Elasticity

Introduction

The knowledge of physical, chemical, mechanical and technological properties of materials are essentially required for selection of right materials for industrial applications. Properties of such as dielectric constant, electrical conduction, thermal conduction etc.. are used to explore newer materials for electronic applications. Similarly, properties such as stress, strain, elastic moduli etc.... are used to explore materials for industrial applications such as ~~constru~~ construction of buildings, bridges and railway wagons.

Elasticity is a branch of physics which deals with the study of elastic property of materials. Elasticity is the property by virtue of which bodies regain original size, & shape after the removal of deforming force. When a deforming force is acts on a body, if there is a change in its length, (or) shape (or) volume. Then the body is said to be strained, so.

Suppose when a suitable force is acted upon a body which undergoes change in the form. This change in the form is called "deformation",

In this chapter we can discuss about elastic properties of materials & their application,

Classification of ~~Elastic~~ Materials

* Perfectly Elastic and * Perfectly plastic

The materials which completely regains its original size (or) shape even after the removal of deforming force. is known as a "perfectly elastic materials"

The materials which donot regain its original size and shape after the removal of deforming force are. Known as " Plastic materials "

In general ~~no~~ materials which is perfectly elastic and perfectly plastic materials

Elasticity : It is the property of material by virtue of which. material bodies regain their original size and shape, after the removal of deforming force,"

Ex: steel, glass quartz, ivory, Phosphor - bronze,

Plasticity : It is the property of the material which donot regain their original size & shape even after the removal of deforming force,

Ex: Rubber, Putty, synthetic materials. [Thermo plastic & thermosetting] , acrylic, Resins.

Importance of Elastic material & Elasticity in Engineering applications

It is important to have a knowledge of elasticity in engineering applications, consider an example of iron, which less elastic than steel, when the tools are made up of such iron material. which are used in an application where. there is a lot of vibrations, say for example, Bridges, Shock absorber blades in heavy vehicles, Etc....,

If a small fracture is formed in the tool., made up of such iron, which are used in application,, The ~~fa~~ fracture (or) crack propagates in the body of the tool due to the dislocation of atoms. which results damage (or) broken the tools in to 2 pieces.

In case the tool is madeup of steel, it springs back the atoms to ~~shape~~ same position (or) shape. repeatedly as steel is more elastic,

Even when the tool made of steel also gets affected by fracture (or) crack, it undergoes plastic deformation.. Hence which will be recognized by the mechanic whose is ~~hand~~-handling the tool and. replace it well. in advance. before the damage takes place,

So pure metals (Iron) are soft by property. they are ductile and have low tensile strength; where as alloys are generally harder than pure metals,

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Stress and strain

In the study of elasticity, the body is subjected to a deforming force. which causes. the stress as a result the body is strained is deformed. Therefore the basic of stress and ~~Stra~~ strain are discussed as follows

Stress : Under the action of the External force, the body changes its dimension because, of the molecules inside its are displaced from their previous positions. which results the relative displacement of various parts of the body.

Thus the applied force 'F' per unit area 'a' is called "Stress"

i.e $\text{Stress} = \frac{\text{Force}}{\text{area}} = \frac{F}{a}$.

It is also defined as "the restoring force per unit area developed inside the body is called 'stress'"

Thus creates pressure (or) Tension exerted on the material object.

S.I Unit is N/m^2 (or) Pa

Thus stress may be Tensile, compressive (or) Tangential stress

Strain : The deformation produced by the external force, accompanies a change in the dimensions. ~~Thus~~

Thus "The ratio of the change in dimensions to the original dimensions is called the "Strain""

Thus change in dimension is Either length, volume (or) Shape.

$\therefore \text{strain} = \frac{\text{Change in dimensions}}{\text{original dimension}}$

It is a dimension-less quantity and has no Unit.

Types of stress and strains

Depending on the type of application, bodies of different shapes are employed and subjected to appropriate type of stress. Under the action of ~~st~~ the perticular type of stress acting, corresponding type of deformation is produced in the body. The deforming force is deformation is broadly classified under 3 types

(i) Tensile stress (or) Longitudinal and strain

(ii) Compressive stress (or) volume stress and strain

(iii) Shear (or) Tangential stress and ~~Strai~~ ~~stra~~ Strain.

(i) (a) Tensile stress (or) Longitudinal stress

It is the stretching force acting per unit area of cross section of the solid wire along its length.

Consider a wire, which is suspended to a rigid support at one end and other end is left free to applied the load, when the deforming force is applied to one end along its length,

The applied force per unit area of cross section along normally which is called "Longitudinal stress"

$\therefore$ Longitudinal stress $= \frac{F}{a}$ ---- N/m^2

(b) Linear (or) Tensile (or) Longitudinal strain

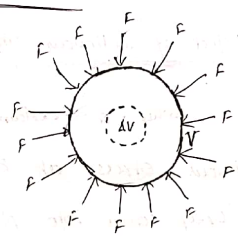
When the deforming force is applied to one end along its length, this force produces a linear strain, (or) the wire undergoes a change in length,

Thus strain is defined as "change in length 'l' (or) 'ΔL' to the original length 'L'"

$$\text{Linear strain} = \frac{\text{change in length}}{\text{original length}} = \frac{l}{L} \text{ (or) } \frac{\Delta L}{L}$$

(ii) Compressive stress (or) Volume stress

It is the uniform force per unit area (or) uniform pressure acting normally all over the body is called "volume stress".



When the deforming force is applied normally and uniformly to the entire surface of a body, per unit area gives the normal stress (or) pressure.

If 'F' is uniform force applied uniformly and normally on a surface area 'a', then the stress (or) pressure is given by

(i) Stress (or) Pressure $(P) = \frac{F}{a}$

i.e. compressive stress $= F/a$ ---- N/m^2

(ii) Compressive strain:

Under the applied force (or) pressure, the body undergoes change in its volume (or) shape in solid bodies.

