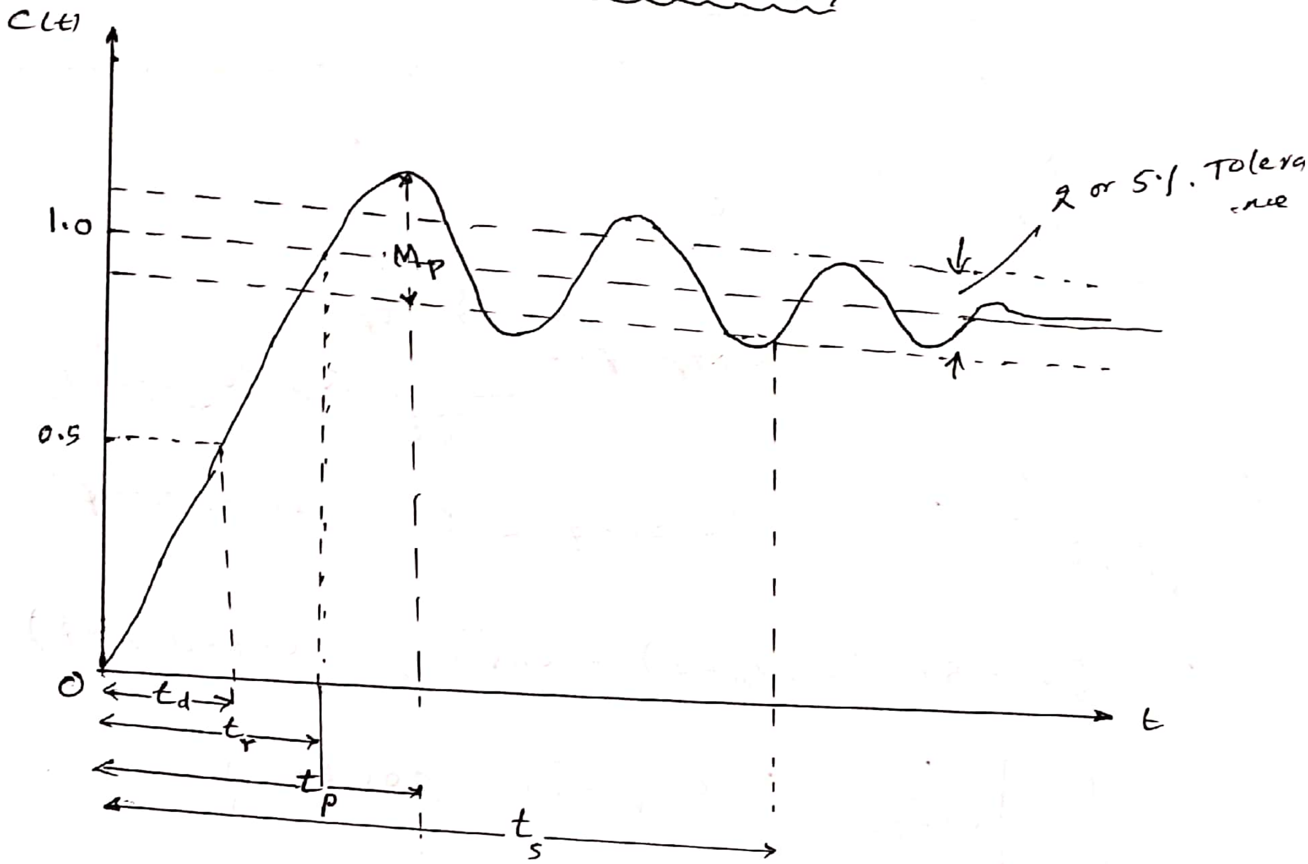
Trasient specification.



$$c(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \sin(\omega_d t + \phi) \quad (1)$$

1. Rise Time

At $t = t_r$ $\quad c(t) = 1.0$.

