Module III

Optical fibers

Total Internal Reflection:

When a ray of light travels from denser to rarer medium it bends away from the normal. As the angle of incidence increases in the denser medium, the angle of refraction also increases. For a particular angle of incidence called the "*critical angle* (θ_c)", the refracted ray grazes the interface of the two media or the angle of refraction becomes equal to 90°. If the angle of incidence increases a little beyond the critical angle, the light ray is reflected back to the same medium. This is called "*Total Internal Reflection*". In total internal reflection, there is no loss of energy. The entire incident ray is reflected back. In the figure, XX¹ is the interface separating medium of refractive index n and medium of refractive index n and medium of refractive index.

medium of refractive index n_1 and medium of refractive index n_2 , with $n_1 > n_2$. AO and OA1 are incident and refracted rays. *i* and *r* are angle of incidence and angle of refraction, with i > r. For the ray BO, θ_c is the critical angle. OB1 is the refracted ray which grazes the interface. The ray CO incident with an angle greater than θ_c is totally reflected back along OC1. For the incidence at A, From Snell's law, we have $n_1 \sin i = n_2 \sin r$ For total internal reflection, $i = \theta_c$ and $r = 90^\circ \therefore n_1 \sin i = n_2$



This is the expression for critical angle.

Optical Fibers:

Optical fibers are thin rods of glass or some other transparent material of high refractive index. If light is admitted at one end of a fiber, it can travel through the fiber with very low loss, even if the fiber is

 $\therefore \theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right)$

curved. A fibre optic cable is made from high refractive index transparent material coated with another transparent material of relatively low refractive index. The low R.I. coat reflects 'escaping' light back into the core, resulting in the light being guided along the fibre through the phenomenon of T,I,R. The outside of the fibre is protected by cladding and may be further protected by additional layers of PVC or polyurethane jacket. Number of such fibers is grouped and moulded to form a cable. This required for protecting the fibre from mechanical deformation. They are used in optical communication. It works on the principle of Total internal reflection (TIR).



Medium 1 (n_1)

Propagation mechanism:

The light entering through one end of core strikes the interface of the core and cladding with angle greater than the critical angle and undergoes total internal reflection. After series of such total internal reflection, it emerges out of the core. Thus the optical fiber works as a waveguide. Care must be taken to avoid very sharp bends in the fiber because at sharp bends, the light ray fails to undergo total internal reflection.



Angle of Acceptance and Numerical Aperture:

Referring to the figure given bellow, the ray which is incident on the core surface along the axis of the fiber, pass through the core without any deviation (:: i = 0, hence r = 0). The ray incident on the core surface

with smaller angle of incidence refracts with smaller angle of refraction and falls on the core-cladding interface at an angle greater than the critical angle and hence ensures TIR. The ray incident on the core surface with greater angle of incidence refracts with greater angle of refraction and falls on the core-cladding interface at an angle lesser than the critical angle and hence refracts in to cladding. There exists a particular angle of incidence, called ' θ_a ' for which the refracted ray falls on the core- cladding interface at an angle equal to critical angle and it grazes the core cladding interface after refraction. This angle is called acceptance angle.



Acceptance angle: It is the maximum limit for the angle of incidence of the incident ray such that the ray refracted in to the core grazes the interface as it falls on the core-cladding interface.

Acceptance cone: The ray incident on the core surface at an angle equal to acceptance angle when rotated about the axis of the fiber forms a cone. This cone is called acceptance cone. The rays which are funnelled through the acceptance cone ensure TIR at the core cladding interface. The rays entering in to the core outside of this acceptance cone are trapped at the cladding because of the refraction at core-cladding interface.

Numerical Aperture: Numerical aperture of an optical fiber is the ability of its light gathering power. It is numerically equal to the sine of the acceptance angle (θ_a).

Expression for Numerical aperture and Condition for propagation:

Consider a ray of light in a medium of RI 'n₀' entering in to a fiber having a core of RI 'n₁' and cladding of RI 'n₂' at a point "O" on the core surface. The ray OA incident at O, at an angle θ_a refracts in to the core at an angle θ_1 and falls on the core-cladding interface at an angle θ_c at B and grazes the interface along BC after refraction.



