quad (1)$$

$$= 33709 \text{ V}$$

(ii) Angle b/w \bar{V}_S and \bar{V}_R

$$\alpha = \tan^{-1} \frac{250}{33708.3} = 0.42^\circ$$

Sending end power factor angle is

$$\phi_S = \phi_R + \alpha$$

$$= 36.87^\circ + 0.42^\circ$$

$$= 37.29^\circ$$

$$\cos \phi_S = \cos 37.29^\circ = 0.795 \text{ lagging}$$

$$(iii) \text{ Line losses} = I^2 R = (41.67)^2 \times 10$$

$$= 17364 \text{ watts} = 17.36 \text{ kW}$$

$$\text{output delivered} = 1100 \text{ kW}$$

$$\text{power sent} = 1100 + 17.36$$

$$= 1117.36 \text{ kW}$$

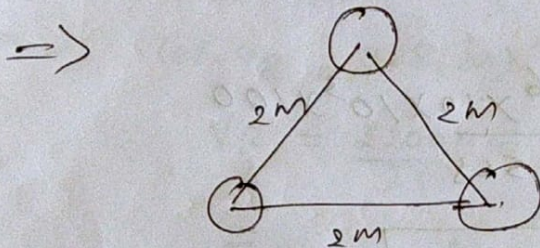
$$\eta = \frac{\text{power delivered}}{\text{power sent}} \times 100$$

$$= \frac{1100}{1117.36} \times 100$$

$$= 98.44 \%$$

(6)

(2) A 3 ϕ line 10km long delivers 5 MW at 11 kV, 50 Hz, 0.8 p.f lagging, the power loss in line is 10% of power delivered, the line conductors are situated at the corners of an equilateral Δ of 2 m side. Take resistivity of conductor material $1.77 \mu\Omega/\text{cm}$. Find V_s and $\cos \phi_s$



$$V_{R/\text{phase}} = \frac{11 \text{ kV}}{\sqrt{3}} = 6350.85 \text{ V}$$

$$I_{\text{ph}} = \frac{P}{3 \times V_{\text{ph}} \times \cos \phi_R}$$

$$= \frac{5 \times 10^6}{3 \times 6350.85 \times 0.8}$$

$$\boxed{I_{\text{ph}} = 328.04 \text{ A}}$$

Let R be the resistance of each conductor

since power loss in line = $3I^2R$

power loss = 10% of power delivered.

$$3I_{\text{ph}}^2 R_{\text{ph}} = \frac{10}{100} \times 5 \times 10^6$$

$$R_{\text{ph}} = \frac{10 \times 5 \times 10^6}{100 \times 3 \times (328.04)^2}$$

$$\boxed{R_{\text{ph}} = 1.548 \Omega}$$

To calculate X_L :

Let r = radius of each conductor.

$$R = \frac{\rho l}{A}$$

$$A = \frac{\rho l}{R}$$

$$= \frac{1.77 \times 10^{-6} \times 10 \times 10^3 \times 100}{1.548}$$

$$A = 1.14729 \text{ cm}^2$$

$$r = \sqrt{\frac{a}{\pi}}$$

$$r = 0.604 \text{ cm}$$

Inductance $L = 2 \times 10^{-7} \log_e \frac{D}{r}$

$$L = 12.1 \times 10^{-3} \text{ H}$$

$$X_L / \text{ph} = 2\pi f L$$

$$= 2\pi \times 50 \times 12.1 \times 10^{-3}$$
$$= 3.801$$

$$V_S = \sqrt{(V_R \cos \phi_R + IR)^2 + (V_R \sin \phi_R + IX_L)^2}$$

$$= 7536 \text{ V}$$

$$V_L = \frac{7536}{\sqrt{3}} = 4328.04 \text{ V}$$

$$\cos \phi_S = \frac{V_R \cos \phi_R + IR}{V_S} = \frac{6350.85 \times 0.8 + (328.04 \times 1.948)}{7536} = 0.741$$

(7)

- ③ An overhead 3 ϕ transmission line delivers 5000 kW at 22 kV at 0.8 pf lagging. The resistance and reactance of each conductor is 4 Ω and 6 Ω . Determine (i) sending end voltage
(ii) % regulation
(iii) transmission efficiency

$$\Rightarrow \cos \phi_R = 0.8 \text{ lag}$$

$$V_R = \frac{22000}{\sqrt{3}} = 12700 \text{ V}$$

$$\bar{Z} = 4 + j6$$

$$I = \frac{5000 \times 10^3}{3 \times 12700 \times 0.8} = 164 \text{ A}$$

$$\text{As } \cos \phi_R = 0.8 \quad \sin \phi_R = 0.6$$

$$\vec{V}_R = V_R + j0 = 12700 \text{ V}$$

$$\bar{I} = I (\cos \phi_R - j \sin \phi_R)$$

$$= 164 (0.8 - j0.6) = 131.2 - j98.4$$

(i) sending end voltage / phase \bar{V}_S

$$\bar{V}_S = \bar{V}_R + \bar{I} \bar{Z} = 12700 + (131.2 - j98.4)(4 + j6)$$

$$= 13815.2 + j393.6$$

$$\text{magnitude of } V_S = \sqrt{(13815.2)^2 + (393.6)^2}$$

$$= 13820.8 \text{ V}$$

$$\text{line } V_s = \sqrt{3} \times 13820.8 = 23938 \text{ V}$$

$$= 23.938 \text{ kV}$$

$$(ii) \% \text{ Regulation} = \frac{V_s - V_R}{V_R} \times 100$$

$$= 8.825 \%$$

$$(iii) \text{ losses} = 3 I^2 R = 3 (164)^2 \times 4$$

$$= 322.752 \text{ kW}$$

$$\eta = \frac{5000}{5000 + 322.752} \times 100$$

$$= 93.94 \%$$

4) estimate the distance over which a load of 15000 kW at a pf 0.8 lagging can be delivered by a 3 ϕ Transmission line having conductors each of resistance 1 Ω /km. The voltage at the receiving end is to be 132 kV and loss in the transmission is to be 5%

$$\Rightarrow \text{Line current } I = \frac{P}{\sqrt{3} V \cos \phi}$$

$$= \frac{15000 \times 10^3}{\sqrt{3} \times 132 \times 10^3 \times 0.8} = 82 \text{ A}$$

Line loss = 5% of power delivered

$$7500 \times 10^3 = 0.05 \times 15000 \times 10^3 = 750 \text{ kW}$$

$$\text{Line losses} = 3 I^2 R$$

$$750 \times 10^3 = 3 \times (82)^2 \times R$$

$$R = 37.18 \Omega$$

length of line = 37.18 km.