$$1.0 = 1 - \frac{e^{-\xi\omega_n t_r}}{\sqrt{1-\xi^2}} \sin(\omega_d t_r + \phi)$$

$$-\frac{e^{-\xi\omega_n t_r}}{\sqrt{1-\xi^2}} \sin(\omega_d t_r + \phi) = 0$$

$$\sin(\omega_d t_r + \phi) = 0$$

$$\sin(\omega_d t_r + \phi) = \sin\pi$$

$$\omega_d t_r + \phi = \pi$$

$$\boxed{t_r = \frac{\pi - \phi}{\omega_d}} \text{ sec.} \quad (2)$$

2. Peak Time (t_p)

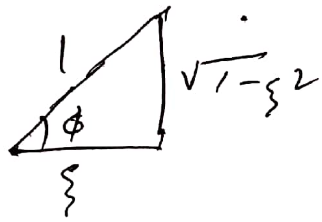
Differentiate E-1 w.r.t 't' & equate to zero

$$\frac{d\, c(t_p)}{dt} = 0$$

$$0 = \frac{\xi\omega_n}{\sqrt{1-\xi^2}} e^{-\xi\omega_n t_p} \sin(\omega_d t_p + \phi) - \frac{e^{-\xi\omega_n t_p}}{\sqrt{1-\xi^2}} \omega_d \cos(\omega_d t_p + \phi)$$

$$0 = \frac{e^{-\xi\omega_n t_p}}{\sqrt{1-\xi^2}} \left\{ \xi\omega_n \sin(\omega_d t_p + \phi) - \omega_d \cos(\omega_d t_p + \phi) \right\}$$

$$0 = \xi\omega_n \sin(\omega_d t_p + \phi) - \omega_n\sqrt{1-\xi^2} \cos(\omega_d t_p + \phi)$$



$\sin\phi = \sqrt{1-\xi^2}$ $\cos\phi = \xi$

$$0 = \sin(\omega_d t_p + \phi)\cos\phi - \sin\phi \cos(\omega_d t_p + \phi)$$

$$\sin(\omega_d t_p + \phi - \phi) = 0$$

$$\sin(\omega_d t_p) = \sin \pi$$

$$t_p = \frac{\pi}{\omega_d} = \frac{\pi}{\omega_n\sqrt{1-\xi^2}} \text{ sec} \quad (3)$$

3. Peak Overshoot (M_p)

$$M_p = c(t_p) - 1$$

$$= 1 - \frac{e^{-\xi \omega_n t_p}}{\sqrt{1-\xi^2}} \sin(\omega_d t_p + \phi) - 1$$

$$M_p = \frac{-e^{-\xi \omega_n \times \pi/\omega_d}}{\sqrt{1-\xi^2}} \sin\left(\cancel{\omega_d} \times \frac{\pi}{\cancel{\omega_d}} + \phi\right)$$

$$= \frac{-e^{-\xi \cancel{\omega_n} \times \frac{\pi}{\cancel{\omega_n}\sqrt{1-\xi^2}}}}{\sqrt{1-\xi^2}} \times -\sin\phi$$

$$= \frac{e^{-\frac{\xi \pi}{\sqrt{1-\xi^2}}}}{\cancel{\sqrt{1-\xi^2}}} \times \cancel{\sqrt{1-\xi^2}}$$

$$\% M_p = e^{-\frac{\pi \xi}{\sqrt{1-\xi^2}}} \times 100 \quad (4)$$

4. Settling Time

$$\frac{e^{-\xi \omega_n t_s}}{\sqrt{1-\xi^2}} \cong 0.02$$

$$e^{-\xi \omega_n t_s} \cong 0.02$$

$$t_s = \frac{4}{\xi \omega_n} \equiv 4T \quad (5) \qquad 2\% \text{ Tolerance}$$

$$t_s = \frac{3}{\xi \omega_n} \equiv 3T \quad (6) \qquad 5\%$$

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Examples

1. A second order system is given by $\frac{C(s)}{R(s)} = \frac{25}{s^2+6s+25}$. Find i) Damped frequency of oscillation ii) Rise time & peak time iii) peak overshoot iv) Settling time at 2 & 5% Tolerance.