$\therefore$ If ΔV is the change in volume & 'V' is its original volume

Then volume strain $= \frac{\text{change in volume}}{\text{original volume}} = \frac{\Delta V}{V}$

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Factors affecting on Elastic materials

Effect of continuous stress and temperature

When a certain elastic materials are subjected to continuous stress at elevated temperature, the phenomenon of creep comes into play,

Creep is the property due to which a material is studied under a steady stress undergoes continuous deformation, It is a type of slow plastic deformation that takes place even below the elastic limit, (proportionality limit).

This plastic deformation is caused due to the slip (or) dislocation of molecule (or) atoms from their mean position, along the crystallographic axis (or) direction in the metals,

The effect of creep is due to the high temperature, is an important factor in the designing of boilers, turbines, ~~jet~~ jet engines, etc,

At room temp Tin, ~~lead~~ lead, zinc and their alloys undergoes creep.,

② Annealing :

Heat treatments are used to alter the physical and mechanical properties of metals (or alloys) without changing its shape, Annealing is also a type of 'heat treatment'

It is a process to make a metal (or) an alloy (or) a glass soft by heating and then cooling it slowly, This process increases the strength, hardness and toughness to meet the requirement of good machinability and casting. It also increases (or) improves the elasticity and ductility.

③ Effect of impurities on Elasticity

The addition of impurities to ~~pu~~ pure metal, results in either increase (or) decrease of elasticity, It is all depends on type of impurities & quantity of impurity that we are adding

If the added impurity is a type, which obstructs, the motion of dislocation in the lattice., This results, increase in the elastic modulus and yield strength.

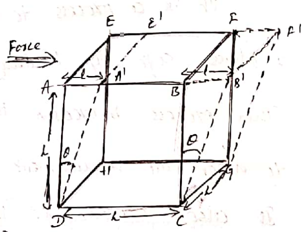
On the other hand, if the impurity enables the movement of dislocation it causes cracks, and thus reduces the strength.

Shear Stress (or) Tangential Stress

It is defined as " The force acting tangentially per unit area on the surface of an object (or) body"

i.e Tangential stress $= F/a = \frac{\text{Tangential force}}{\text{Unit area}}$.

Shear / Tangential strain



It is defined as the ratio of the relative displacement b/w the two layers, under the action of a tangential force, to the distance b/w them, the distance being measured at right angles to the direction of stress.

So, here from the fig 'The shearing angle' itself is measure the ratio of change in dimension to the original dimension,

i.e $\theta = \frac{BB'}{BC} = \frac{\Delta L}{L}$ (or) $\frac{l}{L}$

$\therefore$ Shear strain $= \frac{l}{L}$

Hooke's law

In 1678 Robert Hooke established the fundamental law of elasticity, i.e it states that " within a elastic limit, the stress is directly proportional to strain"

i.e stress $\propto$ strain — ①

It is also defined as " The ratio of stress to strain is some constant and is known as" coefficient of elasticity (or) modulus of elasticity"

①=) i.e stress = E strain

$\Rightarrow E = \frac{\text{stress}}{\text{strain}}$ — ②

'E' is the characteristic of given material and is different for different types of strain in the same material.

Here 'E' is an some constant, is called " The modulus of elasticity of the material."

The S.I. Unit of modulus of elasticity is N/m^2

The dimensions are $[M^1, L^{-1}, T^{-2}]$

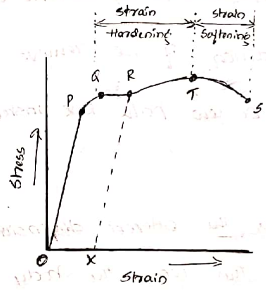
Irrespective of the type of deformation that takes place for a body under the action of an external force, Hookes found that stress is always directly proportional to strain but only when ~~the~~ a small deforming force is applied. Because for a small deforming force, the molecular displacement will be ~~of~~ negligible, so that the molecules can returns to their previous positions, after the removal of force.

By knowing the Elastic limit of the material one can find that the maximum deforming force, that the given body can be ~~stretching~~ subjected to without undergoing permanent deformation, (set). This can be explained by using stress - strain diagram, as follows.

*** Stress - strain Diagram

Let a body is subjected to a Uniform Stress, such that it is to be clamped at one end to rigid support and loaded at the other end., gradually from zero value untill the wire breaks. When the stress increases results in a change in its dimension, ~~The~~ relation b/w the applied stress and ~~strain~~ ~~is~~ resulting strain is studied by drawing the curve b/w stress and strain which is as shown in the fig. This curve is called

** "Stress - strain curve"



O-P- Elastic behavior.
P- proportionality limit.
PQ - Elastic limit.
Q - Yield point.
QS - plastic limit
R - Plastic behaviour/ Upper yield point
S- Fracture point.
O-X - Residual strain

Following points are the observation made by Hookes

* OP is the portion of linearity of the curve, where stress is directly ~~po~~ proportional to the strain. This is the only region of ~~Point~~ plot where the Hookes law is valid.
* ~~within on Elastic~~
* Above the point 'P' the body continues to exhibit perfect elastic property even up to the point 'Q', in such a way that, ~~if~~ the curve will be retraced, if the applied force on the body is removed at any point b/w P and 'Q'.

but we can not observe the linear dependence of stress and strain in 'P and Q' region.

* When the stress is applied beyond the elastic limit PQ (or) yield point 'Q', an exponential growth in strain takes place, which is represented as Q.R. in the curve.

* In this region QR, the material is partly elastic and partly plastic. Under this condition if we remove the deforming force, material takes a new path RX instead of RQ.

* That means the body retains the altered dimension (or) change in size and shape. In this way the body has small strain "OX" which is known as "Residual strain" (or) "Permanent stress"

* At any point b/w Q and S, the body fails to regain its original size & shape, on removal of deforming force. Such a deformation is ~~also~~ called "Plastic deformation"

* Here the region RT is known as 'Plastic region'. In this region a steady increase in strain with increase in stress is noticed, with the decrease in the area of ~~cor~~ cross-section of the body.