For the refraction at 'O', Snell's law can be written as $n_0 \sin \theta_a = n_1 \sin \theta_1$

Similarly For the refraction at 'B', Snell's law becomes $n_1 \sin \theta_c = n_2 \sin 90^\circ$ But we have $\theta_c = (90^\circ - \theta_1)$ \therefore $n_1 \sin (90^\circ - \theta_1) = n_2$

Or
$$\cos \theta_1 = \left(\frac{n_2}{n_1}\right)$$

 $\Rightarrow \sin \theta_1 = \sqrt{1 - \left(\frac{n_2}{n_1}\right)^2} = \sqrt{\frac{(n_1^2 - n_2^2)}{n_1^2}} \dots (2)$
 \therefore Eqn. (1) becomes $\sin \theta_a = \left(\frac{n_1}{n_0}\right) \cdot \sqrt{\frac{(n_1^2 - n_2^2)}{n_1^2}}$
 $\Rightarrow \text{ N. A. = } \sin \theta_a = \frac{\sqrt{(n_1^2 - n_2^2)}}{n_0}$ (This is the expression for the Numerical aperture.)
If the surrounding medium is air then N . A . = $\sin \theta_a = \sqrt{(n_1^2 - n_2^2)}$ (3)
Note: (i) The condition for the effective propagation of light through the fiber is $\theta_1 \leq \theta_a$
Or $\sin \theta_1 \leq \sin \theta_a$
Or $\sin \theta_1 \leq \sin \theta_a$

(ii) **Fraction Index Changes** (Δ): The fractional index change is the ratio of the difference in the refractive index between the core and the cladding to the refractive index of core of the optical fiber.

Therefore,
$$\Delta = \frac{(n_1 - n_2)}{n_1}$$

(iii) **Relation between N.A. and** Δ :

From the definition of fractional index change, $(n_1 - n_2) = = n_1 \Delta$ ------ (1) From the expression for numerical aperture N.A. = $\sqrt{(n_1^2 - n_2^2)} = \sqrt{(n_1 + n_2)(n_1 - n_2)}$ ----- (2)

Since $n_1 \cong n_2$, $n_1 + n_2 = 2n_1$, **N**. **A**. = $\sqrt{2n_1\Delta}$ ------(3)

This is the relation between N.A. and Δ . Though an increase in the value of Δ increase NA (and thus one can enhance the light gathering capacity of the fiber); we cannot increase Δ to a very large value, since it leads to what is called "intermodal dispersion" which causes signal distortion.

Modes of propagation and V-number

The number of modes of propagation refers to the number of possible paths for the rays propagating through the core. The number of modes that supports for propagation in the fibre is determined by a parameter called V – number.

$$V = \left(\frac{\pi d}{\lambda}\right) \left[\frac{\sqrt{(n_1^2 - n_2^2)}}{n_0}\right]$$

where λ is the wavelength of light propagating through the fibre and **n**₀ is the RI of surrounding medium, *d* is core diameter; n₁ and n₂ are the RI of the core and cladding respectively.

If the surrounding medium is air, then the V –numberis given by

$$V = \left(\frac{\pi d}{\lambda}\right) \sqrt{(n_1^2 - n_2^2)} = \left(\frac{\pi d}{\lambda}\right) N.A.$$

For V \gg 1, the number of modes supported by the fibre is approximately $N = \frac{V^2}{2}$



- Step index single mode fibers have a thin core of diameter $8 10\mu m$ with a thick cladding of diameter $60 70\mu m$.
- All the Single mode fibers are step index only. The RI of the core and cladding remain constant with a small percentage of fractional index change (Δ).
- These fibers support the transmission by single mode ie. V number < 2.4.
- Since the core diameter is small, the NA is small (NA < 0.3).
- The attenuation is relatively low. Since they transmit the signals in single mode, the transmission is free from intermodal dispersion. Therefore the o/p signal is almost identical to that of input pulse. ie. There is no pulse stretching.
- Because of their low NA these fiber require laser as light source and are used in long distance communications. Eg. In submarine cables.
- They are cheaper compared to other fibers. Almost 80% of the fibers manufactured are SMF. But because of their low core diameter, **splicing** is difficult.

(b) Step index Multimode fibers (MMF):



- Step index Multimode fibers have a core of diameter of about50 100μm with a cladding of diameter 100 - 250μm.
- The RI of the core and cladding remain constant with a small percentage of fractional index change (Δ).
- These fibers support the transmission by multimode ie. V number >2.4.
- Since the core diameter is large, the NA is small (NA > 0.3).
- The attenuation increases with the increase in core diameter. Pulse stretching arises as a result of intermodal dispersion.

