Bapuji Educational Association (Regd.)

BAPUJI INSTITUTE OF ENGINEERING AND TECHNOLOGY, DAVANAGERE-04

DEPARTMENT OF MATHEMATICS

Question Bank

Advanced Calculus and Numerical Methods (18MAT21)

1) Find the directional derivative of $\emptyset = yzx^2 + 4xz^2$ at $(1,-2,-1)$ along $2i - j - 2k$.

2) Find the angle between the surfaces $x^2 + y^2 + z^2 = 9$ and $z = x^2 + y^2 - 3$ at $(2,-1,2)$

3) Find div$\vec{F}$ and Curl$\vec{F}$ if $\vec{F} = grad(x^3 + y^3 + z^3 - 3xyz)$.

4) Find the values of constants a, b, c such that $\vec{F} = (x + y + az)i + (bx + 2y - z)j + (x + cy + 2z)k$ is irrotational.
Find $\emptyset$ such that $\vec{F} = \nabla\emptyset$.

5) Find the constants a and b such that the vector field $f = (axy + z^3)i + (3x^2 - z)j + (bxz^2 - y)k$ is irrotational. Find $\emptyset$ such that $\vec{F} = \nabla\emptyset$.

6) Show that $\vec{F} = (y + z)i + (z + x)j + (x + y)k$ is irrotational. Also find a scalar function ϕ such that $\vec{F} = \nabla\phi$.

7) Evaluate $\int_c \overrightarrow{F}\cdot\overrightarrow{dr}$ where $\vec{F} = xyi + (x^2 + y^2)j$ along the path of straight line from (0,0) to (1,0) and then to (1,1).

8) Verify Green's theorem for $\oint_c (xy + y^2)dx + x^2dy$ where c is the closed curve of the region bounded by $y = x$ and $y = x^2$.

9) Verify Stoke's theorem for $\vec{F} = yi + zj + xk$ where S is the upper half of the sphere $x^2 + y^2 + z^2 = 1$ and C is the boundary.

10) Use divergence theorem to evaluate $\iint \overrightarrow{F}.\,\hat{n}\,ds$ over the entire surface of the region above xyplane bounded by $x^2 + y^2 = z^2$ the plane $z = 4$ where $\vec{F} = 4xzi + xyz^2j + 3zk$.

11) Solve $(4D^4 - 8D^3 - 7D^2 + 11D + 6)y = 0$

12) Solve $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 3y = e^{-x}$.

13) Solve $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 2x^2 + sin2x$.

14) Solve $\frac{d^2y}{dx^2} - 4y = \cosh(2x-1) + 3^x$.

15) Solve $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = e^{2x} + \sin x$

16) Solve $(D^2 + 4)y = tan2x$ by the method of variation of parameter.

17) Solve by the method of variation of parameters $(D^2 - 6D + 9)y = \frac{e^{3x}}{x^2}$

18) Solve $x^2\frac{d^2y}{dx^2} + 2x\frac{dy}{dx} - 2y = (x+1)^2$ **.**

19) Solve the Cauchy's homogeneous linear equation $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = \log x\sin(\log x)$

20) Solve the Legendre's form of linear equation $(1+x)^2\frac{d^2y}{dx^2} + (1+x)\frac{dy}{dx} + y = \sin 2[\log(1+x)]$

21) The DE of a simple pendulum is Solve $\frac{d^2x}{dt^2} + {W_0}^2x = F_0Sinnt$ where W_0 and F_0 are constants. Also initially $x = 0, \frac{dx}{dt} = 0$ solve it.

22) Form PDE by eliminating Arbitrary function from relation

$\emptyset(x + y + z,\ x^2 + y^2 + z^2) = 0$.

23) Form the PDE by eliminating the arbitrary function for $f(x^2 + 2yz, y^2 + 2xz) = 0$

24) Solve $\frac{\partial^2 z}{\partial x \partial y} = sinxsiny$ for which $\frac{\partial z}{\partial y} = -2siny$ when $x = 0$ and $z = 0$ if $y = (2n+1)\frac{\pi}{2}$.