* Beyond the point 'T', an increase in strain without any increase in stress is noticed up to 'S'. i.e from 'T' to S. (TS)

* At 'S' the deformation becomes large enough to the pull the molecules far apart so that the binding force is breaks, hence beyond 'S' the body is no more a single piece. i.e the body ~~brea~~ breaks in to 2 pieces.

∴ the point 'S' is called "Fracture point" (or) "breaking point". The force per unit area for which the wire breaks is called breaking stress"

* Between 'Q' and S, it is the plastic range which is subdivided into 2 region namely "strain hardening" i.e Q to T and "strain softening" i.e T to S.

Based on ~~these~~ observation Hookes stated that "with in an elastic limit the stress is directly proportional to strain"

~~Classification~~ of Elastic Modulus

According to the nature of strain, there are three moduli of elasticity of an isotropic material.

They are

(i) Young's modulus (Y (or) q)

(ii) Bulk modulus (K)

(iii) Rigidity modulus (η)

Youngs modulus (Y (or) q)

It is defined as "The ratio of longitudinal stress to the linear strain with in an elastic limit is called "Youngs modulus"

If 'L' is the original length of the wire and 'l' be the change in length is caused by a force,

then force per unit area is called - stress and change in length to the original length - strain

Clearly Youngs modulus $= Y = \frac{\text{Longitudinal stress}}{\text{Linear strain}}$

$$Y = \frac{F/a}{l/L}$$

$$\Rightarrow \boxed{Y = \frac{FL}{al}} \quad \ldots \; N/m^2 \text{ (or) } Pa$$

(ii) Bulk modulus (K)

The ratio of the compressive stress (or) pressure to the volume strain without a change in shape of the body within the elastic limits is called "Bulk modulus"

It is also defined as " It is the ratio of compressive stress to the volumetric strain"

When force is applied normally to the surface of a body, a change in volume takes place., that strain is known as "Volumetric strain"

Hence Bulk modulus $K = \frac{\text{Compressive stress}}{\text{Volumetric strain}}$

$$K = \frac{F/a}{v/V} \quad (\text{Here } F/a = \text{Pressure})$$

$$\boxed{\therefore K = \frac{PV}{v}} \quad \ldots \; N/m^2 \text{ (or) } Pa.$$

When a very small pressure dP is applied, the change in volume is also very small. i.e dv

then $K = \frac{-dP}{dv/V}$ (or) $\frac{-dP\,V}{dv}$ (or) $\frac{-\Delta P V}{\Delta v}$

negative sign indicates that, an increase in applied pressure causes a decrease in volume,

Rigidity Modulus : (η)

It is defined as " the ratio of tangential stress to the shearing strain" with in an elastic limit.

Consider the unit cube on which a force 'F' is applied tangentially. Here each horizontal section of the cube suffers the sliding effect. Such that, the sliding is maximum for topmost section, and minimum for bottom section. hence the body experience the turning effect and changes its shape. This is called shearing and the angle through which the turning is taken place is called "Shearing angle" (θ)

If 'x' is the maximum slide and 'θ' is shearing angle then Rigidity modulus $= \eta = \frac{\text{Sharing stress}}{\text{Shearing strain}} = \frac{F/A}{(x/L)}$

$$\boxed{\eta = \frac{T}{\theta}} \quad (\text{or}) \quad \boxed{\eta = \frac{FL}{Ax}} \ N/m^2$$

Longitudinal strain coefficient (α)

The longitudinal strain produced per unit stress is called "longitudinal strain coefficient".

w.k.t longitudinal strain $= \frac{l}{L} = \frac{\text{Change in length}}{\text{original length}}$

Now longitudinal strain coefficient $\Big\} = \alpha = \frac{\text{longitudinal strain}}{\text{Unit stress}}$

i.e $\boxed{\alpha = \frac{l}{LT}}$ (or) $\boxed{\alpha = \frac{l/L}{T}}$

Lateral deformation :

In case of any deformation taking place along the length of a body like a wire due to a deforming force. there is always some change in the thickness of the body this phenomenon is called "lateral deformation".

This change occurs in a direction perpendicular to the direction along which the deforming force acting (or) to the direction of elongation is called "lateral strain"

Lateral strain (contraction strain)

If a deforming force acting on a wire assumed to be having a circular cross-section produces a change in 'd' to its original diameter 'D'

the lateral strain $= \frac{d}{D} = \frac{\text{changes in diameter}}{\text{original Diameter}}$

Lateral strain coefficient (β)

The lateral strain produced per unit stress is called "lateral strain coefficient"

i.e $\beta = \frac{\text{Lateral strain}}{\text{Unit stress}} = \frac{d/D}{T} = \frac{d}{TD}$

Poisson's Ratio (σ)

With in the elastic limit of a body "the ratio of lateral strain to the longitudinal strain is a constant. and is called "Poisson's ratio"

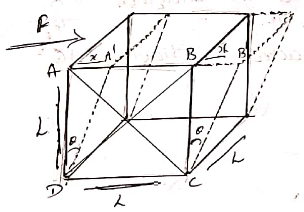
i.e Poisson ratio $\sigma = \dfrac{\text{Lateral Strain}}{\text{longitudinal Strain}}$

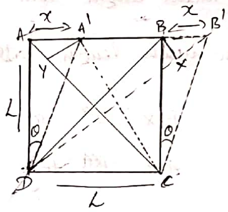
$$\boxed{\sigma = \frac{\beta}{\alpha}} \quad (or) \quad \boxed{\sigma = \frac{Ld}{xD}}$$

Relation b/w the elastic constants

When the body undergoes an elastic deformation it is studied under any one the three elastic moduli, that is depending upon the type of deformation However, these moduli are related to each other, their relation can be understand by knowing how one type of deformation could be equated to a combination of other types of deformation.