5) In 11 kV, 3 ϕ transmission line has a resistance of 1.5 Ω and reactance of 4 Ω /phase. Calculate the percentage regulation and efficiency of the line when a total load of 5000 kW at 0.8 lagging power factor is supplied at 11 kV at the distant end

$$\Rightarrow R = 1.5 \Omega$$

$$X_L = 4 \Omega$$

$$V_R = \frac{11 \times 10^3}{\sqrt{3}} = 6351 \text{ V}$$

$$\text{load current } I = \frac{\text{power delivered in KVA} \times 1000}{3 \times V_R}$$

$$= \frac{5000 \times 1000}{3 \times 6351}$$

$$= 262.43 \text{ A}$$

$$V_S = V_R + I R \cos \phi_R + I X_C \sin \phi_R$$

$$= 7295.8 \text{ V}$$

$$\% \text{ Reg} = \frac{V_S - V_R}{V_R} \times 100 = \frac{7295.8 - 6351}{6351} \times 100 = 14.88\%$$

$$\text{line loss} = 3 I^2 R = 3 (262.43)^2 \times 1.5$$

$$= 310 \text{ kW}$$

$$\text{o/p power} = 5000 \times 0.8 = 4000 \text{ kW}$$

$$\text{I/p power} = \text{o/p} + \text{losses} = 4000 + 310 = 4310 \text{ kW}$$

$$\eta = \frac{\text{o/p}}{\text{I/p}} \times 100 = \frac{4000}{4310} \times 100 = 92.8\%$$

6] A 3φ 50Hz, 16km long overhead line supplies 1000kW at 11kV 0.8 pf lagging. The line resistance is 0.03Ω per phase per km and line conductor inductance is 0.7mH per phase per km. Calculate sending end voltage; voltage regulation & η of transmission

⇒ R = 0.03 × 16 = 0.48Ω

X_L = 2πfL = 2π × 50 × 0.7 × 10⁻³ × 16 = 3.52Ω

V_p = $\frac{11 \times 10^3}{\sqrt{3}}$ = 6351V

load power factor cos φ_p = 0.8 lagging

Line current I = $\frac{1000 \times 10^3}{3 \times V_p \times \cos \phi} = \frac{1000 \times 10^3}{3 \times 6351 \times 0.8}$

I = 65.6A

V_s = V_p + IR cos φ_p + I X_L sin φ_p

V_s = 6515V

∴ Reg = $\frac{V_s - V_p}{V_p} \times 100 = \frac{6515 - 6351}{6351} \times 100 = 2.58\%$

Line losses = 3I²R = 3(65.6)² × 0.48 = 6.2kW

I/p power = o/p power + losses = 1000 + 6.2 = 1006.2kW

η = $\frac{o/p}{I/p} \times 100 = \frac{1000}{1006.2} \times 100 = 99.38\%$

Medium Transmission Lines

(10)

As the length and voltage of the line increases, the capacitance becomes equally important. Since medium transmission lines have sufficient length (50-150 km) and usually operate at voltages greater than 20 kV, effects of capacitance can't be neglected.

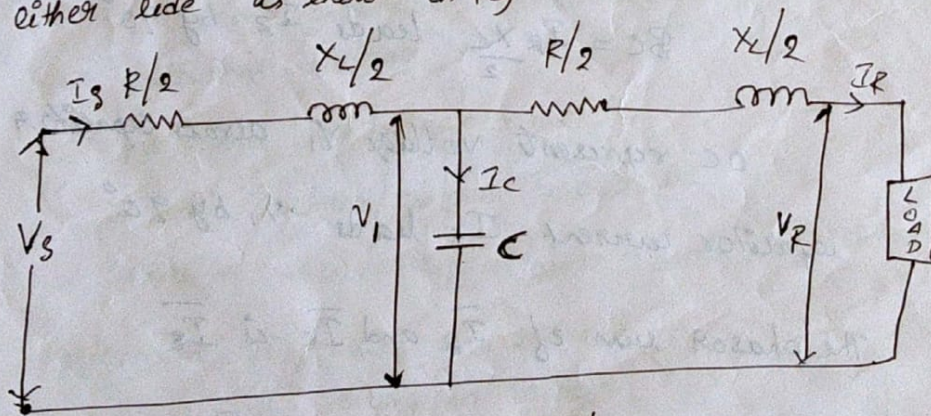
∴ In order to obtain reasonable accuracy in medium transmission line calculations, line capacitance must be taken into consideration.

The most commonly used methods are

- (i) end condenser method
- (ii) Nominal T method
- (iii) nominal π method.

• Nominal T method:

In this method, the whole line capacitance is assumed to be concentrated at the middle point of the line and half the resistance & reactance are lumped on it either side as shown in fig



let I_R = load current per phase

X_L = Inductive reactance per phase

$\cos \phi_R$ = Receiving end power factor

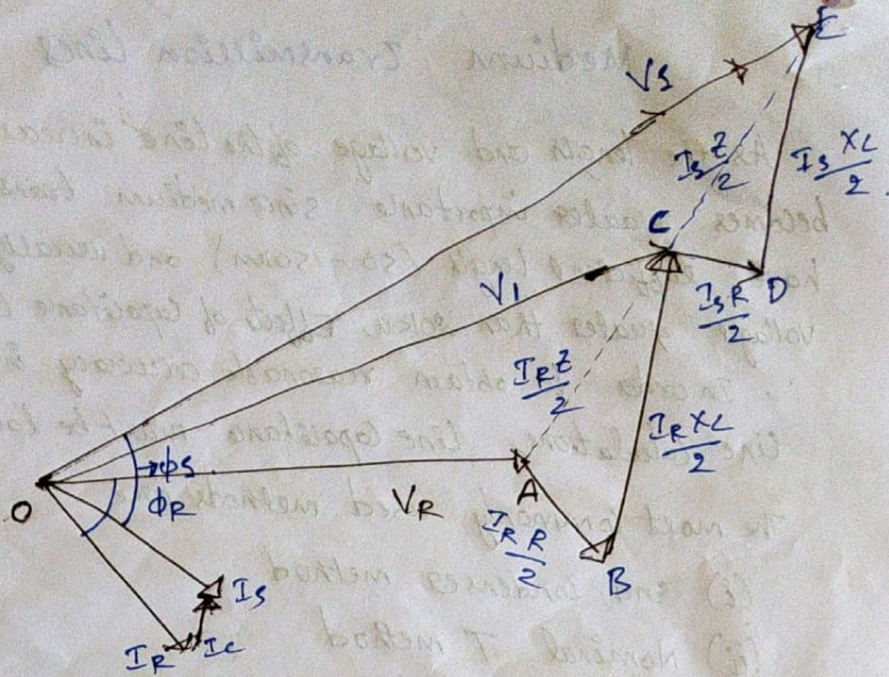
V_1 = voltage across capacitor C

R = resistance per phase

C = capacitance per phase

V_S = sending end voltage

V_R = receiving end voltage



$\vec{V}_R = I_r$ is taken as reference

The load current I_r lags V_r by angle ϕ_R
 I_s lags V_s by angle ϕ_S

The drop $AB = \frac{I_r R}{2}$ is in phase with I_r

$BC = \frac{I_r X_L}{2}$ leads I_r by 90°

OC represents voltage V_1 across capacitor
 capacitor current I_c leads V_1 by 90°