- Since NA is better, LED can be used as light source and are used in short distance communications. Eg. In LAN network cables, and CCTV and CATV(Community Antenna Television eg.Cable TV network)
- Connectors are Cheaper because of large core diameter. Hence **splicing** is easy.

Graded index Multimode fibers (GRIN):



- GRIN fibers have a core of diameter of about $50 100 \mu m$ with a cladding of diameter $100 250 \mu m$.
- The RI of the core is maximum at the axis and falls gradually away from the axis of the fiber, however cladding remains constant.
- All the GRIN fibers are multimode fibers only ie. V number > 2.4.
- NA is a function of RI of core and fractional index change. The refractive index of the core vary with

the radial distance according to the relation $n(r) = n_1 \sqrt{2\Delta \left[1 - \left(\frac{r}{a}\right)^2\right]}$

- The attenuation is relatively high and pulse stretching arises as a result of intermodal dispersion.
- Either LED or a laser can be used as light source.
- They are the costliest among all the fibers.
- Connectors are costlier because of the variation in the RI of the core along the radial distance. Hence **splicing** is difficult.

Attenuation in Optical fibers:

"Loss of power suffered by the signal as it propagate along the length of the fiber is called attenuation." Attenuation in optical fibers may be due to either absorption or scattering or radiation.

1. Absorption losses:-

- Any homogeneous transparent medium have a tendency to absorb photons from the incident radiation. Such an absorption by a homogeneous medium which is free from all the impurities is called **intrinsic absorption**.
- Absorption of photons by impurities like metal ions such as iron, chromium, cobalt and copper in the silica glass, of which the fiber is made of, is called **absorption due to impurities.**

During signal processing photons interact with electrons of impurity atoms and excite the atoms. During the d de-excitation, the photons may be emitted either spontaneously or stimulatively. Such spontaneously emitted photons interfere destructively. Hence it is a loss of energy.

The other impurity such as hydroxyl ions (OH) causes significant absorption loss.

2. Scattering losses:

- When the wavelength of the photon is comparable to the size of the particle then the Rayleigh scattering takes place.(whose cross-section is inversely proportional to λ^4 .)
- Because of the non-uniformity in manufacturing, sharp variations in the refractive index leads to scattering.
- Scattering of photons also takes place due to trapped gas bubbles, some unreacted starting materials and any impurity such as iron, chromium, cobalt and copper present in the core.

3. Radiation losses: Radiation losses occur due to macroscopic bends and microscopic bends.

• **Macroscopic bending:** All optical fibers are having critical radius of curvature provided by the manufacturer. Such macroscopic bending arises due to wrapping of the fiber over a spool, turning the fiber

around the corners. If the fiber is bent below that critical radius of curvature, the light ray incident on the core cladding interface may not satisfy the condition of TIR and hence enter in to cladding. This causes loss of optical power.

• **Microscopic bending:** Optical power loss in optical fibers is due to non-uniformity of the axis of the optical fibers when they are laid. Non uniformity arises due to manufacturing defects or mechanical strain or lateral pressure built up on the fiber. Light rays falling on such non linear core cladding interface may not satisfy the condition of TIR and hence may get trapped in the cladding. The non-uniformity of the core cladding interface due to mechanical injuries can be overcome by introducing optical fiber inside a polyurethane jacket.

Expression for the attenuation coefficient:

The attenuation of light signal in the optical fiber takes place according to Lambert's Law. The law states

that the loss in the power of light signal as it propagate along the length of the fiber is proportional to its initial

intensity. ie
$$\frac{-dP}{dL} \propto P$$

Or $-\frac{dP}{dL} = \alpha P$

Where, the proportionality constant α is called the attenuation coefficient.

On rearranging the above equation, we have
$$-\frac{dP}{P} = \alpha \, dL$$

Integrating on both sides $\int_{P_{in}}^{P_{out}} \frac{dP}{P} = -\alpha \int_{0}^{L} dL$

$$\therefore \qquad \log\left(\frac{P_{out}}{P_{in}}\right) = -\alpha \ L$$

Or
$$\alpha = -\frac{1}{L} \log \left(\frac{P_{out}}{P_{in}} \right)$$
 Bell/Km

Bell / Km is a lrge unit to express attenuation. Since attenuation shold be very small for the efficient transmission of signals, it is expressed in decibel per Kilometer

$$\therefore \qquad \alpha = -\frac{10}{L} \log \left(\frac{P_{out}}{P_{in}} \right) \quad dB/Km$$

This is the expression for attenuation coefficient.