Sol: Standard S.O.S $\frac{C}{R} = \frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}$

Std. C.E $\rightarrow$ $s^2+2\xi\omega_n s+\omega_n^2 = 0$

C.E $\rightarrow$ $s^2+6s+25 = 0$

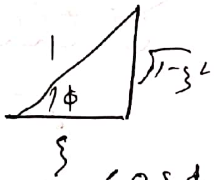
$\omega_n^2 = 25$ $\therefore \boxed{\omega_n = 5\ r/s}$ Undamped frequency of oscillation

& $2\xi\omega_n = 6$ $\xi = \frac{6}{2\times\omega_n} = \frac{6}{2\times5} = 0.6\ (<1)$

Damping Factor $\xi = 0.6$ (Underdamped)

$\omega_d = \omega_n\sqrt{1-\xi^2} = 5\sqrt{1-0.6^2} = 4\ r/s$

Rise time $t_r = \frac{\pi-\phi}{\omega_d}$



$t_r = \frac{\pi-0.93}{4} = 0.55$ sec

$\cos\phi = \xi$

$\phi = \cos^{-1} 0.6$

$\phi = 0.93$ rad

Peak Time $t_p = \frac{\pi}{\omega_d} = \frac{\pi}{4} = 0.785$ sec

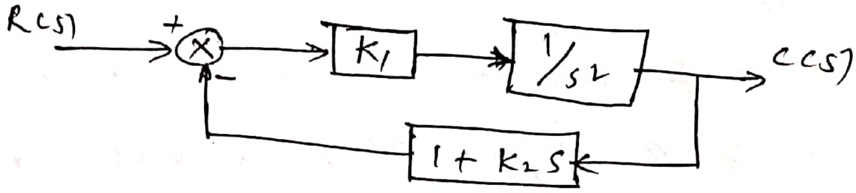
Maximum overshoot $\%M_p = e^{\frac{-\xi\pi}{\sqrt{1-\xi^2}}} \times 100$

$= 9.5\%$

Settling Time $t_s = \frac{4}{\xi\omega_n} = 1.33$ sec @ 2%

$= \frac{3}{\xi\omega_n} = 1.0$ sec @ 5%

3. For a control system shown in B.D, find K_1 & K_2 so that $M_p = 25\%$ and $t_p = 4$ sec.



$G(s) = K_1/s^2 \qquad H(s) = 1+K_2 s$

CE → $1+G(s)H(s)=0 \qquad 1+\frac{K_1}{s^2}(1+K_2 s) = 0$

$s^2 + K_1 K_2 s + K_1 = 0$

$s^2 + 2\xi\omega_n s + \omega_n^2 = 0$

$K_1 = \omega_n^2$ ① $\qquad 2\xi\omega_n = K_1 K_2$

$2\xi\omega_n = \omega_n^2 K_2 \qquad \xi = \frac{\omega_n K_2}{2}$

$K_2 = \frac{2\xi}{\omega_n}$ ②

Given $t_p = 4 = \frac{\pi}{\omega_d} \qquad \therefore \omega_d = \pi/4$ r/s $= \omega_n\sqrt{1-\xi^2}$

$M_p = e^{-\frac{\xi\pi}{\sqrt{1-\xi^2}}} = 0.25 \qquad \xi = 0.403$

$\therefore K_1 = 0.737$ & $K_2 = 0.94$.

4. A system has 30% overshoot & $t_s = 5$ sec. for an input of step, Find i) T.F ii) t_p iii) Response c(t). Assume 2% tolerance

Sol: $M_p = 0.3 = e^{-\frac{\xi\pi}{\sqrt{1-\xi^2}}} \qquad \xi = 0.358$

$T_s = \frac{4}{\xi\omega_n} \qquad 5 = \frac{4}{\xi\omega_n} \qquad \omega_n = 2.234$ r/s

$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2} = \frac{5}{s^2+1.6s+5} = T.F$

$\boxed{t_p = \frac{\pi}{\omega_d} = 1.5 \text{ sec}}$

2. A Servo motor is represented by D.E

$$\frac{d^2c}{dt^2} + 8\frac{dc}{dt} = 64\, e(t)$$ where $e(t)$ is the displacement of the output shaft (r) is the displacement of the input shift & $e(t) = r - c$

Determine trasient Specification for unit i/p.