The phasor sum of \vec{I}_r and \vec{I}_c is \vec{I}_s

$CD = \frac{I_s R}{2}$ in phase with I_s

$DE = \frac{I_s X_L}{2}$ leads I_s by 90°

$OE =$ represents sending end voltage

Receiving end voltage $\bar{V}_R = V_R + j0$

load current $\bar{I}_R = I_R (\cos \phi_R - j \sin \phi_R)$

voltage across C $\bar{V}_1 = \bar{V}_R + \bar{I}_R \frac{Z}{2}$

$$\bar{V}_1 = V_R + I_R (\cos \phi_R - j \sin \phi_R) \left(\frac{R}{2} + j \frac{X_L}{2} \right)$$

capacitive current $\bar{I}_C = j \omega C \bar{V}_1$

$$= j 2\pi f C \bar{V}_1$$

sending end current $\bar{I}_S = \bar{I}_R + \bar{I}_C$

sending end voltage $\bar{V}_S = \bar{V}_1 + \bar{I}_S \frac{Z}{2}$

$$\bar{V}_S = \bar{V}_1 + \bar{I}_S \left(\frac{R}{2} + j \frac{X_L}{2} \right)$$

problems

① A 3φ 50Hz overhead transmission line 100km long has following constants

resistance / km / phase = 0.1 Ω

inductive reactance / km / phase = 0.2 Ω

capacitive susceptance / km / phase = 0.04 × 10⁻⁴ siemen

determine (i) sending end current

(ii) sending end voltage

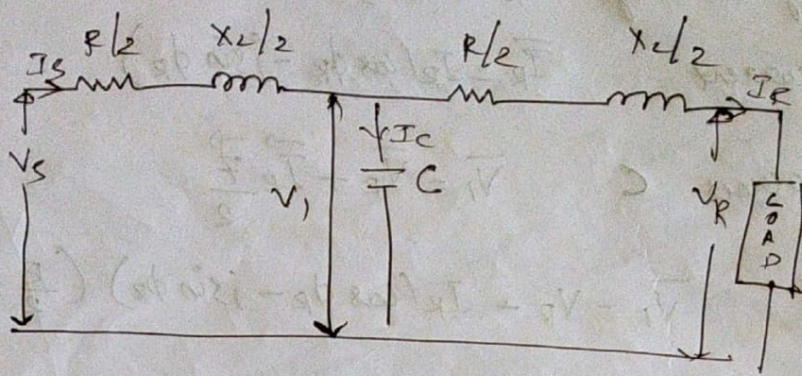
(iii) sending end power factor

(iv) Transmission Efficiency when supplying

a balanced load of 10000kW at 66kV pf 0.8 lagging

use nominal T method





total resistance / phase $R = 0.1 \times 100 = 10 \Omega$
 total reactance / phase $X_L = 0.2 \times 100 = 20 \Omega$
 capacitive susceptance $Y = 0.04 \times 10^{-4} \times 100$
 $= 4 \times 10^{-4}$ Siemens

Receiving end voltage $V_R = \frac{66 \text{ k}}{\sqrt{3}} = 38105 \text{ V}$

load current $= I_R = \frac{10000 \times 10^3}{\sqrt{3} \times 66 \times 10^3 \times 0.8} = 109 \text{ A}$

$\cos \phi_R = 0.8$ $\sin \phi_R = 0.6$

Impedance / phase $\bar{Z} = R + jX_L = 10 + j20 \Omega$

receiving end voltage $\bar{V}_R = V_R + j0 = 38105 \text{ V}$

load current $\bar{I}_R = I_R (\cos \phi_R - j \sin \phi_R)$

$= 109 (0.8 - j0.6)$

$= 87.2 - j65.4$

voltage across C $\bar{V}_1 = \bar{V}_R + \bar{I}_R \cdot \bar{Z}$

$= 38105 + (87.2 - j65.4)(5 + j10)$

$= 39195 + j545$

charging current $\bar{I}_C = jY\bar{V}_1$

$$= j 4 \times 10^{-4} (39195 + j545)$$

$$= -0.218 + j15.06$$

sending end current $\bar{I}_S = \bar{I}_R + \bar{I}_C$

$$= (87.2 - j65.4) + (-0.218 + j15.06)$$

$$= 87 - j49.8 = 100 \angle -29^{\circ}47' \text{ A}$$

sending end current = 100 A

(ii) sending end voltage

$$\bar{V}_S = \bar{V}_1 + \bar{I}_S \bar{Z}_2$$

$$= (39195 + j545) + (87.0 - j49.8)(5 + j0)$$

$$= 40128 + j1170$$

$$= 40145 \angle 1^{\circ}40' \text{ V}$$

line value of sending end voltage

$$= 40145 \sqrt{3}$$

$$= 69533 \text{ V}$$

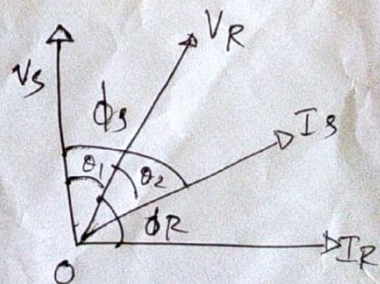
$$= 69.53 \text{ kV}$$

(iii) $\theta_1 = \text{angle b/w } \bar{V}_R \text{ and } \bar{V}_S = 1^{\circ}40'$