Note: (i) The length of the fiber is always measured in Km.

(ii) The logarithm is in base 10.

Block diagram of point to point fiber optic communication system:

- The microphone in the telephone receiver converts voice in to equivalent analog electrical signals.
- The analog signals are converted in to digital signals using a coder.
- A transmitter is a semiconductor diode laser which emits light according to the digital binary input. The light emitted from the laser source is launched on the optical fiber.
- The information in the form of light can be transmitted over large distances. If necessary, repeaters can be used. A repeater receives the signal and amplifies it and transmit again.
- At the receiving end the photodiode convert the received light in to equivalent binary electrical signals.
- The decoder converts binary electrical signal in to analog electrical signals.
- The loud speaker in the handset produces sound waves to convey the voice information.



Module V

DIELECTRIC AND MAGNETIC PROPERTIES OF MATERIALS

Dielectrics are those materials, which have the ability to get polarized in the presence of the electric field thus increasing the ability of charge storing capacity of capacitor. They do not allow the electric current to flow through them but they transmit electric effects without conduction . Examples Wood, Glass, Rubber, Paper, Ceramics

Capacitance of a capacitor is given by $Co = \frac{Qo}{Vo}$ when a capacitor is charged to a potential V_o, Q_o is the magnitude of charge on either plate when the dielectric is air or vacuum.



In the presence of the dielectric having dielectric constant ε_r capacitance increases from C_o to C, where $Co = \frac{Qo}{V}$ where V <<<V_o.

Say
$$V = \frac{V_o}{\varepsilon_r}$$

$$Co = \frac{Qo}{Vo}$$
. ε_{r}
C = C₀ $\varepsilon_{r.}$

Dielectric increases the charge storing ability of the capacitor. It permits the capacitor to store ϵ_r times more charges for the same potential V_o .

Let σ be the surface charge density on the plates of the capacitor with air or vacuum between plates.

From Gauss theorem
$$E_o = \frac{\sigma}{\varepsilon_o}$$

Where E_o is the electric field between the plates. Also $E_o = \frac{V_o}{d}$, where d is the separation between the plates.

Let σ_p be the induced surface charge density

$$E_i = \frac{\sigma_p}{\varepsilon_o}$$

Resultant field in the dielectric is given by

Relative permitivity of the medium ε_r is given by

$$\varepsilon_r = \frac{C}{C_o} = \frac{E_o}{E}$$
Now $E = \frac{E_o}{\varepsilon_r} = \frac{\sigma}{\varepsilon_o} * \frac{1}{\varepsilon_r} = \frac{\sigma}{\varepsilon_o \varepsilon_r} = \frac{\sigma}{\varepsilon_o}$

$$\varepsilon = \varepsilon_0 \varepsilon_r$$

from equation (a),

$$\frac{\sigma - \sigma_p}{\varepsilon_o} = \frac{\sigma}{\varepsilon_o \varepsilon_r}$$
$$\sigma_p = \sigma \left[1 - \frac{1}{\varepsilon_r} \right].$$

 ϵ_r is the dimensionless constant which is greater than unity. Hence the $\sigma_p \ll \sigma$. Implies that induced charges decrease the field within the dielectric. Hence $E \ll E_0$.

A dipole is an entity in which equal positive and negative charges are separated by a small distance. The dipole is induced in the atom due to the action of external electric field. Though a dipole as a whole is electrically neutral the induced dipole sets up its own electric field which is opposite in direction to the external field. The product of magnitude of the either charge and distance of their separation is called the dipole moment μ of the electric field. $\mu = qd$, μ is a vector directed along the axis of the dipole from negative to positive charge. When an atom or molecule is placed in electric field, the field tends to displace the equilibrium position of the bound charges as a result of which dipole moment is induced in the molecule. The amount of induced dipole moment is proportional to the field strength. The larger the field greater the displacement of charges and hence larger the induced dipole moment. Hence $\mu = \alpha E$. where, α is a constant of proportionality called as polarizability of the molecule. It characterizes the capacity of the electric charges in a molecule to suffer displacement in an external field. Unit of polarizability α is F.m² in SI system. The induced dipole moment vanishes as soon as the electric field is switched off.