Relation b/w shearing strain, Elongation strain and compression strain:





* ~~Let us consider a unit cube whose lower surface is fixed to rigid support.~~
* Let ABCD be one of the face of the cube with the bottom DC' is fixed plane as shown in the fig.
* When a tangential stress is applied to its upper face along 'AB'. it causes relative displacement of different planes through a small angle.
* Such that, the vertex 'A' moves to A' and B to B' also one can notice that, the diagonal AC is shrunk (or) compress to A'C and BD is stretched (or) Elongation to B'D taking place right angle to each other.
* Let 'θ' be the angle of shearing which is very small in magnitude.

If we draw the perpendicular from B to DB'

and A' to AC then we can easily determine

how much length will elongated in diagonal DB.

and how much length will contracted in diagonal AC

i.e $BX \perp^{r} \text{ to } DB'$

$A'Y \perp^{r} \text{ to } AC$

Now we can approximated that

B'X is the Elongation in an original length BD

and AY is the contraction in our original length AC.

WKT

$$\left.\begin{array}{l}\text{Elongation}\\ \text{Strain}\end{array}\right\} = \frac{\text{change in length}}{\text{Original length}} = \frac{B'X}{BD} \quad \text{--- (1)}$$

and ||ly

$$\left.\begin{array}{l}\text{Compressive}\\ \text{Strain}\end{array}\right\} = \frac{AY}{AC} = \frac{\text{change in volume}}{\text{Original volume}} \quad \text{--- (2)}$$

If 'L' is the length of each side of the cube

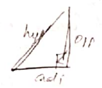
then The Diagonal AC = DB

from pythagorus theorem we get

$$AC = BD = (\sqrt{2})L \quad \text{--- (3)}$$

Now from isosceles right angle Δ^{le} $BB'X$

$$\cos\theta = \frac{Adj}{Hyp} = \frac{B'X}{BB'}$$



$\cos\theta = \frac{A}{H}$

$\Rightarrow B'X = BB' \cos 45°$

$\Rightarrow B'X = x\left(\frac{1}{\sqrt{2}}\right) \Rightarrow B'X = \frac{x}{\sqrt{2}} \quad \text{--- (4)}$

||ly for $AA'Y$ it is $AY = \frac{x}{\sqrt{2}} \quad \text{--- (5)}$

Substitute eq (3) (4) (5) in eq (1) & (2) we get

$$\text{Elongation strain} = \frac{B'X}{BD} = \frac{x/\sqrt{2}}{(\sqrt{2})L} = \frac{x}{2L}$$

$$\Rightarrow \quad = \frac{\theta}{2} \quad \text{--- (6)}$$

||ly

$$\text{Compressive strain} = \frac{AY}{AC} = \frac{x/\sqrt{2}}{(\sqrt{2})L} = \frac{x}{2L}$$

$$= \frac{\theta}{2} \quad \text{--- (7)}$$

Adding (6) & (7) we get

$$\frac{\theta}{2} + \frac{\theta}{2} = \theta$$

i.e $\boxed{\text{Elongation strain} + \text{compressive strain} = \text{Shearing strain}}$

Relation b/w Y, K, η and σ

Relation b/w Y and α

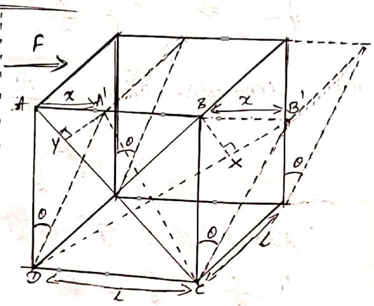
Consider a unit cube and let a unit stress is act along one side, then the extension produced is called "longitudinal strain". and its coefficient is denoted as 'α'

$$Y = \frac{\text{Stress}}{\text{Longitudinal strain}} \quad \text{If } T = 1 \text{ (Unit Stress)}$$

$$\boxed{Y = \frac{1}{\alpha}} \quad \alpha - \text{longitudinal strain coefficient.}$$

Relation b/w Y, η and σ

* Let us consider a cube with each side of the length will be 'L'.
* Let ABCD be one of its face, with DC is fixed.
* Let a tangential force is applied to its upper face, it results that the faces turns through an angle 'θ'



$AC = DB$ - original diagonal

$A'C = DB'$ - change in diagonal.

θ - shearing angle

$AA' = x = BB'$

* As a result, the vertex 'A' moves to A', and B to moves to B' also diagonals DB extends to DB' and AC contracts to A'C
* We know that shearing strain occurs along the plane AB can be treated as equivalent to elongation strain along DB, to DB', and compressive strain along the AC to A'C
* If 'α' and β are the elongation (longitudinal) strain and compression (lateral) strain coefficients produced ~~along~~ per unit stress applied on AB.

Since 'T' is applied stress.

Elongation produced in the diagonal due to tensile stress along 'AB' plane $\Big\} = DB = T\,DB\,\alpha$ — ①

IIly

Compression produced in diagonal due to compressive stress along AB plane $\Big\} = AC = T\,AC\,\beta$ — (ii)

Since $AC = DB$

(ii) $\Rightarrow$ $DB = T\,DB\,\beta$ — ②

Total Extension in the diagonal $DB = DB\,T(\alpha+\beta)$ — ③

Now if we draw the perpendicular

i.e from B to DB' i.e BX $\perp^{r}$ to DB'

& from A' to AC i.e A'Y $\perp^{r}$ to AC

we can easily get how much extends and compress in the diagonals,

For diagonal DB' it is clear that the total extension in the diagonal DB' is approximately equal to B'X

$$③ \Rightarrow \quad B'X = DB\,T(\alpha+\beta) \quad — ④$$

Now from isosceles Δ^{le} BB'X

$$\cos\theta = \frac{B'X}{BB} \qquad \cos\theta = \frac{Adj}{Hyp}$$

$$\Rightarrow B'X = BB' \cos 45^\circ$$

$$B'X = \frac{x}{\sqrt{2}} \quad — ⑤ \qquad \left[\because \cos 45^\circ = \frac{1}{\sqrt{2}}\right]$$

