Bapuji Educational Association (Regd.)
BAPUJI INSTITUTE OF ENGINEERING AND TECHNOLOGY, DAVANAGERE-04
DEPARTMENT OF MATHEMATICS

Question Bank

Calculus and Linear Algebra (18MAT11)

1. With usual notation, prove that $tan\emptyset = r\left(\frac{d\theta}{dr}\right)$.

2. Find the angle between two curves $r = a(1-\cos\theta)$ and $r = 2a\cos\theta$.

3. Show that the curves $r^n = a^n cosn\theta$ and $r^n = b^n sinn\theta$ are intersect orthogonally.

4. Find the Pedal equation of the curve $r^m = a^m cosm\theta$.

5. Find the Pedal equation of the curve $\frac{2a}{r} = (1 + cos\theta)$.

6. Find the radius of curvature for the curve $x^2 y = a(x^2 + y^2)$ at the point (-2a,2a)

7. Find the Radius of curvature of the curve $y^2 = \frac{4a^2(2a-x)}{x}$ where the curve meets x-axis.

8. Show that for the curve $(1 - cos\theta) = 2a$, ρ^2 varies as r^3 .

9. Find the Centre of Curvature for the parabola $y^2 = 4ax$ and hence find its Evolute.

10. Find the Circle of Curvature for the curve $\sqrt{x} + \sqrt{y} = \sqrt{a}$ at $\left(\frac{a}{4}, \frac{a}{4}\right)$.

11. Expand $tan^{-1}x$ in powers of $(x - 1)$ upto fourth degree terms.

12. Using Maclaurin's expansion, prove that $\sqrt{1+\sin 2x} = 1 + x - \frac{x^2}{2} - \frac{x^3}{6} + \ldots\ldots$.

13. Using Maclaurin's series expand $\log(1 + e^x)$ upto term containing x^4.

14. Evaluate a) $\lim_{x\to 0}\left(\frac{a^x+b^x+c^x+d^x}{4}\right)^{\frac{1}{x}}$ b) $\lim_{x\to 0}\left(\frac{sinx}{x}\right)^{\frac{1}{x^2}}$ c) $\lim_{x\to 0}\left(\frac{(1+x)^{1/x} - e}{x}\right)$

15. If $z = e^{ax+by} f(ax - by)$ then P.T. $b\frac{\partial z}{\partial x} + a\frac{\partial z}{\partial y} = 2abz$.

16. If $u = f(x - y,\ y - z, z - x)$ then Show that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$.

17. If $u = f\left(\frac{x}{y}, \frac{y}{z}, \frac{z}{x}\right)$ then prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = 0$

18. If $u = x^2 + y^2 + z^2$, $v = xy + yz + zx$, $w = x + y + z$,then find $J\left(\frac{u,v,w}{x,y,z}\right)$.

19. Find the extreme values of the function $f(x,y) = x^3y^2(1 - x - y)$ for $x, y \neq 0$.

20. Find the stationary values of $x^2 + y^2 + z^2$ subject to the condition $xy + yz + zx = 3a^2$.

21. Evaluate $\int_0^1 \int_x^{\sqrt{x}} xy\, dydx$ by changing order of integration**.**

22. Evaluate $\int_0^a \int_0^{\sqrt{a^2-y^2}} (y\sqrt{x^2+y^2})\, dxdy$ by changing into Polar coordinates.

23. Evaluate $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)} dxdy$ by changing to polar coordinates**.**

24. Evaluate $\int_0^a \int_0^x \int_0^{x+y} e^{(x+y+z)}\, dz\, dy\, dx.$

25. Evaluate $\int_{-c}^c \int_{-b}^b \int_{-a}^a (x^2 + y^2 + z^2)\, dz\, dy\, dx.$

26. Find the area bounded between parabolas $y^2 = 4ax$ and $x^2 = 4ay$ using double integration.

27. Find the area of the Ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ by double integration.

28. Prove that $\beta(m,n) = \dfrac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$.

29. Prove that $\int_0^{\frac{\pi}{2}} \sqrt{\sin\theta}\, d\theta\ X\ \int_0^{\frac{\pi}{2}} \frac{1}{\sqrt{\sin\theta}}\, d\theta = \pi$.

30. Solve $(y^3 - 3x^2y)dx - (x^3 - 3xy^2)dy = 0$

31. Solve $y(2x - y + 1)dx + x(3x - 4y + 3)dy = 0$.

32. Solve $(ylogx - 2)ydx = xdy$.

33. Solve $\dfrac{dy}{dx} = \dfrac{x+y+1}{2x+2y+3}$.

34. Solve $\left(x^2y^3 + xy\right)\dfrac{dy}{dx} = 1$.

35. Solve $\dfrac{dy}{dx} + \dfrac{y\cos x + \sin y + y}{\sin x + x\cos y + x} = 0$.

36. Solve $\frac{dy}{dx} + xsin2y = x^3cos^2y$.

37. Find the orthogonal trajectories of the family $\dfrac{x^2}{a^2} + \dfrac{y^2}{a^2+\lambda} = 1$ where λ is parameters.

38. Find the orthogonal trajectory of the family $r^n cosn\theta = a^n$.

39. A body originally at 80^0C cools to 60^0C in 20 minutes the temperature of air being 40^0C . What will be the temperature of the body after 40 minutes.

40. Solve $p^2 - 2psinhx - 1 = 0.$

41. Obtain the General solution and Singular solution of equation $xp^3 - yp^2 + 1 = 0$ as Clairaut's equation.

42. Solve $(px - y)(py + x) = 2p$ by reducing into Clairaut's form taking substitution $X = x^2$,$Y = y^2$.

iii) No solution.

43. Find the Rank of the matrix $A = \begin{bmatrix} 1 & 0 & 2 & 1 \\ 0 & 1 & -2 & 1 \\ 1 & -1 & 4 & 0 \\ -2 & 2 & 8 & 0 \end{bmatrix}$ by reducing it into Echelon form.

44. For what values of λ and μ the system of equations $x + y + z = 6, x + 2y + 3z = 10,$ $x + 2y + \lambda z = \mu$ may have i) Unique solution ii) Infinite number of solutions

45. Solve the system of equations $x + y + z = 9, x - 2y + 3z = 8$, $2x + y - z = 3$ by Gauss elimination method.

46. Solve by using Gauss-Jordon method

$x + y + z = 9, 2x + y - z = 0, 2x + 5y + 7z = 52.$

47. Solve the system of equations by Gauss Seidel Method

$x + y + 54z = 110,\ 27x + 6y - z = 85,\ 6x + 15y + 2z = 72.$

48. Find the Largest Eigen value and the corresponding Eigen vector of A= $\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ using power method .Take $[1,1,1]^T$ as intial vector .

49. Find the Largest Eigen value and the corresponding Eigen vector of

A= $\begin{bmatrix} 25 & 1 & 2 \\ 1 & 3 & 0 \\ 2 & 0 & -4 \end{bmatrix}$ using power method .Take $[1,0,0]^T$ as initial vector.

50. Reduce the matrix A = $\begin{bmatrix} 4 & 1 \\ 2 & 3 \end{bmatrix}$ into Diagonal form.