Permanent dipoles:

In some molecules the centers of gravity of the positive and negative charges of opposite sign are separated even in the absence of an external electric field. Such molecules are said to have intrinsic dipole moment and carry permanent dipoles. When such molecule is placed in uniform electric field exerts a force +qE on the charge +q and -qE on -q.



The net force on the dipole is zero. Since the two forces acting on it are equal and opposite to each other. Therefore there is no translational force on the dipole in a uniform field. However the two forces are ant parallel and constitute a couple which tands to constitute the dipole. The torque on the dipole is given by

$$\tau = qEd\sin\theta$$
$$\tau = \mu^* E$$

Hence dipole in a uniform electric field does not undergo translational motion but rotates in attempt to align with the field direction. Infact a free dipole aligns its axis with field direction.

Polar and Nonpolar dielectrics:

Dielectrics are broadly classified in to two major groups on the basis of dipole moment.

Non polar dielectrics: In the molecules of some materials the effective center of the negative charge distribution coincides with the effective center of the positive charges thus neutralizing each other effect. Such materials are called nonpolar dielectrics.

Example : All symmetric molecules.

Monoatomic molecules He, Ne, Ar, and Xe molecules consisting of two identical atoms linked by a covalent bond such as H_2 , N_2 and Cl_2 .

Polar dielectrics : Dielectric materials in which the effective centers of the negative and positive charges in the molecules do not coincide with each other even in the absence of any external field. Each molecule behaves as though it consists of a pair of negative and positive charges separated by a small constant distance. Molecule is said to have permanent intrinsic dipole. Materials comprising of such dipoles are called polar dielectrics.

Examples : The molecules of ionic compounds with polar bond such as KI, HI, HBr.

Dipole moment of CO_2 is zero. It is possible only if the molecule is linear (O = C = O) and the dipole momenta of the two C = O bonds being equal and opposite cancel each other. Dipole moment of $H_2O = 1.85$ D. This shows that molecule is not linear. It has bent structure H_2O is example for polar dielectric. SO_2 ; NH_3 polar.

Polarization :

_ Displacement of charges in the molecules of a dielectric under the action of applied electric field leading to the development of dipole moment is known as dielectric polarization.

Consider an electrically neutral dielectric slab inserted between the plates of a parallel plate capacitor. Dielectric is imagined to be divided into a large number of identical cells of volume dV. Under the action of external electric field charges are induced in each cell and cell acquires a dipole moment $d\mu$.

Then intensity of polarization P is defined as the total dipole moment per unit volume of the material

$$P = \Sigma \frac{d\mu}{dV} = \frac{\mu}{V}$$

Magnitude of Polarisation P is directly proportional to the intensity of electric field.

 $P = \chi \varepsilon_o E$

 χ is known as dielectric susceptibility of the material which charesterises the ease with which a dielectric can be polarized under the action of the external field.

 χ is dimensionless constant.



Polarized dielectric is equivalent to a big dipole consisting of polarization charges separated by a distance d which is the thickness of the slab.

The dipole moment of the entire slab is given by $\mu = (A\sigma_p)d$ Where A is the area of the slab in m². d is the thickness in meters. V is the volume of the slab in m³. $\mu = \sigma_p V$

 $P = \sigma_p$

Polarization is equal to surface density of induced charges in a dielectric. Polarization is vector directed from negative charges to positive charges. Polarization produces in the dielectric a field E_i opposite to external field.

Relation between P,χ & ε₀

Resultant field within dielectric after polarization $E = E_o - E_i$

 $E = \frac{\sigma}{\varepsilon_0} - \frac{\sigma_p}{\varepsilon_0}$ Where σ free surface charge density σ_p induced surface charge density