Since cube having equal length in all the sides

diagonal AC = DB

From pythagoras theorem

$$AC = DB = (\sqrt{2})L \quad — ⑥$$

Substitute eq ⑤ & ⑥ in eq ④

$$④ \Rightarrow \frac{x}{\sqrt{2}} = (\sqrt{2})L\,T(\alpha+\beta)$$

$$\frac{x}{TL} = 2(\alpha+\beta) \qquad \left[\because \theta = \frac{x}{L}\right]$$

$$\Rightarrow \frac{\theta}{T} = 2(\alpha+\beta)$$

$$\Rightarrow \eta = \frac{1}{2(\alpha+\beta)} \qquad \left[\because \eta = \frac{T}{\theta}\right]$$

$$\Rightarrow \eta = \frac{1}{2\alpha(1+\frac{\beta}{\alpha})} \qquad \left[\text{WKT } Y = \frac{1}{\alpha},\ \sigma = \frac{\beta}{\alpha}\right]$$

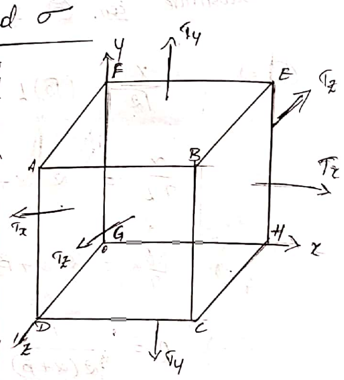
$$\eta = \frac{\frac{1}{\alpha}}{2(1+\beta/\alpha)}$$

$$\Rightarrow \boxed{\eta = \frac{Y}{2(1+\sigma)}} \quad (or) \quad \boxed{Y = 2\eta(1+\sigma)}$$

This is the relation b/w Y, η and σ

Relation b/w Y, K and σ

* consider a unit cube ABCD EFGH, which is homogeneous and isotropic



* let T_x T_y T_z be the stretching force acting $\perp^{re}$ to the faces of the cube, i.e Face, BEHC, AFGD, EFGH, ABCD, ABEF and CDGH respectively

* If α - is the ~~Ela~~ coefficient of Elongation (elongation) i.e increase in length per unit length per unit tension along the direction of force,

Then the elongation produced in the edges AB, BE and BC will be $T_x\alpha$, $T_y\alpha$ & $T_z\alpha$.

11ly If β is coefficient of compression i.e contraction produced per unit length per unit tension in the direction perpendicular to the forces,

Then contraction produces in the perpendicular to the edges AB, BE and BC will be $T_x\beta$, $T_y\beta$ and $T_z\beta$.

If the stress is applied individually to x, y & z direction, then length of the edges will be

Along x-axis $AB = 1 + T_x\alpha - T_y\beta - T_z\beta$

y-axis $BE = 1 + T_y\alpha - T_x\beta - T_z\beta$

z-axis $BC = 1 + T_z\alpha - T_x\beta - T_y\beta$ —— (1)

the volume of the cube can be written as

$AB \times BE \times BC = V_1 \times V_2 \times V_3 = V$

$$\Rightarrow V = (1 + T_x\alpha - T_y\beta - T_z\beta) \times (1 + T_y\alpha - T_x\beta - T_z\beta) \times (1 + T_z\alpha - T_x\beta - T_y\beta) \quad (2)$$

Since α and β are very small ~~here~~ and negligible, the terms which contains either the power of α and β and their products can be neglected.

then (2) $\Rightarrow$ $V = 1 + \alpha(T_x + T_y + T_z) - 2\beta(T_x + T_y + T_z)$ —— (3)

~~Since we applying unit deforming force acting on 3 faces of the cube, $T_x = T_y = T_z = T$. Then eq (3) becomes~~

~~(4) $\Rightarrow$ $V = 1 + 3T(\alpha - 2\beta)$~~

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$$V = 1 + (\alpha - 2\beta)(T_x + T_y + T_z) \quad -③$$

Since we are applying the unit deforming force, acting 3 faces of the cube then ③ becomes

③ ⇒ $$\Delta V = 1 + 3T(\alpha - \beta) \quad -④$$

Here we consider the cube of unit volume. Hence, the increase in the volume is taken by

Volume strain $\Delta V = (\cancel{1} + 3T(\alpha - \beta) - \cancel{1})$

$$\Delta V = 3T(\alpha - \beta) \quad -⑤$$

WKT

Bulk modulus = $\dfrac{\text{compressive stress}}{\text{volumetric strain}} = \dfrac{T}{\Delta V}$

$$\Rightarrow K = \frac{T}{3T(\alpha - 2\beta)} = \frac{1}{3\alpha(1 - 2\beta/\alpha)}$$

$$K = \frac{1/\alpha}{3(1 - 2(\beta/\alpha))} \qquad \because Y = \frac{1}{\alpha},\ \sigma = \beta/\alpha$$

$$\boxed{K = \frac{Y}{3(1-2\sigma)}} \quad (or) \quad \boxed{Y = 3K(1-2\sigma)}$$

This is the relation b/w Y K and σ

Relation b/w K; η and Y

WKT the relation b/w Y η σ and Y K σ

i.e $$\eta = \frac{Y}{2(1+\sigma)} \Rightarrow \frac{Y}{\eta} = 2(1+\sigma) \quad -①$$

||ly $$K = \frac{Y}{3(1-2\sigma)} \Rightarrow \frac{Y}{3K} = (1-2\sigma) \quad -②$$