25) Solve $\frac{\partial^3 z}{\partial x^2 \partial y} + 18xy^2 + sin\,(2x - y) = 0$

26) Solve $\frac{\partial^3 z}{\partial x^2 \partial y} = cos\,(2x + 3y)$.

27) Solve $\frac{\partial^2 z}{\partial x^2} + z = 0$ given that when $x = 0, z = e^y$ and $\frac{\partial z}{\partial x} = 1$.

28) Solve $\frac{\partial^2 z}{\partial y^2} - z = 0$ given that when $y = 0, z = e^x$ and $\frac{\partial z}{\partial y} = e^{-x}$.

29) Solve $(y - z)p + (z - x)q = (x - y)$.

30) Derive one-dimensional Heat equation as $\frac{\partial u}{\partial t} = c^2\frac{\partial^2 u}{\partial x^2}$.

31) Derive one-dimensional Wave equation in usual notations.

32) Obtain the various possible solutions of Heat equation $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$ by method of separation of variables.

33) Find the solutions of Wave equation $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$ by method of separation of variables.

34) Test for Convergence and Divergence $\frac{1}{2\sqrt{1}} + \frac{x^2}{3\sqrt{2}} + \frac{x^4}{4\sqrt{3}} + \cdots \quad (x > 0).$

35) Discuss the nature of the series $\frac{x}{1.2} + \frac{x^2}{2.3} + \frac{x^3}{3.4} + \cdots$

36) If α and β are two distinct roots of $J_n(x) = 0$ then prove that

$$\int_0^1 x\, J_n(\alpha x)\, J_n(\beta x)\, dx = 0 \quad if\ \alpha \neq \beta.$$

37) With usual notation prove that $J_{\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}} sinx$.

38) Obtain the series solution of Legendre`s differential equation of the form

$$y = a_0 u(x) + a_1 v(x).$$

39) Express $f(x) = x^3 + 2x^2 - x - 3$ in terms of Legendre polynomials.

40) Express $f(x) = x^4 + 3x^3 - x^2 + 5x - 2$ in terms of Legendre polynomials.

41) Use appropriate interpolating formula to compute $y(82)$ *and* $y(98)$ for the data

x	80	85	90	95	100
y	5026	5674	6362	7088	7854

42) The area of a Circle corresponding to diameter is given below

D	80	85	90	95	100
A	5026	5674	6362	7088	7854

Find the area corresponding to diameter 82 and 105 by using appropriate interpolation formula.

43) Using suitable interpolation formula find the number of students who obtained marks between 40 and 45

Marks	30-40	40-50	50-60	60-70	70-80
No. of Students	31	42	51	35	31

44) If $f(1) = 4, f(3) = 32, f(4) = 55, f(6) = 119$. Find interpolating polynomial by Newton's divided difference formula. Hence find (4.8) .

45) Using the Lagrange's formula, find interpolating polynomial from the data

x	0	1	2	3	4
$f(x)$	3	6	11	18	27

Hence find $f(0.5)$ and (3.1)

46) Use Regula - Falsi method to find the real root of the equation $x\log_{10} x - 1.2 = 0$. Carry out four iterations**.**

47) Use Newton –Raphson method to find the real root of the equation $xsinx + cosx = 0$ near $= \pi$. Carryout three iterations.

48) Evaluate $\int_0^5 \frac{dx}{4x+5}$ by using Simpson`s 1/3rd rule, taking 10 equal parts. Hence find $\log_e 5$.

49) Evaluate $\int_0^1 \frac{x}{1+x^2}$ by using Simpson`s 1/3rd rule by taking 6 equal parts. Hence find the value of $\log_e 2$.

50) Evaluate $\int_4^{5.2} \log_e x \, dx$ by using Weddle's rule by taking 6 equal parts.