Sol: $\ddot{c} + 8\dot{c} = 64(r - c)$

$$s^2 C(s) + 8s\,C(s) + 64\,C(s) = 64\,R(s)$$

$$\frac{C(s)}{R(s)} = T.F = \frac{64}{s^2 + 8s + 64}$$

C.E → $s^2 + 8s + 64 = 0$

Std C.E → $s^2 + 2\xi\omega_n s + \omega_n^2 = 0$

$\omega_n = 8\ r/s$ $\quad 2\xi\omega_n = 8 \quad \xi = 0.5$

$\omega_d = \omega_n\sqrt{1-\xi^2} = 8\sqrt{1-0.25}$

$t_r = \frac{\pi - \phi}{\omega_d} =$ $\qquad t_s = \frac{4}{\xi\omega_n} =$

$t_p = \frac{\pi}{\omega_d} =$ $\qquad t_s = \frac{3}{\xi\omega_n} =$

$\%M_p = e^{-\frac{\xi\pi}{\sqrt{1-\xi^2}}} \times 100 = 16.3\%$

$$c(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \sin(\omega_d t + \phi)$$

$$c(t) = 1 - 1.0708 \sin(2.08 + 1.2)$$

Q.4. OLTF of a UFBCS is $G(s) = \frac{K}{s(1+sT)}$

Find the factor in which the gain should be multipled so that the overshoot of the unit step response be reduced from 75% to 25%.

Sol: CE → $1 + G(s) = 0$ $\quad 1 + \frac{K}{s(1+sT)} = s^2T + s + K = 0$

$$s^2 + \frac{s}{T} + \frac{K}{T} = 0 \quad (1)$$

Let K_1 = initial gain & K_2 = Final gain

When gain is K_1; $\% M_{p1} = 75\%$.

K_2; $\% M_{p2} = 25\%$.

$$\left.\begin{aligned} M_{p1} &= e^{-\frac{\pi\xi_1}{\sqrt{1-\xi_1^2}}} = 0.75 \\ M_{p2} &= e^{-\frac{\pi\xi_2}{\sqrt{1-\xi_2^2}}} = 0.25 \end{aligned}\right\} \quad (2)$$

Standard CE → $s^2 + 2\xi\omega_n s + \omega_n^2 = 0$ (3)

comparing (1) & (3)

$\omega_n = \sqrt{K/T}$ (4) $\qquad 2\xi\omega_n = \frac{1}{T}$

$$\xi = \frac{1}{2T \times \sqrt{K}/\sqrt{T}} = \frac{1}{\sqrt{4TK}} \quad (5)$$

$$\therefore \xi_1 = \frac{1}{\sqrt{4k_1 T}} \quad \& \quad \xi_2 = \frac{1}{\sqrt{4k_2 T}} \quad (5)$$

From E-2

$$\frac{\pi \xi_1^2}{\sqrt{1-\xi_1^2}} = 0.28 \quad \therefore \xi_1^2 = \frac{0.28}{3.14} = 0.089$$

$$\text{and} \quad \frac{\pi \xi_2^2}{\sqrt{1-\xi^2}} = 1.4 \quad \therefore \xi_2^2 = 0.408$$

$\Bigg\}$ (6)

From E-5 & E-6

$$\frac{\xi_2^2}{\xi_1^2} = \frac{0.408}{0.089} = \frac{1/\sqrt{4k_2 T}}{1/\sqrt{4k_1 T}} = \frac{k_1}{k_2} = \underline{\underline{21}}$$

Gain has to be increased by 21 Times

Q-5 The step response of a UFBCS is

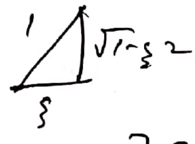
$c(t) = 1 - 1.66 e^{-8t} \sin(6t + 37^\circ)$. Find

i) CLTF ii) OLTF iii) t_r, t_p, t_s & M_p.