By adding eq ① & ② we get

$$\frac{Y}{\eta} + \frac{Y}{3K} = 2 + \cancel{2\sigma} + 1 - \cancel{2\sigma}$$

$$\frac{Y(3K+\eta)}{3K\eta} = 3$$

$$\Rightarrow \boxed{Y = \frac{9K\eta}{3K+\eta}}$$ This is relation b/w K, η & Y

Relation b/w K η σ

From above eq ① & ② can also written as

$$Y = 2\eta(1+\sigma) \quad -③ \qquad \& \qquad Y = 3K(1-2\sigma) \quad -④$$

Equating eq no ③ & ④ we get

$$2\eta(1+\sigma) = 3K(1-2\sigma)$$

$$2\eta + 2\eta\sigma = 3K - 6K\sigma$$

$$6K\sigma + 2\eta\sigma = 3K - 2\eta$$

$$\Rightarrow \sigma(6K + 2\eta) = 3K - 2\eta$$

$$\boxed{\sigma = \frac{3K - 2\eta}{6K + 2\eta}}$$

This is the relation b/w K, η & σ.

Limiting value of σ

w.r.t. Limiting value of 'σ' is discussed as follows

w.r.t $Y = 2\eta(1+\sigma)$ — ① and $Y = 3K(1-2\sigma)$ — ②

If we substitute Eq ① in ② (or) ② in ①

we get $2\eta(1+\sigma) = 3K(1-2\sigma)$ — ③

Suppose in the above Eq if 'σ' gives any positive value, then left hand side (LHS) of the above Eq will becomes positive. In above Eq if LHS is positive its RHS should also must be positive. But (RHS will be negative)

~~Suppose~~ RHS is suppose to be positive iff 'σ' does not take a value not more than $\frac{1}{2}$, ~~because~~ $\frac{1}{2}$.

because when 'σ' takes a value more than $\frac{1}{2}$, the RHS of the above Eq ③ will becomes negative.

Therefore 'σ' can take the values less than $\frac{1}{2}$ (or) 0.5

On the other hand, if a negative values is assigned to the 'σ' for example $\sigma = -1$. Then RHS becomes positive, ~~but~~ for LHS σ may equal to '-1' but ~~not~~ σ does not take lesser than -1 [$\sigma = -2, -3, -4$]. if so LHS becomes negative.

Thus the limiting values for σ always lies b/w -1 and 0.5 (or) $\frac{1}{2}$. [-1 to 0.5]

Practical limitation [Not for all materials]

A negative value of 'σ' means that, on being extended, a body a would also expand ~~latter~~ laterally.

Ex (i) Auxetics (Auxetic materials),

* Auxetic polyurethane foam, α-cristobalite.

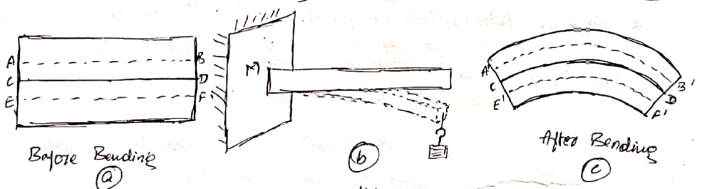
(ii) Rubber ~~cork~~ cork

Bending of Beams

A homogeneous body of uniform cross section whose length is large compared to its other dimensions is called "a Beam"

whenever a beam is subjected to any bending shearing stresses b/w different layers of an atoms comes into play, However, since the beam is long, bending moment becomes too large as compared to shearing stress, Here shearing stresses become negligible,

If an arrangement is made to fix one end of the beam to a rigid support and its other end is loaded, the arrangement is called "Single cantilever"



fig

Neutral Surface:-

"Neutral surface is that layer of a uniform beam which does not undergo any change in its dimensions when the beam is subjected to bending within its Elastic limit",

which is as shown in fig (a) & (c)

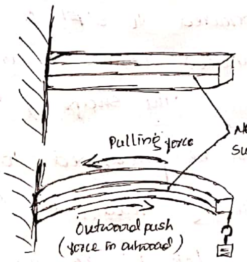
The layer CD is the Neutral surface

Neutral axis

Neutral axis is a longitudinal line along which the neutral surface is intercepted by any longitudinal plane considered in the plane of bending'

When the beam is bent, all its layers which are above the neutral surface CD undergo Elongation, where as those below the neutral surface are subjected to compression, which is as shown in the fig (c)

At this ~~mom~~ situation, the beam develops an inward pull towards the fixed end for all the layers above the neutral surface, and, an outward push directed away from the fixed end for all the layers below the neutral surface.



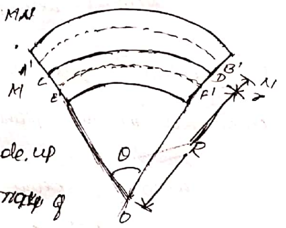
These two groups of forces result in a restoring couple which balances the applied couple, acting on the beam.

The moment of ~~of~~ the restoring couple is called the "Restoring moment".

The moment of applied couple subjected to which the beam undergoes bending longitudinally is called the "Bending moment"

The bending moment and restoring moments are Equal only when the beam is equillibrium position,

Expression for Bending moment



Consider a long uniform beam, MN whose one end is fixed at 'M' and other end is subjected loaded. The beam can be thought of made up of no. of parallel layers (made by array of atoms) AB, CD, EF,

If we loaded at 'N' the successive layers are strained. Such that AB is elongated to A'B' & EF is contracted to E'F' & CD is remains unchanged (neutral surface) which is as shown in the fig.

The shape of different layers of the bent beam can be imagined to form part of concentric circles of varying radii as shown in the fig.

Let 'R' be the radius of the circle from the neutral surface., its length can be expressed as

Arc length of CD $= R\theta$. θ - common angle for all the layers

Now the layer AB is elongated to A'B'

then change in length $= A'B' - AB$

The arc length of $A'B' = (R+r)\theta$

Before we apply the load $AB = CD = EF$.