$\theta_2 = \text{angle b/w } \bar{V}_R \text{ and } \bar{I}_S = 29^{\circ}47'$

$\theta_3 = \text{angle b/w } \bar{V}_S \text{ and } \bar{I}_S$

$$= \theta_1 + \theta_2 = 31^{\circ}27'$$

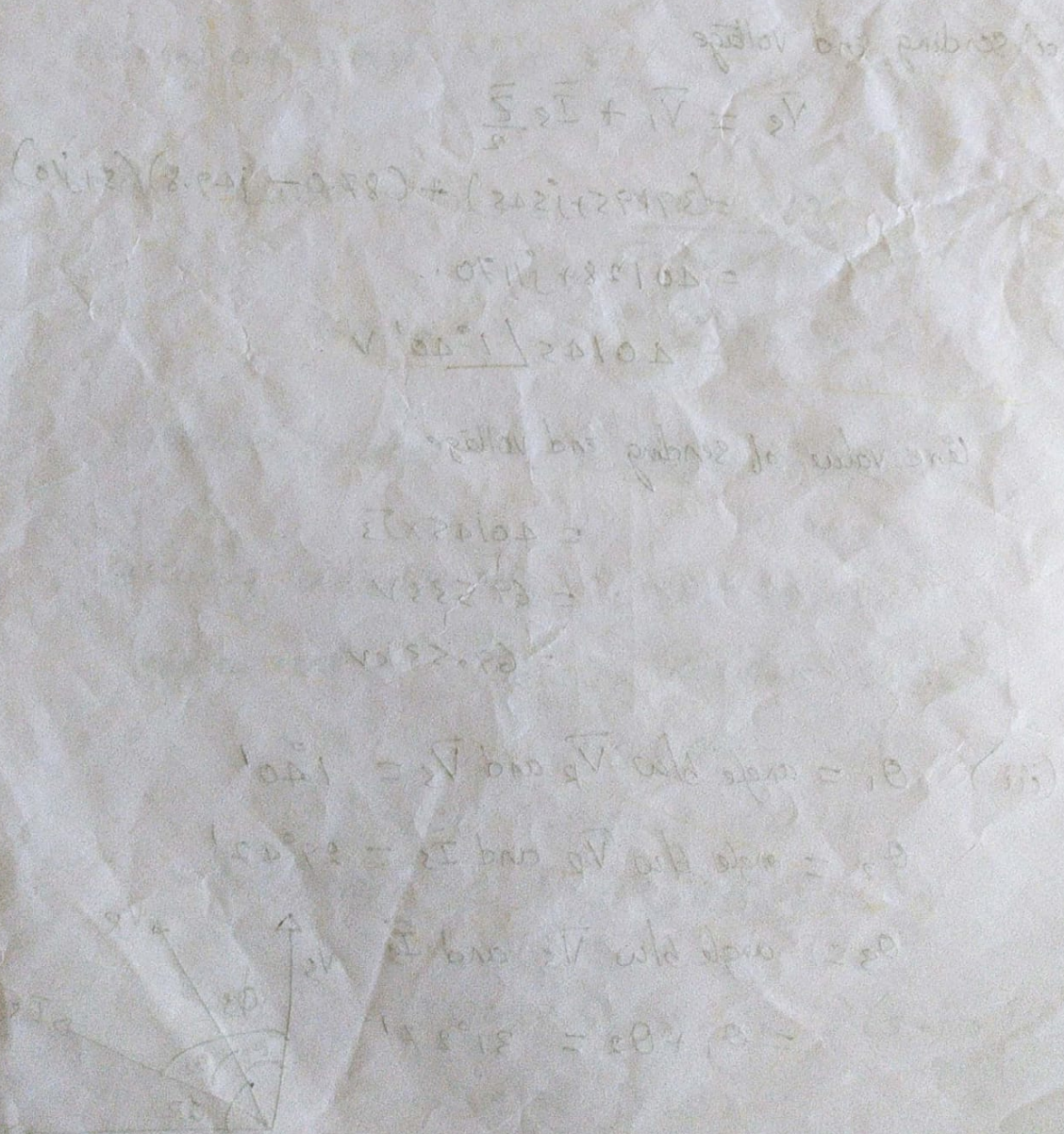


sending end power factor $\cos \phi_s = \cos 31.27'$
 $= 0.853 \text{ lag}$

(iv) sending end power = $3 \sqrt{3} I_s \cos \phi_s$
 $= 3 \times 40145 \times 100 \times 0.853$
 $= 10273.10 \text{ kW}$

power delivered = 10000 kW

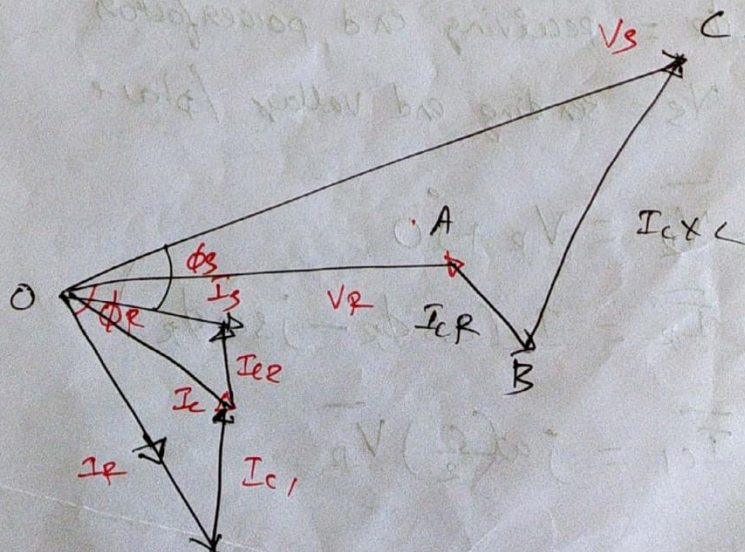
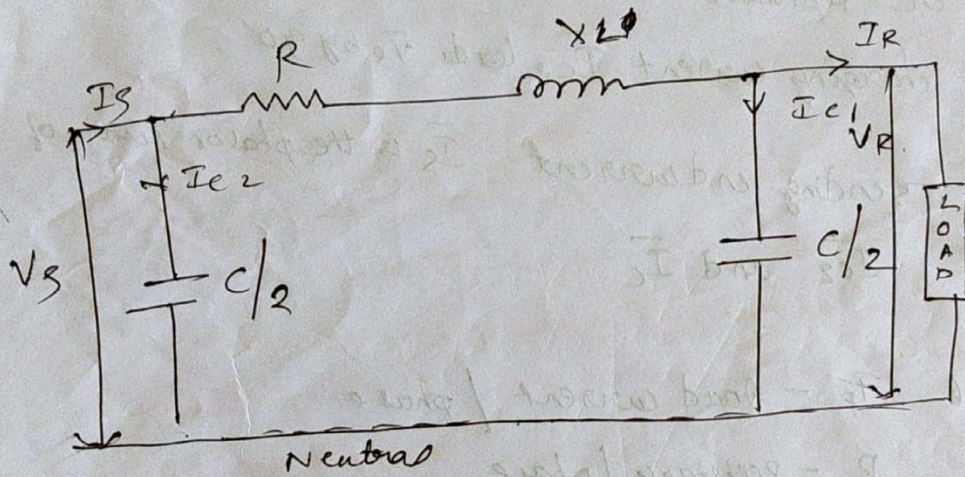
$\eta = \frac{10000}{10273.105} \times 100 = 97.34\%$



Nominal T method:

In this method, capacitance of each conductor is divided into two halves; one half being lumped at the sending end and other half is at receiving end.

It is obvious that capacitance at sending end has no effect on line drop. However charging current must be added to line current in order to obtain the total sending end current



V_R = Reference represented by OA

\bar{I}_R = lags \bar{V}_R by ϕ_R

\bar{I}_{C1} leads \bar{V}_R by 90°

Line current \bar{I}_L phasor sum of \bar{I}_R and \bar{I}_{C1}

The drop $AB = I_L R$ is in phase with \bar{I}_L

$BC = I_L X_L$ leads \bar{I}_L by 90°

OC represents the sending end voltage \bar{V}_S

charging current \bar{I}_{C2} leads \bar{V}_S by 90°

\therefore sending end current \bar{I}_S is the phasor sum of

\bar{I}_{C2} and \bar{I}_L

Let I_R = load current / phase

R = Resistance / phase

X_L = Inductive reactance / phase

C = capacitance / phase

$\cos \phi_R$ = receiving end power factor

V_S = sending end voltage / phase

$$\bar{V}_R = V_R + j0$$

$$\bar{I}_R = I_R (\cos \phi_R - j \sin \phi_R)$$

$$\bar{I}_{C1} = j \omega \left(\frac{C}{2} \right) \bar{V}_R$$

$$\bar{I}_{C1} = j\pi f C \bar{V}_R$$

$$\bar{I}_L = \bar{I}_R + \bar{I}_{C1}$$

$$\bar{V}_S = \bar{V}_R + \bar{I}_L \bar{Z} = \bar{V}_R + \bar{I}_L (R + jX_L)$$

$$\bar{I}_{C2} = j\omega C_2 \bar{V}_S$$

$$= j\pi f C \bar{V}_S$$

$$\bar{I}_S = \bar{I}_L + \bar{I}_{C2}$$

problem

- * A 3 phase 50 Hz, 150 km line has a resistance, inductive reactance & capacitive shunt admittance of 0.1Ω , 0.5Ω and $3 \times 10^{-6} \text{ S/km/phase}$ If the line delivers 50 MW at 110 kV and 0.8 pf lagging, determine the sending end voltage and current. Assume Nominal π method