 $E = \frac{\sigma - \sigma_p}{\varepsilon_o}$ E\varepsilon_0 = \sigma - \sigma_p where P = \sigma_p (proved)

$$P = \sigma - E\varepsilon_{o} - - - - - (A) \quad \varepsilon_{r} = \frac{\varepsilon_{0}}{E}$$

$$P = \varepsilon_{o}\varepsilon_{r}E - E\varepsilon_{o}$$

$$P = \varepsilon_{o}(\varepsilon_{r} - 1)E \qquad E = \frac{E_{0}}{\varepsilon_{r}}$$

$$E = \frac{\sigma}{\varepsilon_{0}} * \frac{1}{\varepsilon_{r}}$$

$$P = \varepsilon_{\chi}E \qquad \sigma = \varepsilon_{0}\varepsilon_{r}E$$
where,
$$\frac{\chi = \varepsilon_{r} - 1}{\varepsilon_{r}} = 1 + \chi.$$

Introduces a displacement vector D, magnitude of D is equal to the surface density of free charges, $D = \sigma$. D is also known as Electric Induction or electric flux density.

$$D = E\varepsilon_o = \sigma$$

From equation (A)
$$P = D - E\varepsilon_o$$

Different Polarization Mechanism

Four types of polarizations are commonly distinguished,

- 1. Electronic Polarization.
- 2. Ionic Polarization.
- **3.** Orientation Polarization.

Electronic Polarization:



It is due to the displacement of electron clouds of atoms, molecules and ions with respect to heavy nuclei to a distance less than atomic dimensions. It sets in over a very short period time of the order of 10^{-14} to 10^{-15} s. it is independent of temperature.

 $P_e = N \alpha_e E$, where α_e is Electronic Polarizability.

Contribution of P_e to ε_r is obtained as follows,

$$\varepsilon_{r} = 1 + \psi$$
$$= 1 + \frac{P_{e}}{\varepsilon_{o}E}$$
$$= 1 + \frac{N\alpha_{e}E}{\varepsilon_{o}E}$$
$$\varepsilon_{r} = 1 + \frac{N\alpha_{e}}{\varepsilon_{o}E}$$

This above expression gives ε_r for Nonpolar gaseous substance. For monoatomic gas

$$\alpha_e = 4\pi\varepsilon_o R^3$$
$$\varepsilon_r = 1 + 4\pi N R^3$$
Where R is atomic radius.

Ionic Polarization:



It occurs in ionic crystals. Example NaCl, CsCl. It is due to elastic displacement of positive and negative ions from their equilibrium positions. In NaCl crystal, Na⁺ ions are bound to Cl⁻ ions through ionic bonds. When field is applied Na⁺ and Cl⁻ ions are displaced in opposite directions until ionic bonding forces stop the process. If the field is reversed, ions come closed and the dipole moment again undergoes a change. $\mu_i = \alpha_i E$

where α_i = ionic polarizability. Ions experience electronic polarization also. $\alpha_i = \frac{1}{10} \alpha_e$. It takes 10⁻¹¹ to 10⁻¹⁴ s to build up. It is independent of temperature.

Orientation Polarisation:

It is a characteristic of polar dielectrics. In the absence of electric field, the orientation of dipoles is random resulting in a complete cancellation of each others effect. In the presence of field molecular dipoles rotate about their axis of symmetry to align with the applied field. Dipole rotation is counteracted by thermal agitation. Greater the temperature, high will be the thermal agitation. So the dipoles can rotate through a small angle.



Even in liquids and gases, where molecules are free to rotate, complete alignment is not achieved due randomizing effect of the temperature. In solids the rotation of polar molecules may be highly restricted by the lattice forces leading to great reduction in their contribution to orientation polarization. Because of this reason ε_r of water is 80

And that of solid is 10. It is strongly temperature dependent, build up time is of the order 10^{-10} s.

$$\alpha_o = \frac{\mu^2}{3kT}$$
$$P_e = \frac{N\mu^2 E}{3kT}$$

Internal fields in solids and liquids.

Internal field or local field E_i is defined as the electric field acting at the location of a given atom & is given by the sum of the electric fields created by the neighbouring atoms and the applied field.

 $E_i = E + E^I$

 E^{I} is the resultant field acting at a dipole due to all other surrounding dipoles. To calculate this E^{I} , consider a one dimensional solid consisting of a string of equidistant identical atoms each of polarizability α_{e} .



Let the inter atomic distance of each of the atomic dipole be 'd' and dipole moment be μ . The resultant field at X due to all other dipoles can be determined as follows.