Sol: $c(t) = 1 - \dfrac{e^{-\xi \omega_n t}}{\sqrt{1-\xi^2}} \sin(\omega_d t + \phi)$ Standard

$c(t) = 1 - 1.66 e^{-8t} \sin(6t + 37^\circ)$ Given

$\therefore \omega_d = 6$ r/s $\quad \phi = 37^\circ$



$\cos\phi = \xi \quad \therefore \xi = \cos 37^\circ = 0.798$

$$\omega_n = \frac{\omega_d}{\sqrt{1-\xi^2}} = \frac{6}{\sqrt{1-0.798^2}} = 10 \text{ r/s}$$

or $\xi \omega_n = 8 \quad \therefore \omega_n = \dfrac{8}{\xi} = 10$ r/s

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$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2} = \frac{100}{s^2+16s+100}$$

$$\div (s^2+16s) = s(s+16)$$

i) $$\therefore \frac{C(s)}{R(s)} = \frac{100/s(s+16)}{1+\frac{100}{s(s+16)}} = \frac{G(s)}{1+G(s)H(s)} = CLTF$$

$$\therefore G(s) = \frac{100}{s(s+16)} \quad \& \quad H(s) = 1.$$

ii) $$OLTF = G(s) = \frac{100}{s(s+16)}$$

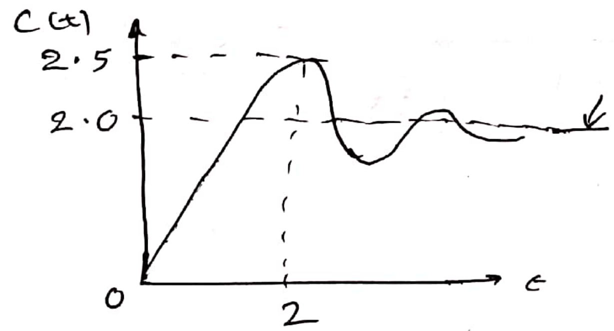
iii) $$t_r = \frac{\pi-\phi}{\omega_d} = \qquad \phi = 37 \times \frac{\pi}{180} \text{ rad.}$$

$$t_p = \pi/\omega =$$

$$M_p = e^{-\frac{\xi\pi}{\sqrt{1-\xi^2}}} \times 100 =$$

$$t_s = \frac{4}{\xi\omega_n} = \frac{3}{\xi\omega_n} =$$

Q.6 The step response of S.O.S is shown for an input of $2u(t)$. Determine the OL & CLTF. Assume UFBCS.



$r(t) = 2$

Sol: input $r(t) = 2$ $\qquad t_p = 2$ sec

~~Mp calculation~~

$$M_p = \frac{c(t_p) - c(\infty)}{c(\infty)} \times 100$$

$$\%M_p = \frac{2.5 - 2.0}{2.0} \times 100 = 25\%$$

$$\%M_p = e^{\frac{-\xi\pi}{\sqrt{1-\xi^2}}} = 25 \qquad \boxed{\xi = 0.4037}$$

$$t_p = \frac{\pi}{\omega_d} \qquad \therefore \boxed{\omega_d = \frac{\pi}{2}} \text{ r/s}$$

$$\boxed{\omega_n = 1.717 \text{ r/s}}$$

$$\frac{C}{R} = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2} = \frac{2.95}{s^2 + 1.386s + 2.95}$$

$$\div \quad s^2 + 1.386s = s(s + 1.386)$$

$$\frac{C}{R} = \frac{2.95/s(s+1.386)}{1 + \frac{2.95}{s(s+1.386)}} = \frac{G(s)}{1 + G(s)H(s)} = \text{CLTF}$$

Since System is UFBCS $H(s) = 1$

$$\therefore G(s) = \frac{2.95}{s(s+1.386)} = \underline{\text{OLTF}}$$