$$\Rightarrow R = 0.1 \times 150 = 15 \Omega$$

$$X_L = 0.5 \times 150 = 75 \Omega$$

$$Y = 3 \times 10^{-6} \times 150 = 45 \times 10^{-5} \text{ S}$$

$$V_R = \frac{110 \times 10^3}{\sqrt{3}} = 63508 \text{ V}$$

$$I_R = \frac{50 \times 10^6}{\sqrt{3} \times 110 \times 10^3 \times 0.8} = 328 \text{ A}$$

$$\cos \phi_R = 0.8 \quad \sin \phi_R = 0.6$$

$$\bar{V}_R = V_R + j0 = 63508 \text{ V}$$

$$\begin{aligned} \bar{I}_R &= I_R (\cos \phi_R - j \sin \phi_R) = 328 (0.8 - j0.6) \\ &= 262.4 - j196.8 \end{aligned}$$

charging current at load end

$$\begin{aligned} \bar{I}_{C1} &= \bar{V}_R j \frac{Y}{2} = 63508 \times j \frac{45 \times 10^{-5}}{2} \\ &= j14.3 \end{aligned}$$

$$\begin{aligned} \bar{I}_L &= \bar{I}_R + \bar{I}_{C1} \\ &= (262.4 - j196.8) + (j14.3) \\ &= 262.4 - j182.5 \end{aligned}$$

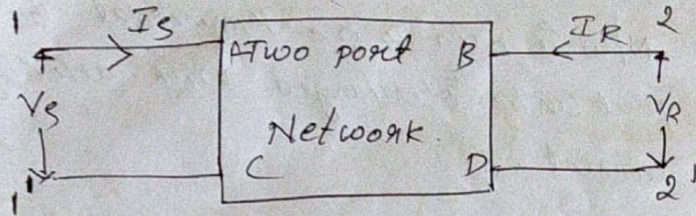
$$\begin{aligned} \bar{V}_S &= \bar{V}_R + \bar{I}_L \bar{Z} \\ &= 81131 + j16922.5 \\ &= 82881 \angle 11^\circ 47' \text{ V} \end{aligned}$$

$$\begin{aligned} \text{line to line sending end voltage} &= 82881 \times \sqrt{3} \\ &= 143.55 \text{ kV} \end{aligned}$$

$$I_{C2} = j \bar{V}_S \frac{Y}{2} = -3.81 + j18.25$$

$$\begin{aligned} \text{sending end current } \bar{I}_S &= \bar{I}_L + \bar{I}_{C2} \\ &= (262.4 - j182.5) + (-3.81 + j18.25) \\ &= 258.6 - j164.25 \\ \bar{I}_S &= 306.4 \angle -32.4^\circ \text{ A} \end{aligned}$$

Generalized circuit constants of transmission line



Transmission line can be assumed by a 2 port N/w with port terminals i.e., 2 input terminals where power enters the N/w and 2 output terminals where power leaves the N/w.

\therefore Sending end voltage (V_S) and sending end current (I_S) of a line can be expressed as

$$V_S = A V_R + B I_R$$

$$I_S = C V_R + D I_R$$

where V_S = sending end voltage

I_S = sending end current

V_R = Receiving end voltage

I_R = Receiving end current

and A, B, C, D are constants known as generalized circuit constants of the transmission line.

values of these constants depends on the method adopted for solving a transmission line. once the values of these constants are known, performance can be easily find out.

(i) The constants A, B, C and D are generally complex numbers.

(ii) The constants A and D are dimensionless where as B and C are ohms and siemen respectively.

(iii) If the Network is symmetrical, then $A=D$

(iv) If N/w is reciprocal then $AD-BC=1$

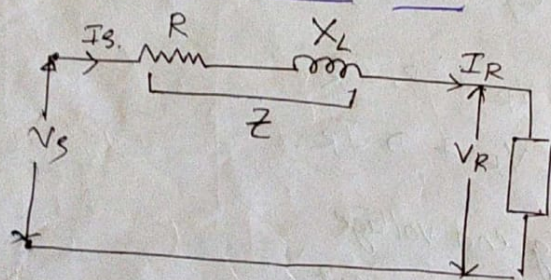
A 2 port N/w is said to be symmetrical if I/p and o/p port can be interchanged with altering port voltage and current.

Determination of Generalized constants of transmission line

$$\vec{V}_S = A \vec{V}_R + B \vec{I}_R$$

$$\vec{I}_S = C \vec{V}_R + D \vec{I}_R$$

(i) short transmission line:



In short transmission line, the effect of line capacitance is neglected. The line is having series impedance. Figure shows the circuit of a 3 ϕ transmission line on a single phase basis.

$$\vec{I}_S = \vec{I}_R \quad \text{--- (1)}$$

$$\vec{V}_S = \vec{V}_R + \vec{I}_R \vec{Z} \quad \text{--- (2)}$$

The defined equations are:

$$\vec{V}_S = A \vec{V}_R + B \vec{I}_R \quad \text{--- (3)}$$

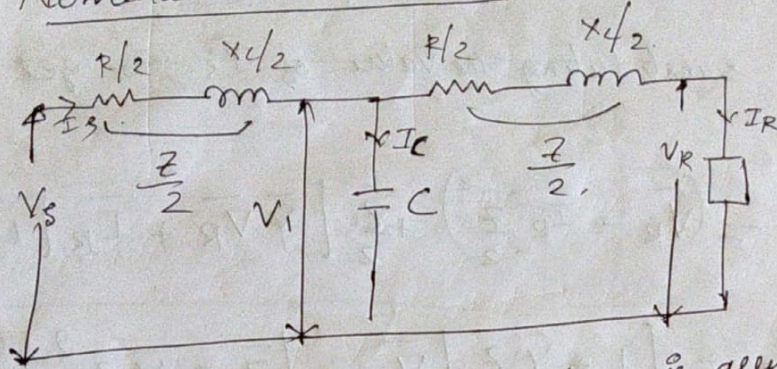
$$\vec{I}_S = C \vec{V}_R + D \vec{I}_R \quad \text{--- (4)}$$

$$A=1, B=\vec{Z}, C=0, D=1$$

we have got $A=D=1$, the network is similar to symmetrical since $AD-BC=1$ the network to be reciprocal also

(ii) medium line :

(*) Nominal T method :