The components of the electric field at P due to the atomic dipole in polar form are given by

$$E_r = \frac{1}{4\pi\varepsilon_o} \frac{2\mu\cos\theta}{r^3}$$
$$E_\theta = \frac{1}{4\pi\varepsilon_o} \frac{\mu\sin\theta}{r^3}$$

Where r is the distance between the points O and P. OP = r and θ is the angle between r and μ . The electric field at X due to the dipole A₁ can be obtained by replacing r = d and θ = 0 in equation of E_r and E_{θ}

$$E_r = \frac{1}{4\pi\varepsilon_o} \frac{2\mu}{d^3} = \frac{\mu}{2\pi\varepsilon_o d^3}$$
$$E_\theta = 0$$

Electric field at X due to $A_1 = E_r + E_{\theta} = \frac{\mu}{2\pi\varepsilon_o d^3}$

Since the dipoles are situated symmetrically, the field seen by all dipoles will be the same. Therefore,

Field at X due to dipole $A_2 = \frac{\mu}{2\pi\varepsilon_a d^3}$

Therefore Field at X due to A₁ and A₂ is $E_1 = \frac{2\mu}{2\pi\varepsilon_o d^3} = \frac{\mu}{\pi\varepsilon_\theta d^3}$

Similarly field at X due to dipoles B1 and B2 and C1 and C2 situated at distances 2d and 3d respectively are given by

$$E_2 = \frac{\mu}{\pi \varepsilon (2d)^3}$$
$$E_3 = \frac{\mu}{\pi \varepsilon_o (3d)^3}$$

The total electric field at P due to all the dipoles in linear array is given by

The total internal field at X is $E_i = E + E^1$

$$E_{local} = E + \frac{1.2\mu}{\pi\varepsilon_o d^3}$$

The dipoles moment induced in each of the atoms of the string is $\mu_{ind} = \alpha E$

Since α , E, ϵ_0 and d are all positive hence $E_i > E$ Therefore, actual field seen by atom in the string is larger than the applied field E

Physically this means that the dipole cooperate with each other in the sense that a large dipole moment on a given atom helps to induce a dipole moment in its neighbours which intern induce a dipole moment in the former etc....

Hence the cooperation becomes stronger as the polarizability of the atom increases, and as the distance between them decreases.

In case of 3 dimensional array of atoms the final formula for internal field is given by

$$E_{local} = E + \frac{\gamma p}{\varepsilon_o}$$

Where Gamma is called internal field constant and P is polarization

For Cubic system (in case of non-polar dielectric material) the value of gamma is found to be 1/3 and the internal field is called Lorentz field.

$$E_{Lorentz} = E + \frac{p}{3\varepsilon_o}$$

Claussius Mosotti Euqation

Let us consider a simple elemental solid dielectric which exhibits only electronic polarizability. Solids like Diamond, silicon and germanium crystals made up of single atoms. If α_e is the electronic polarizability per atom, it is related to the bulk polarization P through the relation

$$\alpha_e = \frac{P}{NE_i}$$
 (we know that $P = N\alpha_e E_i$)

Where N is the number of atoms per m^3 and E_i is the local field. Using the expression for internal field in 3 dimension we can write

$$\alpha_e = \frac{P}{N\left[E + \frac{\gamma P}{\varepsilon_o}\right]}$$

According to the relation between P,χ and E

$$E = \frac{P}{\varepsilon_0(\varepsilon_r - 1)}$$

If the internal field is assumed to be Lorentz field, $\gamma = 1/3$ and the above equation becomes.

$$\alpha_e = \frac{P}{N\left[E + \frac{P}{3\varepsilon_0}\right]}$$

Using the relation for E and substituting in the above equation

$$\alpha_{e} = \frac{P}{N\left[\frac{P}{\varepsilon_{0}(\varepsilon_{r}-1)} + \frac{P}{3\varepsilon_{0}}\right]}$$
$$\frac{N\alpha_{e}}{\varepsilon_{0}} = \frac{1}{\left[\frac{1}{\varepsilon_{r}-1} + \frac{1}{3}\right]} = \frac{1}{\left[\frac{\varepsilon_{r}+2}{3(\varepsilon_{r}-1)}\right]}$$

 $\frac{\varepsilon_r - 1}{\varepsilon_r + 2} = \frac{N\alpha_e}{3\varepsilon_0}$ is the Clausius-Mosotti equation.

The above equation is known as Clausius-Mosotti equation which is valid for non-polar solids having cubic crystal structure.