...Since $CD = R\theta$ ∴ original length $AB = R\theta$

Now change in length becomes $\Big\} = (R+r)\theta - R\theta$

$$= r\theta$$

WKT linear strain $= \dfrac{\text{change in length}}{\text{original length}}$

$$\text{linear strain} = \frac{r\theta}{R\theta} = \frac{r}{R} \quad \text{— ①}$$

also WKT young's modulus $= Y = \dfrac{\text{longitudinal stress}}{\text{linear strain}}$

on rearranging

$$\Rightarrow \text{longitudinal stress} = Y\left(\frac{r}{R}\right) \quad \text{— ②}$$

we can define stress $= \dfrac{F}{a}$ — ③

F - force acting on the beam
a - area of the layer AB.

Eq ③ $\Rightarrow \dfrac{F}{a} = Y\left(\dfrac{r}{R}\right) \Rightarrow F = \dfrac{Yar}{R}$

Now moment of force about the neutral axis $\Big\} =$ Force $\times$ {$\perp^{r}$ distance from neutral axis}

$$= F \times r$$

$$= \frac{Yar^2}{R} \quad \text{for one layer.}$$

∴ the moment of force acting on the entire beam $\Big\} = \Sigma \dfrac{Yar^2}{R}$

$\Rightarrow$ Moment of force on Entire beam $\Big\} = \frac{Y}{R} \Sigma a r^2$ — (4)

WKT moment of Inertia of a body about given axis is given by $I = \Sigma m r^2$, Σm mass of the body

$^{||}$ly $\Sigma a r^2$ is called geometric moment of inertia "I_g"

$\therefore I_g = a r^2 = A k^2$ — A - area, k - radius of gyration

$\therefore$ $\boxed{\text{Moment of force} = \frac{Y}{R} I_g}$ (5)

$\boxed{\text{Bending moment} = \frac{Y}{R} I_g}$

I_g varies with the geometrical shape of the beam

(i) Bending moment for a beam of rectangular cross section

$$BM = \frac{Y}{R}\left(\frac{bd^3}{12}\right)$$

b - Breadth & d - Thickness

(ii) Bending moment for a circular ~~(…)~~ cross section

$$BM = \frac{\pi Y}{4R} \rho^4$$

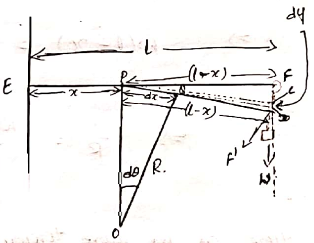
ρ - radius of the beam

Single cantilever

A cantilever is a beam which is fixed horizontally at one end and a load is applied at other end. Which is as shown in the fig.

One can determine the youngs modulus of the materials of a cantilever by knowing the value of depression produced in the cantilever.

Theory of the cantilever



~~(…)~~ Let us consider a cantilever, 'EF' represents the equilibrium position of the cantilever, of length 'l' fixed at one end 'E' and loaded ~~at~~ with a load 'W' at 'F' as shown in the fig.

Let the end "F" be depressed to the position "F'" under the action of the load

Consider a section of beam 'PQ' i.e 'P' at a distance 'x' from the fixed end 'E' then the moment of couple at this point 'P' due to the load 'W' is given by Bending moment,

i.e

Bending moment = Force × perpendicular distance.

Bending moment } = $w \times PF'$

$BM = w(l-x)$ — ①

For a small negligible depression made by F to F',

PF' is almost ⊥le to FW; ∴ The beam looks at Equilibrium.

WKT under Equilibrium condition

Bending couple = Restoring couple.

$$\Rightarrow \quad w(l-x) = \frac{YI}{R}.$$

R - radius of curvature of neutral axis 'P'

$$\Rightarrow \quad \frac{1}{R} = \frac{w(l-x)}{YI_g} \quad \text{— ②}$$

I_g - geometric MI.

Here as we move towards the fixed end 'E', the moment of couples increases, Hence the radius of curvature will be different at different point and it decreases as we approaches to the Point 'E'.

Now consider another point 'Q' at a distance 'dx' from 'P'. The radius of curvature at 'Q' is same as that of point 'P'.

Since the point 'Q' is very closer to the point 'P'

Its length will becomes

Arc length of PQ = $R\,d\theta$

(or) $\frac{dx}{R} = d\theta$ — ③

Substitute eq ② in ③

$$③ \Rightarrow \quad d\theta = \frac{w(l-x)\,dx}{YI} \quad \text{— ④}$$

Now to know about the depression we have to draw the tangent from 'P' and 'Q' meeting the verticle line FF' at C & D respectively

Then depression 'dy' of 'Q' below 'P' is given by

Arc length CD = $dy_0 = (l-x)\,d\theta$ — ⑤

$d\theta$ - angle b/w the tangents.

Now substitute eq ④ in ⑤.

$$dy_0 = (l-x)\left[\frac{w(l-x)}{YI}\right]dx$$

$$dy_0 = \frac{w}{YI}(l-x)^2\,dx$$

Now we can get the total depression of 'F' by Integrating the above eq b/w the limits zero (0) to 'l'

∴ Total depression = $FF' = y_0 = \int_0^l \frac{w}{YI}(l-x)^2\,dx$

$$y_0 = \frac{w}{YI}\int_0^l (l^2+x^2-2xl)\,dx$$

$$Y_0 = \frac{W}{YI}\left[l^2x + \frac{x^3}{3} - \frac{2lx^2}{2}\right]_0^l$$

$$= \frac{W}{YI}\left[\cancel{l^3} + \frac{l^3}{3} - \cancel{l^3}\right]$$

$$\boxed{Y_0 = \frac{Wl^3}{3YI}}$$ This is Expression for depression made by the cantilever.

If we rearrange the above eqn we get

$$\boxed{Y = \frac{Wl^3}{3Y_0 I}}$$ This is the Expression for Young's modulus of the material.