In this method, the total capacitance is assumed to be lumped or concentrated at the centre point of a transmission line.

$$\text{Here } \bar{V}_s = \bar{V}_1 + \bar{I}_s \frac{\bar{Z}}{2} \quad \rightarrow (1)$$

$$\bar{V}_1 = \bar{V}_r + \bar{I}_r \frac{\bar{Z}}{2} \quad \rightarrow (2)$$

$$\bar{I}_s = \bar{I}_r + \bar{I}_c$$

$$\bar{I}_c = \bar{V}_1 \bar{Y}$$

$$= \bar{Y} \left[\bar{V}_r + \frac{\bar{I}_r \bar{Z}}{2} \right]$$

$$\bar{I}_s = \bar{I}_r + \bar{Y} \left[\bar{V}_r + \frac{\bar{I}_r \bar{Z}}{2} \right]$$

$$= \bar{I}_r + \bar{Y} \bar{V}_r + \bar{Y} \frac{\bar{I}_r \bar{Z}}{2}$$

$$\bar{I}_s = \bar{Y} \bar{V}_r + \bar{I}_r \left[1 + \frac{\bar{Y} \bar{Z}}{2} \right] \quad \rightarrow (3)$$

Substituting the value of V_1 in eqn (1)

$$\bar{V}_S = \bar{V}_R + \frac{\bar{I}_R \bar{Z}}{2} + \frac{\bar{I}_S \bar{Z}}{2} \quad (*)$$

Substituting the value of \bar{I}_S ; we get

$$\bar{V}_S = \left(\bar{V}_R + \bar{I}_R \frac{\bar{Z}}{2} \right) + \frac{\bar{Z}}{2} \left[\bar{Y} \bar{V}_R + \bar{I}_R \left[1 + \frac{\bar{Y} \bar{Z}}{2} \right] \right]$$

$$\bar{V}_S = \left[1 + \frac{\bar{Y} \bar{Z}}{2} \right] \bar{V}_R + \left[\bar{Z} + \frac{\bar{Y} \bar{Z}^2}{4} \right] \bar{I}_R \rightarrow (4)$$

compare (3) and (4) with standard equation

$$A = 1 + \frac{\bar{Z} \bar{Y}}{2}$$

$$B = \bar{Z} + \frac{\bar{Z}^2 \bar{Y}}{4}$$

$$C = \bar{Y}$$

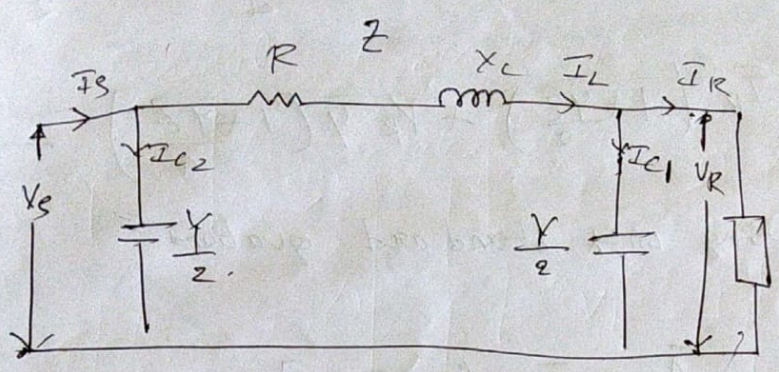
$$D = \frac{\bar{Y} \bar{Z}}{2} + 1$$

Network seems to be $\bar{A}\bar{D} - \bar{B}\bar{C} = 1$

$$= \left(1 + \frac{\bar{Y} \bar{Z}}{2} \right)^2 - \bar{Z} \left[1 + \frac{\bar{Y} \bar{Z}}{4} \right] \bar{Y}$$

$$= 1$$

(*) medium line - nominal π method:



In this method capacitance is divided into two halves, one half at receiving end and other half at sending end.

Here $\bar{Z} = R + jX_L = \text{series impedance/phase}$

$\bar{Y} = j\omega C = \text{shunt admittance}$

$$\bar{I}_S = \bar{I}_L + \bar{I}_{C2} \quad \text{--- (1)}$$

$$\bar{I}_L = \bar{I}_R + \bar{I}_{C1}$$

$$\bar{I}_{C1} = \frac{V_R \bar{Y}}{2} \quad \text{--- (2)}$$

Now $\bar{V}_S = \bar{V}_R + \bar{I}_L \bar{Z}$
 $\bar{V}_S = \bar{V}_R + \left[\bar{I}_R + \bar{V}_R \frac{\bar{Y}}{2} \right] \bar{Z}$ [substitute value of \bar{I}_L]

$$\bar{V}_S = \bar{V}_R \left[1 + \frac{\bar{Y} \bar{Z}}{2} \right] + \bar{I}_R \bar{Z} \quad \text{--- (3)}$$

$$\bar{I}_S = \bar{I}_L + \bar{V}_S \frac{\bar{Y}}{2}$$

$$\bar{I}_S = \left[\bar{I}_R + \bar{V}_R \frac{\bar{Y}}{2} \right] + \bar{V}_S \frac{\bar{Y}}{2}$$

substitute (3) in above equation

$$\bar{I}_S = \bar{I}_R + \bar{V}_R \frac{\bar{Y}}{2} + \frac{\bar{Y}}{2} \left\{ \bar{V}_R \left(1 + \frac{\bar{Y} \bar{Z}}{2} \right) + \bar{I}_R \bar{Z} \right\}$$

$$= \bar{I}_R + \bar{V}_R \frac{\bar{Y}}{2} + \frac{\bar{V}_R \bar{Y}}{2} + \bar{V}_R \frac{Y^2 \bar{Z}}{4} + \frac{\bar{Y} \bar{I}_R \bar{Z}}{2}$$

$$\bar{I}_S = \bar{I}_R \left(1 + \frac{\bar{Y} \bar{Z}}{2} \right) + \bar{V}_R \bar{Y} \left(1 + \frac{\bar{Y} \bar{Z}}{4} \right) \quad \text{--- (4)}$$

comparing with standard equations

$$\bar{A} = \bar{D} = \left[1 + \frac{\bar{Y} \bar{Z}}{2} \right]$$

$$\bar{B} = \bar{Z}$$

$$\bar{C} = \bar{Y} \left[1 + \frac{\bar{Y} \bar{Z}}{4} \right]$$

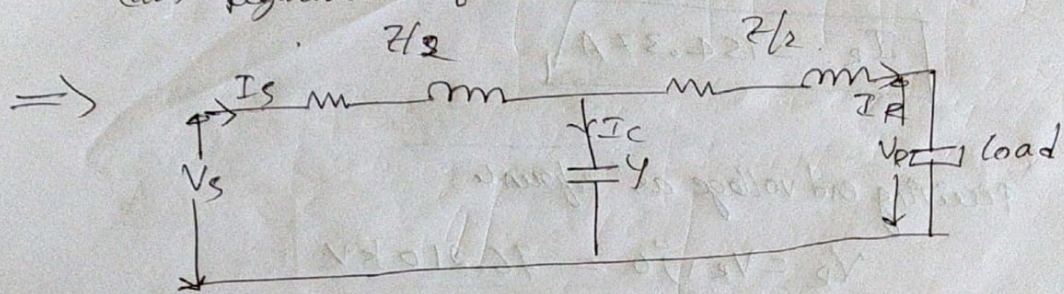
$$\bar{A} \bar{D} - \bar{B} \bar{C} = \left(1 + \frac{\bar{Y} \bar{Z}}{2} \right)^2 - \bar{Z} \bar{Y} \left[1 + \frac{\bar{Y} \bar{Z}}{4} \right]$$

$$= 1 + \frac{\bar{Y}^2 \bar{Z}^2}{4} + \bar{Y} \bar{Z} - \bar{Z} \bar{Y} + \frac{\bar{Y}^2 \bar{Z}^2}{4}$$

$$\bar{A} \bar{D} - \bar{B} \bar{C} = 1$$

① A Balanced 3 ϕ load of 30 MW is supplied at 132 kV 50 Hz and 0.85 pf lagging by means of transmission line. The series impedance of single conductor is $(20 + j52)\Omega$ and total phase neutral admittance is 315×10^{-6} Siemens using Nominal T method.

- (i) determine A, B, C and D constants of line
- (ii) Sending end voltage
- (iii) regulation of TL



$$Z = 20 + j52 \Omega$$

$$Y = j315 \times 10^{-6} \text{ S}$$

(i) constants

$$\bar{A} = \bar{D} = 1 + \frac{YZ}{2}$$

$$\bar{B} = Z \left(1 + \frac{ZY}{4} \right)$$

$$\bar{C} = Y$$

$$\bar{A} = \bar{D} = \left[1 + (20 + j52)(1.575 \times 10^{-9}) \right]$$

$$= 0.99 \angle 0.1819^\circ$$

$$= 0.9981 + j3.15 \times 10^{-3}$$

$$\bar{B} = 19.84 + j51.82$$

$$\bar{C} = \bar{Y} = 0.00315 \angle 90^\circ$$

(ii) sending end voltage

$$V_{p/\text{phase}} = \frac{132 \times 10^3}{\sqrt{3}} = 76.21 \text{ kV}$$

$$I_R = \frac{30 \times 10^6}{\sqrt{3} \times 132 \times 10^3 \times 0.85}$$

$$I_R = 154.37 \text{ A}$$

Receiving end voltage as reference.

$$\bar{V}_R = V_R + j0 = 76.210 \text{ kV}$$

$$\bar{I}_R = I_R (\cos \phi_R - j \sin \phi_R)$$
$$= 154.37 (0.85 - j0.53)$$

$$= 131.21 - 81.81j$$

$$= 131.21 - 81.81j$$

$$\bar{V}_S = \bar{A} \bar{V}_R + \bar{B} \bar{I}_R$$

$$= 82.427 + j5.416$$

$$\bar{V}_S = 82.605 \text{ kV}$$

$$\bar{V}_S \text{ line} = 82.6 \times \sqrt{3}$$

$$\bar{V}_S = 143 \text{ kV}$$

(iii) Regulation

$$V_s = A \bar{V}_R + B \bar{I}_R$$

At no load $\bar{I}_R = 0$

$$V_s = A \bar{V}_{R0}$$

$$\bar{V}_{R0} = \frac{V_s}{A}$$

$$\text{regulation} = \frac{V_s - V_R}{V_R} \times 100$$

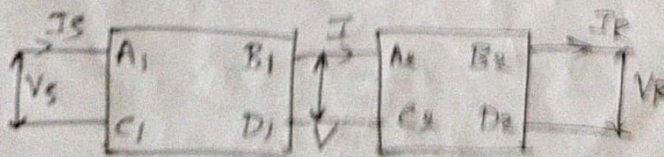
$$= \frac{82.6k - 76.21k}{76.21k} \times 100$$

$$\boxed{\%R = 9.25\%}$$

(2) Two transmission lines having generalized circuit constants A_1, B_1, C_1 and D_1 and A_2, B_2, C_2 and D_2 are connected (a) series (b) parallel

derive the expression for overall ABCD constant of the resulting network

⇒ series



$$V_s = A_1 V + B_1 I \longrightarrow (1)$$

$$I_s = C_1 V + D_1 I \longrightarrow (2)$$

$$V = A_2 V_R + B_2 I_R \longrightarrow (3)$$

$$I = C_2 V_R + D_2 I_R \longrightarrow (4)$$

substituting (3) and (4) in (1) and (2) respectively
values of I and V in (1) and (2)

$$V_s = A_1 [A_2 V_R + B_2 I_R] + B_1 [C_2 V_R + D_2 I_R]$$

$$V_s = A_1 A_2 V_R + A_1 B_2 I_R + B_1 C_2 V_R + B_1 D_2 I_R$$

$$V_s = (A_1 A_2 + B_1 C_2) V_R + (A_1 B_2 + B_1 D_2) I_R \longrightarrow (5)$$

$$V_s =$$

$$I_s = C_1 (A_2 V_R + B_2 I_R)$$

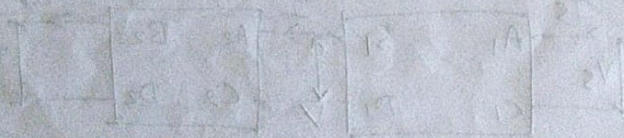
$$I_s = C_1 (A_2 V_R + B_2 I_R) + D_1 (C_2 V_R + D_2 I_R)$$

$$I_s = V_R (C_1 A_2 + C_2 D_1) + I_R (C_1 B_2 + D_1 D_2)$$

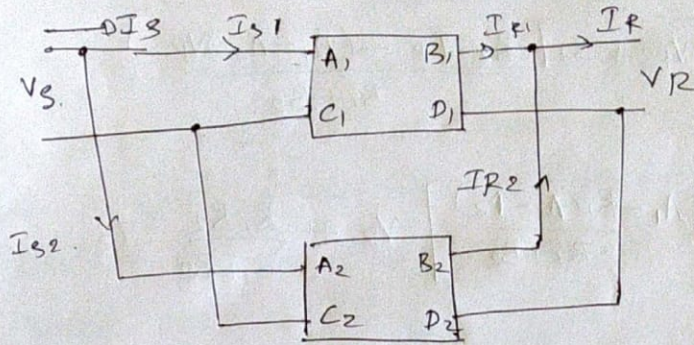
overall ABCD constants of respective network are

$$A = A_1 A_2 + B_1 C_2 \quad B = A_1 B_2 + B_1 D_2$$

$$C = C_1 A_2 + C_2 D_1 \quad D = C_1 B_2 + D_1 D_2$$



• parallel :



$$V_S = A_1 V_R + B_1 I_{R1} \quad \text{--- (1)}$$

$$V_S = A_2 V_R + B_2 I_{R2} \quad \text{--- (2)}$$

$$I_{S1} = C_1 V_R + D_1 I_{R1} \quad \text{--- (3)}$$

$$I_{S2} = C_2 V_R + D_2 I_{R2} \quad \text{--- (4)}$$

Equating (1) and (2) & (3) and (4)