Case (i) For rectangular crossection $I = \frac{bd^3}{12}$

then above eqn $Y = \frac{Wl^3}{3Y_0}\left(\frac{12}{bd^3}\right)$ $[\because W = mg]$

$$\boxed{Y = \frac{4mgl^3}{Y_0 bd^3}}$$

Case 2 For circular crossection $I = \frac{\pi r^4}{4}$

$$\boxed{Y = \frac{4mgl^3}{3Y_0 \pi r^4}}$$ r - radius of circular rod

Different types of beams and their Engineering applications

There are four types of beams

1) Simple beam :- A bar resting up on supports at its Ends

2) continuous beam :- A bar resting upon more than two supports

3) Cantilever beam :- A bar whose one End is fixed, & other End is free.

4) Fixed beam :- A bar whose both the Ends are fixed.

Applications

i) In the fabrication of trolley ways

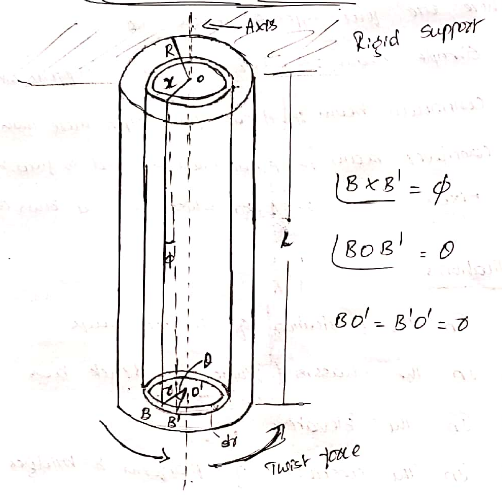
2) In the chassis / frame as truck beds

3) In the Elevators

4) In the construction of platform & bridges

5) Construction of Railway tracks

Torsion of a Cylinder



Consider a long cylindrical rod of length 'L' and radius 'R', rigidly fixed at its upper end. Let OO' be axis of cylindrical rod.

Imagine that this cylindrical rod is made up of concentric hollow cylindrical layer of thickness "dr".

If the rod is twisted at its lower end then the concentric layers slides one over the other.

Let us consider one concentric circular layer of radius 'r' and its thickness is 'dr'.

Now a point 'x' on top portion which is fixed to the rigid support. and the point 'B' at the bottom end which is shifted to 'B'' due to twist force.

$\therefore$ Here $\angle BxB' = \phi$ ϕ - Shearing angle & is very small.

Here we can get the arc length i.e BB'

$\therefore$ Arc length $= BB' = L\phi$ — ①

Also if $\angle BO'B' = \theta$

Then Arc length $= BB' = r\theta$ — ②

from eq ① & ② we get

$\Rightarrow L\phi = r\theta \Rightarrow \phi = \frac{r\theta}{L}$ — ③

W.K.T the cross-sectional area of this circular layer will be

$a = 2\pi r\, dr$.

Now shearing stress is defined as

$$\text{Shearing stress} = T = \frac{\text{Force}}{\text{area}} = \frac{F}{2\pi r\, dr}$$

$\Rightarrow F = T(2\pi r\, dr)$ — ④

W.K.T Regidity modulus $= \eta = \frac{\text{Shearing stress}}{\text{Shearing strain}} = \frac{T}{\phi}$

$\Rightarrow T = \eta\phi$

from Eq ③ $\tau = \frac{\eta r \theta}{L}$ — ⑤

Now Eq ⑤ in ④ we get.

④ ⇒ $F = \frac{\eta r \theta}{L} (2\pi r\, dr)$

$F = \frac{2\pi\eta\theta}{L} r^2 dr$

Now moment of force about OO' is given by

$$\left.\begin{matrix}\text{moment of}\\ \text{force}\end{matrix}\right\} = \text{Force} \times \perp^{\text{le}} \text{ distance}$$

$$= \frac{2\pi\eta\theta}{L} r^2 dr \times r$$

This moment of force is also known as Twisting couple.

⇒ Twisting couple $= \frac{2\pi\eta\theta}{L} r^3 dr$

This is twisting couple acting on only one layer.

Suppose this twisting couple acting on the entire cylinder., for this we have integrate above Eq b/w the limits '0' to 'R'

$$\therefore \left.\begin{matrix}\text{Twisting}\\ \text{couple}\end{matrix}\right\} = \int_0^R \frac{2\pi\eta\theta}{L} r^3 dr \Rightarrow \frac{2\pi\eta\theta}{L}\int_0^R r^3 dr$$

$$= \frac{2\pi\eta\theta}{L}\left(\frac{r^4}{4}\right)_0^R$$

$$\Rightarrow \left.\begin{matrix}\text{Twisting}\\ \text{couple}\end{matrix}\right\} = \tau = \frac{\pi\eta R^4\theta}{2L}.$$

Finally couple per unit twist is given by

$$\left.\begin{matrix}\text{Couple per}\\ \text{unit twist } (C)\end{matrix}\right\} = \frac{\text{Total twisting couple } (\tau)}{\text{Angle of twist } (\theta)}$$

$$C = \frac{\tau}{\theta} = \frac{\frac{\pi\eta R^4\phi}{2L}}{\phi}$$

$$\boxed{C = \left[\frac{\pi\eta R^4}{2L}\right]}$$

This is the expression for couple per unit twist

Torsional Pendulum

Consider a straight uniform wire whose end is fixed to a ~~sto~~ rigid support and to its other end a rigid body is attached. under the application of twist force, the body executes to & fro turning motion about its axis, ~~which~~ this setup is called "Torsional pendulum" and the oscillations are called "Torsional oscillation".

The time period of the torsional pendulum is given by

$$\boxed{T = 2\pi\sqrt{\frac{I}{C}}}$$

I - moment of inertia of the rigid body

C - couple per unit twist.

Applications of Torsional Pendulum

* One can find the moment of inertia of irregular bodies using torsional pendulum experiment

* The rigidity modulus of a material of wire can be found by setting up a torsional pendulum experiment