$$A_1 V_R + B_1 I_{R1} = A_2 V_R + B_2 I_{R2}$$

$$V_R (A_1 - A_2) = B_2 I_{R2} - B_1 I_{R1}$$

$$I_R = I_{R1} + I_{R2}$$

$$I_{R2} = I_R - I_{R1}$$

$$V_R (A_1 - A_2) = B_2 (I_R - I_{R1}) - B_1 I_{R1}$$

$$V_R (A_1 - A_2) = B_2 I_R - I_{R1} (B_1 + B_2)$$

$$I_{R1} (B_1 + B_2) = B_2 I_R - V_R (A_1 - A_2)$$

$$I_{R1} = \frac{B_2 I_R - V_R (A_1 - A_2)}{B_1 + B_2}$$

$$V_S = A_1 V_R + B_1 I_R$$

$$= A_1 V_R + B_1 \left[\frac{B_2 I_R - (A_1 - A_2) V_R}{B_1 + B_2} \right]$$

$$= \left[A_1 - \frac{B_1 (A_1 - A_2)}{B_1 + B_2} \right] V_R + \frac{B_1 B_2}{B_1 + B_2} I_R$$

$$V_S = \left[\frac{A_1 B_1 + A_1 B_2 - A_1 B_1 + B_1 A_2}{B_1 + B_2} \right] V_R + \frac{B_1 B_2}{B_1 + B_2} I_R$$

$$V_S = \left[\frac{A_1 B_2 + A_2 B_1}{B_1 + B_2} \right] V_R + \frac{B_1 B_2}{B_1 + B_2} I_R$$

Now $I_{S1} + I_{S2} = (C_1 + C_2) V_R + D_1 I_{R1} + D_2 I_{R2}$

$$I_S = (C_1 + C_2) V_R + D_1 I_{R1} + D_2 (I_R - I_{R1})$$

$$= (C_1 + C_2) V_R + D_2 I_R + (D_1 - D_2) I_{R1}$$

$$= (C_1 + C_2) V_R + D_2 I_R + (D_1 - D_2) \left[\frac{B_2 I_R - V_R (A_1 - A_2)}{B_1 + B_2} \right]$$

$$I_S = \left[C_1 + C_2 - \frac{(D_1 - D_2) (A_1 - A_2)}{B_1 + B_2} \right] V_R + \left[\frac{B_1 D_2 + D_1 B_2}{B_1 + B_2} \right] I_R$$

$$A = \frac{A_1 B_2 + A_2 B_1}{B_1 + B_2}$$

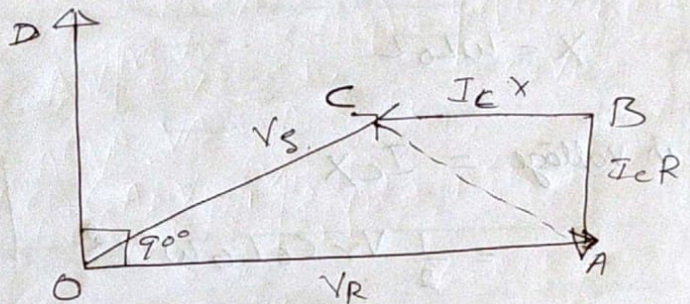
$$B = \frac{B_1 B_2}{B_1 + B_2}$$

$$C = C_1 + C_2 - \frac{(A_1 - A_2) (D_1 - D_2)}{B_1 + B_2}$$

$$D = \frac{B_1 D_2 + D_1 B_2}{B_1 + B_2}$$

• Ferranti Effect :

The phenomenon of rise in voltage at receiving end of line is called as Ferranti Effect. This effect is due to voltage drop across the line inductance due to charging current being in phase with applied voltage.



In long transmission line the effect of resistance is less compared to reactance, neglecting resistor drop ($I_C R$) we get rise in voltage.

$$\text{rise in voltage} = OA - OC.$$

$$= \text{Inductive drop}$$

$$= I_C X.$$

$$I_C = \frac{1}{\omega L C_0} \quad \text{Capacitance of TL / km}$$

$$L_0 = \text{Inductance of TL / km}$$

$$L = \text{length of line / km}$$

$$I_C = \frac{1}{\omega L C_0}$$

As the capacitance of line is uniformly distributed over the entire length of line.

∴ Average current flowing through the line is given by

$$I_c = \frac{1}{2} \frac{V_r}{X_c} \quad I_c \text{ leads } V_r \text{ by } 90^\circ$$

Inductive reactance of line

$$X = \omega L_0 L$$

$$\text{rise in voltage} = I_c X$$

$$= \frac{1}{2} V_r C_0 L_0 \omega^2 L^2$$

$$[X = \omega L L_0]$$

$$\text{rise in voltage at receiving end} = \frac{1}{\sqrt{L_0 C_0}}$$

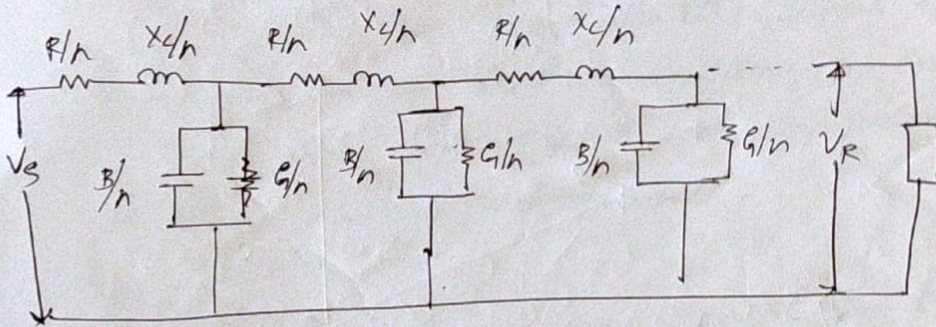
$$= \frac{1}{2} \omega^2 L^2 V_r \frac{1}{3 \times 10^5}$$

∴ excess of voltage at receiving end of a open circuited line proportional to square of length of line.

∴ For long HV and EHV lines, shunt reactors are provided to absorb the part of charging current of line in order to prevent over voltage on line

long transmission line

In order to obtain fair degree of accuracy in the performance calculation of line transmission line, the constants are considered as uniformly distributed throughout the length of line.



The whole length is divided into n sections, each section having line constants $\frac{1}{n}$ of those for whole line.

The following points are noted.

- (i) The line constants are uniformly distributed over the entire length of line as is actually the case.
- (ii) resistance and inductive reactance are the series elements
- (iii) leakage susceptance (B) and leakage conductance (G) are shunt elements. The leakage susceptance is due to the fact that capacitance exists b/w line and neutral. The leakage conductance takes into account the energy losses occurring through leakage over the insulator or due to corona effect b/w conductors

$$\text{Admittance} = \sqrt{G^2 + B^2}$$

- (iv) The leakage current through shunt admittance is maximum at the sending end of the line and decreases continuously as the receiving end of the circuit is approached at which point it becomes zero